# *Surprises and Challenges in the QCD Phase Diagram*

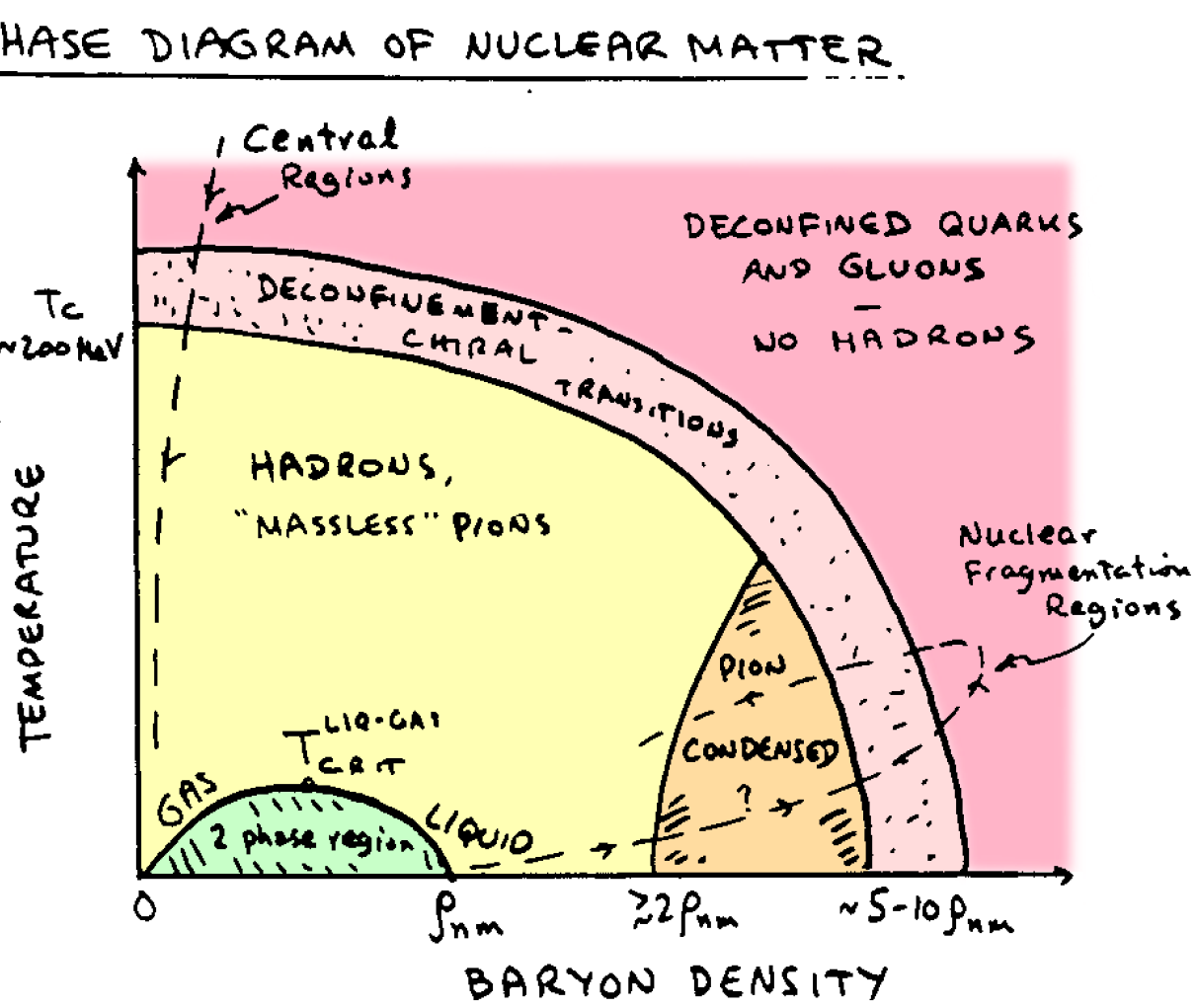
Kenji Fukushima

The University of Tokyo

— Nuclear Physics Across Energy Scales —

# Prototype Phase Diagram

**Baym (1986)**



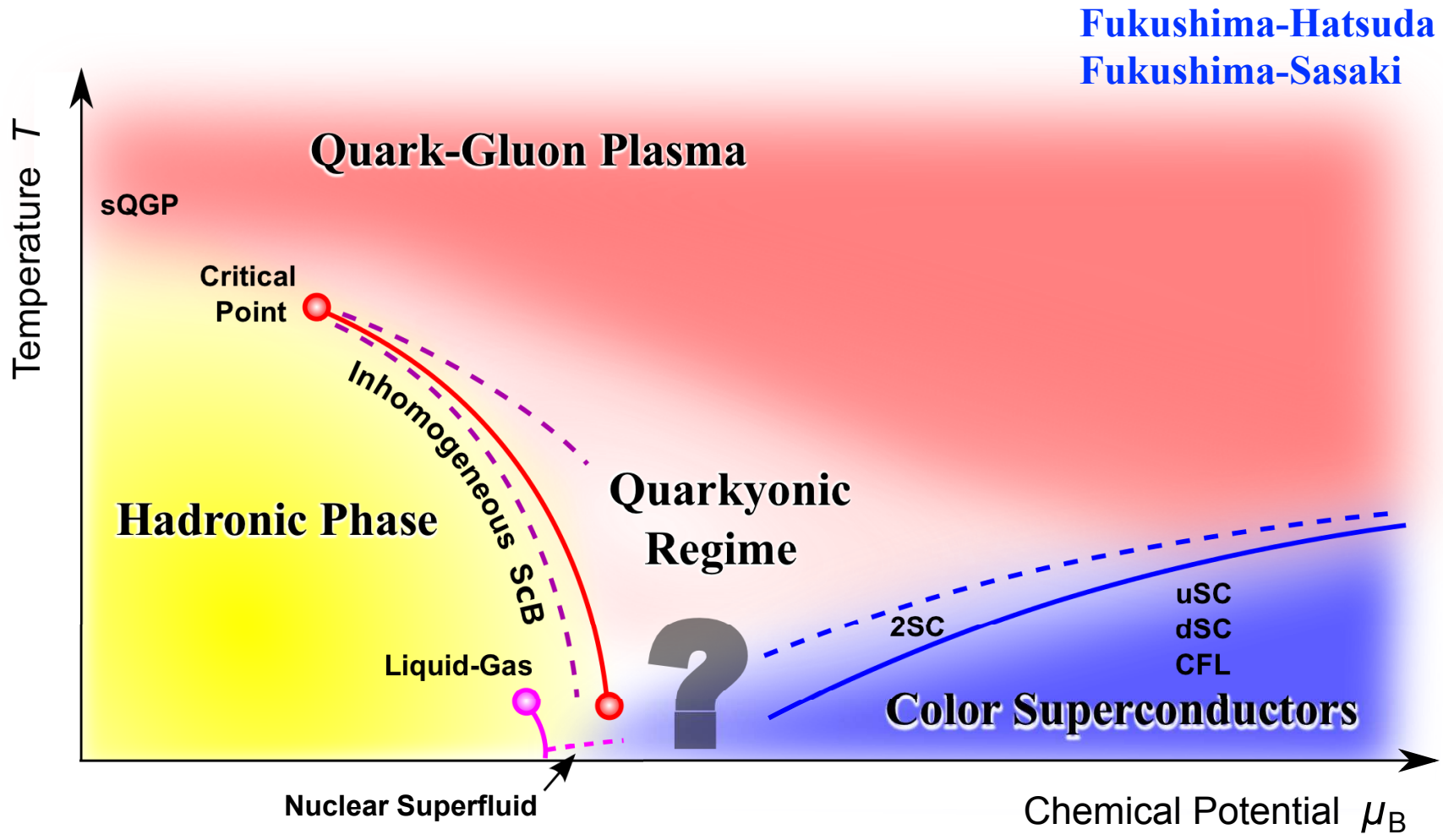

**Missing —**

**1st-order Phase Transition (QCD CP)**

**Color-super Conductors**

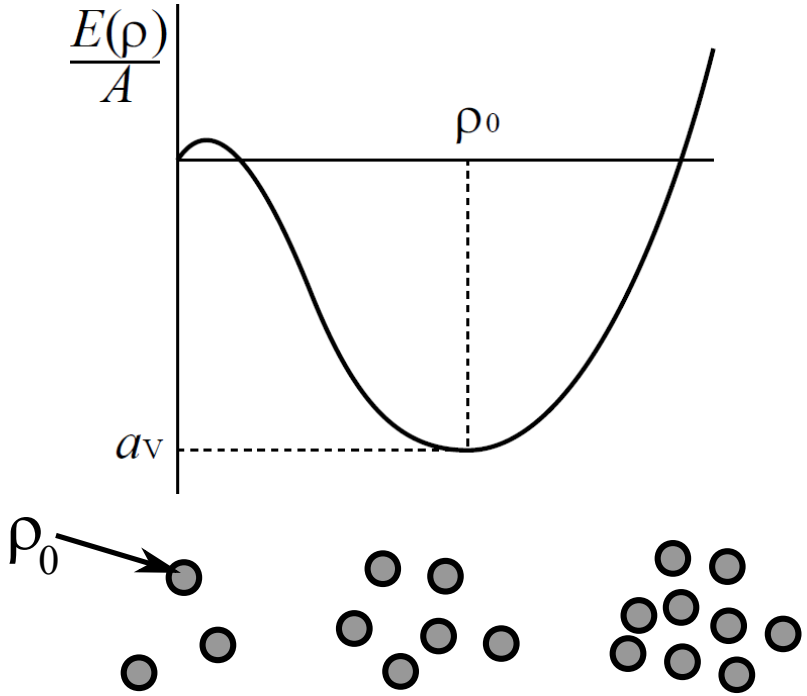
**Quarkyonic**

# Typical Phase Diagram



# First-order Phase Transition?

## Nuclear Saturation




**Self-bound fermionic systems have a preferred density. Diluteness is realized as a "mixed phase" of nuclei.**

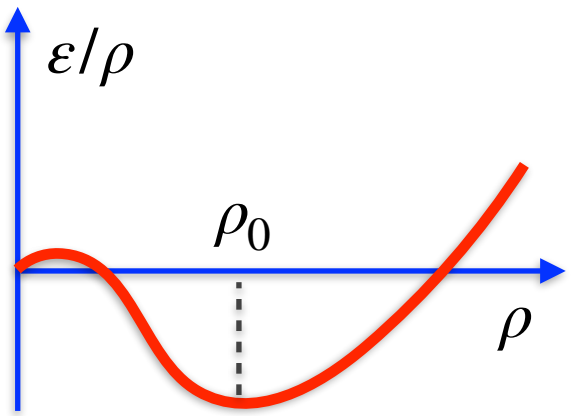
**This is how this world is like what we know.**

# First-order Phase Transition?

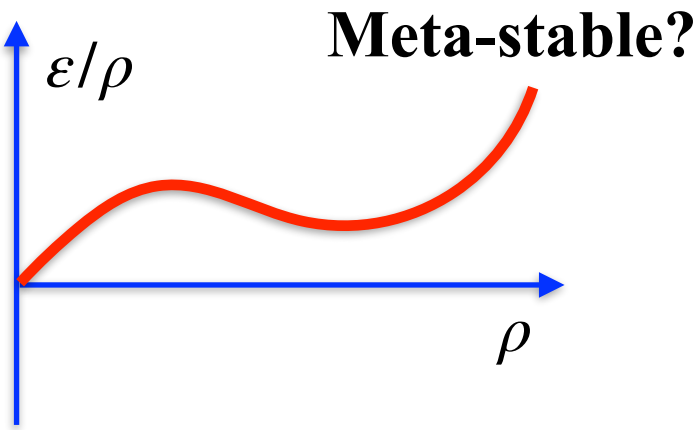
**Extremal point → 1st-order PT**

$$\frac{d}{d\rho}\left(\frac{\varepsilon}{\rho}\bigg|_{\text{gas}} - \frac{\varepsilon}{\rho}\bigg|_{\text{liquid}}\right) = \frac{p_{\text{gas}} - p_{\text{liquid}}}{\rho^2} = 0$$

**Stationary cond.** **PT cond.**



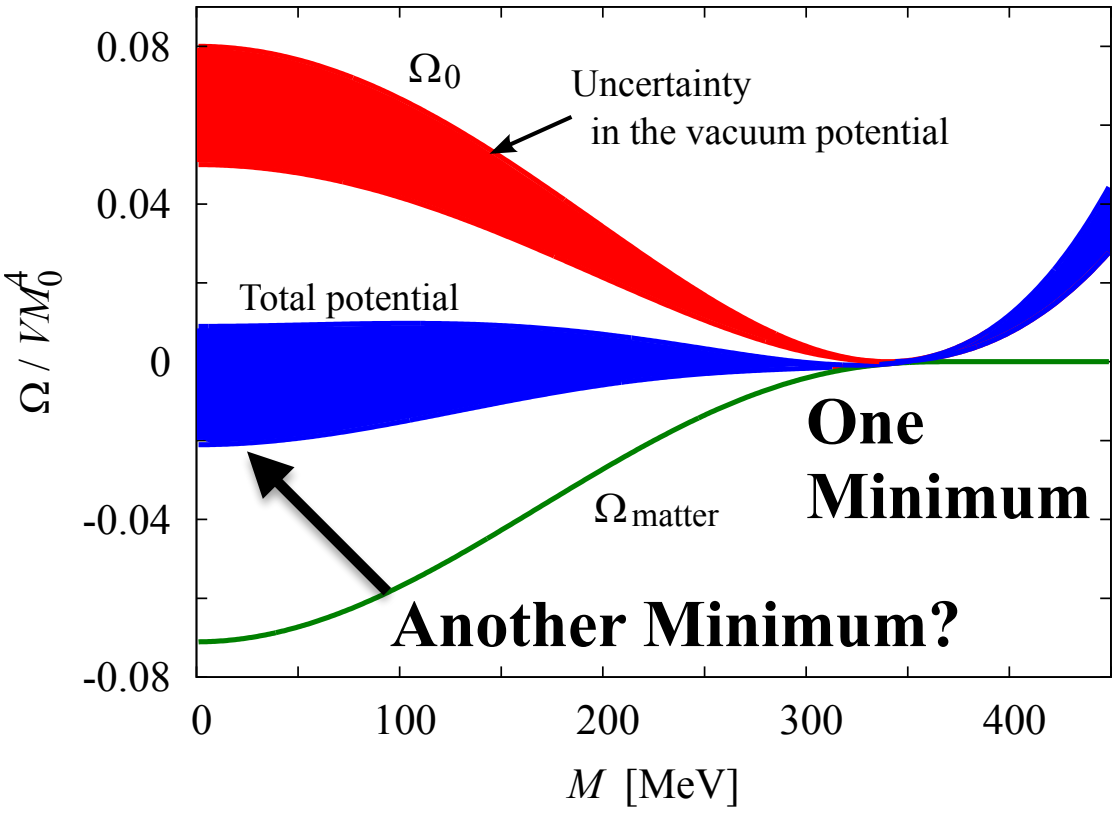

**Nuclear Matter**




**Self-bound? Quark Star?**

# First-order Phase Transition?

## General Tendency to 1st-order PT




The grand potential is the sum of the vacuum part (symmetry breaking) and the matter part

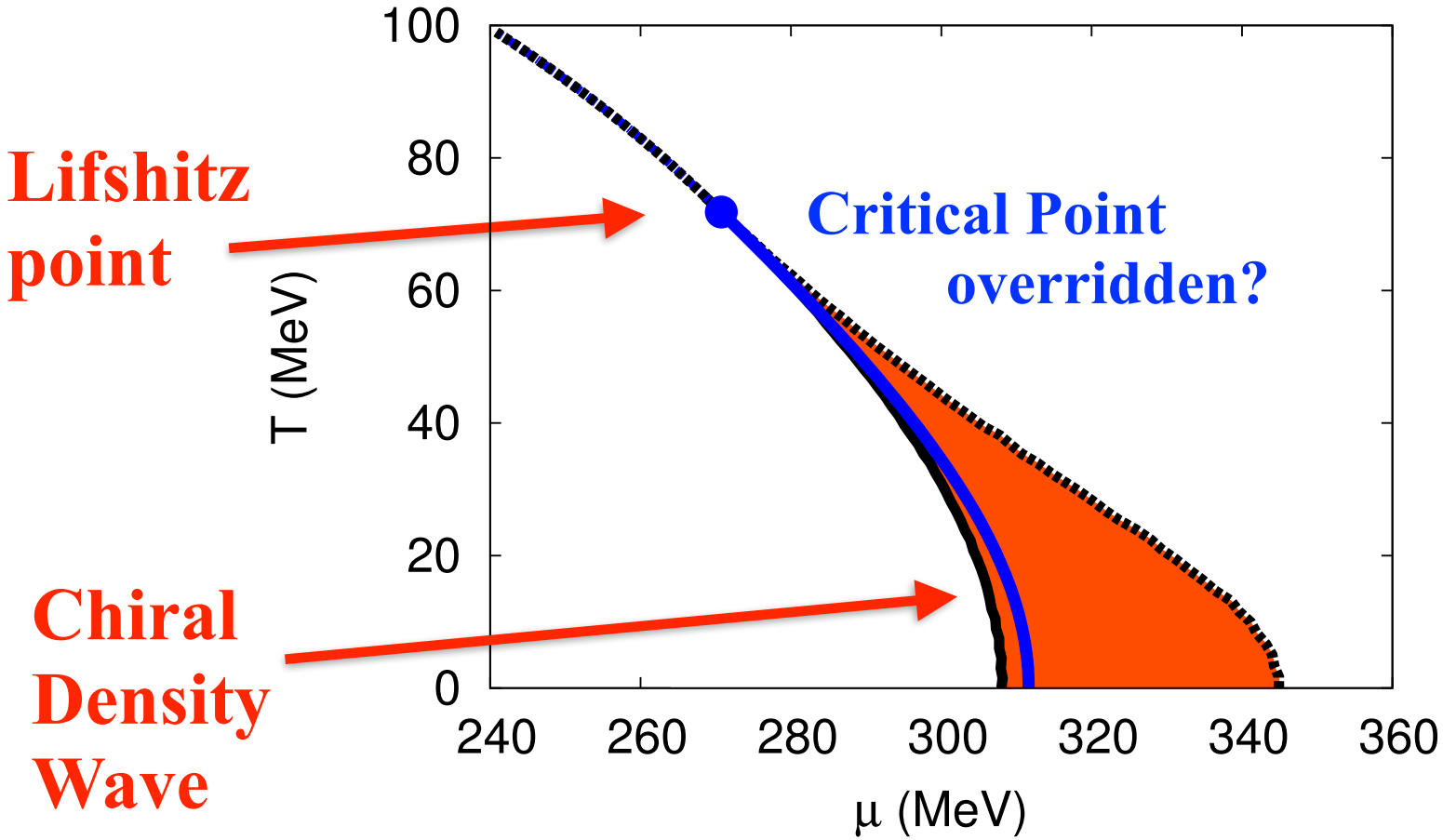
$$\Omega_{\rm mat} = -\int_m^\mu d\mu' n(\mu)$$

The sum may have a double-well structure (1st-order PT) or may not (crossover only).

# *First-order Phase Transition?*

## Inhomogeneous States?

**Review: Buballa-Carignano (2014)**



# *First-order Phase Transition?*

## Fluctuation effects

**It is known by now that phonon fluctuations wash out the 1-dimensional modulation but a remnant remains**
**= Quasi Long-Range Order**

**Hidaka-Kamikado-Kanazawa-Noumi (2015)**
**Lee-Nakano-Tsue-Tatsumi-Friman (2015)**

**Finite *B* (magnetic) / $\omega$ (rotation) breaks 3d symmetry**
**→ Pseudo-1d Helical Structures**

**Chiral Magnetic Spirals / Kharzeev-Dunne (2010)**

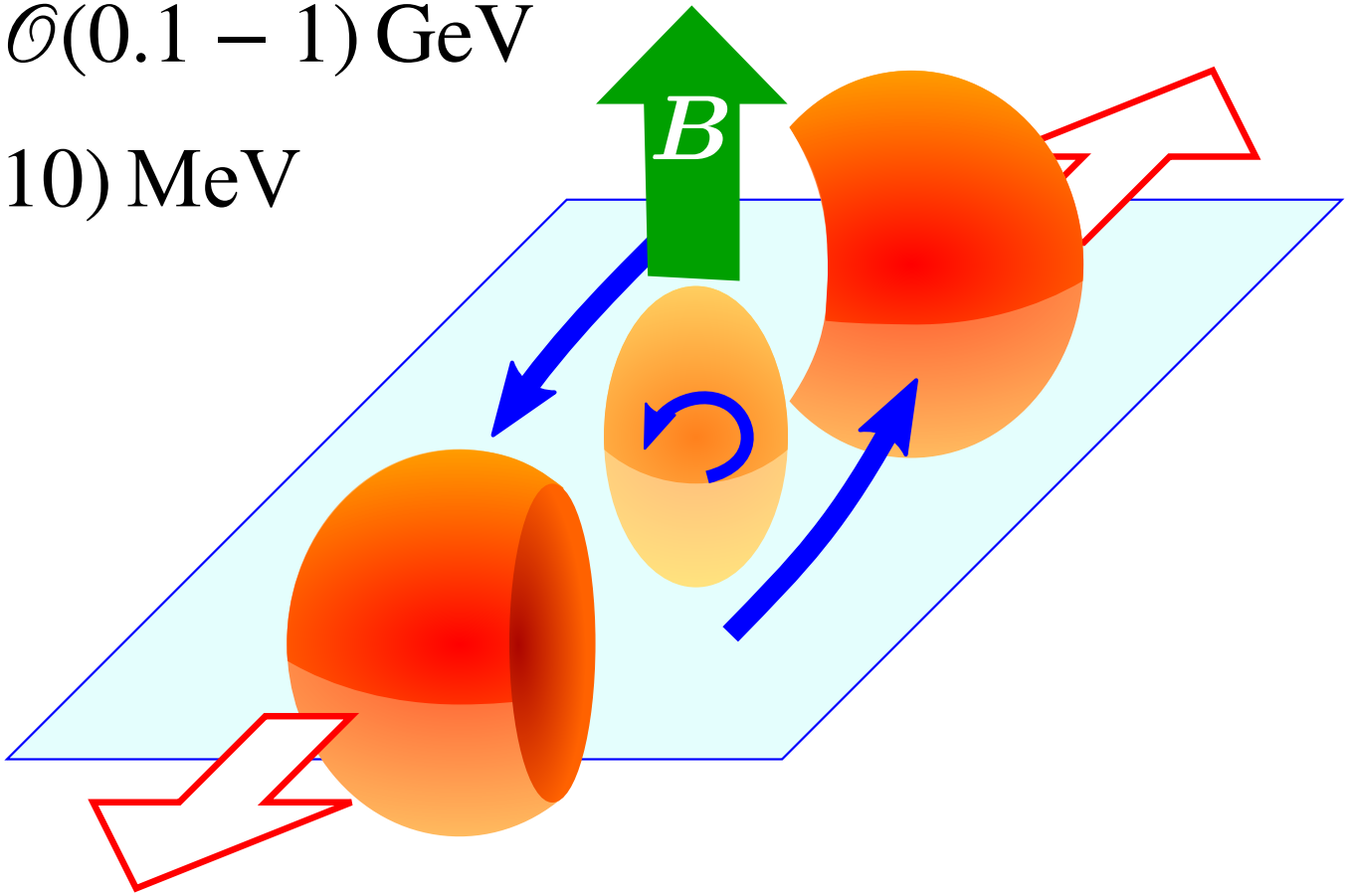
**Chiral Soliton Lattice / Brauner-Yamamoto (2016)**

**Rotating CSL / Nishimura-Yamamoto (2020)**

# *Magnetized Rotating Plasma*

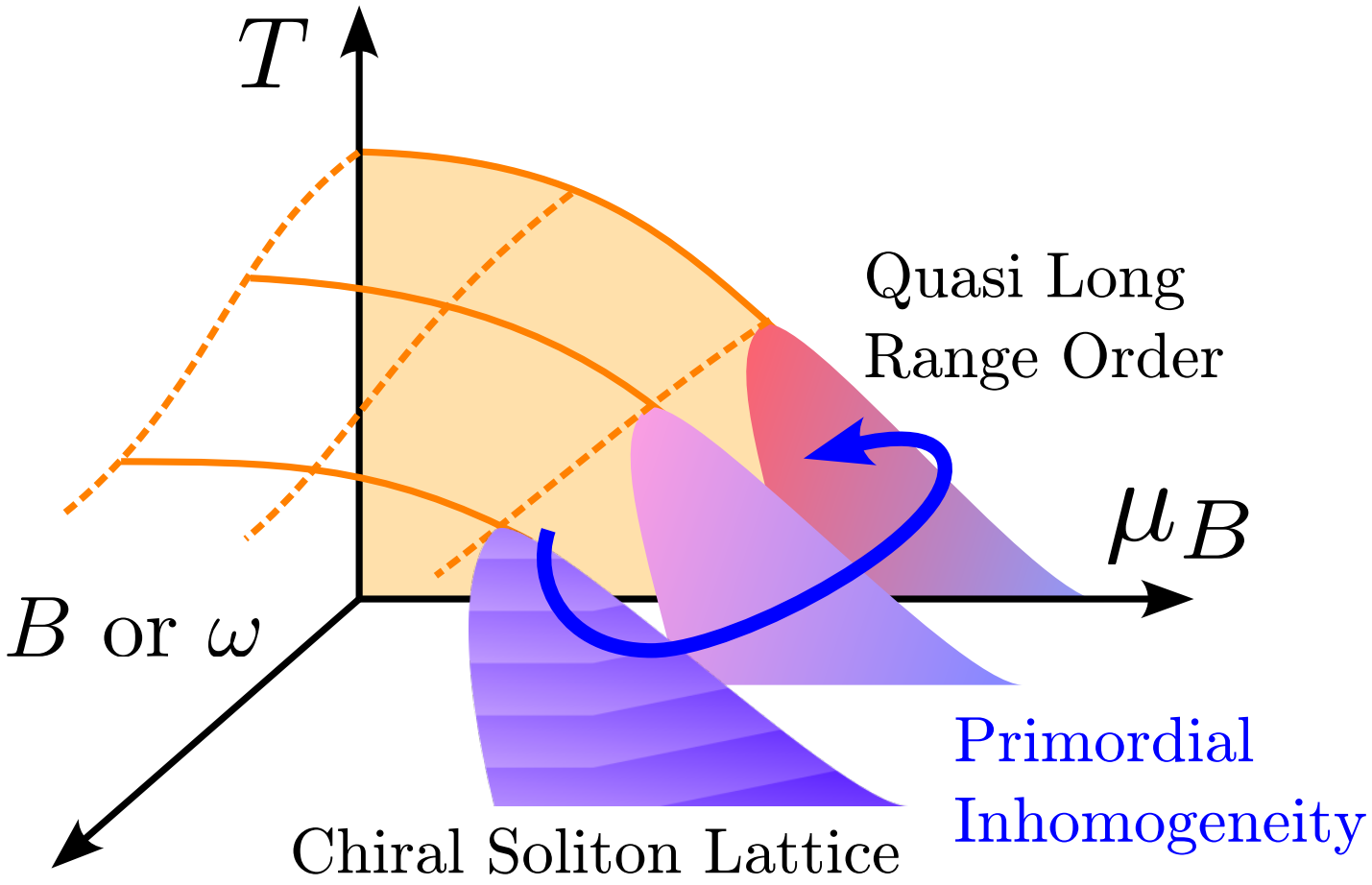
$$\sqrt{eB} \sim \mathcal{O}(0.1 - 1)\,\mathrm{GeV}$$

$$\omega \sim \mathcal{O}(10)\,\mathrm{MeV}$$



# *Primordial Inhomogeneity*

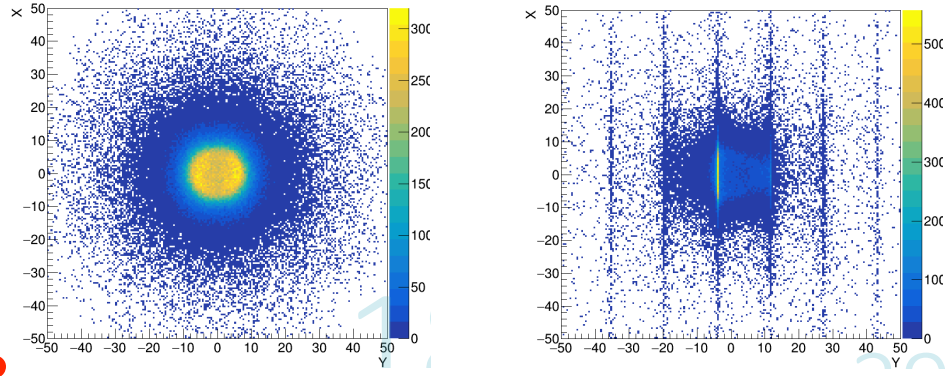
**Fukushima-Hidaka-Inoue-Shigaki-Yamaguchi (2023)**



# *Primordial Inhomogeneity*

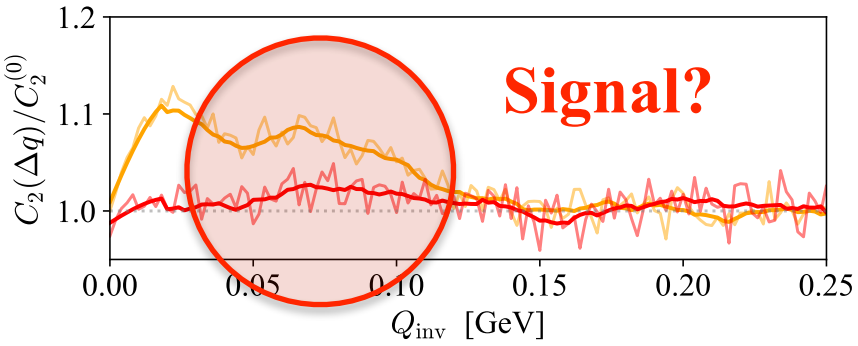
**Recent analysis by Yamauchi (Hiroshima U.)**

**Source distribution: generated by AMPT modulated by hand**

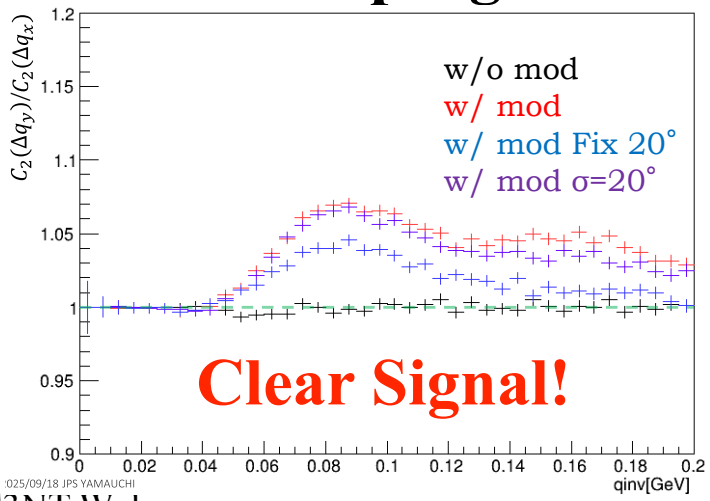



**HBT detects clusters?**

**Previous (2023)**

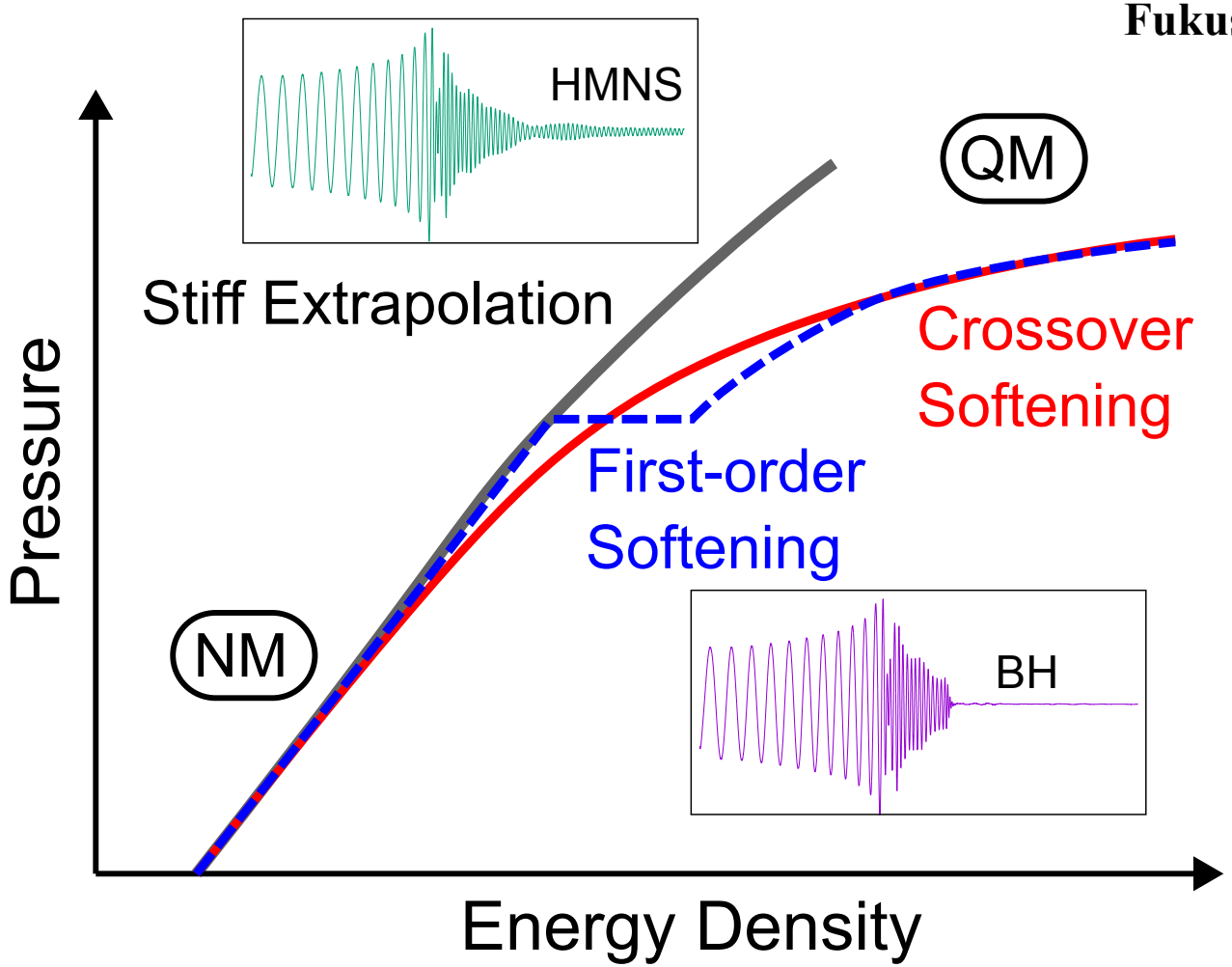



**Now in progress**



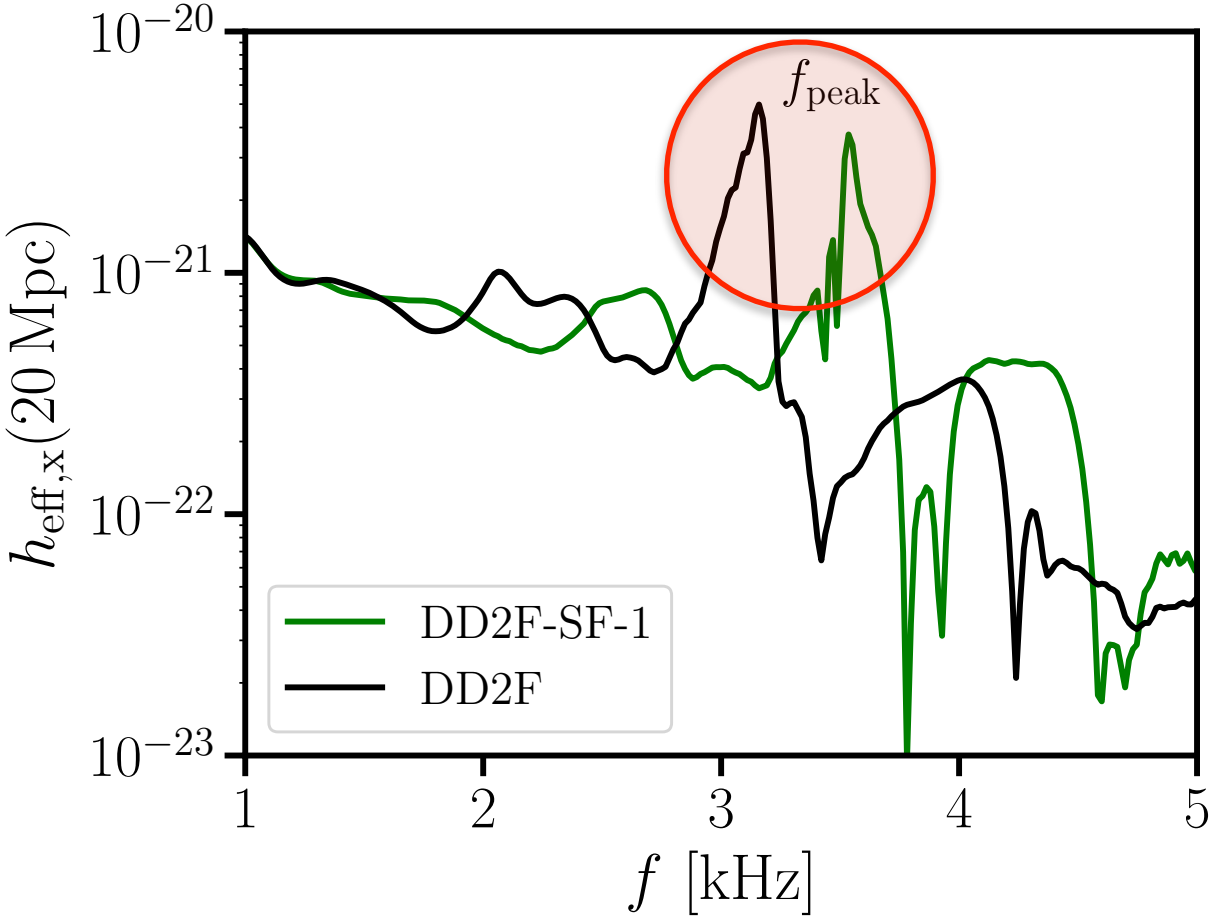
# *First-order in Astro-nuclear Physics*

**Fukushima (2025)**



# *First-order in Astro-nuclear Physics*

**Bauswein-Bastian-Blaschke-Chatziioannou-Clark-Fischer-Oertel (2018)**




**Evidence of First-Order?**

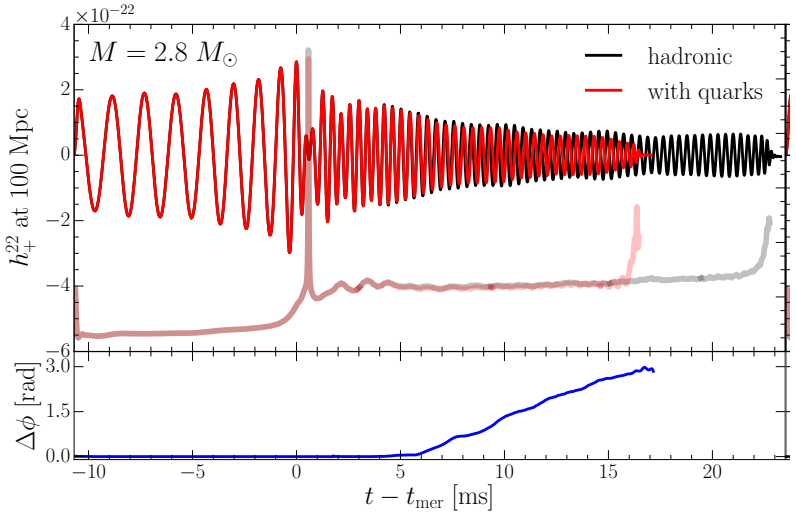
# *First-order in Astro-nuclear Physics*

## Can we see the phase transition with the GW signal?

**Most-Papenfort-Dexheimer-Hanauske-Schramm-Stocker-Rezzolla (2018)**

$CMF_Q$ : EOS with a strong-1st PT to Quark Matter (3~4 times $n_{sat}$)

$CMF_H$ : EOS without quarks




**Quark matter shortens the lifetime of post-merger supramassive/hypermassive ( uniform / differential ) neutron star.**

## What if the transition is only a smooth crossover?

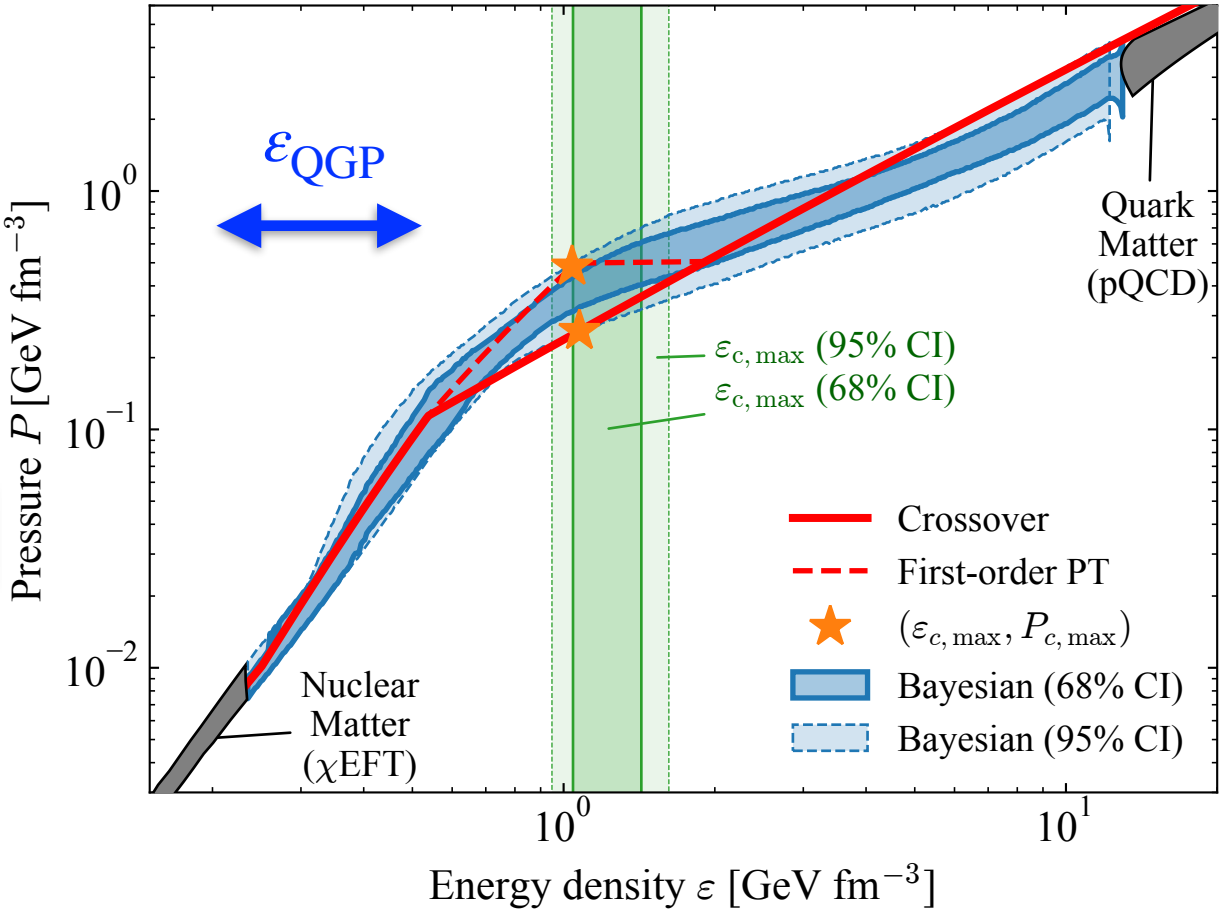
# *Crossover in Astro-nuclear Physics*

## Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)

**Crossover at lower density**

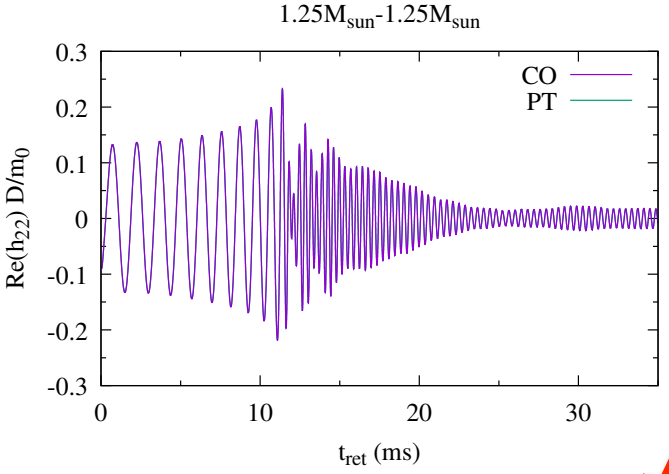
**VS**

**1st-order at higher density**



# *Crossover in Astro-nuclear Physics*

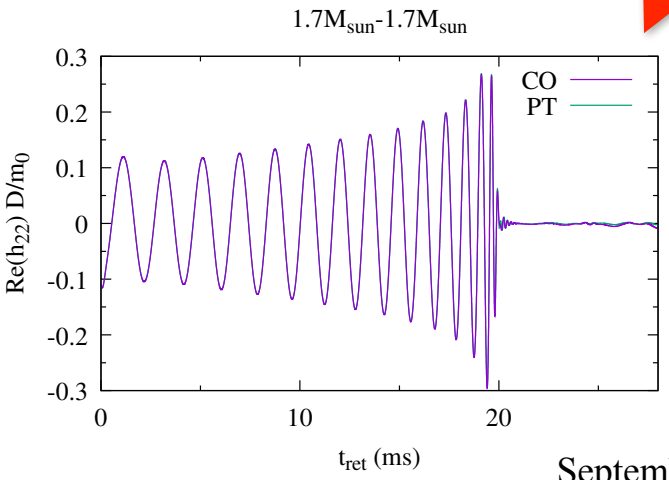
## Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)

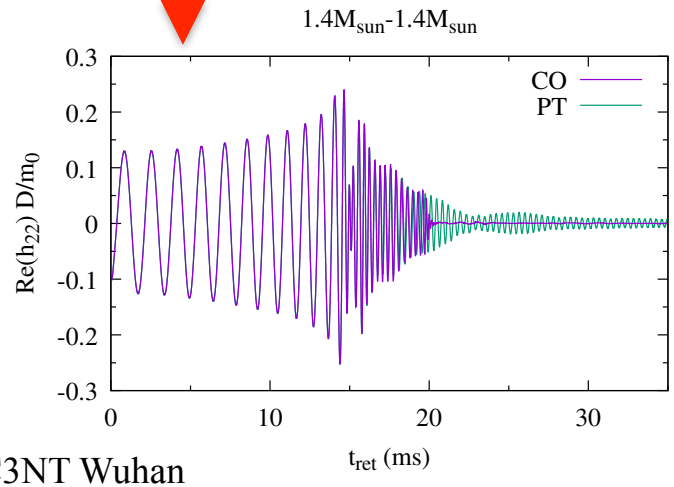



**Light System (No BH)**

**Heavy System (Both BH)**

**Like GW170817 (Discriminable!)**

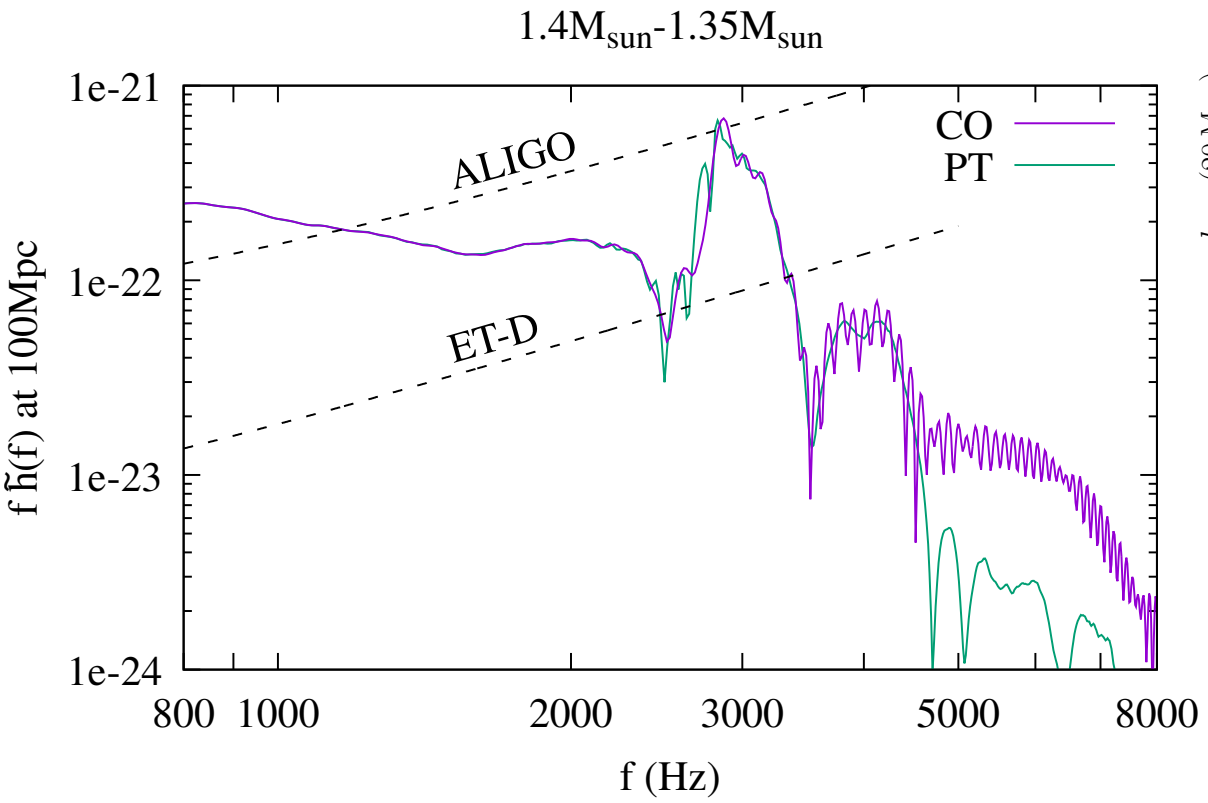


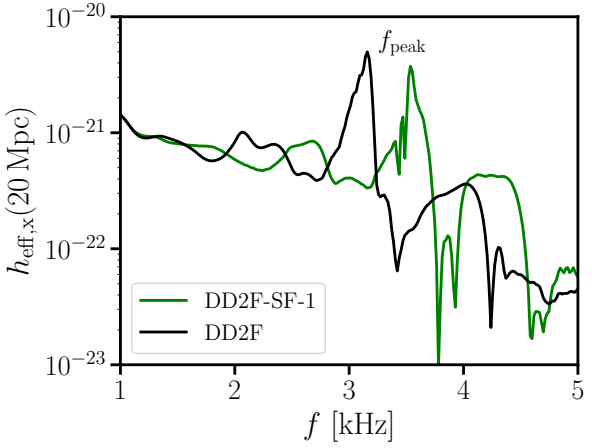



# *Crossover in Astro-nuclear Physics*

**Post-merger stage is very challenging to see:**

**↓ Our crossover case… no difference?**

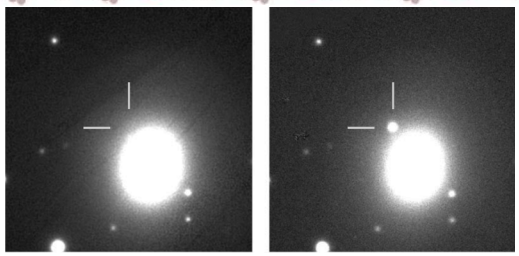



**Maybe the stiffness can be constrained by the peak position.**

# *Multi-messanger Era*

**Kilonova brightness: ejected mass** $> 0.05 M_{\odot}$



**AT 2017 gfo**

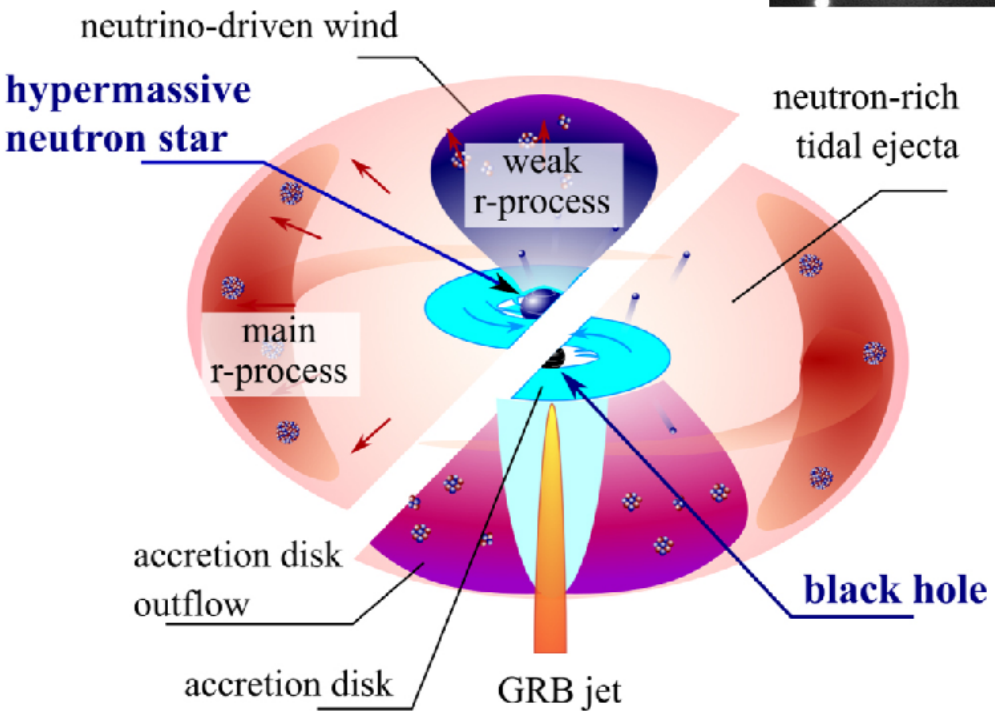
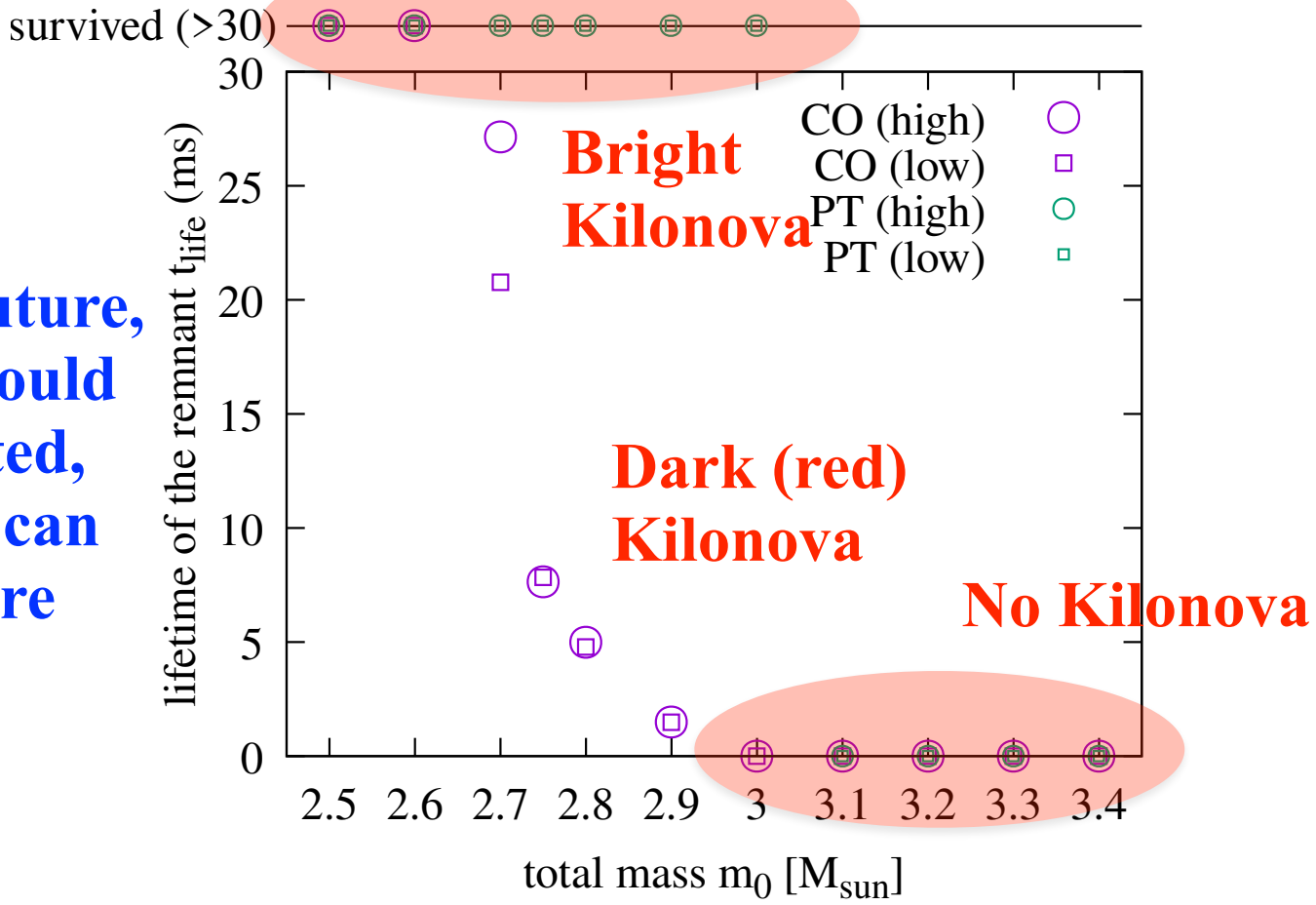



**Illustration from Korobkin+ (2021)**

**Brightness and "color" depend on the EoS and the total mass.**

# *Multi-messanger Era*

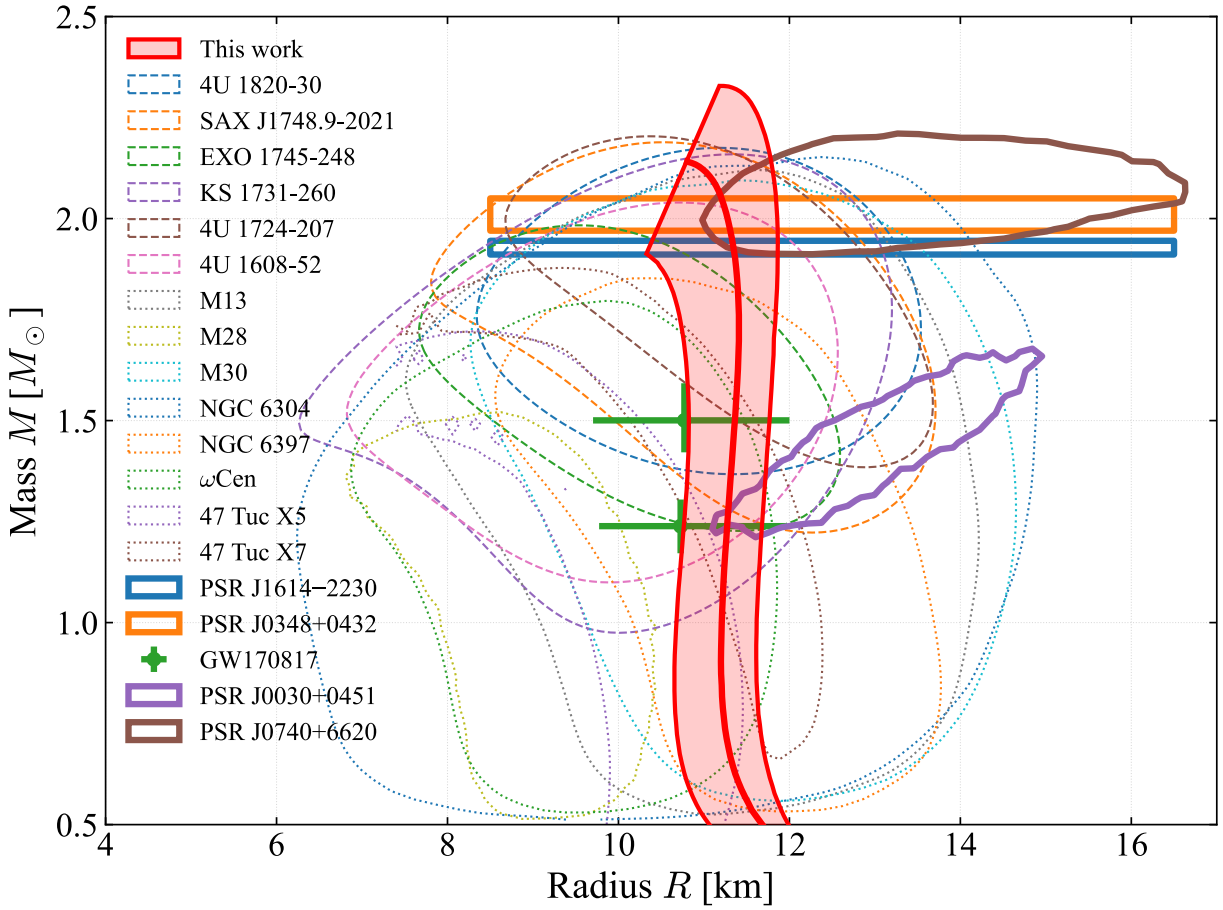
**Fujimoto-Fukushima-Kyutoku-Hotokezaka (2022-2024)**




**In the near future, more data should be accumulated, and then, we can say much more about PT !**

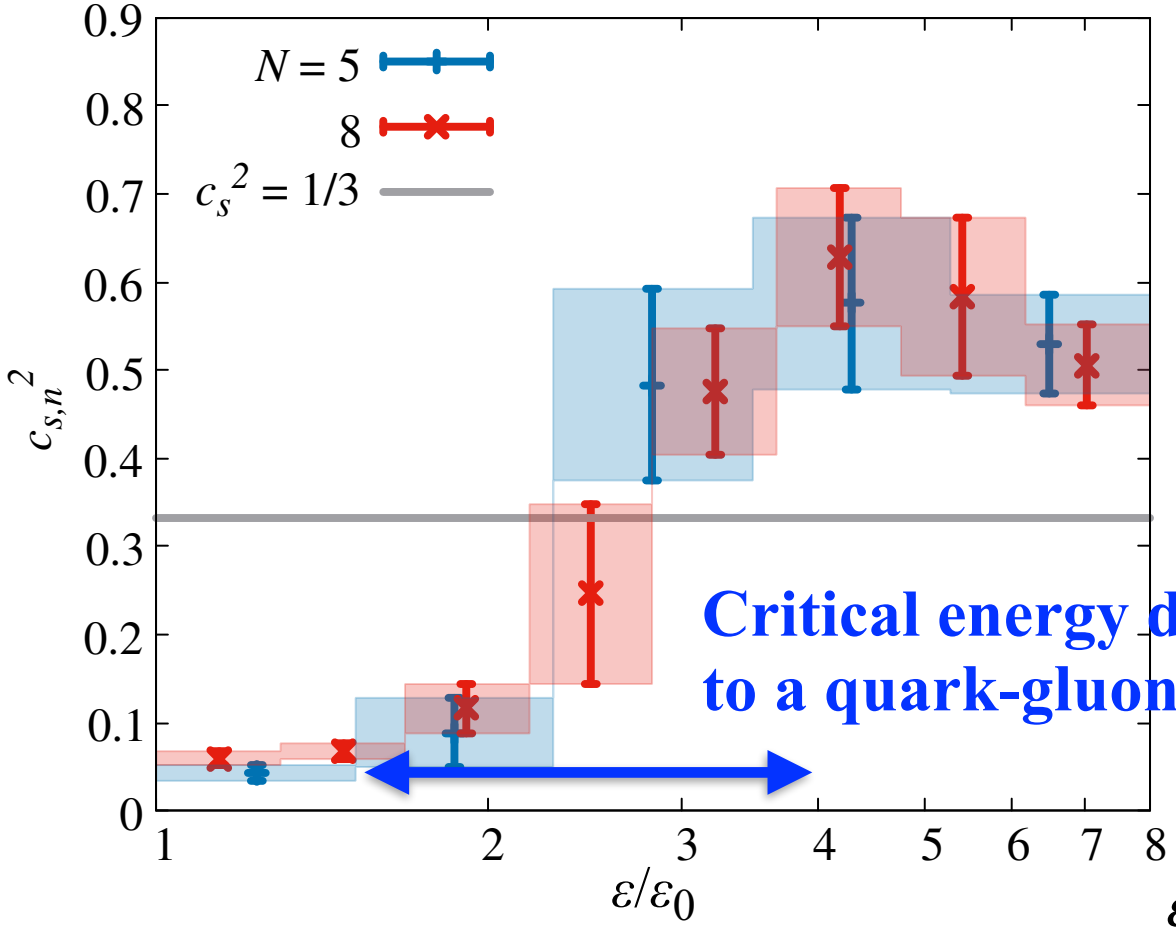
# *Most-likely EoS from NS*

## Fujimoto-Fukushima-Kamata-Murase (2018-2024)



# *Most-likely EoS from NS*

**Fujimoto-Fukushima-Kamata-Murase (2018-2024)**



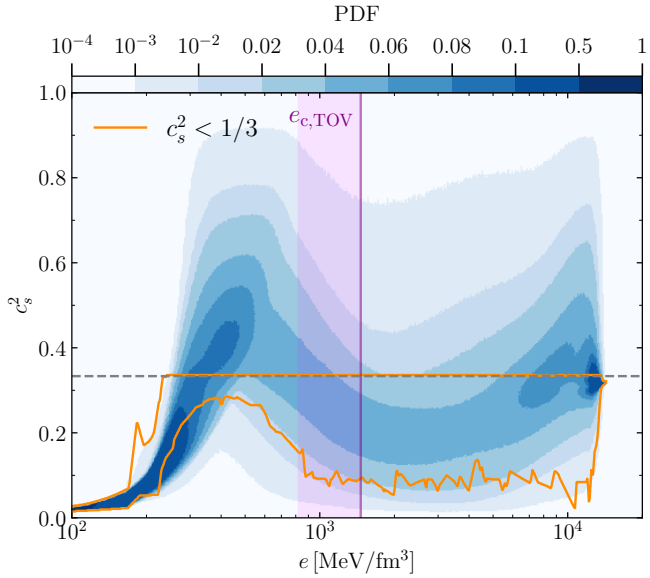

**Not minimum, but maximum!?**

**Critical energy density to a quark-gluon plasma**

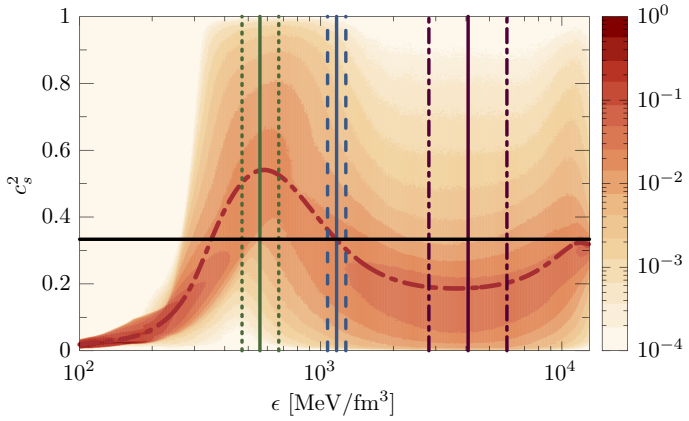
$\varepsilon_0 = 150\,\mathrm{MeV\ fm}^{-3}$

# *Most-likely EoS from NS*

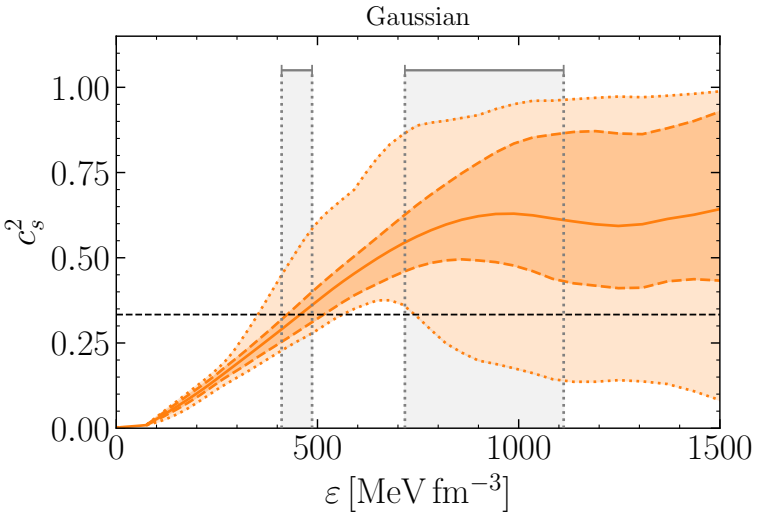
**Altiparmak-Ecker-Rezzolla (2022)**




**Marczenko-McLerran-Redlich-Sasaki (2022)**



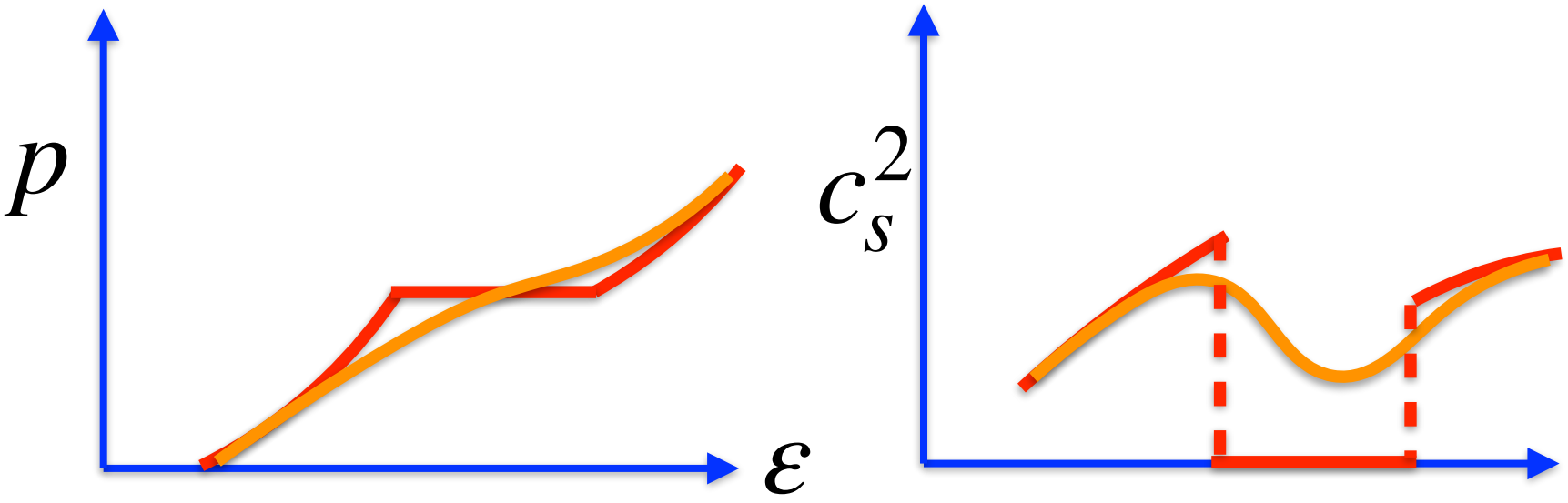

**Brandes-Weise-Kaiser (2022)**




**Supporting the peak!**

# *Interpretation of Sound Speed*

**[1st-order-like EoS]**

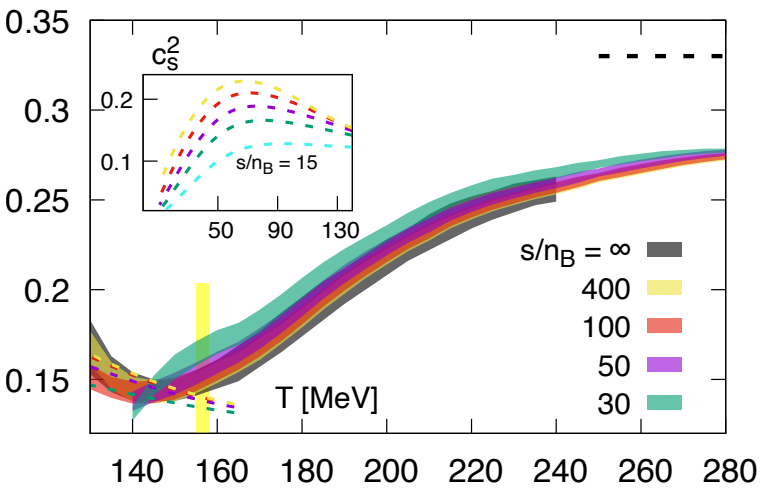


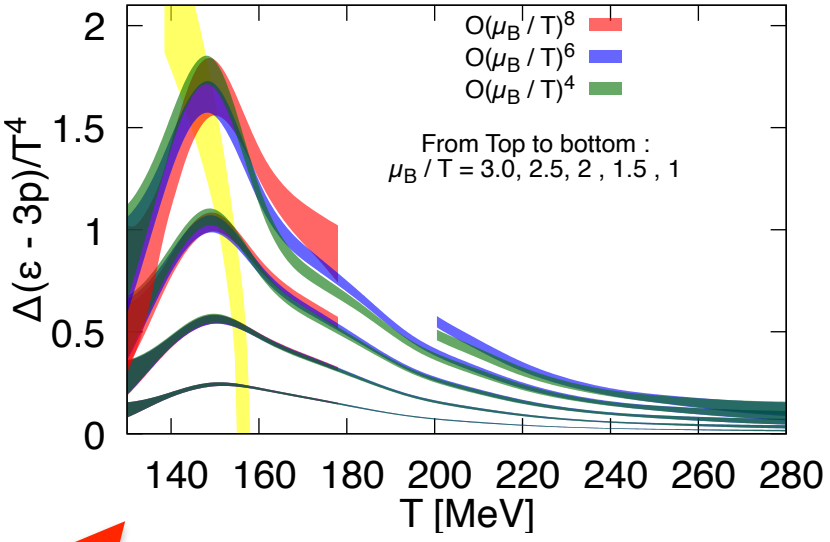

**Phase transition is manifested by a minimum in the speed of sound.**

# *Interpretation of Sound Speed*

**Energy-momentum tensor → Trace anomaly**

$$\Theta^{\mu}{}_{\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$




**Any relation? Yes!!**

**HotQCD Collab. (2022)**

# *Interpretation of Sound Speed*

**Fujimoto-Fukushima-McLerran-Praszalowicz (2022)**

**Measure of conformality:** $\Delta = \frac{1}{3} - \frac{p}{\varepsilon}$

**Gavai-Gupta-Mukherjee (2004)**

$$c_s^2 = \frac{dp}{d\varepsilon} = c_{s,\,\mathrm{deriv}}^2 + c_{s,\,\mathrm{non-deriv}}^2$$

$$c_{s,\,\mathrm{deriv}}^2 = -\varepsilon \frac{d\Delta}{d\varepsilon} \qquad c_{s,\,\mathrm{non-deriv}}^2 = \frac{1}{3} - \Delta$$

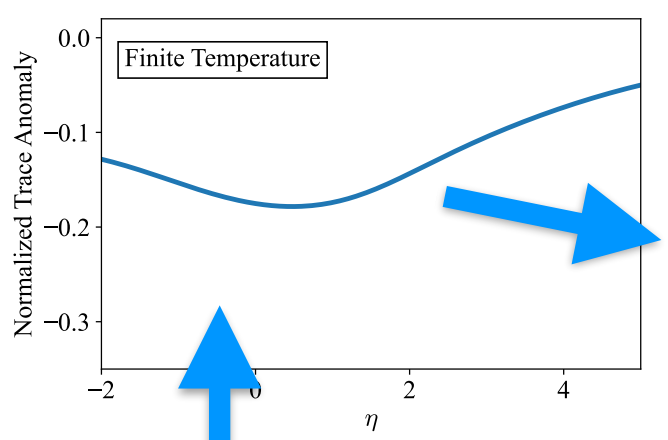
**Derivative** **Non-Derivative**

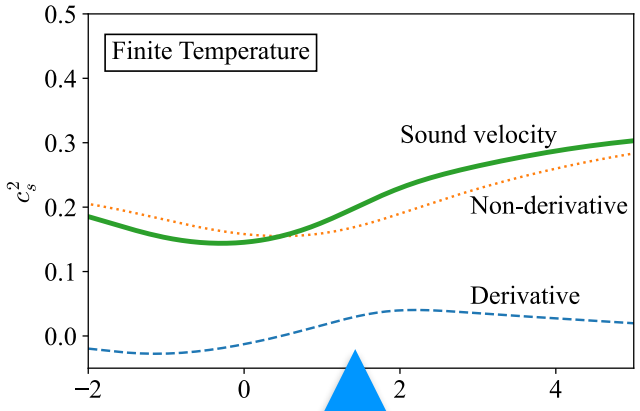
**Dominant at high density making a peak!**

# *Interpretation of Sound Speed*

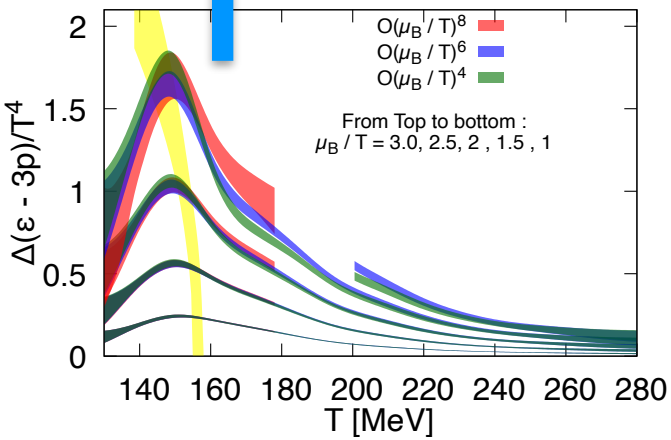
## High-$T$ — Non-Derivative Dominant $c_s^2 \simeq p/\varepsilon$

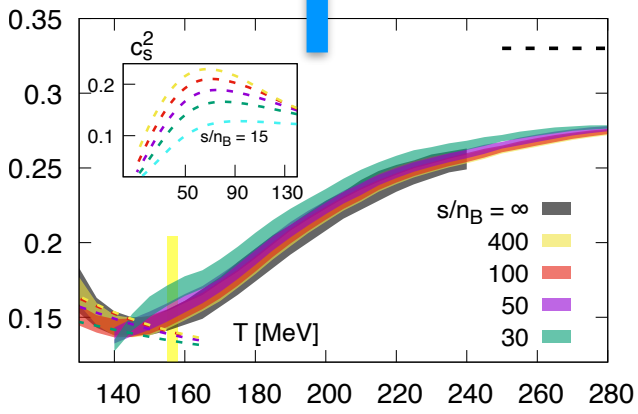
**Sign Flipped** $-\Delta$




$\eta = \ln(\varepsilon/\varepsilon_0)$



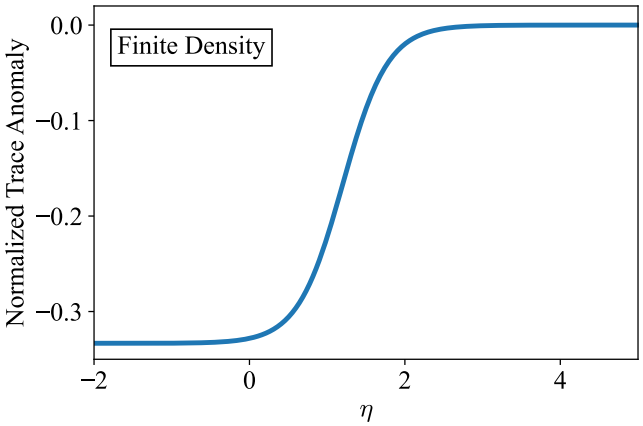



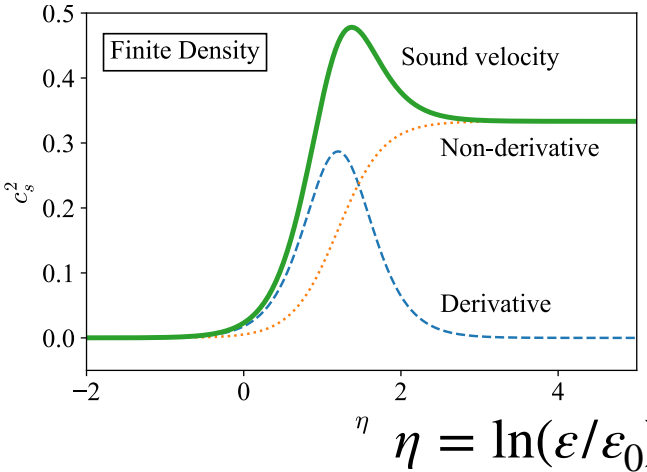

**Speed of Sound**

# *Interpretation of Sound Speed*

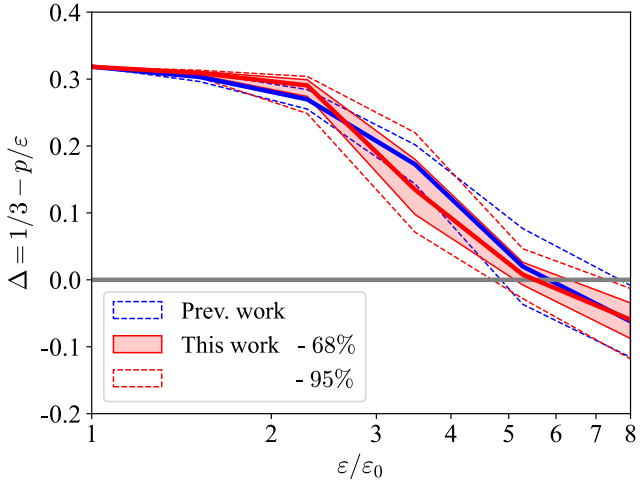
## High Density — Derivative Peak

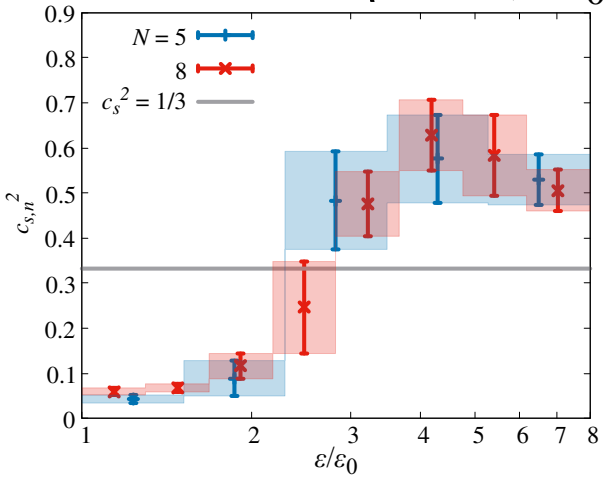
**Sign Flipped $-\Delta$**




$\eta = \ln(\varepsilon/\varepsilon_0)$

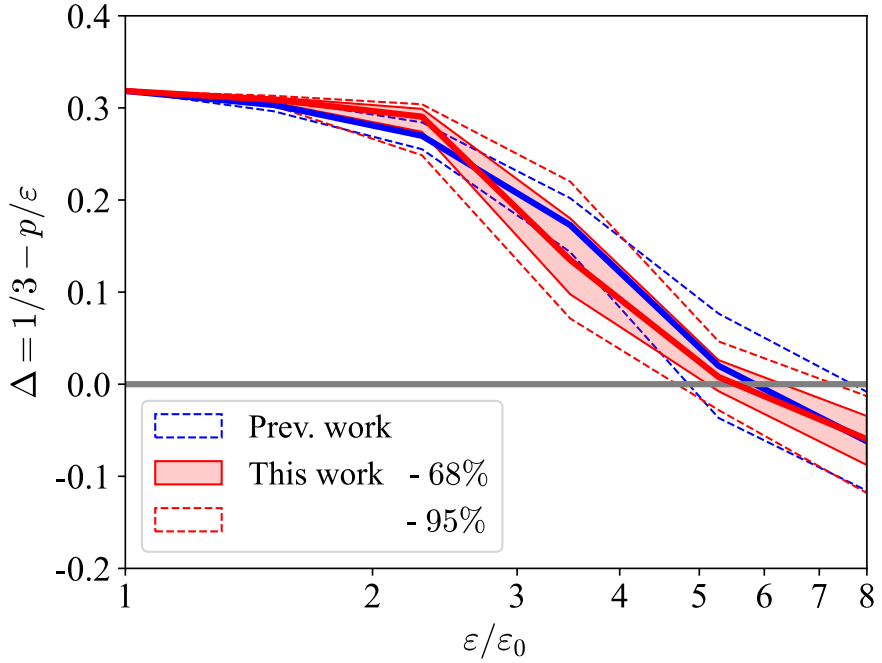



**Speed of Sound**

# *Interpretation of Sound Speed*

## Physical interpretation of $\Delta < 0$ ???




$$\Delta \propto \varepsilon - 3p$$

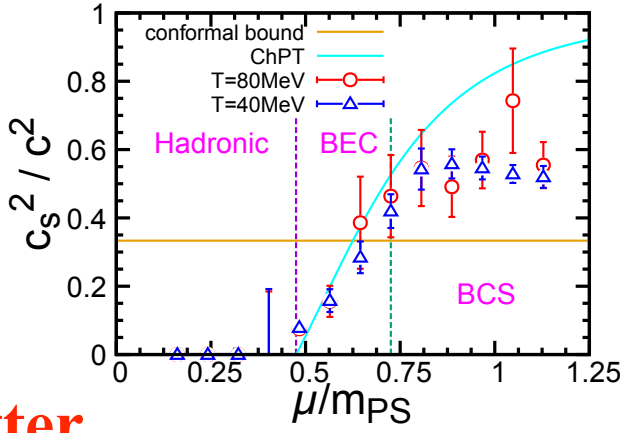
$$\propto \frac{d}{d\mu}\left(\frac{p}{\mu^4}\right)$$

**Thermodynamic degrees of freedom**

**Negative trace anomaly implies the presence of "condensates"!?**

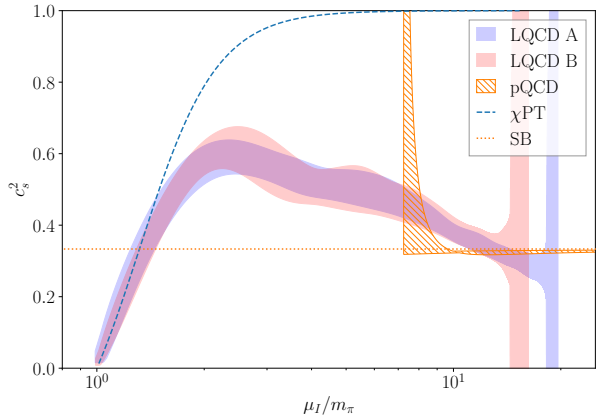
# *Interpretation of Sound Speed*

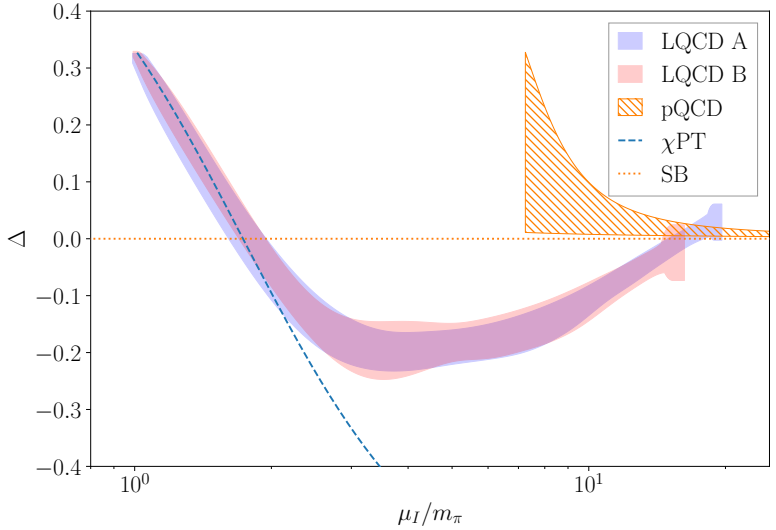
**Lattice results for QCD-like theories**




* **Diquark superfluid in $QC_2D$**
  **To be compared with**
  **Lattice: Itou+ (2023-2024)**

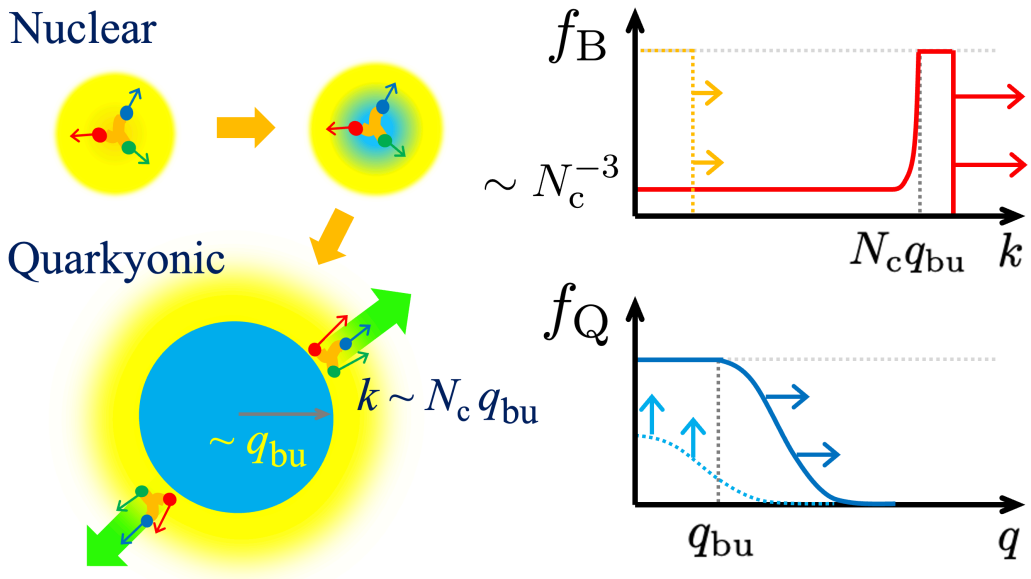
* **Pion-condensed high-isospin matter**
  **To be compared with**
  **Lattice: Abbott+ (2023)**



# *Recent Quarkyonic Scenario*

**Idyllic** **Fujimoto-Kojo-McLerran (2024) / Tajima-Iida-Kojo-Liang (2025)**

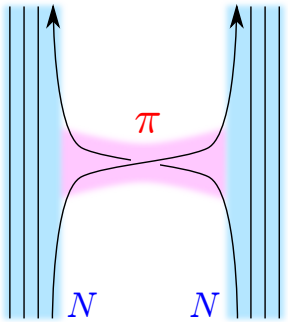



$$n_B \sim \mu_B^3 \sim N_c^3 \mu_q^3$$

**Dual**

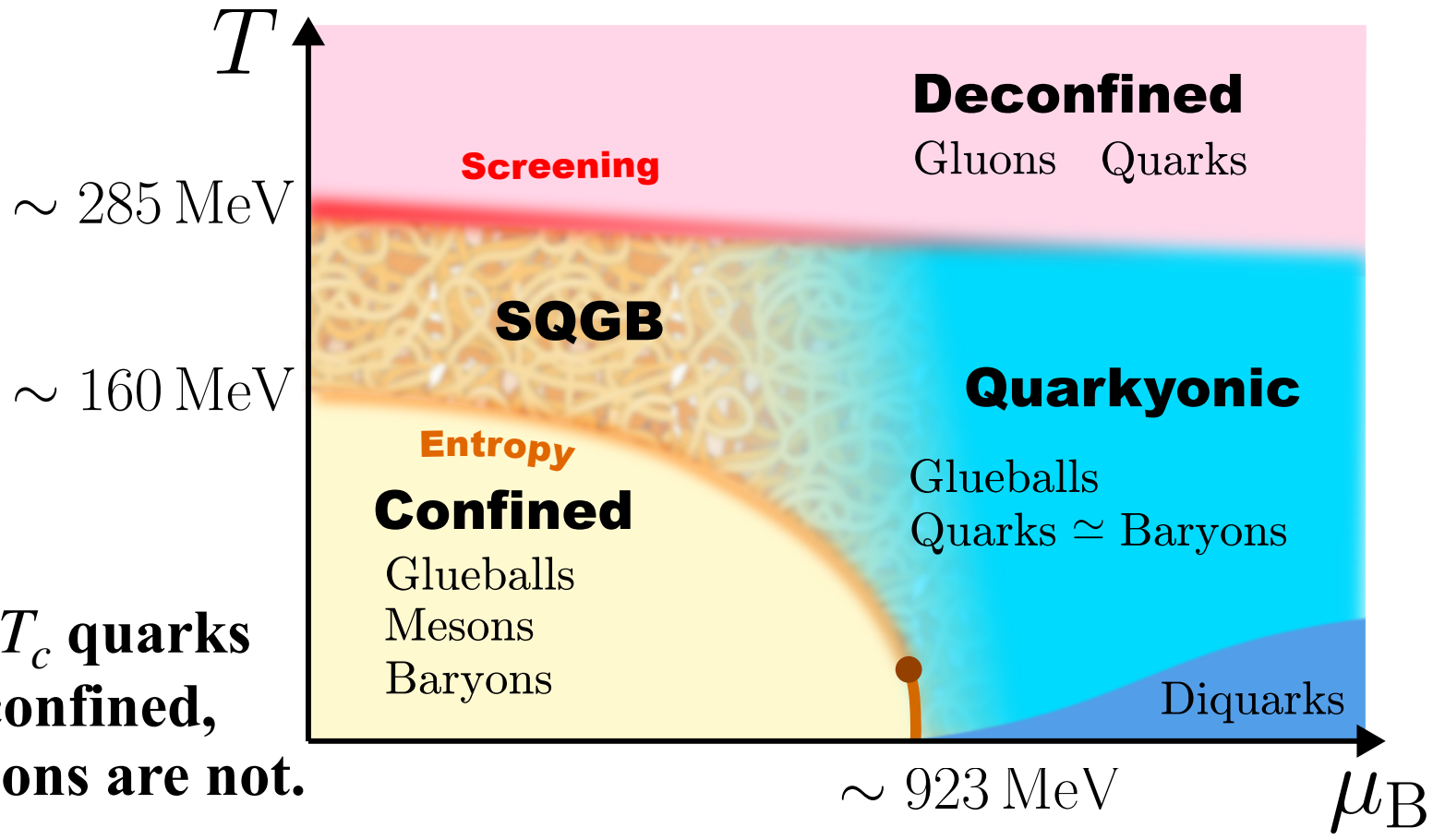
$$n_q \sim \mu_q^3$$

**Suppression of nucleon distribution should be caused by quark saturation at short range due to quark exchanges.**



# *More Recent Spaghetti Scenario*

**Fujimoto-Fukushima-Hidaka-McLerran (2025)**




**Above $T_c$ quarks are deconfined, but gluons are not.**

# *Summary*

## Surprises

- Speed of sound at high density exceeds the conformal value making a peak.
- Conformal symmetry is rapidly restored already at intermediate densities.

## Challenges

- Can we estimate the cluster size by HIC-HBT?
- Can we confirm (exclude) the color superconducting states in NS matter?
- Non Fermi liquid nature of dense QCD matter?