Nuclear physics across energy scales, C3NT Wuhan Sep 18-21, 2025

Understanding Xe isotopes near A=130 through the prolate-oblate shape phase transition

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Outline

I Introduction

II A QPT perspective on Xe nuclei

V Conclusion

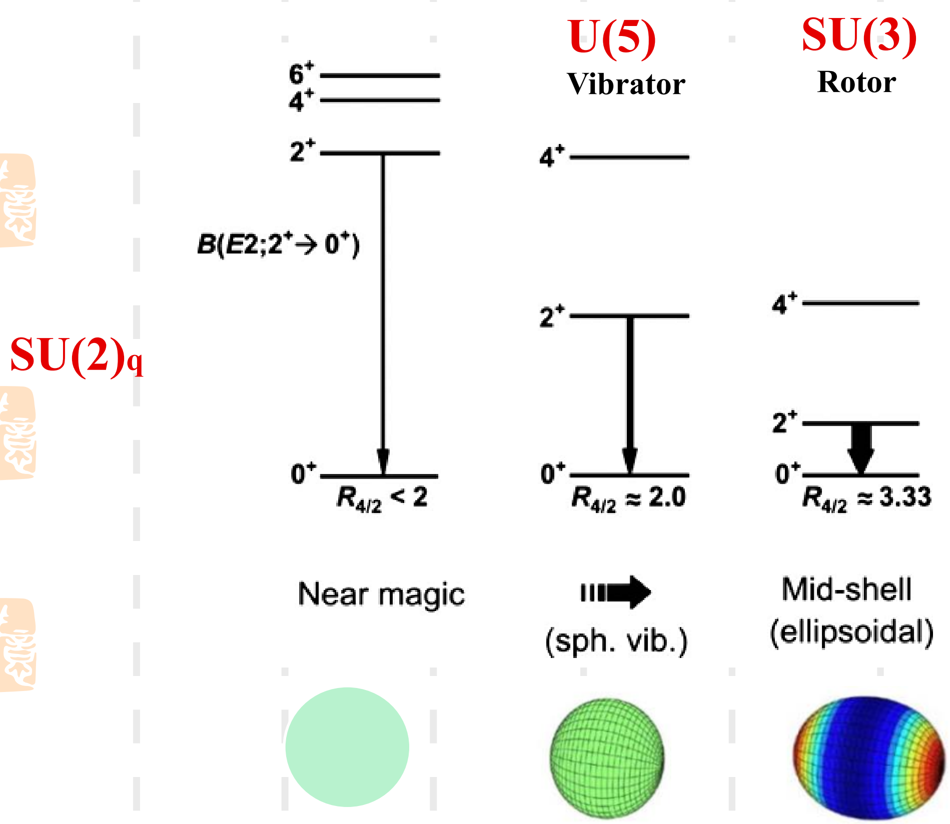
Nuclear shape (deformation) manifested by spectra

A common feature of systems that have rotational spectra is the existence of a "deformation", by which is implied a feature of anisotropy that makes it possible to specify an orientation of the system as a whole. In a molecule, as in a solid body, the deformation reflects the highly anisotropic mass distribution, as viewed from the intrinsic coordinate frame defined by the equilibrium positions of the nuclei. In the nucleus, the rotational degrees of freedom are associated with the deformations in the nuclear equilibrium shape that result from the shell structure. (Evidence for

NUCLEAR STRUCTURE
Volume II: Nuclear Deformations

AAGE BOHR
The Niels Bohr Institute, University of Copenhagen

BEN R. MOTTELSON
Nordita, Copenhagen



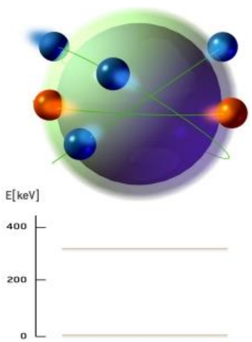
0p-0h, 2p-2h, 4p-4h …

$$H = H_0 + H_{\text{pair}} + H_{QQ}$$

Collective Shapes

PP+QQ interaction for heavy nuclei

Shell Model (Fermions) ⟶ IBM (Bosons) ⟶ Collective Model (shape)



D pair ⟶ d boson

S pair ⟶ s boson

$N_B \to \infty$, $\hbar \sim 1/N$ ⟶ Geometry (β, γ deformation)

Pairing force:

$$V^{\mathrm{IBM}}_{\mathrm{pairing}} = \varepsilon_s s^\dagger s + \varepsilon'_s (s^\dagger s)^2 + \cdots$$

Quadrupole force:

$$V^{\mathrm{IBM}}_{QQ} = f_2 \hat{Q}\cdot\hat{Q} + f_3 (\hat{Q}\times\hat{Q}\times\hat{Q})^{(0)} + f_4 (\hat{Q}\cdot\hat{Q})^2 + \cdots$$

$$\hat{Q}_u = (d^\dagger \tilde{s} + s^\dagger \tilde{d})^{(2)}_u + \chi (d^\dagger \times \tilde{d})^{(2)}_u$$

introducing N-body operators in the boson space even if the fermion operator was only one and/or two body. If one has to do this, the introduction of the boson space

NUCLEAR SHELL MODEL AND INTERACTING BOSONS

T. OTSUKA and A. ARIMA

Department of Physics, University of Tokyo, Tokyo, Japan

and

F. IACHELLO

Shape Phase Diagram for identical bosons system

at leading order

$$\hat{H} = \varepsilon_s s^\dagger s + f \hat{Q} \cdot \hat{Q}$$

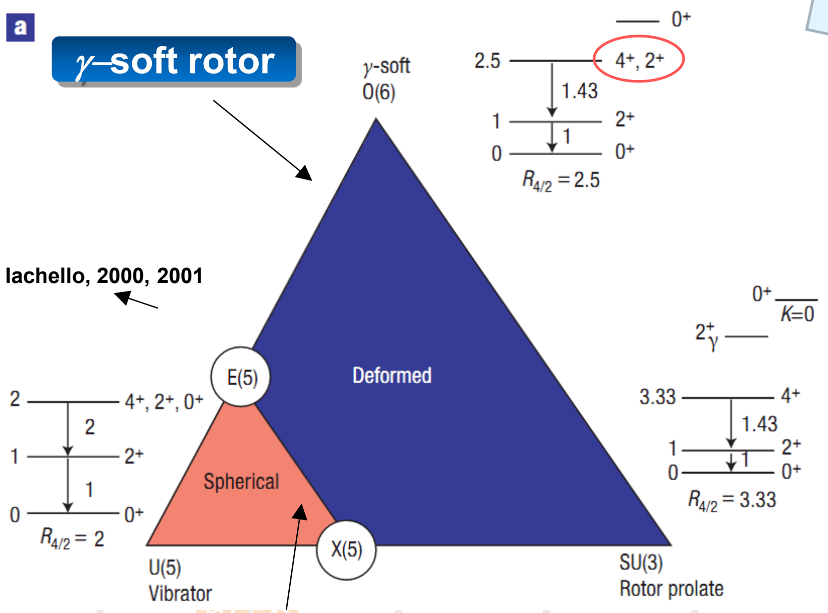
$$|\beta, \gamma, N\rangle = [s^\dagger + \beta \cos\gamma d_0^\dagger + \frac{1}{\sqrt{2}} \beta \sin\gamma (d_2^\dagger + d_{-2}^\dagger)]^N |0\rangle$$

$$V(\beta, \gamma, \varepsilon_s, f) = \frac{1}{N} \langle N, \beta, \gamma | \hat{H} | N, \beta, \gamma \rangle_{N \to \infty} = \frac{E_0(\varepsilon_s, f) + A\beta^2 + B\beta^3 \cos 3\gamma + C\beta^4}{(1+\beta^2)^2}$$

Order parameters (β, γ)

γ–soft rotor

by Iachello, 2000, 2001

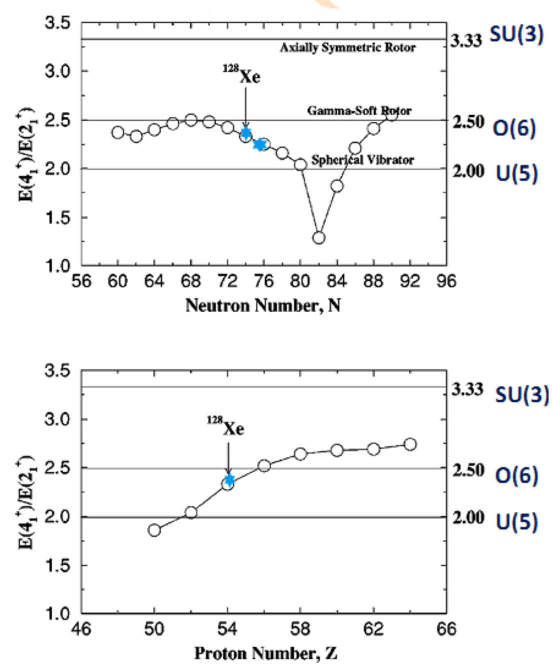


β–soft rotor

Cejnar P, Jolie J and Casten R F 2010 *Rev. Mod. Phys.* **82** 2155

PHYSICAL REVIEW C **69**, 064322 (2004)

Searching for E(5) behavior in nuclei



An updated SPT views on Xe nuclei near A=130

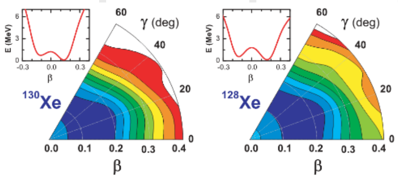
Therefore, we conclude that ^{128}Xe is not a close realization of E(5) symmetry, leaving ^{130}Xe as the most likely candidate among the Xe isotopes. Our analysis highlights the importance

Not perfect E(5) ←

PHYSICAL REVIEW C **80**, 061304(R) (2009)

Robust test of E(5) symmetry in ^{128}Xe

L. Coquard,[1] N. Pietralla,[1] T. Ahn,[1,2] G. Rainovski,[3] L. Bettermann,[4] M. P. Carpenter,[5] R. V. F. Janssens,[5] J. Leske,[1] C. J. Lister,[5] O. Möller,[1] W. Rother,[4] V. Werner,[2] and S. Zhu[5]



Ok for Spectra ! ←

PHYSICAL REVIEW C **81**, 034316 (2010)

Microscopic description of spherical to γ-soft shape transitions in Ba and Xe nuclei

Z. P. Li,[1,2] T. Nikšić,[2] D. Vretenar,[2,*] and J. Meng[1,3]

		^{128}Xe		
J_i^π	$<J^\pi\|E2\|J^\pi>_{exp}$ [eb]	Q_{exp} [eb]	Q_{PMMU} [eb]	Q_{DF} [eb]
2_1^+	-0.58 (-15 +12)	-0.44 (-12 +9)	-0.37	-0.49
4_1^+	-1.38 (13)	-1.04 (10)	-0.45	-0.28
2_2^+	+0.01 (-10 +9)	+0.008 (-0.08 +0.07)	+0.33	+0.49

in the Q_0-Q_2 plane. In other words, for a nucleus with no well-defined shape there can be rich possibilities to develop shape(s) including unusual ones when it gets excited. This seems to be what we have encountered in the present examples.

New puzzles ←

PHYSICAL REVIEW C **106**, 034311 (2022)

Structure of ^{126,128}Xe studied in Coulomb excitation measurements

S. Kisyov,[1,*] C. Y. Wu,[1] J. Henderson,[2] A. Gade,[3,4] K. Kaneko,[5] Y. Sun,[6] N. Shimizu,[7] T. Mizusaki,[8] D. Rhodes,[3,4,†] S. Biswas,[3] A. Chester,[3] M. Devlin,[9] P. Farris,[3,4] A. M. Hill,[3,4] J. Li,[3] E. Rubino,[3] and D. Weisshaar[3]

With one more neutron:

Phys. Rev. Lett. 128, 082301 (2022)

Evidence of the triaxial structure of ^{129}Xe at the Large Hadron Collider

Benjamin Bally,[1] Michael Bender,[2] Giuliano Giacalone,[3] and Vittorio Somà[4]

the average intrinsic moments $(\bar{\beta}_v, \bar{\gamma}_v) = (0.19, 23.6°)$. ^{129}Xe appears to be, hence, a rigid triaxial spheroid. ←

Phys. Rev. Lett. 133, 192301 (2024)

Exploring the Nuclear Shape Phase Transition in Ultra-Relativistic ^{129}Xe+^{129}Xe Collisions at the LHC

Shujun Zhao,[1,2] Hao-jie Xu,[2,3] You Zhou,[4] Yu-Xin Liu,[1,5,6] and Huichao Song[1,5,6]

previously interpreted as the evidence for the rigid triaxial deformation of ^{129}Xe, can also be well explained by the γ-soft deformation of ^{129}Xe. These two correlators ←

γ–soft rotor modes manifested in energy spectrum

ANNALS OF PHYSICS 123, 468–492 (1979)

Interacting Boson Model of Collective Nuclear States IV. The $O(6)$ Limit

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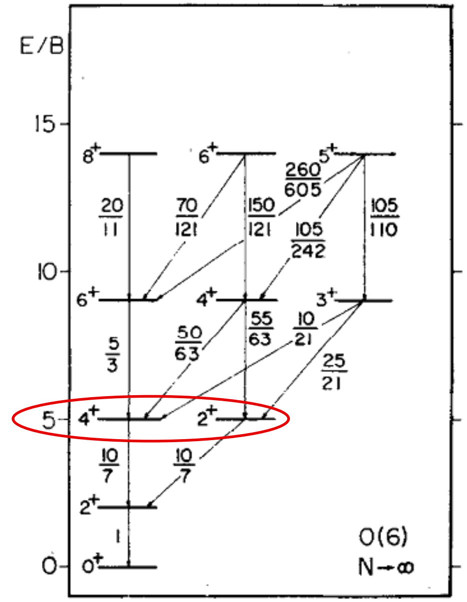
AND

F. IACHELLO



What have we missed?

The consequence is that level schemes are not a sensitive enough probe, and $B(E2)$ values can provide more detailed comparisons, but quadrupole moments are the most stringent test of theoretical models [11].

Phys. Rev. Lett. 87, 163501 (2001)

Quantum Phase Transition for γ-Soft Nuclei

J. Jolie,[1] R. F. Casten,[2] P. von Brentano,[1] and V. Werner[1]
[1]*Institute of Nuclear Physics, University of Cologne, Zülpicherstrasse 77, D-50937 Cologne, Germany*
[2]*Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520-8124*
(Received 20 July 2001; published 2 October 2001)

We examine a quantum phase transition in γ-soft nuclei, where the O(6) limit is simultaneously a dynamical symmetry of the U(6) group of the interacting boson model and a critical point of a prolate-oblate phase transition. This is the only example of phase transitional behavior that can be described analytically for a finite s, d boson system.

Clues

Phys. Rev. Lett. 130, 052501 (2023)

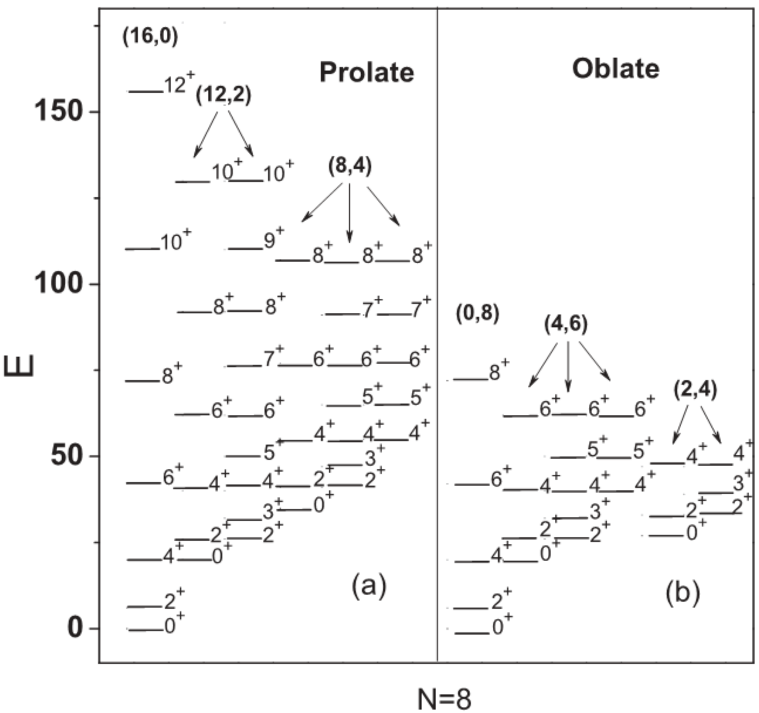
Quasi-SU(3) coupling induced oblate-prolate shape phase transition in the Casten triangle

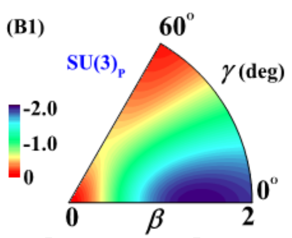
K. Kaneko[1*], Y. Sun[2†], N. Shimizu[3], and T. Mizusaki[4]

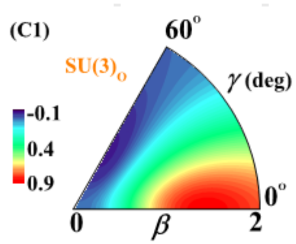
ered that while the QQ force acting on the $(1g_{9/2}, 2d_{5/2})$ orbit pairs tends driving the evolution toward the O(6) dynamical limit, that acting on $(1h_{11/2}, 2f_{7/2})$ robustly changes the shape from oblate to prolate at $N = 76$, moving the deformed

An SU(3) scheme for rotor modes

PHYSICAL REVIEW C **85**, 064312 (2012)







$$\langle -\hat{Q}\cdot\hat{Q}\rangle \qquad \langle -(\hat{Q}\times\hat{Q}\times\hat{Q})^{(0)}\rangle$$

Quadrupole force:

$$V_{QQ}^{\mathrm{IBM}} = f_2\hat{Q}\cdot\hat{Q} + f_3(\hat{Q}\times\hat{Q}\times\hat{Q})^{(0)} + f_4(\hat{Q}\cdot\hat{Q})^2 + \cdots$$

$$[\hat{L}_u,\hat{L}_v] = -\sqrt{2}\langle 1u,1v|1u+v\rangle\hat{L}_{u+v},$$
$$[\hat{L}_u,\hat{Q}_v] = -\sqrt{6}\langle 1u,2v|2u+v\rangle\hat{Q}_{u+v},$$
$$[\hat{Q}_u,\hat{Q}_v] = \frac{3}{8}\sqrt{10}\langle 2u,2v|1u+v\rangle\hat{L}_{u+v}.$$

for SU(3)

$$\hat{C}_2[\mathrm{SU}(3)] = 2\hat{Q}\cdot\hat{Q} + \tfrac{3}{4}\hat{L}^2, \qquad (1)$$
$$\hat{C}_3[\mathrm{SU}(3)] = -\tfrac{4\sqrt{35}}{9}(\hat{Q}\times\hat{Q}\times\hat{Q})^{(0)} - \tfrac{\sqrt{15}}{2}(\hat{L}\times\hat{Q}\times\hat{L})^{(0)} \qquad (2)$$

$$\langle\hat{C}_2[\mathrm{SU}(3)]\rangle = \lambda^2+\mu^2+3\lambda+3\mu+\lambda\mu,$$
$$\langle\hat{C}_3[\mathrm{SU}(3)]\rangle = \tfrac{1}{9}(\lambda-\mu)(2\lambda+\mu+3)(\lambda+2\mu+3)$$

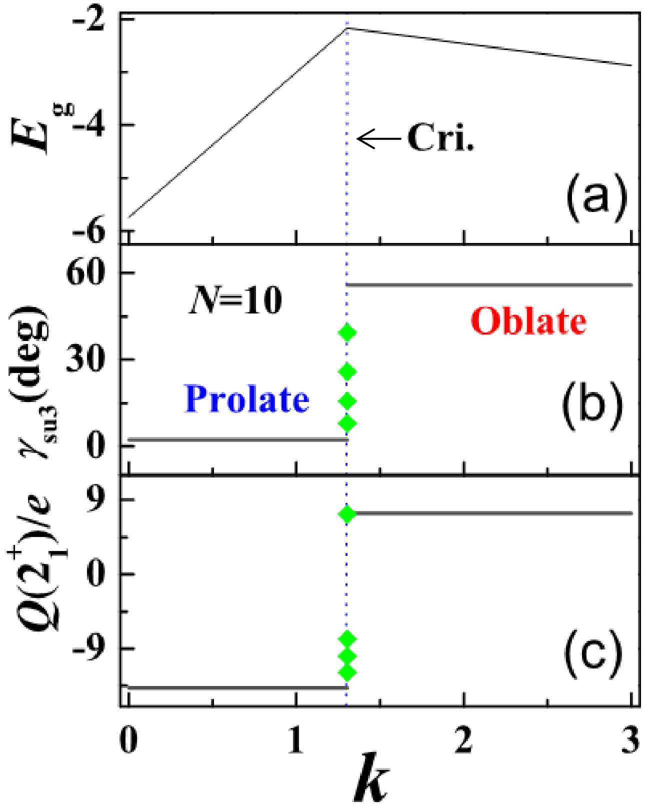
γ deformation vs su3 irrep (λ, μ)

$$\gamma = \tan^{-1}\left(\frac{\sqrt{3}(\mu+1)}{2\lambda+\mu+3}\right)$$

Prolate-oblate SPT in the SU(3) IBM

$$\hat{H}_{SU(3)} = -\frac{1}{N}\hat{C}_2[SU(3)] + \frac{k}{N^2}\hat{C}_3[SU(3)]$$

1st order SPT



$$\hat{T}^{E2} = e\hat{Q}$$

In IBM

$$\begin{array}{ccccccc} U(6) & \supset & SU(3) & \supset & & & O(3) \\ \downarrow & & \downarrow & & & & \downarrow \\ [N] & & (\lambda,\mu) & & K & & L \end{array}$$

$$(\lambda,\mu) = (2N,0),(2N-4,2),\ldots,(0,N) \text{ or } (2,N-1)$$
$$(2N-6,0),(2N-10,2),\ldots$$

$$\hat{\gamma}_{su3} = \mathrm{Tan}^{-1}\left(\frac{\sqrt{3}(\hat{\mu}+1)}{2\hat{\lambda}+\hat{\mu}+3}\right)$$

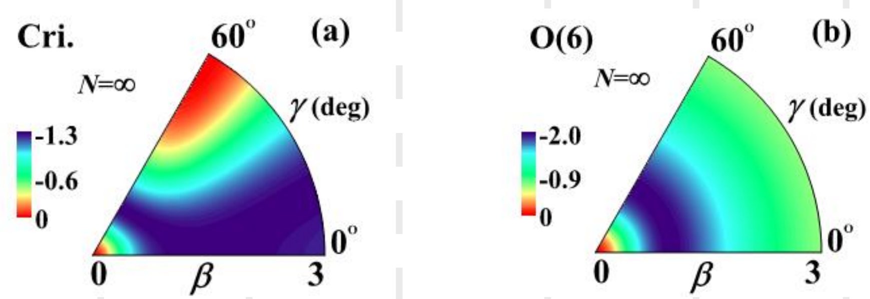
$$E_g(k_c,2N,0) = E_g(k_c,0,N) \Rightarrow k_c = \frac{3N}{2N+3}$$

$$||$$

$$E_g(k_c,\lambda,\mu) \quad \lambda+2\mu = 2N$$

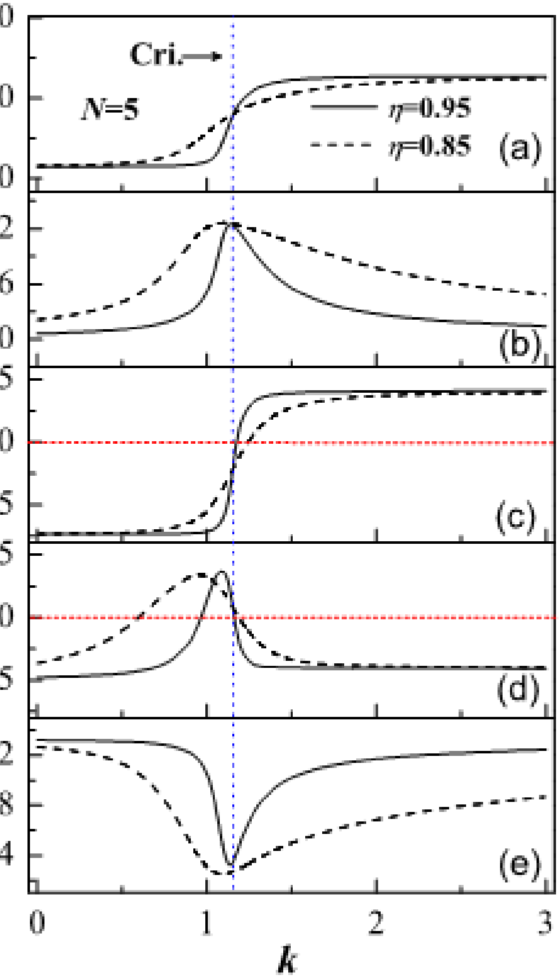
Finite *N*: γ deformations coexistence

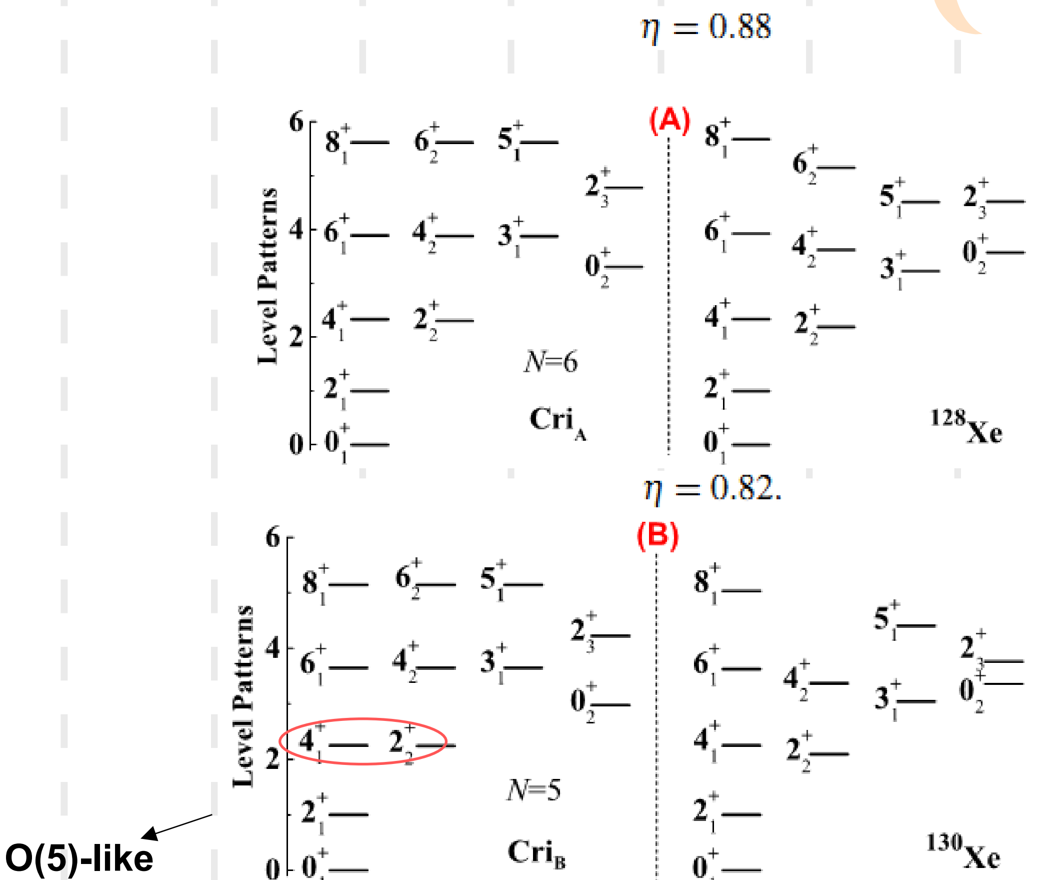
Infinite *N*: γ soft



Critical behaviors corrected by pairing plus finite *N*

$$\hat{H} = \varepsilon\left[\frac{\eta}{8}\hat{H}_{\mathrm{SU(3)}} + (\eta - 1)s^{\dagger}s\right]$$





Application to Xe128,130

The $B(E2)$ values (in W.u.) $\hat{T}^{E2} = e\hat{Q}$

$L_i^\pi \to L_f^\pi)$	^{128}Xe	Cri$_A$	SM	DF	CBS	E(5)	^{130}Xe	Cri$_B$	SM	DF	CBS	E(5)
$2_1^+ \to 0_1^+$	46.1(15)	46.1	44	46.1	46.1	46.1	32(3)	32	34	32	32	32
$4_1^+ \to 2_1^+$	55.4(32)	63	66	65	72	77	47(4)	44	54	45	53	54
$6_1^+ \to 4_1^+$	76(10)	69	80	82	92	102	60^{+14}_{-12}	45	68	56	71	71
$2_2^+ \to 2_1^+$	44(4)	66	54	44	72	77	37(3)	44	48	37	53	54
$2_2^+ \to 0_1^+$	0.58(9)	0.59	0.24	1.84	0	0	0.23(2)	0.16	0.2	0.74	0	0
$0_2^+ \to 2_1^+$	3.7(6)[a]	16	-	-	0	40	$18(^{+17}_{-11})$[b]	15	-	-	24	28
$0_2^+ \to 2_2^+$	52.8(76)[a]	108	-	-	92	0	$120(^{+110}_{-70})$[b]	78	-	-	0	0

the spectroscopic quadrupole moments (unit in eb)

$Q(L^+)$	^{128}Xe	Cri$_A$	SM	DF(25.4°)	^{130}Xe	Cri$_B$	SM	DF(26.8°)
$Q(2_1^+)$	-0.44^{+9}_{-12}	-0.26	-0.37	-0.52	-0.38^{+17}_{-14}	-0.30	-0.26	-0.33
$Q(4_1^+)$	-1.04(10)	-0.61	-0.45	-0.31	-0.41(12)	-0.61	-0.32	-0.17
$Q(2_2^+)$	**$0.008^{+0.07}_{-0.08}$**	**0.066**	0.33	0.52	**0.1(1)**	**0.11**	0.25	0.33
$Q(4_2^+)$	-	-0.25	-	-0.98	-	-0.22	-	-0.81

Fine !

Conclusion

1, Even Xe nuclei near A=130 are gamma soft and lie close to the critical point of the shape transition from prolate to oblate.

2, Odd Xe nuclei?

Thanks for your attention!