SHANDONG UNIVERSITY

Massive antenna functions in N3LO QCD top-pair production @ electron-positron colliders

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Based on on-going works with Xuan Chen

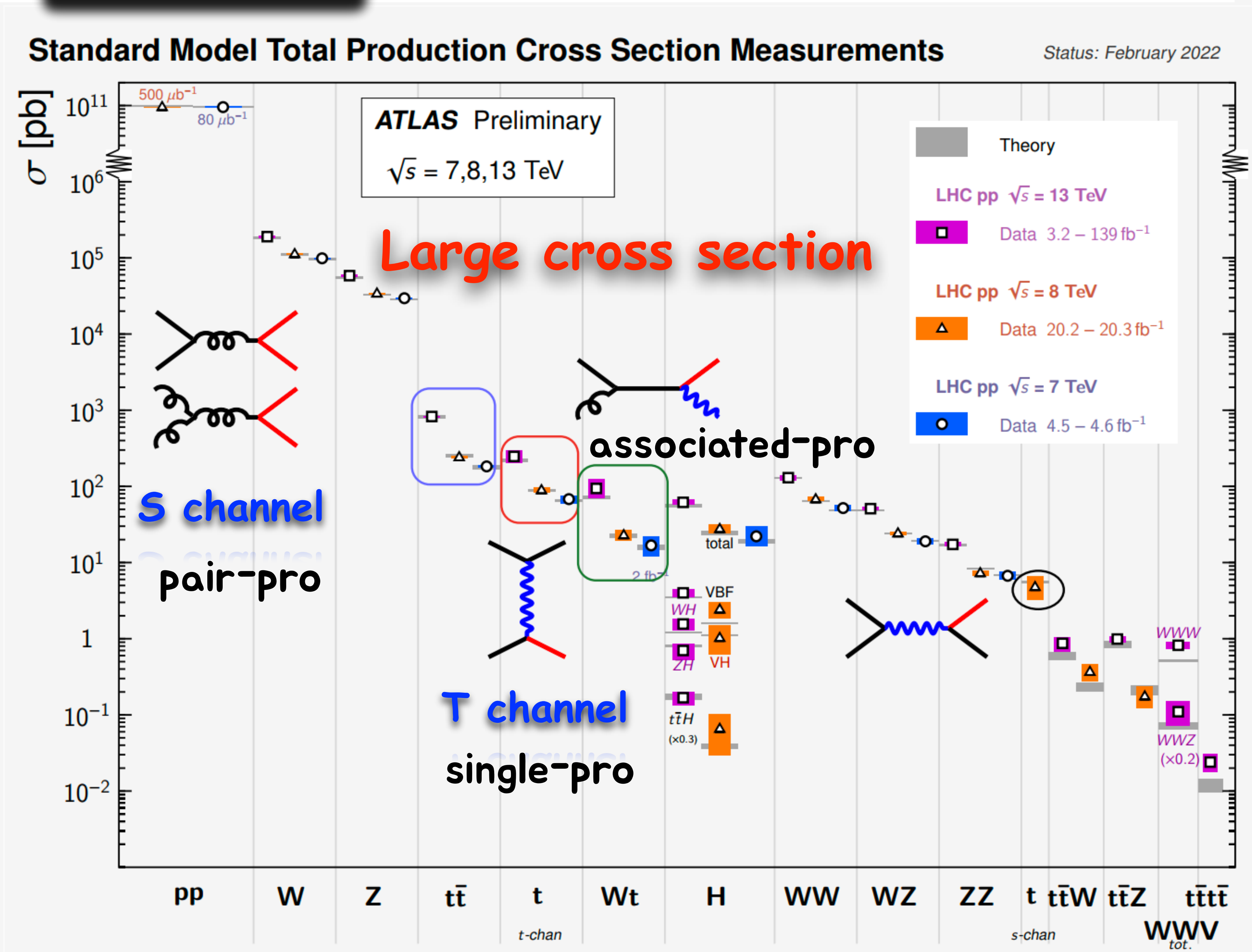
第五届量子场论及其应用研讨会

2025.10.31-11.2, Beijing

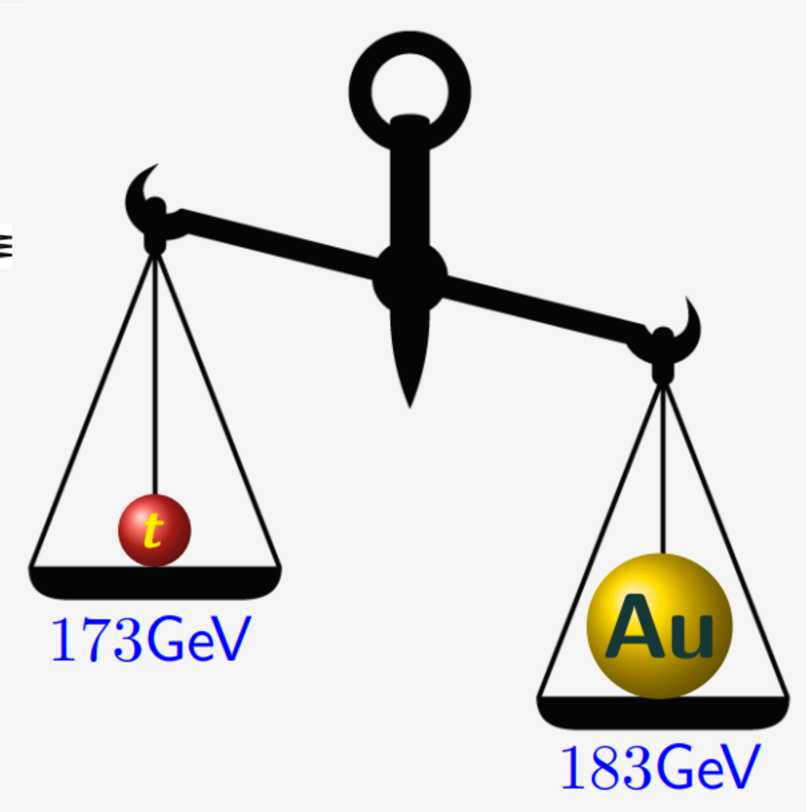
Outline

Research background

Top production



- **Heaviest particle**



- **Top-mass**
- **Decay width**
- **Yukawa coupling**

Research background

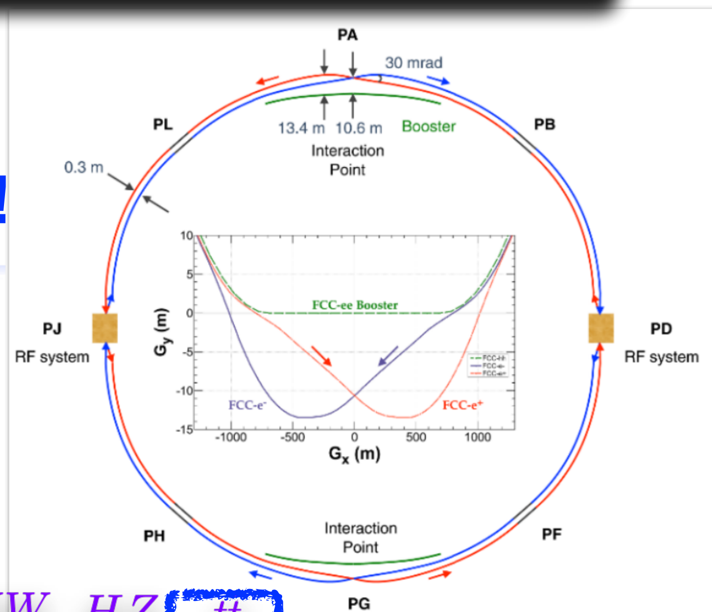
Future Lepton Colliders

Cleaner background than LHC

FCC-ee

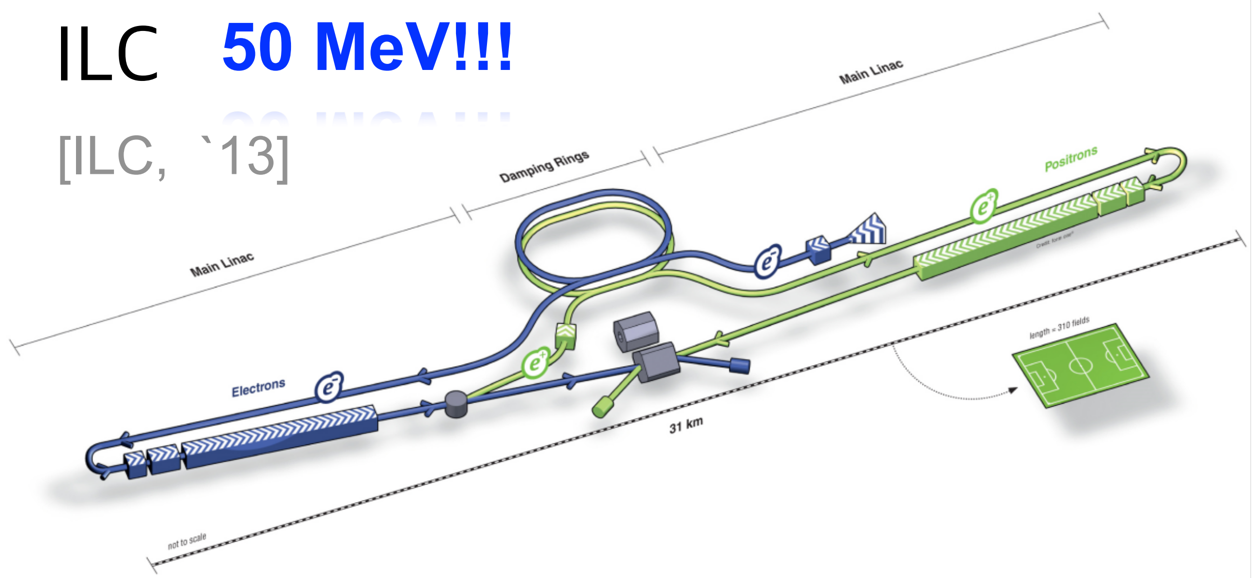
17 MeV!!!

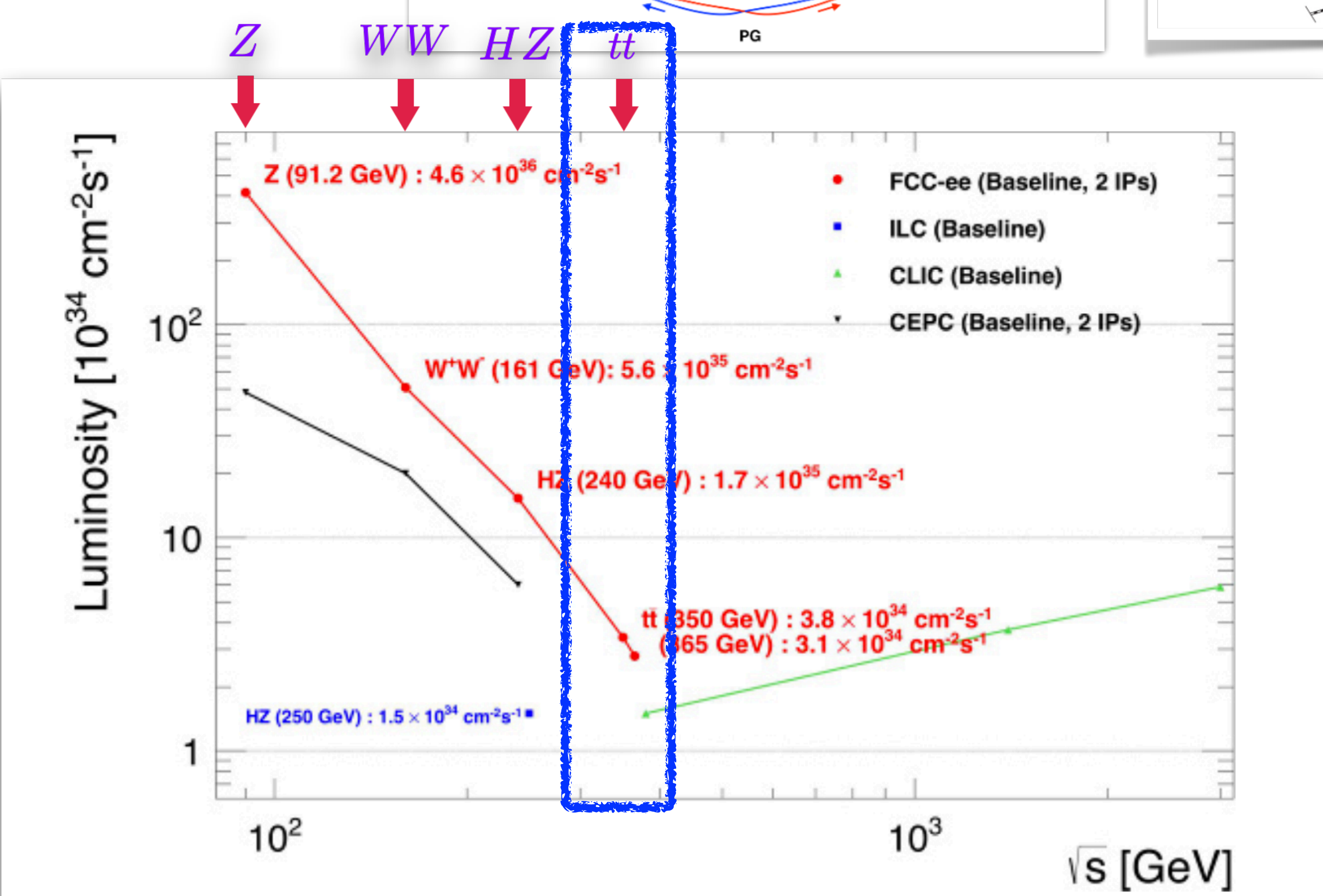
[FCC, `19]



ILC **50 MeV!!!**

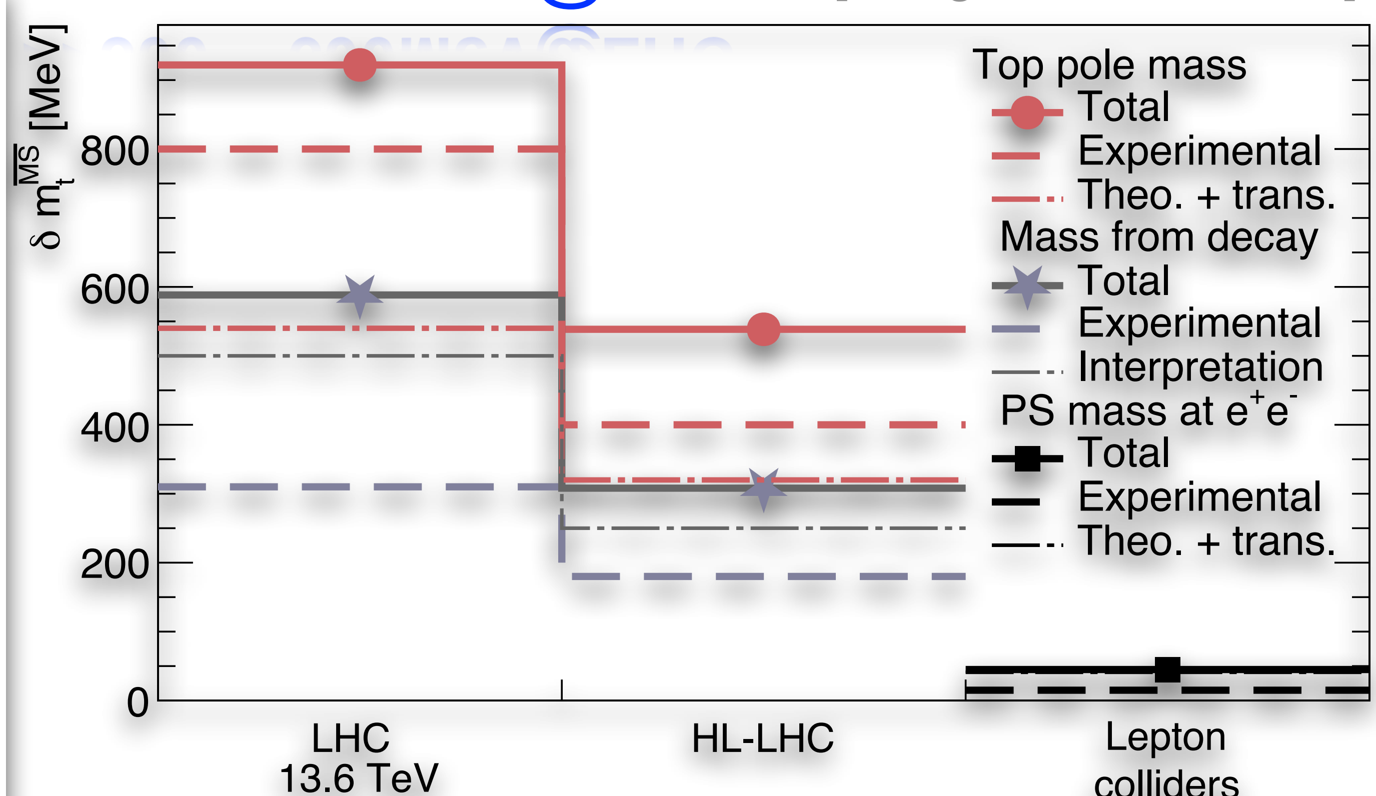
[ILC, `13]





>>500 – 600MeV@LHC

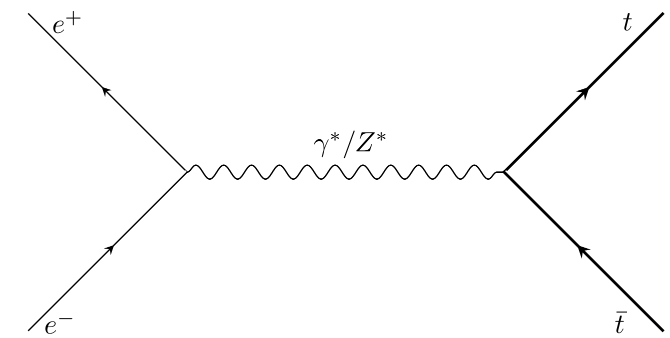
[K. Agashe et al., `22]



Research background

State-of-the-art

$$e^+e^- \to \gamma^*/Z^* \to t\bar{t}$$



✤ NLO

- **NLO QCD:** [J. Jersak, E. Laermann `82]
- **NLO EW :** [W. Beenakker, W. Hollik `91]

✤ NNLO QCD

Total xSec, distributions, AFB

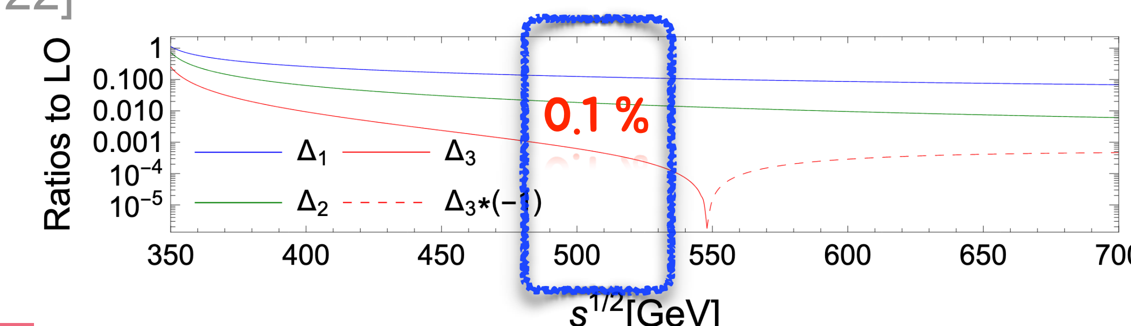
- **Phase-space slicing:** [J. Gao, H.X. Zhu `14]
- **Antenna subtraction :** [L. Chen, W. Bernreuther `16]

$\sqrt{s}$ [GeV]	360	381.3	400	500
Δ_1	0.627	0.352	0.266	0.127
Δ_2	0.281	0.110	0.070	0.020

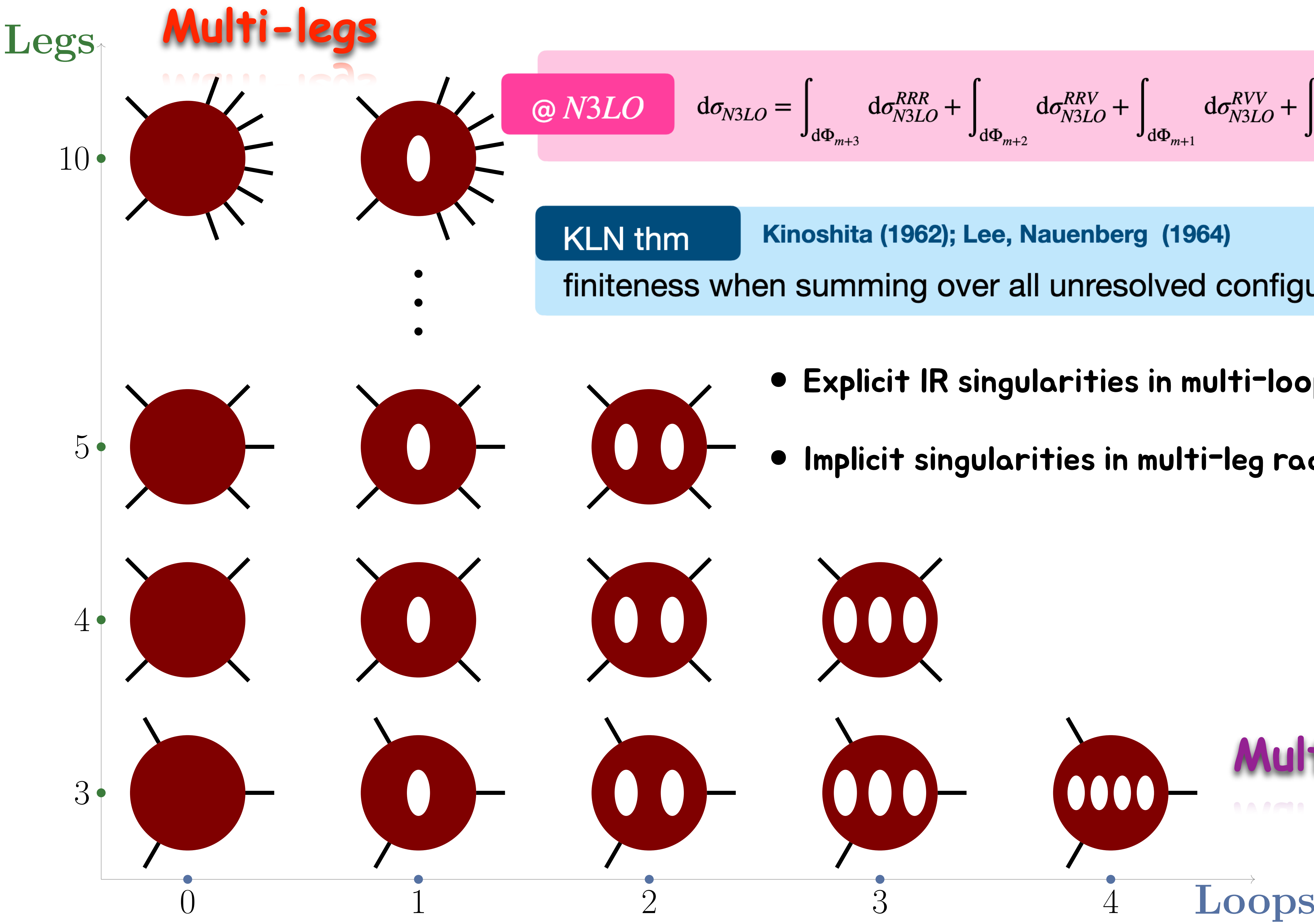
2 %

✤ N3LO QCD

- **Form factor :** [J.M. Henn `16, R. N. Lee `18, M. Fael, `22]
- **Total xSec & $M_{t\bar{t}}$-distribution :** [X. Chen, Y.-Q Ma `24]



Research background



@ N3LO

$$d\sigma_{N3LO} = \int_{d\Phi_{m+3}} d\sigma^{RRR}_{N3LO} + \int_{d\Phi_{m+2}} d\sigma^{RRV}_{N3LO} + \int_{d\Phi_{m+1}} d\sigma^{RVV}_{N3LO} + \int_{d\Phi_{m}} d\sigma^{VVV}_{N3LO}$$

KLN thm Kinoshita (1962); Lee, Nauenberg (1964)

finiteness when summing over all unresolved configurations

- Explicit IR singularities in multi-loop integrals
- Implicit singularities in multi-leg radiation

Research background

IR subtraction methods

Subtraction methods

- NLO : CS dipole [S. Catani `96] FKS sutraction [S. Frixione, Z. Kunszt, A. Signer `96]
- NNLO :
  - Antenna subtraction [A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover `05]
    $e^+e^- \to t\bar{t}$ [L. Chen, W. Bernreuther `16]
  - SecToR-Improved residue subtraction STRIPPER $t\bar{t}$ [M. Czakon `10, 11`…]
  - Nested soft–collinear subtraction [F. Caola `17]
    NNLO massive extension [K. Melnikov, M.-M. Long `24, `25]
  - Local analytic sector subtraction, ColorFul subtraction …

Slicing methods

- q_T-subtraction $t\bar{t}H$ [S. Catani `23, S. Devoto `25] $t\bar{t}W$ [L. Buonocore `25]
- N -jettiness

See R.-J. Fu's talk

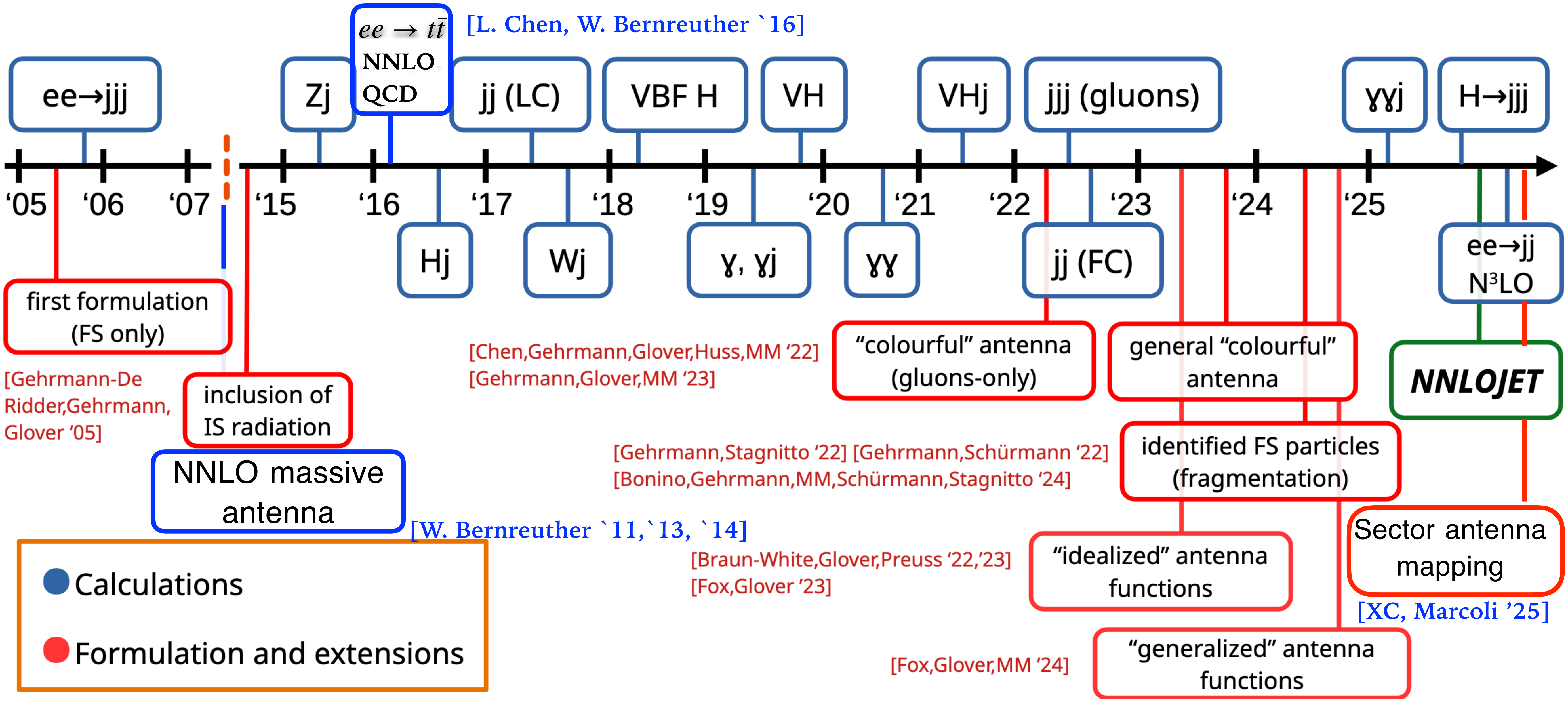
Research background

Based on Marcoli's slide @ Loop Summit 2

History of antenna subtraction

[NNLOJET collaboration: Huss *et al.* '25]

Successfully applied at NNLO to a variety of processes within the ***NNLOJET*** Monte Carlo framework



NNLOJET: Parton Level Event Generator

Based on Xuan's slide @ SPCS2025

NNLO JET

Still missing massive quark production process @ public version NNLOJET

A parton-level event generator for jet cross sections at NNLO QCD accuracy

About NNLOJET is a parton-level event generator for jet cross sections using the antenna subtraction method. It can be used to compute a large number of jet cross sections and related observables in e^+e^-, ep and pp collisions at next-to-next-to-leading order in QCD. NNLOJET contains routines for Monte Carlo phase-space integration, event handling and analysis.

Citation If you are using NNLOJET for a scientific paper, please cite:

A. Huss et al. (NNLOJET Collaboration)
NNLOJET: a parton-level event generator for jet cross sections at NNLO QCD accuracy
arXiv:2503.22804 [INSPIRE]

Please also cite the revelant references for each processes (as included in the .bib file which is automatically written when running NNLOJET through the automatic workflow)

License GNU General Public License (GPL) v3.0

Contact Please send comments, questions and suggestions to nnlojet-support@cern.ch

https://nnlojet.hepforge.org/index.html

➤ Processes implemented:

e^+e^- scattering Jet production

- $e^+e^- \to 2$jets
- $e^+e^- \to 3$jets

***ep* scattering** Jet production

- $ep \to$ lepton + 1jet
- $ep \to$ lepton + 2jets

***pp* scattering** Jet production

- $pp \to 1\text{jet} + X$
- $pp \to 2$jets

Vector boson (+ jet) production

- $pp \to (\gamma^* Z) + 0$jet
- $pp \to (\gamma^* Z) + 1$jet
- $pp \to W^\pm + 0$jet
- $pp \to W^\pm + 1$jet

Photon (+ jet) production

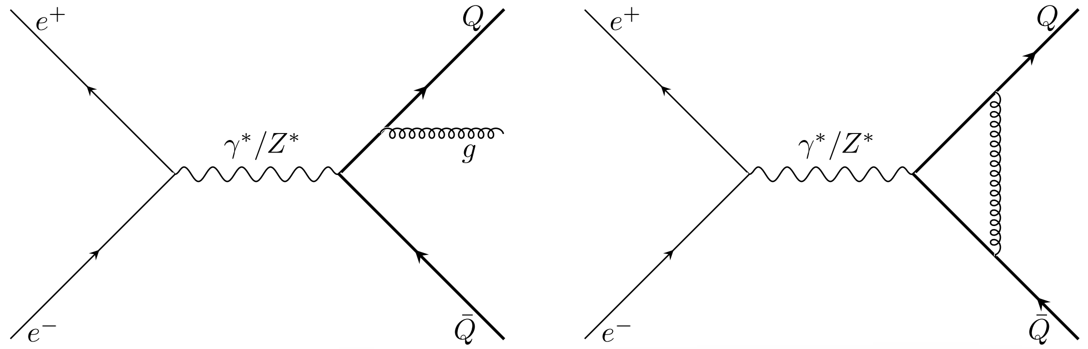
- $pp \to \gamma + X$
- $pp \to \gamma + 1$jet
- $pp \to \gamma\gamma$

Higgs (+ jet) production

- $pp \to H + X$
- $pp \to H + 1$jet

Framework of antenna

NLO antenna subtractions



✤ NLO cross sections

$$\sigma_{\mathrm{NLO}} = \int_{\mathrm{d}\Phi_3} \mathrm{d}\,\sigma^{R}_{\mathrm{NLO}} + \int_{\mathrm{d}\Phi_2} \mathrm{d}\,\sigma^{V}_{\mathrm{NLO}}\,,$$

Difficultly to solve analytically

✤ Subtraction schemes

$$\sigma_{\mathrm{NLO}} = \int_{\mathrm{d}\Phi_3} \underbrace{\left(\mathrm{d}\sigma^{R}_{\mathrm{NLO}} - \mathrm{d}\sigma^{S}_{\mathrm{NLO}}\right)}_{\text{Finite}} + \int_{\mathrm{d}\Phi_2} \underbrace{\left[\int_1 \mathrm{d}\,\sigma^{S}_{\mathrm{NLO}} + \mathrm{d}\sigma^{V}_{\mathrm{NLO}}\right]}_{\text{Finite}}$$

- $\mathrm{d}\sigma^{S}_{NLO}$: Mimics singular behaviour in IR-limits of $\mathrm{d}\sigma^{R}_{NLO}$
- $\int_1 \mathrm{d}\sigma^{S}_{NLO}$: Analytically integration over $\mathrm{d}\Phi_1$
- Building block: NLO massive antenna function A_3^0

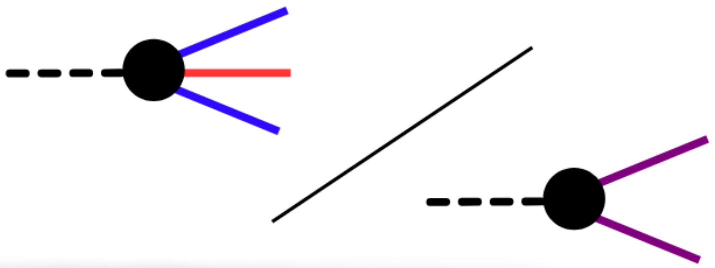
✤ Antenna subtraction

$$\mathrm{d}\sigma^{S}_{\mathrm{NLO}} = \mathcal{N}\,\mathrm{d}\,\Phi_3(p_1, p_2, p_3; q) A_3^0 |\mathcal{M}_2(\widetilde{p_{13}}, \widetilde{p_{23}})|^2 J_2^{(2)}(\widetilde{p_{13}}, \widetilde{p_{23}})$$

Framework of antenna

Antenna functions

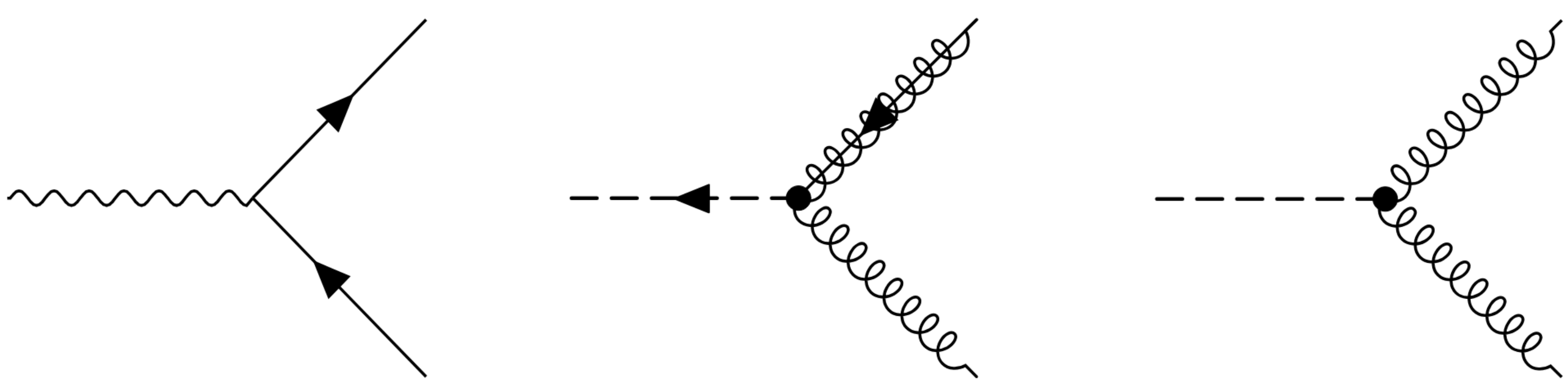
$$X_3^0(i,j,k) = \frac{|\mathcal{M}_3^0(i,j,k)|^2}{|\mathcal{M}_2^0(\tilde{I},\tilde{K})|^2} \quad \propto$$



- Built from simple matrix elements
- Mimic the divergent behaviour in singular limits
- Easily integrated over phase space

✤ **Colour-singlet decay MEs**

Photon decay: $\gamma^* \to q\bar{q}$ | Neutralino decay: $\chi \to \tilde{g}g$ | Higgs decay: $H \to gg$



$$\mathcal{L} = i\eta\bar{\psi}^a_{\tilde{g}}\sigma^{\mu\nu}\psi_{\tilde{\chi}}F^a_{\mu\nu} + \text{h.c.}$$

$$\mathcal{L} = -\frac{\lambda}{4}HF^{a,\mu\nu}F^a_{\mu\nu}$$

Quark-Antiquark antenna functions

Quark-Gluon antenna functions

Gluon-Gluon antenna functions

N3LO massless antenna

[P. Jakubčík et al `22]

[X. Chen et al `23]

[X. Chen et al `23]

Massive antenna

N3LO massive antenna

Local subtraction @ N3LO

[Chen,Jakubcik,Marcoli,Stagnitto `25]

$$d\sigma_{N^3LO} = \int_n [d\sigma^{VVV} - d\sigma^{W}] + \int_n [d\sigma^{RVV} - d\sigma^{U}] + \int_{n+1} [d\sigma^{RRV} - d\sigma^{T}] + \int_{n+2} [d\sigma^{RRR} - d\sigma^{S}]$$

triple-virtual subtraction term

double-virtual real subtraction term

double-real-virtual subtraction term

triple-real subtraction term

[X. Chen,M. Marcoli `25]

✤ Subtraction terms

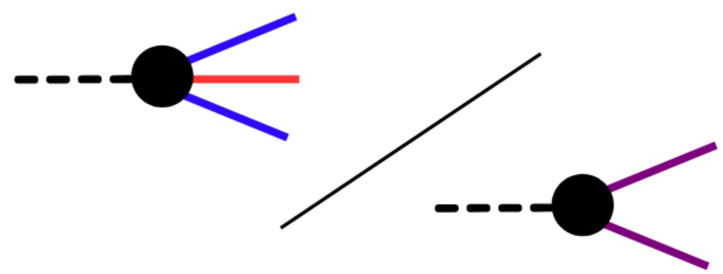
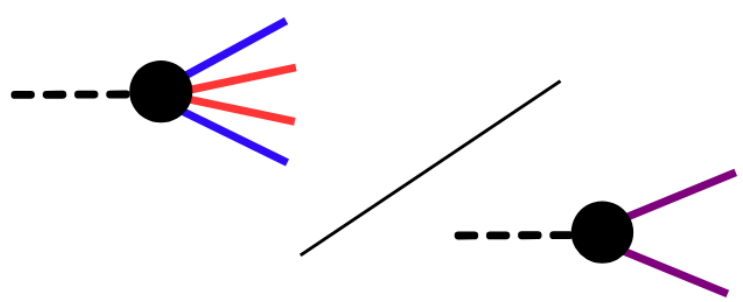
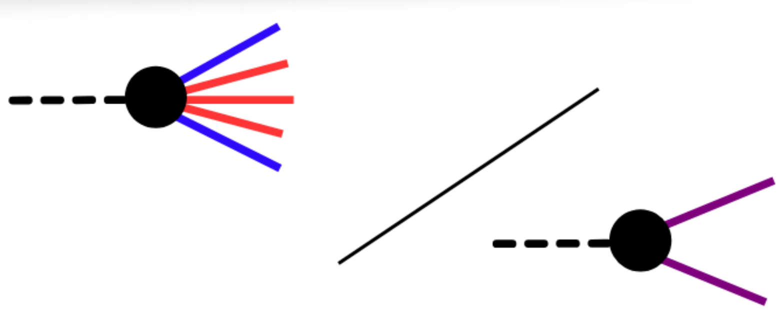
- **RRR:** $d\sigma^{S} = d\sigma^{S_1} + d\sigma^{S_2} + d\sigma^{S_3}$
- **RRV:** $d\sigma^{T} = d\sigma^{V_1S_1} + d\sigma^{V_1S_2} - \int_1 d\sigma^{S_1}$
- **RVV:** $d\sigma^{U} = d\sigma^{V_2S_1} - \int_1 d\sigma^{V_1S_1} - \int_2 d\sigma^{S_2}$
- **VVV:** $d\sigma^{W} = -\int_1 d\sigma^{V_2S_1} - \int_2 d\sigma^{V_1S_2} - \int_3 d\sigma^{S_3}$

- $d\sigma^{V_lS_m}$: counterterm for the divergent behavior of m unresolved partons as well as the explicit IR poles in a l-loop matrix element.
- NNLO-like terms: $d\sigma^{S_1}, d\sigma^{S_2}, d\sigma^{V_1}, \sigma^{V_1S_1}, \sigma^{V_2}$
- N3LO terms: $d\sigma^{S_3}, d\sigma^{V1S_2}, \sigma^{V_2S_1}, \sigma^{V_3}$

N3LO massive antenna

Based on Marcoli's slide @ HP 2

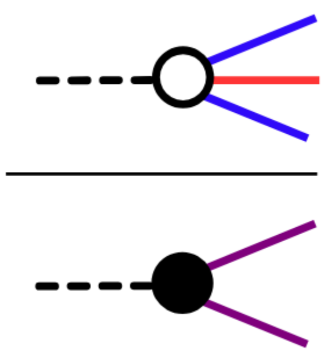
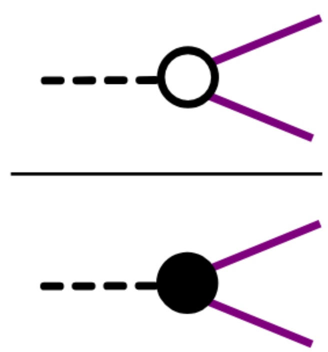
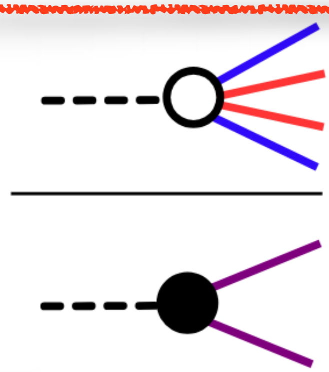
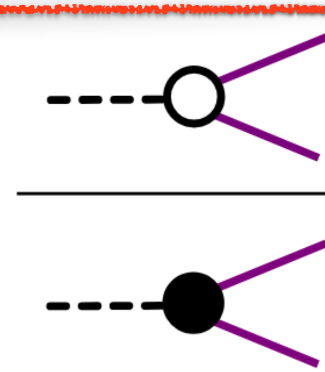
Tree-level antenna functions

		Antenna	Integrated antenna
NLO:		$X_3^0 = \frac{M_3^0}{M_2^0}$	$\mathcal{X}_3^0 \propto \int \mathrm{d}\Phi_3\, X_3^0$
NNLO:		$X_4^0 = \frac{M_4^0}{M_2^0}$	$\mathcal{X}_4^0 \propto \int \mathrm{d}\Phi_4\, X_4^0$
N³LO:		$X_5^0 = \frac{M_5^0}{M_2^0}$	$\mathcal{X}_5^0 \propto \int \mathrm{d}\Phi_5\, X_5^0$

N3LO massive antenna

Based on Marcoli's slide @ HP 2

One-loop antenna functions

		Antenna	Integrated antenna
NLO:			
NNLO:	 $-X_3^0 \cdot$ 	$X_3^1 = \frac{M_3^1}{M_2^0} - X_3^0 \frac{M_2^1}{M_2^0}$	$\mathcal{X}_3^1 \propto \int \mathrm{d}\Phi_3 \, X_3^1$
N³LO:	 $-X_4^0 \cdot$ 	$X_4^1 = \frac{M_4^1}{M_2^0} - X_4^0 \frac{M_2^1}{M_2^0}$	$\mathcal{X}_4^1 \propto \int \mathrm{d}\Phi_4 \, X_4^1$

N3LO massive antenna

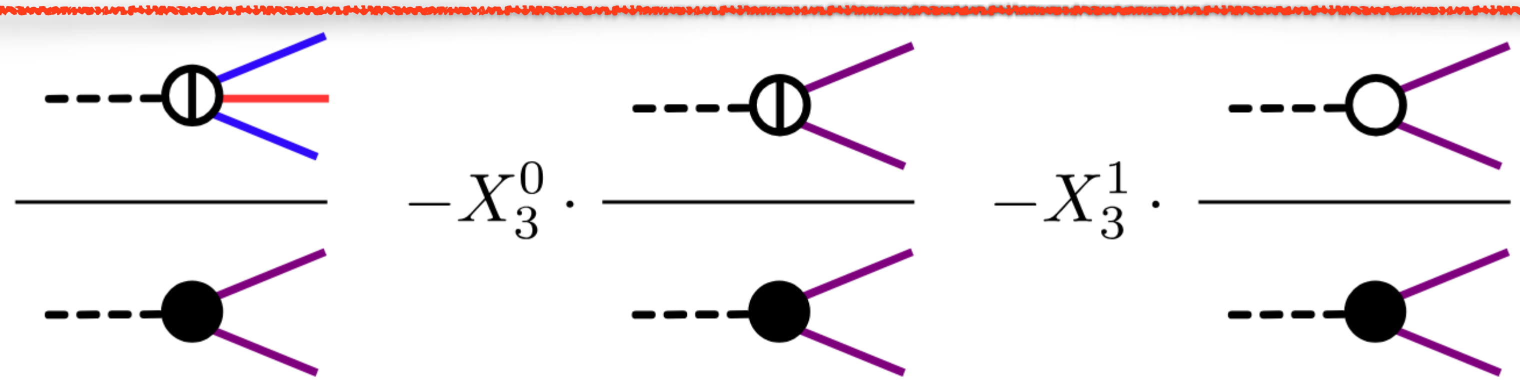
Based on Marcoli's slide @ HP 2

Two-loop antenna functions

NLO: ✗

NNLO: ✗

N³LO:



Antenna $X_3^2 = \frac{M_3^2}{M_2^0} - X_3^0 \frac{M_2^2}{M_2^0} - X_3^1 \frac{M_2^1}{M_2^0}$

Integrated antenna $\mathcal{X}_3^2 = \int \mathrm{d}\Phi_3 X_3^2$

N3LO massive antenna

Workflow

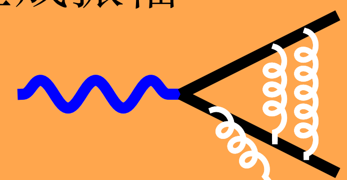
天线函数

$X_3^2 \propto M_3^2 / M_2^0$

天线因子化

→

生成振幅



FeynArts QGRAF

→

振幅约化

$\mathcal{M}^{\mu\nu} = \sum C_i T_i^{\mu\nu}$

CalcLoop FeynCalc

→

IBP 约化

$C_i = \sum c_j I_j$

有限域 块三角

↓ Bottleneck Blade

主积分计算

$I_j = \sum \epsilon^k I_j^{(k)}$

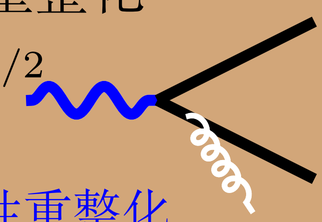
AMFlow 渐进展开

[X. Liu and Y.-Q. Ma `23]

←

紫外重整化

$Z_2 Z_3^{1/2}$



相乘性重整化

[K. Melnikov `03]

←

相空间积分

$\mathcal{X}_3^2 \propto \int \mathrm{d}\Phi_3 X_3^2$

逆幺正性

$$2\pi i \delta^+(p^2) \to \frac{1}{p^2 - i0} - \frac{1}{p^2 + i0}$$

[Cutkosky `60] [Anastasiou, Melnikov `02,`03]

← . . .

N3LO massive antenna

Generalized Power-Log Series Expansion

- **Differential equations**

$$\frac{\partial}{\partial x_i} I(\vec{x},\epsilon) = B_i(\vec{x},\epsilon) I(\vec{x},\epsilon) \qquad x = \frac{4m_t^2}{s}$$

- **Boundary: AMFlow**

$$I(\boldsymbol{\alpha},\vec{x},d,\eta) = \int \prod_{i=1}^{L} \frac{\mathrm{d}^d l_i}{i\pi^{d/2}} \frac{1}{(D_1 + i\eta)^{\alpha_1} \cdots (D_n + i\eta)^{\alpha_n}} \qquad I(\boldsymbol{\alpha},\vec{x},d) = \lim_{\eta \to 0^+} I(\boldsymbol{\alpha},\vec{x},d,\eta)$$

[X. Liu, Y-Q. Ma `17]

- **Asymptotic expansion**

Asy behavior

$$I(x) = \sum_{\mu \in S} \sum_{k=0}^{k_\mu} \sum_{n=0}^{N} (x - x_0)^\mu \log^k (x - x_0) c_{\mu,k,n} (x - x_0)^n$$

Expansion point

[R. Lee, Smirnovs `17]

- **Segment lines**

$$[\max(x_{start}, x_0 - r_0),\ x_0,\ \min(x_{end}, x_0 + r_0)] \qquad r_0 = k \times \min(\mathrm{Abs}[x_0 - x_j]|_{x_j \in X_{sing} \wedge x_j \neq x_0})$$

N3LO massive antenna

Renormalization

- **Multiplicatively Renormalization**

$$S_\epsilon \alpha_s^b = \mu^{2\epsilon} Z_{\alpha_s} \alpha_s^r, \qquad m^b = Z_m m^r, \qquad \psi^b = \sqrt{Z_2}\psi^r, \qquad A_b^{a\mu} = \sqrt{Z_3} A_r^{a\mu},$$

- **Renormalized amplitude:**

renormalized mass

$$|\hat{\mathcal{M}}\rangle_{Q\bar{Q}} = Z_2 |\mathcal{M}_b(m^b)\rangle_{Q\bar{Q}}$$

$$= Z_2 \sqrt{4\pi\alpha} e_Q \left[\boxed{|\mathcal{M}_b^{(0)}(m^r)\rangle_{Q\bar{Q}}} + \left(\frac{\alpha_s^b}{2\pi}\right) |\mathcal{M}_b^{(1)}(m^b)\rangle_{Q\bar{Q}} + \left(\frac{\alpha_s^b}{2\pi}\right)^2 |\mathcal{M}_b^{(2)}(m^b)\rangle_{Q\bar{Q}} + \mathcal{O}(\alpha_s^3) \right],$$

- **Expansion of quark mass renormalization constant**

$$\frac{\mathrm{d}\hat{M}_n^l}{\mathrm{d}\alpha_s^r}\alpha_s^r \supset \boxed{\frac{\partial \hat{M}_n^l}{\partial m_b}} \frac{\partial Z_m}{\partial \alpha_s^r} m^r\, \alpha_s^r$$

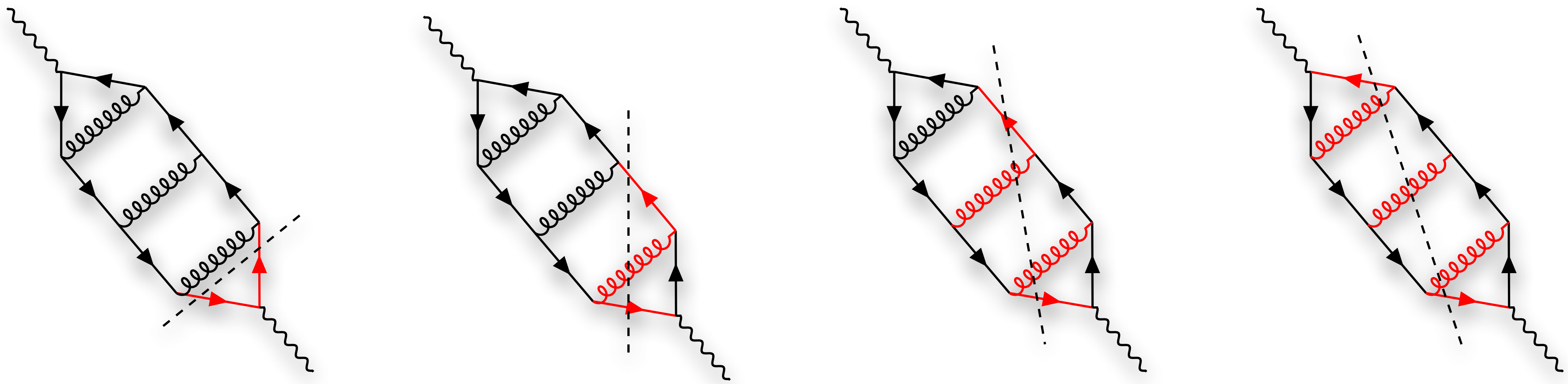
- Integrand level: Amplitude
- Integral level: $\frac{\partial |\mathscr{M}|^2}{\partial m_b} = \frac{\partial c_i}{\partial m_b} I_i + c_i \frac{\partial I_i}{\partial m_b},$

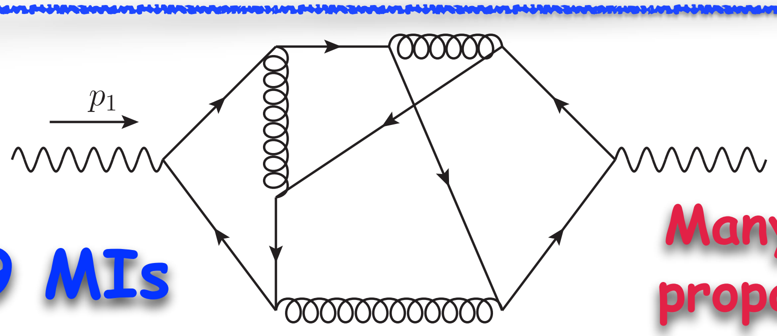
bare mass

N3LO massive antenna

N3LO integral families

- VVV: 893 diagrams, 46 families, $\sim \mathcal{O}(1\times 10^5)$ FIs
- VVR: 659 diagrams, 104 families, $\sim \mathcal{O}(2\times 10^5)$ FIs $\mathscr{A}_3^2$
- VRR: 407 diagrams, 257 families, $\sim \mathcal{O}(4\times 10^5)$ FIs $\mathscr{A}_4^1$ $\mathscr{B}_4^1$
- RRR: 86 diagrams, 201 families, $\sim \mathcal{O}(3\times 10^5)$ FIs $\mathscr{A}_5^0$ $\mathscr{B}_5^0$





369 MIs

Many massive propagators!!!

$$D_1 = l_1^2,\quad D_2 = l_2^2,\quad D_3 = l_3^2,\quad D_4 = l_4^2 - m_t^2$$
$$D_5 = (p_1+l_4)^2 - m_t^2,\quad D_6 = (l_1+l_4)^2 - m_t^2$$
$$D_7 = (p_1+l_2+l_4)^2 - m_t^2,\quad D_8 = (l_1+l_2+l_4)^2 - m_t^2$$
$$D_9 = (p_1+l_2+l_3+l_4)^2 - m_t^2,\quad D_{10} = (l_1+l_2+l_3+l_4)^2 - m_t^2$$
$$D_{11} = (p_1+l_1+l_2+l_3+l_4)^2 - m_t^2$$
$$D_{12} = (p_1+l_1)^2,\quad D_{13} = (p_1+l_2)^2,\quad D_{14} = (l_2+l_3)^2,$$

[X. Guan,X. Liu Y.-Q. Ma, et al `24]

N3LO massive antenna

Validation of antenna @ NNLO

Quark-antiquark antenna:

$$A_3^0\,(1_Q,3_g,2_{\bar{Q}}) = \frac{\left|\mathcal{M}_3^{0,\gamma^*}\,(1_Q,3_g,2_{\bar{Q}})\right|^2}{\left|\mathcal{M}_2^{0,\gamma^*}\,(1_Q,2_{\bar{Q}})\right|^2} \qquad B_4^0(1_Q,3_q,4_{\bar{q}},2_{\bar{Q}}) = \frac{|\mathcal{M}_4^{Q,\gamma^*}(1_Q,3_q,4_{\bar{q}},2_{\bar{Q}})|^2}{|\mathcal{M}_2^{0,\gamma^*}(1_Q,2_{\bar{Q}})|^2}\,,$$

$$A_4^0(1_Q,3_g,4_g,2_{\bar{Q}})| \;=\frac{|\mathcal{M}_4^{g,\gamma^*}(1_Q,3_g,4_g,2_{\bar{Q}})|^2}{|\mathcal{M}_2^{0,\gamma^*}(1_Q,2_{\bar{Q}})|^2} \qquad \tilde{A}_4^0(1_Q,3_g,4_g,2_{\bar{Q}})| \;= \frac{\left|\mathcal{M}_4^{g,\gamma^*}(1_Q,3_g,4_g,2_{\bar{Q}})+\mathcal{M}_4^{g,\gamma^*}(1_Q,4_g,3_g,2_{\bar{Q}})\right|^2}{|\mathcal{M}_2^{0,\gamma^*}(1_Q,2_{\bar{Q}})|^2}$$

Expressions:

$$\begin{aligned}
\mathcal{B}^0_{Qq\bar{q}\bar{Q}}(q^2,y,\mu,\epsilon) = \left(\frac{\mu^2}{q^2}\right)^{2\epsilon}\Bigg[&\frac{1}{\epsilon^2}\left\{-\frac{1}{6}+\left(\frac{1}{6}-\frac{1}{6(1-y)}-\frac{1}{6(1+y)}\right)H(0;y)\right\}\\
&+\frac{1}{\epsilon}\left\{\left(-\frac{8}{9}+\frac{23}{36(1-y)}-\frac{13}{36(1+y)}+\frac{1+2y}{6\,(1+4y+y^2)}\right)H(0;y)\right.\\
&-\frac{4}{3}H(1;y)+\left(\frac{4}{3}-\frac{4}{3(1-y)}-\frac{4}{3(1+y)}\right)H(2;y)\\
&+\left(\frac{4}{3}-\frac{4}{3(1-y)}-\frac{4}{3(1+y)}\right)H(-1,0;y)\\
&-\left(\frac{1}{3}-\frac{1}{3(1-y)}-\frac{1}{3(1+y)}\right)H(0,0;y)\\
&-\left(\frac{2}{3}-\frac{2}{3(1-y)}-\frac{2}{3(1+y)}\right)H(1,0;y)\\
&\left.-\frac{43}{36}-\frac{2\pi^2}{9}+\frac{2\pi^2}{9(1-y)}+\frac{2\pi^2}{9(1+y)}-\frac{y}{1+4y+y^2}\right\}
\end{aligned}$$

[0904.3297]

[1105.0530]

[1309.6887]

HPL:

- **Integral kernel**

$$f_{-1}(z)=\frac{1}{z+1},\qquad f_0(z)=\frac{1}{z},\qquad f_1(z)=\frac{1}{1-z}.$$

- **Weight 1:**

$$H(\pm1;z)\equiv\mp\log(1\mp z)=\int_0^z \mathrm{d}t\, f_{\pm1}(t),$$

$$H(0;z)\equiv\log(z)=\int_1^z \mathrm{d}t\, f_0(t).$$

- **Recursive definition:**

$$H(a_1,\cdots,a_n;z)=\int_0^z \mathrm{d}t\; f_{a_1}(t)H(a_2,\cdots,a_n;z),$$

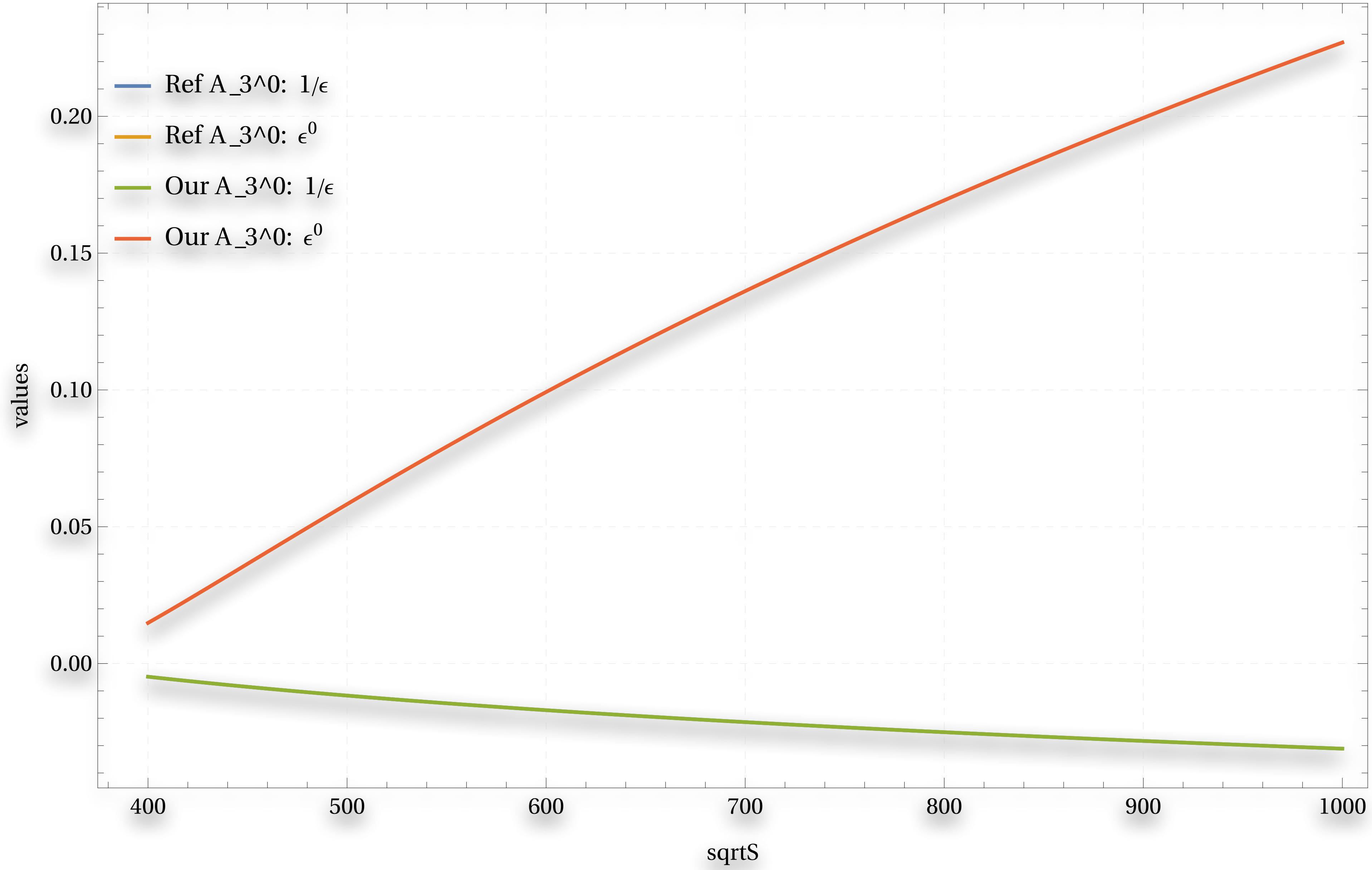
- **Package: HPL/PolyLogTools**

N3LO massive antenna

Validation of antenna @ NNLO

- $\mathscr{A}_3^0$:

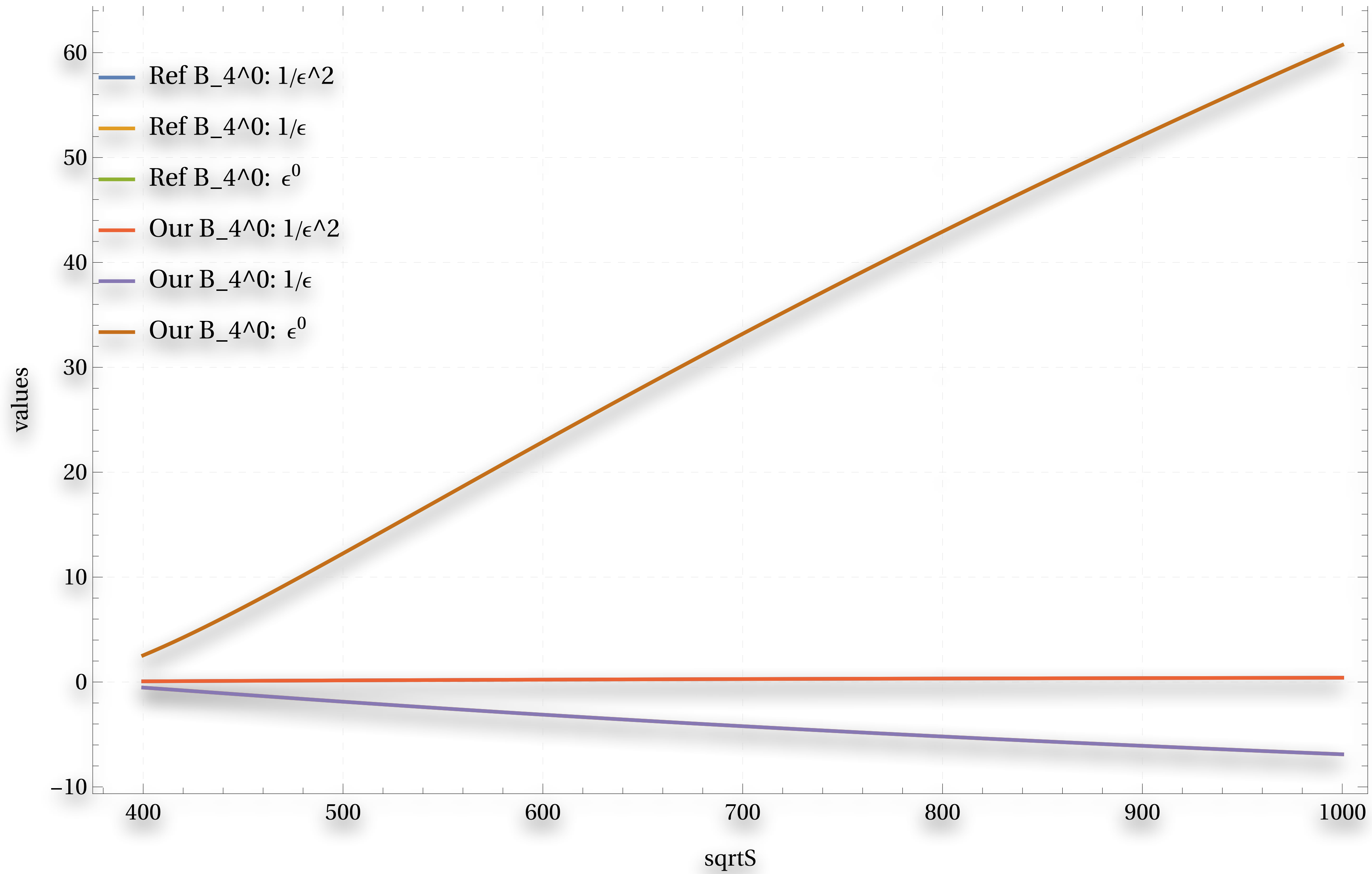
$$\int d\Phi_3 \left[A_3^0\left(1_Q, 3_g, 2_{\bar{Q}}\right) = \frac{\left|\mathcal{M}_3^{0,\gamma^*}\left(1_Q, 3_g, 2_{\bar{Q}}\right)\right|^2}{\left|\mathcal{M}_2^{0,\gamma^*}\left(1_Q, 2_{\bar{Q}}\right)\right|^2} \right]$$



N3LO massive antenna

- $\mathscr{B}_4^0$:

$$\int \mathrm{d}\Phi_4 \left[B_4^0\left(1_Q, 3_q, 4_{\bar{q}}, 2_{\bar{Q}}\right) = \frac{\left|\mathcal{M}_4^{Q,\gamma^*}\left(1_Q, 3_q, 4_{\bar{q}}, 2_{\bar{Q}}\right)\right|^2}{\left|\mathcal{M}_2^{0,\gamma^*}\left(1_Q, 2_{\bar{Q}}\right)\right|^2} \right]$$

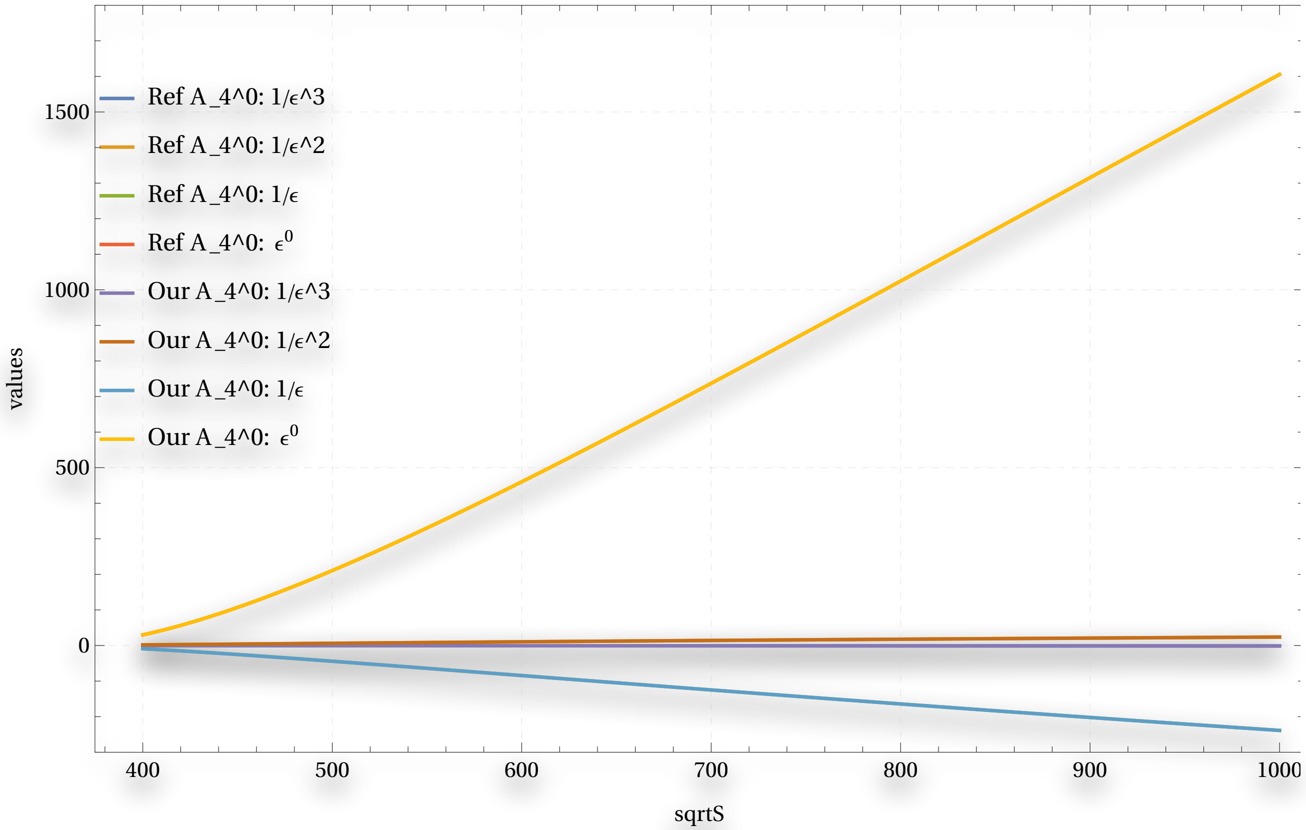


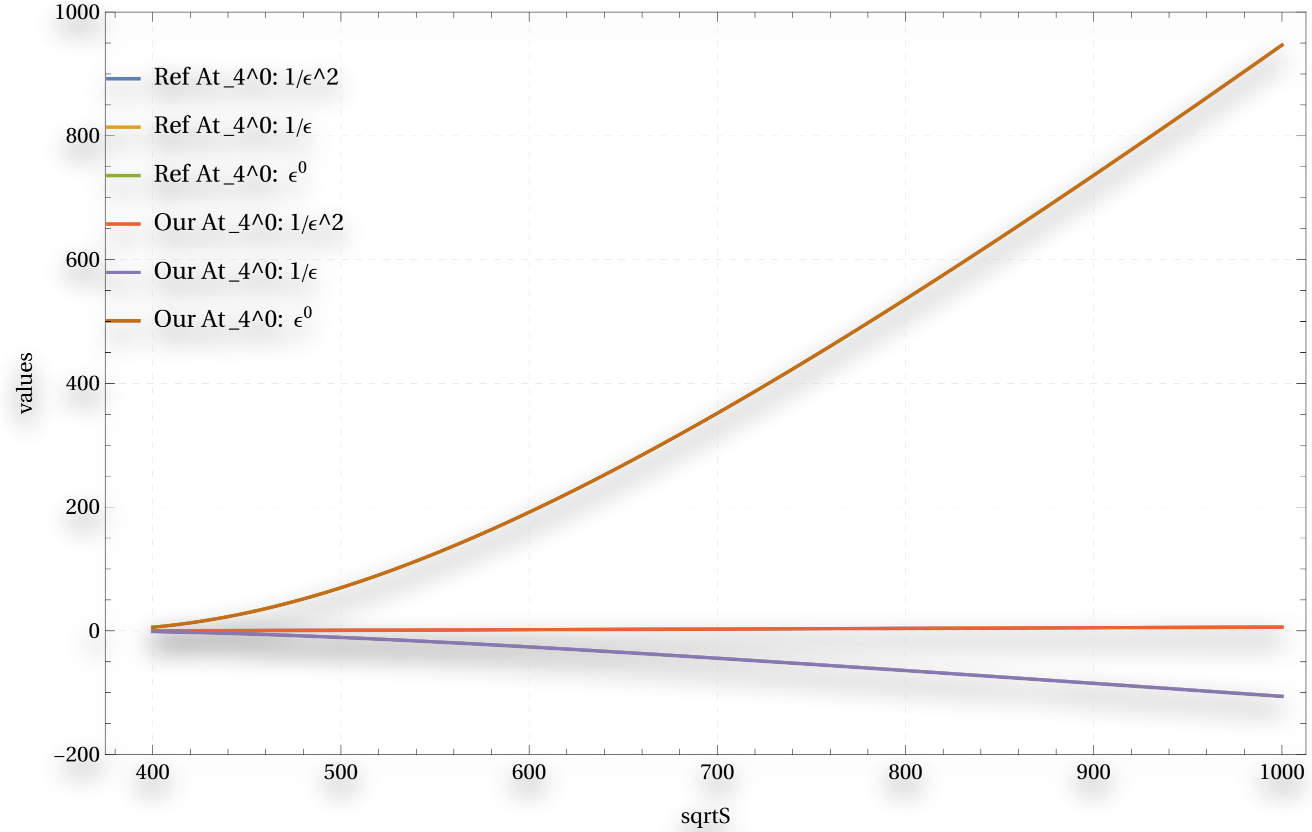
N3LO massive antenna

- $\mathscr{A}_4^0$ & $\tilde{\mathscr{A}}_4^0$:

$$\int \mathrm{d}\Phi_4 \left[A_4^0\left(1_Q, 3_g, 4_g, 2_{\bar{Q}}\right) = \frac{\left|\mathcal{M}_4^{g,\gamma^*}\left(1_Q, 3_g, 4_g, 2_{\bar{Q}}\right)\right|^2}{\left|\mathcal{M}_2^{0,\gamma^*}\left(1_Q, 2_{\bar{Q}}\right)\right|^2} \right]$$

$$\int \mathrm{d}\Phi_4 \left[\tilde{A}_4^0\left(1_Q, 3_g, 4_g, 2_{\bar{Q}}\right) = \frac{\left|\mathcal{M}_4^{g,\gamma^*}\left(1_Q, 3_g, 4_g, 2_{\bar{Q}}\right) + \mathcal{M}_4^{g,\gamma^*}\left(1_Q, 4_g, 3_g, 2_{\bar{Q}}\right)\right|^2}{\left|\mathcal{M}_2^{0,\gamma^*}\left(1_Q, 2_{\bar{Q}}\right)\right|^2} \right]$$

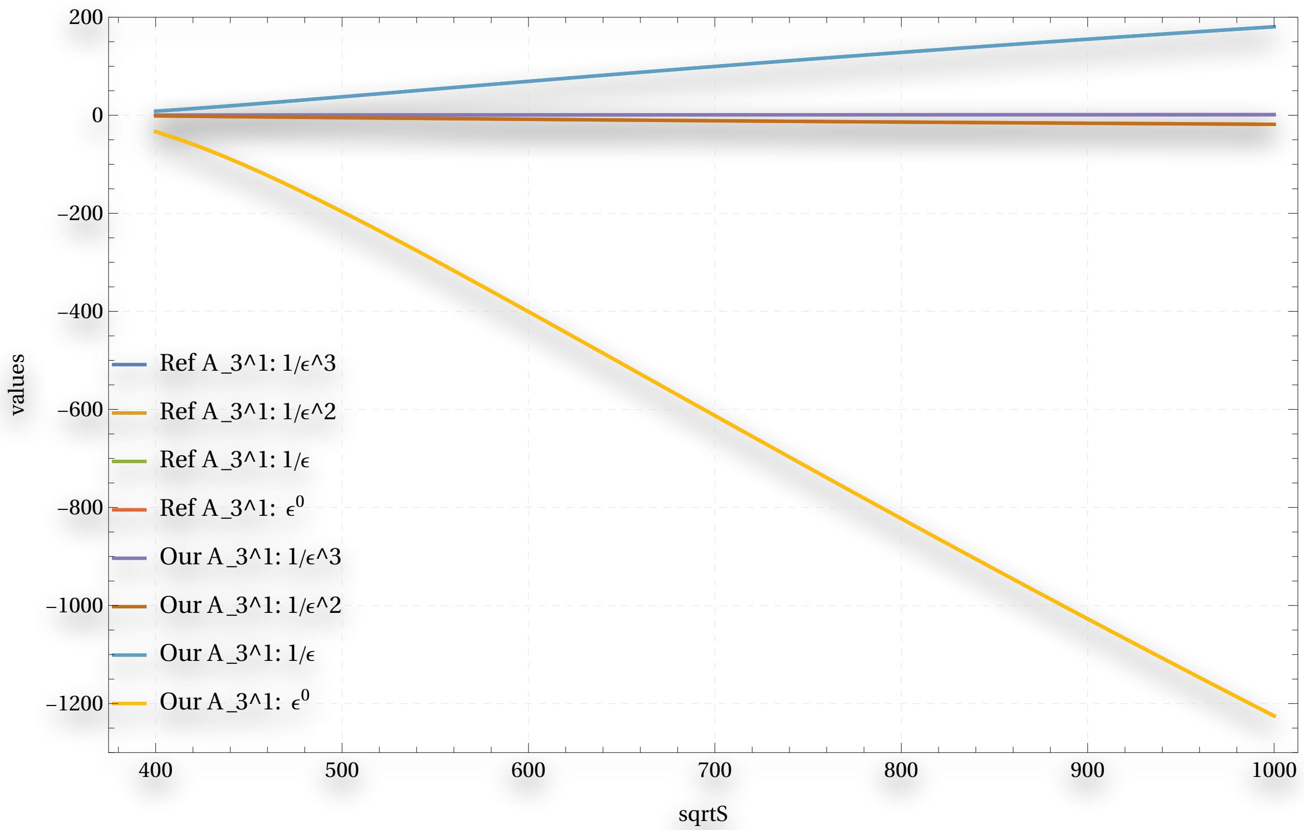


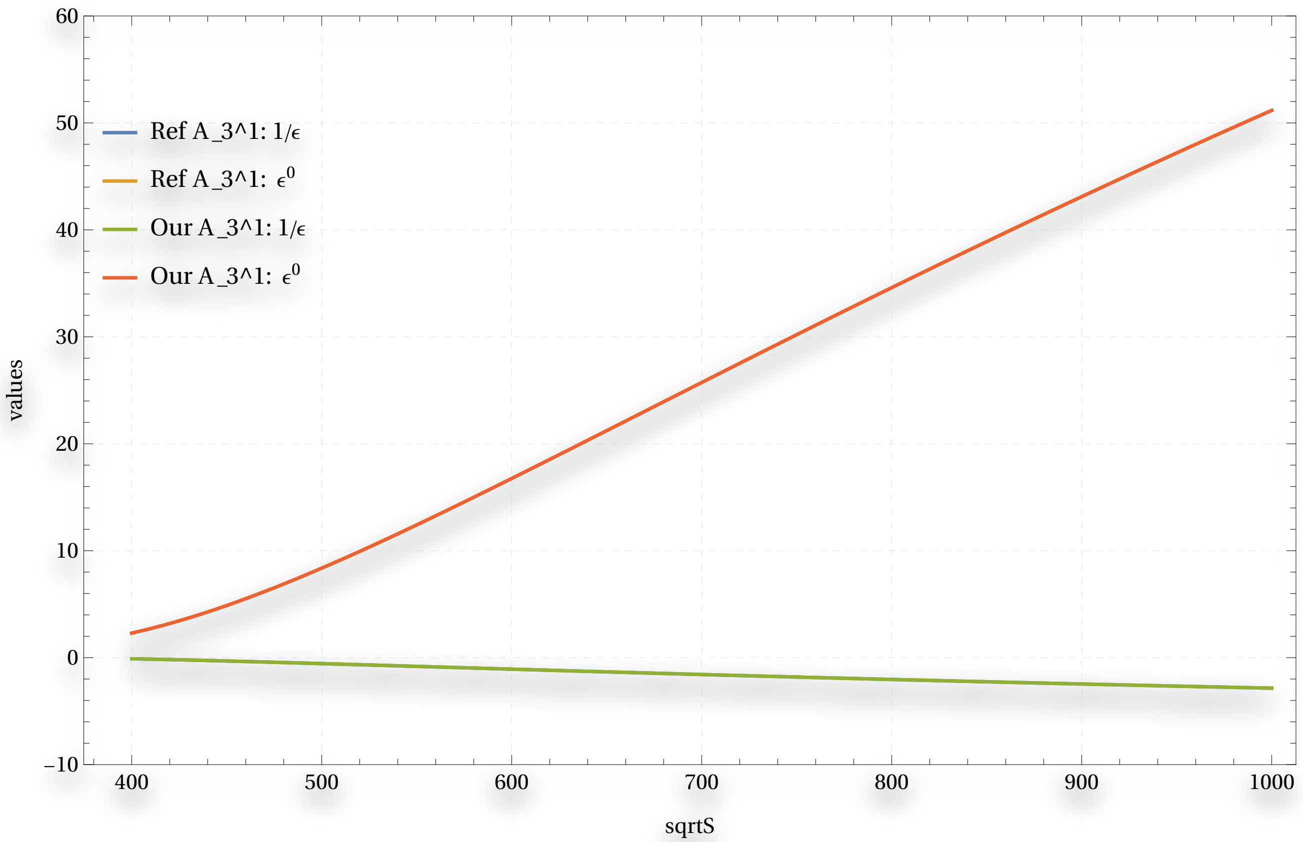


N3LO massive antenna

✤ $\mathscr{A}_3^1$ & $\tilde{\mathscr{A}}_3^1$:

$$\int \mathrm{d}\Phi_3 \left[A_3^1(p_1,p_3,p_2) = \frac{\delta \mathcal{M}^{\gamma^*,\mathrm{lc}}_{(3,1)}}{\left|\mathcal{M}^{\gamma^*}_{(2,0)}\right|^2} - A_3^0(p_1,p_3,p_2) \frac{\delta \mathcal{M}^{\gamma^*}_{(2,1)}}{\left|\mathcal{M}^{\gamma^*}_{(2,0)}\right|^2} \right]$$



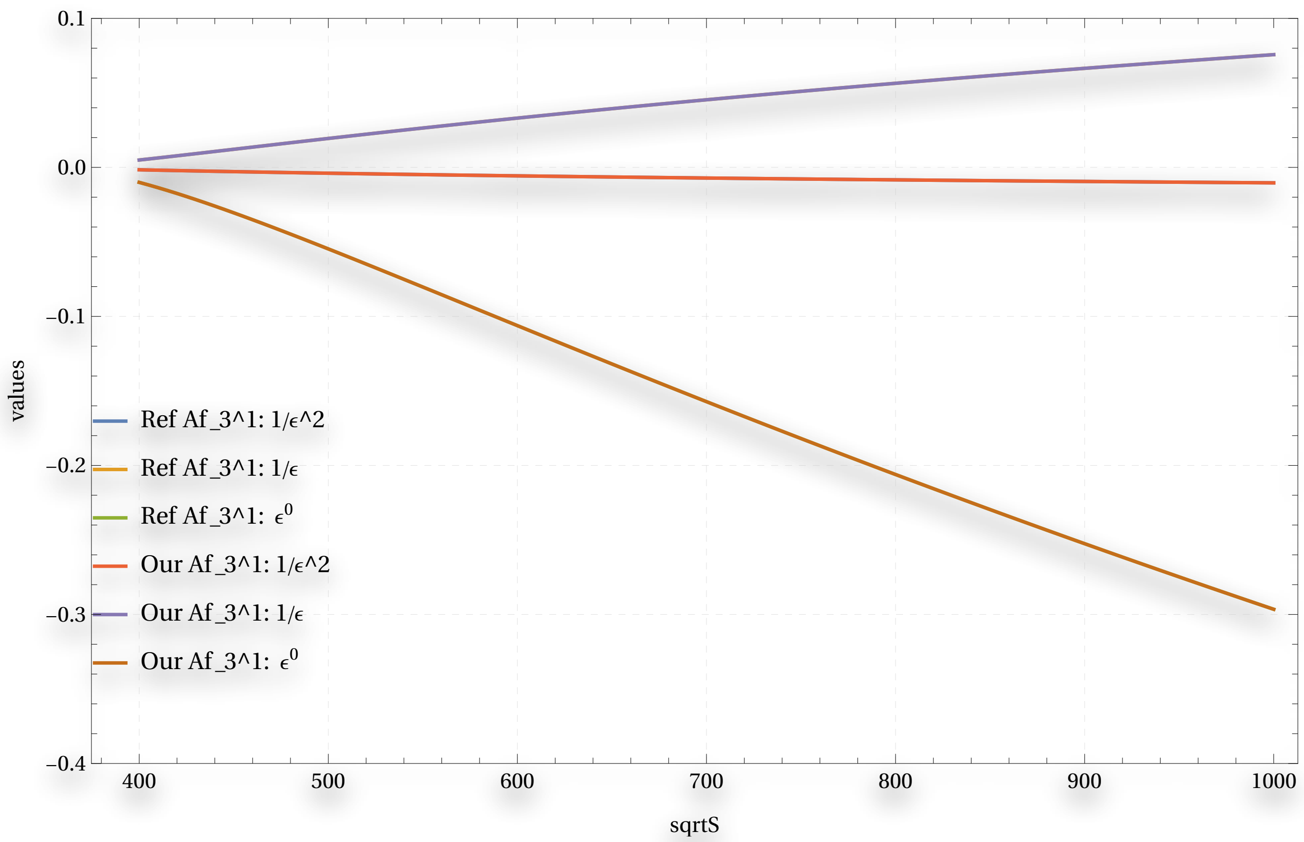


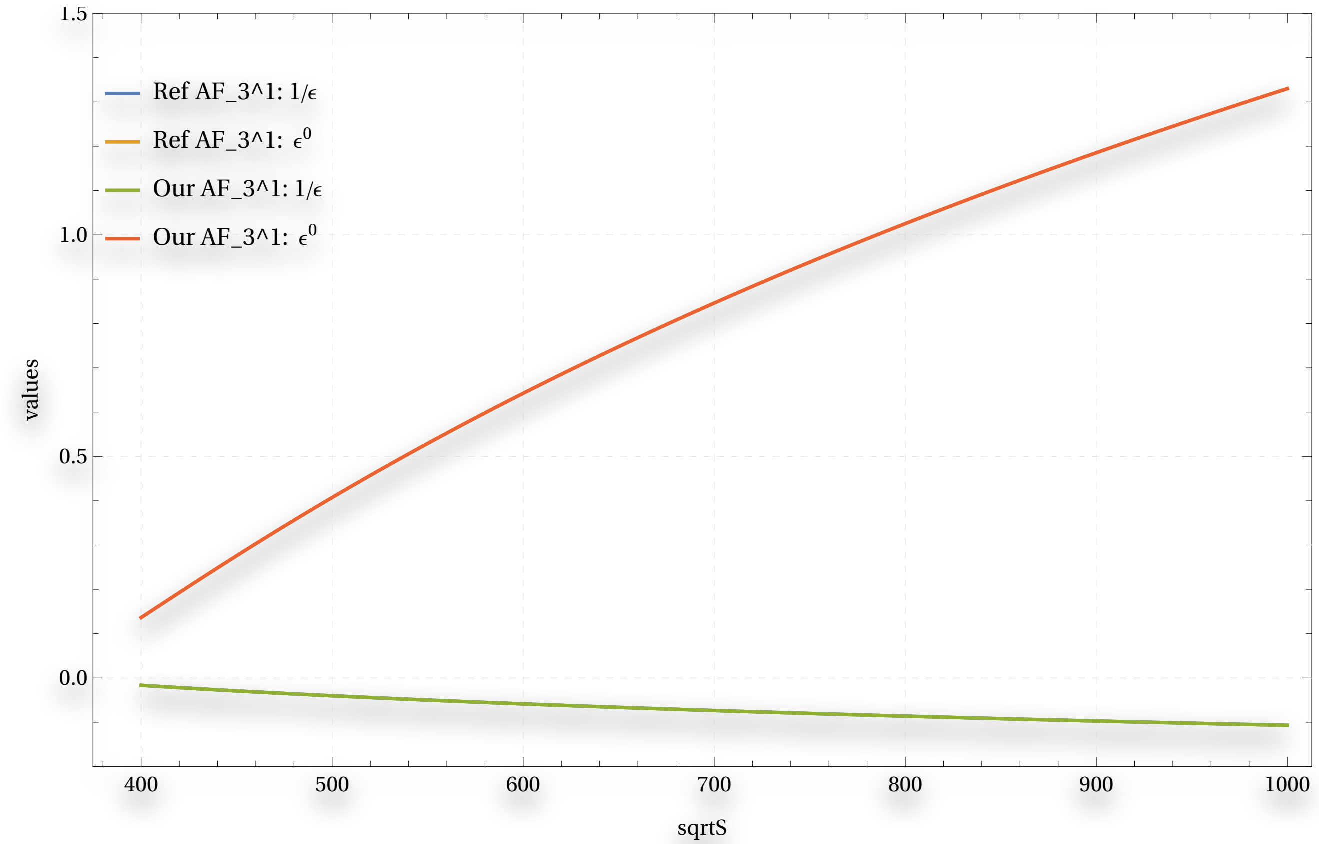
N3LO massive antenna

- $\hat{\mathscr{A}}^1_{3f}$ & $\hat{\mathscr{A}}^1_{3F}$:

$$\int \mathrm{d}\Phi_3 \left[\hat{A}^1_{3,f} = \delta\mathcal{M}^{\gamma^*,f}_{(3,1)} / \left|\mathcal{M}^{\gamma^*}_{(2,0)}\right|^2\right]$$

$$\int \mathrm{d}\Phi_3 \left[\hat{A}^1_{3,F} = \delta\mathcal{M}^{\gamma^*,F}_{(3,1)} / \left|\mathcal{M}^{\gamma^*}_{(2,0)}\right|^2\right]$$





Summary & outlook

Summary

- Validate massive antenna @ NNLO
- N3LO massive antenna $\mathscr{B}_5^0$ available now

Outlook

- Full N3LO massive antenna functions
- Massive antenna implemented in NNLOJET
- Fully differential cross section for $e^+e^- \to t\bar{t} + X$ @ N3LO QCD

Thanks for your attention