tW production with soft gluon resummation at next-to-next-to-next-to-leading logarithmic level

丁家乐

山东大学

In collaboration with
Chong Sheng Li , Hai Tao Li and Jian Wang

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Contents

Introduction

- Among the various production modes of a single top quark, the associated production of a top quark with a W boson has the second-largest cross section at Large Hadron Collider (LHC).
- The cross section is directly proportional to the square of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{tb}.
- Inclusive and differential cross sections for tW production have been measured extensively.

Progress toward complete NNLO QCD correction for tW production:

- ✓ Two-loop hard function. [2106.12093][2111.14172][2211.13713][2204.13500][2208.08786][2212.07190]
- ✓ Two-loop N-jettiness soft function. [1611.02749][1804.06358]
- ✓ Two-loop threshold soft function. [2502.18648]
- ✓ NNNLL soft gluon resummation.

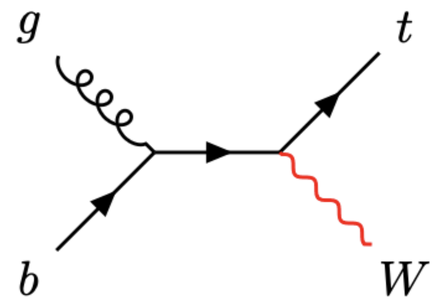
Introduction

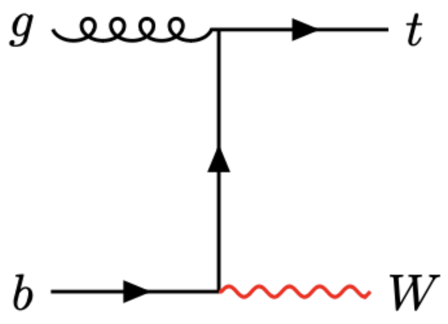
The inclusive associated production of top quark and W boson at the LHC is

$$p(P_1) + p(P_2) \to t/\bar{t}\,(p_3) + W^-/W^+\,(p_4) + X(p_X).$$

The LO partonic scattering channel for this process is

$$b/\bar{b}(p_1) + g(p_2) \to t/\bar{t}(p_3) + W^-/W^+(p_4).$$





$s = (P_1 + P_2)^2$ hadronic energy of the collider

$\hat{s} = (p_1 + p_2)^2$ partonic invariance mass

$Q^2 = (p_3 + p_4)^2$ tW invariance mass

$\tau = Q^2/s$ hadronic threshold variable

$z = Q^2/\hat{s}$ partonic threshold variable

Introduction

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In the hadronic threshold limit, the cross section takes a factorization form:

$$\frac{d\sigma}{dQ^2 d\Phi_2} = \frac{1}{s}\int_\tau^1 \frac{dz}{z}\mathcal{L}\left(\frac{\tau}{z},\mu\right)\frac{1}{2Q^2}H(\mu,\beta_t,y)\mathcal{S}(1-z,\mu,\beta_t,y)$$

$$\mathcal{L}\left(\frac{\tau}{z},\mu\right) = \int_{\tau/z}^1 \frac{dx}{x}\left[f_b(x,\mu)f_g\left(\frac{\tau}{xz},\mu\right) + f_g(x,\mu)f_b\left(\frac{\tau}{xz},\mu\right)\right].$$

H: hard function

$\mathcal{S}$: soft function

f_i: parton distribution functions

β_t: the velocity of the top quark

θ: the polar angle of the momentum of the top quark

$y = \cos\theta$

Fixed-order

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Performing the Laplace transformation, which converts the cross section to a product of the PDFs, hard function and soft function:

$$\frac{2Q^2 s d\tilde{\sigma}}{dQ^2 d\Phi_2} = \tilde{\mathcal{L}}(t, \mu_f) H(\mu_f, \beta_t, y) \tilde{S}(t, \mu_f, \beta_t, y).$$

The approximate N^{3}LO results for hard and soft functions are obtained by solving the Renormalization Group (RG) evolution equations:

$$\frac{d\tilde{S}}{d\ln\mu} = \gamma_s \tilde{S}, \quad \frac{dH}{d\ln\mu} = \gamma_h H.$$

The RG evolution of the PDFs in the threshold limit is diagonal

$$\frac{d\tilde{f}_i(t,\mu)}{d\ln\mu} = \left(2\mathbf{T}_i^2 \gamma_{\text{cusp}} \ln t + 2\gamma_f^i\right) \tilde{f}_i(t,\mu).$$

Fixed-order

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The soft anomalous dimension is

$$\gamma_s = -\left(\mathbf{T}_1^2+\mathbf{T}_2^2\right)\gamma_{\text{cusp}}\ln\left(\frac{\hat{s}t^2}{\mu^2}\right)+\mathbf{T}_1\cdot\mathbf{T}_3\gamma_{\text{cusp}}\ln\frac{(1-\beta_t y)^2}{1-\beta_t^2}+\mathbf{T}_2\cdot\mathbf{T}_3\gamma_{\text{cusp}}\ln\frac{(1+\beta_t y)^2}{1-\beta_t^2}$$
$$-2\gamma^q-2\gamma^g-2\gamma^Q-2\gamma_f^q-2\gamma_f^g-2\mathcal{T}_{1233}\left(\frac{\alpha_s}{4\pi}\right)^3\mathcal{F}_{h2}\left(\frac{1-\beta_t^2}{1-\beta_t^2y^2}\right).$$

$$\mathcal{T}_{1233}=\frac{1}{2}f^{ade}f^{bce}\mathbf{T}_1^a\mathbf{T}_2^b(\mathbf{T}_3^c\mathbf{T}_3^d+\mathbf{T}_3^d\mathbf{T}_3^c)$$

The total physical cross section must be independent of the factorization scale μ :

$$\frac{d\ln H}{d\ln\mu}+\frac{d\ln\tilde{f}_q}{d\ln\mu}+\frac{d\ln\tilde{f}_g}{d\ln\mu}+\frac{d\ln\tilde{\mathcal{S}}}{d\ln\mu}=0,$$

and hard anomalous dimension can be driven as

$$\gamma_h = \left(\mathbf{T}_1^2+\mathbf{T}_2^2\right)\gamma_{\text{cusp}}\ln\frac{\hat{s}}{\mu^2}-\mathbf{T}_1\cdot\mathbf{T}_3\gamma_{\text{cusp}}\ln\frac{(1-\beta_t y)^2}{1-\beta_t^2}-\mathbf{T}_2\cdot\mathbf{T}_3\gamma_{\text{cusp}}\ln\frac{(1+\beta_t y)^2}{1-\beta_t^2}$$
$$+2\gamma^q+2\gamma^g+2\gamma^Q+2\mathcal{T}_{1233}\left(\frac{\alpha_s}{4\pi}\right)^3\mathcal{F}_{h2}\left(\frac{1-\beta_t^2}{1-\beta_t^2y^2}\right).$$

Fixed-order

The cross section in momentum space can be written as

$$\frac{d\sigma}{dQ^2 d\Phi_2} = \frac{1}{s}\int_\tau^1 \frac{dz}{z}\mathscr{L}\left(\frac{\tau}{z},\mu\right)\frac{1}{2Q^2}H\left(\mu,\beta_t,y\right)\frac{z^{-\eta}}{(1-z)^{1-2\eta}}$$
$$\times\tilde{\mathcal{S}}\left(\ln\frac{Q^2(1-z)^2}{z\mu^2}+\partial_\eta,\beta_t,y\right)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\Bigg|_{\eta=0}$$

The fixed-order partonic cross section in the threshold limit contains a series of logarithmically enhanced terms arising from soft and collinear gluon radiation, which can be written as

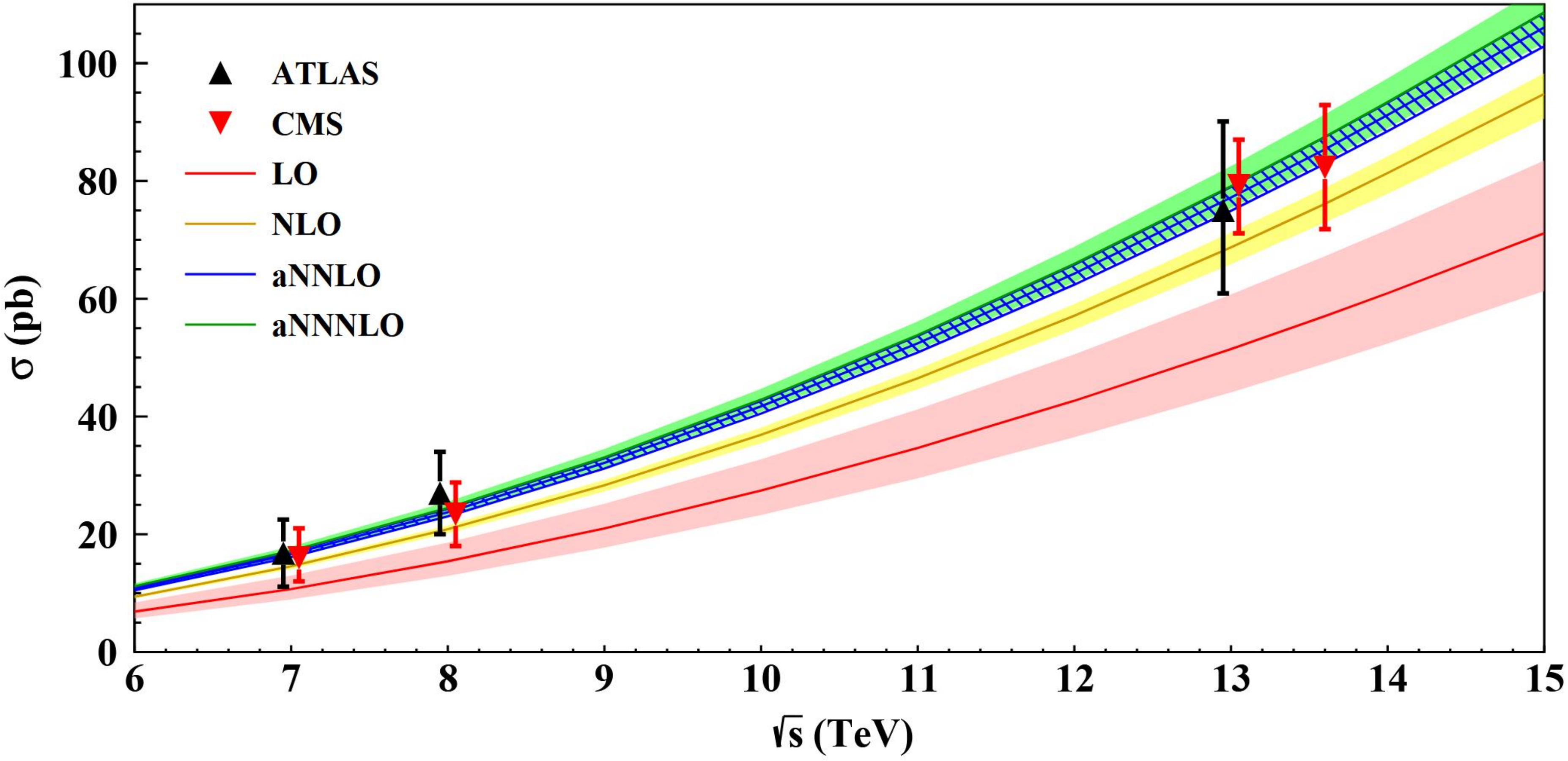
$$\frac{d\hat{\sigma}}{dQ^2 d\Phi_2} = \frac{d\hat{\sigma}_{\rm LO}}{dQ^2 d\Phi_2}\Bigg\{\delta(1-z)+\frac{\alpha_s(\mu_r)}{4\pi}\Big(C_{1,1}P_1+C_{1,0}P_0+C_{1,-1}\delta(1-z)\Big)$$
$$+\left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^2\Big(C_{2,3}P_3+C_{2,2}P_2+C_{2,1}P_1+\cdots\Big)$$
$$+\left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^3\Big(C_{3,5}P_5+C_{3,4}P_4+C_{3,3}P_3+\cdots\Big)+\mathcal{O}\left[\left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^4\right]\Bigg\}$$

$$P_n = \left[\frac{1}{1-z}\ln^n\frac{(1-z)^2Q^2}{z\mu_f^2}\right]_+$$

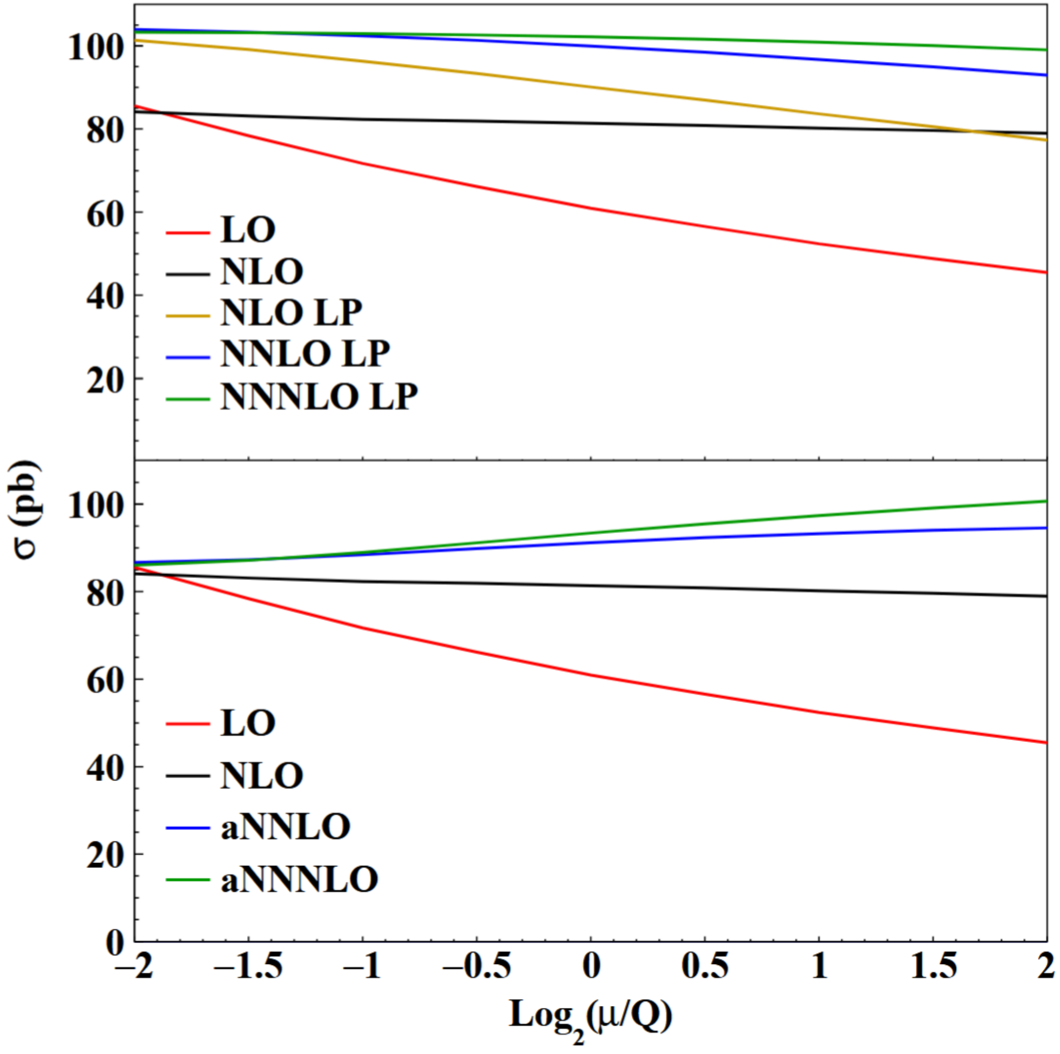
Fixed-order

The approximate N^{n}LO fixed-order cross section are obtained by matching it with full NLO corrections:

$$d\sigma(\mathrm{aN^nLO}) = d\sigma(\mathrm{N^nLO_{LP}}) + d\sigma(\mathrm{NLO}) - d\sigma(\mathrm{NLO_{LP}}).$$



Fixed-order



$\mu = \mu_f = \mu_r$

$$d\sigma(\mathrm{aN^nLO}) = d\sigma(\mathrm{N^nLO_{LP}}) + d\sigma(\mathrm{NLO}) - d\sigma(\mathrm{NLO_{LP}})$$

$d\sigma(\mathrm{N^nLO_{LP}})$ Scale uncertainty $\sim O(\alpha_s^{n+1})$

$d\sigma(\mathrm{NLO_{LP}})$ Scale uncertainty $\sim O(\alpha_s^2)$

$d\sigma(\mathrm{NLO})$ Scale uncertainty $\ll O(\alpha_s^2)$

gb channel's scale uncertainty $\sim O(\alpha_s^2)$

cancel with

gg and $d\bar{d}$ channels' scale uncertainty $\sim O(\alpha_s^2)$

Resummation

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Mellin transform and its inverse are defined by

$$\tilde{f}(N)=\mathcal{M}[f](N)=\int_0^1 d\,x x^{N-1}f(x),\quad f(x)=\mathcal{M}^{-1}[\tilde{f}](x)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} d\,x x^{-N}\tilde{f}(N).$$

Then the differential cross section in Mellin space can be written as

$$\begin{aligned}\frac{2Q^2 s d\tilde{\sigma}(N,\mu_f)}{dQ^2 d\Phi_2}&=\tilde{\mathcal{L}}(N,\mu_f)H(\mu_f,\beta,y)\tilde{S}(N,\mu_f,\beta,y)\\&=\tilde{\mathcal{L}}(N,\mu_f)H(\mu_h,\beta,y)\widetilde{U}(\mu_h,\mu_s,\mu_f)\tilde{S}(N,\mu_s,\beta,y).\end{aligned}$$

where

$$\begin{aligned}\tilde{S}(\mu_f)=\exp\Bigg\{\int_{\alpha_s(\mu_s)}^{\alpha_s(\mu_f)}\frac{d\alpha_s}{\beta(\alpha_s)}\Bigg[&-\left(\mathbf{T}_1^2+\mathbf{T}_2^2\right)\gamma_{\text{cusp}}\left(2\ln\frac{\mu_h}{\mu}+\ln\frac{\hat{s}}{\mu_h^2}-2\ln\bar{N}\right)\\&+\mathbf{T}_1\cdot\mathbf{T}_3\gamma_{\text{cusp}}\ln\frac{(1-\beta_t y)^2}{1-\beta_t^2}+\mathbf{T}_2\cdot\mathbf{T}_3\gamma_{\text{cusp}}\ln\frac{(1+\beta_t y)^2}{1-\beta_t^2}-2\gamma^q-2\gamma^g\\&-2\gamma^Q-2\gamma_f^q-2\gamma_f^g-2\mathcal{T}_{1233}\left(\frac{\alpha_s}{4\pi}\right)^3\mathcal{F}_{h2}\left(\frac{1-\beta_t^2}{1-\beta_t^2y^2}\right)\Bigg]\Bigg\}\tilde{S}(\mu_s).\end{aligned}$$

Resummation

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$\widetilde{U}(\mu_h, \mu_s, \mu_f)$ is the evolution factor and absorbs all the large logarithms caused by soft gluon effects,

$$\widetilde{U}(\mu_h, \mu_s, \mu_f) = \exp\left\{\frac{4\pi}{\alpha_s(\mu_h)} g_1(\lambda_f, \lambda_s) + g_2(\lambda_f, \lambda_s) + \frac{\alpha_s(\mu_h)}{4\pi} g_3(\lambda_f, \lambda_s) + \left[\frac{\alpha_s(\mu_h)}{4\pi}\right]^2 g_4(\lambda_f, \lambda_s) + \cdots\right\}.$$

$$\lambda_i = \frac{\alpha_s}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_i} \qquad g_1(\lambda_s, \lambda_f) = \frac{(\mathrm{C_A} + \mathrm{C_F})\gamma_{\mathrm{cusp}}^{(0)}}{2\beta_0^2}\left[\lambda_s + \lambda_s \ln(1-\lambda_f) + (1-\lambda_s)\ln(1-\lambda_s)\right]$$

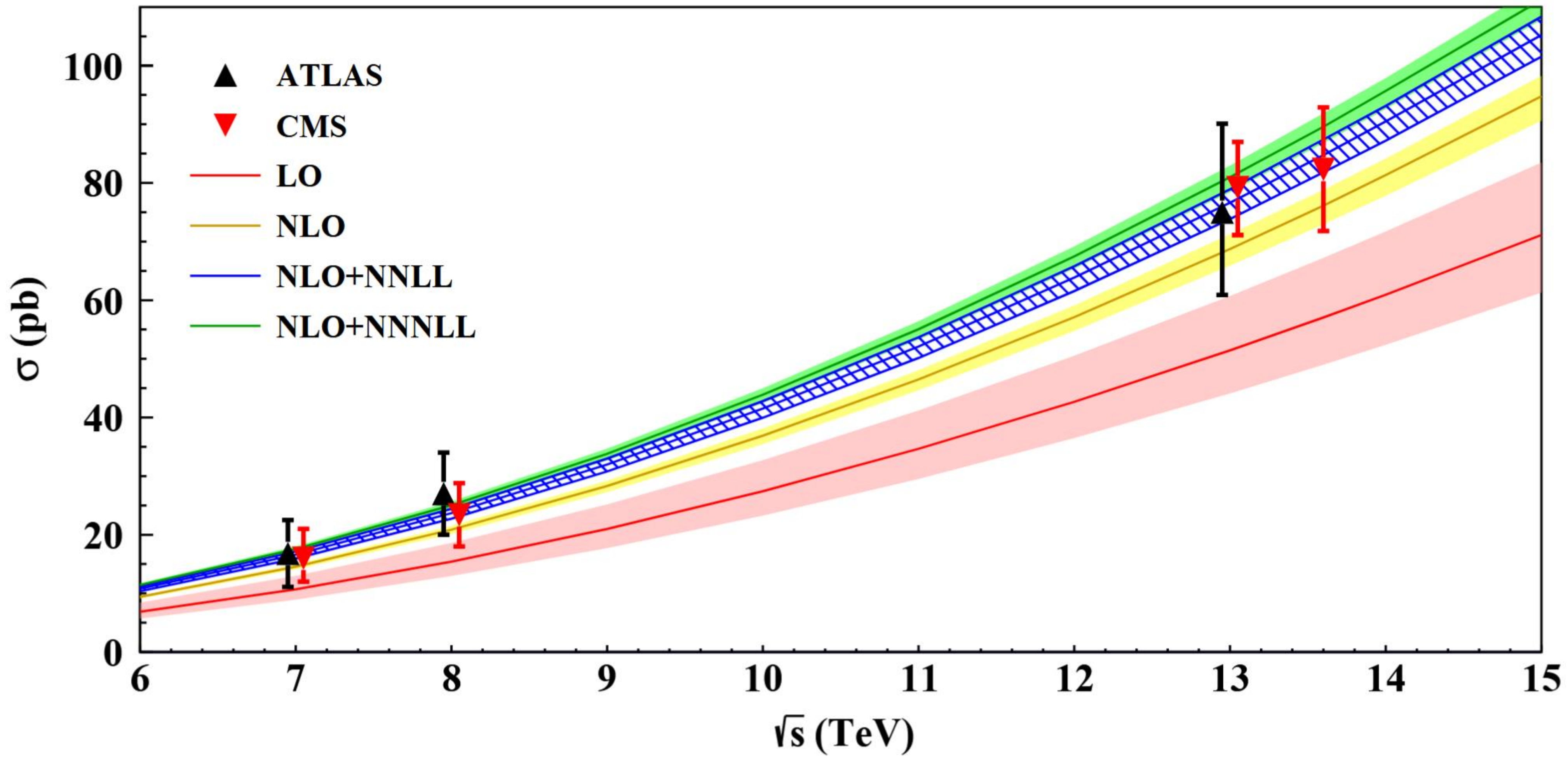
The differential cross section in momentum space can be obtain by Mellin inverse

$$\frac{2Q^2 s d\sigma}{dQ^2 d\Phi_2} = \frac{1}{2\pi i}\int_\tau^\infty \frac{dz}{z}\mathcal{L}\left(\frac{Q^2}{sz}, \mu_f\right)\int_{c-i\infty}^{c+i\infty} dN\, z^{-N} H(\mu_f, \beta, y)\tilde{S}(N, \mu_f, \beta, y).$$

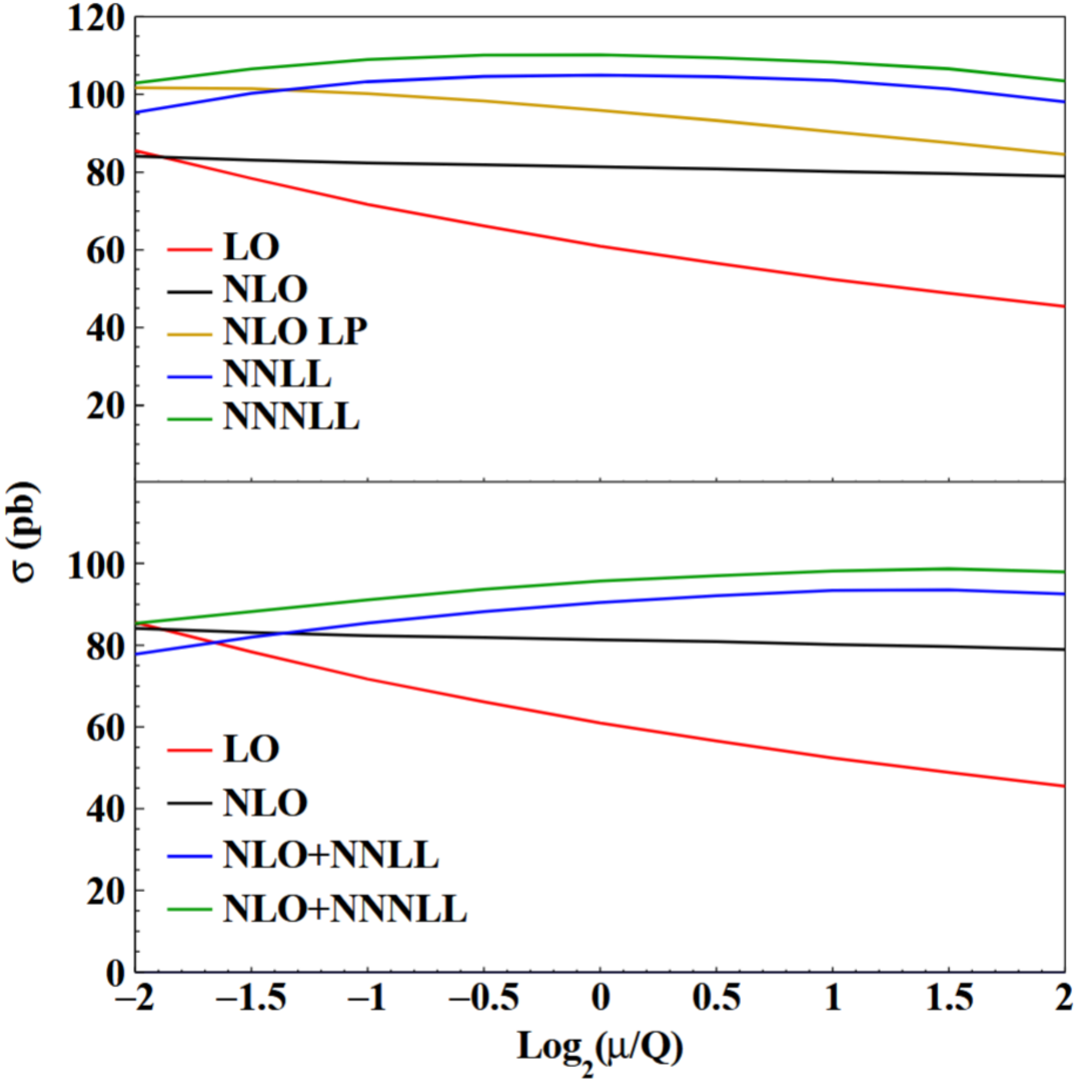
Obtaining NLO + N^nLL resummed cross section by performing phase space integration in large-N limit, and then matching it to full NLO result

$$d\sigma(\mathrm{NLO} + \mathrm{N^nLL}) = d\sigma(\mathrm{N^nLL}) + d\sigma(\mathrm{NLO}) - d\sigma(\mathrm{NLO_{LP}}).$$

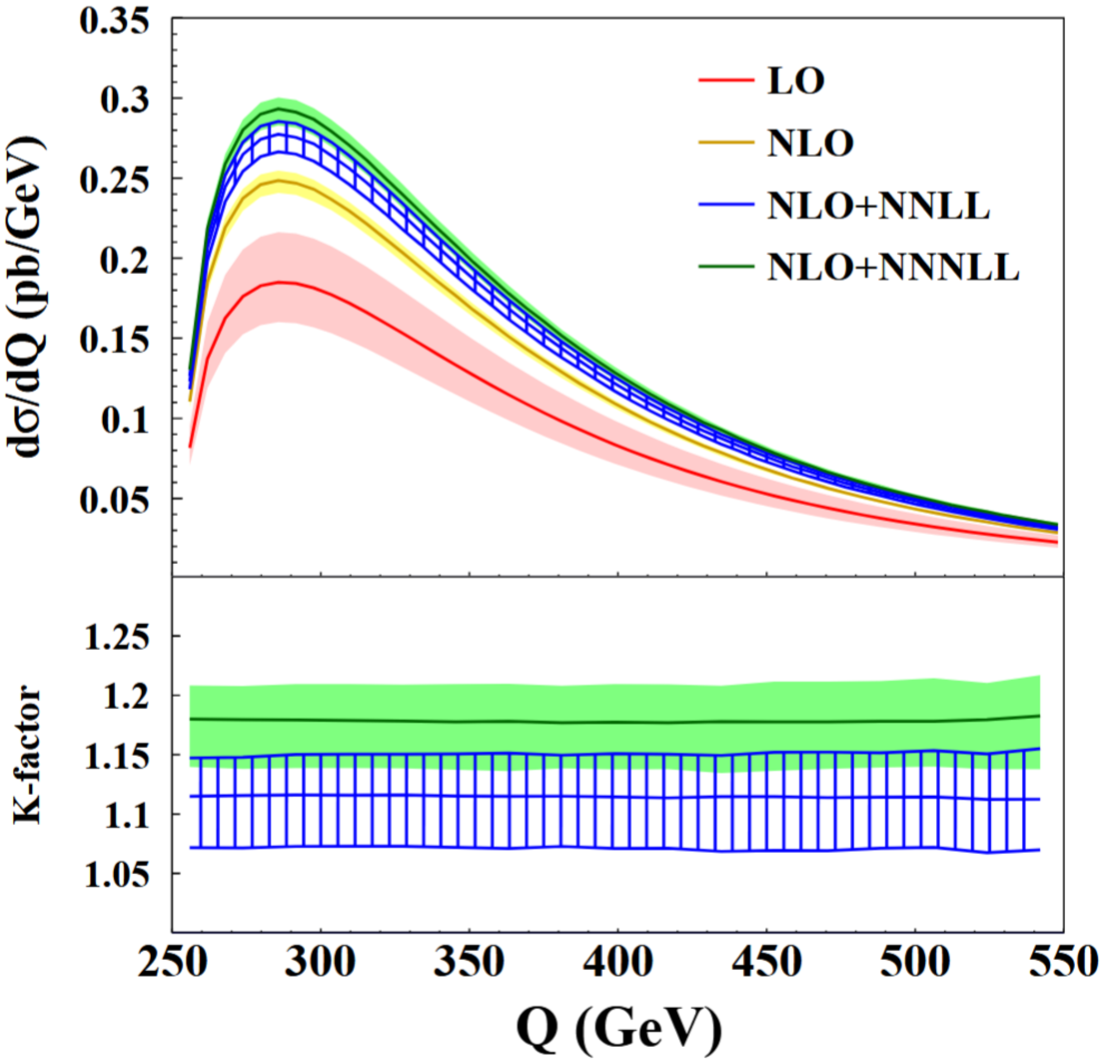
Resummation



Resummation



$$\mu = \overline{N}\mu_s = \mu_h = \mu_f$$



Summary

- Calculated the approximate NNNLO cross section for tW production at LHC.
- Compared the approximate fixed-order predictions with measured cross sections.
- Analyzed the main source of scale uncertainty in approximate fixed-order predictions.
- Calculated the NNNLL soft gluon resummation.
- Compared the RG-improved predictions with measured cross sections.

Thanks for your attention!