Precision Predictions for Three-Dimensional Nucleon Tomography

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第五届量子场论及其应用研讨会

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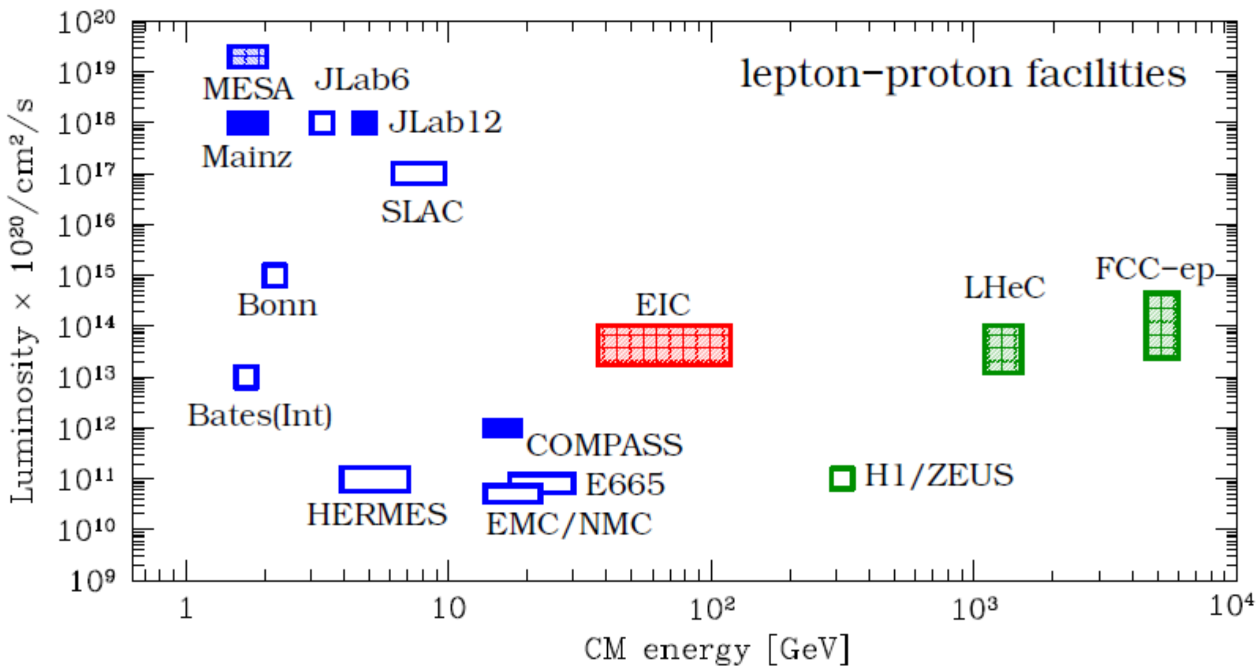
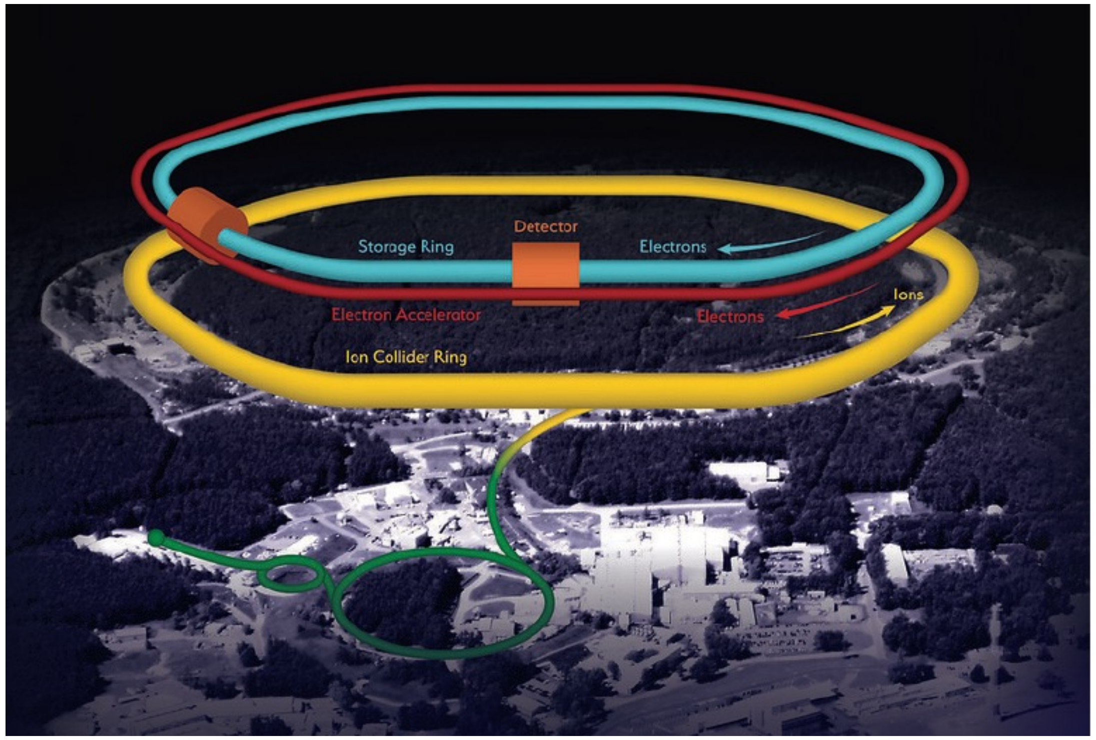
Reference: JHEP05(2024)066, JHEP01(2025)029

Electron-ion collider (EIC)

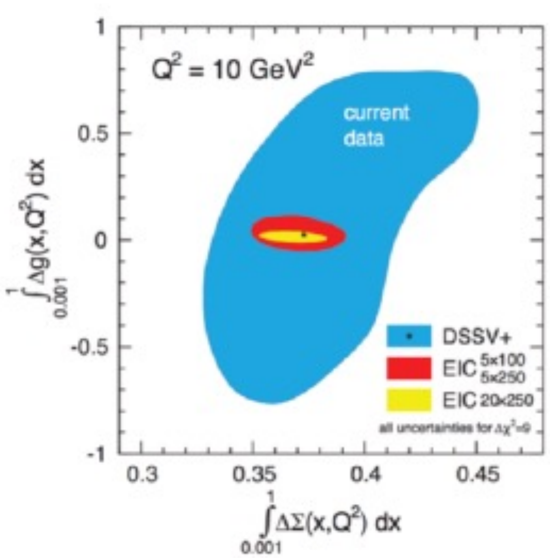


Abdul et al. ’22

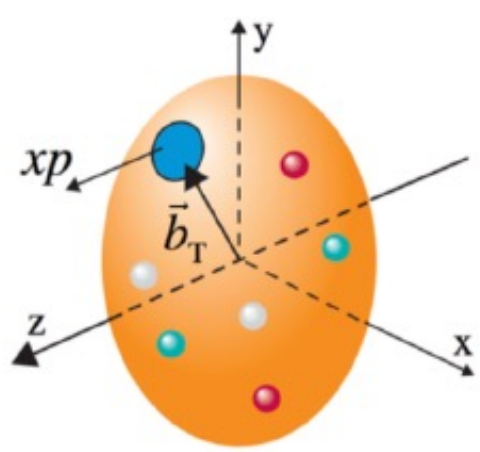


Abdul et al. ’22

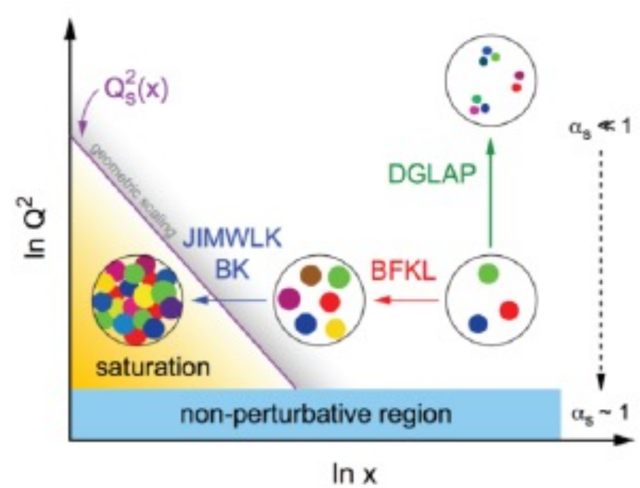
Proton spin



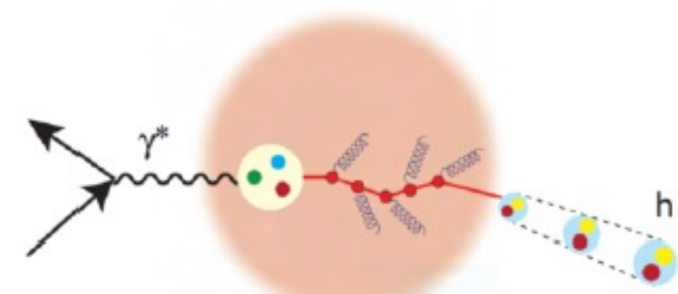
3D nucleon tomography



Gluon saturation



Hadronization in the nucleus

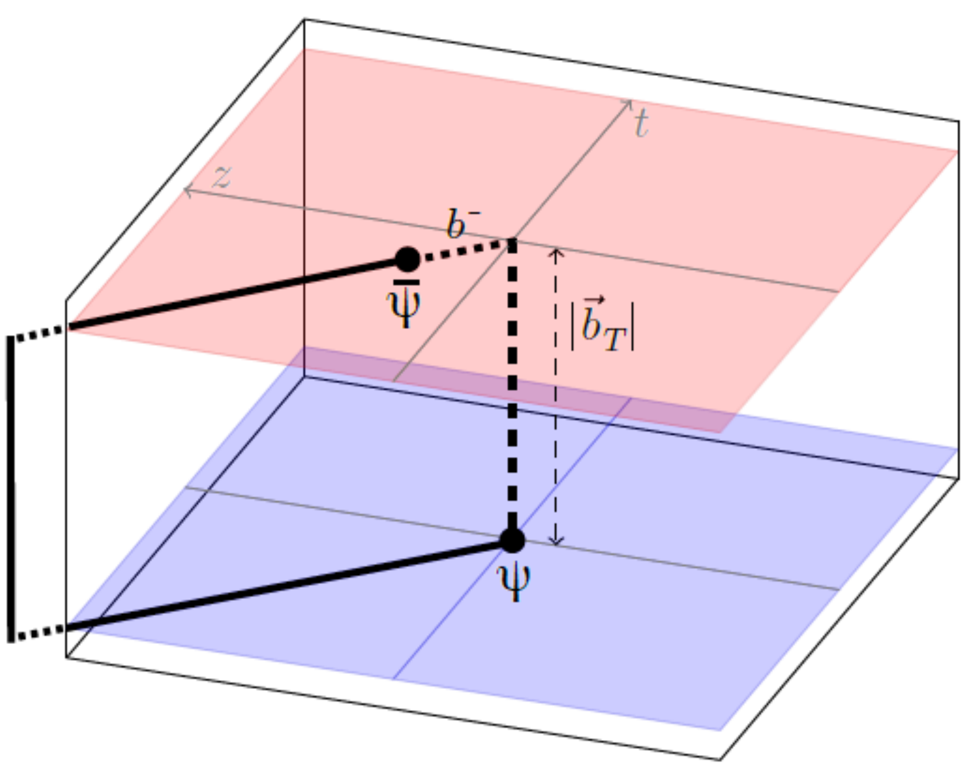


3D imaging of the nucleons

- **Both longitudinal and transverse motion**
- **Correlation between nucleon spin with parton(quark, gluon) orbital angular momentum**

$$\tilde{f}_{i/ps}^{[\Gamma]0(u)}(x, \mathbf{b}_T, \epsilon, \tau, xP^+) = \int \frac{\mathrm{d}b^-}{2\pi} e^{-ib^-(xP^+)} \Big\langle p(P,S) \Big| \Big[\bar{\psi}^i(b^\mu) W_{\sqsubset}(b^\mu, 0) \frac{\Gamma}{2} \psi^i(0) \Big]_\tau \Big| p(P,S) \Big\rangle$$

- **Dirac structures** $\Gamma \in \{ \gamma^+ \, , \, \gamma^+\gamma_5 \, , \, i\sigma^{\alpha+}\gamma_5 \}$



Boussarie et al. '23

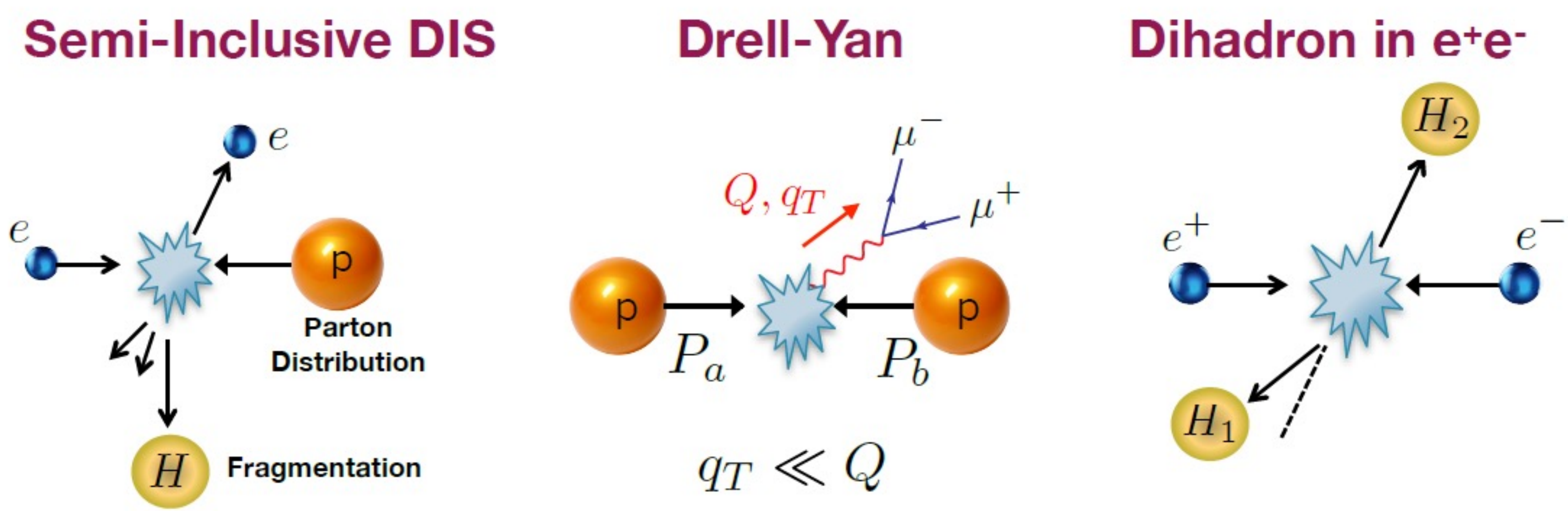
Leading Quark TMDPDFs — Nucleon Spin — Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

Transverse momentum distributions of quarks

- **Three classical processes used to probe quark TMDs:**

Boussarie et al. ’23



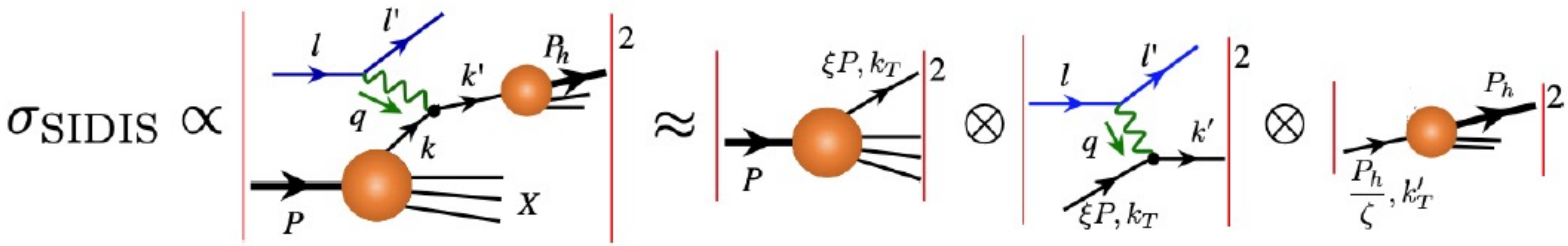
- **Typical multi-scale problems**
- **Theory tools: TMD factorization**

Collins, Soper & Sterman 1985
Florian & Grazzini ’01

(Semi-inclusive) deep inelastic scattering

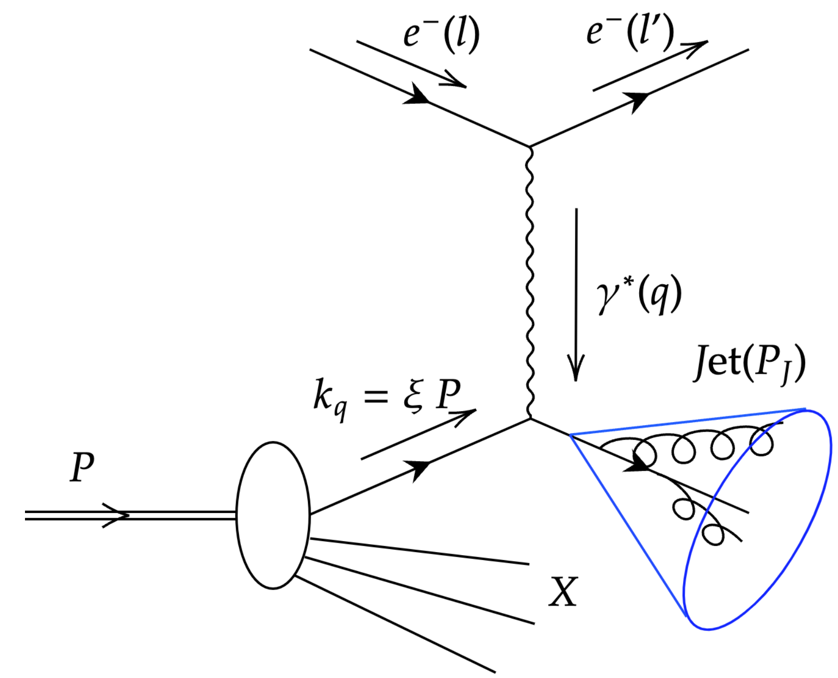
Factorized SIDIS cross section in the parton model:

Boussarie et al. ’23



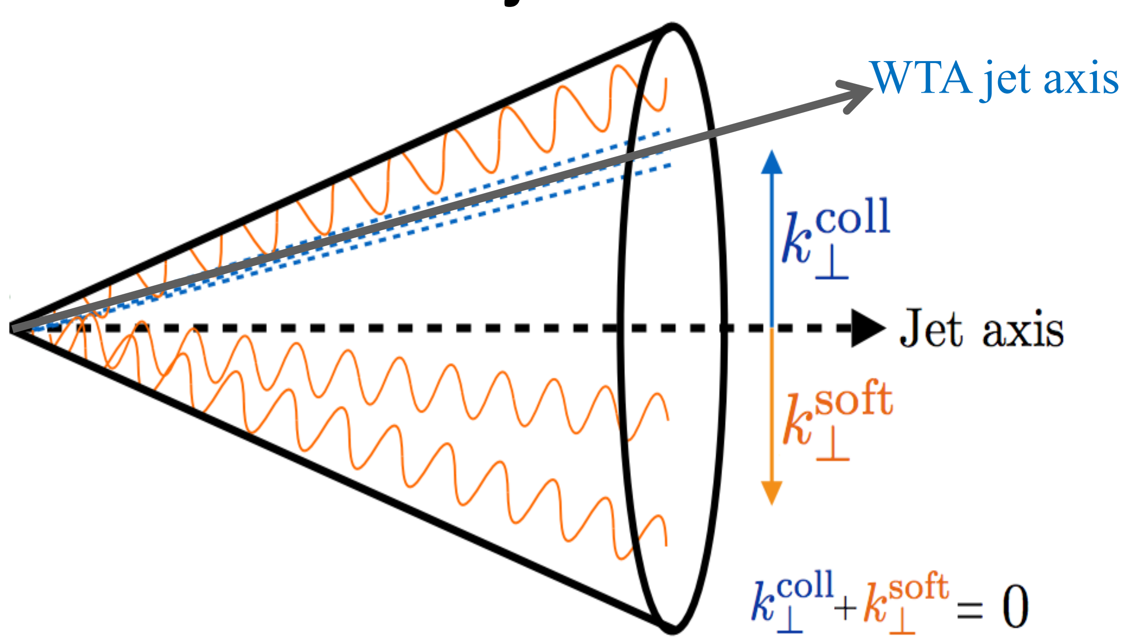
Liu, Ringer, Vogelsang & Yuan ’18,20

New ideas: $e(\ell) + N(P) \to e(\ell') + J(P_J) + X$



Winner-Take-All jets/ Recoil-free jets

Bertolini, Chan & Thaler ’14



$$\text{SJA}: E_{12} = E_1 + E_2\,,\qquad \boldsymbol{P}_{12} = \boldsymbol{p}_1 + \boldsymbol{p}_2\,.$$

$$\text{WTA}: E_{12} = E_1 + E_2\,,\qquad \boldsymbol{P}_{12} = E_{12}\left[\frac{\boldsymbol{p}_1}{|\boldsymbol{p}_1|}\theta(E_1 - E_2) + \frac{\boldsymbol{p}_2}{|\boldsymbol{p}_2|}\theta(E_2 - E_1)\right]\,.$$

EIC wish list

Regarding DATA

- Measure cross-sections instead of ratios for a more dedicated analysis
- Release both QED corrected and uncorrected data
- Develop method for unbinned cross-sections

Regarding PDFs, FFs and other distributions

- Replication of PDF4LHC and HERA efforts for EIC : PDF4EIC
- Perform global NNLO analysis of polarized PDFs
- Impact of QED corrections on polarized PDFS
- Perform global analysis of DVCS
- Generate threshold resummed PDFS and FFs
- New set of photon PDFs (existing are outdated)

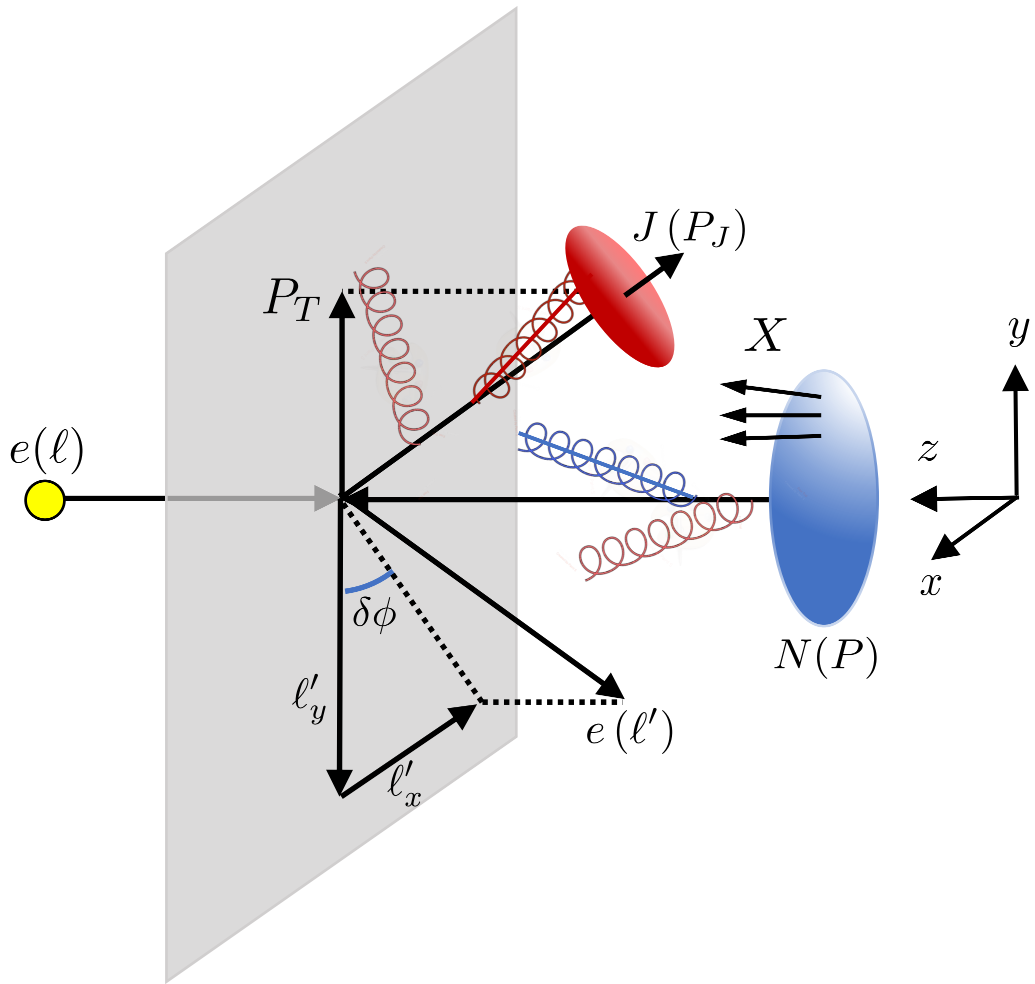
Regarding Perturbative corrections (QCD/QED)

- Jets in DIS: matched NNLO + q_T resummation
- DIS with QED/EW corrections
- Calculations for dihadron production

Regarding Theoretical Issues

- Discuss (non)universality of TMDs
- Search for ideal observables to measure Wigner distribution
- Role of lattice in PDFs (in two slides!)
- Studies for Λ polarization at EIC
- N*, Δ electro-couplings at $Q^2 > \mathrm{GeV}^2$
- Proton structure functions from transition regime to DIS
- Small-x dipole and quadrupole amplitudes

QCD factorization



$$e(\ell) + N(P) \to e(\ell') + J(P_J) + X$$

- **Factorization formula :**

$$\frac{d\sigma}{d^2\ell'_T\, dy\, dq_x} = \frac{\sigma_0}{1-y} H\,(Q,\mu)\; \mathcal{C}\,[B\,\mathcal{J}\,S]\;,$$

$$\mathcal{C}\,[B\,\mathcal{J}\,S] = \sum_q e_q^2 \int \frac{db}{2\pi} \cos\,(b\,q_x)\; B_{q/N}\left(x_B, b, \mu, \zeta_B/\nu^2\right)$$

$$\times\; \mathcal{J}_q(b,\mu,\zeta_{\mathcal{J}}/\nu^2)\; S(b, n\cdot n_J, \mu, \nu)\;.$$

- **EFT modes :**

$$\text{hard}: \quad p_h^\mu \sim \ell'_T(1,1,1),$$

$$n\text{-collinear}: \quad p_c^\mu \sim \ell'_T\,(\delta\phi^2, 1, \delta\phi),$$

$$\text{soft}: \quad p_s^\mu \sim \ell'_T\,(\delta\phi, \delta\phi, \delta\phi),$$

$$n_J\text{-collinear}: \quad p_J^\mu \sim \ell'_T\,(\delta\phi^2, 1, \delta\phi)_J\,.$$

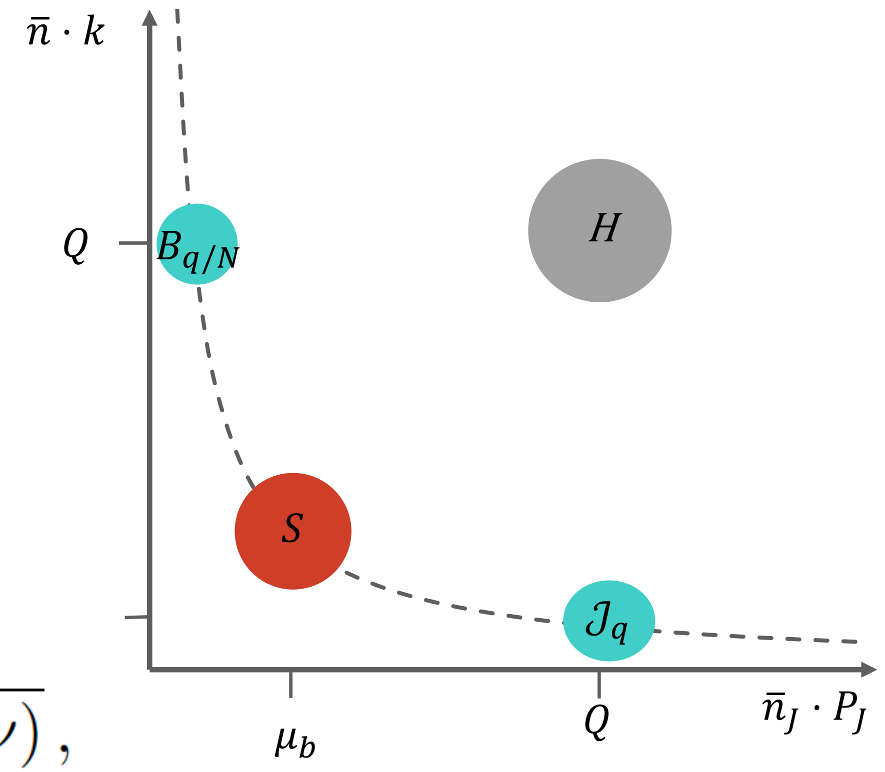
- **Observables:**

$$y = 1 - P\cdot\ell'/P\cdot l \qquad\qquad q_x = \ell'_T\delta\phi$$

- **EFT parameters:**

$$\delta\phi \ll 1$$

RG evolution



Hierarchy Problem:

$$\mu_b = \frac{2e^{-\gamma_E}}{b} \ll Q \qquad L = \ln\frac{Q^2}{\mu_b^2} \gg 1$$

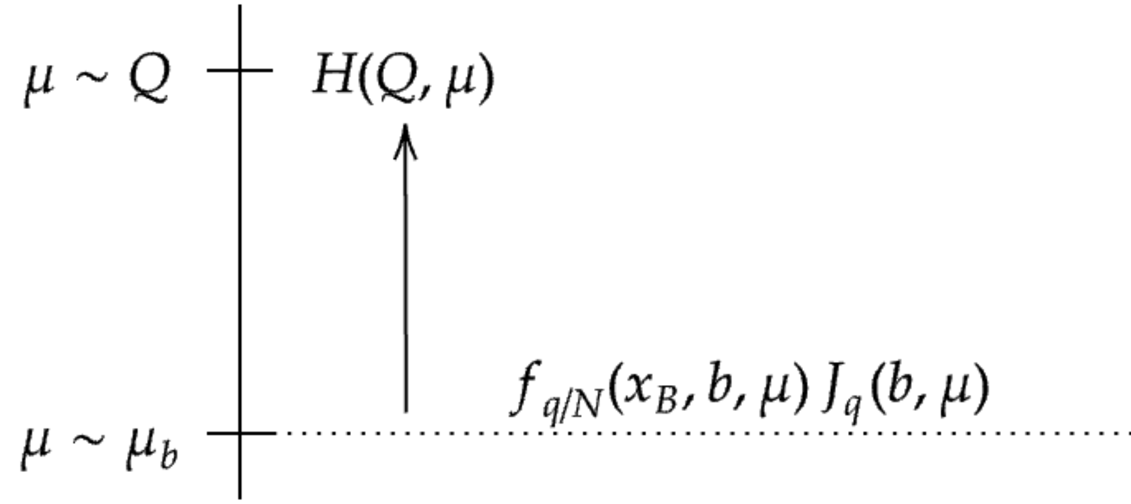
Standard CSS formalism: Collins '13

$$f_{q/N}\left(x_B, b, \mu, \zeta_f\right) = B_{q/N}\left(x_B, b, \mu, \zeta_B/\nu^2\right)\sqrt{S_{n\bar{n}}\left(b, \mu, \nu\right)}\,,$$

$$J_q\left(b, \mu, \zeta_J\right) = \mathcal{J}_q\left(b, \mu, \zeta_{\mathcal{J}}/\nu^2\right)\frac{S\left(b, n\cdot n_J, \mu, \nu\right)}{\sqrt{S_{n\bar{n}}\left(b, \mu, \nu\right)}}\,,$$

Scale independence:

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln\mu}\ln\,\sigma_{\text{phsy}}(Q, \mu_b, \mu) = 0$$



Predictions in e-A at NNLL

SF, Ke, Shao, Terry '23

non-perturbative model

$$U_{\rm NP}^{f}(x,b,A,Q_0,Q)=\exp\left[-g_1^A b^2-\frac{g_2}{2}\ln\frac{Q}{Q_0}\ln\frac{b}{b_*}\right],$$

Sun, Isaacson, Yuan & Yuan '14

We apply modified nuclear TMD PDFs

$$g_1^A = g_1^f + a_N(A^{1/3}-1) \quad a_N = 0.016 \pm 0.003\ \mathrm{GeV}^2$$

Collinear dynamics using EPPS16

Alrashed, Anderle, Kang, Terry & Xing '22

We include LO momentum broadening of the jet within SCET$_G$

$$J_q^A(b,\mu,\zeta_J)=J_q(b,\mu,\zeta_J)e^{\chi[\xi b K_1(\xi b)-1]}$$

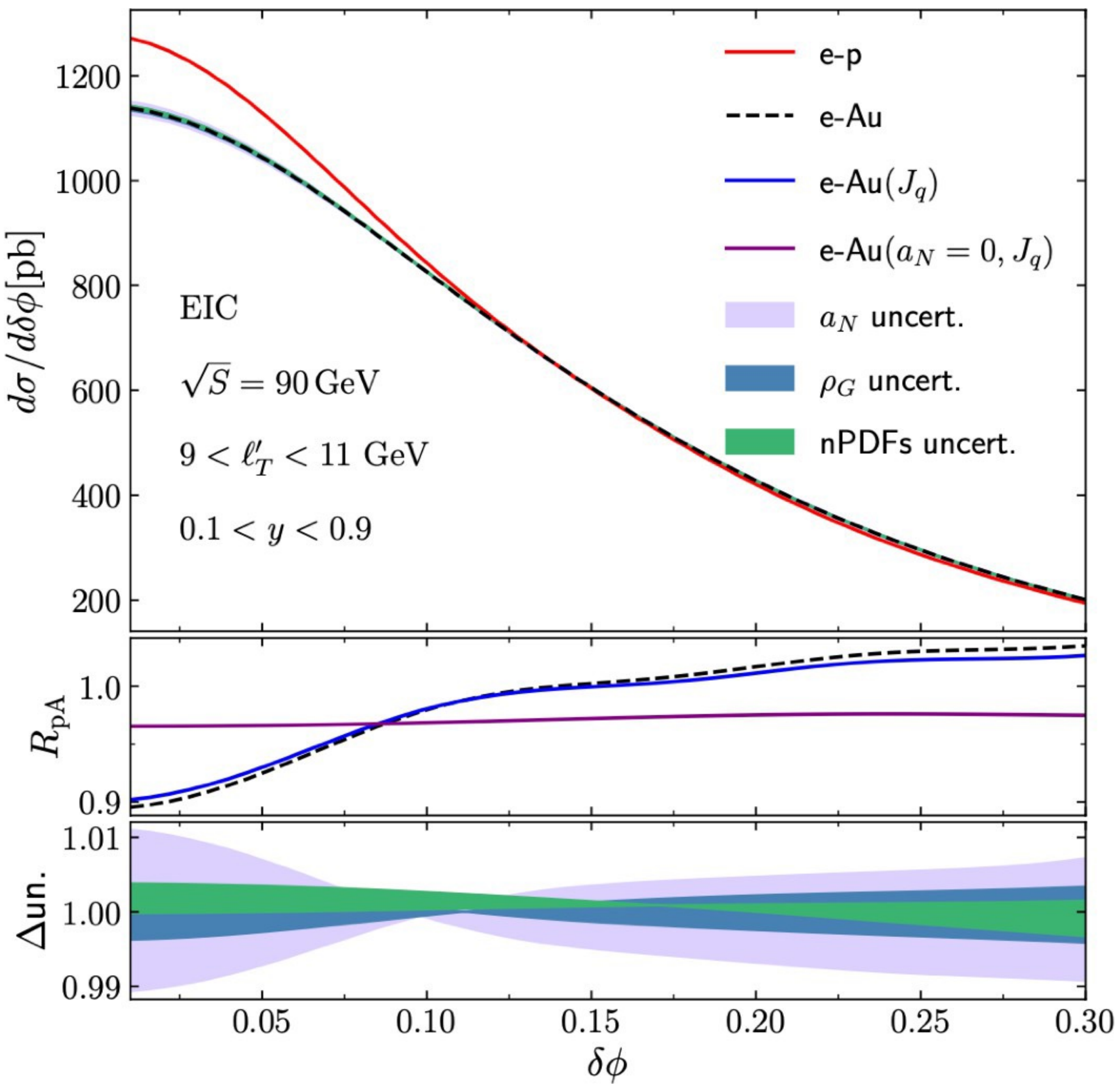
Opacity parameter $\chi = \dfrac{\rho_G L}{\xi^2}\alpha_s(\mu_{b_*})C_F$

Gyulassy, Levai & Vitev '02

ρ_G **: density of the medium**

ξ **: the screening mass**

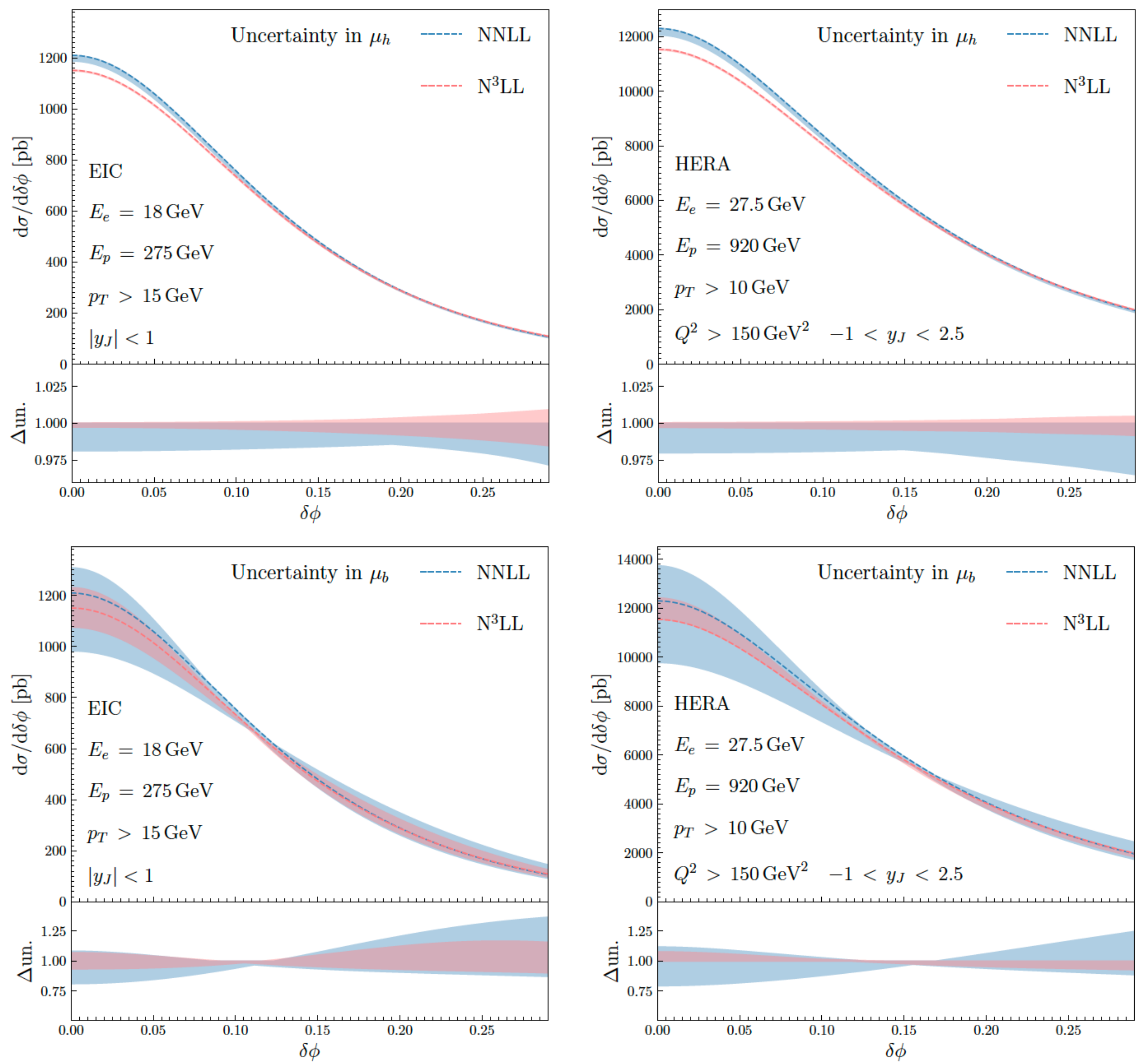
L**: the length of the medium**



The process is primarily sensitive to the initial state's broadening effects, thereby serving as a clean probe of nTMD PDFs.

Comparison of resummation results at NNLL and N^{3}LL

SF, Gao, Li, Shao ’24



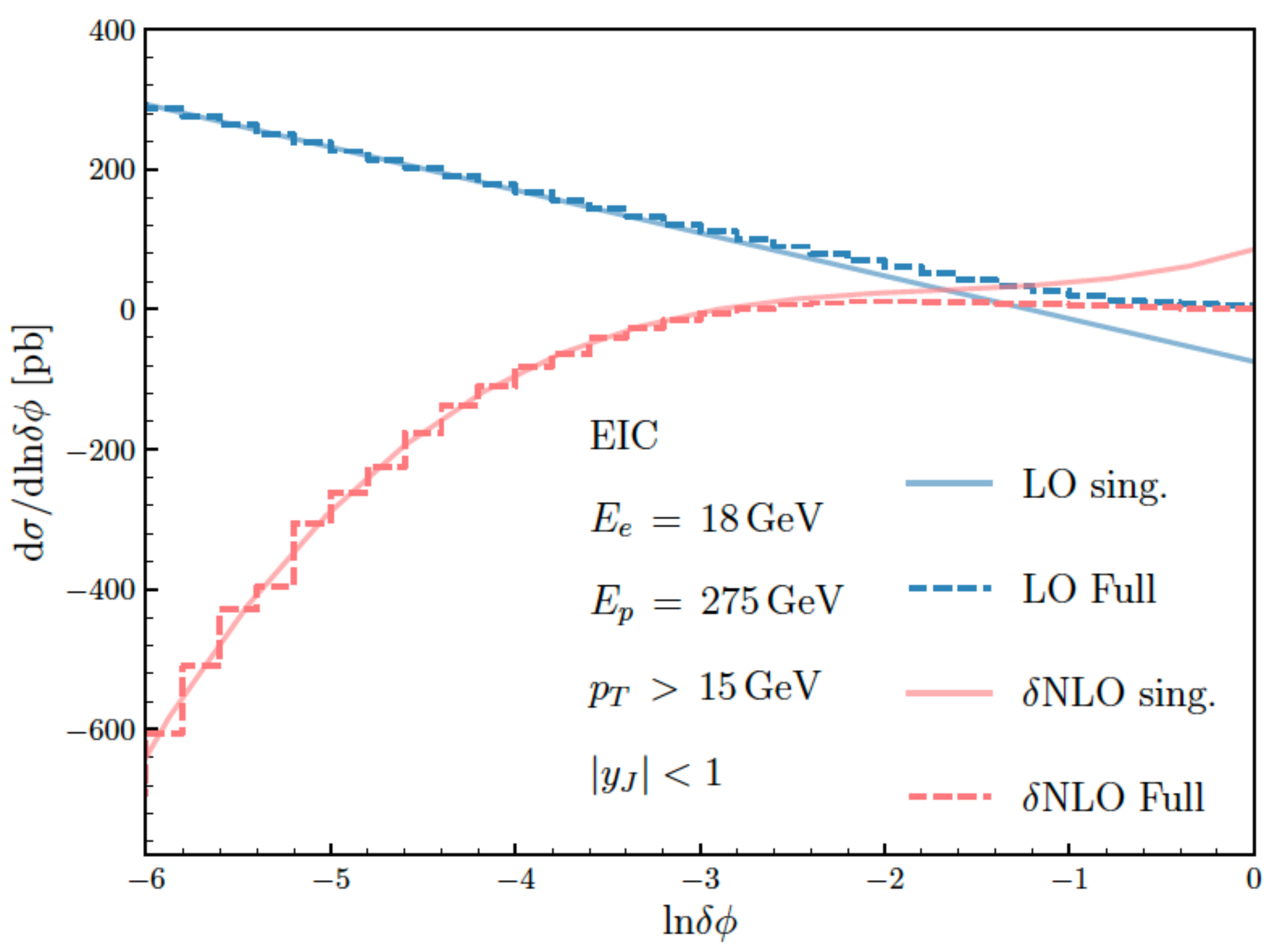
- **The uncertainty bands are narrower at N^{3}LL (red) compared to NNLL (blue)**
- **At N^{3}LL the dominant scale uncertainties are from μ_b variation**

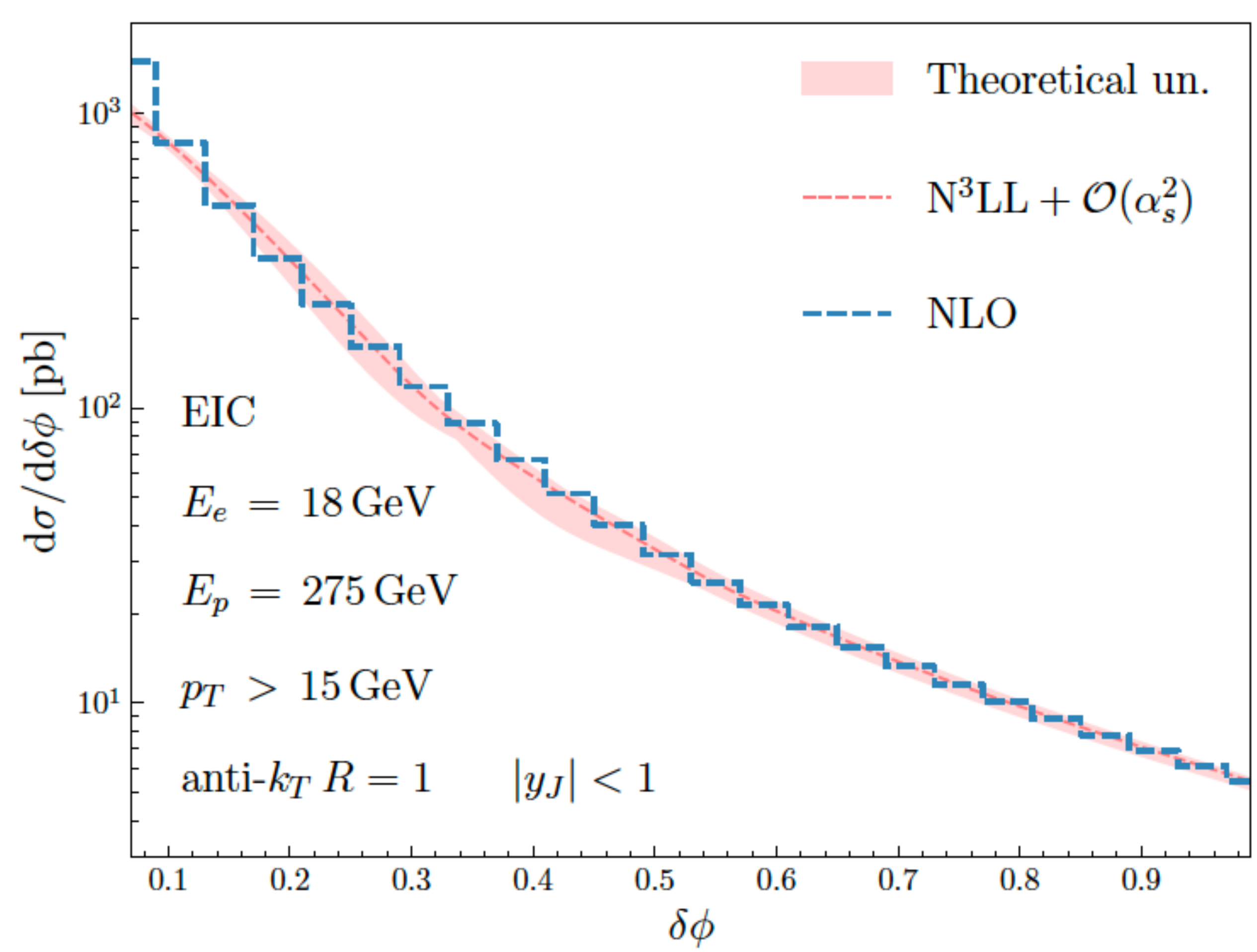
$\mathrm{N^3LL} + \mathcal{O}(\alpha_s^2)$ predictions on lepton-jet azimuthal correlation

SF, Gao, Li, Shao '24

- **In the back-to-back limit ($\delta\phi \to 0$) the singular contributions in DIS are consistent with the fixed-order results from NLOJET++ up to $\mathcal{O}(\alpha_s^2)$**
- **In the large $\delta\phi$ region the resummation formula receives significant matching corrections**
- **It is necessary to switch off the resummation and instead employ fixed-order calculations**

$$\mathrm{d}\sigma_{\text{add}}\left(\text{NNNLL} + \mathcal{O}(\alpha_s^2)\right) \equiv \mathrm{d}\sigma(\text{NNNLL}) + \underbrace{\mathrm{d}\sigma(\text{NLO}) - \mathrm{d}\sigma(\text{NLO singular})}_{\mathrm{d}\sigma(\text{NLO non-singular})}$$





Single spin asymmetry for 1-jettiness

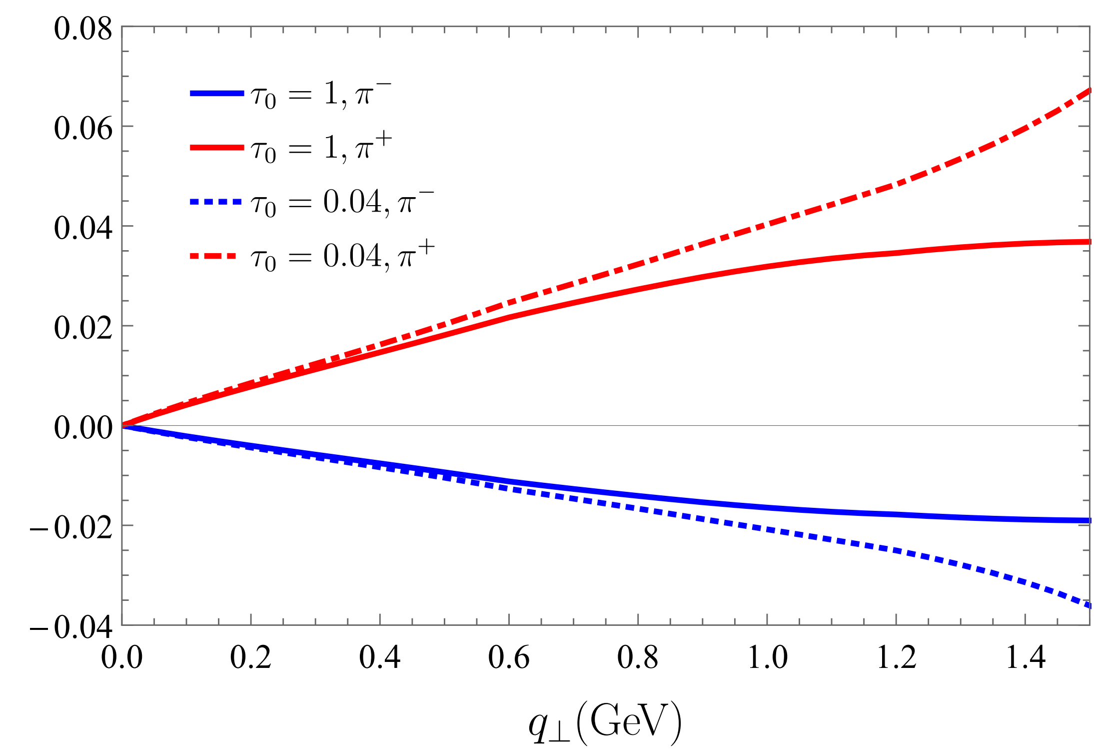
SF, Lin, Shao, Zhou ’25

- **1-jettiness:**

$$\tau_1 \equiv \frac{2}{Q^2} \sum_k \min\{q_B \cdot p_k, q_D \cdot p_k\}$$

- **Single spin asymmetry (SSA) at NLL:**

$$A_N = \frac{F_{UT}}{F_{UU}}$$
$$= \frac{\int_0^\infty \frac{b^2 \mathrm{d}b}{4\pi} J_1(\frac{bq_\perp}{z_h}) \sum_q e_q^2 \, T_F(x_B, x_B, \mu_b) D_{h/q}(z_h, \mu_b) e^{-S_{\mathrm{P}}(b)}}{\int_0^\infty \frac{b \mathrm{d}b}{2\pi} J_0(\frac{bq_\perp}{z_h}) \sum_q e_q^2 \, f_q(x_B, \mu_b) D_{h/q}(z_h, \mu_b) e^{-S_{\mathrm{P}}(b)}}$$



- **Qiu-Sterman function:**

$$T_{F,q}(x, x, \mu) = \mathcal{N}_q(x) f_q(x, \mu)$$

Echevarria, Kang, Terry ’14,20

Summary

- **We have studied on the lepton-jet correlation in both e-p and e-A collisions. Utilizing SCET, we derived a factorization theorem for back-to-back lepton-jet configurations.**
- **In e-A collisions, we discussed the utility of our approach in disentangling intrinsic non-perturbative contributions from nTMDs and dynamical medium effects in nuclear environments. We find the process is primarily sensitive to the initial state's broadening effects.**
- **TMD resummation accuracy has been improved to $N^3LL + \mathcal{O}(\alpha_s^2)$ accuracy in e-p collisions. It is good to have the measurement at the HERA to make a comparison.**
- **Our work sets the groundwork for future experiments at the EIC, offering a robust framework for measuring nTMDs and polarized TMDs.**

Thank you!