

Evgeny Epelbaum, Ruhr University Bochum

Nuclear physics across energy scales, C3NT, Wuhan, China
September 18-21, 2025

Can nuclear interactions be determined using femtoscopy?

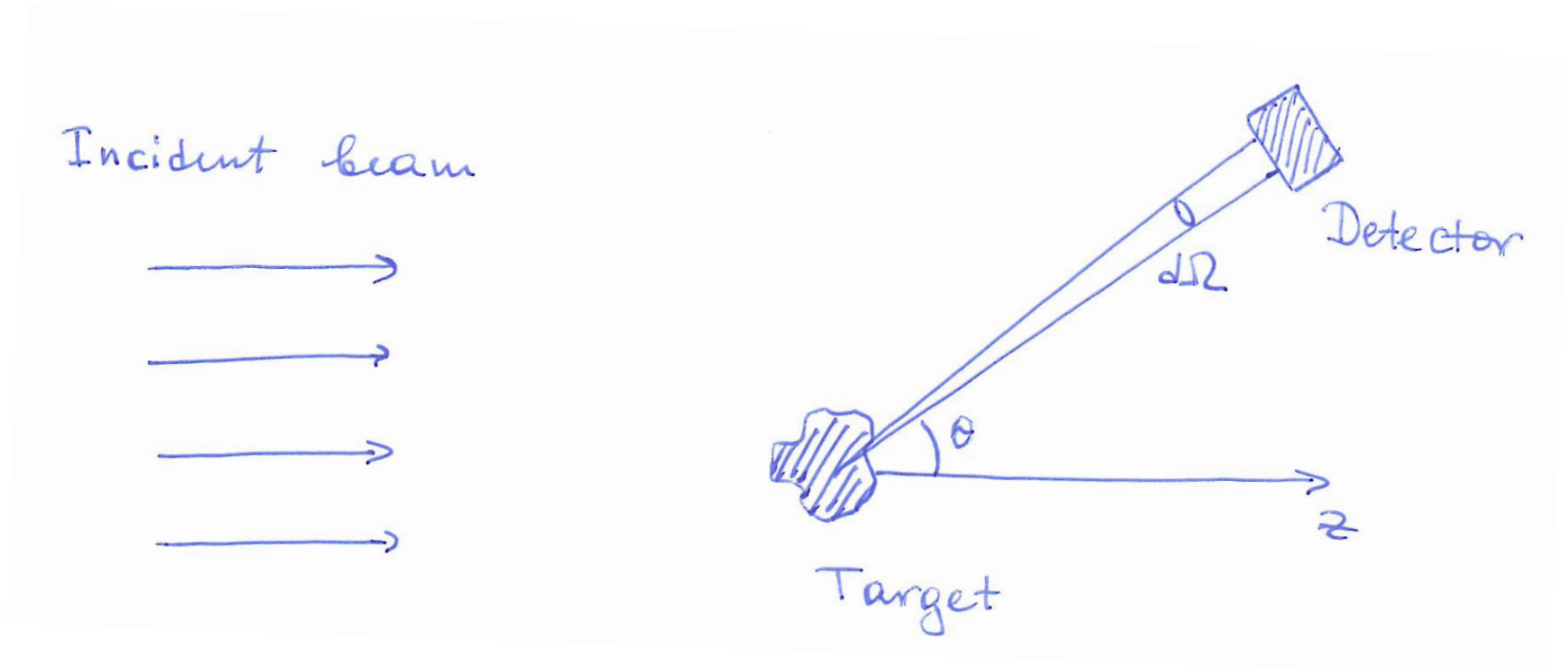
based on EE, S. Heihoff, U.-G. Meißner, A. Tschewon, arXiv:2504.08631

CENTRAL CHINA
CENTER FOR
NUCLEAR THEORY
华中核理论中心

„Admitting to a problem is the first step toward finding a solution.“

— John Perkins

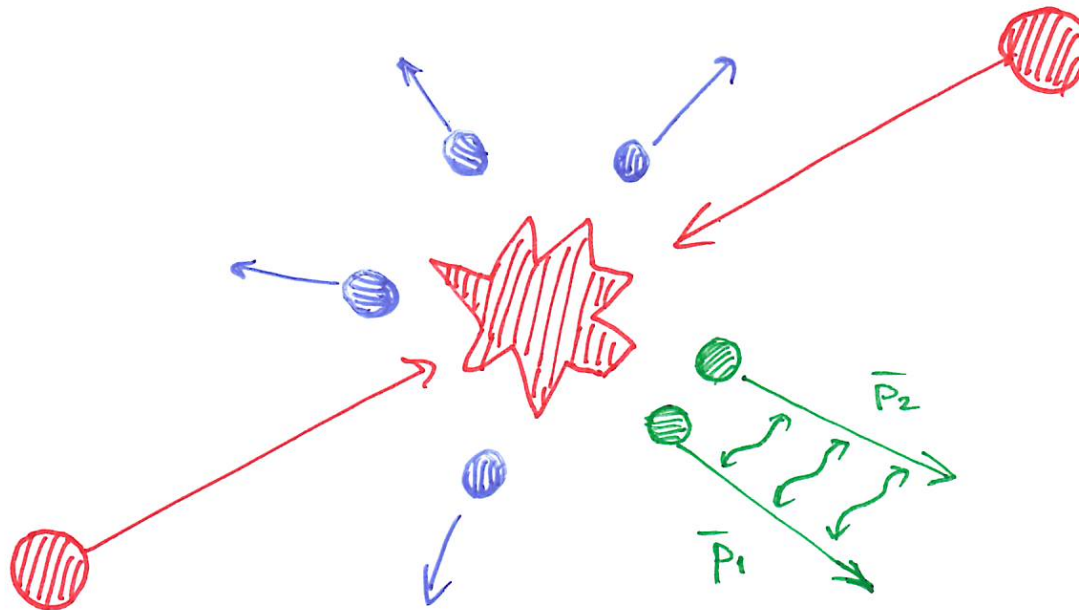
Scattering experiments



Measure S-matrix (i.e., the asymptotic form of the scattering wave function):

$$\Psi_{\vec{p}}^{(+)}(\vec{r}) \longrightarrow e^{i\vec{p}\cdot\vec{r}} + f_p(\hat{r})\frac{e^{ipr}}{r}$$

Femtoscscopy experiments



Measure correlation functions. The Koonin-Pratt model:

$$C(p) = \int \underbrace{S(\vec{r})}_{\text{Source term}} \left| \Psi_{\vec{p}}^{(+)}(\vec{r}) \right|^2 d^3r$$

Source term: Needs to be modeled (!)

Goal: Extract the strong interaction between hadrons by measuring $C(p)$

ALICE probes the strong interaction three-body problem

With new measurements of hadron–deuteron correlations, the ALICE collaboration explores the strong interaction of three-body systems at the LHC.

25 SEPTEMBER, 2024 | By ALICE collaboration

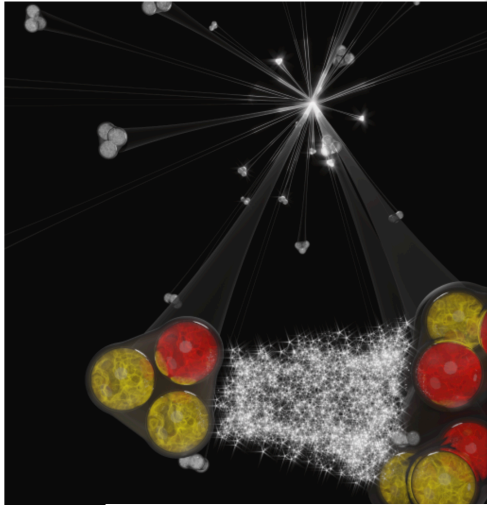


Illustration of the strong interaction three-body problem. (Image: CERN)

nature

Explore content ▾ About the journal ▾ Publish with us ▾ Subscribe

[nature](#) > [news & views](#) > article

NEWS AND VIEWS | 09 December 2020

Proton collisions probe frontier of the standard model of particle physics

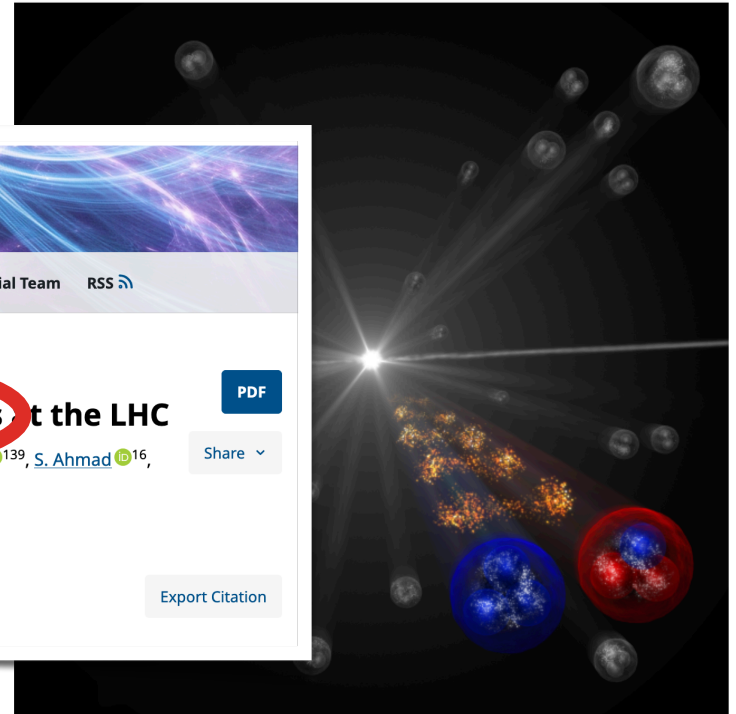
The nuclear forces that act on short-lived subatomic particles are a frontier of the standard model of particle physics. This problem has now been solved by smashing high-energy protons together and measuring the momenta of the unstable particles produced.

[Voir en français](#)

ALICE collaboration opens avenue for high-precision studies of the strong force

The ALICE collaboration has shown how proton–proton collisions at the Large Hadron Collider can reveal the strong interaction between composite particles called hadrons.

9 DECEMBER, 2020



An artist's impression of the ALICE study of the interaction between the rarest of the hyperons, Omega (Ω) hyperon (left), which contains three strange quarks, and a proton (right). (Image: CERN)

Physical Review X

Highlights Recent Subjects Accepted Collections Authors Referees Press About Editorial Team RSS

OPEN ACCESS

Exploring the Strong Interaction of Three-Body Systems at the LHC

S. Acharya¹³¹, D. Adamová¹⁹⁰, G. Aglieri Rinella¹³⁵, M. Agnello¹³⁵, J. Alcaraz¹³⁵, S. Alkhalaf¹³⁹, S. Ahmad¹¹⁶, S. U. Ahn¹⁷⁵, I. Ajuja¹⁴⁰ et al. (ALICE Collaboration)

Show more ▾

Phys. Rev. X **14**, 031051 – Published 24 September, 2024

DOI: <https://doi.org/10.1103/PhysRevX.14.031051>

PDF

Share ▾

Export Citation

Measuring unmeasurable?

Measurable effects of the strong interaction: S-matrix, nuclear spectra, resonances, EoS,... 😊

On the other hand: Nuclear forces themselves are not observable and scheme-dependent 😞

Measuring unmeasurable?

Measurable effects of the strong interaction: S-matrix, nuclear spectra, resonances, EoS,... 😊

On the other hand: Nuclear forces themselves are not observable and scheme-dependent 😞

Some examples of non-observable quantities in nuclear physics:

- deuteron D-state probability J. Friar PRC 20 (1979)
- matter radii, i.e., $\langle \Psi | r_{ij}^2 | \Psi \rangle$ (but: charge, axial, ... radii are, of course, observable)
- strong proton-proton scattering length P. Sauer PRL 32 (74), J. Gegelia EPJA 19 (2004)
- nuclear wave functions
- off-shell effects
- ...

Measuring unmeasurable?

Measurable effects of the strong interaction: S-matrix, nuclear spectra, resonances, EoS,... 😊

On the other hand: Nuclear forces themselves are not observable and scheme-dependent 😞

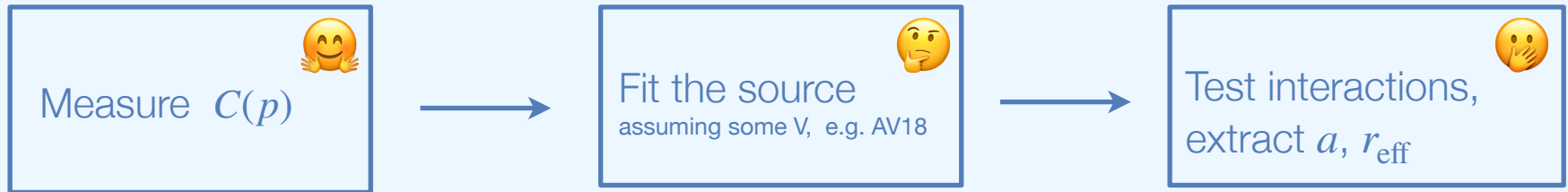
Some examples of non-observable quantities in nuclear physics:

- deuteron D-state probability J. Friar PRC 20 (1979)
- matter radii, i.e., $\langle \Psi | r_{ij}^2 | \Psi \rangle$ (but: charge, axial, ... radii are, of course, observable)
- strong proton-proton scattering length P. Sauer PRL 32 (74), J. Gegelia EPJA 19 (2004)
- nuclear wave functions
- off-shell effects
- ...

Non-observable quantities can be still very useful if one can control scheme dependence:

- nuclear forces and currents in chiral EFT EE, Hammer, Meißner, RMP 81 (2009)
- LECs in the effective pion Lagrangian FLAG Review
- quark masses PDG
- ...

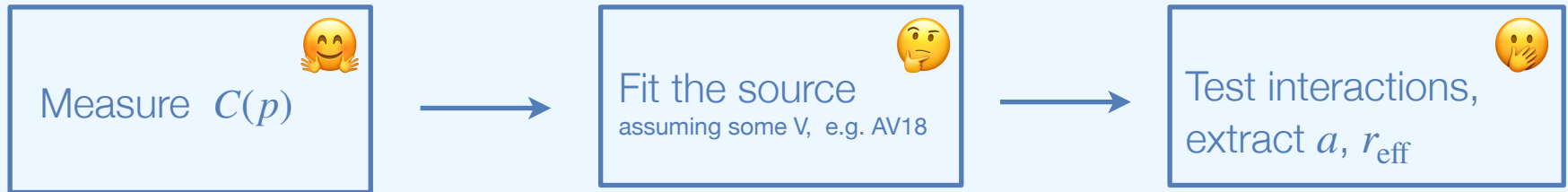
Workflow in femtoscopy experiments



The Koonin-Pratt formula:
$$C(p) = \int S(\vec{r}) \left| \Psi_{\vec{p}}^{(+)}(\vec{r}) \right|^2 d^3r$$

The source function $S(\vec{r})$ is assumed universal (otherwise cannot probe the interaction...)

Workflow in femtoscopy experiments

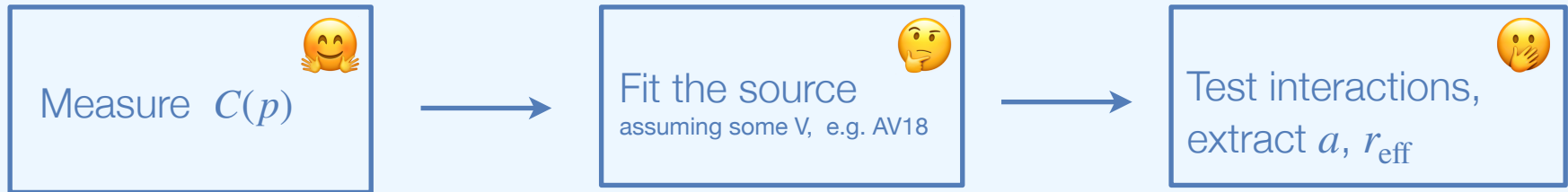


The Koonin-Pratt formula:
$$C(p) = \int S(\vec{r}) \left| \Psi_{\vec{p}}^{(+)}(\vec{r}) \right|^2 d^3r = \langle \Psi_{\vec{p}}^{(+)} | \hat{S} | \Psi_{\vec{p}}^{(+)} \rangle$$

for $S(\vec{r}', \vec{r}) \equiv \langle \vec{r}' | \hat{S} | \vec{r} \rangle = \delta(\vec{r}' - \vec{r}) S(\vec{r})$ ———— ↗

The source function $S(\vec{r})$ is assumed universal (otherwise cannot probe the interaction...)

Workflow in femtoscopy experiments



The Koonin-Pratt formula:
$$C(p) = \int S(\vec{r}) \left| \Psi_{\vec{p}}^{(+)}(\vec{r}) \right|^2 d^3r = \langle \Psi_{\vec{p}}^{(+)} | \hat{S} | \Psi_{\vec{p}}^{(+)} \rangle$$

for $S(\vec{r}', \vec{r}) \equiv \langle \vec{r}' | \hat{S} | \vec{r} \rangle = \delta(\vec{r}' - \vec{r}) S(\vec{r})$ ———

The source function $S(\vec{r})$ is assumed universal (otherwise cannot probe the interaction...)

One can always change a basis on the Hilbert space by means of a UT:

$$\langle \Psi_{\vec{p}}^{(+)} | \hat{S} | \Psi_{\vec{p}}^{(+)} \rangle = \left(\langle \Psi_{\vec{p}}^{(+)} | \hat{U}^\dagger \right) (\hat{U} \hat{S} \hat{U}^\dagger) (\hat{U} | \Psi_{\vec{p}}^{(+)} \rangle) = \langle \Psi_{\vec{p}}'^{(+)} | \hat{S}' | \Psi_{\vec{p}}'^{(+)} \rangle$$

Observable quantities ($C(p)$, asymptotics of the w.f.) do not depend on the basis, while not-observable ones (wave functions, nuclear forces, the source term) generally do...

⇒ assuming a universal source, $\hat{S}' = \hat{S}$, introduces potentially large model dependence!

Gedankenexperiment

How severe is this model dependence in practice?

Let's do Gedankenexperiment: ALICE & Bob analyze a femtoscopy measurement...
(two distinguishable spin-less particles, no Coulomb)



Gedankenexperiment

How severe is this model dependence in practice?



Let's do Gedankenexperiment: ALICE & Bob analyze a femtoscopy measurement...
(two distinguishable spin-less particles, no Coulomb)

Scattering wave functions can be obtained from the half-shell K/T-matrix. With little algebra, one finds:

$$C(k) = \langle \Psi_{-\mathbf{k}}^{(+)} | \hat{S}_{12} | \Psi_{-\mathbf{k}}^{(+)} \rangle = \sum_l \frac{2l+1}{4\pi} \cos^2 \delta_l(k) \left[S_{kk}^l + K_{kp}^l \circ S_{pk}^l + S_{kp}^l \circ K_{pk}^l + K_{kp}^l \circ S_{pp'}^l \circ K_{p'k}^l \right]$$

$$\text{where } S_{pp'}^l \equiv \langle pl | \hat{S}_{12} | p'l \rangle, \quad K_{pk}^l = K_{kp}^l \equiv \langle pl | \hat{K} | kl \rangle, \quad A_{pp'}^l \circ B_{p'p''}^l \equiv \int_0^\infty p'^2 dp' A_{pp'}^l \frac{2\mu}{k^2 - p'^2} B_{p'p''}^l$$

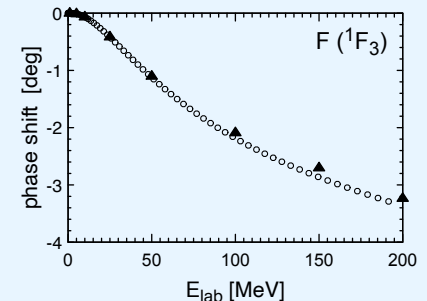
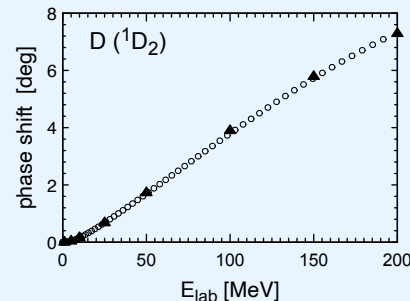
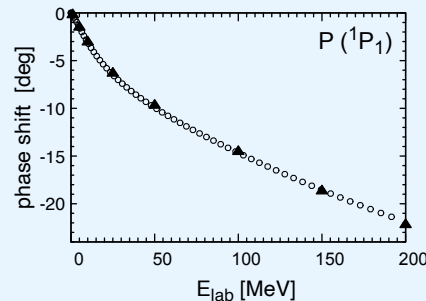
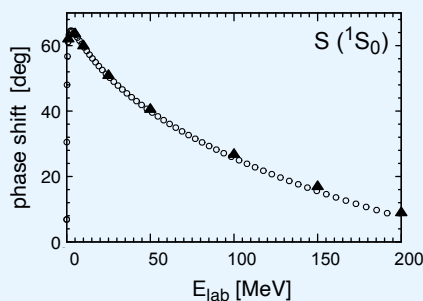
Gedankenexperiment

How severe is this model dependence in practice?

Let's do Gedankenexperiment: ALICE & Bob analyze a femtoscopy measurement...
(two distinguishable spin-less particles, no Coulomb)



The **interaction** is taken as the chiral EFT@N⁴LO⁺ projected on spin-0 states:



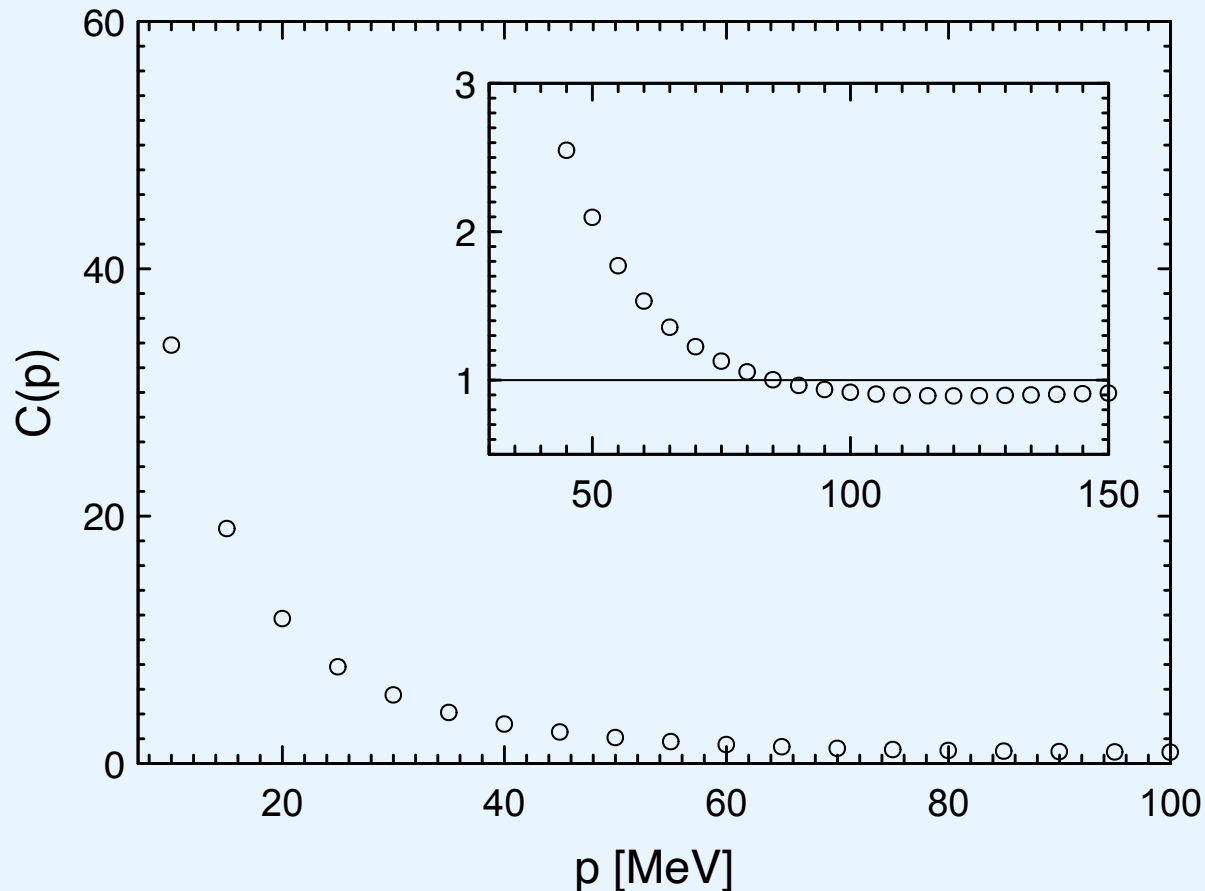
For the **source term**, the (usual) static local Gaussian form is chosen:

$$S(\vec{r}) = \frac{1}{(2\pi r_0^2)^{3/2}} e^{-r^2/(4r_0^2)} \xrightarrow{\text{FT}} S(\vec{q}) \equiv \underbrace{\langle \vec{p}' | \hat{S} | \vec{p} \rangle}_{= \vec{p}' - \vec{p}} = e^{-q^2 r_0^2} \quad \text{choose: } r_0 = 1.5 \text{ fm}$$

Gedankenexperiment

How severe is this model dependence in practice?

Let's do Gedankenexperiment: ALICE & Bob analyze a femtoscopy measurement...
(two distinguishable spin-less particles, no Coulomb)



Gedankenexperiment



Bob knows that physics is independent on the choice of basis in the Hilbert space.

He uses states $|\Psi_{\text{Bob}}\rangle = \hat{U} |\Psi_{\text{Alice}}\rangle$, where $\hat{U} = 1 - 2|g\rangle\langle g|$, $\langle g|g\rangle = 1$.

Gedankenexperiment

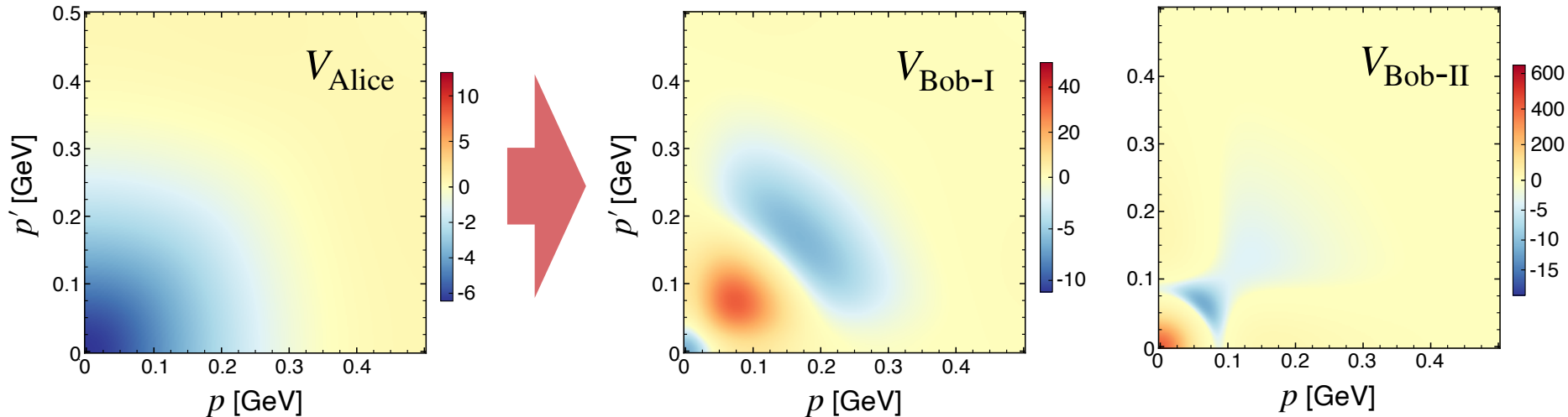


Bob knows that physics is independent on the choice of basis in the Hilbert space.

He uses states $|\Psi_{\text{Bob}}\rangle = \hat{U} |\Psi_{\text{Alice}}\rangle$, where $\hat{U} = 1 - 2|g\rangle\langle g|$, $\langle g|g\rangle = 1$.

In Bob's notation, the potential takes the form: $\hat{V}_{\text{Bob}} = \hat{U}(\hat{H}_0 + \hat{V}_{\text{Alice}})\hat{U}^\dagger - \hat{H}_0$

Specifically, choose $g(\vec{r}) \equiv \langle \vec{r} | g \rangle = C r (1 - \beta r) e^{-\alpha r}$ Peter Sauer, PRL 32 (74)



$$\alpha = 1 \text{ fm}^{-1}$$

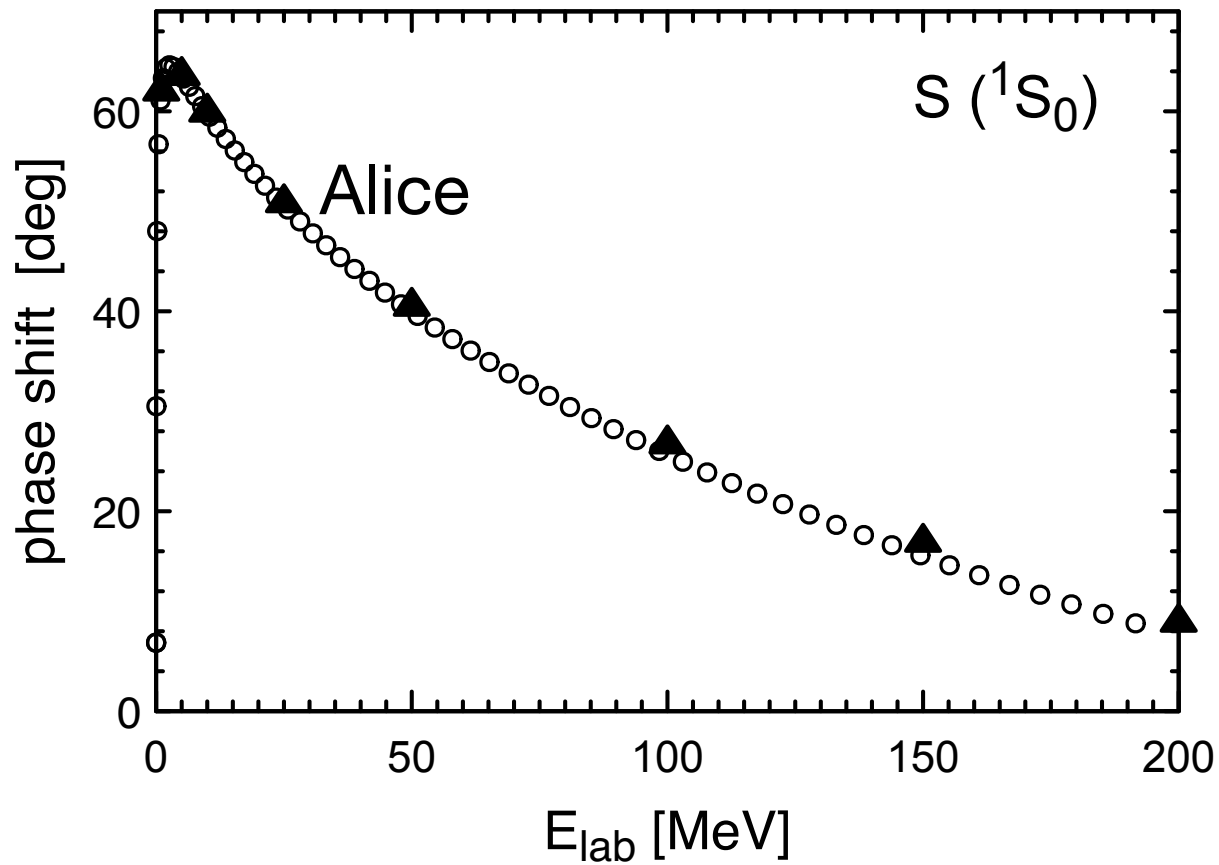
$$\beta = 0.25 \text{ fm}^{-1}$$

$$\alpha = 0.7 \text{ fm}^{-1}$$

$$\beta = 2.0 \text{ fm}^{-1}$$

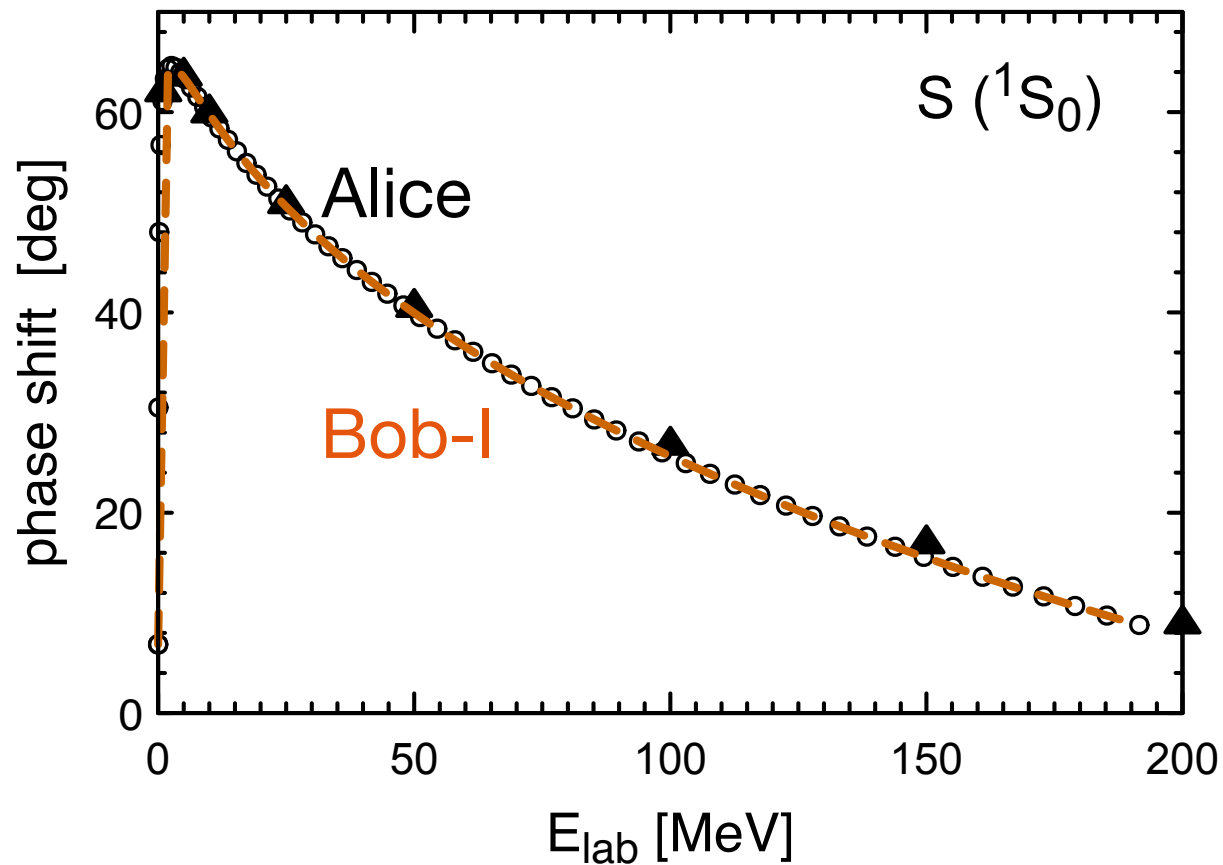
Gedankenexperiment

Yet, observables are not affected by UTs, e.g.:



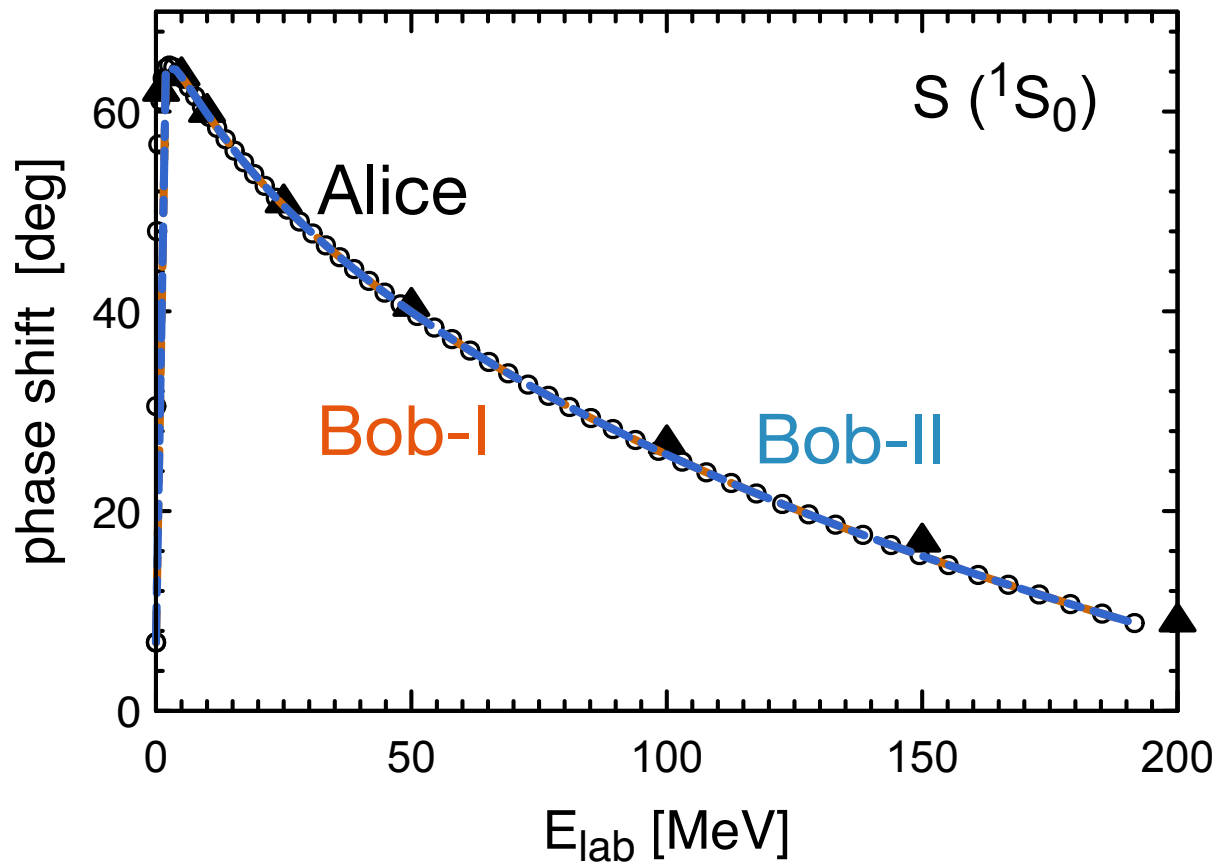
Gedankenexperiment

Yet, observables are not affected by UTs, e.g.:



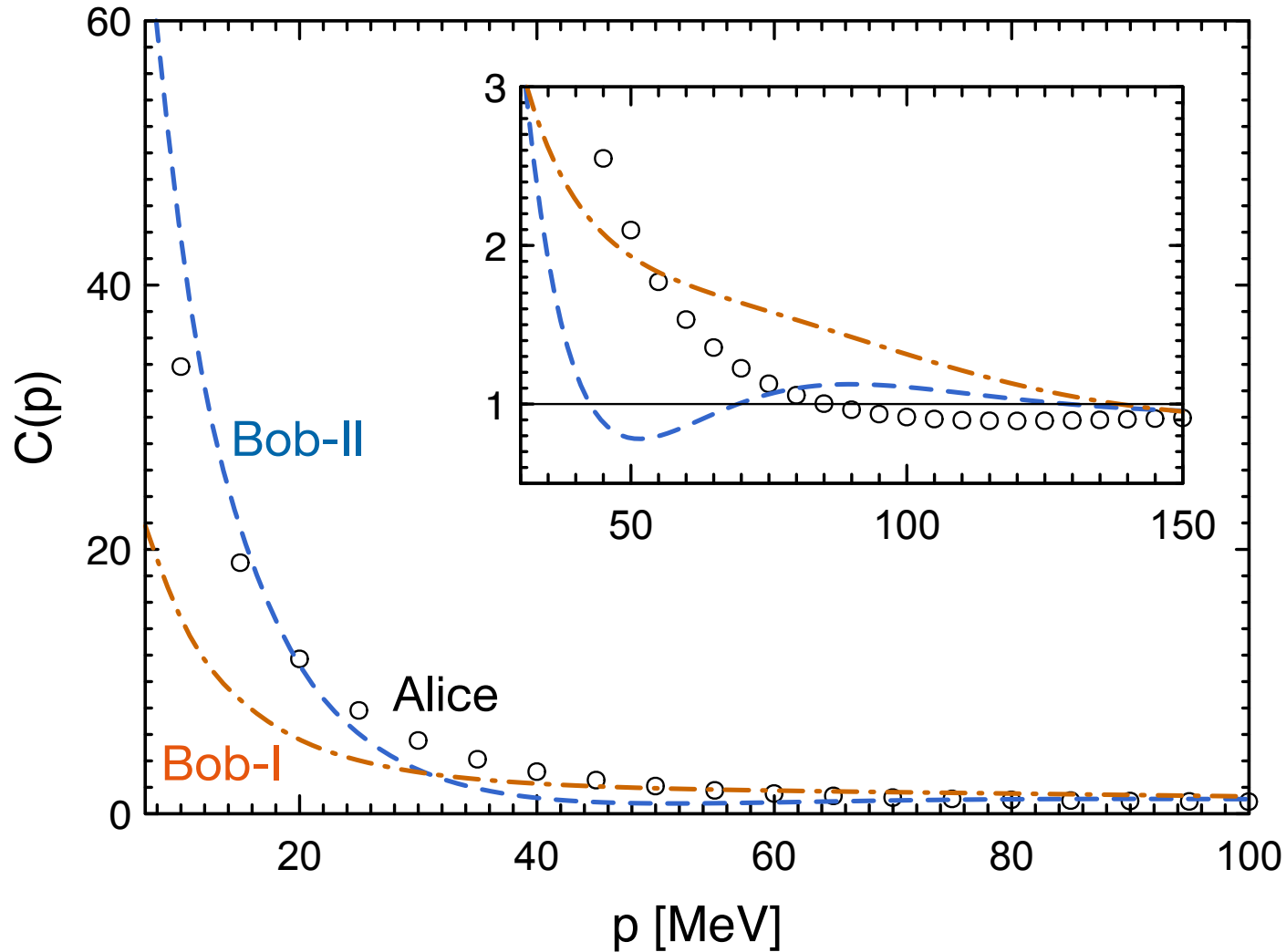
Gedankenexperiment

Yet, observables are not affected by UTs, e.g.:



Gedankenexperiment

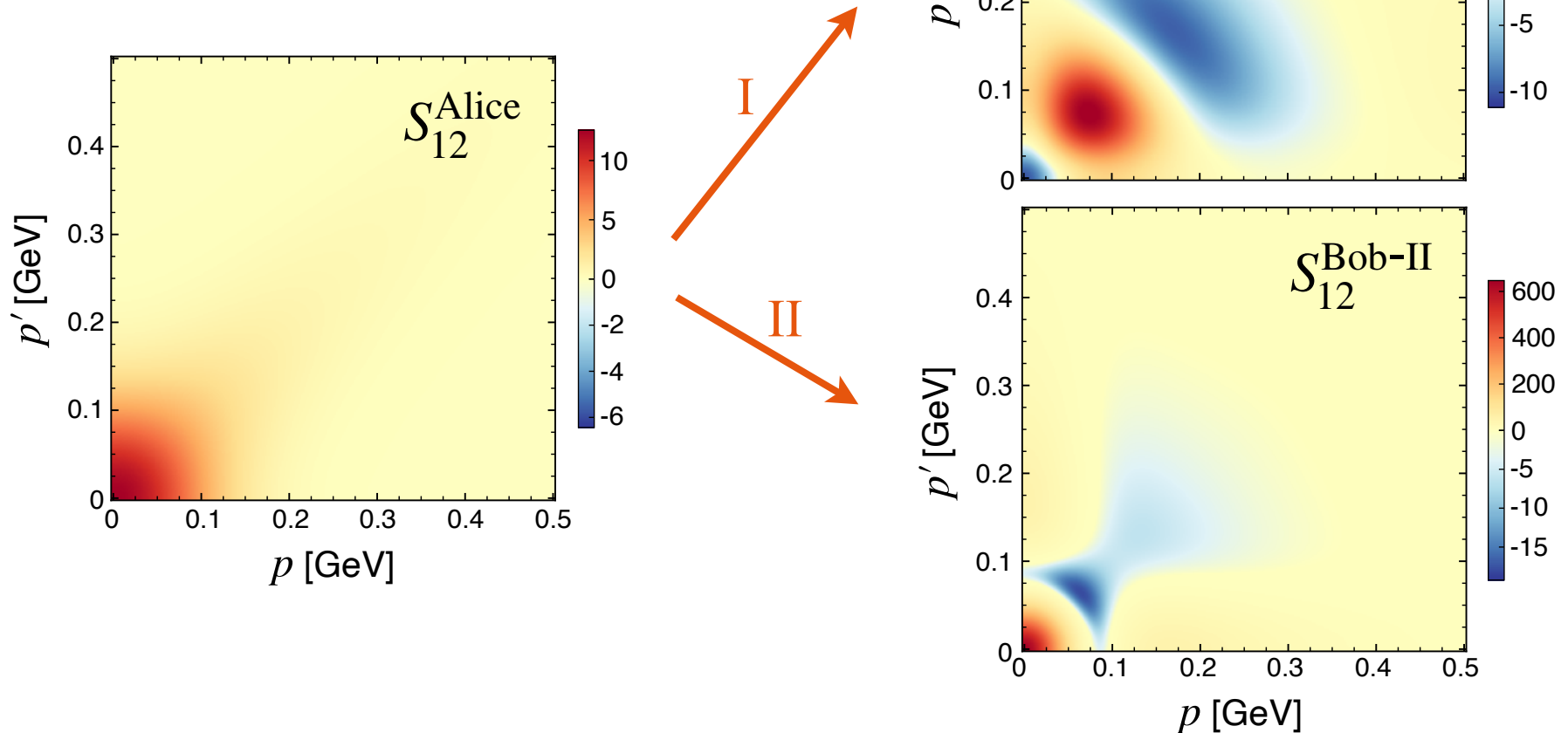
However, the correlation functions calculated by Bob look differently
when using the same Gaussian source



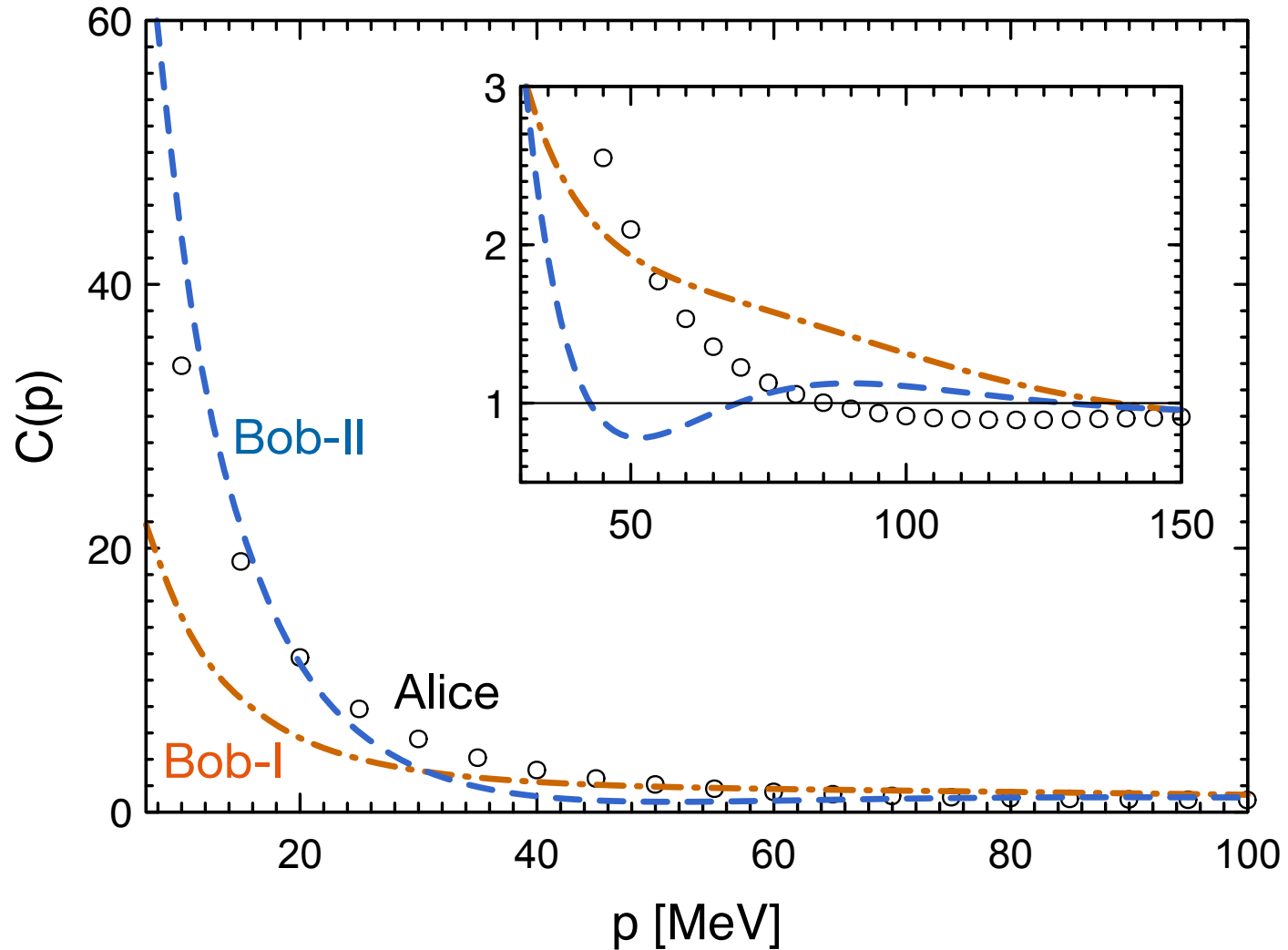
Gedankenexperiment

The solution is, of course, trivial:

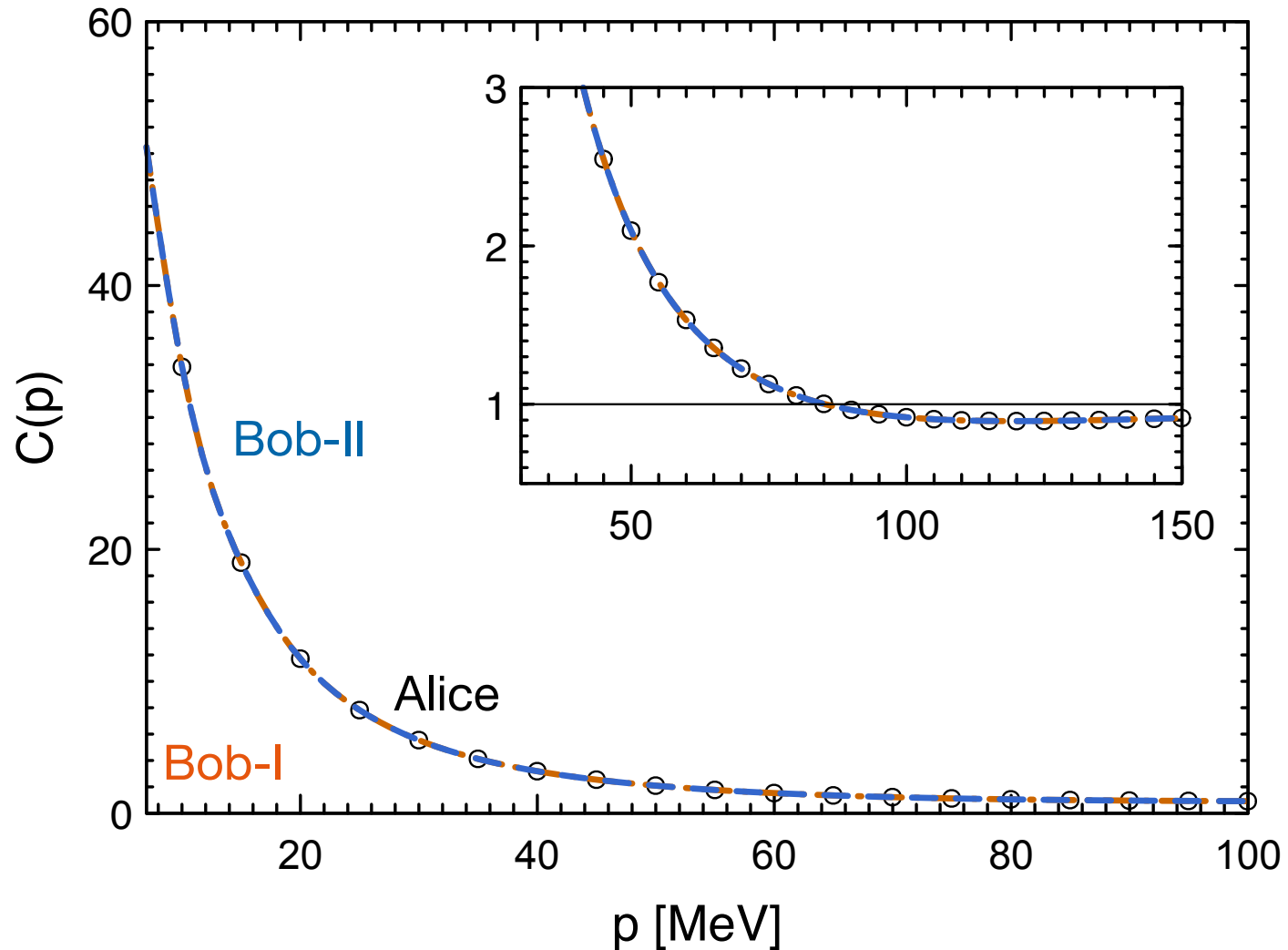
The source term needs to be translated
to Bob's convention!



Gedankenexperiment



Gedankenexperiment



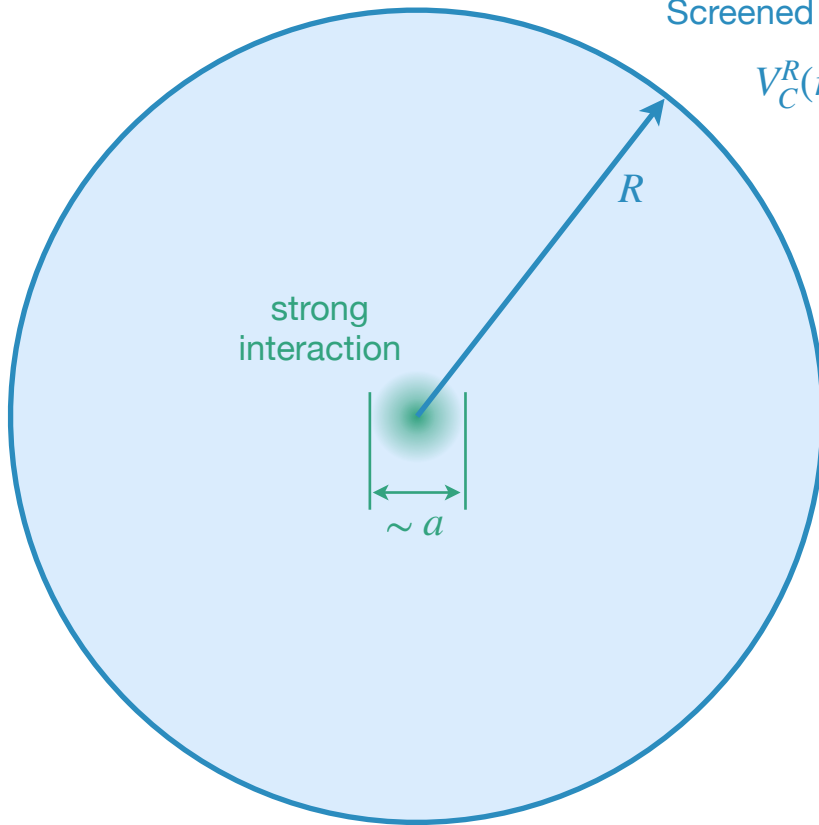
Realistic example:
Proton-proton correlation functions

Proton-proton scattering wave function

The scattering wave function can be obtained using the Vincent-Phatak method:

Screened Coulomb potential $V_C^R(r)$ with $R \gg a$:

$$V_C^R(r) = V_C(r) \text{ for } r \leq R, \text{ while } V_C^R(r) = 0 \text{ otherwise}$$

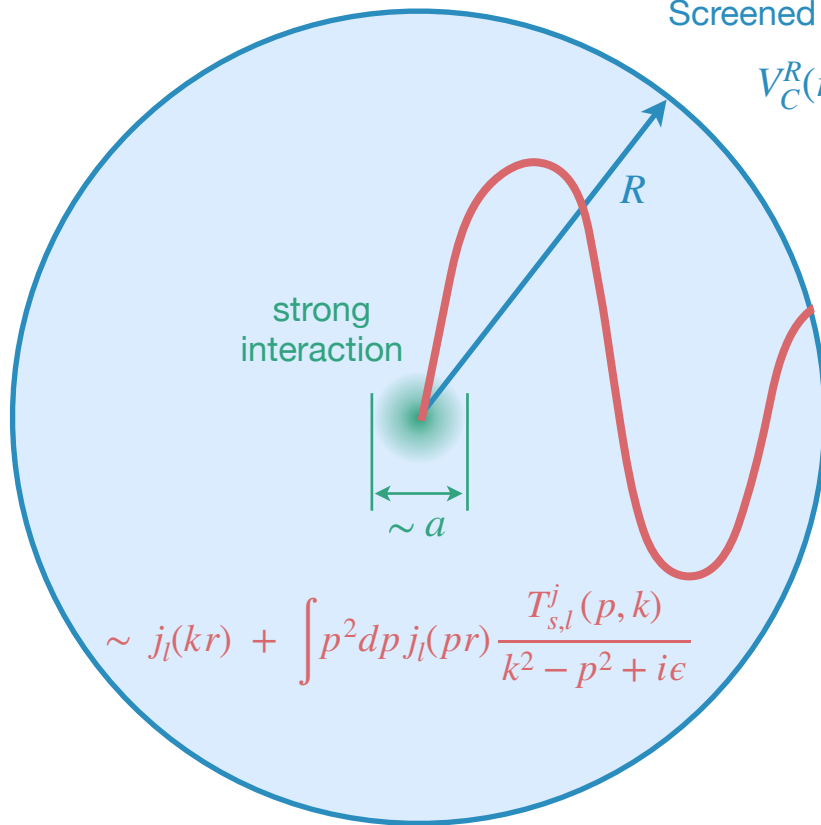


Proton-proton scattering wave function

The scattering wave function can be obtained using the Vincent-Phatak method:

Screened Coulomb potential $V_C^R(r)$ with $R \gg a$:

$V_C^R(r) = V_C(r)$ for $r \leq R$, while $V_C^R(r) = 0$ otherwise

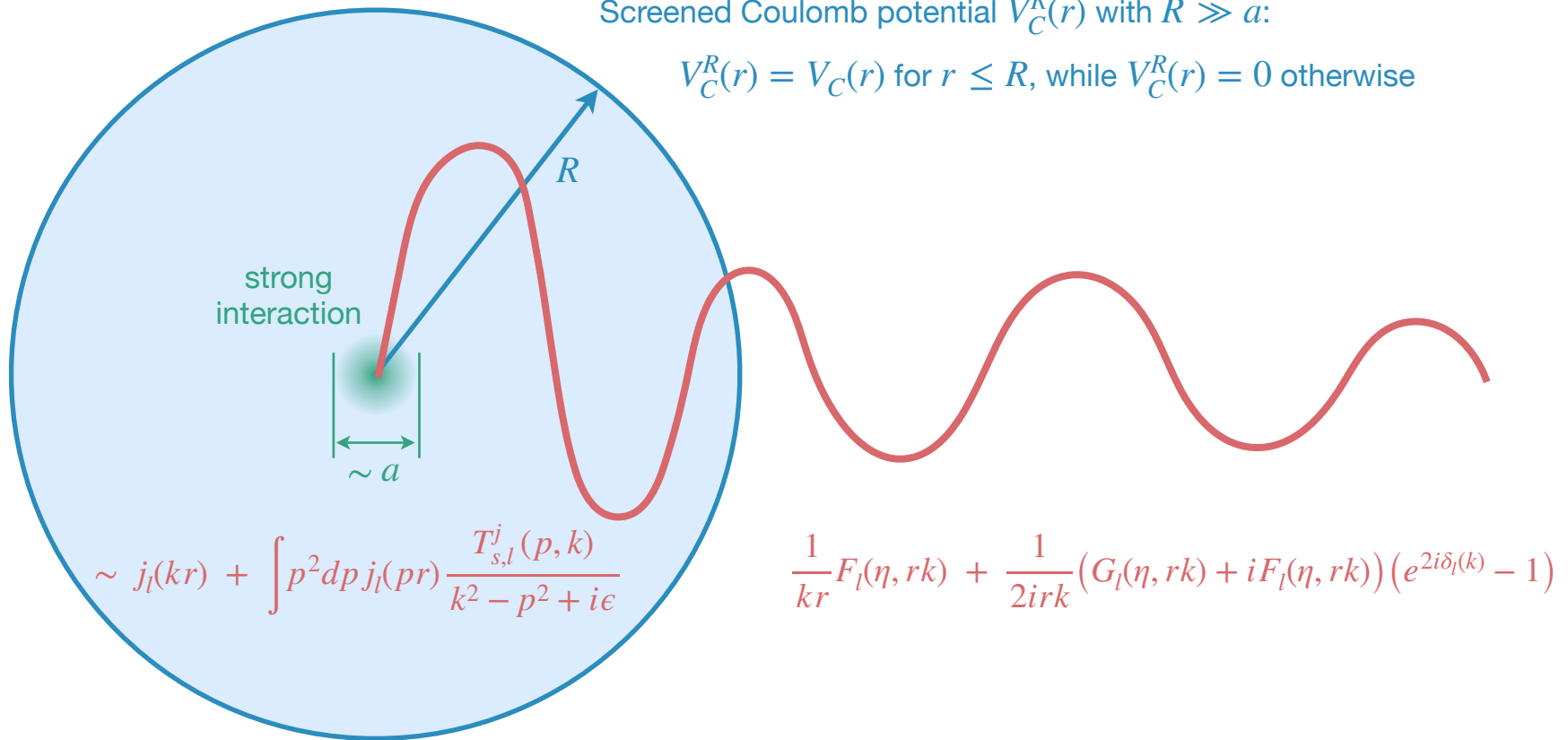


Proton-proton scattering wave function

The scattering wave function can be obtained using the Vincent-Phatak method:

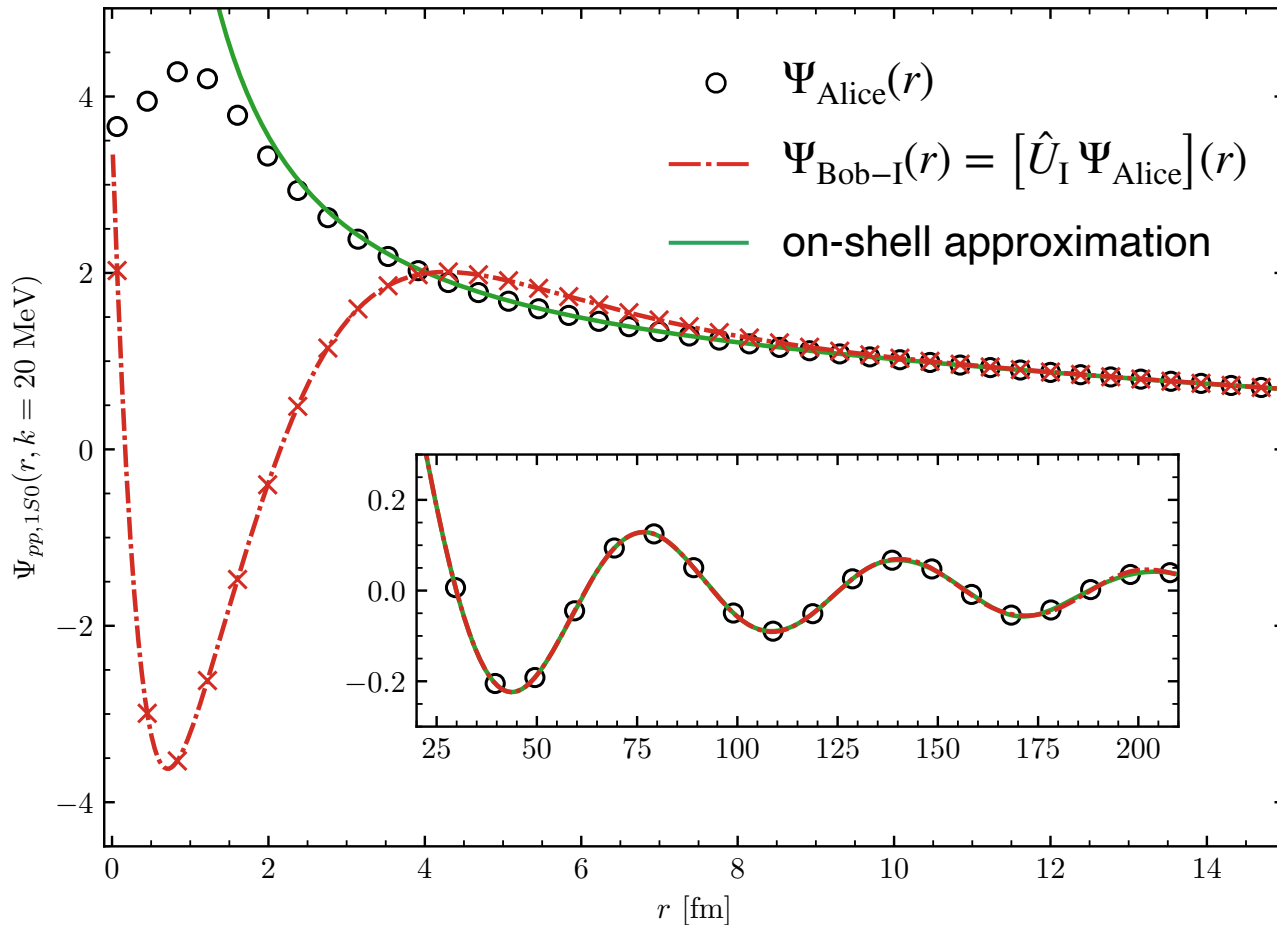
Screened Coulomb potential $V_C^R(r)$ with $R \gg a$:

$$V_C^R(r) = V_C(r) \text{ for } r \leq R, \text{ while } V_C^R(r) = 0 \text{ otherwise}$$



Matching the two solutions at $r = R$ yields the full scattering wave function

Proton-proton scattering wave function

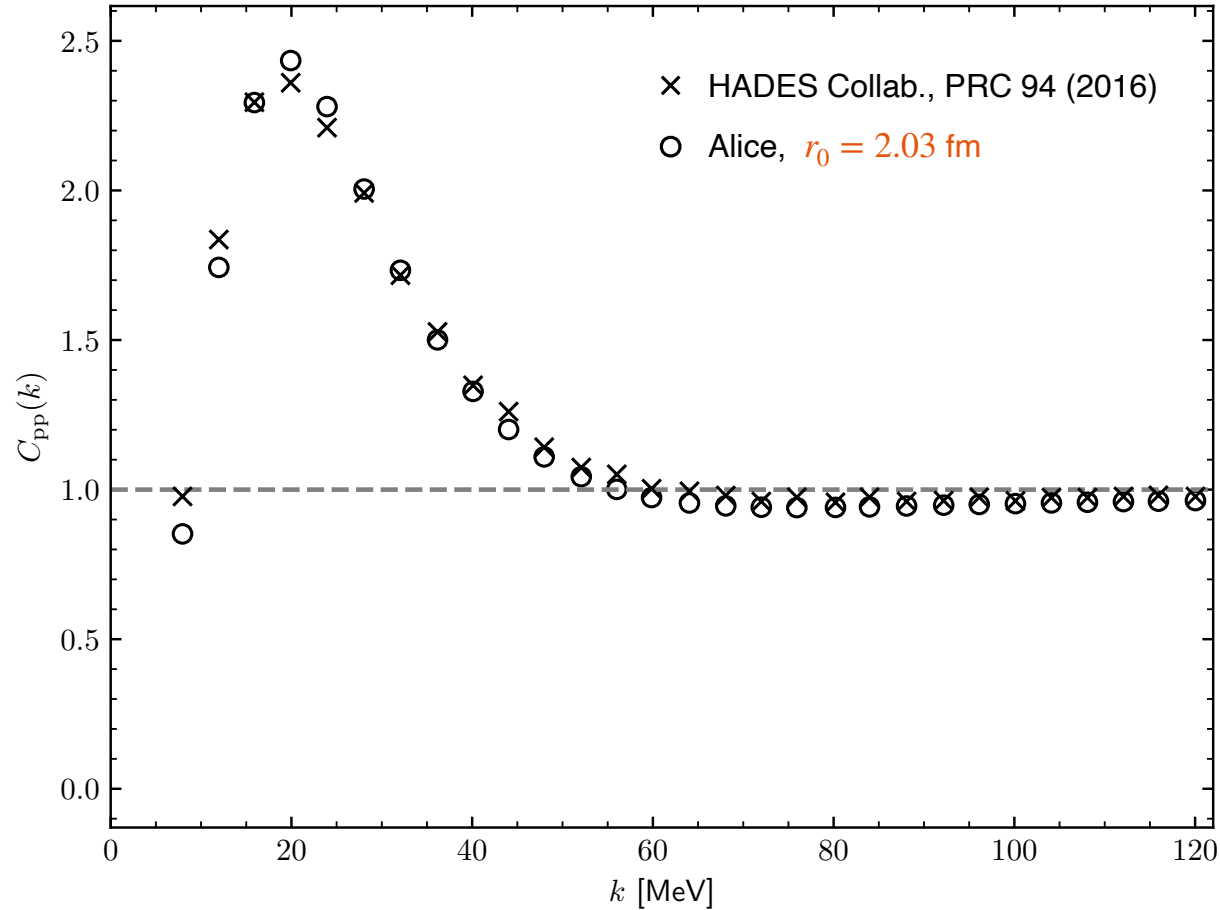


$\Psi_{\text{Bob}}(r)$ can also be calculated from the transformed potential:

$$\hat{V}_{\text{Bob}} = \hat{U} \left(\frac{\hat{p}^2}{2m} + \hat{V}_{\text{Alice}} + \hat{V}_C^R \right) U^\dagger - \frac{\hat{p}^2}{2m} - V_C^R$$

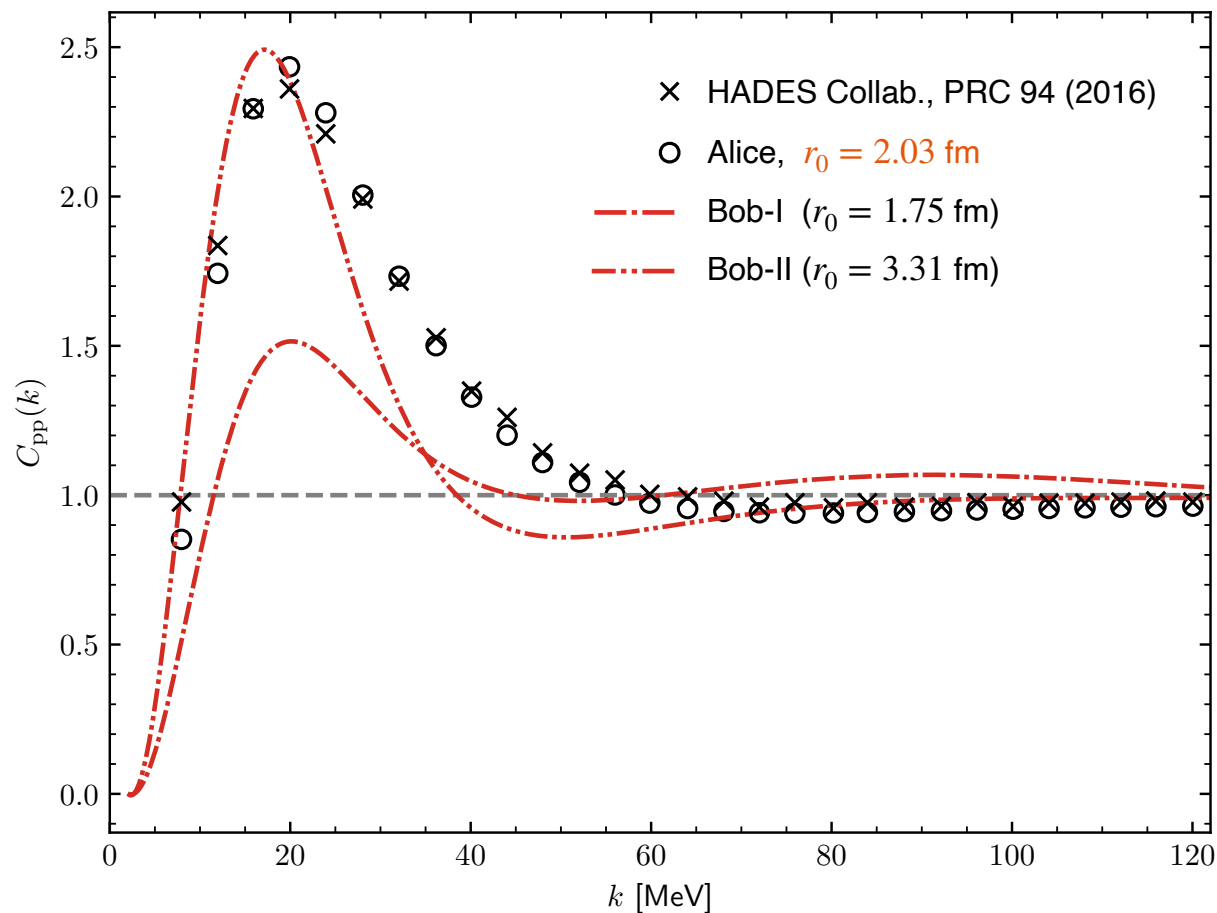
Proton-proton correlation functions

Use HADES proton-proton data to extract the radius of the Gaussian source term:



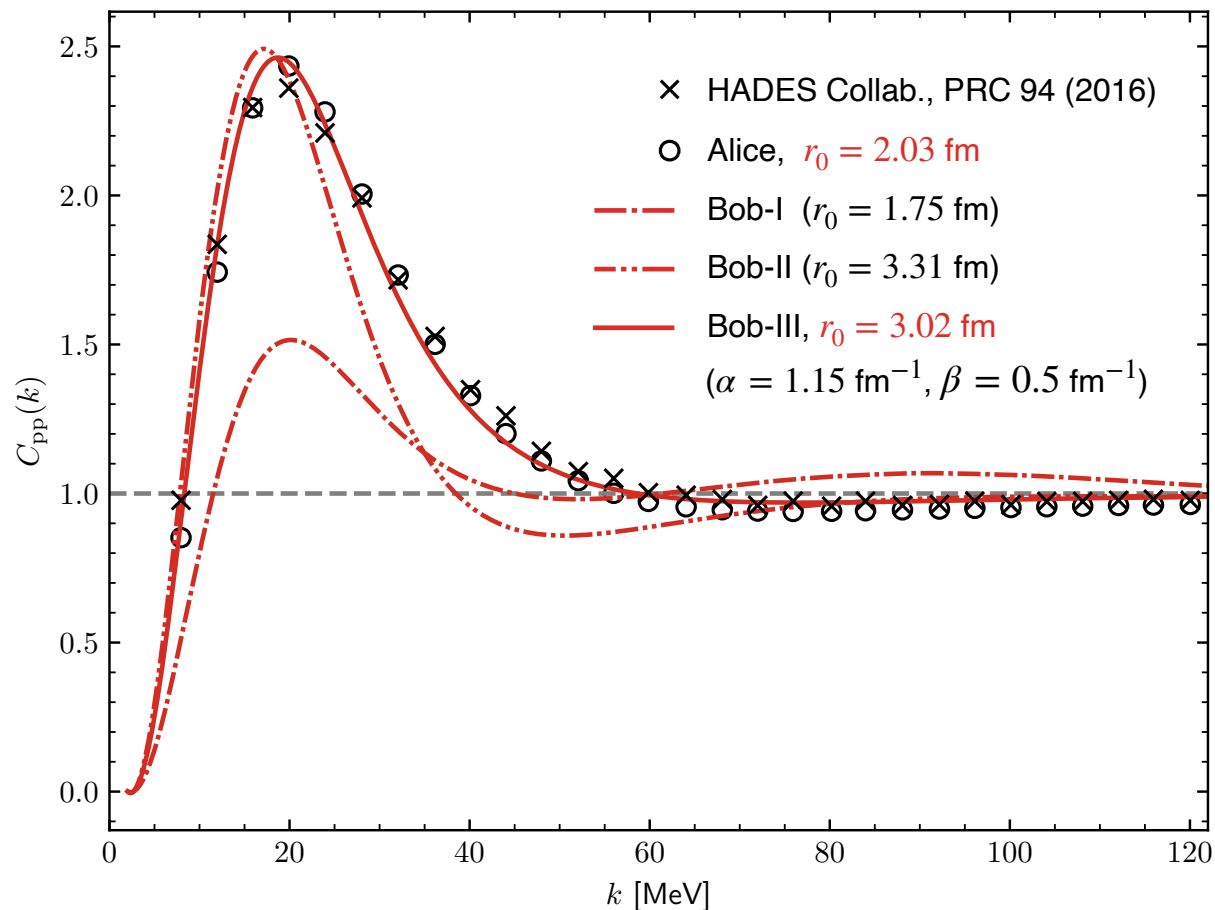
Proton-proton correlation functions

Use HADES proton-proton data to extract the radius of the Gaussian source term:



Proton-proton correlation functions

Use HADES proton-proton data to extract the radius of the Gaussian source term:

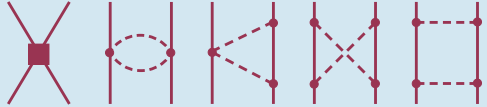
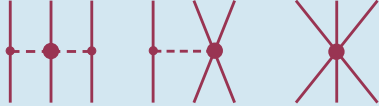
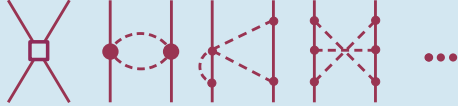
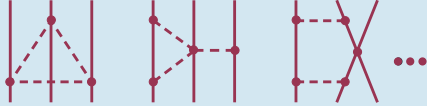
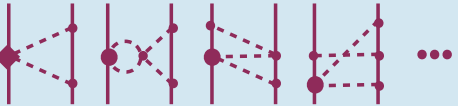


Alice and Bob-III provide good description of the data, but yield rather different radii...

Scheme dependence in chiral EFT

Scheme dependence in chiral EFT

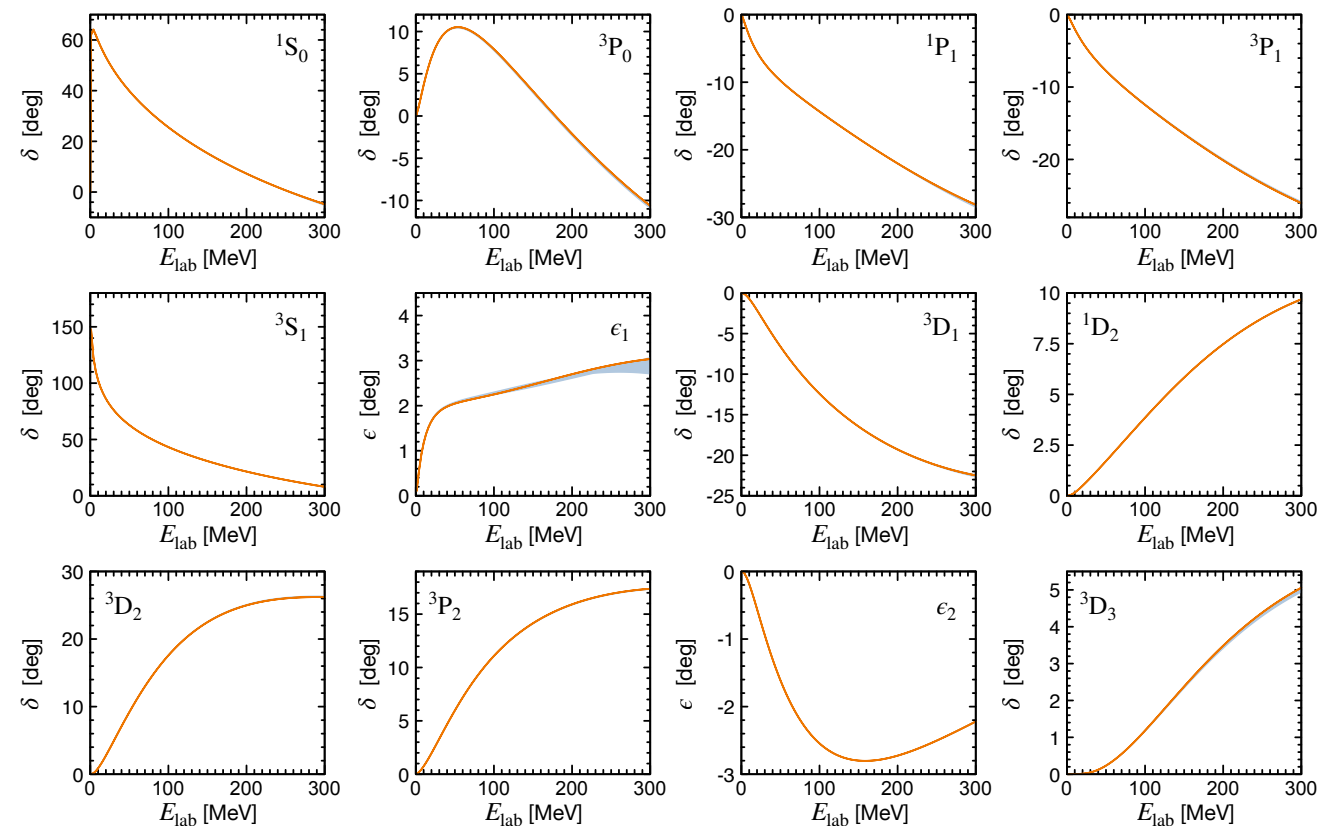
Where do unitary ambiguities start showing up in chiral EFT?

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO:			
NLO:			
N ² LO:			
N ³ LO:			
N ⁴ LO:			

Concern: Potentially large scheme dependence in the 3N force!

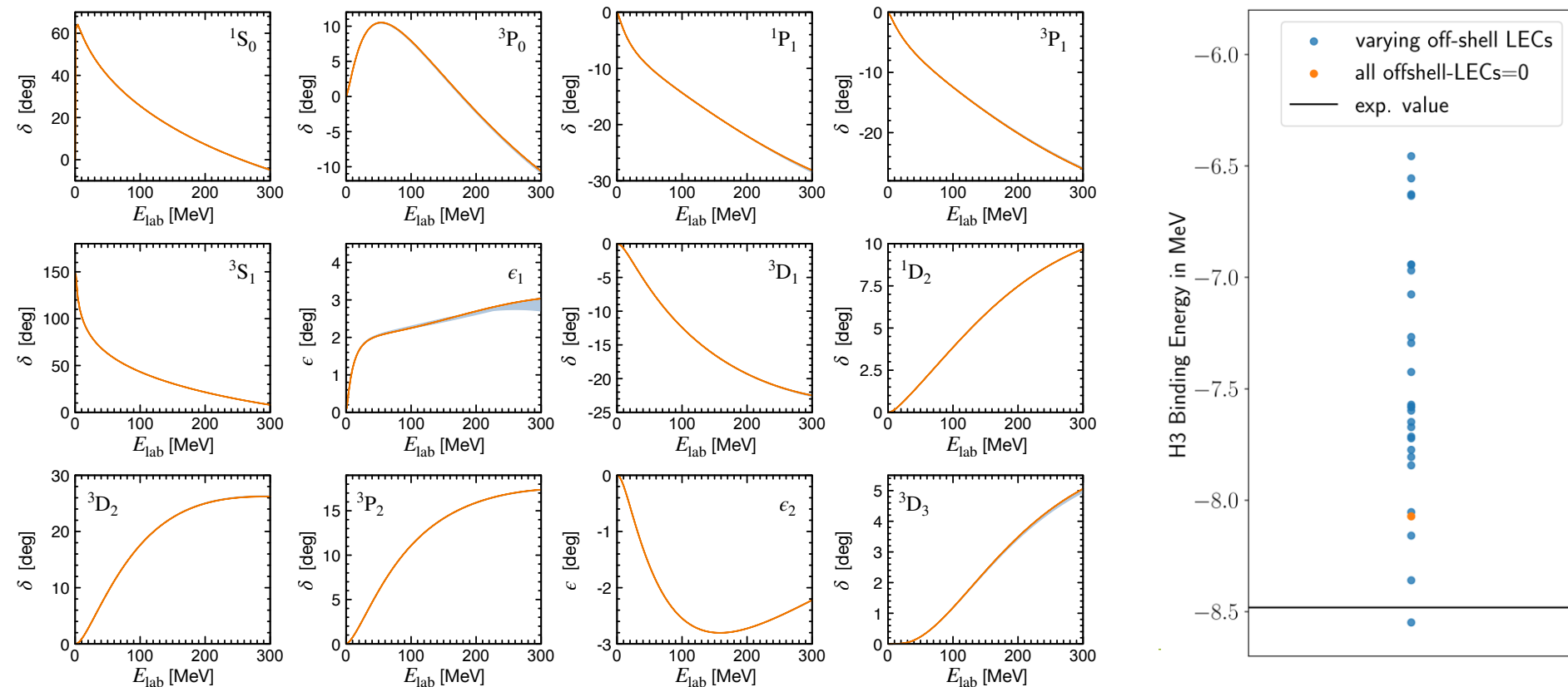
Scheme dependence in chiral EFT

Generated a set of 27 $N^4\text{LO}^+$ potentials with $D_{1S0,3S1}^{\text{off}} = \{\pm 3, 0\}$, $D_{\epsilon 1}^{\text{off}} = \{\pm 1, 0\}$



Scheme dependence in chiral EFT

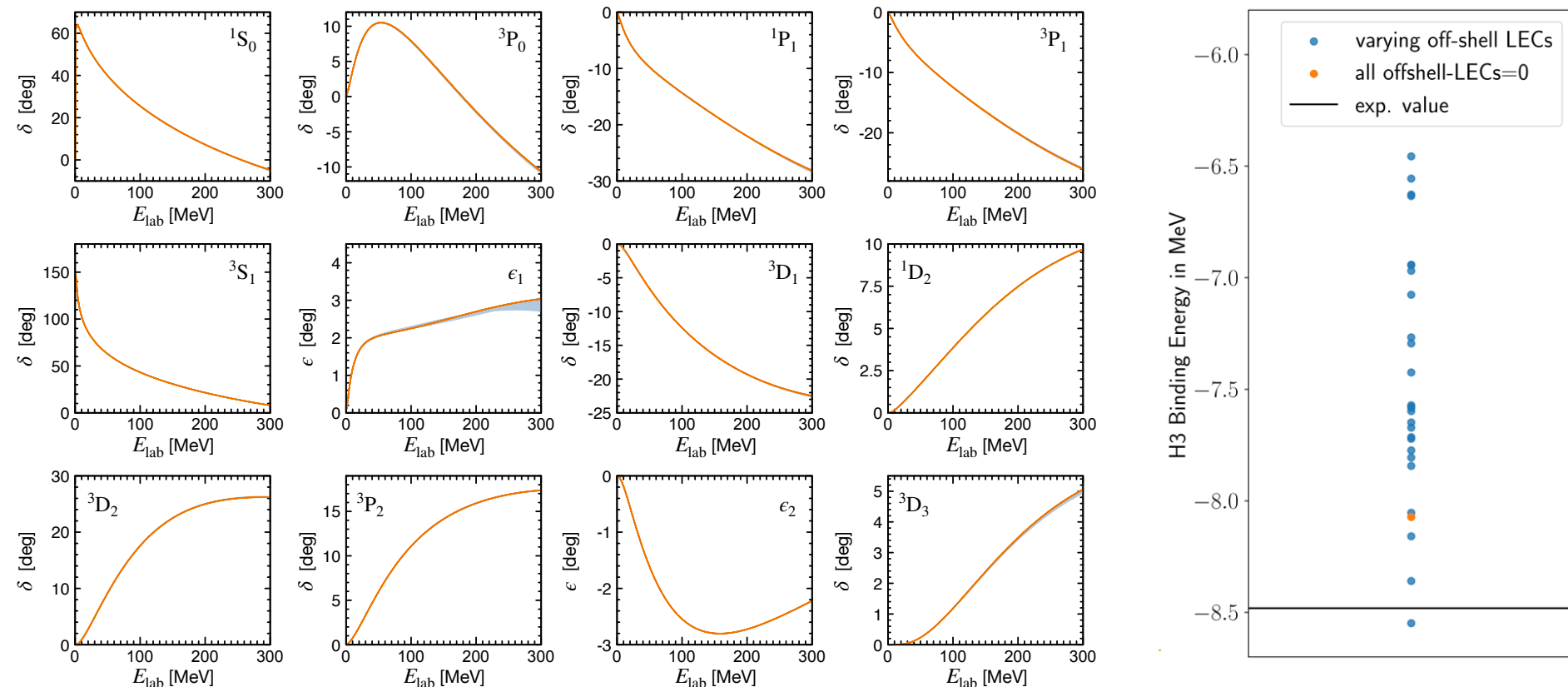
Generated a set of 27 $N^4\text{LO}^+$ potentials with $D_{1S0,3S1}^{\text{off}} = \{\pm 3, 0\}$, $D_{\epsilon_1}^{\text{off}} = \{\pm 1, 0\}$



(...an illustration of the Polyzou-Glöckle Theorem, FBS 9 (1990) 97...)

Scheme dependence in chiral EFT

Generated a set of 27 $N^4\text{LO}^+$ potentials with $D_{1S_0, 3S_1}^{\text{off}} = \{\pm 3, 0\}$, $D_{\epsilon_1}^{\text{off}} = \{\pm 1, 0\}$



(...an illustration of the Polyzou-Glöckle Theorem, FBS 9 (1990) 97...)

So, which particular three-body force is going to be measured in femtoscopy experiments?

Summary and outlook

- Nuclear interactions are **intrinsically scheme dependent**.
- They can be determined/measured **provided one fixes the convention and can keep it consistently in all applications as done in EFT**. (Can this be achieved in femtoscopy?)
- The **source is not universal** and depends on the off-shell behavior of the interaction. Off-shell inconsistent treatment of the source and interaction leads to **significant model dependence** that should be taken into account.
- This is especially problematic for **three-body interactions**.

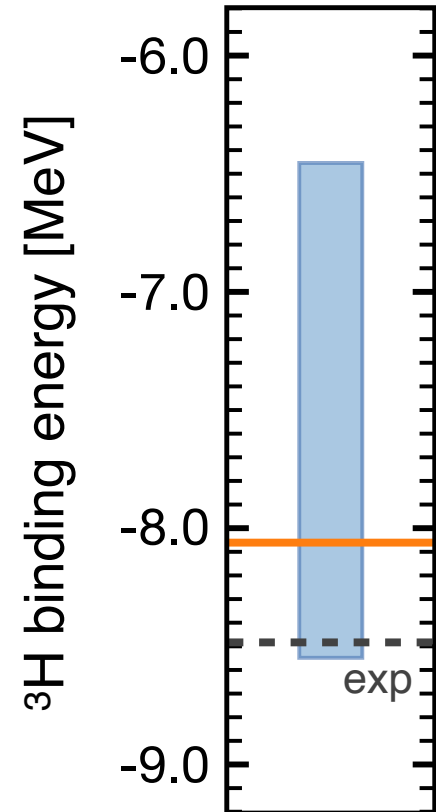
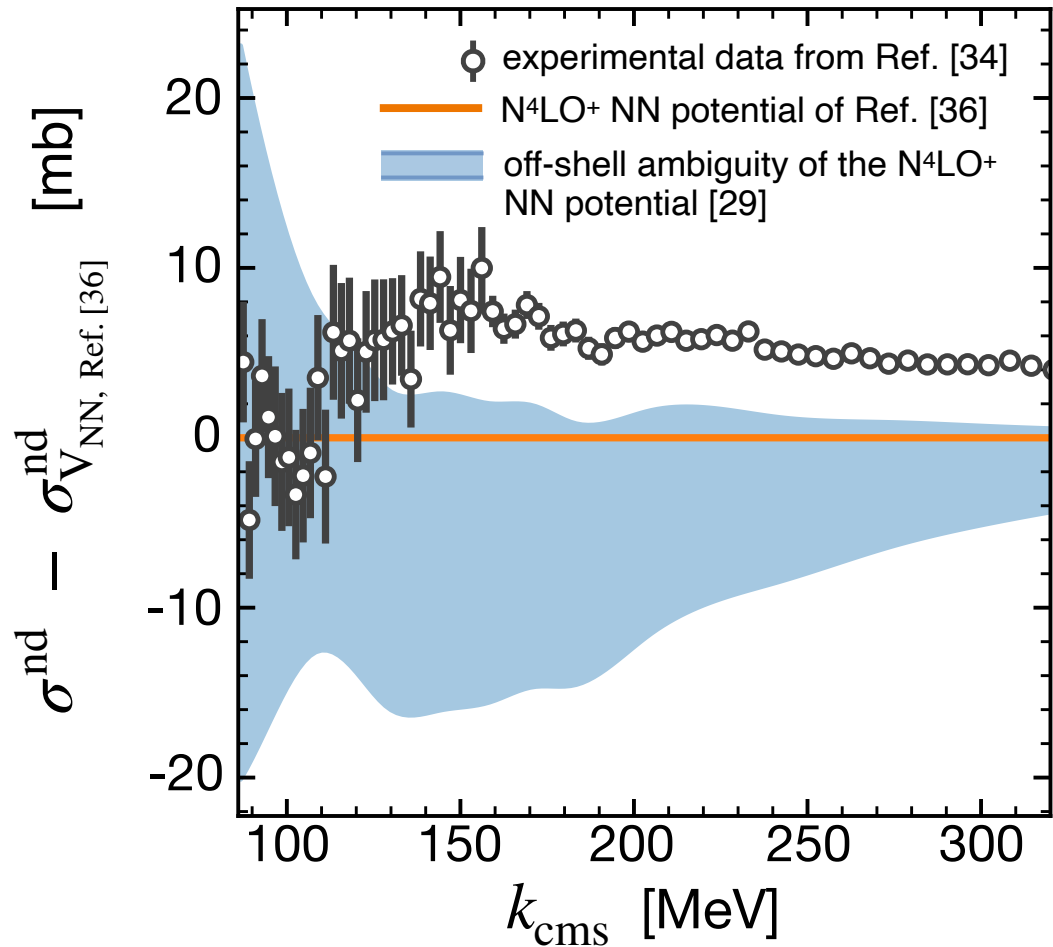
Future

- More effort needed to quantitatively estimate uncertainty from modeling the source, perhaps using EFT-inspired methods. Can model dependence be reduced by imposing specific constraints on the interaction (e.g., pion dynamics)?

Thank you for your attention

Spares

Scheme dependence in chiral EFT



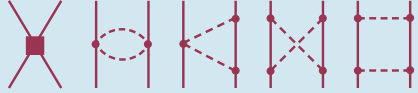
Back to reality...

But where does scheme-dependence (off-shell effects) starts showing up in chiral EFT?

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO:			
NLO:			
N ² LO:			
N ³ LO:			
N ⁴ LO:			

Back to reality...

But where does scheme-dependence (off-shell effects) starts showing up in chiral EFT?

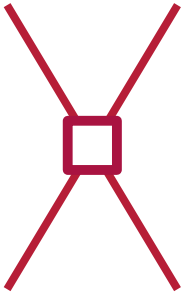
	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO:			
NLO:			
N ² LO:			
N ³ LO:			
N ⁴ LO:			

Some UTs are fixed from renormalizability requirement, but a lot of ambiguity remain:

- 2 phases in the relativistic corrections
- 3 phases in the short-range interactions (+ 2 more that depend on the total momentum)

Concern: Potentially large scheme dependence in the 3N force!

Scheme dependence in chiral EFT

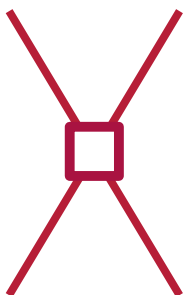


Off-shell ambiguities in the contact interactions at N³LO (¹S₀, ³S₁, ³S₁-³D₁):

$$\text{E.g.: } \langle p', {}^1S_0 | V_{\text{cont}} | p, {}^1S_0 \rangle = \underbrace{\tilde{C}_{1S_0}}_{\text{tuned to the scatt. length}} + \underbrace{C_{1S_0}(p'^2 + p^2)}_{\text{tuned to the effective range}} + \underbrace{D_{1S_0}p^2p'^2 + D_{1S_0}^{\text{off}}(p'^2 - p^2)^2}_{\text{tuned to the first shape parameter}}$$

⇒ $D_{1S_0}^{\text{off}}$ cannot be fixed from NN data Hammer, Furnstahl '00; Beane, Savage '01; Reinert, Krebs, EE '18

Scheme dependence in chiral EFT



Off-shell ambiguities in the contact interactions at N³LO (¹S₀, ³S₁, ³S₁-³D₁):

$$\text{E.g.: } \langle p', {}^1S_0 | V_{\text{cont}} | p, {}^1S_0 \rangle = \underbrace{\tilde{C}_{1S_0}}_{\text{tuned to the scatt. length}} + \underbrace{C_{1S_0}(p'^2 + p^2)}_{\text{tuned to the effective range}} + \underbrace{D_{1S_0}p^2p'^2 + D_{1S_0}^{\text{off}}(p'^2 - p^2)^2}_{\text{tuned to the first shape parameter}}$$

⇒ $D_{1S_0}^{\text{off}}$ cannot be fixed from NN data Hammer, Furnstahl '00; Beane, Savage '01; Reinert, Krebs, EE '18

Thus, it should be possible to eliminate the off-shell contacts via a suitable UT. Indeed:

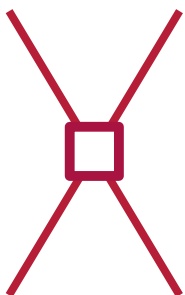
$$U = e^{\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3} \quad \text{with} \quad T_1 \propto (p_1'^2 + p_2'^2 - p_1^2 - p_2^2), \quad T_2 \propto (p_1'^2 + p_2'^2 - p_1^2 - p_2^2) \vec{\sigma}_1 \cdot \vec{\sigma}_2, \quad T_3 = \dots$$

$$\Rightarrow \delta H = U^\dagger H U - H = \sum_i \gamma_i \left[\left(\boxed{H_{\text{kin}}} + \boxed{V_{1\pi}^{(0)} + V_{\text{cont}}^{(0)}} + \mathcal{O}(Q^2) \right), T_i \right] + \mathcal{O}(\gamma_i^2)$$

└─ induce off-shell NN contact interactions
└─ induce N⁴LO 3NFs

(Notice: these UT will also induce contributions to the exchange currents.)

Scheme dependence in chiral EFT



Off-shell ambiguities in the contact interactions at N³LO (¹S₀, ³S₁, ³S₁-³D₁):

$$\text{E.g.: } \langle p', {}^1S_0 | V_{\text{cont}} | p, {}^1S_0 \rangle = \underbrace{\tilde{C}_{1S_0}}_{\text{tuned to the scatt. length}} + \underbrace{C_{1S_0}(p'^2 + p^2)}_{\text{tuned to the effective range}} + \underbrace{D_{1S_0}p^2p'^2 + D_{1S_0}^{\text{off}}(p'^2 - p^2)^2}_{\text{tuned to the first shape parameter}}$$

$\Rightarrow D_{1S_0}^{\text{off}}$ cannot be fixed from NN data Hammer, Furnstahl '00; Beane, Savage '01; Reinert, Krebs, EE '18

Thus, it should be possible to eliminate the off-shell contacts via a suitable UT. Indeed:

$$U = e^{\gamma_1 T_1 + \gamma_2 T_2 + \gamma_3 T_3} \quad \text{with} \quad T_1 \propto (p_1'^2 + p_2'^2 - p_1^2 - p_2^2), \quad T_2 \propto (p_1'^2 + p_2'^2 - p_1^2 - p_2^2) \vec{\sigma}_1 \cdot \vec{\sigma}_2, \quad T_3 = \dots$$

$$\Rightarrow \delta H = U^\dagger H U - H = \sum_i \gamma_i \left[\left(\underbrace{H_{\text{kin}}}_{\text{induce off-shell NN contact interactions}} + \underbrace{V_{1\pi}^{(0)} + V_{\text{cont}}^{(0)}}_{\text{induce N}^4\text{LO 3NFs}} + \mathcal{O}(Q^2) \right), T_i \right] + \mathcal{O}(\gamma_i^2)$$

(Notice: these UT will also induce contributions to the exchange currents.)

In the SMS chiral interactions, we **made a choice** by setting $D_{1S_0}^{\text{off}} = D_{3S_1}^{\text{off}} = D_{\epsilon_1}^{\text{off}} = 0$.

But we do not **have to** make this choice. Any natural-sized values of these off-shell LECs are just as good as setting $D_{1S_0}^{\text{off}} = D_{3S_1}^{\text{off}} = D_{\epsilon_1}^{\text{off}} = 0$!

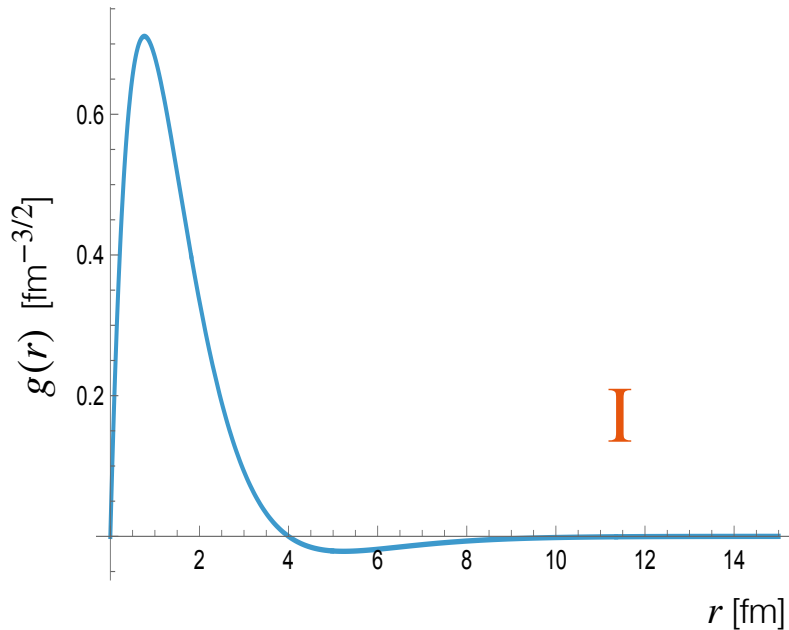
Gedankenexperiment



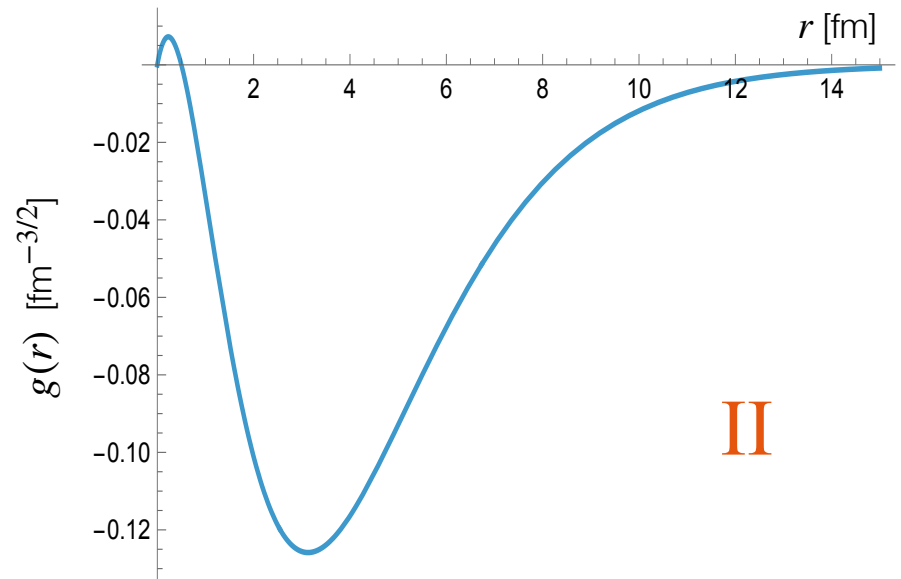
Bob knows that physics is independent on the choice of basis in the Hilbert space.

He uses states $|\Psi_{\text{Bob}}\rangle = \hat{U} |\Psi_{\text{Alice}}\rangle$, where $\hat{U} = 1 - 2|g\rangle\langle g|$, $\langle g|g\rangle = 1$.

Specifically, choose $g(\vec{r}) \equiv \langle \vec{r} | g \rangle = C r (1 - \beta r) e^{-\alpha r}$ Peter Sauer, PRL 32 (74)



$$\alpha = 1 \text{ fm}^{-1}, \quad \beta = 0.25 \text{ fm}^{-1}$$



$$\alpha = 0.7 \text{ fm}^{-1}, \quad \beta = 2.0 \text{ fm}^{-1}$$

Gedankenexperiment



Bob knows that physics is independent on the choice of basis in the Hilbert space.

He uses states $|\Psi_{\text{Bob}}\rangle = \hat{U} |\Psi_{\text{Alice}}\rangle$, where $\hat{U} = 1 - 2|g\rangle\langle g|$, $\langle g|g\rangle = 1$.

Phenomenological:

AV18, CD-Bonn, Nijm-I,II,
 $V_{\text{low-k}}$, V_{SRG} , INOY,
JISP-16,...

Lattice QCD:

HAL QCD baryon-baron
potentials

potential models
used to analyze
femtoscopia data

Chiral EFT:

Bochum, Idaho, Chalmers,
Jülich, Norfolk, ...

Phenomenological:

AV18, CD-Bonn, Nijm-I,II,
 $V_{\text{low-k}}$, V_{SRG} , INOY,
JISP-16,...

Lattice QCD:

HAL QCD baryon-baryon
potentials

potential models
used to analyze
femtoscopia data

Chiral EFT:

Bochum, Idaho, Chalmers,
Jülich, Norfolk, ...