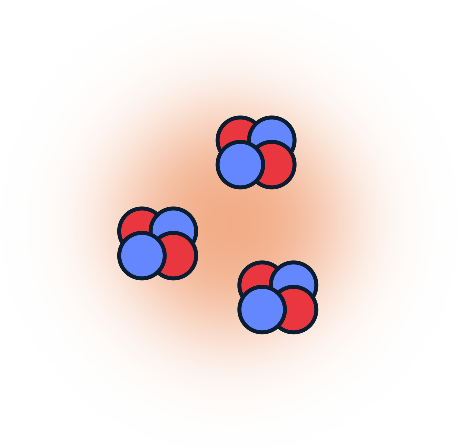
# Emergent Cluster Structures in Light Nuclei



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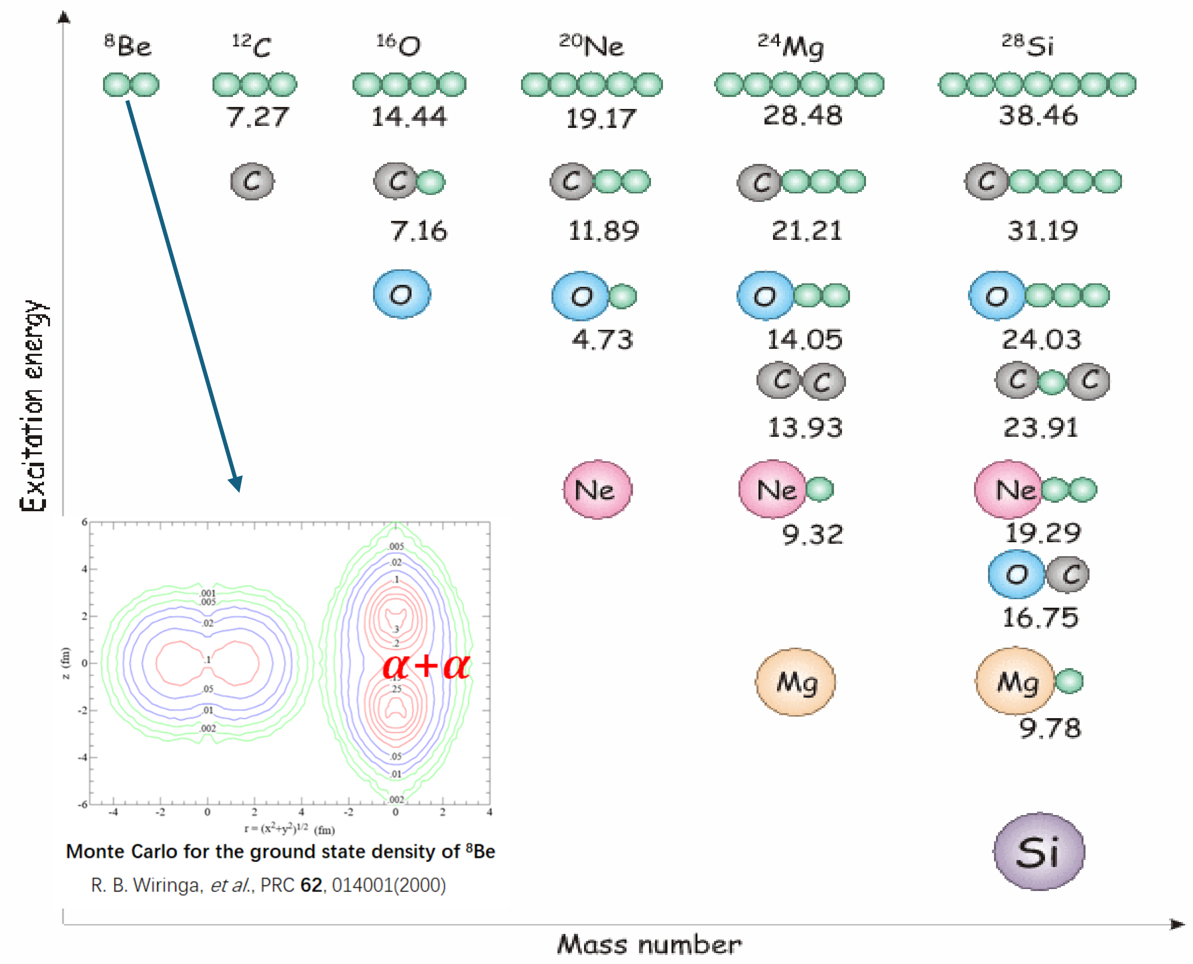
**Central China Center for Nuclear Theory (C3NT)**

# Outline

- **Introduction**
- **N$\alpha$ nuclei**: From the Holy state to Condensate State
- **N$\alpha$+X nuclei**:  Novel clustering structure in nuclei
- **Summary**

# Nuclear Cluster Physics

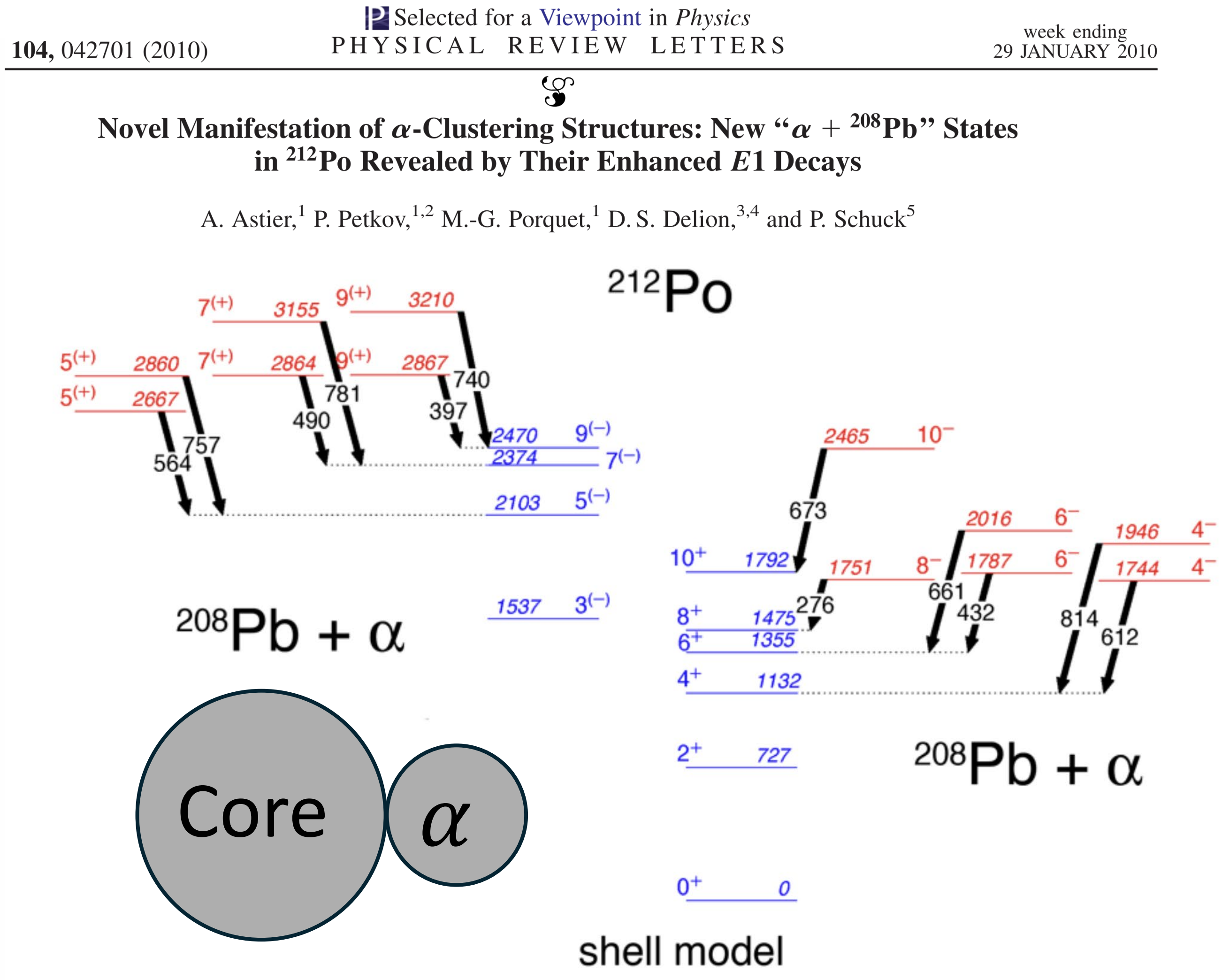
## Ikeda diagram of light nuclei




Monte Carlo for the ground state density of 8Be

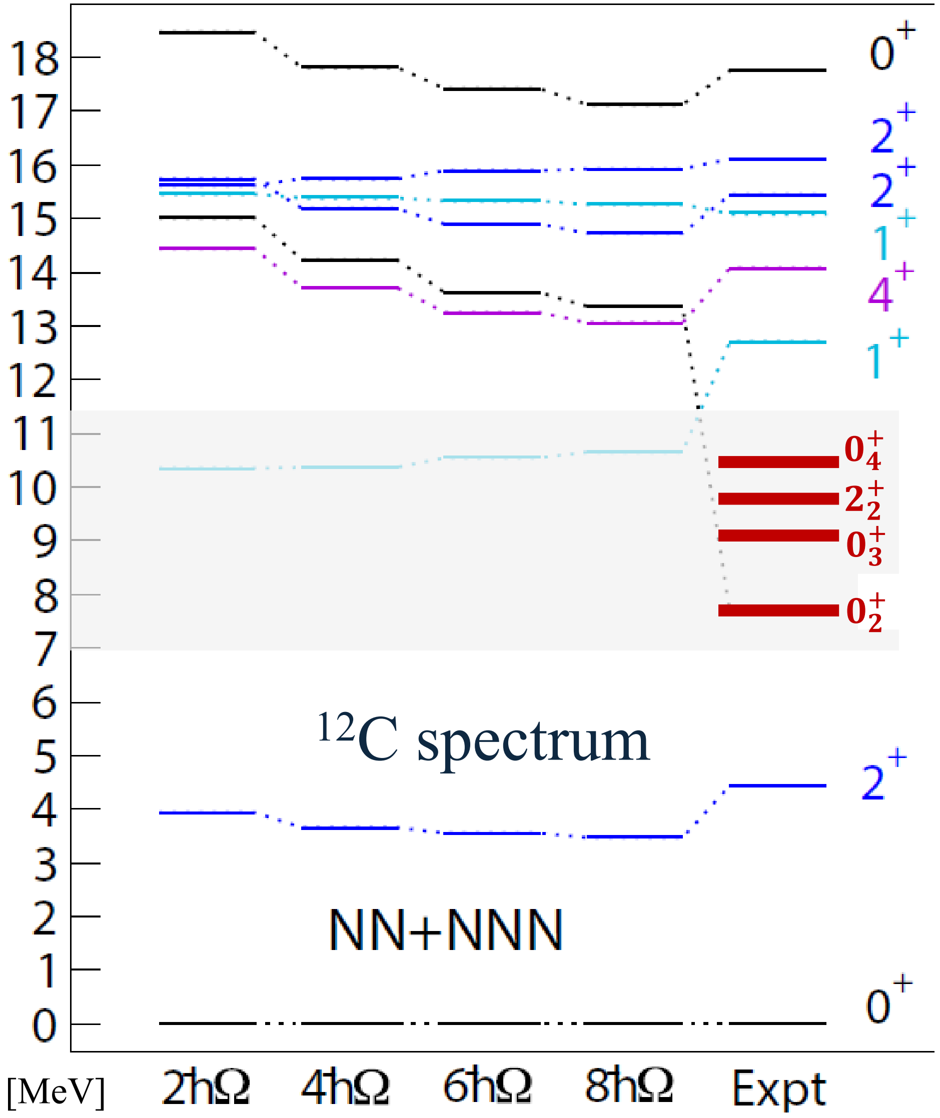
R. B. Wiringa, *et al.*, PRC **62**, 014001(2000)

## Clustering in heavy nuclei ?



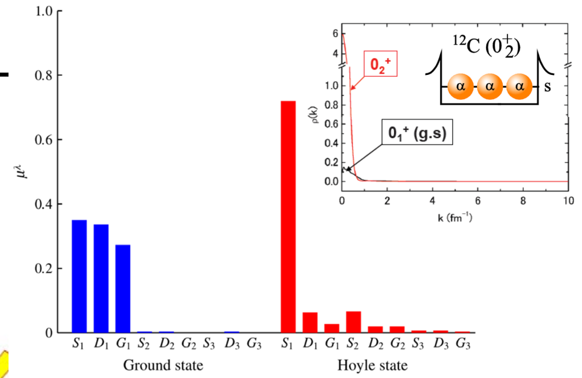

https://physics.aps.org/articles/v3/8

# Cluster states of $^{12}C$

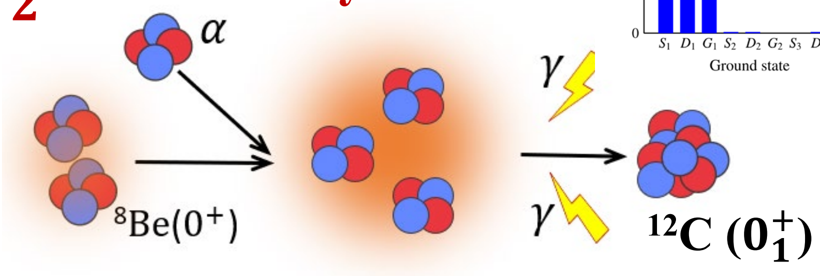



Recent No-Core-Shell-Model calculations

V.Somà,P.Navrátil, *et al.* PRC,101,014318 (2020)




$0_2^+$ **Hoyle State**




Hoyle state & Bose-Einstein Condensate.

Rev. Mod. Phys. **89**, 011002 (2017)

$0_{3,4}^+$

Two broad resonance states with large decay width

Phys. Rev. C **84**, 054308 (2011)

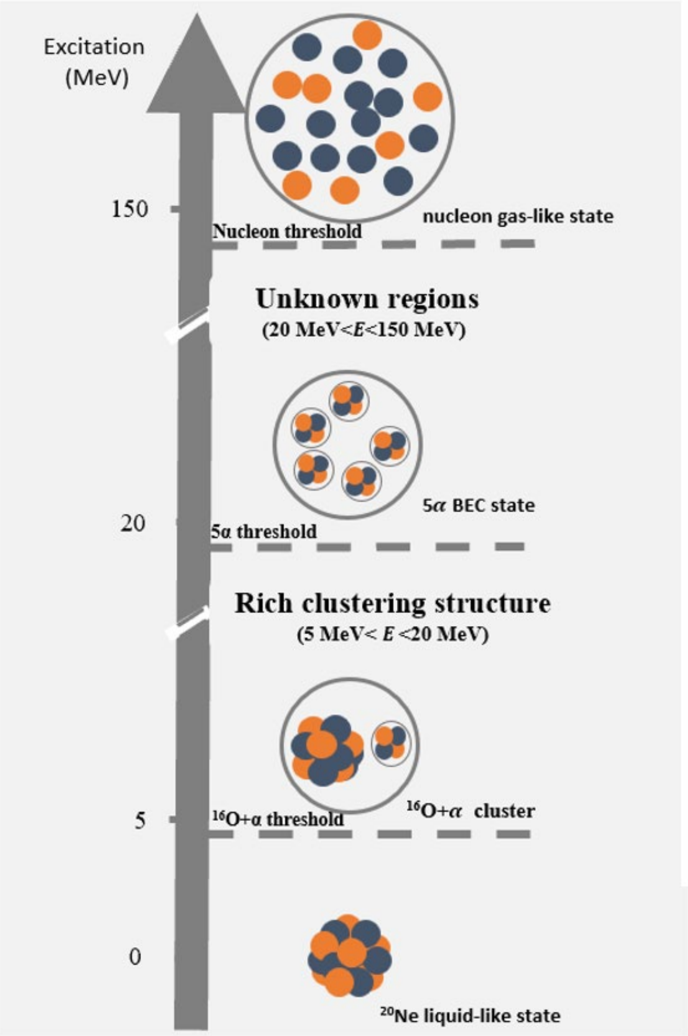
$2_2^+$

Long puzzle and it now has been confirmed for its existence.

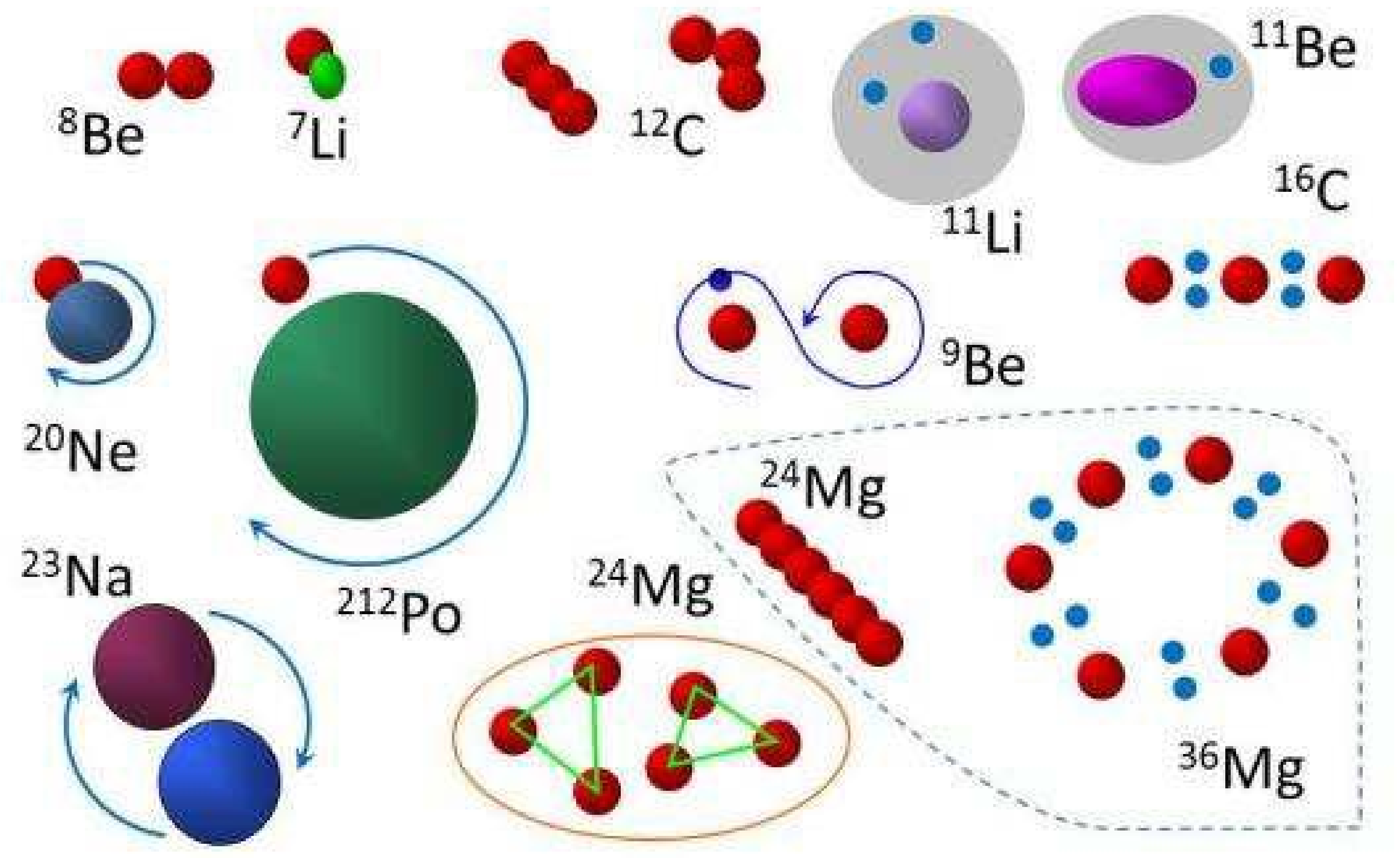
Phys. Rev. Lett. **110**, 152502 (2013)

**well-developed clustering states**

# Rich clustering structure in nuclear systems.




**Evolution of structure of $^{20}$Ne**




DOI:10.1088/1742-6596/863/1/012002

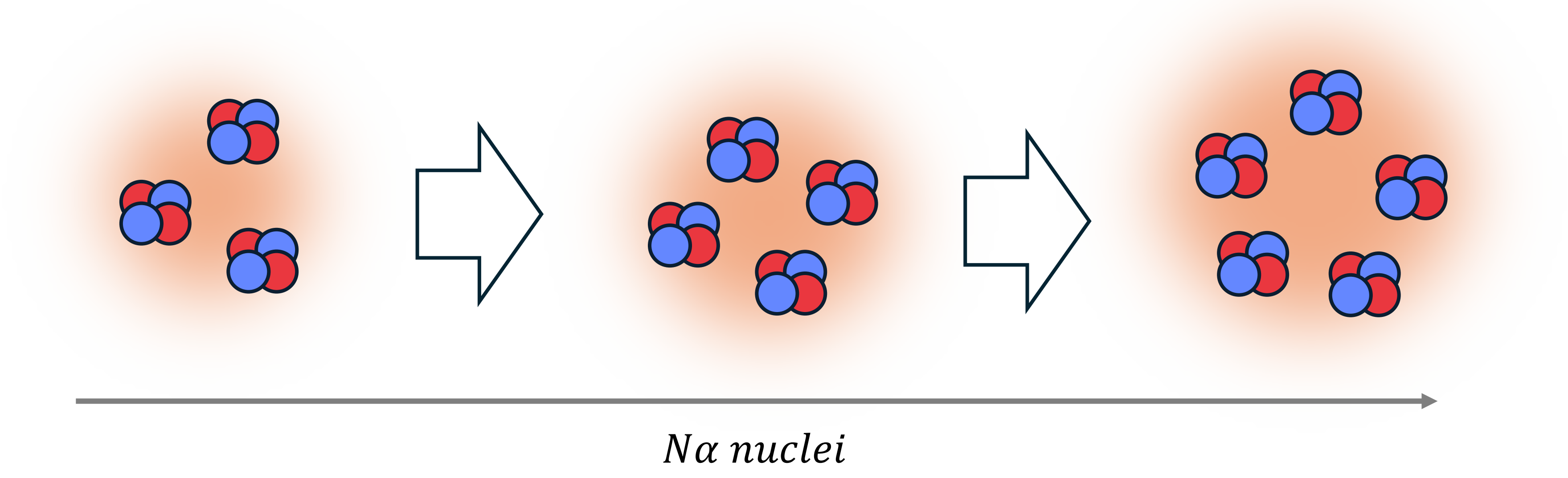
$$\Psi = \mathcal{A}\{\chi(\xi_1, \dots, \xi_{n-1})\phi_1 \dots \phi_n\}$$

**OO and NeNe collisions**

ATLAS arXiv:2509.05171

ALICE arXiv:2509.06428

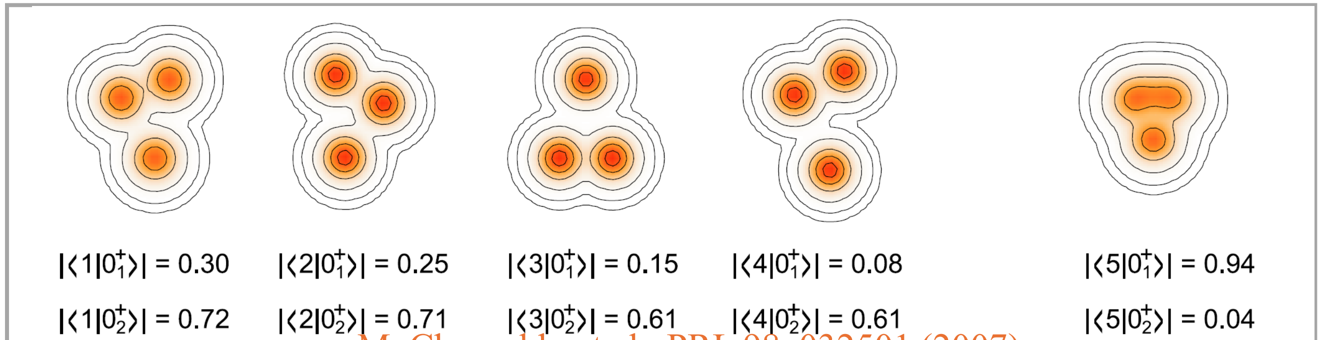
CMS PAS: https://cds.cern.ch/record/2942004



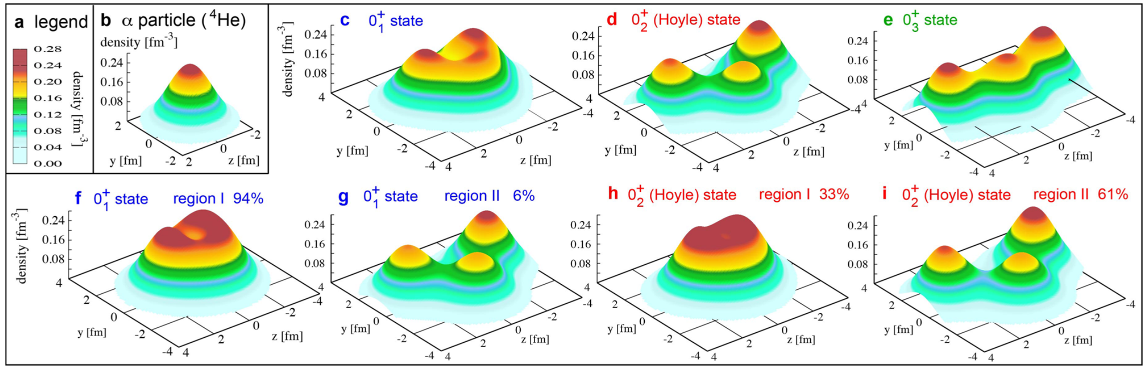

## Search for the Nα novel cluster states

- gas-like cluster state
- no-geometry shape
- excited states

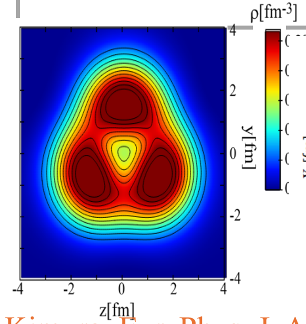
# Shape/Structure of the $^{12}$C



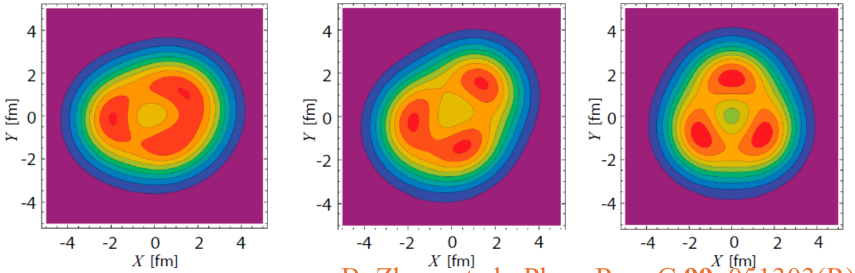

M. Chernykh, et al., PRL 98, 032501 (2007)



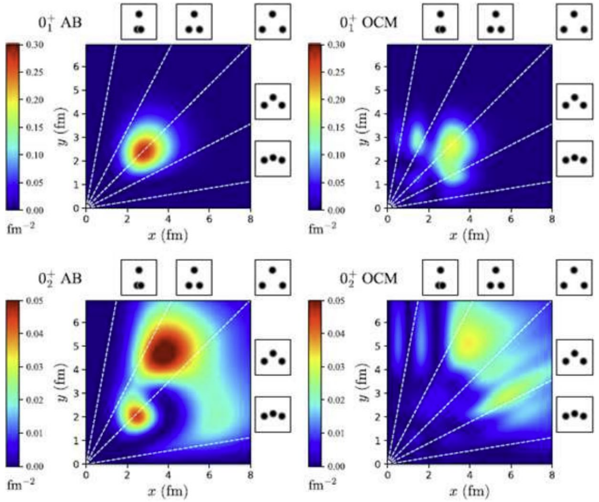

Otsuka, et al., Nat Commun 13, 2234 (2022)



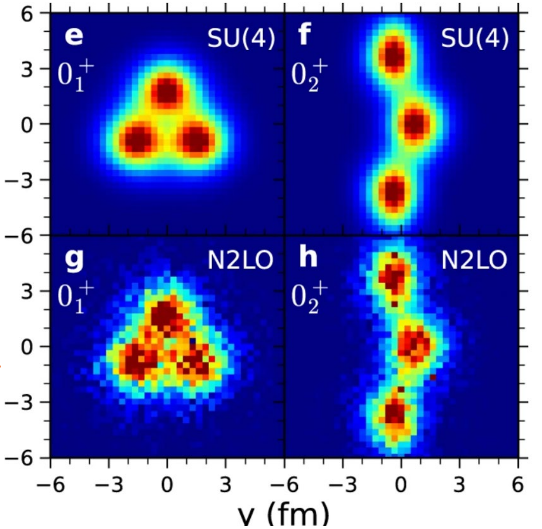

M.Kimura, Eur. Phys. J. A 60, 77 (2024)



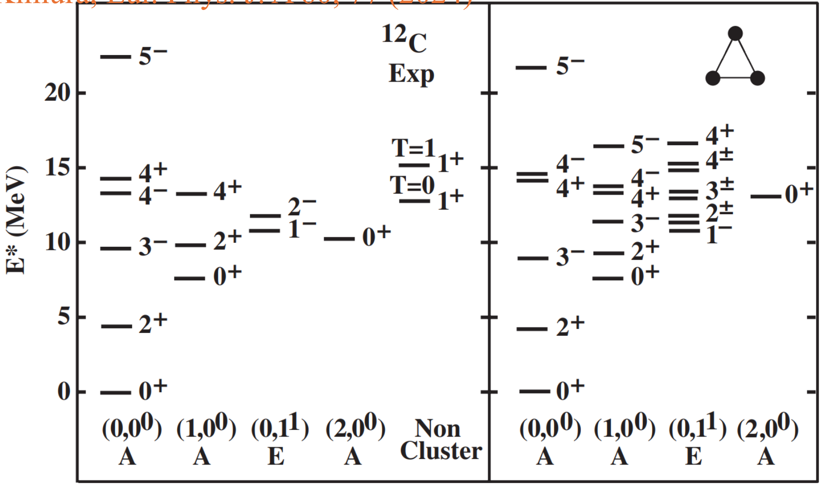

B. Zhou, et al., Phys. Rev. C **99**, 051303(R) (2019).



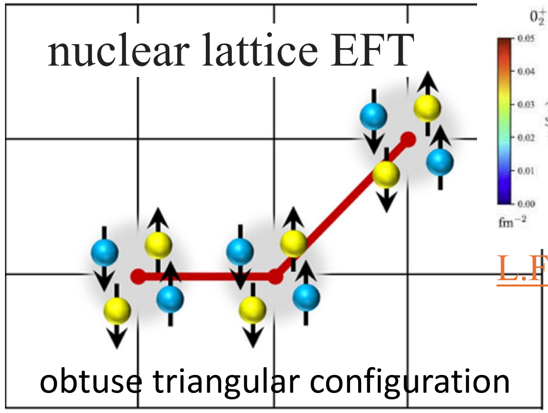

L.Fortunato, *Few-Body Syst* **65**, 1 (2024).




S.Shen, et al., *Nat Commun* **14**, 2777 (2023)

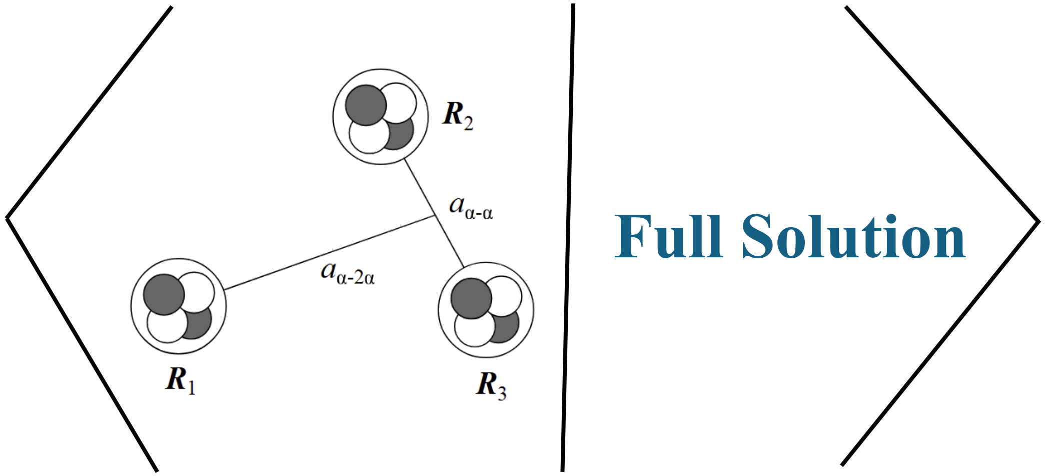



D J Marín-Lámbarri,et al., PRL 113, 012502 (2014)




E.Epelbaum,et al,,PRL 109,252501 (2012)

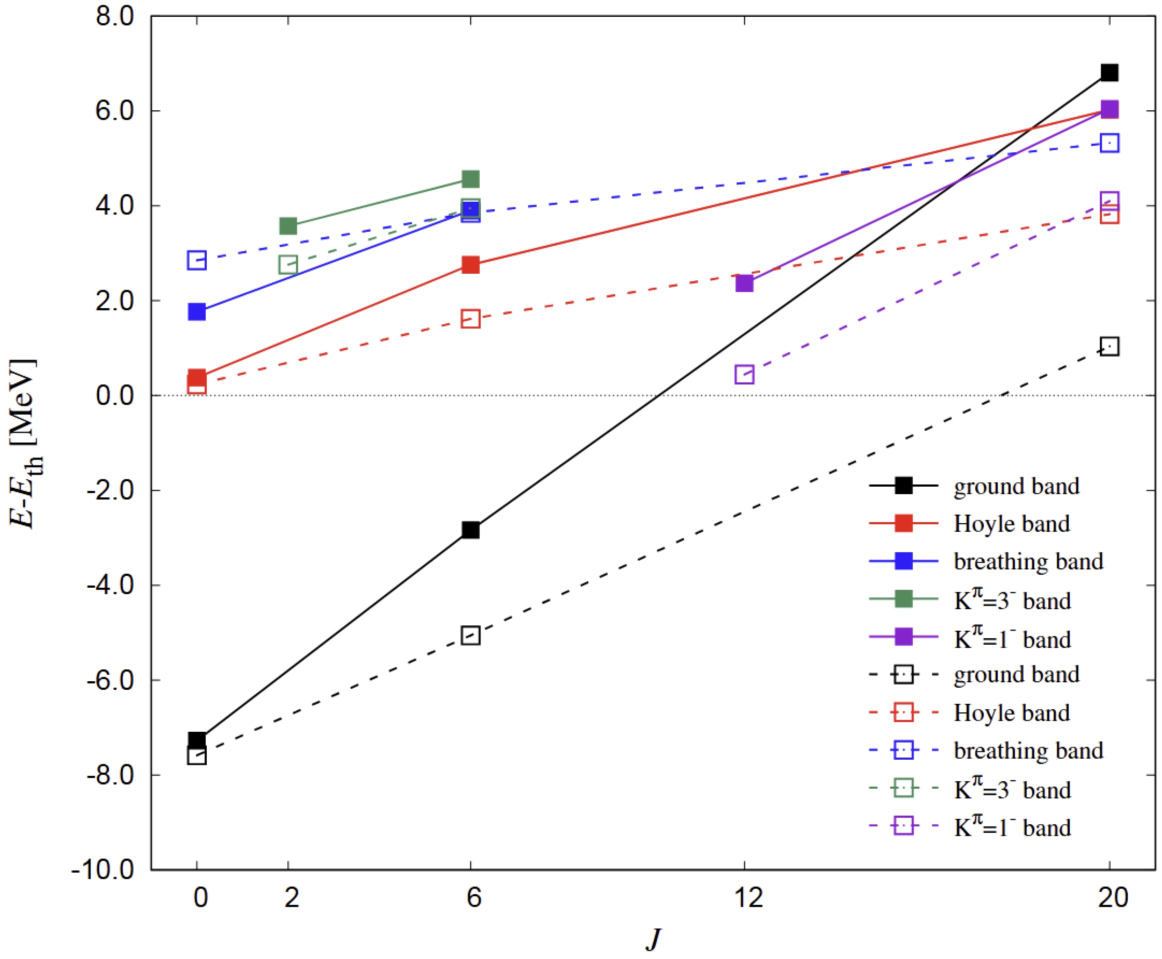
# Two-body overlap function (Two-body RWA)




$$\mathcal{Y}^{J\pi}_{L_{\alpha\text{-}2\alpha}L_{\alpha\text{-}\alpha}L}(a_{\alpha\text{-}2\alpha}, a_{\alpha\text{-}\alpha}) = \sqrt{\frac{12!}{4!4!4!}}$$

$$\times \left\langle \frac{\delta(r_{\alpha\text{-}2\alpha} - a_{\alpha\text{-}2\alpha})\delta(r_{\alpha\text{-}\alpha} - a_{\alpha\text{-}\alpha})}{r^2_{\alpha\text{-}2\alpha} r^2_{\alpha\text{-}\alpha}} [[Y_{L_{\alpha\text{-}2\alpha}}(\hat{r}_{\alpha\text{-}2\alpha}) \otimes Y_{L_{\alpha\text{-}\alpha}}(\hat{r}_{\alpha\text{-}\alpha})]_L \otimes [\Phi_\alpha \otimes \Phi_\alpha \otimes \Phi_\alpha]_0]_{JM} \middle| \Psi^{J\pi}_M \right\rangle$$

$$S^2_{3\alpha} = \int_0^\infty \int_0^\infty \left| a_{\alpha\text{-}2\alpha} a_{\alpha\text{-}\alpha} \mathcal{Y}^{J\pi}_c(a_{\alpha\text{-}2\alpha}, a_{\alpha\text{-}\alpha}) \right|^2 da_{\alpha\text{-}2\alpha} da_{\alpha\text{-}\alpha}.$$




How to classify the rotational bands in light nuclei ?

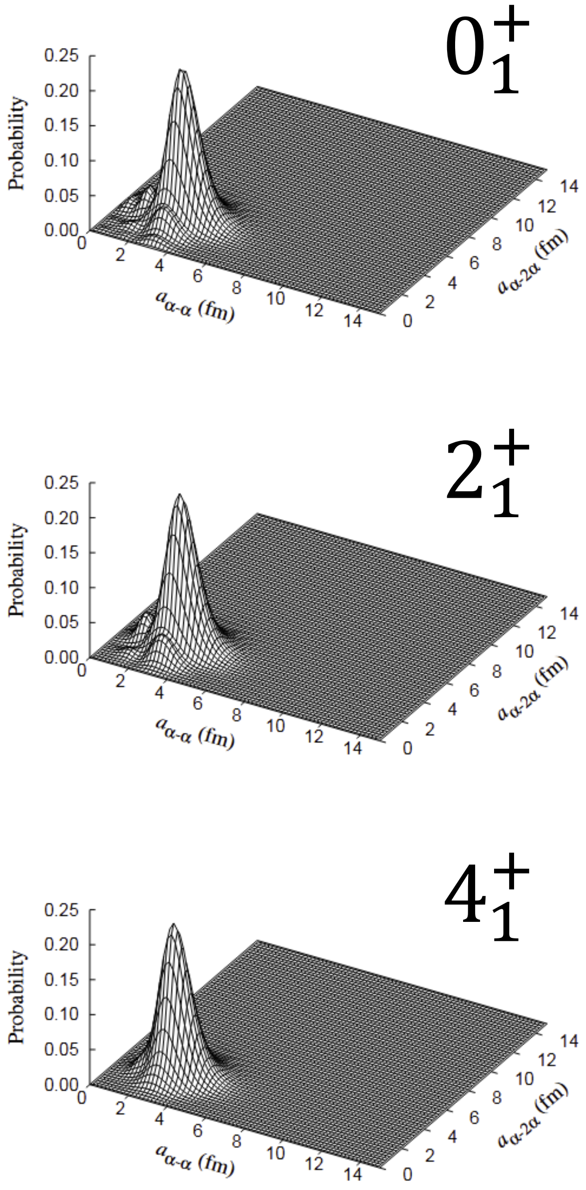



FIG. 1. The two-cluster probabilities for the $0_1^+$, $2_1^+$, and $4_1^+$ states.

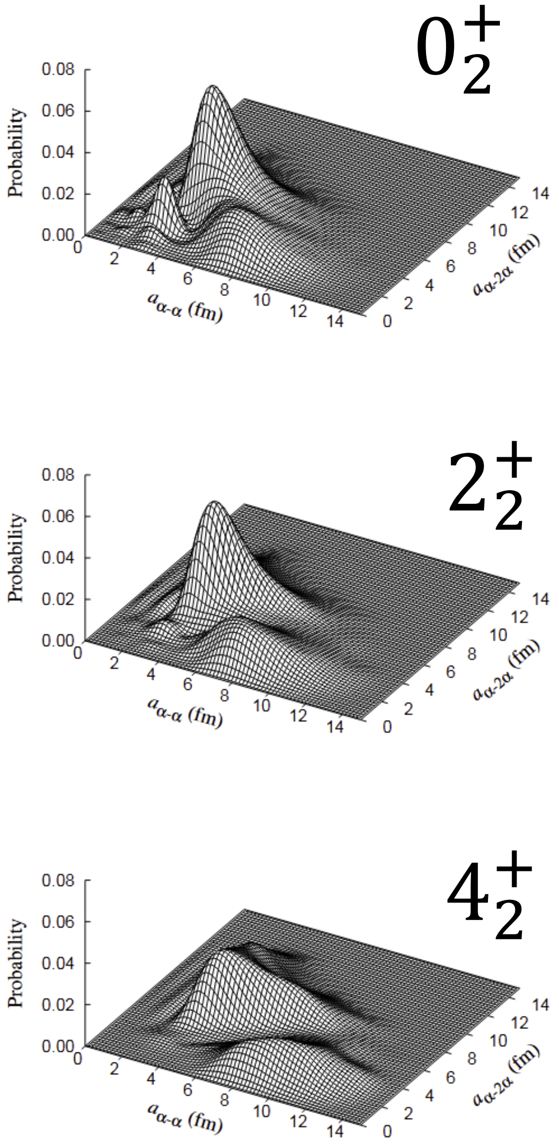



FIG. 2. The two-cluster probabilities for the $0_2^+$, $2_2^+$, and $4_2^+$ states.

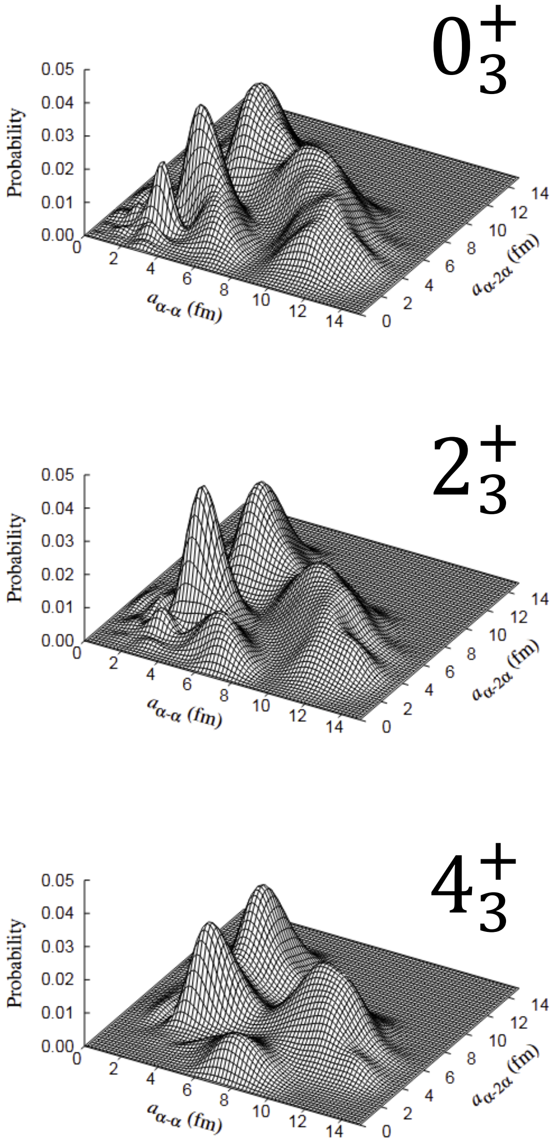
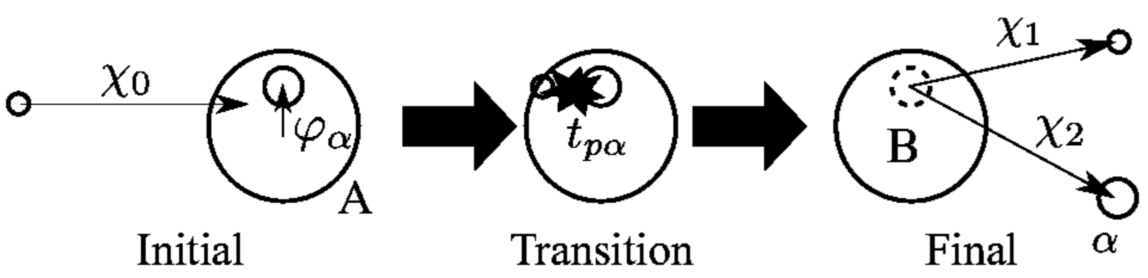



FIG. 3. The two-cluster probabilities for the $0_3^+$, $2_3^+$, and $4_3^+$ states.

Visualization of rotational bands of $^{12}C$ (to be submitted)

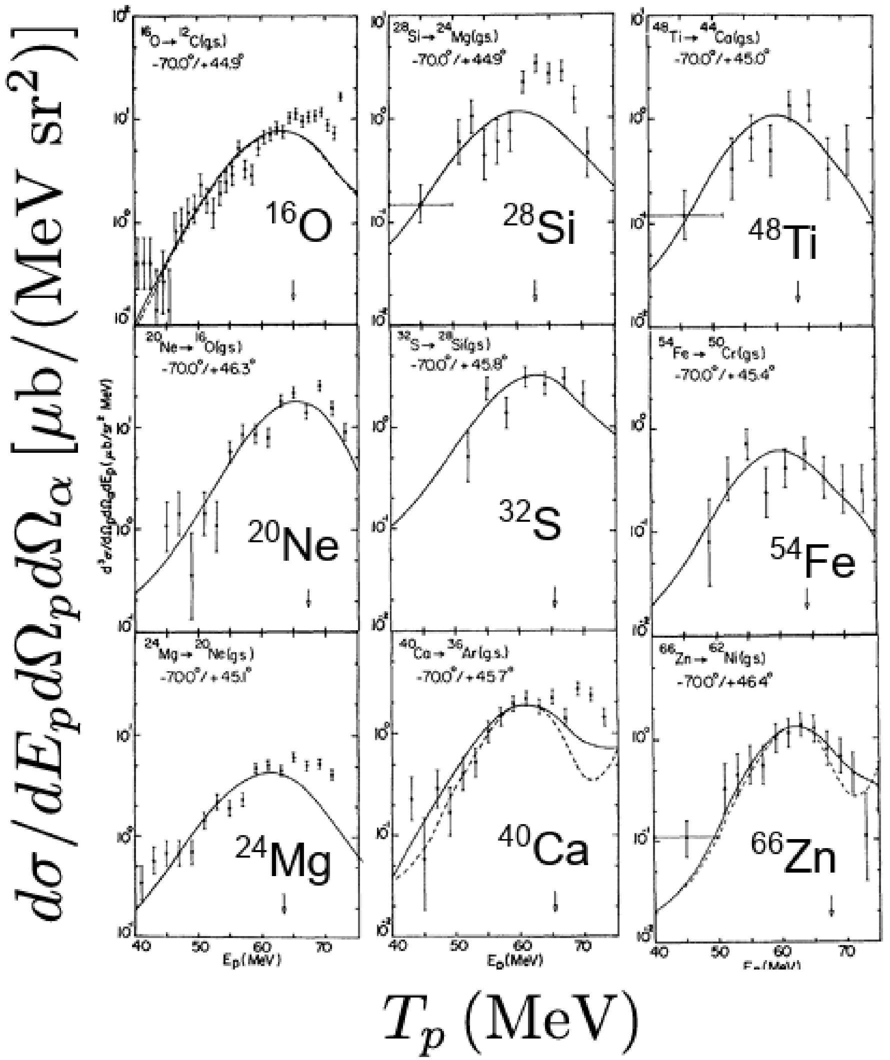
# The clustering structure from experiments




$$T = \langle \chi_1 \chi_2 \Phi_\alpha \Phi_\mathrm{B} \,|\, t_{p\alpha} \,|\, \chi_0 \Phi_\mathrm{A} \rangle = \langle \chi_1 \chi_2 \,|\, t_{p\alpha} \,|\, \chi_0 \varphi_\alpha \rangle$$

$\varphi_\alpha$: Cluster wave function $\langle [\Phi_\alpha \otimes \Phi_\mathrm{B}] \,|\, \Phi_\mathrm{A} \rangle$

$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} \propto |T|^2$$

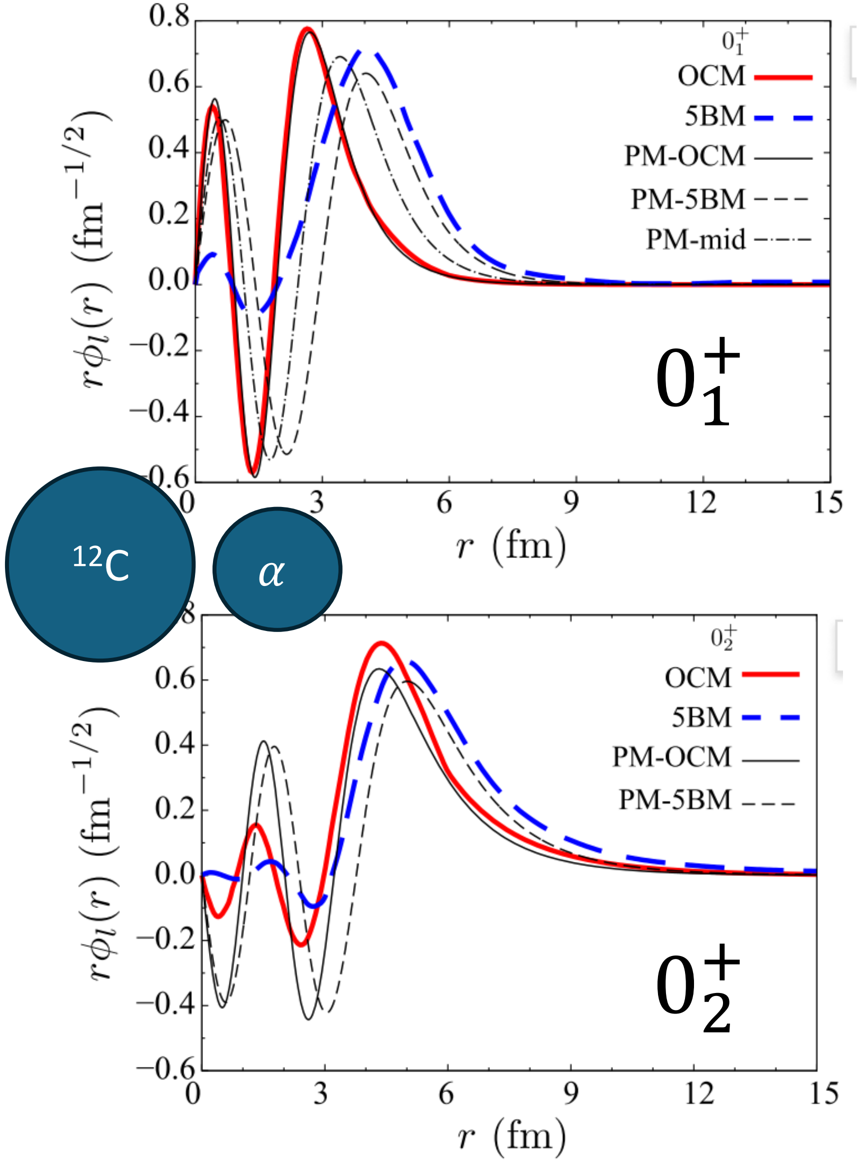


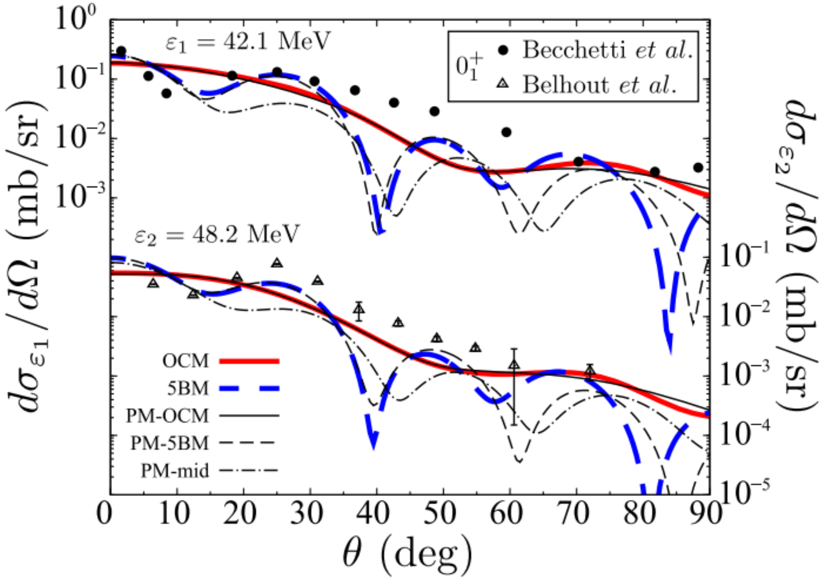

T. A. Carey et al., Phys. Rev. C 29, 1273 (1984).

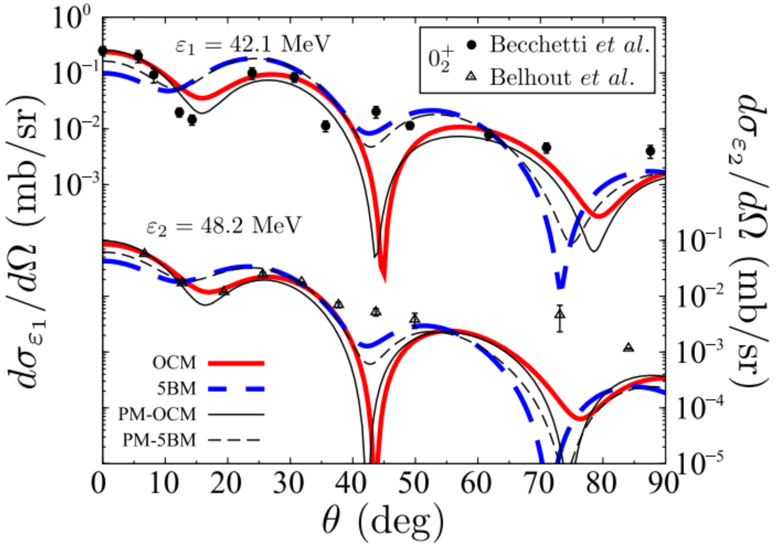
K.Yoshida et al., PRC100(2019): Quantitative description of the $^{20}Ne(p, p\alpha)^{16}$O reaction as a means of probing the surface $\alpha$ amplitude

# The clustering structure of $^{16}O$

$^{12}C(^{6}Li, d)^{16}O$

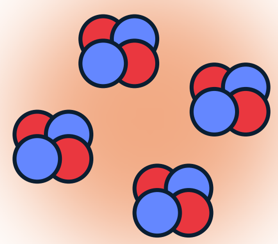



Fukui et al., NPA983 (2019): Investigation of spatial manifestation of α clusters in $^{16}O$ via α-transfer reactions

# The clustering structure of $^{16}$O



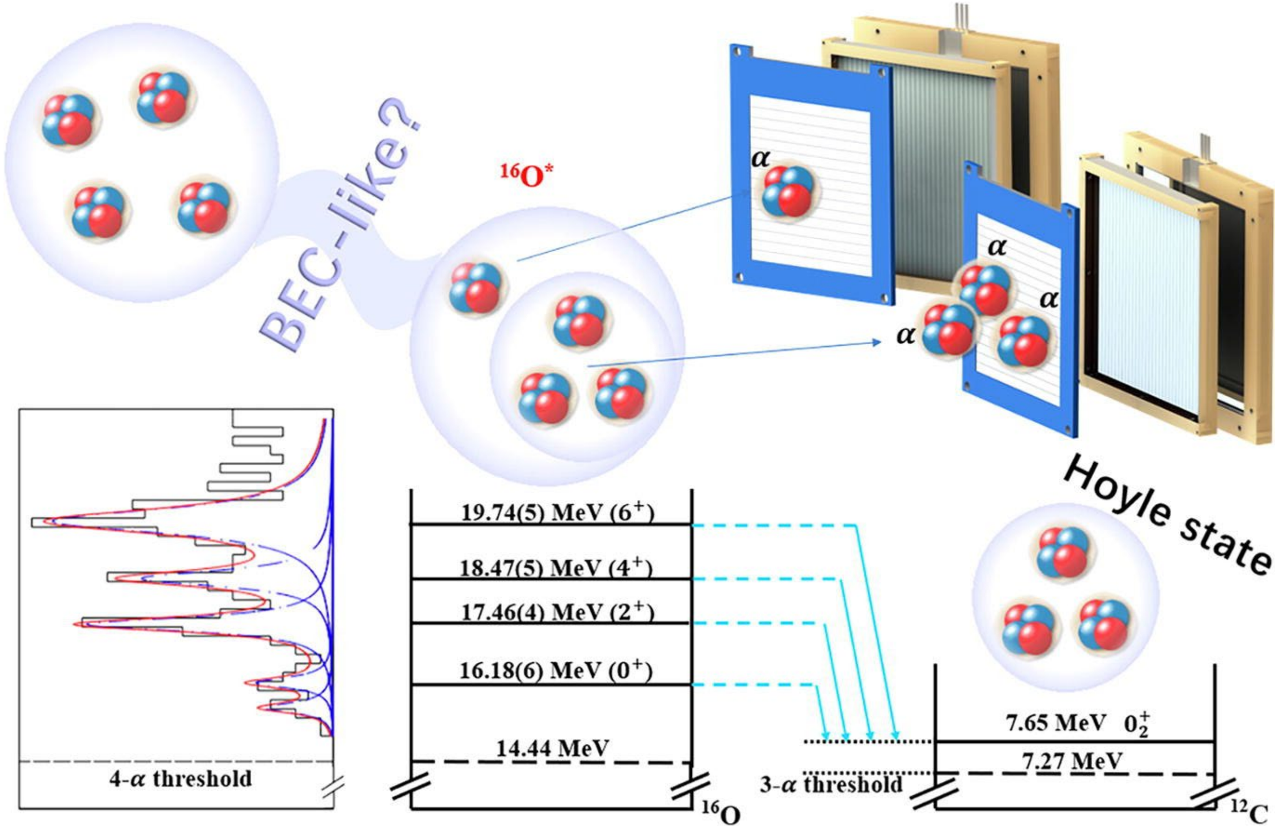
$$\Phi^{\mathrm{B}}(\boldsymbol{R}_1, \boldsymbol{R}_2, \boldsymbol{R}_3, \boldsymbol{R}_4) = \mathcal{A}[\phi_1(\boldsymbol{R}_1)\cdots\phi_4(\boldsymbol{R}_1)\phi_5(\boldsymbol{R}_2)\cdots\phi_{16}(\boldsymbol{R}_4)],$$

$$\phi_k(\boldsymbol{R}_j) = \frac{1}{(\pi b^2)^{3/4}} \exp\left[-\frac{1}{2b^2}(\boldsymbol{r}_k - \boldsymbol{R}_j)^2\right]\chi_k\tau_k$$

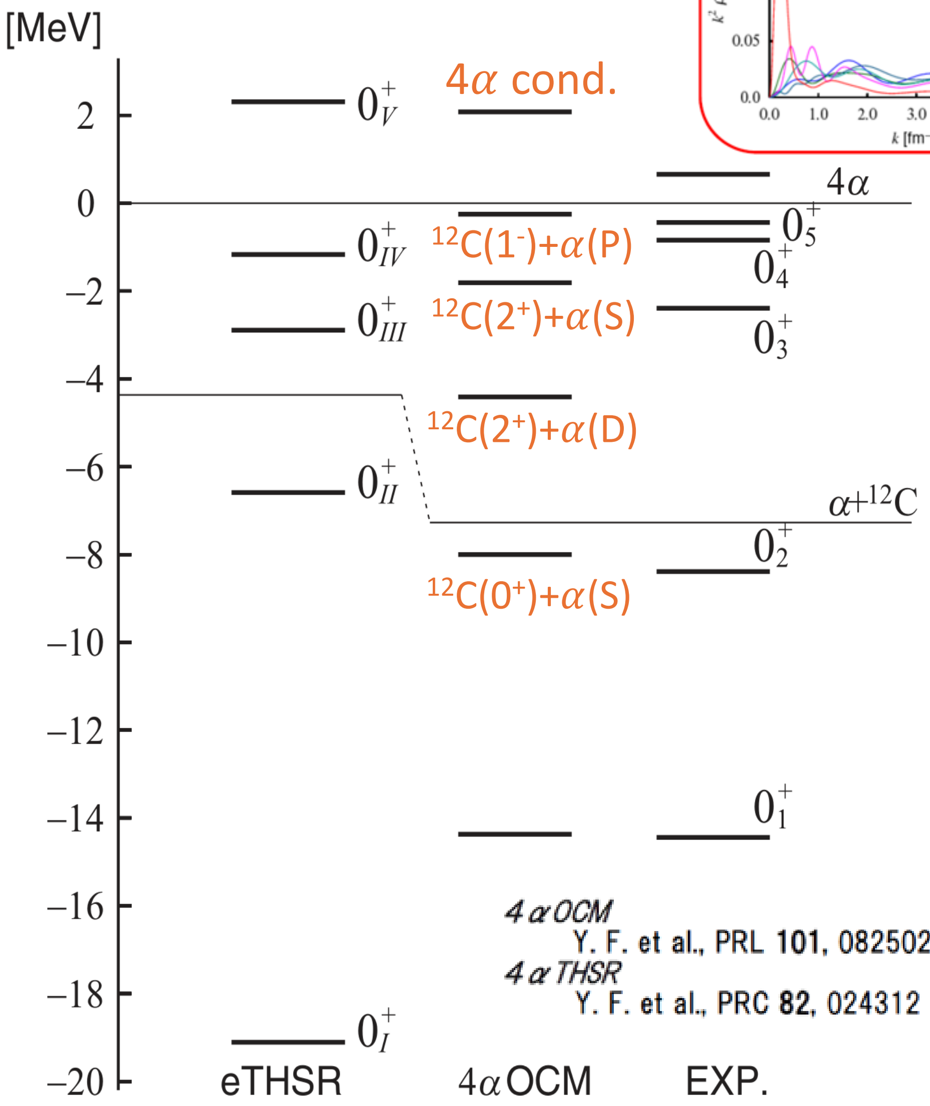
$$\Psi^{J\pi}_{M} = \sum_{i,K} c_{i,K}\hat{P}^{J}_{MK}\hat{P}^{\pi}\Phi^{\mathrm{B}}(\{\boldsymbol{R}\}_i),$$

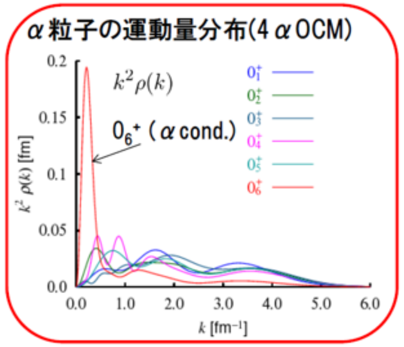
$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 - T_{\mathrm{c.m.}} + \hat{V}_{NN} + \hat{V}_{\mathrm{C}}.$$

$$\hat{V}_{NN} = \frac{1}{2}\sum_{i\neq j} V^{(2)}_{ij} + \frac{1}{6}\sum_{i\neq j, j\neq k, i\neq k} V^{(3)}_{ijk},$$



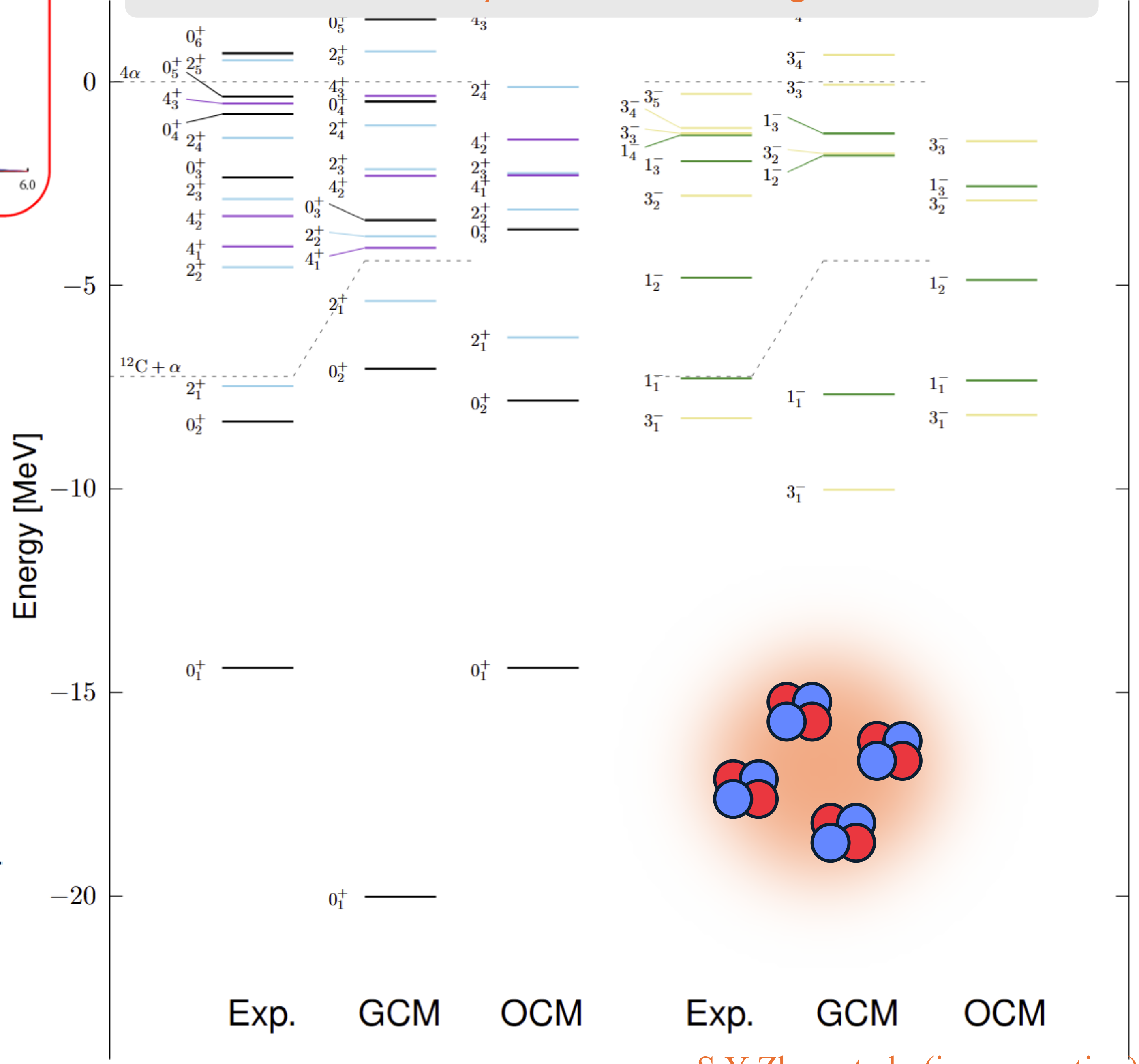

*Science Bulletin, 68*(11), 1119-1126(2023)



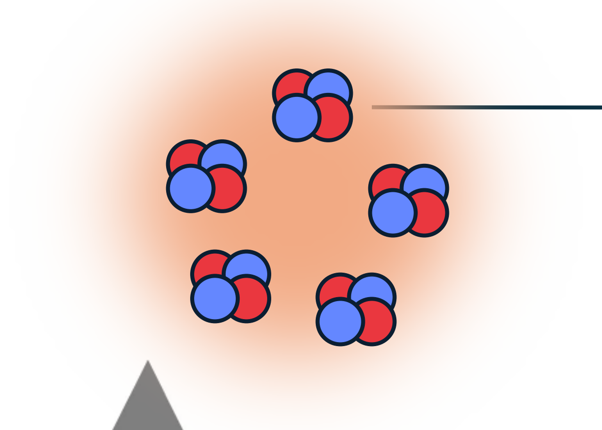

*4αOCM*
Y. F. et al., PRL **101**, 082502 (2008).
*4αTHSR*
Y. F. et al., PRC **82**, 024312 (2010).

still very difficult in this resgion

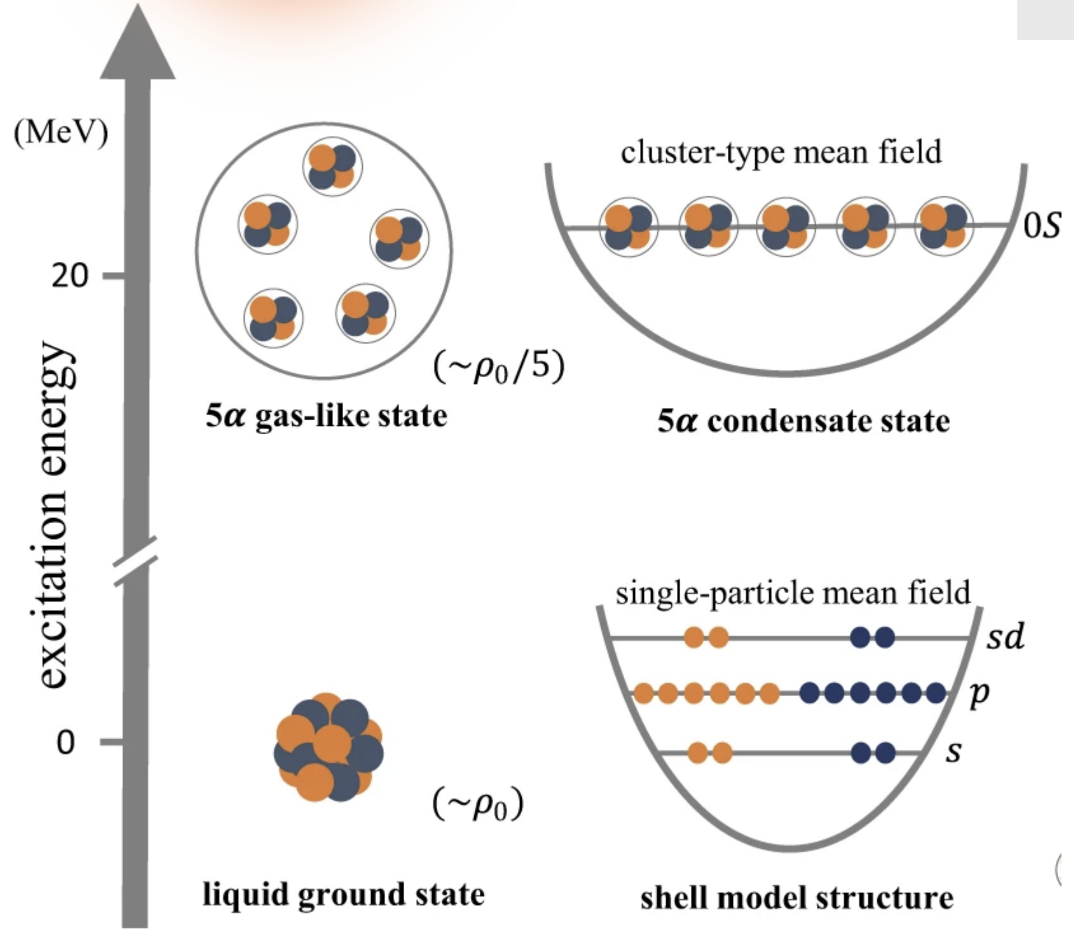


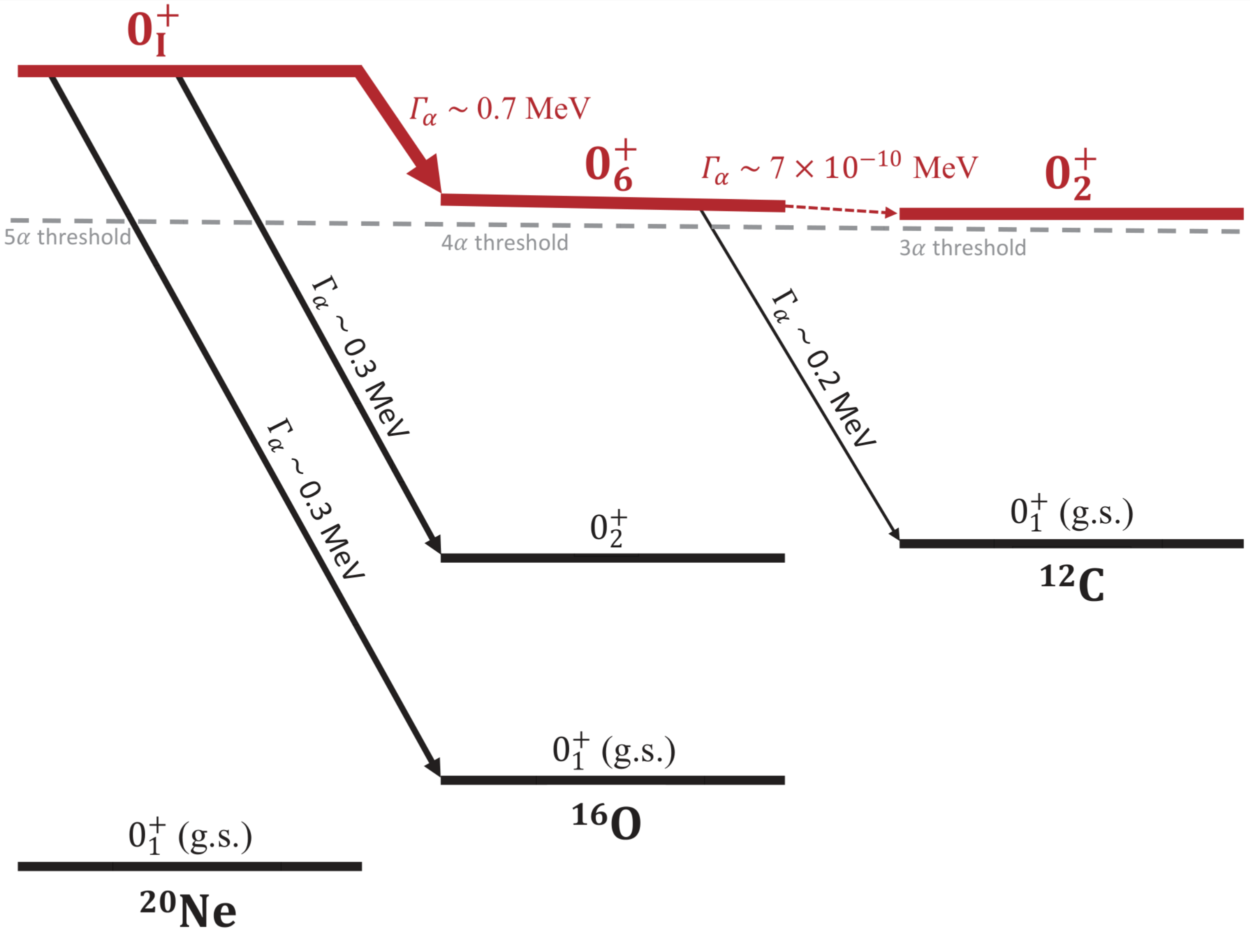

S.Y Zhou et al. (in preparation)

# The decay scheme and connections



Exotic clustering structure ?

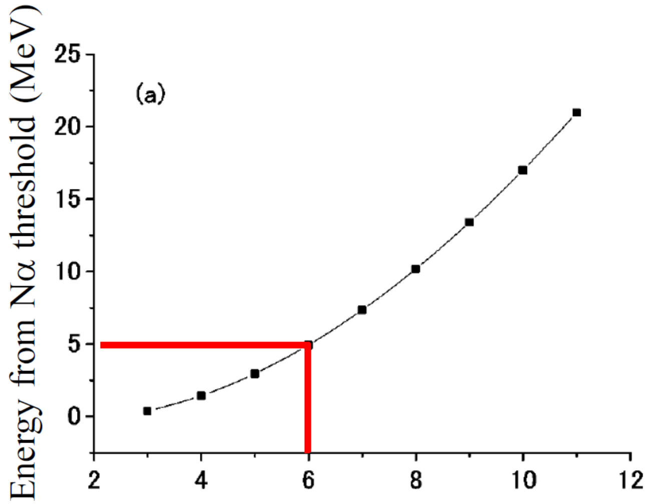



B. Zhou, et al., *Nat. Commun.* **14**, 8206 (2023).

# The 6$\alpha$ clustering structure probed by Inelastic Scattering

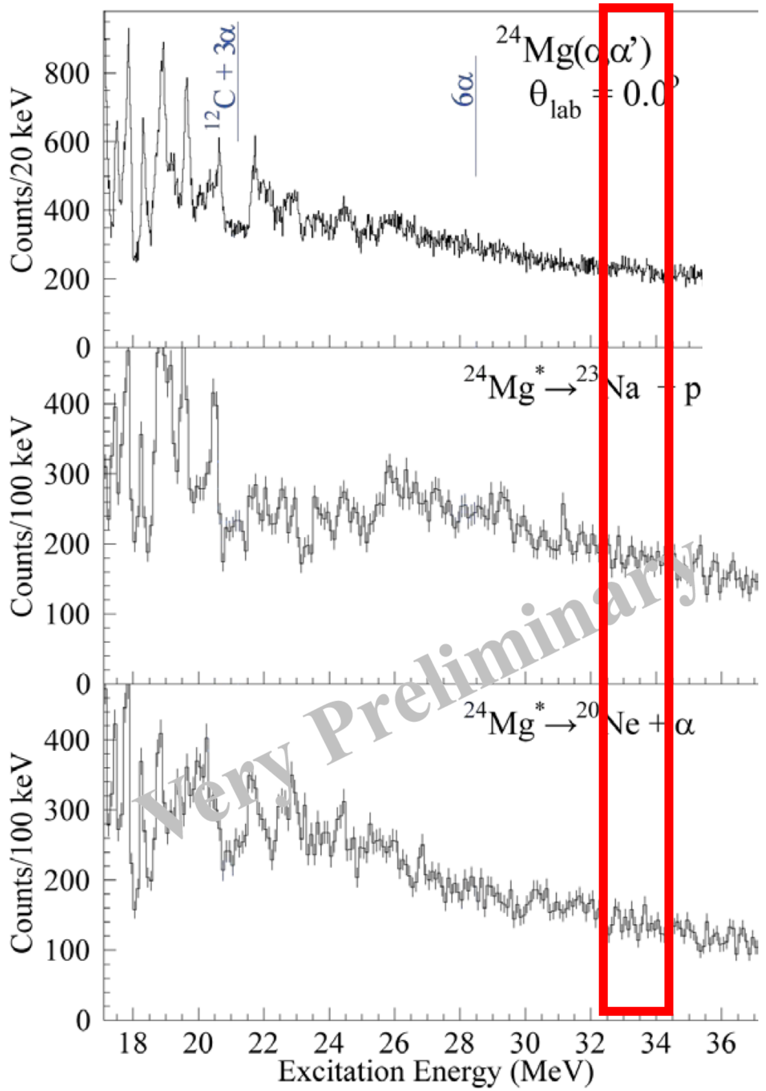
6α condensed state was searched for in the highly excited region.




T. Yamada and P. Schuck, Phys. Rev. C **69**, 024309 (2004).

- 6α condensed state is expected at 5 MeV above the 6α threshold.
  - $E_x \sim 28.5 + 5 = 33.5$ MeV
- No significant structure suggesting the 6α condensed state.
  - Several small structures indistinguishable from the statistical fluctuation. → Need more statistics.

by measuring the 12C+12C scattering




*PLB,848 (2024)*

*A. Tohsaki et al. / Nuclear Physics A738 (2004) 259–263* 261

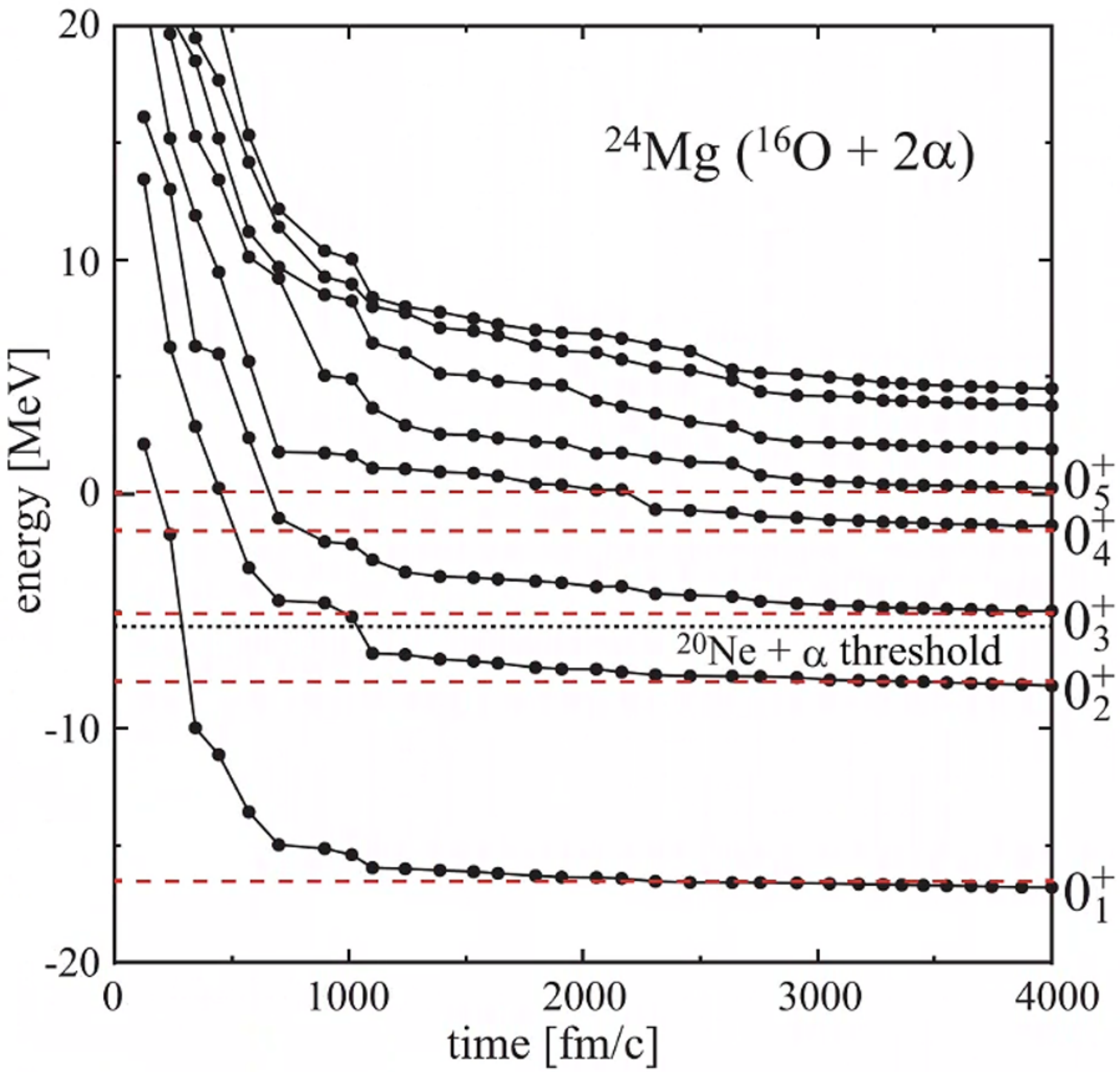
Table 1
The independent number of permutations for each kernel. Here, the case of the norm kernel for $^{24}$Mg is added. The final row shows a full number of permuations without any reduction for the norm kernel.

| | $^{8}$Be(2α) | $^{12}$C(3α) | $^{16}$O(4α) | $^{20}$Ne(5α) | $^{24}$Mg(6α) |
|---|---|---|---|---|---|
| norm | 3 | 9 | 35 | 185 | 1614 |
| kinetic | 7 | 34 | 242 | 2546 | |
| two-body | 9 | 58 | 669 | 10912 | |
| three-body | 40 | 366 | 6773 | 156617 | |
| $(n!)^4$ | 16 | 1296 | $3.32\times10^5$ | $2.07\times10^8$ | $2.79\times10^{11}$ |

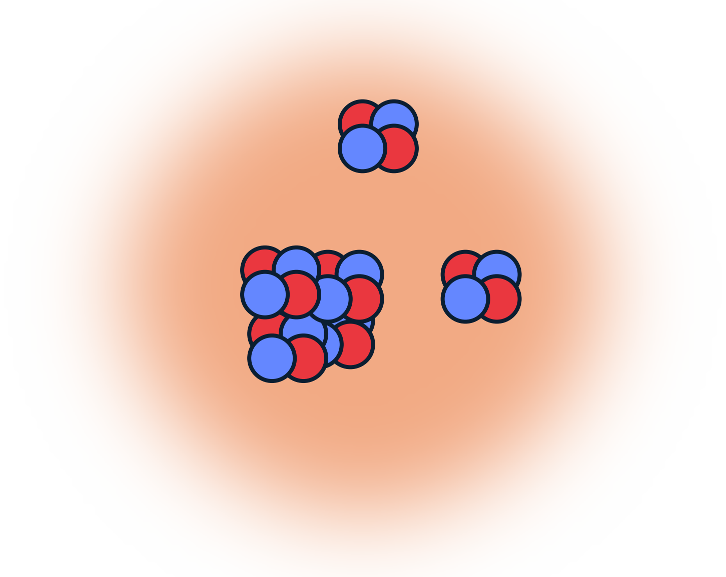
$$\left\langle \Psi_{n\alpha}^{\mathrm{THSR}}(\beta)\,|\mathcal{O}|\,\Psi_{n\alpha}^{\mathrm{THSR}}(\beta')\right\rangle = \sum_{p=0}^{m_p^{(1)}-1} W_p^{(1)} I_p^{(1)}$$

Remains challenging in theoretical calculations

# Candidates of a 2$\alpha$ condensate surrounding the $^{16}$O nucleus

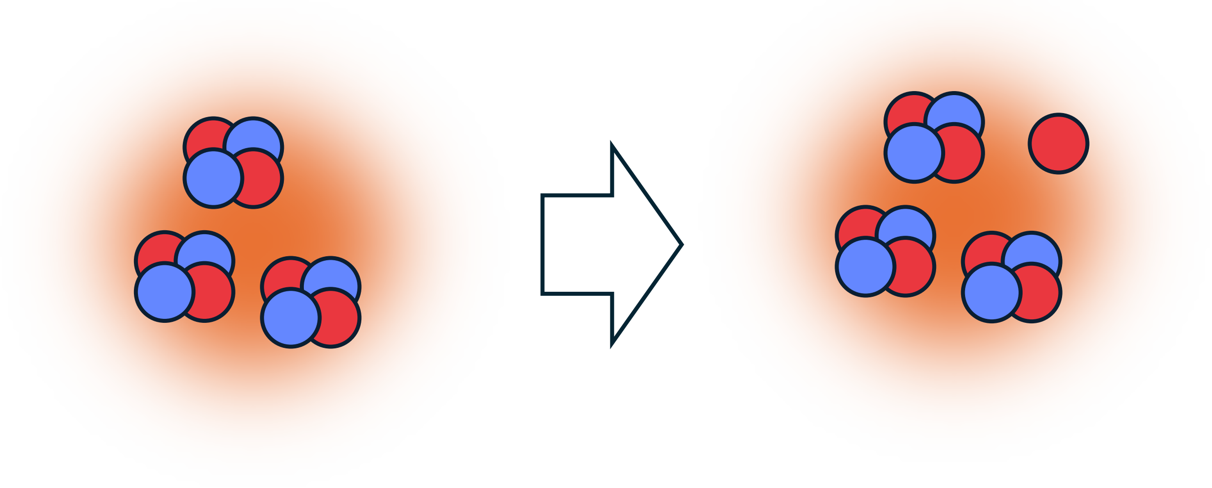



| State | $E$ | $\Gamma$ | $B$(IS0) | $\sqrt{\langle r^2 \rangle}$ | $\sqrt{\langle r^2 \rangle_{\mathrm{val}}}$ |
|---|---|---|---|---|---|
| $0_1^+$ | −16.78 | | | 2.67 | 3.47 |
| $0_2^+$ | −8.16 | | 28.4 | 2.70 | 3.53 |
| $0_3^+$ | −4.95 | ⩽0.01 | 145.0 | 2.97 | 4.13 |
| $0_4^+$ | −1.26 | 0.17 | 160.7 | 2.94 | 4.08 |
| $0_5^+$ | 0.37 | | 4.28 | 3.03 | 4.28 |



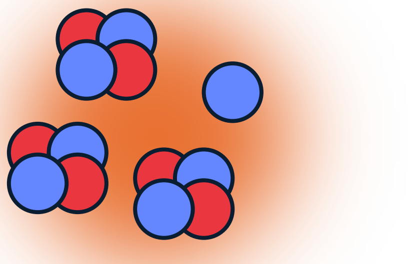
2α condensation around the core nucleus 16O

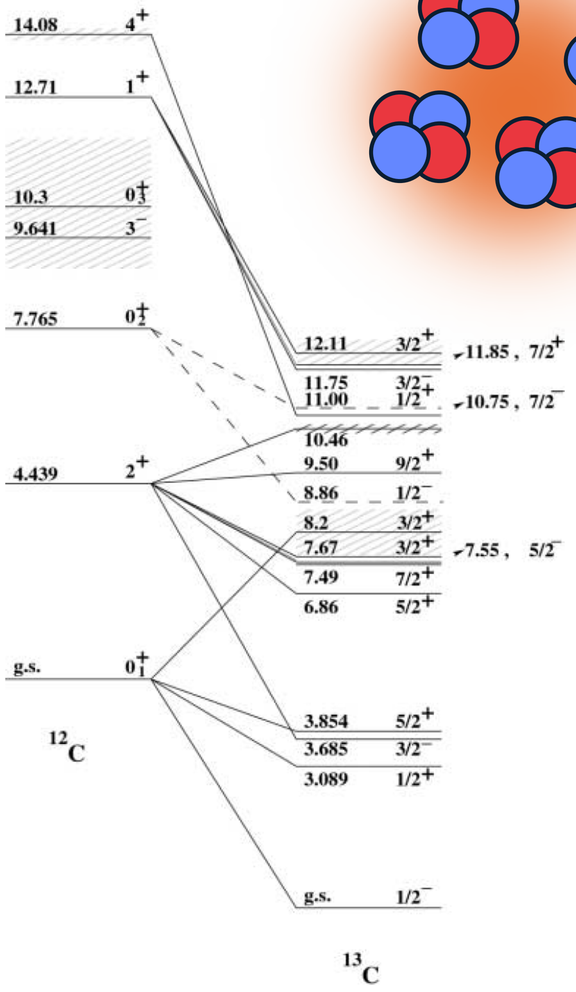
Htet, et al., PRC112, 014322 (2025)



Clustering structure of $3\alpha+p$ in $^{13}N$

# Search for the Hoyle-analogy state in $^{13}C$

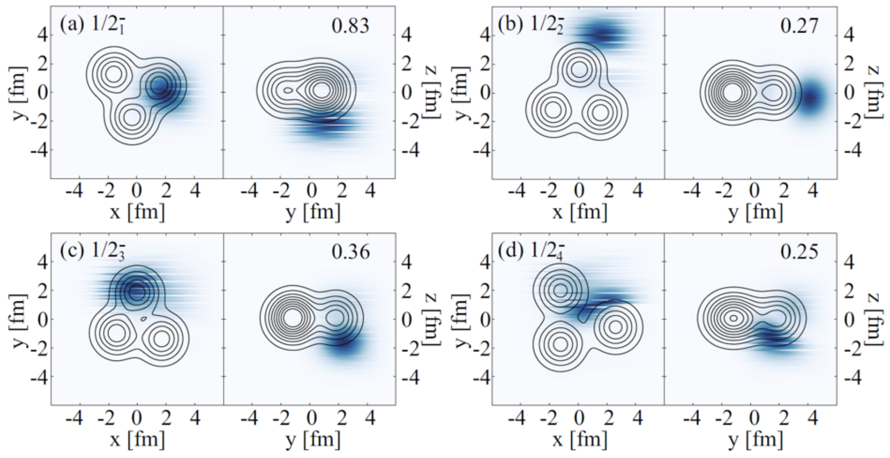


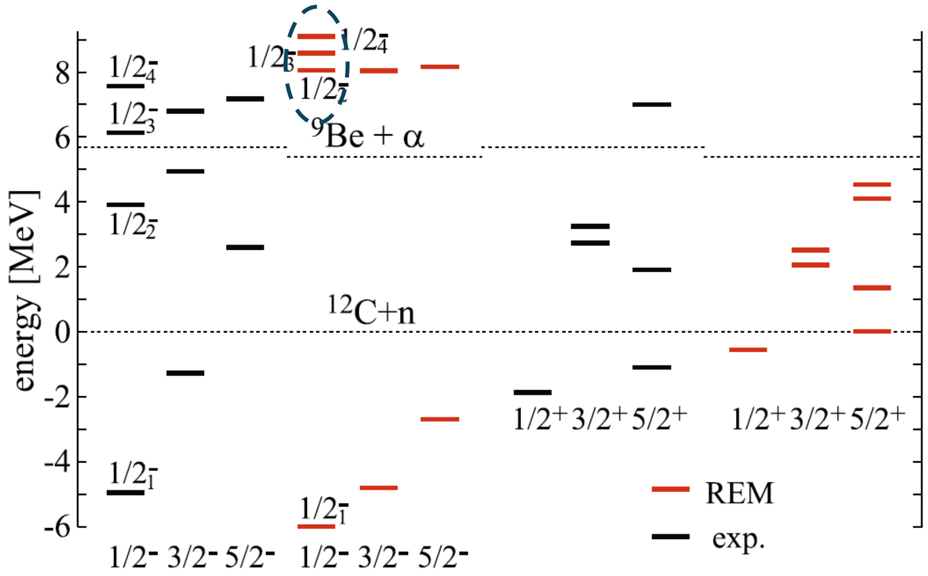



$^{12}C$(states) +
single particle states

$^{13}C$ (states)

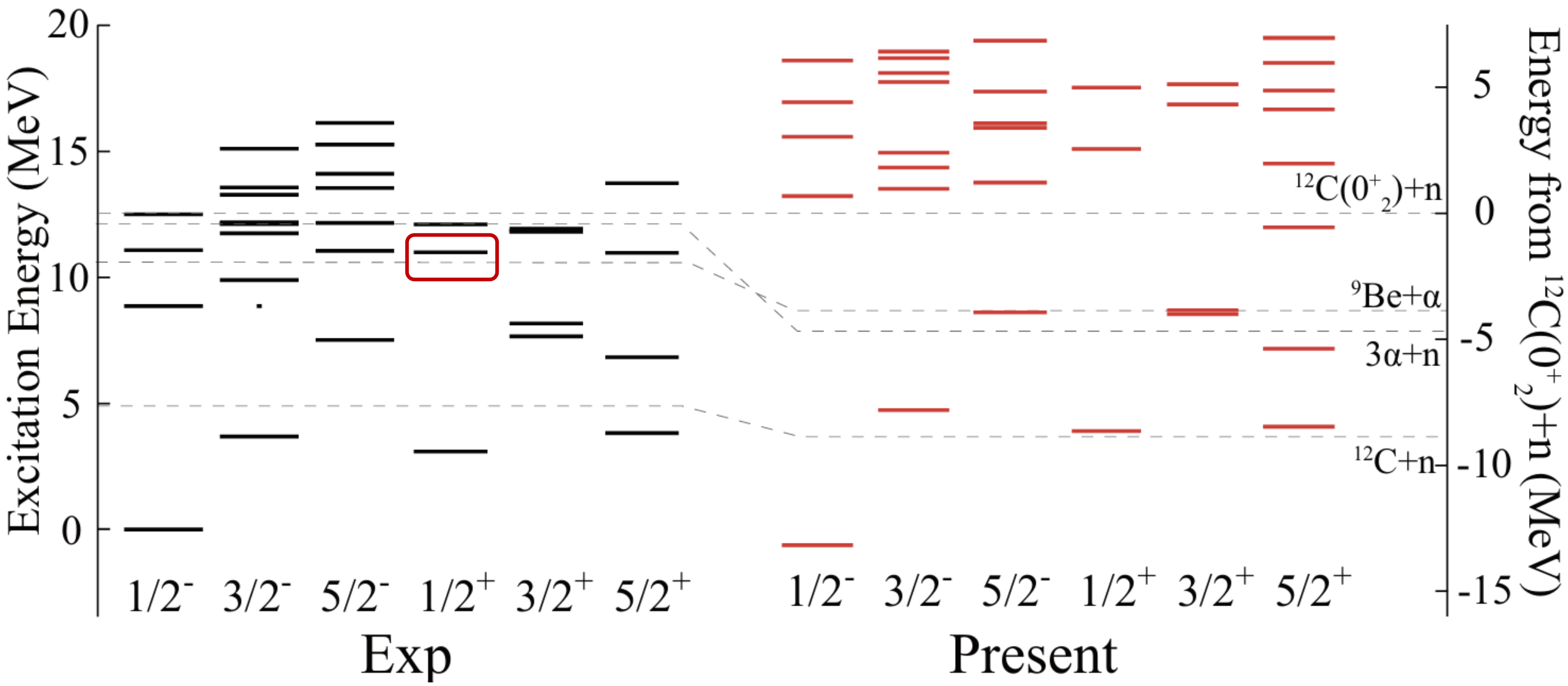
M. Milin and W. von Oertzen, Eur Phys J A **14**, 295 (2002).

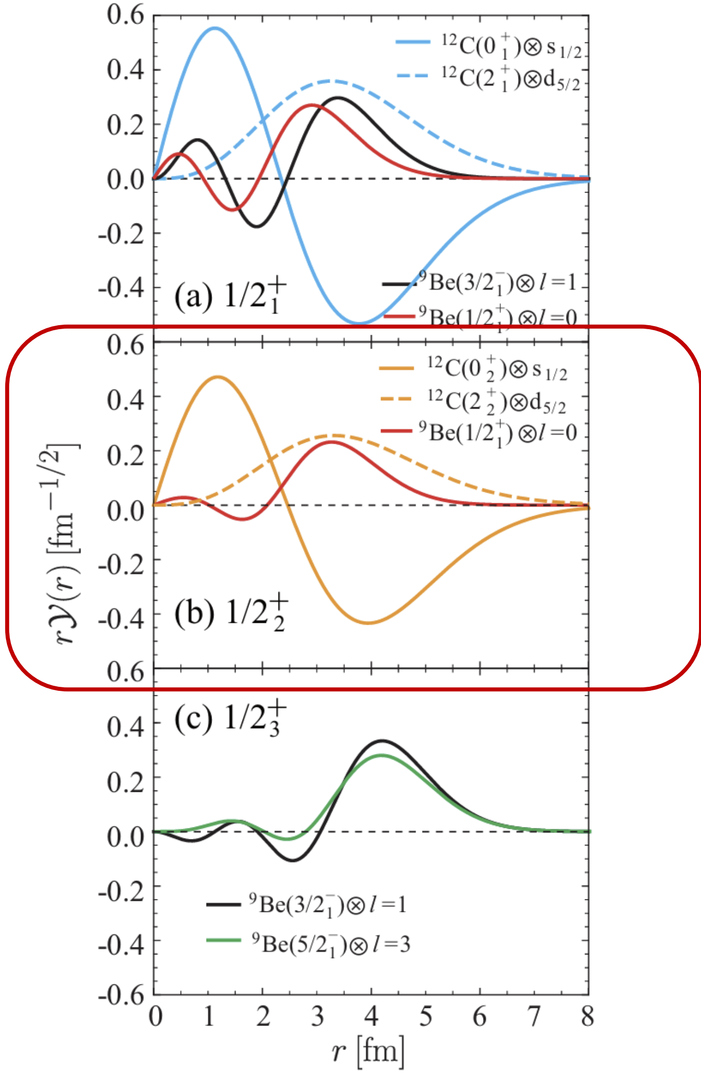



S. Shin, et al., Phys. Rev. C **103**, 054313 (2021).

# Search for the Hoyle-analogy state in $^{13}$C

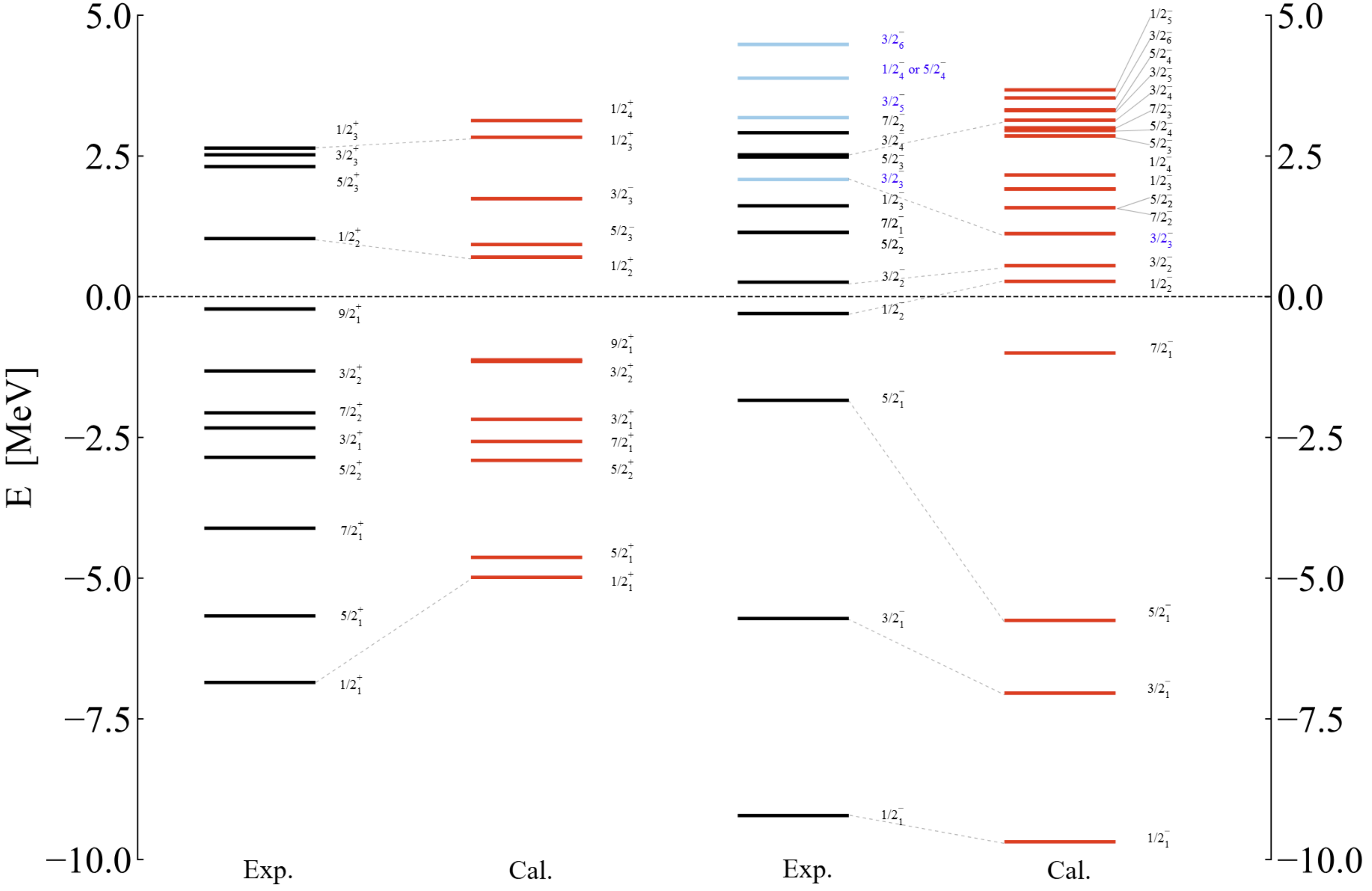


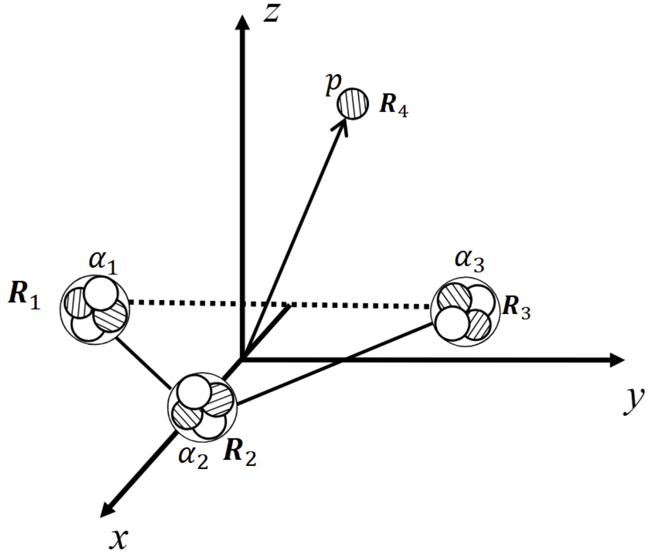




It is found that the $1/2_2^+$ state is dominantly composed of the Hoyle state configuration and can be regarded as a Hoyle analogue state.

Chiba et al.,Phys. Rev. C 101, 024317 (2020)

# Hoyle-analog state in $^{13}N$




| Transition | Present | Experiment |
|---|---|---|
| $B(E1, 3/2_1^- \to 1/2_1^+)$ | 0.016 | 0.036 |
| $B(E1, 1/2_1^+ \to 1/2_1^-)$ | 0.0007 | 0.036 ± 0.004 |
| $B(E2, 3/2_1^- \to 1/2_1^-)$ | 4.87 | |
| $B(E2, 5/2_1^- \to 1/2_1^-)$ | 3.71 | |
| $B(E2, 1/2_1^- \to 3/2_1^+)$ | 21.58 | |

Y.Y Cao et al. Phys. Rev. C 112, 034313 (2025)

# Hoyle-analog state in $^{13}$N

PHYSICAL REVIEW C **109**, 054308 (2024)

## Cluster structure of $3\alpha + p$ states in $^{13}$N

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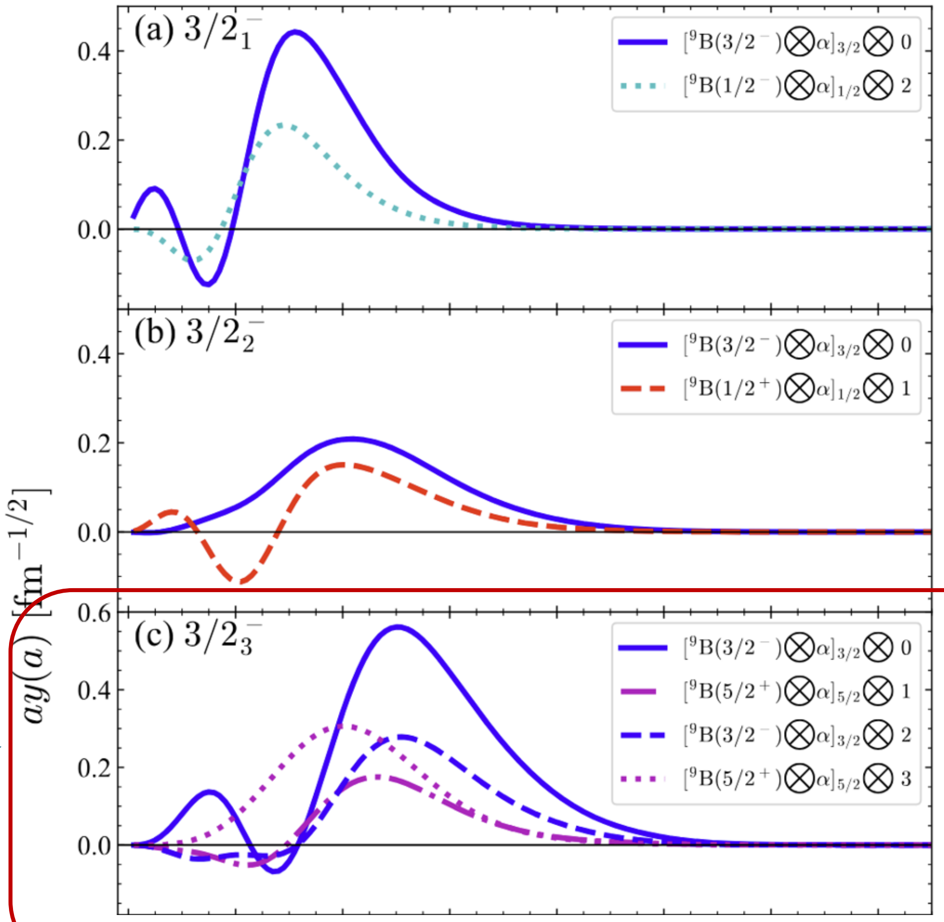
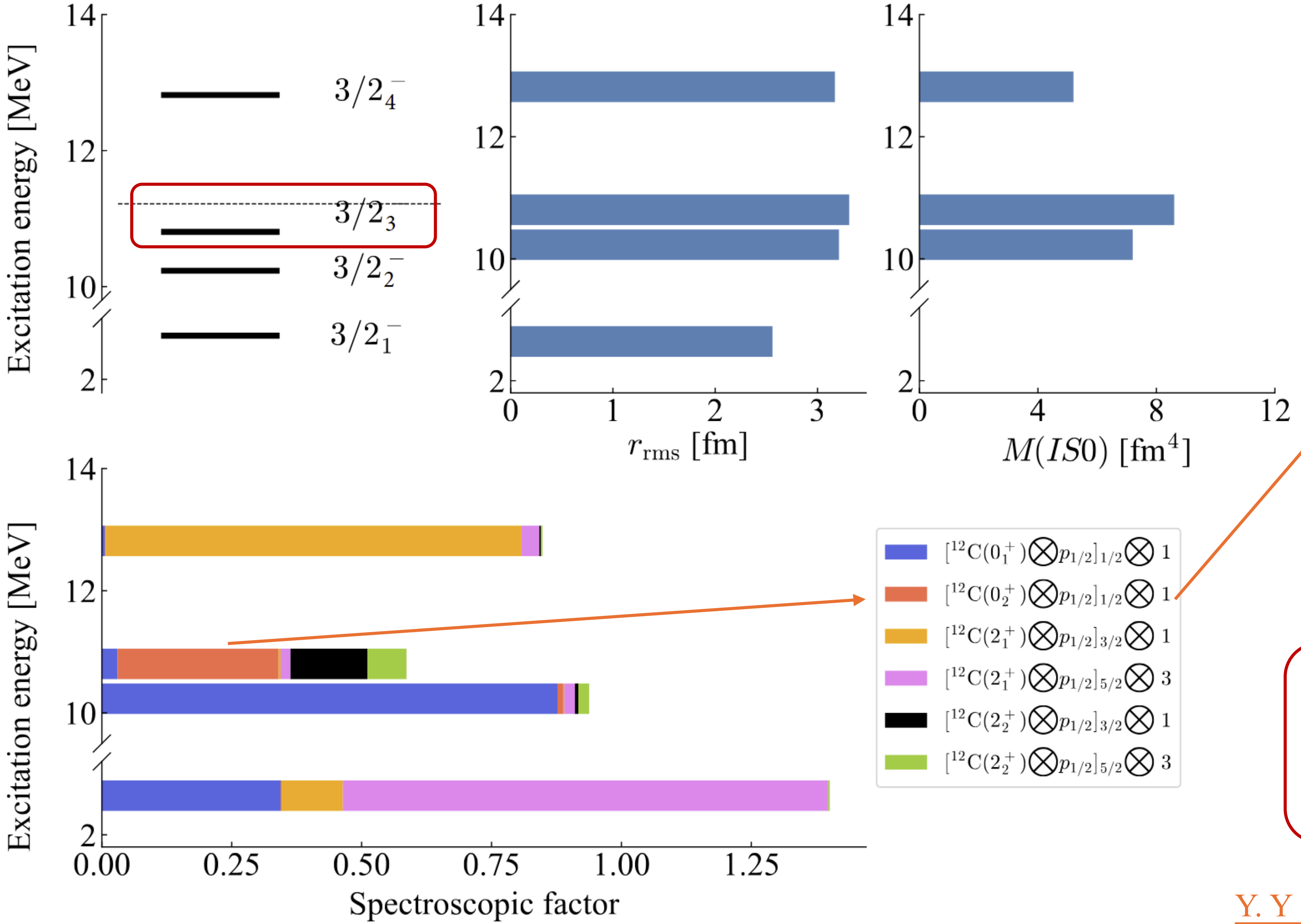
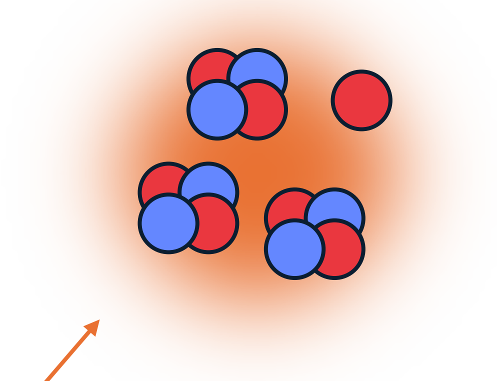
**Background:** Cluster states in $^{13}$N are extremely difficult to measure due to the unavailability of $^{9}$B $+\alpha$ elastic-scattering data.
**Purpose:** Using $\beta$-delayed charged-particle spectroscopy of $^{13}$O, clustered states in $^{13}$N can be populated and measured in the $3\alpha + p$ decay channel.
**Methods:** One-at-a-time implantation and decay of $^{13}$O was performed with the Texas Active Target Time Projection Chamber. 149 $\beta 3\alpha p$ decay events were observed and the excitation function in $^{13}$N reconstructed.
**Results:** Four previously unknown $\alpha$-decaying excited states were observed in $^{13}$N at an excitation energy of 11.3, 12.4, 13.1, and 13.7 MeV decaying via the $3\alpha + p$ channel.
**Conclusions:** These states are seen to have a $[^{9}\mathrm{B(g.s)} \otimes \alpha / p + {}^{12}\mathrm{C}(0_2^+)]$, $[^{9}\mathrm{B}(\frac{1}{2}^+) \otimes \alpha]$, $[^{9}\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$, and $[^{9}\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$ structure, respectively. A previously seen state at 11.8 MeV was also determined to have a $[p + {}^{12}\mathrm{C(g.s.)} / p + {}^{12}\mathrm{C}(0_2^+)]$ structure. The overall magnitude of the clustering is not able to be extracted, however, due to the lack of a total width measurement. Clustered states in $^{13}$N (with unknown magnitude) seem to persist from the addition of a proton to the highly $\alpha$-clustered $^{12}$C. Evidence of the $\frac{1}{2}^+$ state in $^{9}$B was also seen to be populated by decays from $^{13}$N$^\star$.




This obtained state corresponds to the state observed at 11.3 MeV

# Hoyle-analog state in $^{13}$N



$\langle \text{Hoyle} + p | {}^{13}\text{N state}\rangle$

$$y^{J\pi}_{j_1\pi_1 j_2\pi_2 j_{12} l}(a) = \sqrt{\frac{A!}{(1+\delta_{C_1C_2})\, C_1! C_2!}} \times \left\langle \frac{\delta(r-a)}{r^2}\left[Y_l(\hat{r})\left[\Phi^{j_1\pi_1}_{C_1}\Phi^{j_2\pi_2}_{C_2}\right]_{j_{12}}\right]_{JM} \middle| \Psi^{J\pi}_M \right\rangle$$

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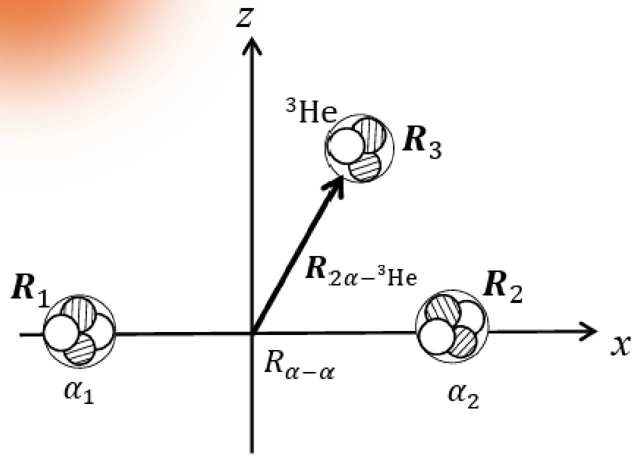
# Gas-like states in $^{11}C$

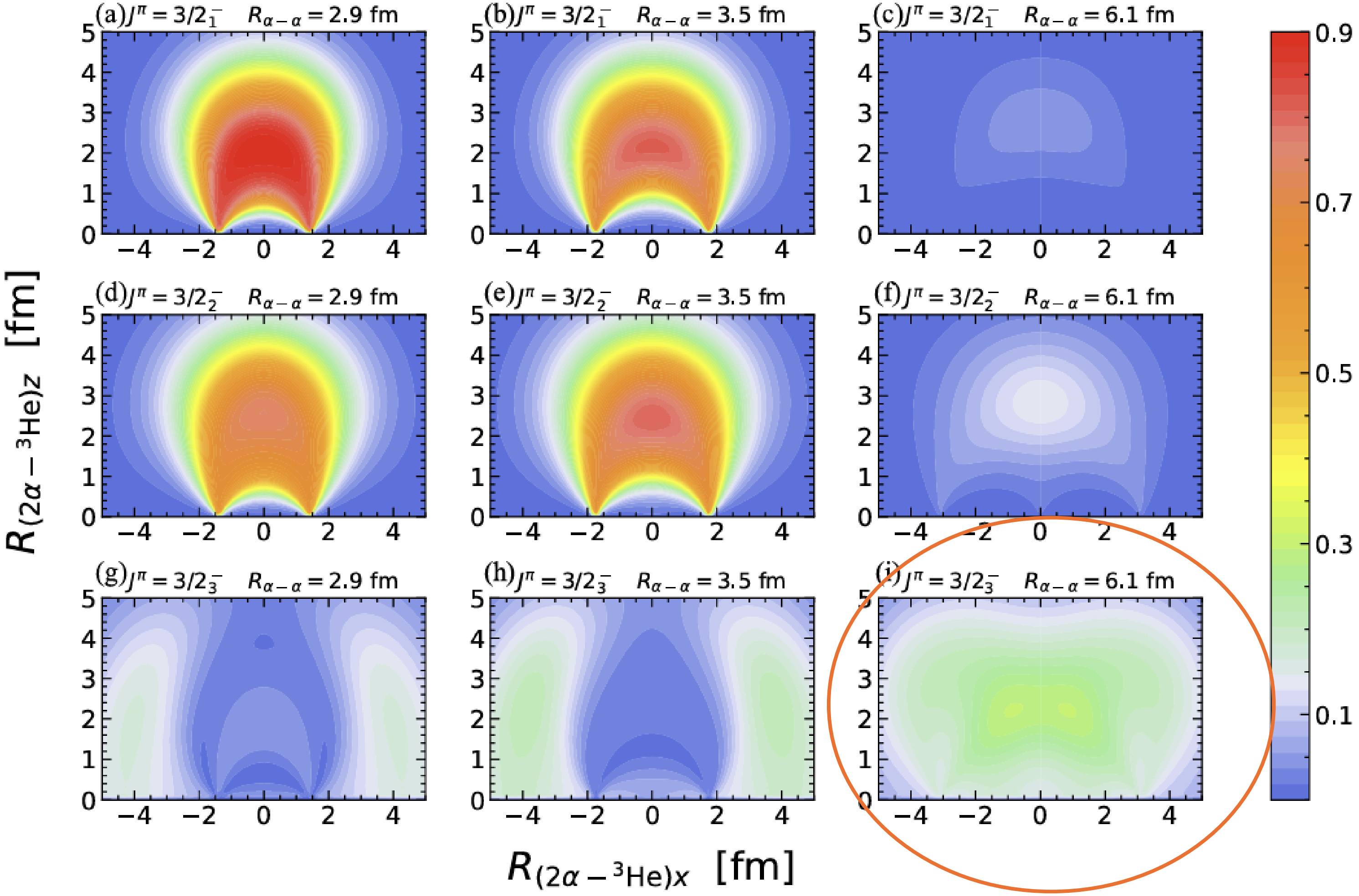



$$\Phi(\boldsymbol{R}_1, \boldsymbol{R}_2, \boldsymbol{R}_3) = \mathcal{A}\{\Phi_\alpha(\boldsymbol{R}_1)\Phi_\alpha(\boldsymbol{R}_2)\Phi_{^3\mathrm{He}}(\boldsymbol{R}_3)\},$$

$$\Phi_\alpha(\boldsymbol{R}) = \mathcal{A}\{\prod_{i=1}^{4}\phi(\boldsymbol{R}, \boldsymbol{r}_i)\chi_i\tau_i\},$$

$$\Phi_{^3\mathrm{He}}(\boldsymbol{R}) = \mathcal{A}\{\prod_{i=1}^{3}\phi(\boldsymbol{R}, \boldsymbol{r}_i)\chi_i\tau_i\},$$

$$\phi(\boldsymbol{R}, \boldsymbol{r}_i) = (\frac{1}{\pi b^2})^{3/4} e^{-\frac{(\boldsymbol{r}_i-\boldsymbol{R})^2}{2b^2}},$$

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PHYSICAL REVIEW LETTERS **131**, 242501 (2023)

Editors' Suggestion | Featured in Physics

# Observation of the Exotic $0_2^+$ Cluster State in $^8$He

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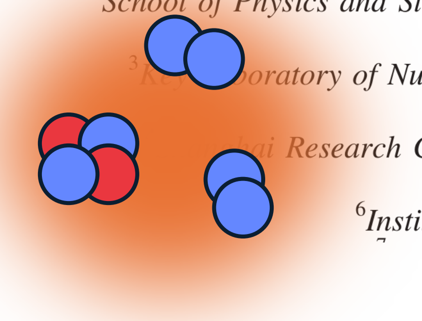
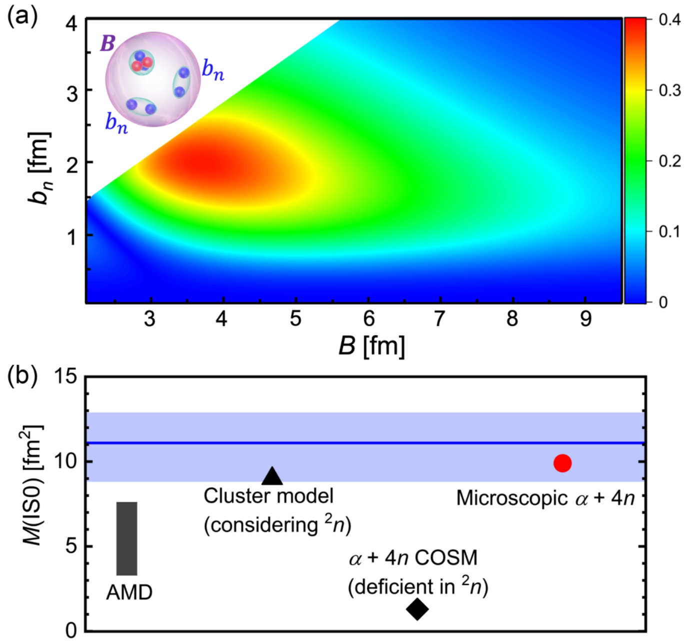


$$\Phi(\boldsymbol{B}, b_n) \propto \mathcal{A}\left\{\exp\left[-\frac{4\boldsymbol{\xi}_1^2}{3B^2} - \frac{3\boldsymbol{\xi}_2^2}{2B^2}\right] \times \phi_\alpha(b_\alpha)\phi_{^2_1n}(b_n)\phi_{^2_2n}(b_n)\right\},$$

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Letter

## New trial wave function for the nuclear cluster structure of nuclei

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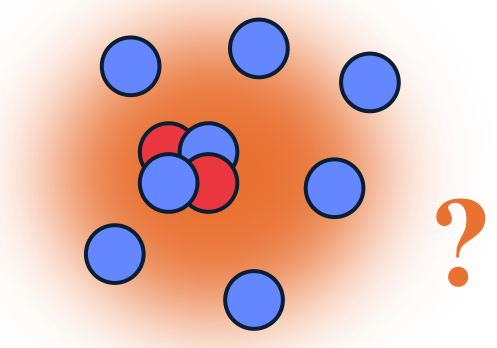
A new trial wave function is proposed for nuclear cluster physics, in which an exact solution to the long-standing center-of-mass problem is given. In the new approach, the widths of the

$$\Psi(\mathbf{r}) = \Phi_{\mathrm{g}}(\mathbf{r}_g)\Phi_{\mathrm{int}}(\mathbf{r}_i - \mathbf{r}_j)$$

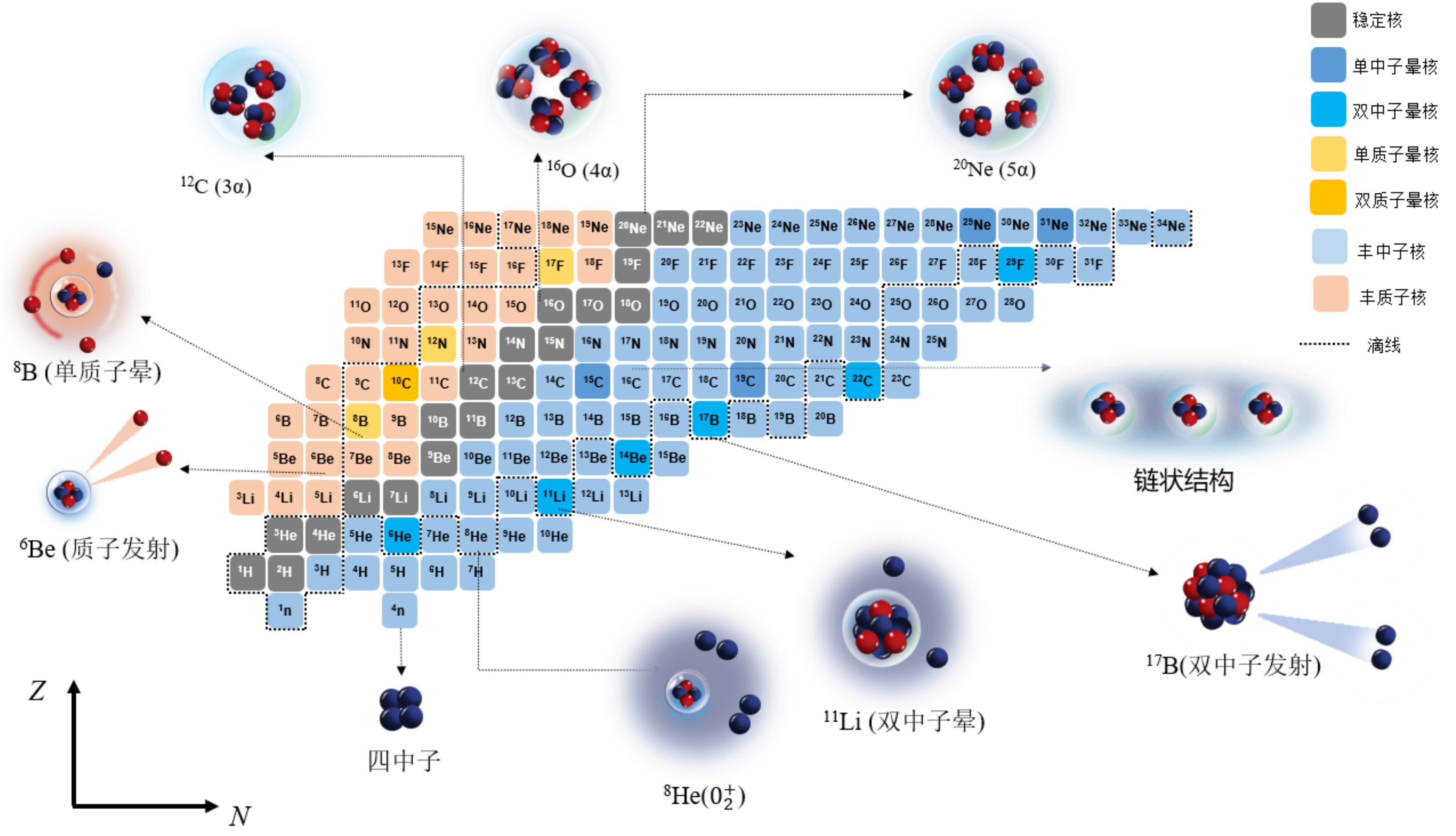
$$\begin{aligned}\Psi_{\rm new} &= \widehat{L}_{n-1}(\beta)\widehat{G}_n(\beta_0)\widehat{D}(\boldsymbol{Z})\Phi_0(\boldsymbol{r})\\ &= \int d^3\widetilde{T}_1\cdots d^3\widetilde{T}_{n-1}\exp[-\sum_{i=1}^{n-1}\frac{\widetilde{T}_i^2}{\beta_i^2}]\int d^3R_1\cdots d^3R_n\exp[-\sum_{i=1}^{n}(\frac{A_i}{\beta_0^2-2b_i^2})(\boldsymbol{R}_i-\boldsymbol{Z}_i-\boldsymbol{T}_i)^2]\Phi_0(\boldsymbol{r}-\boldsymbol{R})\\ &= n_0\exp[-\frac{A}{\beta_0^2}X_g^2]\,\mathcal{A}\{\prod_{i=1}^{n-1}\exp[-\frac{1}{2B_i^2}(\boldsymbol{\xi}_i-\boldsymbol{S}_i)^2]\prod_{i=1}^{n}\phi_i^{\rm int}(b_i)\}.\end{aligned}$$

a tool for studying the cluster correlations



# Summary and Prospect

- High excited states, around the cluster threshold, could show novel clustering structures.
- For the ground states of light nuclei, how to identify the clustering components ?
- New methods should be developed for treating resonance states and new effective interactions are required.




# Thanks for your attentions.