Probing Non-global Logarithm with hemisphere energy correlates

YuXuan Sun
FuDan U.

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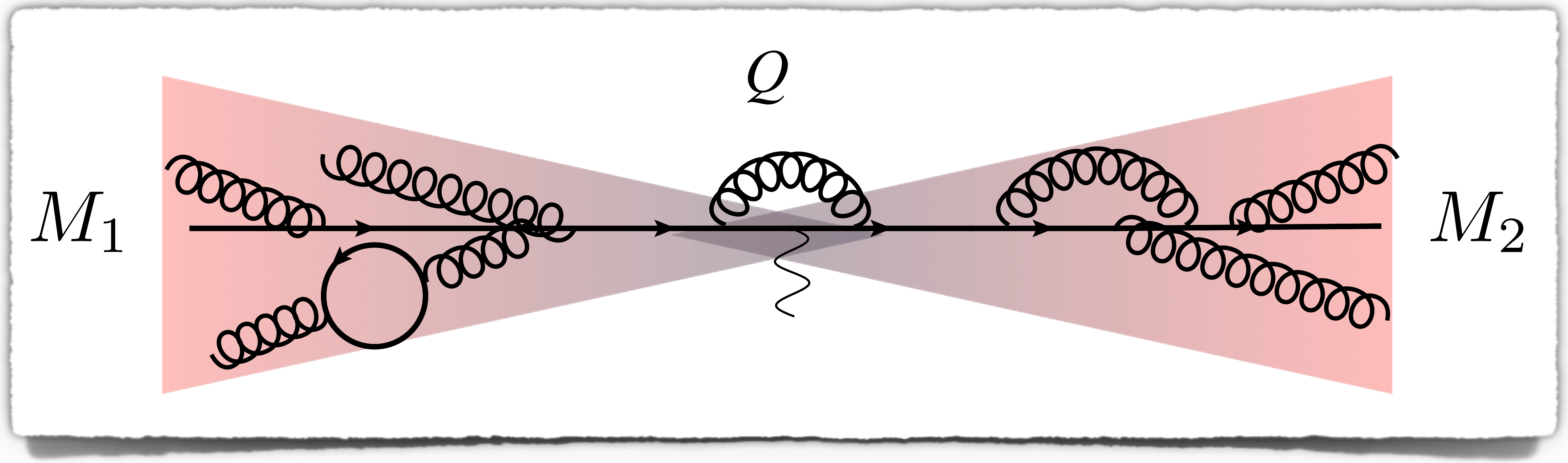
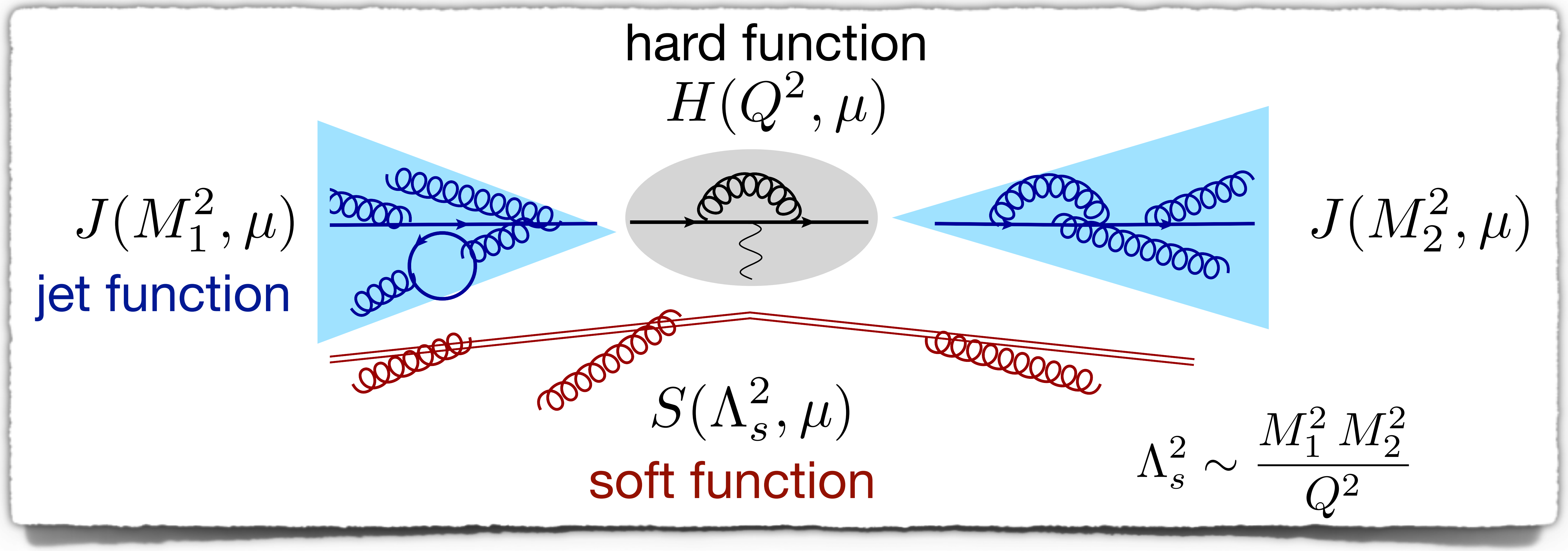
Soft-Collinear Factorization

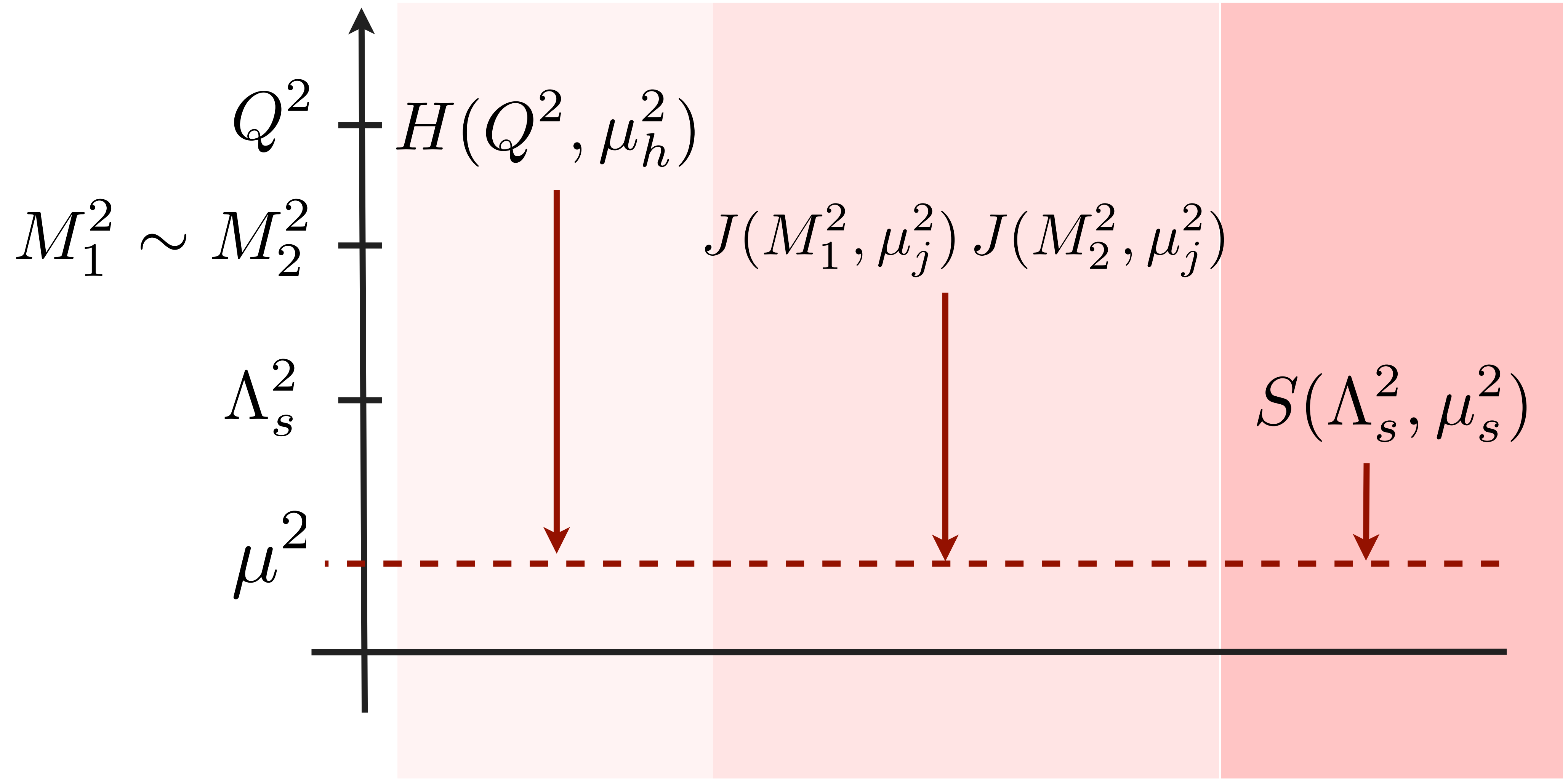


For $M_1 \sim M_2 \ll Q$ the cross section $\sigma_{\text{phys}}(Q, M_1, M_2)$ factorizes:



Resummation by RG evolution

Evaluate each part at its characteristic scale, evolve to common reference scale μ

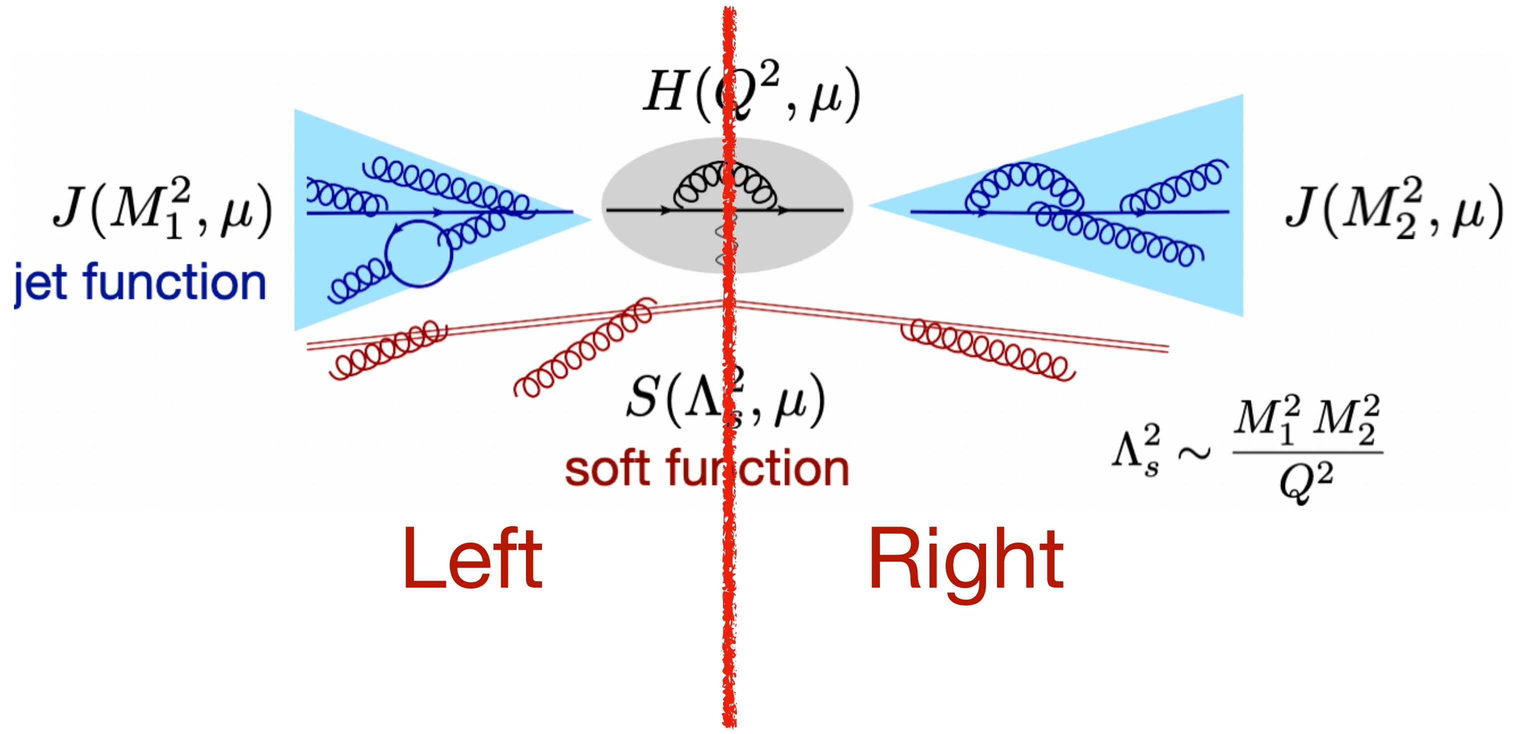


Each contribution is evaluated at its natural scale. No large perturbative logarithms.

But for region $M_L \ll M_R \ll Q$, large $\log(M_L/M_R)$ can not be resumed by RGE.

- Jet masses distribution

(Fleming, Hoang, Mantry & Stewart 2008)

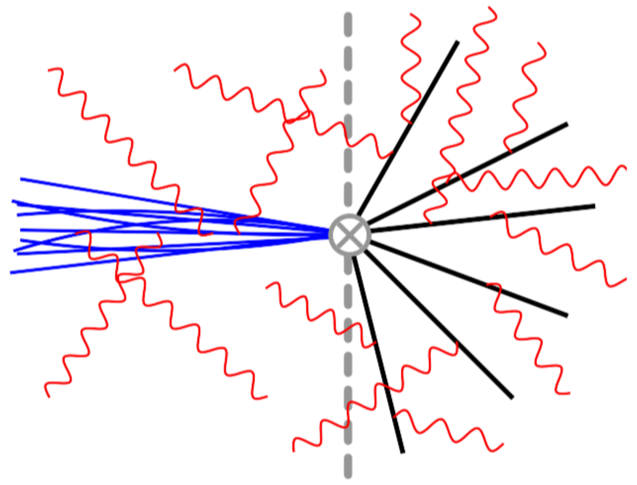


$$\frac{d\sigma}{dM_L^2 dM_R^2} = \sigma_0 \overbrace{H(Q^2)}^{\text{Hard scale}} \int_0^\infty d\omega_L \int_0^\infty d\omega_R \underbrace{J_q(M_L^2 - Q\omega_L) J_q(M_R^2 - Q\omega_R)}_{\text{Jet mass}} \overbrace{S(\omega_L, \omega_R)}^{\text{Soft scale}}$$

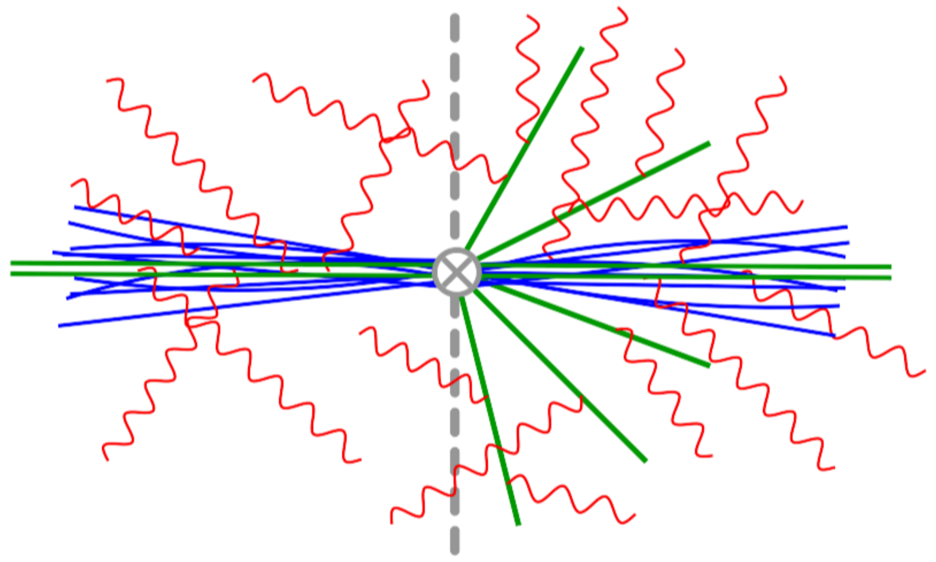
- Three regions

$M_L \sim M_R \ll Q$, Standard soft function

$M_L \ll M_R \sim Q$ Left jet mass



$M_L \ll M_R \ll Q$ Can be refactorized again!



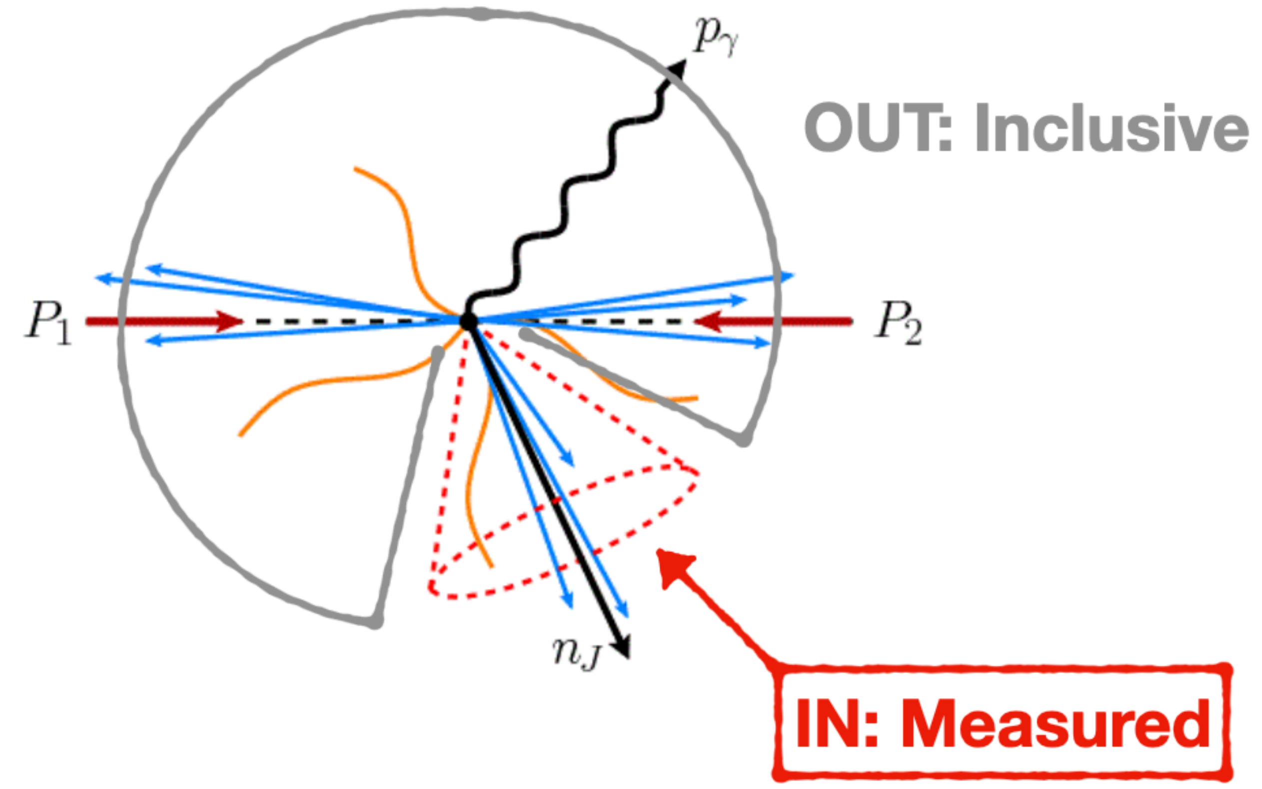
hard: $p_h \sim \omega_R\,(1,1,1)$

soft: $p_s \sim \omega_R\,(\kappa,\kappa,\kappa)$

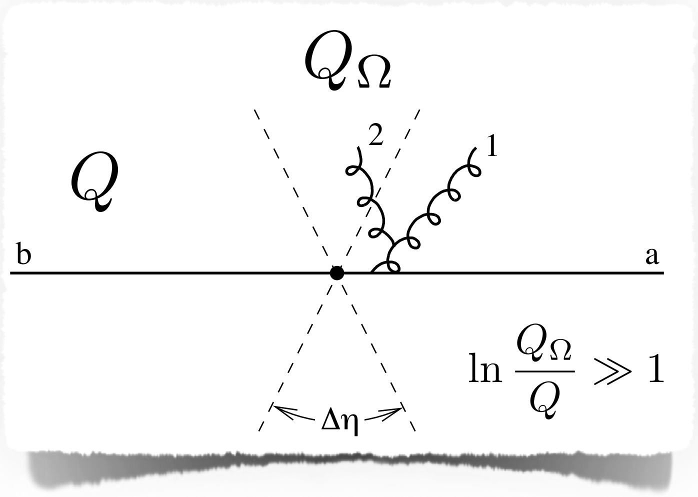
NGLs! $\kappa \equiv \omega_L/\omega_R \ll 1$

Non-global Logarithm

Observables which are insensitive to emissions into certain regions of phase space involve additional NGLs not captured by the usual resummation formula



- from jets of intermediate energy
- reflect color flow at all scales
- do not exponentiate in a simple manner



(Dasgupta & Salam 2001)

Resummation

$$\exp\left[-4\,C_F\Delta\eta\int_{\alpha(Q_\Omega)}^{\alpha(Q)}\frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{2\pi}\right] = 1+4\frac{\alpha_s}{2\pi}C_F\Delta\eta\ln\frac{Q_\Omega}{Q} + \left(\frac{\alpha_s}{2\pi}\right)^2\left(8\,C_F^2\Delta\eta^2-\frac{22}{3}\,C_FC_A\Delta\eta+\frac{8}{3}C_FT_Fn_f\Delta\eta\right)\ln^2\frac{Q_\Omega}{Q}$$

NGLs

$$\left(\frac{\alpha_s}{2\pi}\right)^2 C_FC_A\left[-\frac{2\pi^2}{3}+4\,\mathrm{Li}_2\left(e^{-2\Delta\eta}\right)\right]\ln^2\frac{Q_\Omega}{Q}$$

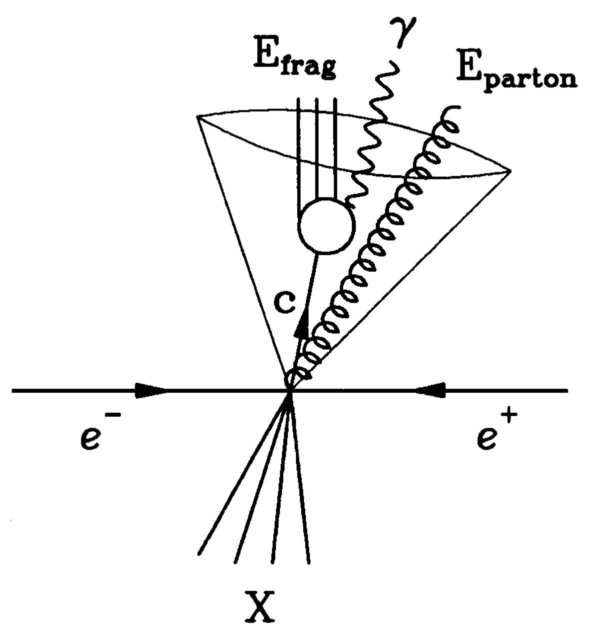
- Non-linear evolution, BMS eq (Banfi, Marchesini & Smye 2002)

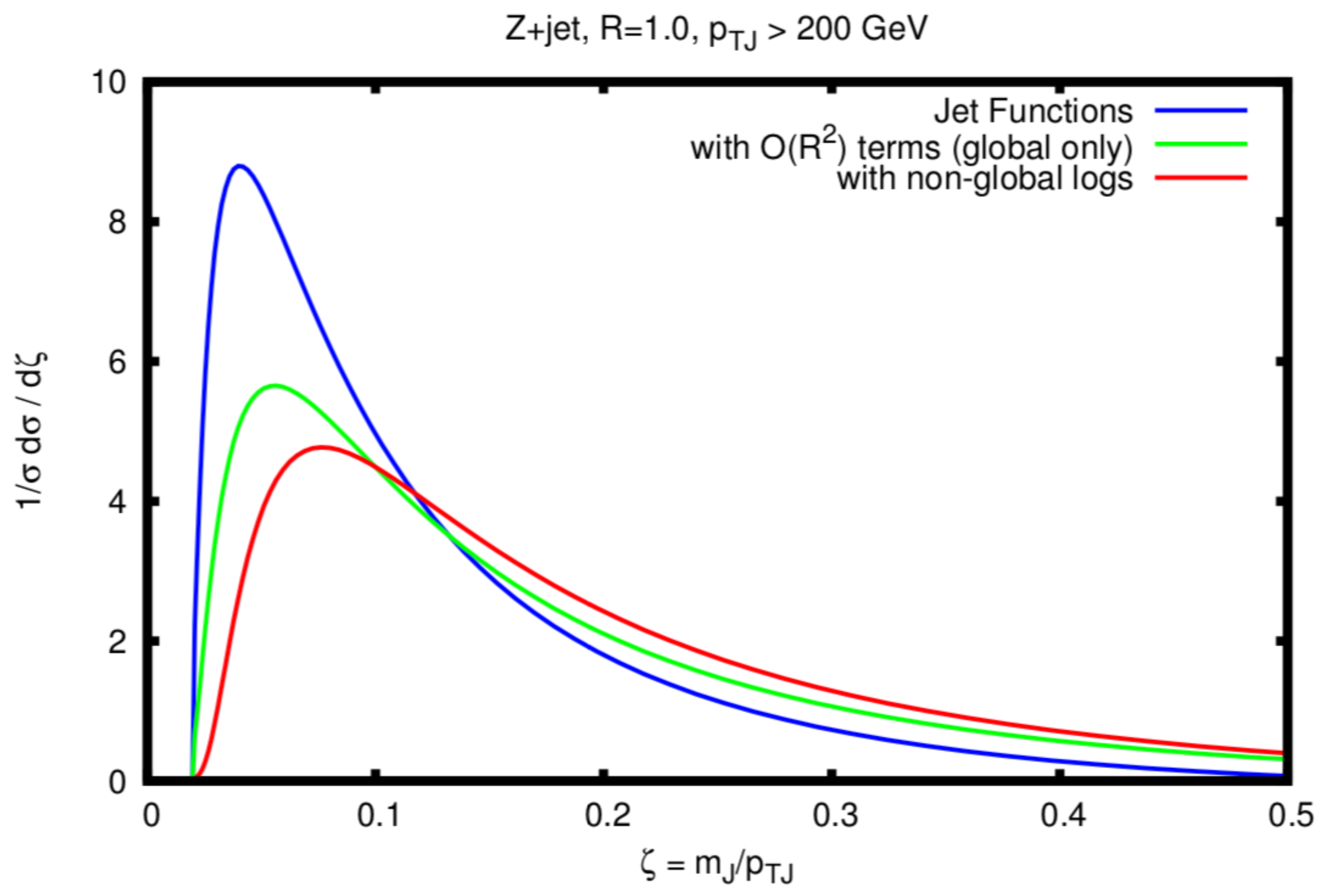
$$\partial_{\hat L}G_{kl}(\hat L) = \int\frac{d\,\Omega(n_j)}{4\pi}\,W_{kl}^{j}\left[\Theta_{\rm in}^{n\bar n}(j)\,G_{kj}(\hat L)\,G_{jl}(\hat L)-G_{kl}(\hat L)\right]$$

$$W_{ij}^{k} = \frac{n_i\cdot n_j}{n_i\cdot n_k\,n_j\cdot n_k}$$

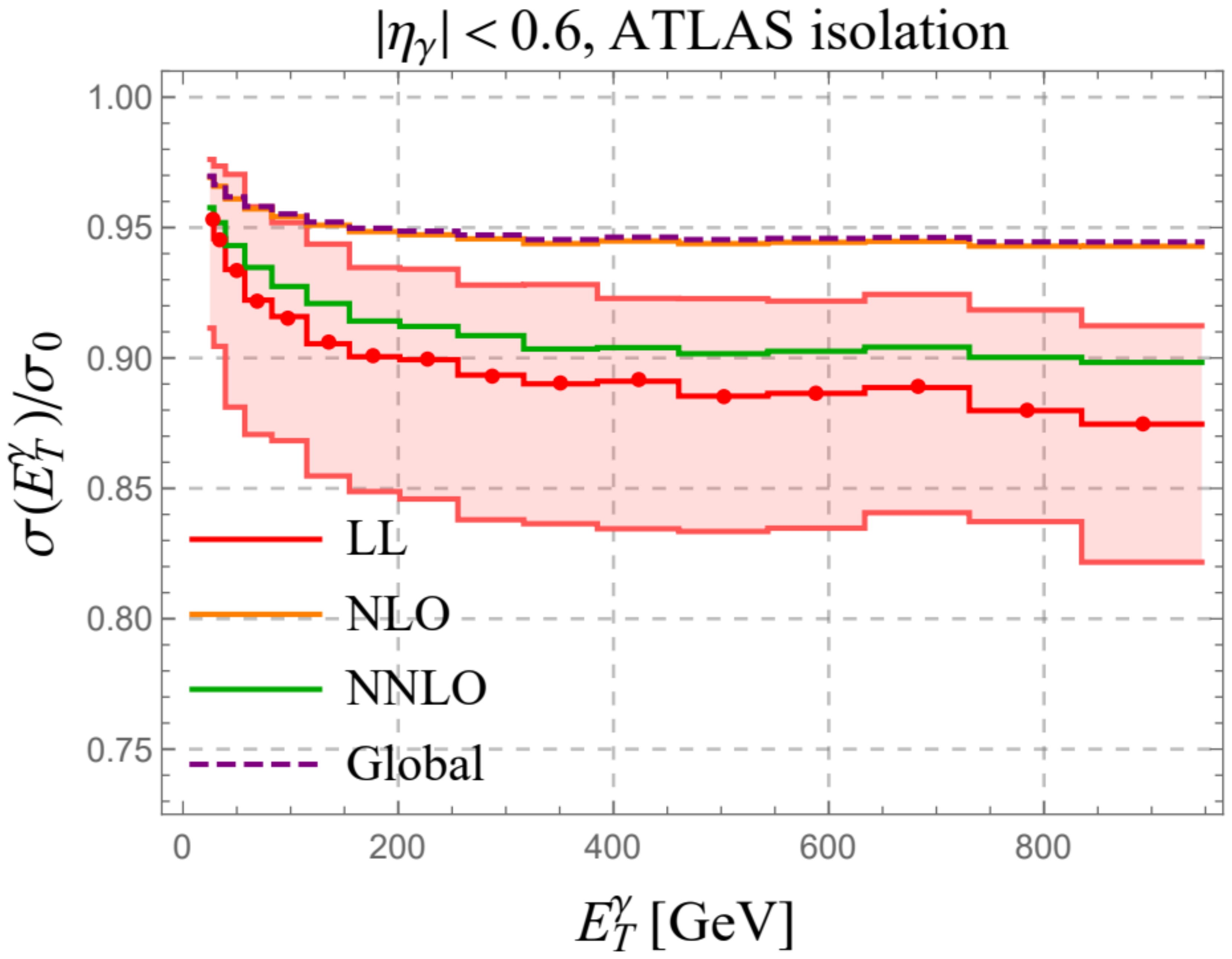
- ee, ep, pp, …

NGLs corrections for Lead log resummation





(Dasgupta, Khelifa-Kerfa, Marzani, Spannowsky ’12)



(Balsiger, Becher, DYS ’17)

Refactorization

$$\text{hard:} \quad p_h \sim \omega_R\,(1,1,1)$$
$$\text{soft:} \quad p_s \sim \omega_R\,(\kappa,\kappa,\kappa)$$

- Hemi-sphere soft function can be factorized to “hard” function and “soft” function

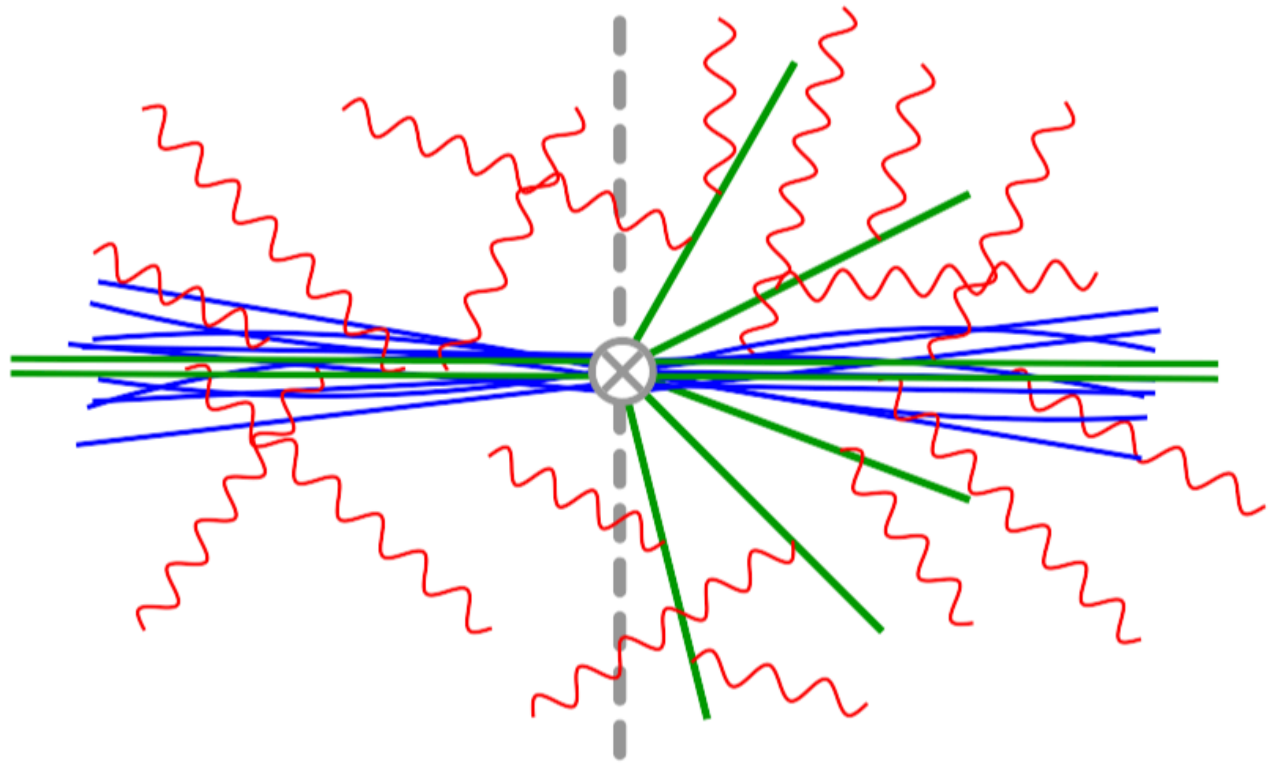
(Becher, Pecjak & Shao 2016)

$$\boxed{S(\omega_L,\omega_R) = \sum_{m=0}^{\infty} \langle \mathcal{H}_m^S(\{\underline{n}\},\omega_R) \otimes \boldsymbol{\mathcal{S}}_{m+1}(\{n,\underline{n}\},\omega_L) \rangle}$$

Integrate the angles for hard partons

$$\mathcal{H}_m^S(\{\underline{n}\},\omega_R) = \prod_{i=1}^{m} \int \frac{dE_i\,E_i^{d-3}}{(2\pi)^{d-2}} |\mathcal{M}_m^S(\{\underline{p}\})\rangle\langle\mathcal{M}_m^S(\{\underline{p}\})|\,\delta(\omega_R - n\cdot P_R)\,\Theta_R(\{\underline{p}\})$$

$$\boldsymbol{\mathcal{S}}_{m+1}(\{n,\underline{n}\},\omega_L) = \sum_{X_s} \langle 0|\,\boldsymbol{S}_a^\dagger(\bar{n})\,\boldsymbol{S}_b^\dagger(n)\,\boldsymbol{S}_1^\dagger(n_1)\ldots\boldsymbol{S}_m^\dagger(n_m)\,|X_s\rangle \times \langle X_s|\,\boldsymbol{S}_a(\bar{n})\,\boldsymbol{S}_b(n)\,\boldsymbol{S}_1(n_1)\ldots\boldsymbol{S}_m(n_m)\,|0\rangle\,\delta(\omega_L - \bar{n}\cdot P_L)$$



- Renormalization separately

$$\boldsymbol{\mathcal{S}}_l(\{\underline{n}\},Q\beta,\delta,\mu) = \sum_{m=l}^{\infty} \boldsymbol{Z}_{lm}^H(\{\underline{n}\},Q,\delta,\epsilon,\mu)\,\hat{\otimes}\,\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q\beta,\delta,\epsilon)$$

$$\mathcal{H}_m(\{\underline{n}\},Q,\delta,\epsilon) = \sum_{l=2}^{m} \mathcal{H}_l(\{\underline{n}\},Q,\delta,\mu)\,\boldsymbol{Z}_{lm}^H(\{\underline{n}\},Q,\delta,\epsilon,\mu)$$

$$\boldsymbol{Z}^H(\{\underline{n}\},Q,\delta,\epsilon,\mu) \sim \begin{pmatrix} 1 & \alpha_s & \alpha_s^2 & \alpha_s^3 & \ldots \\ 0 & 1 & \alpha_s & \alpha_s^2 & \ldots \\ 0 & 0 & 1 & \alpha_s & \ldots \\ 0 & 0 & 0 & 1 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

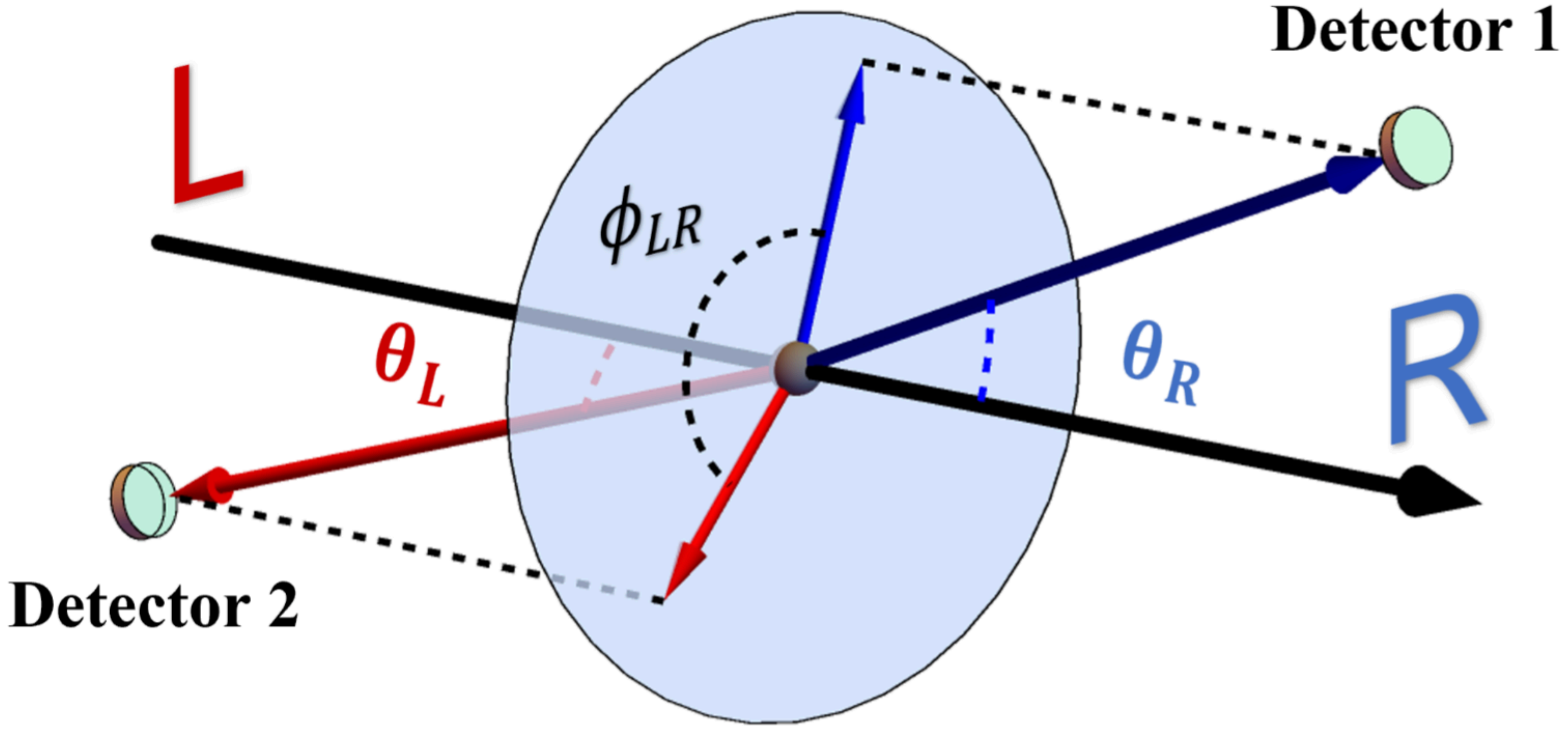
How to observe the NGLs

- NGLs usually appears as a correction to the global logarithms.
- Are there any dynamic effects of NGLs?
- Can we probe NGLs directly?

TMD hemi-sphere EEC

$$\mathrm{EEC}^{\mathrm{hemi}}(\theta_L,\,\theta_R,\,\phi)=\frac{\mathrm{d}\Sigma}{\mathrm{d}\theta_L\,\mathrm{d}\theta_R\,\mathrm{d}\phi}=\int\frac{\mathrm{d}\phi_L\mathrm{d}\phi_R\,\mathrm{d}^2\boldsymbol{b}_L\mathrm{d}^2\boldsymbol{b}_R}{(2\pi)^4}\delta(\phi-\phi^q_{LR})H_{q\bar q}(Q)\times e^{\mathrm{i}\hat{\boldsymbol{q}}_L\cdot\boldsymbol{b}_L\theta_L Q/2}e^{\mathrm{i}\hat{\boldsymbol{q}}_R\cdot\boldsymbol{b}_R\theta_R Q/2}\,\mathcal{S}\left(\boldsymbol{b}_L,\boldsymbol{b}_R\right)J_q(b_L)\,J_{\bar q}(b_R)$$

- Conventional observables where NGLs enter as higher-order corrections
- Isolates the azimuthal structure arising solely from the recoil of soft-gluon emissions between the two hemispheres
- Thereby providing a clean and direct probe of non-global logarithm dynamics.



$$J_q(b,\mu)=\sum_h\int_0^1\mathrm{d}z\,z\,D_{h/q}(z,b,\mu)$$

$$\tilde{S}\left(\boldsymbol{q}_L,\boldsymbol{q}_R\right)=\frac{1}{N_c}\sum_{X_s}\mathrm{Tr}\left\langle 0\left|Y_n^\dagger Y_{\bar n}\right|X_s\right\rangle\left\langle X_s|Y_{\bar n}^\dagger Y_n|0\right\rangle\times\delta\left(\boldsymbol{q}_L-\sum_{i\in L}\boldsymbol{k}_i^T\right)\delta\left(\boldsymbol{q}_R-\sum_{j\in R}\boldsymbol{k}_j^T\right),$$

$$\mathcal{S}\left(\boldsymbol{b}_L,\boldsymbol{b}_R\right)=\int\mathrm{d}^2\boldsymbol{q}_L\mathrm{d}^2\boldsymbol{q}_R e^{i\boldsymbol{q}_R\cdot\boldsymbol{b}_R}e^{i\boldsymbol{q}_L\cdot\boldsymbol{b}_L}\tilde{S}\left(\boldsymbol{q}_L,\boldsymbol{q}_R\right)$$

Soft function Calculation

- At NLO soft function there three color structure

$$\mathcal{S}_2\left(\boldsymbol{b}_L,\boldsymbol{b}_R,\epsilon\right) = g^4 \sum_{X=C_A,n_f,C_F} C^{(X)}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{2\epsilon}\int \frac{d^dq}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{\left|\mathcal{A}^{(X)}\right|^2}{(2q_0 2k_0)^\alpha}\boxed{\mathcal{F}\left(\boldsymbol{b}_L,\boldsymbol{b}_R\right)}$$

$$\begin{aligned}\mathcal{F}\left(\boldsymbol{b}_L,\boldsymbol{b}_R\right) = {} & \frac{1}{2!}(-2\pi i)^2\,\delta^+\left(k^2\right)\delta^+\left(q^2\right)\\ & \times\Big[\theta\left(k^- - k^+\right)\theta\left(q^+ - q^-\right)\boxed{e^{i\boldsymbol{k}_T\cdot\boldsymbol{b}_R}e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_L}}\\ & +\theta\left(q^- - q^+\right)\theta\left(k^+ - k^-\right)\boxed{e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_R}e^{i\boldsymbol{k}_T\cdot\boldsymbol{b}_L}}\\ & +\theta\left(k^- - k^+\right)\theta\left(q^- - q^+\right)e^{i(\boldsymbol{q}_T+\boldsymbol{k}_T)\cdot\boldsymbol{b}_R}\\ & +\theta\left(k^+ - k^-\right)\theta\left(q^+ - q^-\right)e^{i(\boldsymbol{q}_T+\boldsymbol{k}_T)\cdot\boldsymbol{b}_L}\Big]\end{aligned}$$

- Parameterization

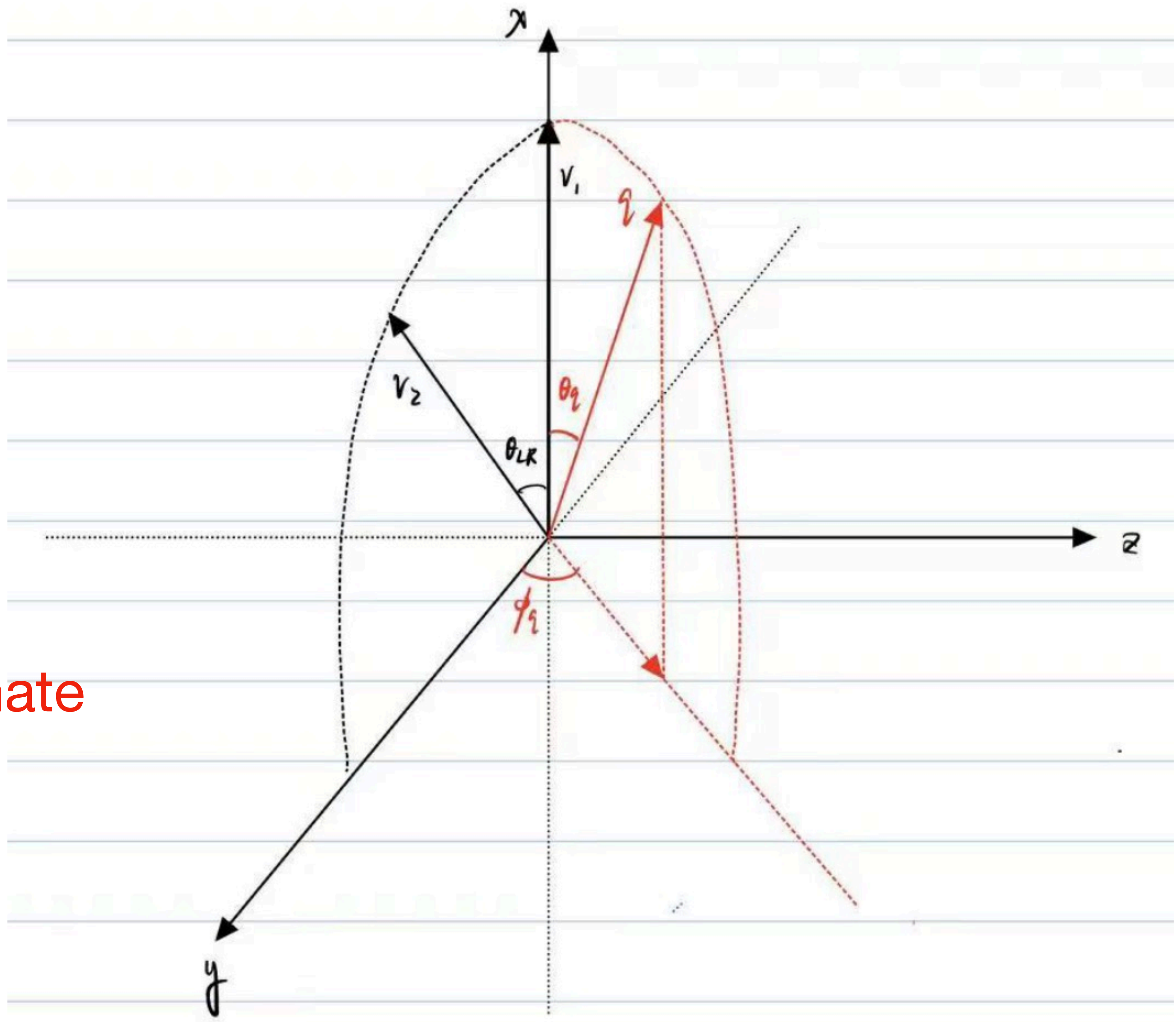
$$p_T = \sqrt{(k_+ + q_+)(k_- + l_-)},\quad y = \frac{k_+ + q_+}{k_- + q_-},\quad a = \sqrt{\frac{k_- q_+}{k_+ q_-}},\quad b = \sqrt{\frac{k_+ k_-}{q_+ q_-}}$$

$$R_x(\phi_q) = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos\phi_q & -\sin\phi_q\\ 0 & \sin\phi_q & \cos\phi_q\end{pmatrix},\quad R_z(\theta_q) = \begin{pmatrix}\cos\theta_q & -\sin\theta_q & 0\\ \sin\theta_q & \cos\theta_q & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\begin{aligned}\boldsymbol{q}_T &= q_T R_x(\phi_q) R_z(\theta_q)\boxed{(1,0,\cdots)}\\ \boldsymbol{k}_T &= k_T R_x(\phi_q) R_z(\theta_q)\boxed{(\cos\theta_{kq},\sin\theta_{kq}\cos\phi_k,\cos\phi_k'\sin\theta_{kq}\sin\phi_k,\cdots)}\\ \boldsymbol{b}_L &= b_L\boxed{(1,0,0)}\\ \boldsymbol{b}_R &= b_R\boxed{(\cos\theta_{LR},\sin\theta_{LR},0)}\\ \boldsymbol{v}_1 &= (1,0,0)\\ \boldsymbol{v}_2 &= (\cos\theta_{LR},\sin\theta_{LR},0)\end{aligned}$$

q, k coordinate

b_L, b_R coordinate



TMD Results

$$\mathcal{S}^{(2)}(b_L, b_R) = \mathcal{S}^{(2)}_{C_F^2} + \mathcal{S}^{(2)}_{C_F C_A} + \mathcal{S}^{(2)}_{C_F n_f T_F}$$

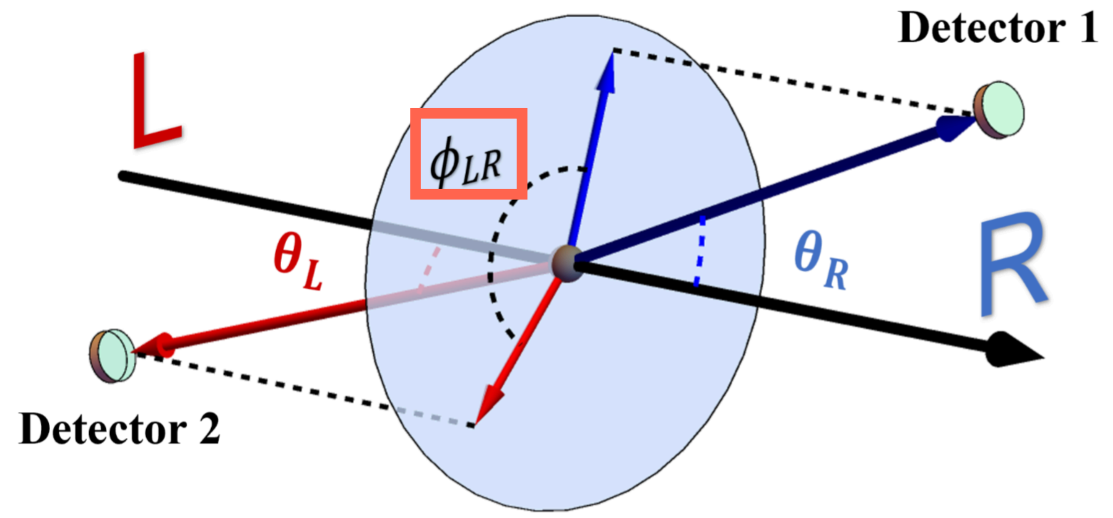
$$\begin{aligned}
\mathcal{S}^{(2)}_{C_F C_A} =& -\frac{22}{9}\left(L_L^3 + L_L^3\right) - 7.470(28)\left(L_L^2 + L_R^2\right) \\
& - 18.25(16)\left(L_L + L_R\right) + \frac{2\pi^2}{3} L_L L_R \\
& + \left(\frac{4}{3} - \frac{44}{9}\pi^2\right)\log r + \Bigg[\frac{22}{3}\left(L_L^2 + L_L^2\right) \\
& -16.68(5)\left(L_L + L_R\right) + 8\zeta_3 \log r \\
& - 16.8(5)\Bigg] \log\left(\frac{\nu}{\mu}\right) + \boxed{F_{C_A}\left(r, \phi^b_{LR}\right)}
\end{aligned}$$

$$\begin{aligned}
\mathcal{S}^{(2)}_{C_F n_f T_F} =& \frac{8}{9}\left(L_L^3 + L_R^3\right) + \frac{20}{9}\left(L_L^2 + L_R^2\right) \\
& + 6.588(11)\left(L_L + L_R\right) + \left(\frac{16}{9}\pi^2 - \frac{8}{3}\right)\log r \\
& + 45.03(8) + \Bigg[-\frac{8}{3}\left(L_L^2 + L_R^2\right) + \frac{80}{9}\left(L_L + L_R\right) \\
& + 25.386(34)\Bigg] \log\left(\frac{\nu}{\mu}\right) + \boxed{F_{n_f}\left(r, \phi^b_{LR}\right)}
\end{aligned}$$

$$\begin{aligned}
\mathcal{S}^{(2)}_{C_F^2} =& \frac{1}{2}\left[L_L^2 + L_R^2 + \frac{\pi^2}{3} + 4\left(L_L + L_R\right)\log\left(\frac{\nu}{\mu}\right)\right]^2 \\
& + \left[40\left(L_L^2 + L_R^2\right) + 16 L_L L_R + \frac{32\pi^2}{3}\right]\log\left(\frac{\nu}{\mu}\right)^2 \\
& + \Bigg[L_L^4 + L_R^4 + \frac{2}{3} L_L L_R\left(L_L^2 + L_R^2 - 2\pi^2\right) \\
& - \frac{7\pi^2}{3}\left(L_L^2 + L_R^2\right) + \frac{16\zeta_3}{3}\left(L_L + L_R\right) - \frac{17\pi^4}{30}\Bigg] \\
& + \Bigg[\frac{28}{3}\left(L_L^3 + L_R^3\right) + 4\left(L_L L_R + \pi^2\right)\left(L_L + L_R\right) \\
& + \frac{64}{3}\zeta_3\Bigg]\log\left(\frac{\nu}{\mu}\right)
\end{aligned}$$

NAE

New structure!



Refactorization results

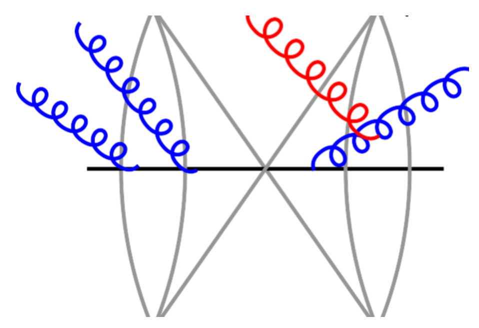
$$\mathcal{S}\left(\boldsymbol{b}_L,\boldsymbol{b}_R\right)=\sum_{m=0}^{\infty}\left\langle\boldsymbol{\mathcal{H}}_m^S\left(\{\underline{n}\},\boldsymbol{b}_R\right)\otimes\boldsymbol{\mathcal{S}}_{m+1}\left(\{\underline{n}\},\boldsymbol{b}_L\right)\right\rangle$$

$$b_L \ll b_R \ll Q$$

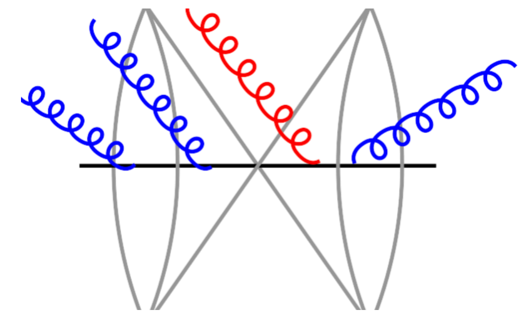
- Hard quark cannot be represented by a Wilson line in the effective theory, such that n_f term do not contain azimuthal structure.

$$\begin{aligned}
&\tilde{s}^{(2)}\left(\boldsymbol{b}_L,\boldsymbol{b}_R,\epsilon\right)\\
=&\left[(\mu b_L)^{4\epsilon}(\nu b_L)^{\alpha}+(\mu b_R)^{4\epsilon}(\nu b_R)^{\alpha}\right]\\
&\cdot\left[(\nu b_{L,R})^{\alpha}C_F^2h_F^2/2+C_FC_A\left(h_A+v_A\right)+C_FT_Fn_fh_f\right]\\
&-\left[(\mu b_L)^{2\epsilon}(\nu b_L)^{\alpha}+(\mu b_R)^{2\epsilon}(\nu b_R)^{\alpha}\right]\frac{\beta_0}{\epsilon}h_F\\
&+(\mu b_L)^{4\epsilon}\left[C_FC_A\boxed{\left(s_A-h_A-v_A\right)}+C_FT_Fn_f\boxed{\left(s_f-n_f\right)}\right]\\
&+(\mu b_L)^{2\epsilon}(\mu b_R)^{2\epsilon}\left[C_FC_Ap_A+C_F^2p_F\right]
\end{aligned}$$

Emissions at same side ,These are exactly the same as the results of TMD



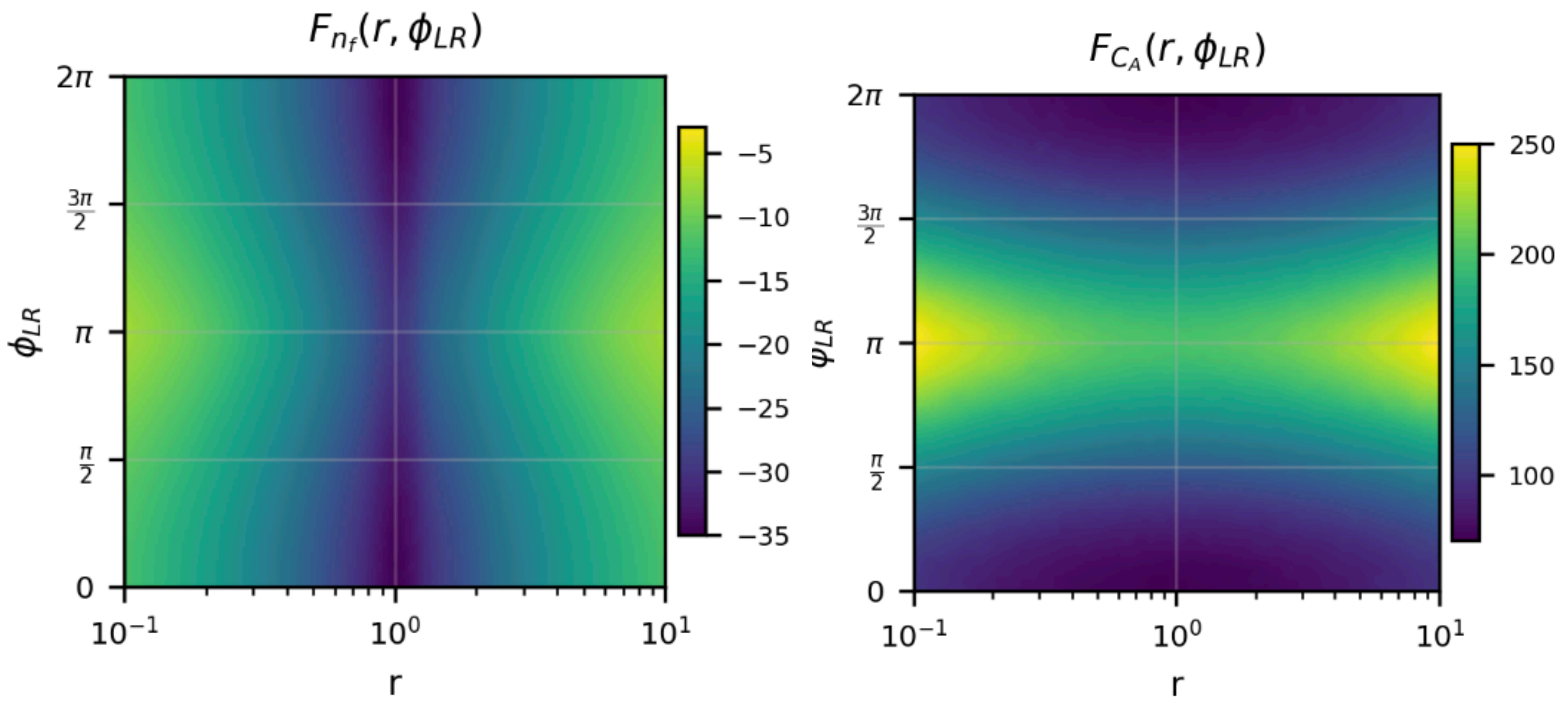
Emissions at different side



$$p_A=\frac{2\pi^2}{3\epsilon^2}+\frac{12\zeta_3}{\epsilon}+p_A^{(0)}\left(\phi_{LR}^b\right).$$

$$\begin{aligned}
s_A-h_A-v_A=&\frac{1}{\epsilon}\left(-\frac{2}{3}+\frac{22\pi^2}{9}-4\zeta_3\right)\\
&+\frac{16}{9}-\frac{134\pi^2}{27}+\frac{2\pi^4}{45}+\frac{176\zeta_3}{3},\\
s_f-n_f=&\frac{1}{\epsilon}\left(\frac{4}{3}-\frac{8\pi^2}{9}\right)-\frac{20}{9}-\frac{64\zeta_3}{3}+\frac{64\pi^2}{27},
\end{aligned}$$

Azimuthal symmetry of NGL



- Different color structure, different azimuthal symmetry mode

- This dynamic (azimuthal structure) effect started at $\mathcal{O}(\alpha_s^2)$, which causes different RG evolutionary effects.

- At $\mathcal{O}(\alpha_s^2)$ only appears in the constant term, which can be extracted using angular projection

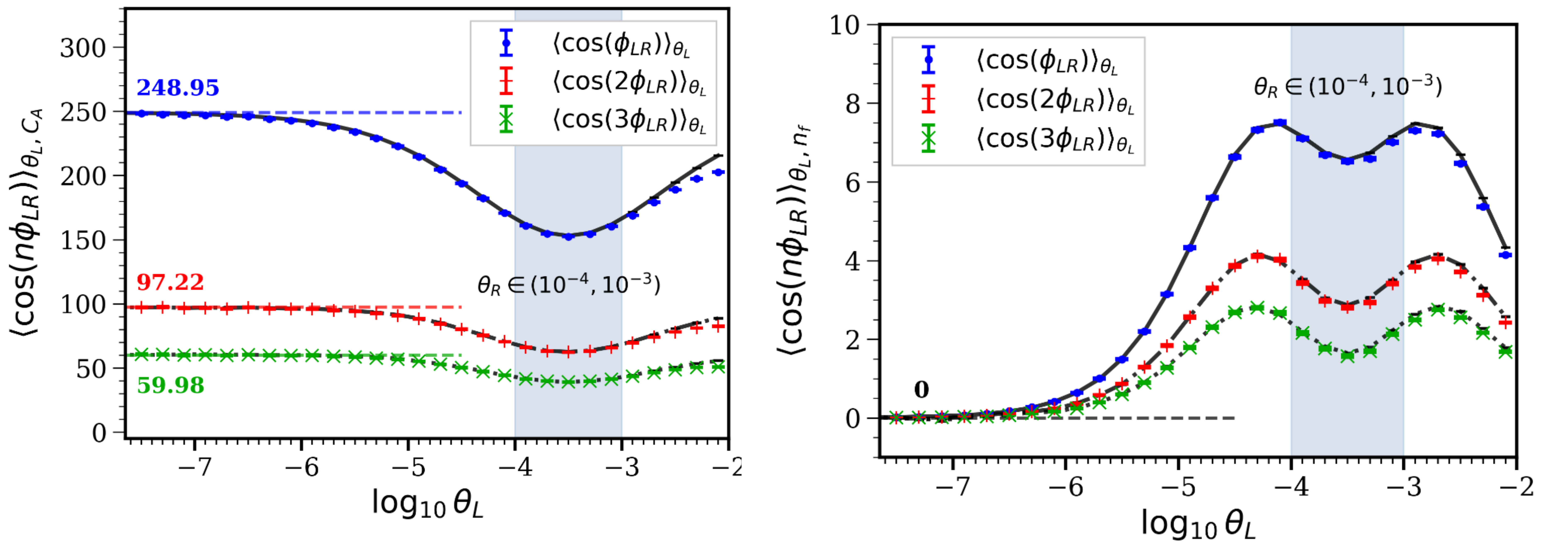
- There are different angular mode $n = 1,2,3,\ldots$

$$\langle\cos(n\phi)\rangle_{\theta_L\theta_R} \equiv \frac{1}{\sigma_0}\int_0^{2\pi} \mathrm{d}\phi_L^q \mathrm{d}\phi_R^q \frac{\mathrm{d}\Sigma}{\mathrm{d}\theta_L \mathrm{d}\theta_R \mathrm{d}\phi} \cos(n\phi_{LR}^q) = (-1)^n \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{Q^4}{16}\int \mathrm{d}b_L^2 \mathrm{d}b_R^2 \, J_n\left(b_L\theta_L\frac{Q}{2}\right) \times J_n\left(b_R\theta_R\frac{Q}{2}\right)\int\frac{\mathrm{d}\phi_{LR}^b}{2\pi}\mathcal{S}^{(2)}\left(\frac{b_L}{b_R},\phi_{LR}^b\right)\cos(n\phi_{LR}^b)$$

$$A_n^b(r) = \int_0^{2\pi}\mathrm{d}\phi_{LR}^b\cos(n\phi_{LR}^b)\mathcal{S}^{(2)}\left(r,\phi_{LR}^b\right)$$

$$\langle\cos(n\phi)\rangle_{\theta_L\theta_R} = \frac{(-1)^n}{2\pi}\left(\frac{\alpha_s}{4\pi}\right)^2\left[\frac{n^2 A_n^b(\theta_L/\theta_R)}{\theta_L\theta_R} - \frac{A_n^{b\prime}(\theta_L/\theta_R)}{\theta_R^2} - \frac{\theta_L A_n^{b\prime\prime}(\theta_L/\theta_R)}{\theta_R^3}\right]$$

$$\langle \cos(n\phi) \rangle_{\theta_L} = \int_{\theta_{\min}}^{\theta_{\max}} \mathrm{d}\theta_R \, \langle \cos(n\phi) \rangle_{\theta_L \theta_R}$$



TMD result can derive refactorization result by taking limit $b_L \ll b_R$

Resummation

$$A_{n,\,\theta_L}^{\text{Hemi}} = \frac{\langle \cos(n\phi) \rangle_{\theta_L}}{\langle \Sigma \rangle_{\theta_L}}$$

$$\begin{aligned}\langle \Sigma \rangle_{\theta_L} = \int_{\theta_{\min}}^{\theta_{\max}} \mathrm{d}\theta_R \frac{1}{\sigma_0} \frac{\mathrm{d}\Sigma}{\mathrm{d}\log\theta_L} \quad &= \int_{\theta_{\min}}^{\theta_{\max}} \mathrm{d}\theta_R \int \mathrm{d}b_L \mathrm{d}b_R \, b_L b_R \, H_{q\bar{q}} J_q(b_L) J_{\bar{q}}(b_R) \\ &\times \mathcal{S}(b_L, b_R) J_0(b_L Q \theta_L/2) e^{-S_{\text{pert}}(b_L,Q) - S_{\text{NP}}(b_L,Q)} \\ &\times J_0(b_R Q \theta_R/2) e^{-S_{\text{pert}}(b_R,Q) - S_{\text{NP}}(b_R,Q)}\end{aligned}$$

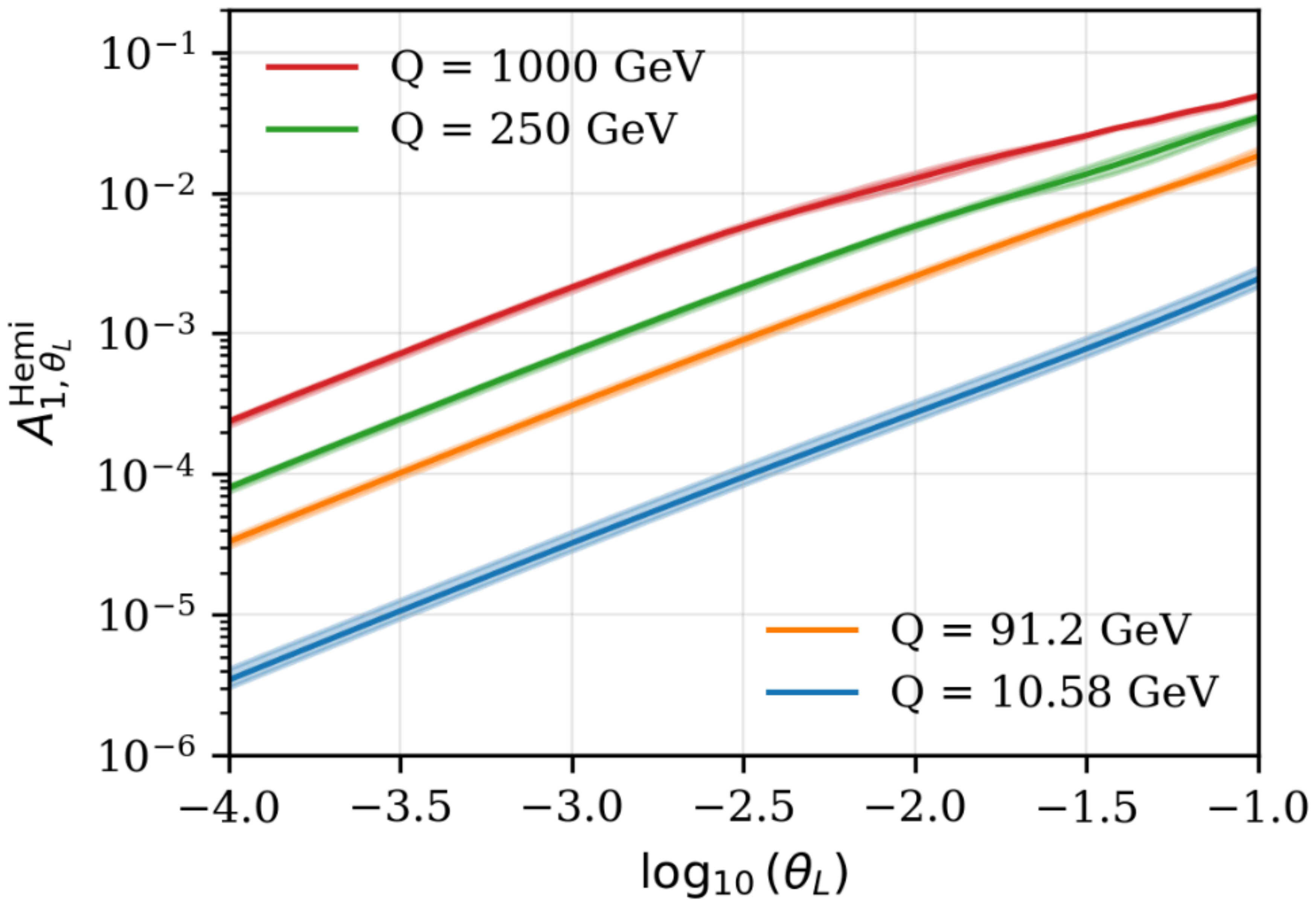
S_{pert} leading-order perturbative Sudakov factor

$S_{\text{NP}}(b,Q) = a_1 b^{a_2} + g_2 b b^* \log\left(\frac{Q}{\mu_b^*}\right)$ non-perturbative contribution

$a_1 = 0.530,\ a_2 = 1.152,$ and $g_2 = 0.076.$

Resummation

- Resummed results: a characteristic linear behavior in θ_L and a systematic growth with the hard scale Q



The approximately linear behavior and its slope arise primarily from the difference between the Fourier transforms involving J_1 and J_0 Bessel functions of an exponentially damped integrand in b space

Summary and outlook

We propose the hemisphere EEC in e^+e^- annihilation as a new observable that directly exposes the non–global structure of QCD radiation.

- Correlating the energy flow between opposite hemispheres
- Isolates azimuthal patterns generated purely by soft–gluon recoil, offering a clean handle on non–global dynamics that have so far been accessible only indirectly.

We computed the two-loop TMD hemisphere soft function, found new azimuthal structure and established its consistency with the refactorized prediction in the asymmetric limit.

- Dynamic (azimuthal structure) effect started at $\mathcal{O}(\alpha_s^2)$
- Only constant terms, no logarithm enhancement, consistents with the jet function
- Resummed results: a characteristic linear behavior in θ_L and a systematic growth with the hard scale Q
- Enhanced soft recoil at larger angles and higher energies.

Providing a quantitatively controlled framework to study non-global logarithms

A direct connection between non-global radiation and measurable angular modulations

Azimuthal logarithm enhancement at $\mathcal{O}(\alpha_s^3)$?

Thank you!!