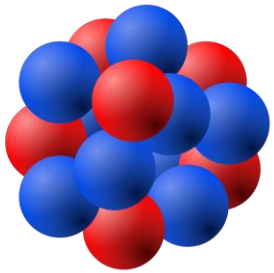
Nuclear physics across energy scales
2025, Wuhan

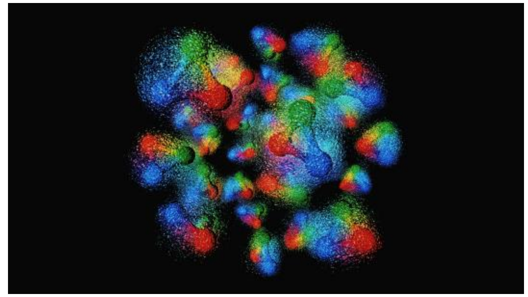
Light nuclei structure by lattice effective field theory

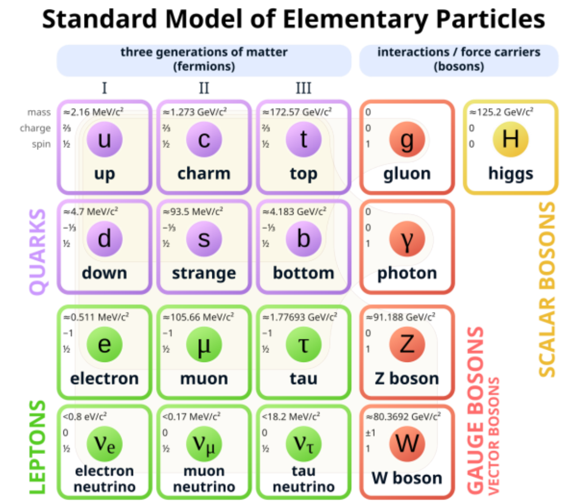
Shihang Shen
Beihang University

Contents

- Introduction
- Nuclear Lattice Effective Field Theory
- Helium-4: puzzle for nuclear forces?
- Beryllium Isotopes: Clustering, Molecular Orbitals, and Halo
- Carbon-12: Hoyle state, single-particle/collective excitations
- Summary







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Proper degrees-of-freedom (nucleons) and energy scale

- Ekström, C. Forssén, G. Hagen, G. R. Jansen, W. Jiang and T. Papenbrock, what is ab initio in nuclear theory? Front. Phys. 11:1129094 (2023)
- R. Machleidt, what is ab initio? Few-Body Syst (2023) 64

*Do the same nuclear forces that explain **free-space scattering** experiments also explain the properties of **finite nuclei** and **nuclear matter** when applied in nuclear many-body theory?*

R. Machleidt, what is ab initio? Few-Body Syst (2023) 64

Nuclear forces

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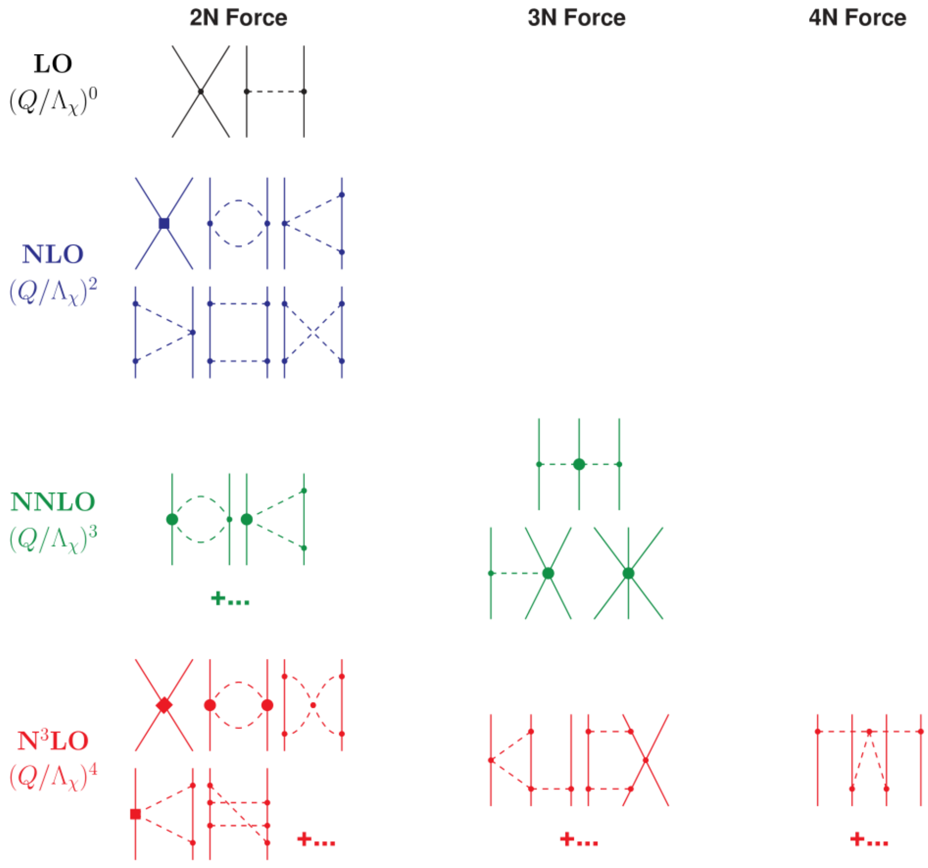
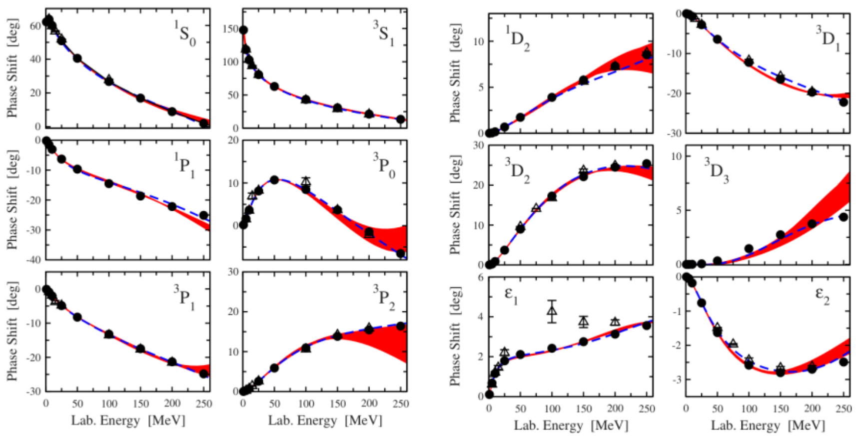
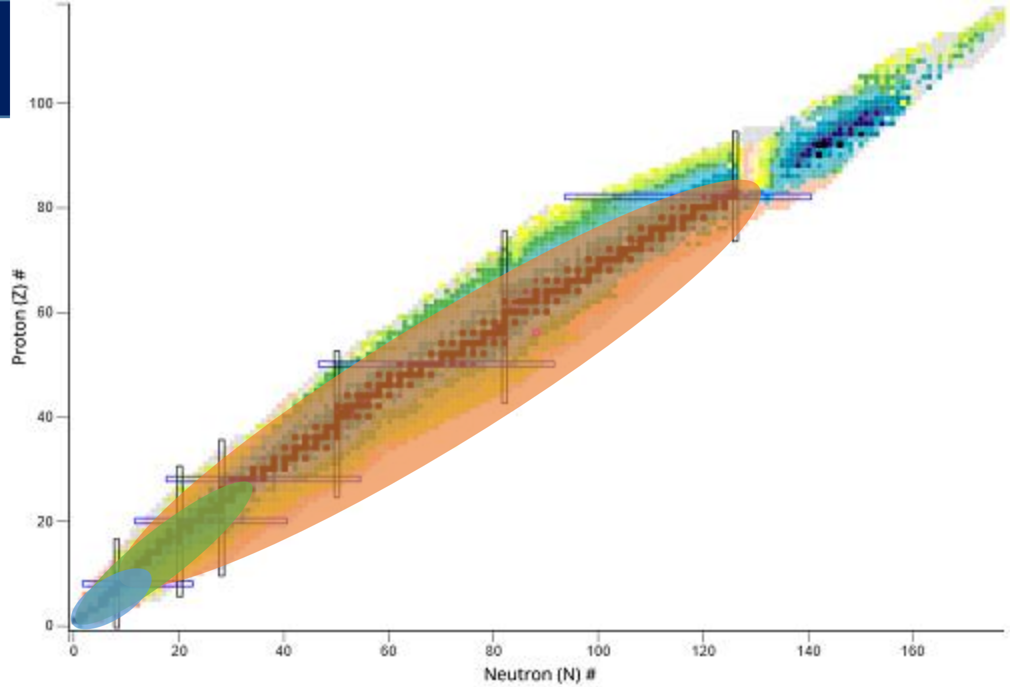




Neutron-proton phase shifts at N^{3}LO

Many-body frameworks



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- Many-body perturbation theory, A. Tichai, R. Roth, and T. Duguet, Front. Phys. 8, 164 (2020)
- Relativistic Brueckner-Hartree-Fock, SS, et al., Prog. Part. Nucl. Phys. 109, 103713 (2019)
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- ... ...

Clustering plays an important role in nuclear physics, especially light nuclei

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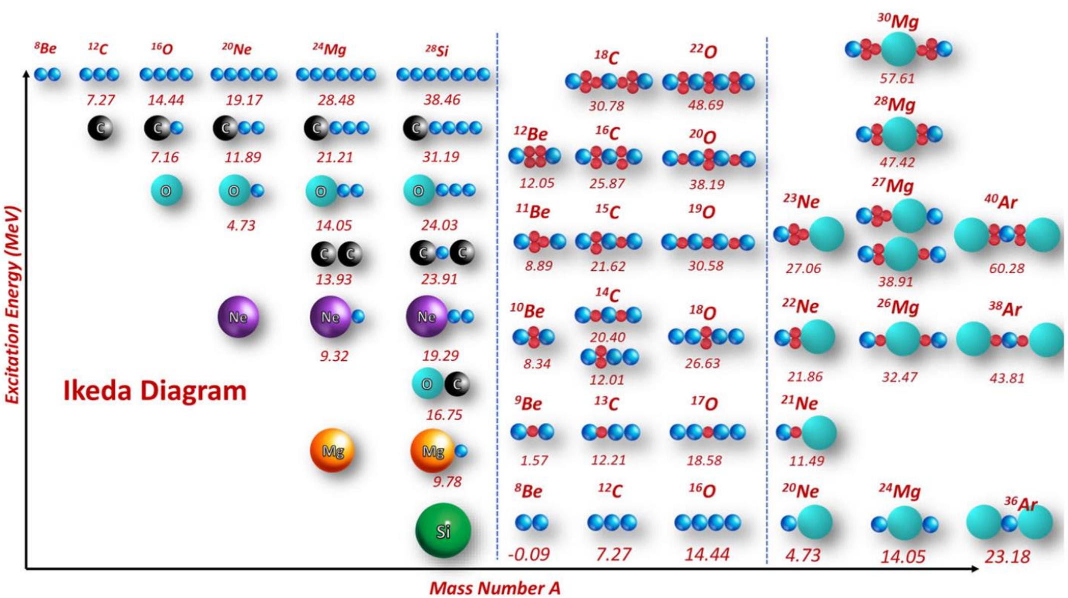
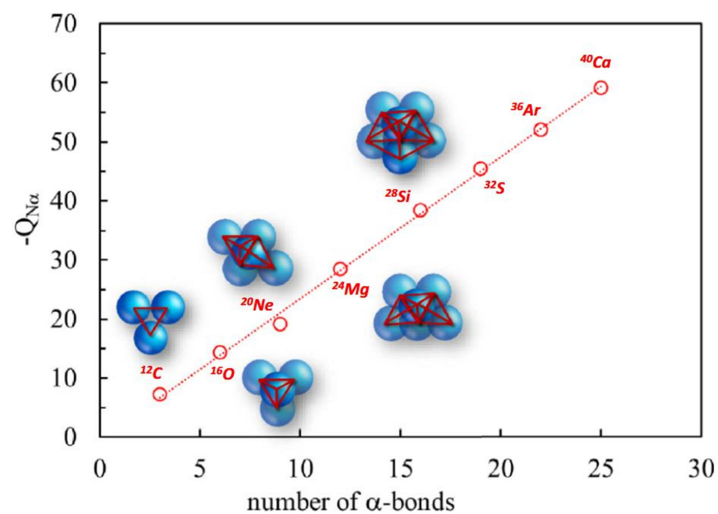
- Y.-G. Ma and S. Zhang, α-clustering effects in relativistic heavy-ion collisions, Sci. Sin. Phys. Mech. Astron. 54, 292004 (2024)
- I. Lombardo and D. Dell'Aquila, Clusters in light nuclei: history and recent developments, Riv. Nuovo Cimento 46, 521 (2023)
- B. Zhou, Y. Funaki, H. Horiuchi, and A. Tohsaki, Nonlocalized clustering and evolution of cluster structure in nuclei, Front. Phys. 15, 14401 (2020)
- M. Freer, H. Horiuchi, Y. Kanada-En'yo et al., Microscopic clustering in light nuclei. Rev. Mod. Phys. 90, 035004 (2018)
- Z. Ren and B. Zhou, Alpha-clustering effects in heavy nuclei, Front. Phys. 13, 132110 (2018)
- ... ...





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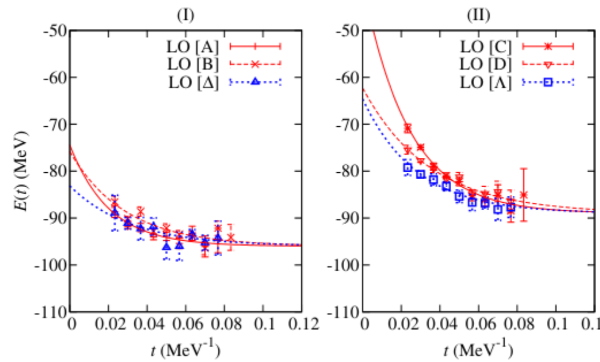
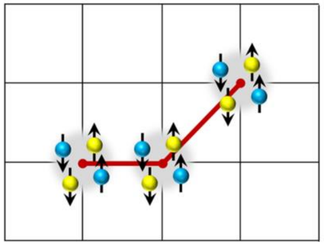
Nuclear lattice effective field theory

- First ab initio description of Hoyle state





PRL 106, 192501 (2011) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 13 MAY 2011

***Ab Initio* Calculation of the Hoyle State**

Evgeny Epelbaum,[1] Hermann Krebs,[1] Dean Lee,[2] and Ulf-G. Meißner[3,4]

PRL 109, 252501 (2012) PHYSICAL REVIEW LETTERS week ending 21 DECEMBER 2012

Structure and Rotations of the Hoyle State

Evgeny Epelbaum,[1] Hermann Krebs,[1] Timo A. Lähde,[2] Dean Lee,[4] and Ulf-G. Meißner[5,2,3]

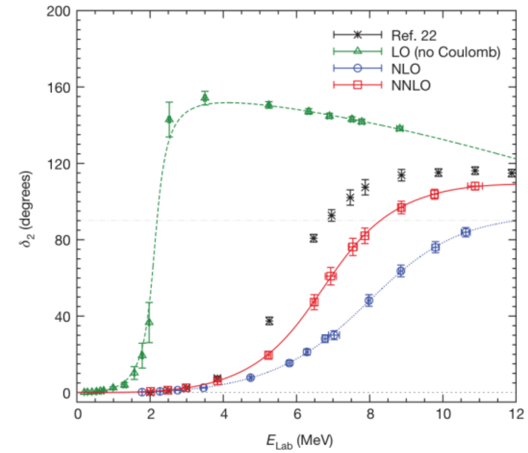
- First ab initio description of α-α scattering

Letter | Published: 02 December 2015

***Ab initio* alpha–alpha scattering**

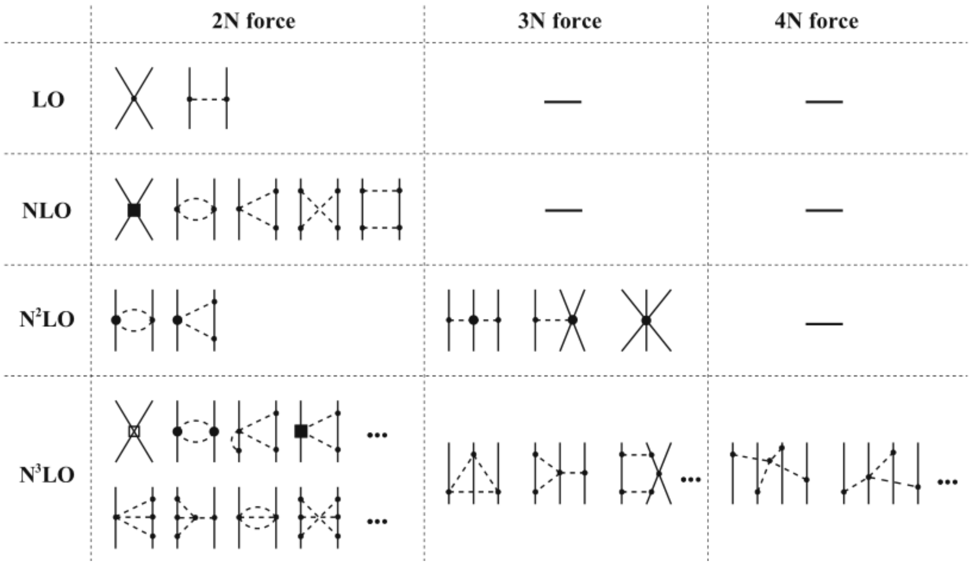
Serdar Elhatisari, Dean Lee, Gautam Rupak, Evgeny Epelbaum, Hermann Krebs, Timo A. Lähde, Thomas Luu & Ulf-G. Meißner

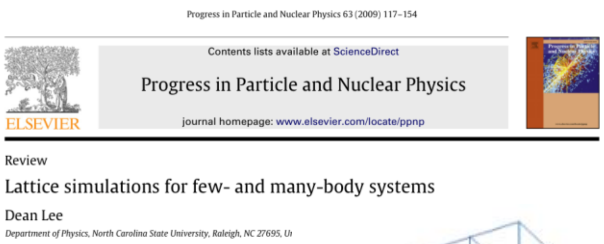
Nature **528**, 111–114 (2015) | Cite this article

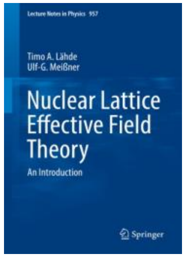


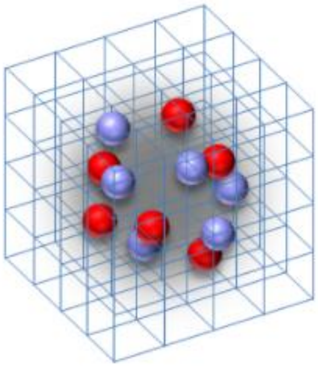
- Nuclear thermodynamics B. N. Lu, et al., Phys. Rev. Lett. 125, 192502 (2020)
- Clustering in hot dilute matter Z. Ren, et al., Phys. Lett. B 850, 138463 (2024)
- Structure factors for hot neutron matter Y. Z. Ma, et al., Phys. Rev. Lett. 132, 232502 (2024)
- Hyperneutron matter H. Tong, S. Elhatisari, and U. G. Meißner, Sci. Bull. 70, 825 (2025)
- … …

Theoretical Framework

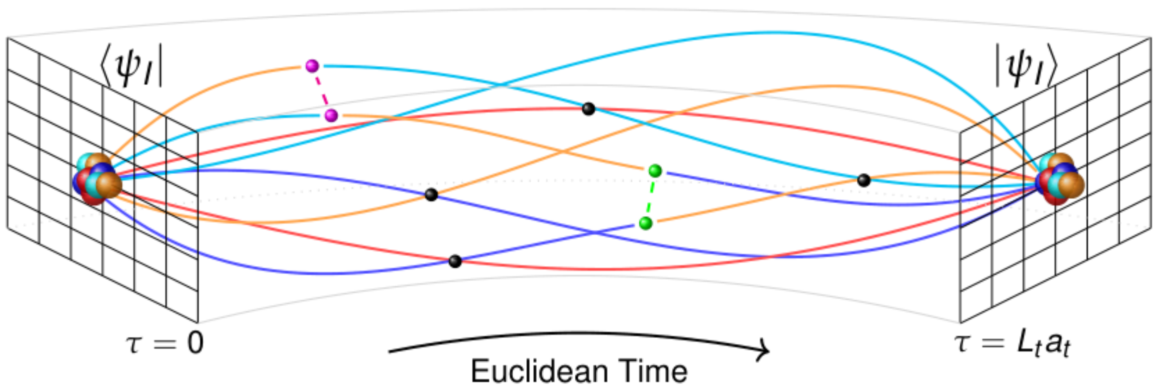








$$M = \exp(-a_t H) \qquad \langle \Psi_I | M M \ldots M O M \ldots M M | \Psi_I \rangle$$



Upper left figure is in courtesy of E. Epelbaum
lattice figure from https://www.physics.ncsu.edu/ntg/leegroup/research.html
Lower figure is in courtesy of S. Elhatisari

Theoretical Framework

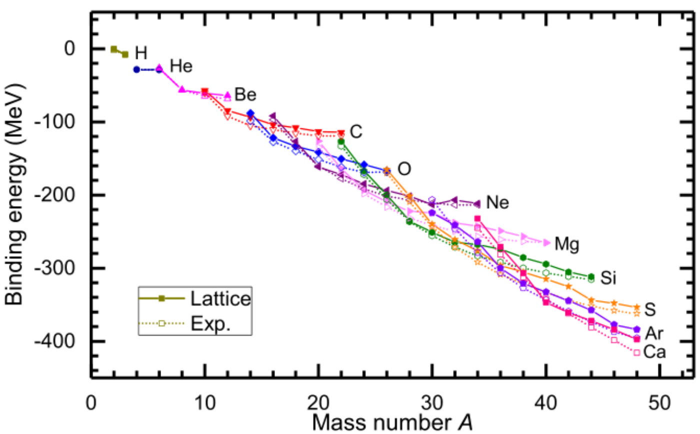
- Wigner SU(4) symmetric interaction (spin and isospin independent):

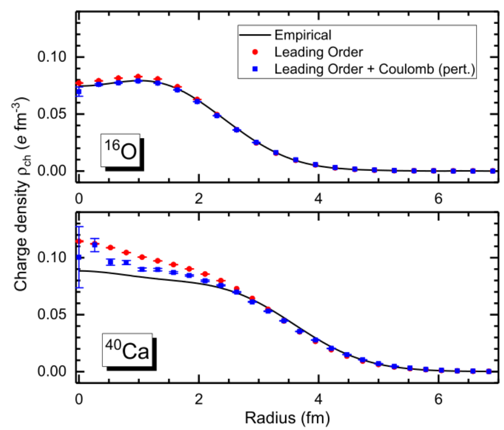
S. Elhatisari et al., PRL 119, 222505 (2017)

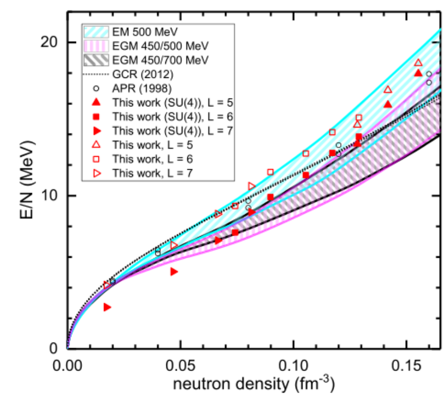
$$V = \frac{C_2}{2!}\sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2 + \frac{C_3}{3!}\sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^3,$$

$$\tilde{\rho}(\mathbf{n}) = \sum_{i=1}^{A} \tilde{a}_i^\dagger(\mathbf{n})\tilde{a}_i(\mathbf{n}) + s_L \sum_{|\mathbf{n}'-\mathbf{n}|=1} \sum_{i=1}^{A} \tilde{a}_i^\dagger(\mathbf{n}')\tilde{a}_i(\mathbf{n}'), \qquad \tilde{a}_i(\mathbf{n}) = a_i(\mathbf{n}) + s_{\rm NL} \sum_{|\mathbf{n}'-\mathbf{n}|=1} a_i(\mathbf{n}').$$

- Sign problem is largely suppressed J.W. Chen, D. Lee, T. Schäfer, PRL, 93, 242302 (2004)
- Ground state properties of light and medium mass nuclei, and neutron matter can be well reproduced B.-N. Luu et al., PLB 797 (2019) 134863







- Higher order (e.g. N3LO) can be built on with perturbation

Elhatisari et al., Nature 630, 59 (2024)

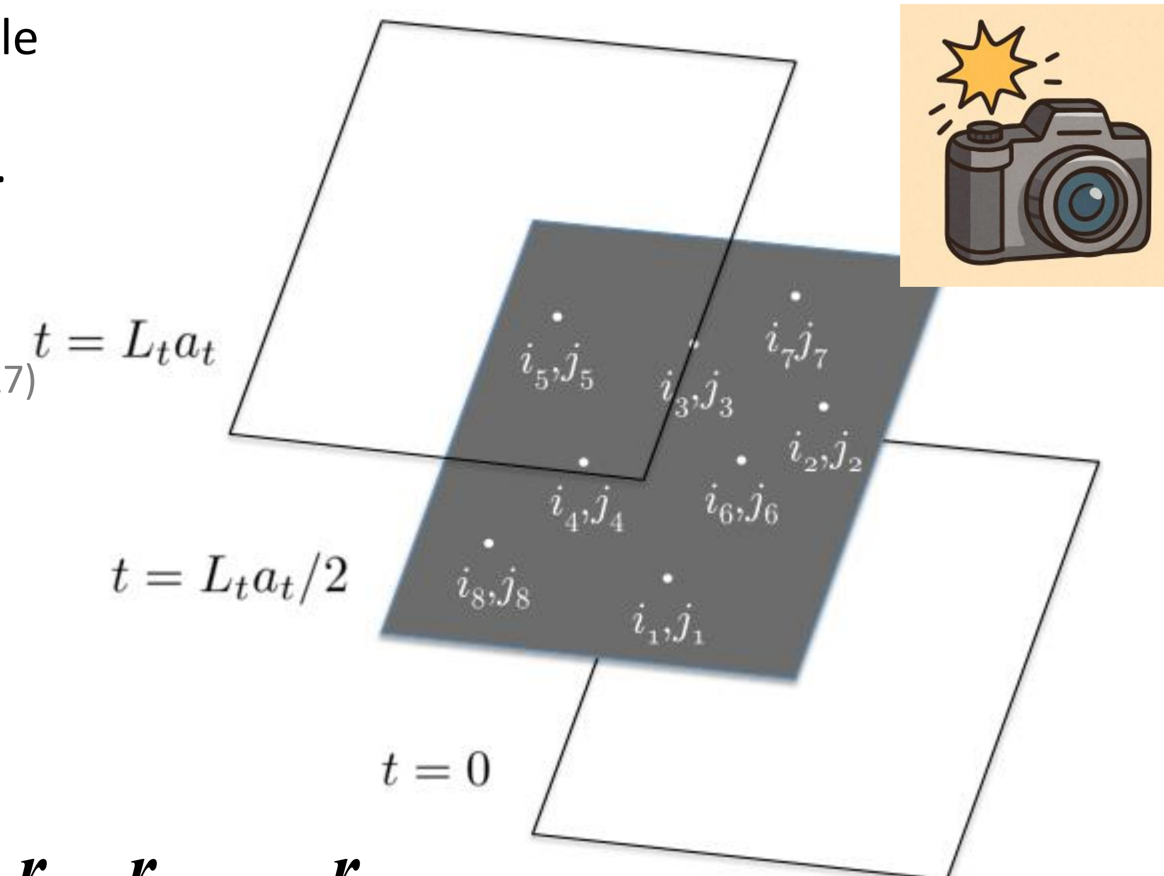
Theoretical Framework

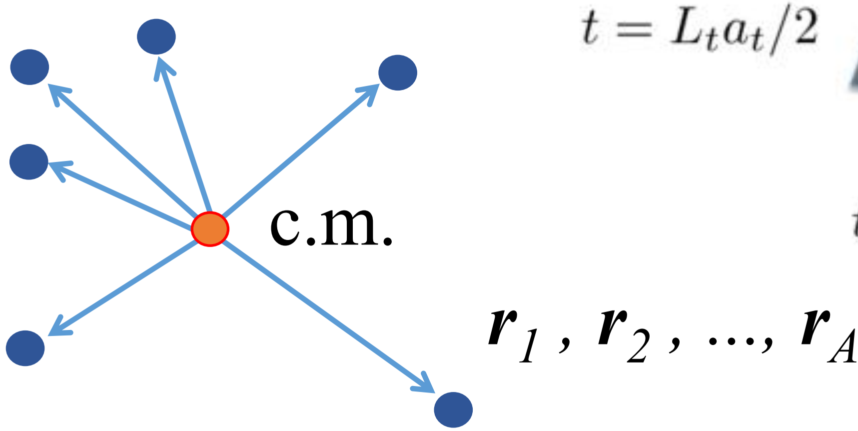
- Pinhole algorithm

$$\langle \Phi_0 | e^{-H\tau/2} \rho(\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_A) e^{-H\tau/2} | \Phi_0 \rangle$$

A time slice is inserted to sample the positions and spin-isospin indices in the middle time step.

S. Elhatisari et al., PRL 119, 222505 (2017)





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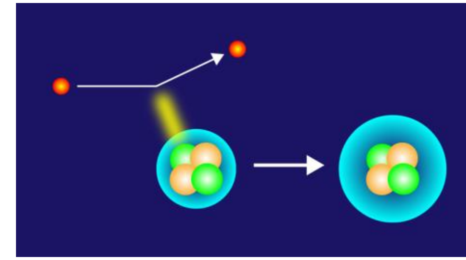
^{4}He: Puzzle for Nuclear Forces?

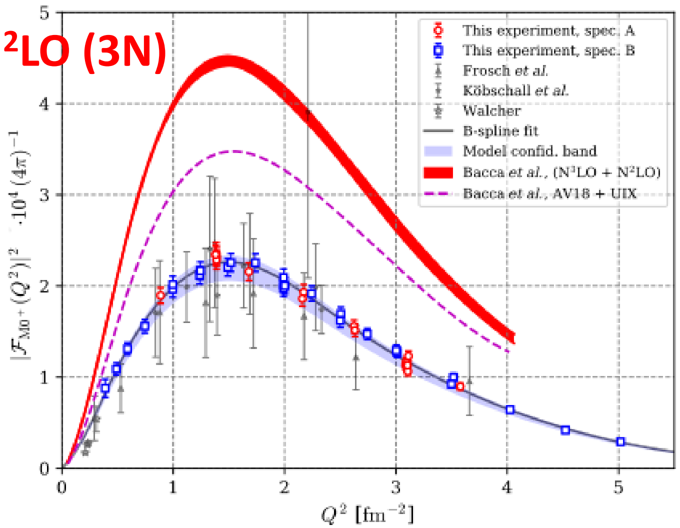
PHYSICAL REVIEW LETTERS **130,** 152502 (2023)

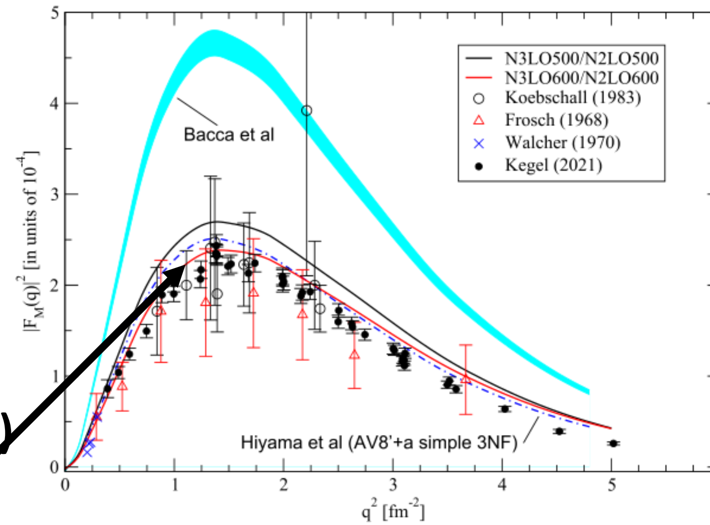
Editors' Suggestion **Featured in Physics**



Measurement of the α-Particle Monopole Transition Form Factor Challenges Theory: A Low-Energy Puzzle for Nuclear Forces?

N^{3}LO (2N) + N^{2}LO (3N)





AV8' (2N) + phenomenological (3N)
N^{3}LO (2N) + N^{2}LO (3N)

E. Hiyama, B. F. Gibson, and M. Kamimura, PRC 70, 031001(R) (2004)
M. Viviani, A. Kievsky, L. E. Marcucci and L. Girlanda, Few Body Syst. 65, 74 (2024)
P. Yin et al., arXiv:2412.18037

Energies and Threshold

- SU(4) interaction from B.-N. Luu et al., PLB 797 (2019) 134863

L [fm]	$E(0_1^+)$ [MeV]	$E(0_2^+)$ [MeV]	ΔE [MeV]
13.2	$-28.32(3)$	$-8.37(14)$	$0.28(14)$
14.5	$-28.30(3)$	$-8.02(14)$	$0.42(14)$
15.7	$-28.30(3)$	$-7.96(9)$	$0.40(9)$

U.-G., Meißner, SS, S. Elhatisari, D. Lee, PRL 132, 062501 (2024)

	NLEFT	Exp.
^{4}He (0_1+)	-28.30(3)	-28.30
^{4}He (0_2+)	-7.96(9)	-8.09
^{3}H	-8.36	-8.48
^{3}He	-7.65	-7.72
ΔE = ^{4}He (0_2+) - ^{3}H	**0.40(9)**	**0.39**

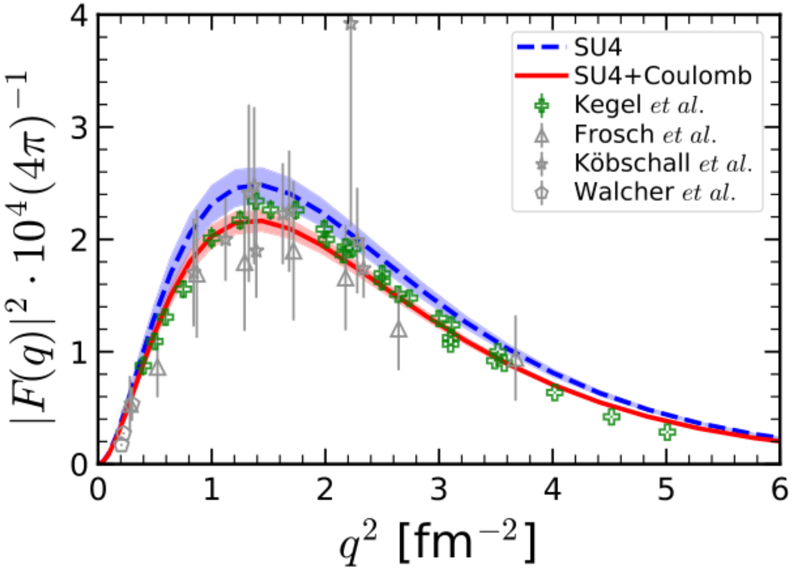
N. Michel, W. Nazarewicz, and M. Plloszajczak, PRL 131, 242502 (2023)

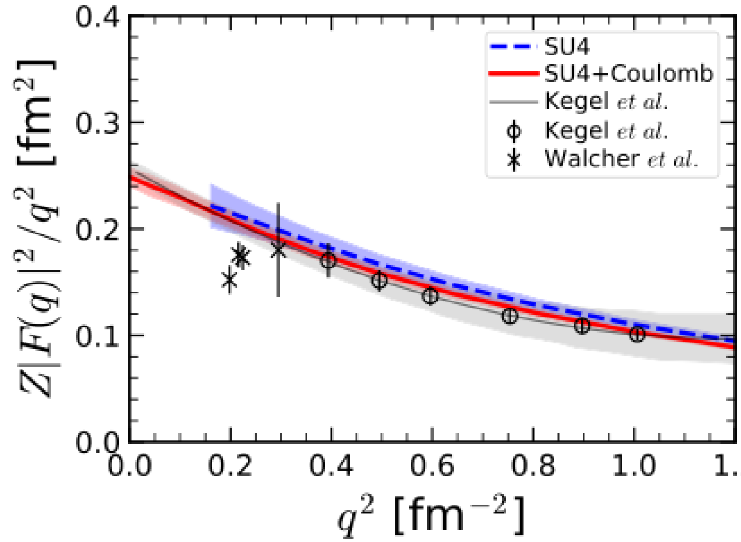
The explicit reproduction of particle-emission thresholds is crucial for the theoretical understanding of the 0_2+ state.

Exact calculation of ^{3}H and ^{3}He in courtesy of S. Elhatisari

Transition form factor

U.-G., Meißner, SS, S. Elhatisari, D. Lee, PRL 132, 062501 (2024)





$$\frac{Z|\mathcal{F}_{\mathrm{M0^+}}(q^2)|}{q^2} = \frac{\langle r^2\rangle_{\mathrm{tr}}}{6}\left[1 - \frac{q^2}{20}\mathcal{R}^2{}_{\mathrm{tr}} + \mathcal{O}(q^4)\right]$$

	<r²>	R
Exp.	1.53 (5)	4.56 (15)
NLEFT	1.49 (1)	4.00 (4)
Bacca et al.	1.83 (1)	3.97 (5)

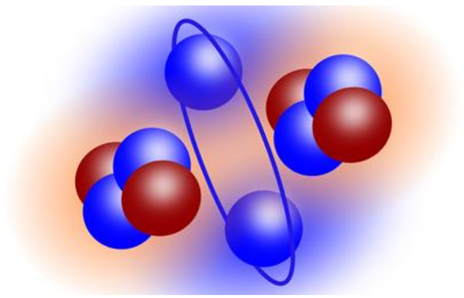
Beryllium Isotopes

Diverse phenomena in beryllium Isotopes: clustering, molecular orbitals, halo, …

- ^{7}Be: significant role in nuclear astrophysics
  E. G. Adelberger et al., Rev. Mod. Phys. 70, 1265 (1998); Rev. Mod. Phys. 83, 195 (2011)

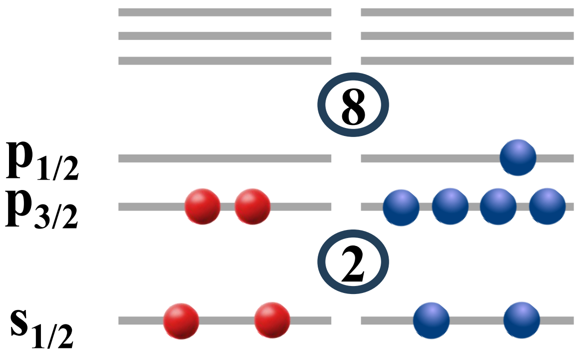
- ^{8}Be: unbound with decaying into two alpha particles

- ^{9}Be, ^{10}Be: clustering and molecular-like structures
  J. Chen, et al., Phys. Rev. Lett. 134, 012502 (2025)
  P. J. Li, et al., Phys. Rev. Lett. 131, 212501 (2023); Physics 16, s167 (2023)
  W. von Oertzen, M. Freer, and Y. Kanada-En'yo, Phys. Rep. 432, 43 (2006)



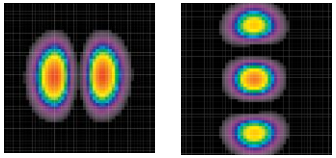
- ^{11}Be: ground-state parity inversion and halo structure
  A. Calci, et al., Phys. Rev. Lett. 117, 242501 (2016)
  W. Nörtershäuser, et al., Phys. Rev. Lett. 102, 062503 (2009)

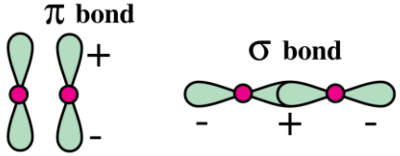
- ^{12}Be: breakdown of the $N = 8$ shell closure
  J. Chen, et al., Phys. Lett. B 781, 412 (2018)
  A. Navin, et al., Phys. Rev. Lett. 85, 266 (2000)



Theoretical Studies

- Antisymmetrized molecular dynamics Y. Kanada-En'yo, Phys. Rev. C 91, 014315 (2015)
- Fermionic molecular dynamics A. Krieger, et al., Phys. Rev. Lett. 108, 142501 (2012)
- Molecular-orbital models N. Itagaki and S. Okabe, Phys. Rev. C 61, 044306 (2000)
  W. Von Oertzen, Z. Phys. A 354, 37 (1996)
- Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function M. Lyu, et al., Eur. Phys. J. A 57, 51 (2021)
- Other cluster models Q. Zhao, M. Kimura, B. Zhou, and S. Shin, Phys. Rev. C 106, 054313 (2022)
  T. Fukui, J. Phys. G 49, 055102 (2022)
- Density functional theory J. Geng, P. W. Zhao, Y. F. Niu, and W. H. Long, Phys. Lett. B 858, 139036 (2024)
- Gamow shell model P. Linares Fernandez, et al., Phys. Rev. C 108, 044616 (2023)
- Green's function Monte Carlo R. B. Wiringa, et al., Phys. Rev. C 62, 014001 (2000)
- Monte Carlo shell model L. Liu, et al., Phys. Rev. C 86, 014302 (2012)
- No-core shell model A. E. McCoy, et al., Phys. Lett. B 856, 138870 (2024)
- Resonating group method K. Kravvaris and A. Volya, Phys. Rev. Lett. 119, 062501 (2017)
- ... ... M. Kimura, T. Suhara, and Y. Kanada-En'yo, Eur. Phys. J. A 52, 373 (2016)
  Y. Funaki, H. Horiuchi, and A. Tohsaki, Prog. Part. Nucl. Phys. 82, 78 (2015)
  W. von Oertzen, M. Freer, and Y. Kanada-En'yo, Phys. Rep. 432, 43 (2006)





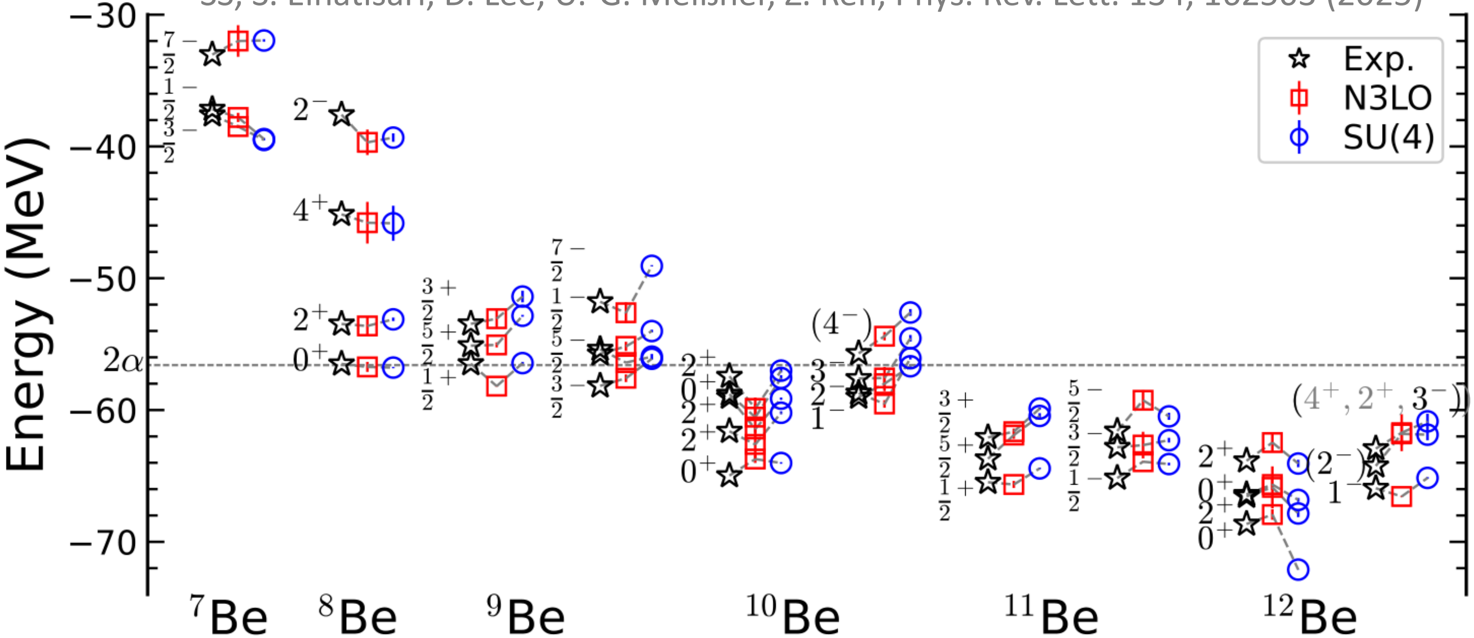
How about in ab initio framework with full many-body correlated wave function $\Psi(r_1, r_2, \ldots, r_A)$, composed of millions of Slater determinant?

Calculation by NLEFT - Spectrum

- N^3LO interaction S. Elhatisari, et al., Nature 630, 59 (2024)
  lattice spacing $a = 1.32$ fm, box size $L = 13.2$ fm

- SU(4) interaction SS, S. Elhatisari, T. Lähde, D. Lee, B. Lu, and U.-G. Meißner, Nat. Commun. 14, 2777 (2023)
  lattice spacing $a = 1.64$ fm, box size $L = 14.8$ fm

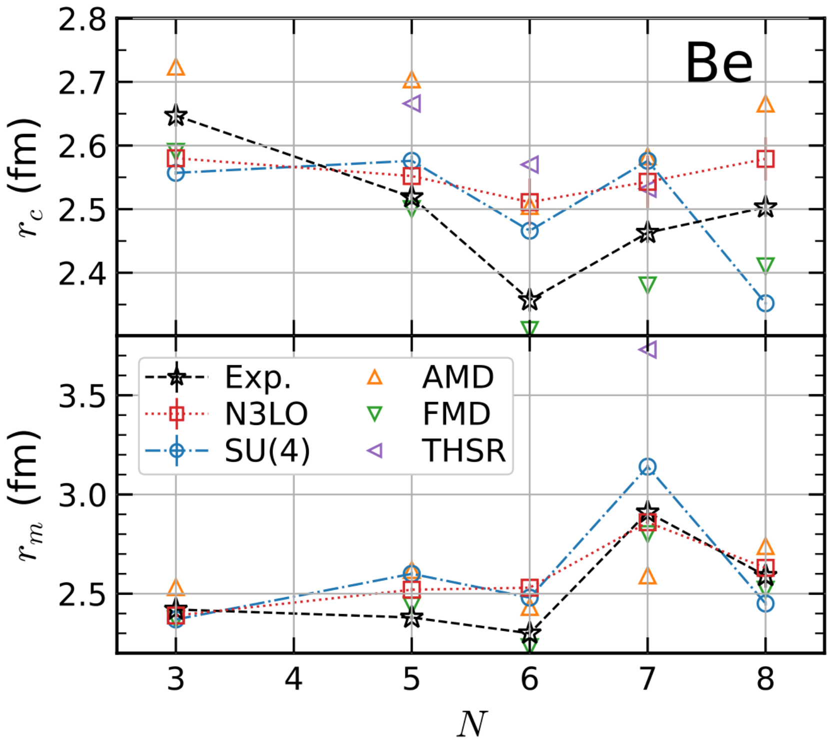
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)



	Exp.	N^3LO	SU(4)
^{11}Be (1/2+)	−65.5	−65.6(3)	−64.6(1)
^{11}Be (1/2-)	−65.2	−63.9(1)	−64.1(1)

Radii and Transitions

Radii and electromagnetic properties can also be well described

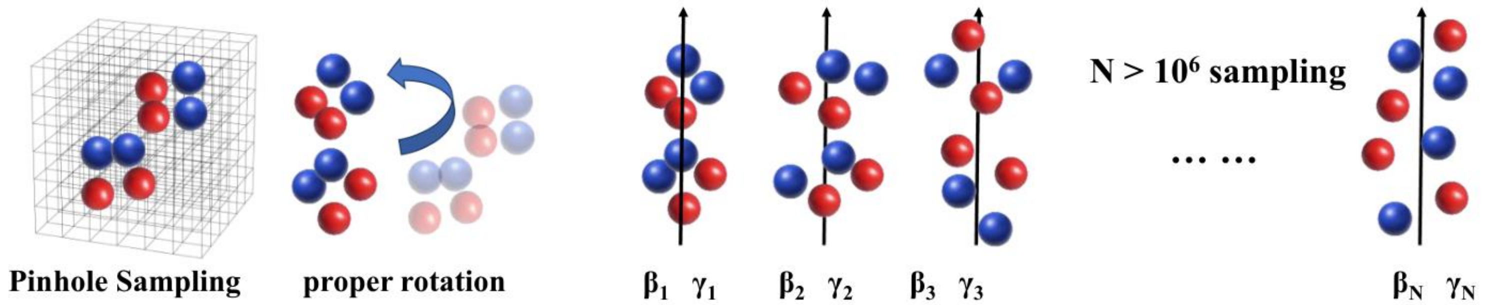


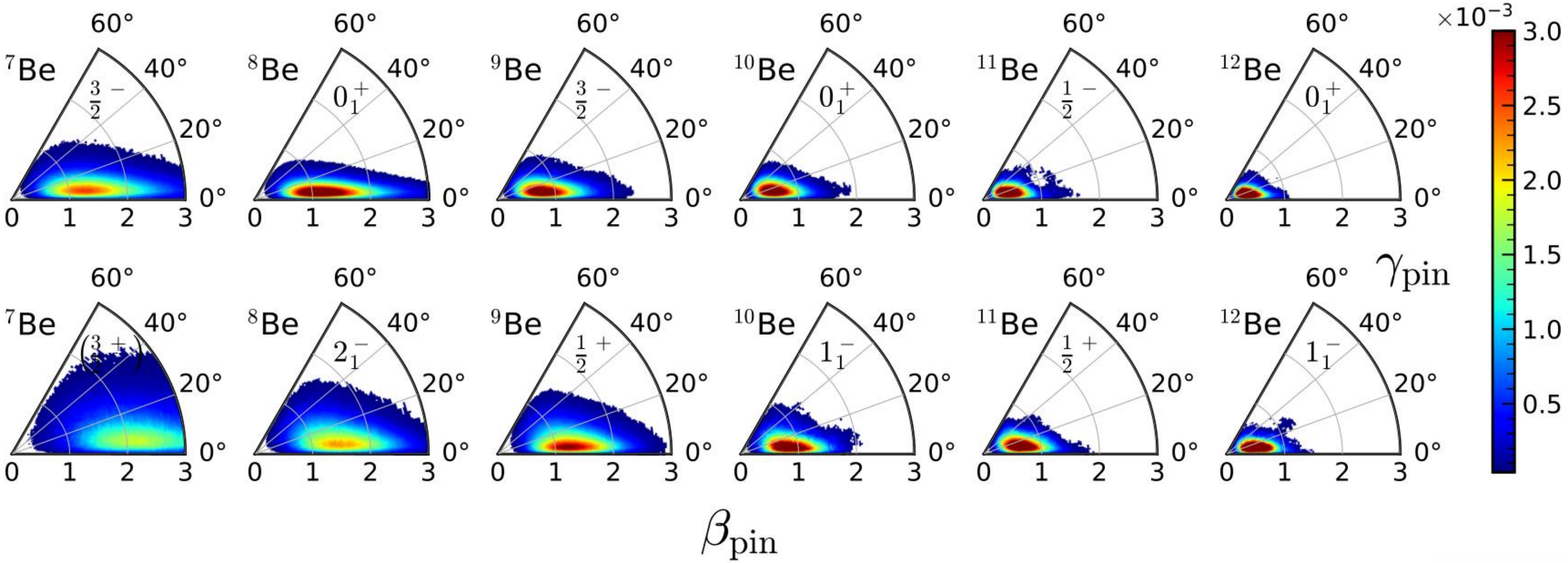
		SU(4)	N^{3}LO	Exp.
^{7}Be	$E2, \frac{3}{2}^- \to \frac{1}{2}^-$	16.0(2)	15.2(5)	26(6)(3) [91]
^{9}Be	$Q(\frac{3}{2}^-)$	7.3(1)	7.4(1.0)	5.29(4) [92]
	$E1, \frac{1}{2}^+ \to \frac{3}{2}^-$	0.131(3)	0.060(15)	0.136(2) [93]
	$E1, \frac{5}{2}^+ \to \frac{3}{2}^-$	0.045(14)	0.049(5)	0.010(8) [67]
	$E2, \frac{5}{2}^- \to \frac{3}{2}^-$	35.7(1.8)	27.8(1.9)	27.1(2.0) [67]
	$E2, \frac{7}{2}^- \to \frac{3}{2}^-$	11.6(2.5)	5.3(8)	9.5(4.1) [67]
^{10}Be	$E1, 3_1^- \to 2_1^+$	0.026(2)	0.004(3)	0.009(1) [67]
	$E2, 2_1^+ \to 0_1^+$	10.6(4)	8.5(9)	9.2(3) [31]
^{11}Be	$E1, \frac{1}{2}^- \to \frac{1}{2}^+$	0.023(3)	0.038(3)	0.102(2) [94]
^{12}Be	$E1, 0_1^+ \to 1_1^-$	0.049(2)	0.056(26)	0.051(13) [95]
	$E2, 2_1^+ \to 0_1^+$	7.8(1.1)	9.0(3.1)	14.2(1.0)(2.0) [6]

SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)

Further refinement of three-body force are needed

Deformation



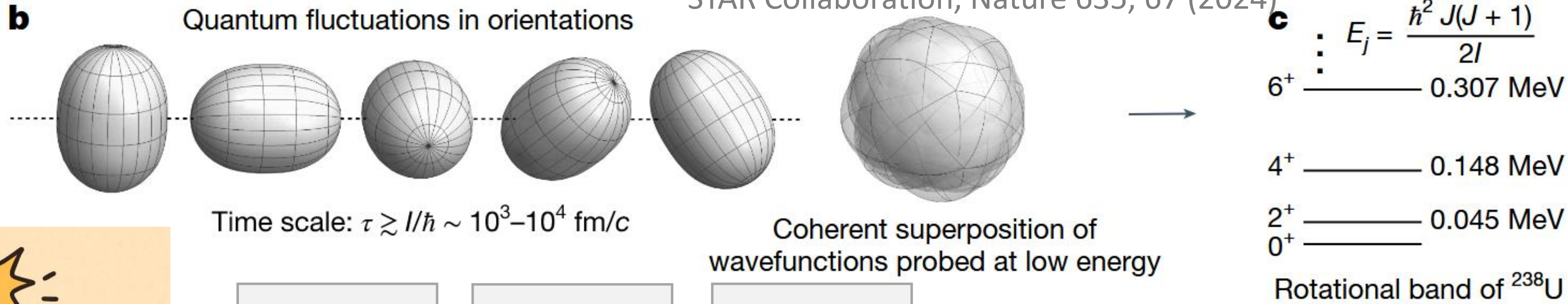


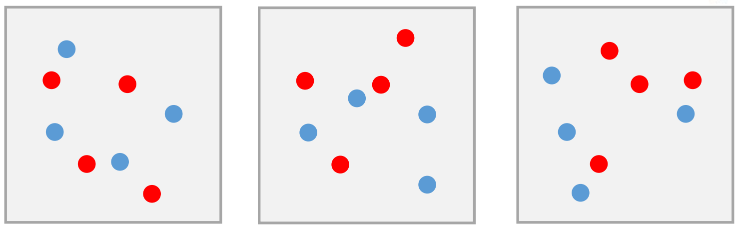
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)

Intrinsic density

What is intrinsic density?

STAR Collaboration, Nature 635, 67 (2024)



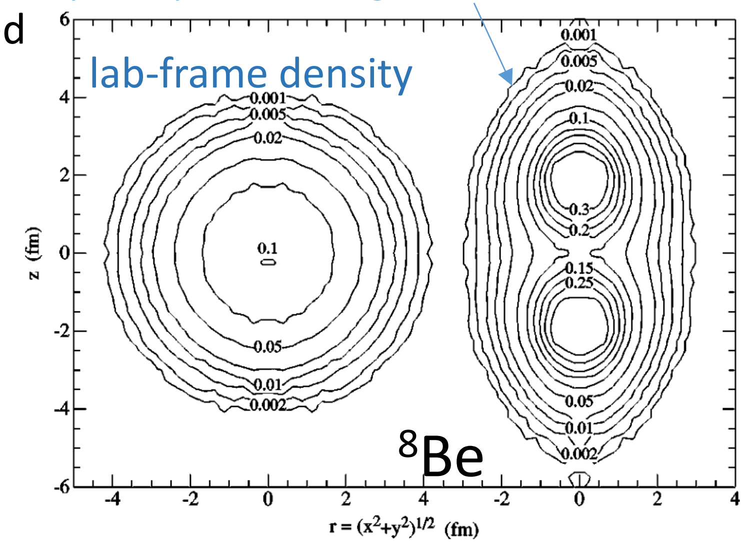


... ...

In QMC study, nucleon coordinates ($r_1, r_2, \ldots, r_A$) are rotated to the (long) principal axis of moment of inertia matrix

$$\mathcal{M} = \sum_{i=1}^{A} \begin{pmatrix} x_i^2 & x_i y_i & x_i z_i \\ y_i x_i & y_i^2 & y_i z_i \\ z_i x_i & z_i y_i & z_i^2 \end{pmatrix}$$

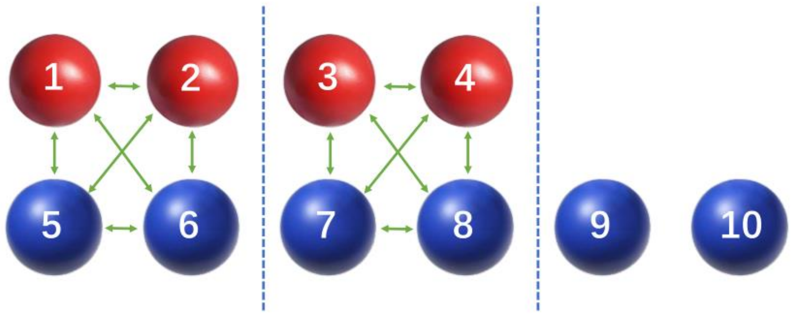
R. Wiringa, S. Pieper, J. Carlson, and V. Pandharipande, Phys. Rev. C 62, 014001 (2000)



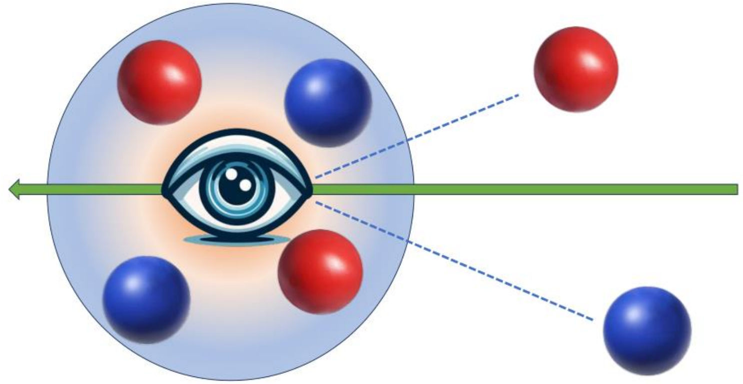
Intrinsic density

α-view intrinsic density

1. Grouping closest 2 protons and 2 neutrons together



2. Rotate the nucleon coordinates (r_1, r_2, …, r_A) such that the cluster is aligned along the symmetry axis

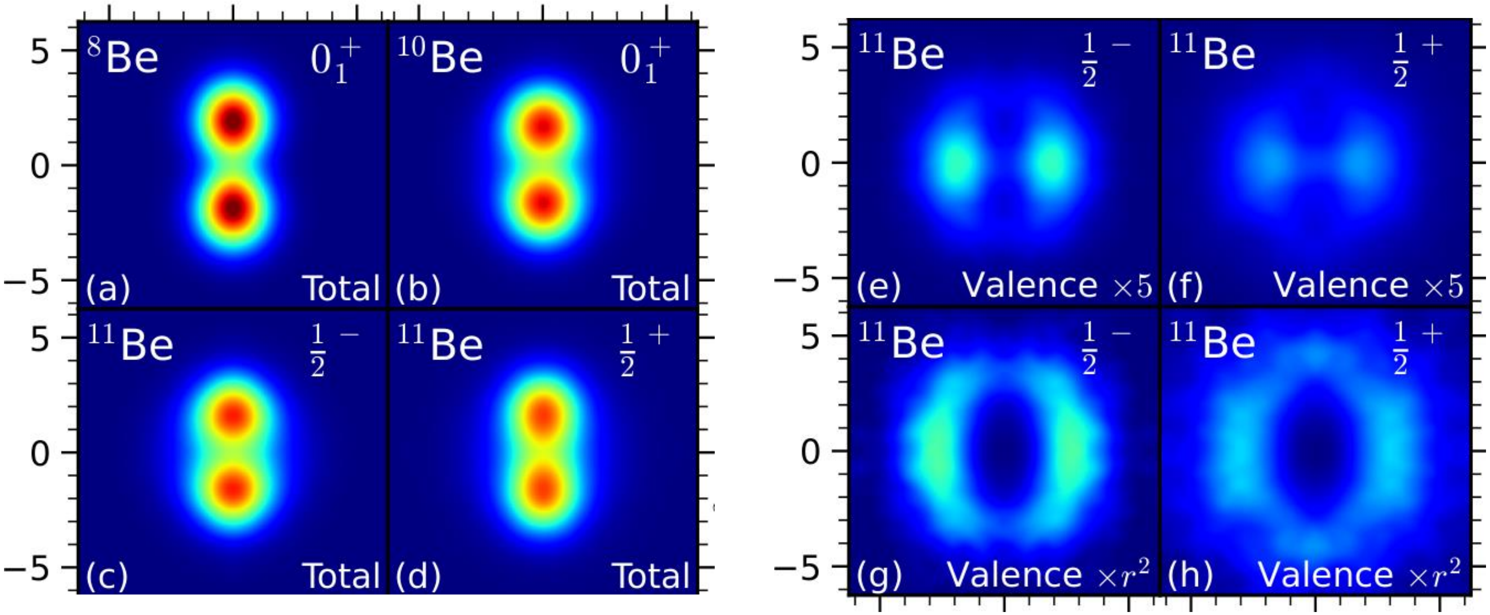


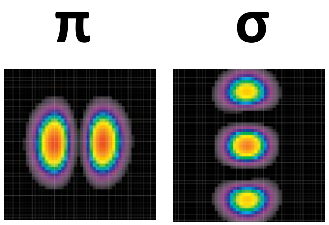
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)

α-view intrinsic density

Intrinsic density calculated by NLEFT using N^{3}LO interaction:

- total density of (a) ^{8}Be; (b) ^{10}Be; (c) ^{11}Be ½-; (d) ^{11}Be ½+ (ground state)
- valence neutron density of (e, g) ^{11}Be ½-; (f, h) ^{11}Be ½+



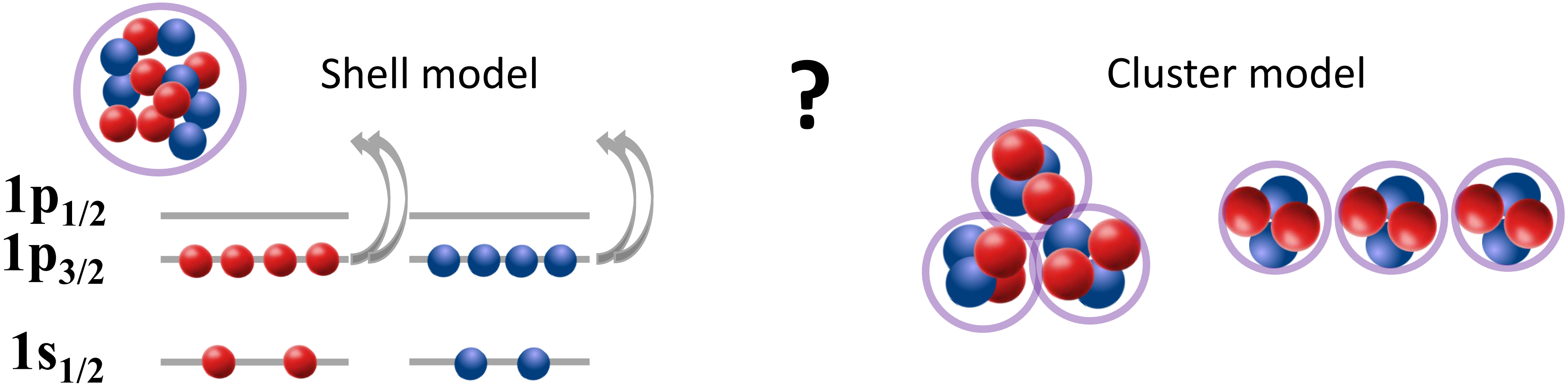


Y. Kanada-En'yo, H. Horiuchi, and A. Dote, J. Phys. G 24, 1499 (1998)

cluster, molecular orbital, and halo from ab initio

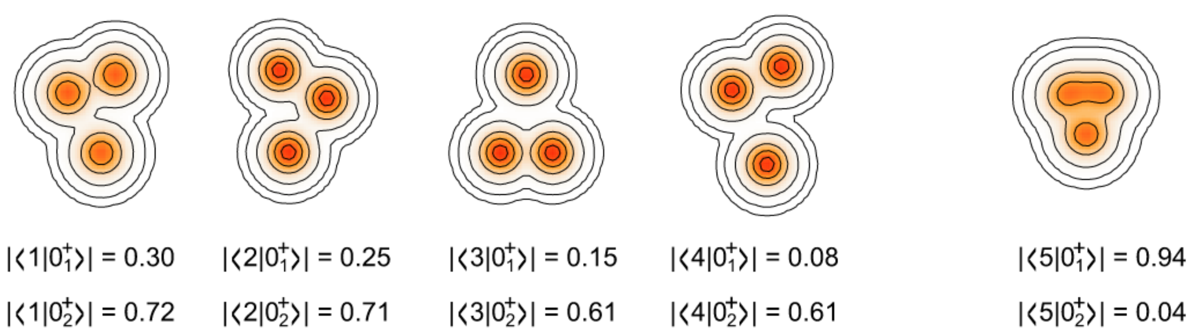
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)

Structure and Shape of Carbon-12



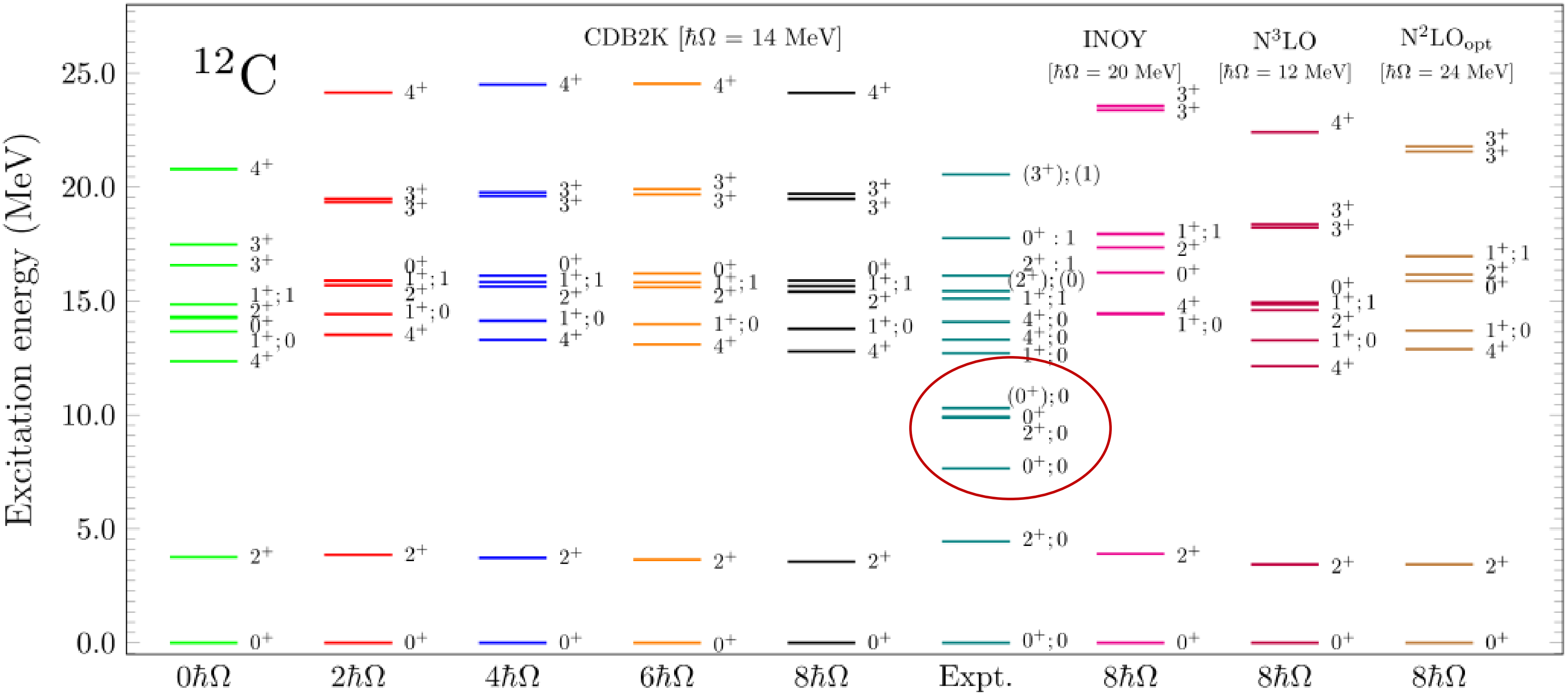
Hoyle State, debate M. Freer and H.O.U. Fynbo., Prog. Part. Nucl. Phys. 78, 1 (2014)

- Linear arrangement of 3α-particles H. Morinaga, Phys. Rev. 101, 254 (1956)
- Vibrational excitation, triangular symmetry R. Bijker and F. Iachello, PRC, 61, 067305 (2000)
- Bose Einstein Condensate of α-particles A. Tohsaki, et al., Phys. Rev. Lett. 87, 192501 (2001)
- Cluster-gas close to an equilateral triangle Y. Kanada-En'yo, Prog. Theor. Phys. 117, 655 (2007)
- Superposition of 3α arrangements M. Chernykh, et al., Phys. Rev. Lett., 98, 032501 (2007)
- … …



Ab initio Study of ^{12}C

Hoyle state is a challenge for ab initio method



No-core shell model

P. Choudhary et al., Phys. Rev. C, 107, 014309 (2023)
P. Navrátil, J. P. Vary, and B. R. Barrett, Phys. Rev. Lett. 84, 5728 (2000)

Ab initio Study of ^{12}C

- Nuclear lattice effective field theory

PRL **106**, 192501 (2011) — Selected for a Viewpoint in *Physics* — PHYSICAL REVIEW LETTERS

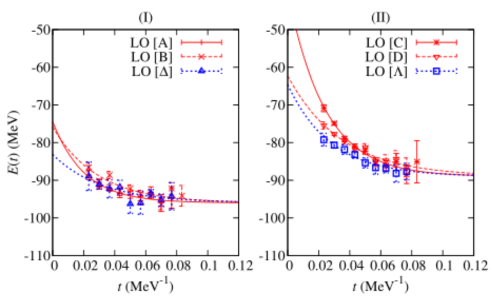
Ab Initio Calculation of the Hoyle State

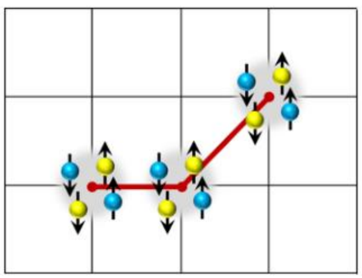
Evgeny Epelbaum,[1] Hermann Krebs,[1] Dean Lee,[2] and Ulf-G. Meißner[3,4]

PRL **109**, 252501 (2012) — PHYSICAL REVIEW LETTERS — week ending 21 DECEMBER 2012

Structure and Rotations of the Hoyle State

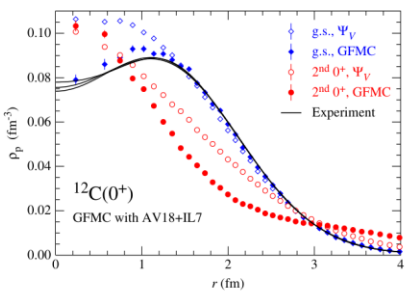
Evgeny Epelbaum,[1] Hermann Krebs,[1] Timo A. Lähde,[2] Dean Lee,[4] and Ulf-G. Meißner[5,2,3]





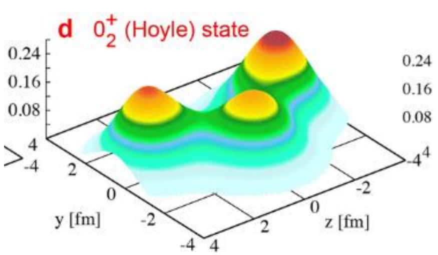
- Green’s function Monte Carlo

J. Carlson, et al., Rev. Mod. Phys., 87, 1067 (2015)



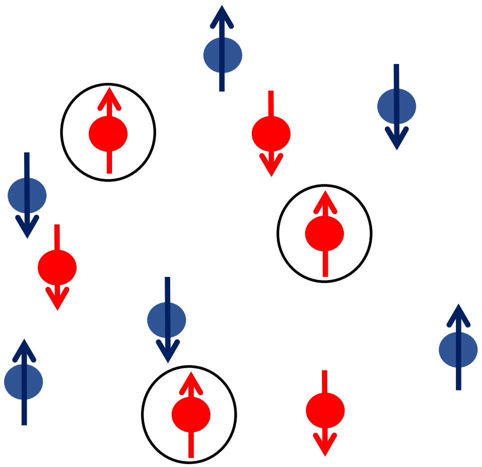
- Monte Carlo Shell Model

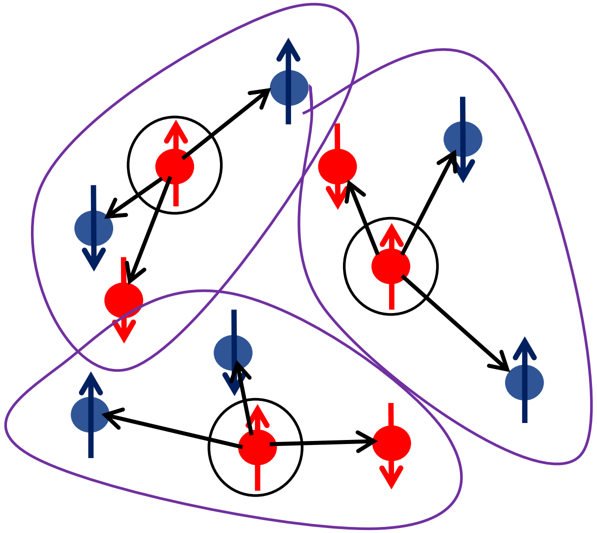
T. Otsuka et al., Nature Commun., 13, 2234 (2022)



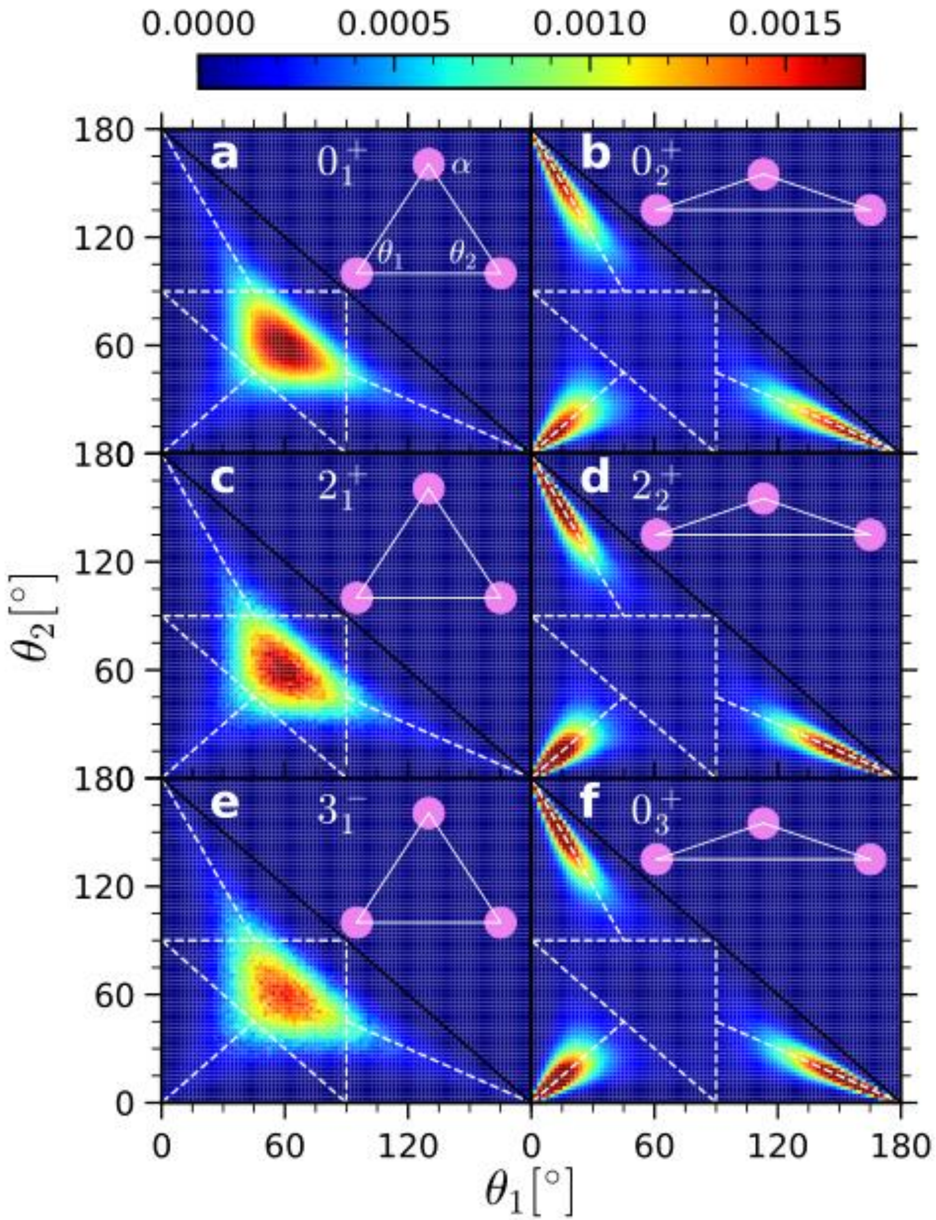
Grouping clusters

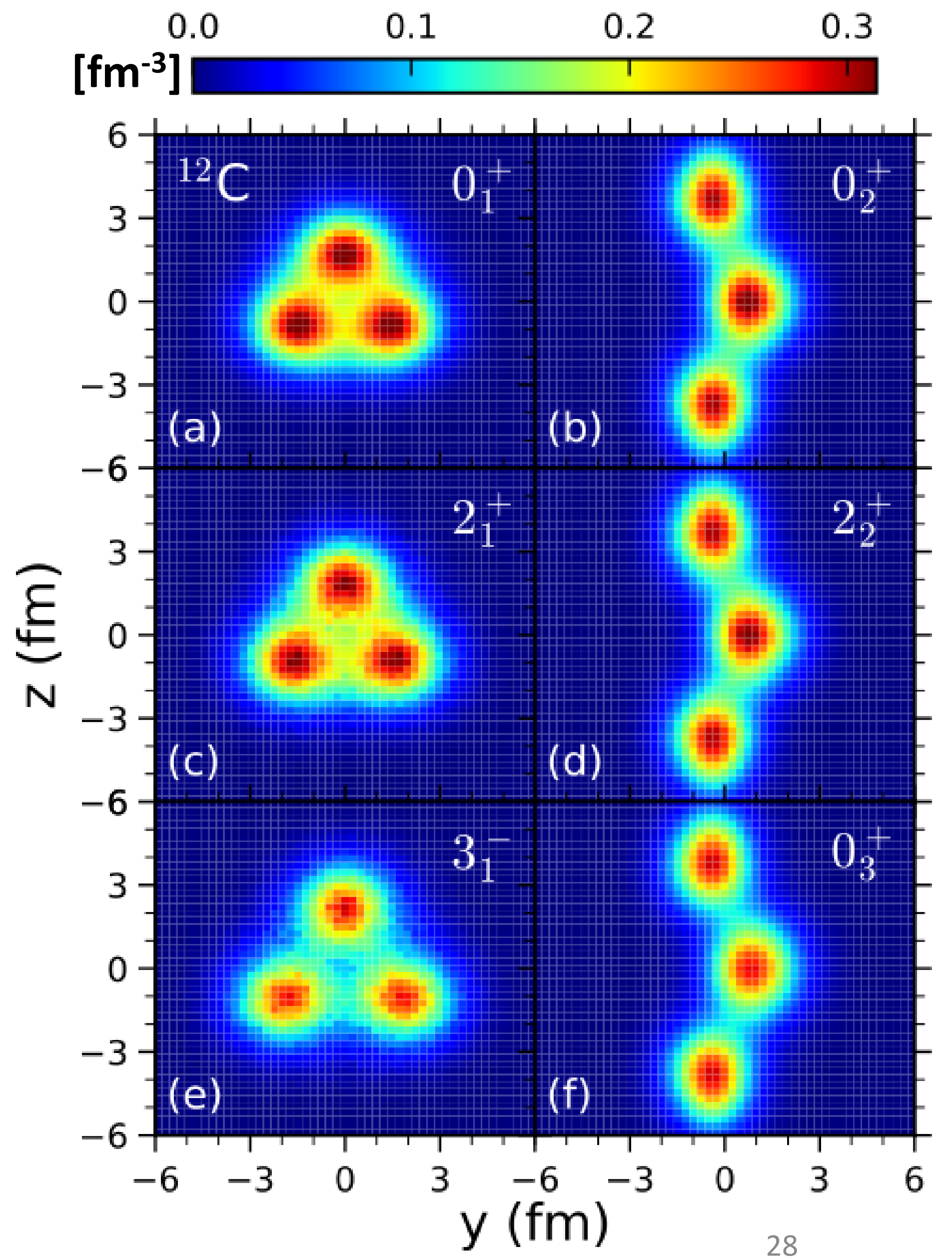
To investigate the geometry shape of different states, we first group 3 clusters



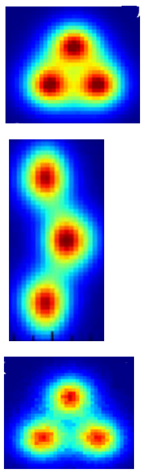


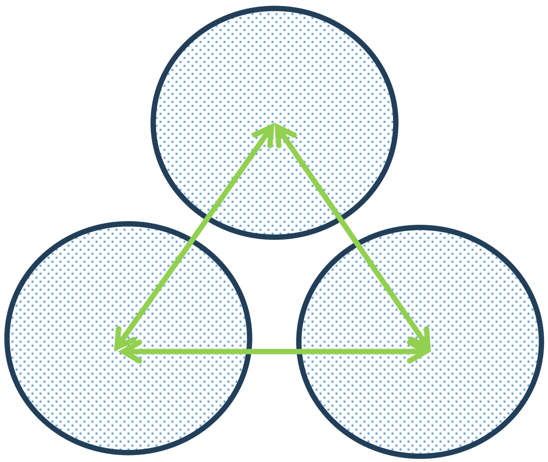
Intrinsic Density Distribution

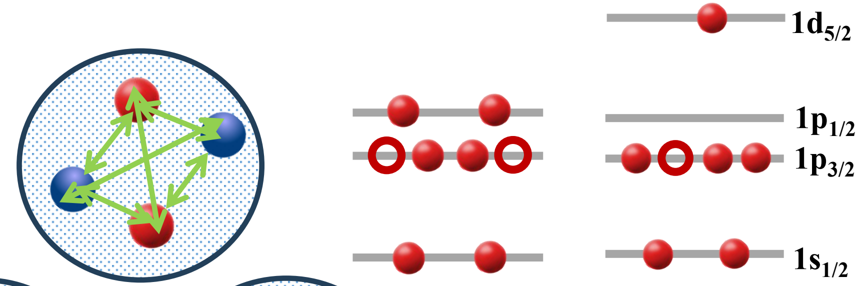


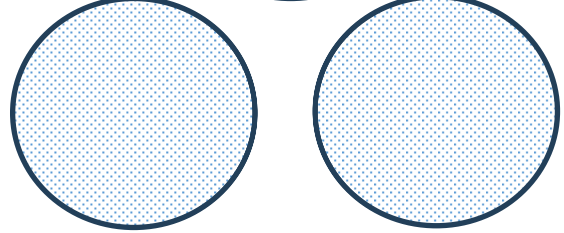


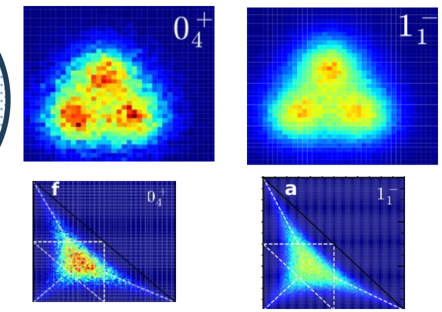
Cluster Excitation? Single-Particle Excitation?



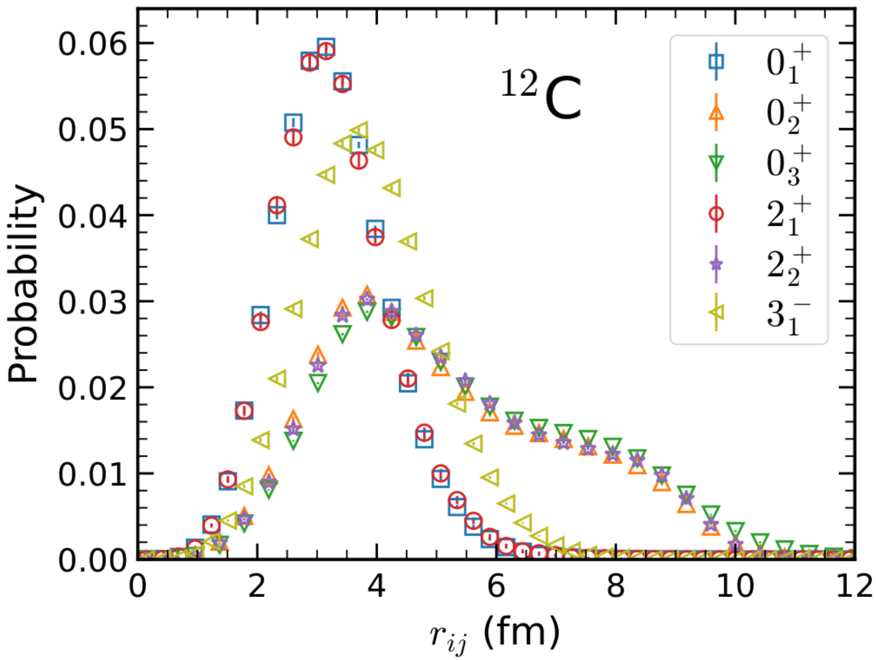


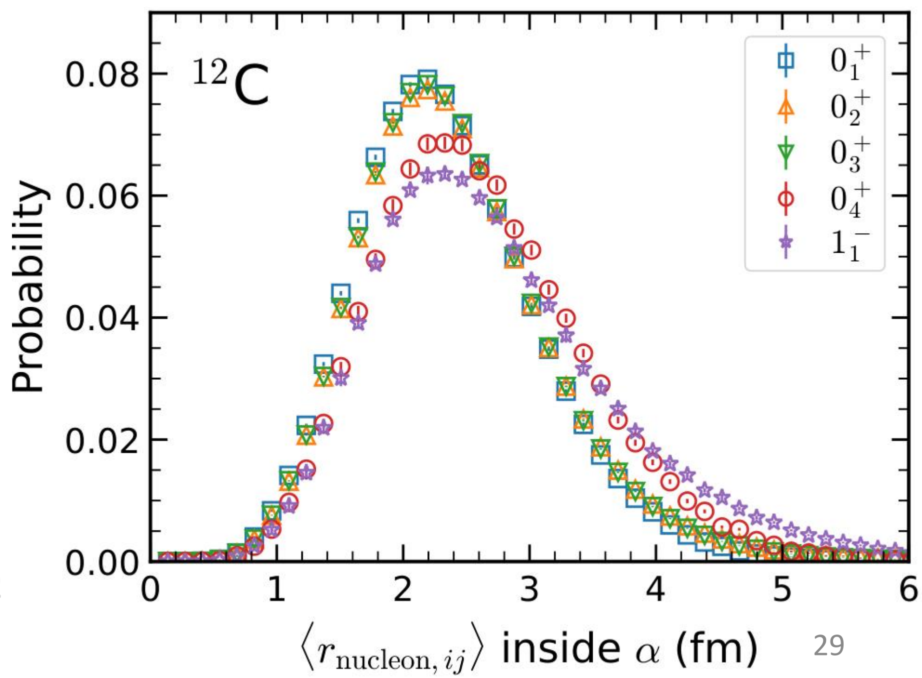






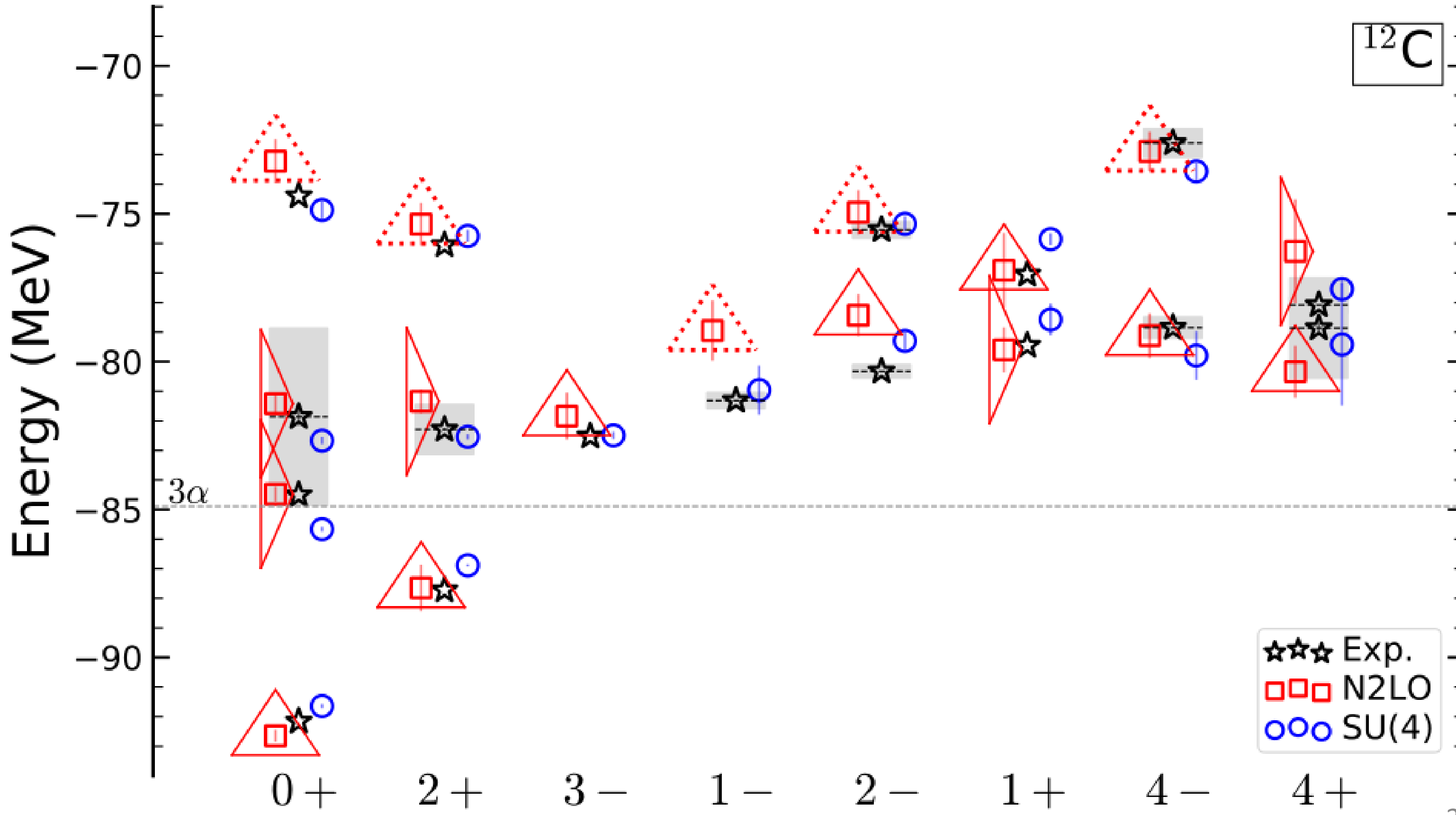
SS, T. Lähde, D. Lee, and U.-G. Meißner, PoS LATTICE2022:241 (2023)





Geometry Information in the Low-Lying Spectrum

- To summarize the geometry properties of each states in the low-lying spectrum of ^{12}C calculated by NLEFT:
  - 2 types of shape: equilateral or large angle obtuse triangle.
  - α cluster is well maintained (solid triangles) or diminished (dashed ones).



Contents

- Introduction
- Nuclear Lattice Effective Field Theory
- Helium-4: puzzle for nuclear forces?
- Beryllium Isotopes: Clustering, Molecular Orbitals, and Halo
- Carbon-12: Hoyle state, single-particle/collective excitations
- **Summary**

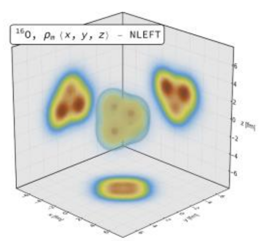
Summary

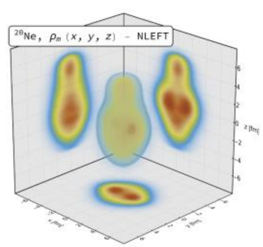
- Recent advancements in nuclear lattice effective field theory (NLEFT) offers a comprehensive ab initio understanding of light nuclei
- Helium-4, Berrylium isotopes, and carbon-12 nucleus have been investigated. Observables such as low-lying spectrum, density profiles, electromagnetic properties can be well described
- Structures such as molecular dynamics, halo, geometry, cluster/single-particle/rotation/vibration excitations can be revealed intuitively

? Cluster in heavier system

? ANC

? … …





PHYSICAL REVIEW LETTERS **135,** 012302 (2025)

Exploiting ^{20}Ne Isotopes for Precision Characterizations of Collectivity in Small Systems

Giuliano Giacalone,[1,*] Benjamin Bally,[2] Govert Nijs,[3] Shihang Shen,[4] Thomas Duguet,[5,6] Jean-Paul Ebran,[7,8] Serdar Elhatisari,[9,10] Mikael Frosini,[11] Timo A. Lähde,[12,13] Dean Lee,[14] Bing-Nan Lu,[15] Yuan-Zhuo Ma,[14] Ulf-G. Meißner,[10,16,17] Jacquelyn Noronha-Hostler,[18] Christopher Plumberg,[19] Tomás R. Rodríguez,[20] Robert Roth,[21,22] Wilke van der Schee,[3,23,24] and Vittorio Somà[5]

PHYSICAL REVIEW LETTERS **134,** 082301 (2025)

Anisotropic Flow in Fixed-Target ^{208}Pb + ^{20}Ne Collisions as a Probe of Quark-Gluon Plasma

Giuliano Giacalone,[1,*] Wenbin Zhao,[2,3,†] Benjamin Bally,[4] Shihang Shen,[5] Thomas Duguet,[6,7] Jean-Paul Ebran,[8,9] Serdar Elhatisari,[10] Mikael Frosini,[11] Timo A. Lähde,[12,13] Dean Lee,[14] Bing-Nan Lu,[15] Yuan-Zhuo Ma,[14] Ulf-G. Meißner,[16,17,5,18] Govert Nijs,[19] Jacquelyn Noronha-Hostler,[20] Christopher Plumberg,[21] Tomás R. Rodríguez,[22] Robert Roth,[23,24] Wilke van der Schee,[19,25,26] Björn Schenke,[27,‡] Chun Shen,[28,29,§] and Vittorio Somà[6]

Collaborators

- Serdar Elhatisari, KFUPM
- Timo A. Lähde, Jülich
- Dean Lee, MSU
- Bing-Nan Lu, GSCAPE
- Ulf-G. Meißner, Bonn/Jülich/Beihang
- Zhengxue Ren, Jülich
- NLEFT Collaboration

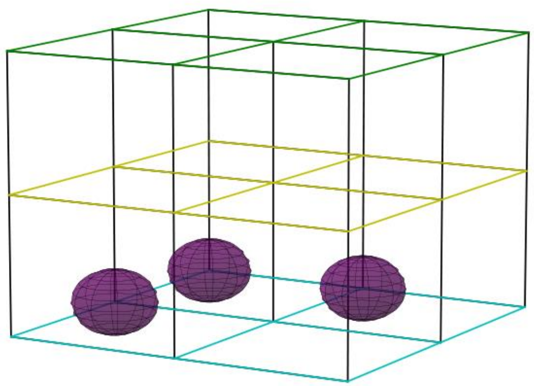
Thank you for your attention!

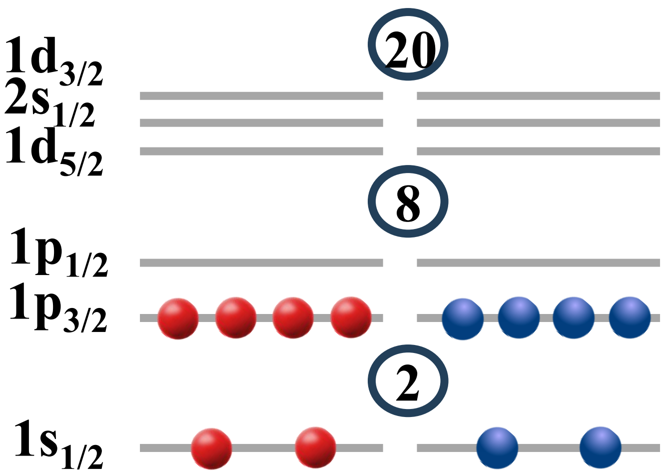
Theoretical Framework

- Starting from an initial many-body wave function:

$$|\Phi_0\rangle = \mathscr{A}\left[\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2)\ldots\phi_A(\mathbf{r}_A)\right]$$

$$\phi(\mathbf{r}) = \exp\left(-(\mathbf{r}-\mathbf{r}_0)^2/2w^2\right)$$



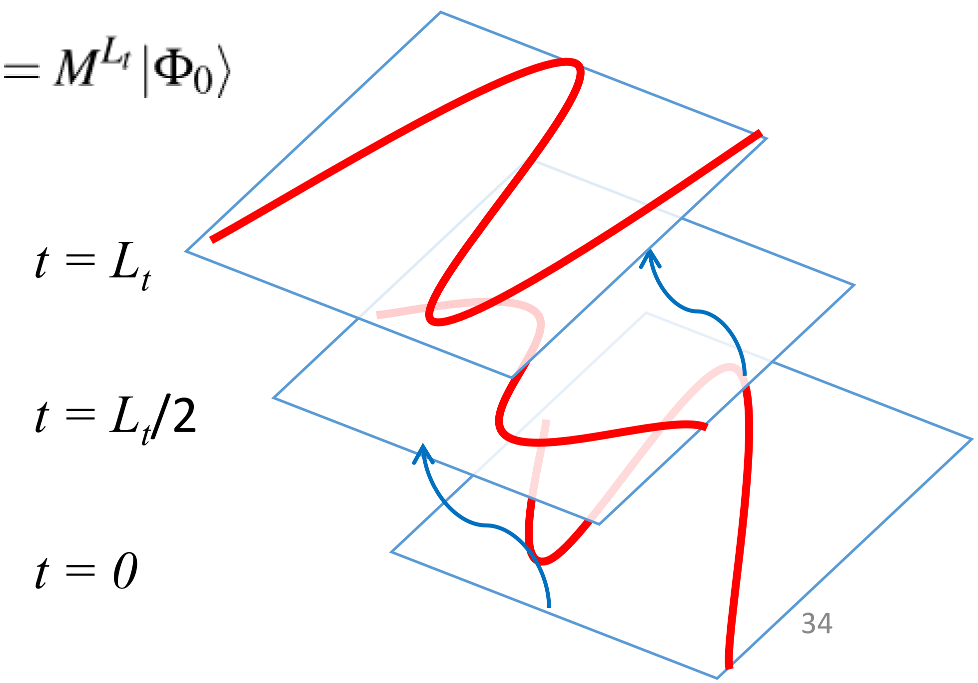


- Euclidean time projection with transfer matrix:

$$M =: \exp(-\alpha_t H) : \qquad \alpha_t = a_t/a$$

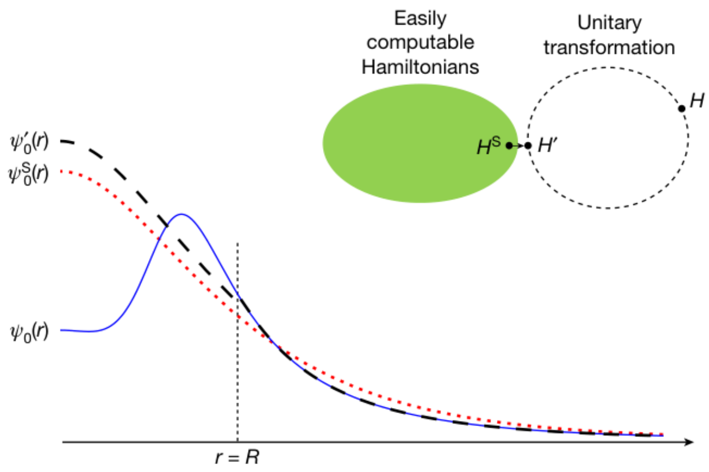
with H the many-body Hamiltonian, a_t and a the temporal and spatial lattice spacing.

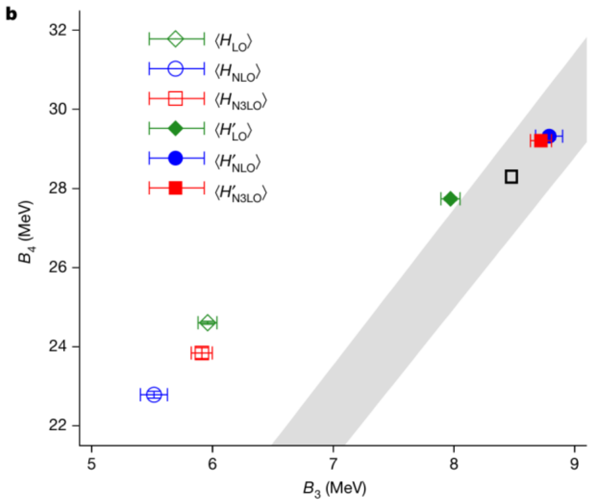
$$|\Phi_{L_t}\rangle = M^{L_t}|\Phi_0\rangle$$

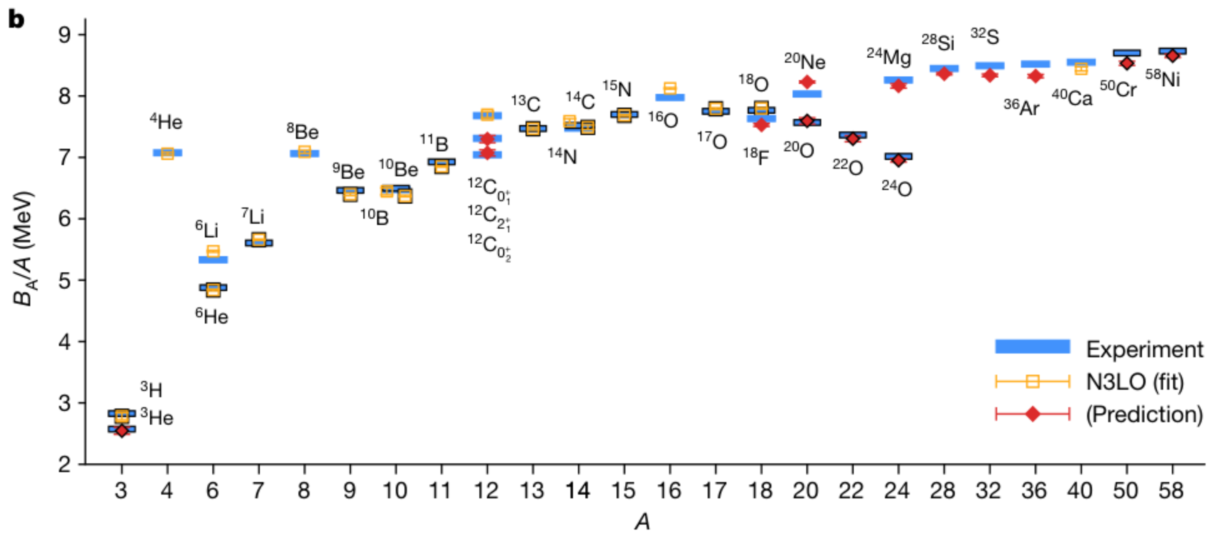


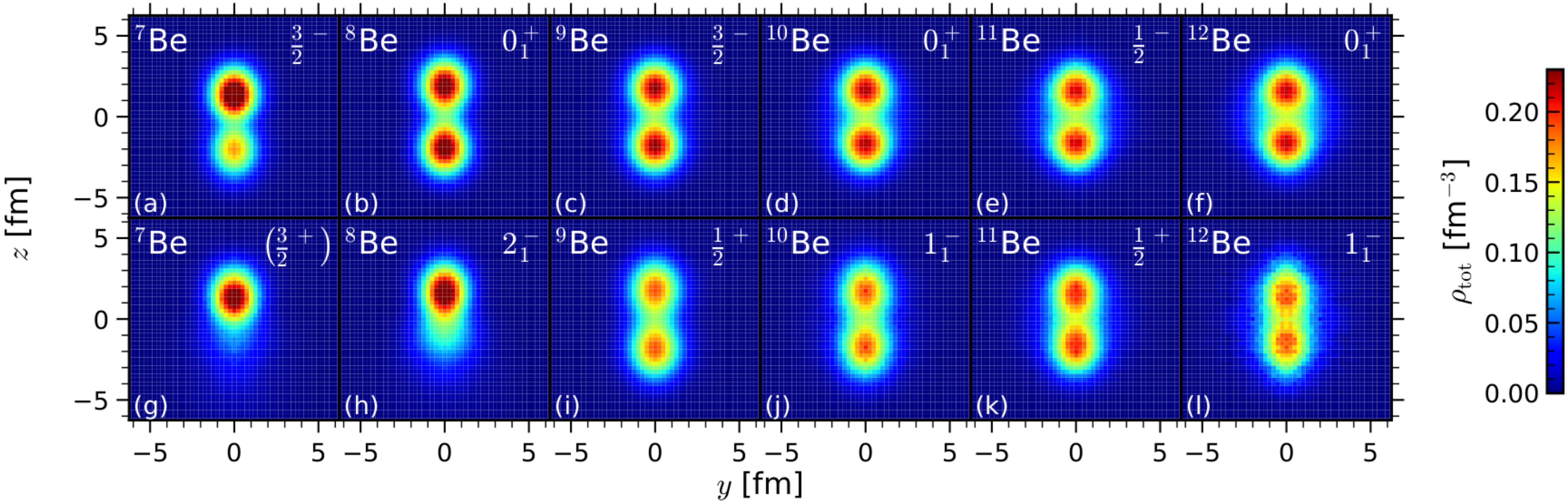
Dealing with strong nuclear forces

- Wave function matching   S. Elhatisari, et al., Nature 630, 59 (2024)

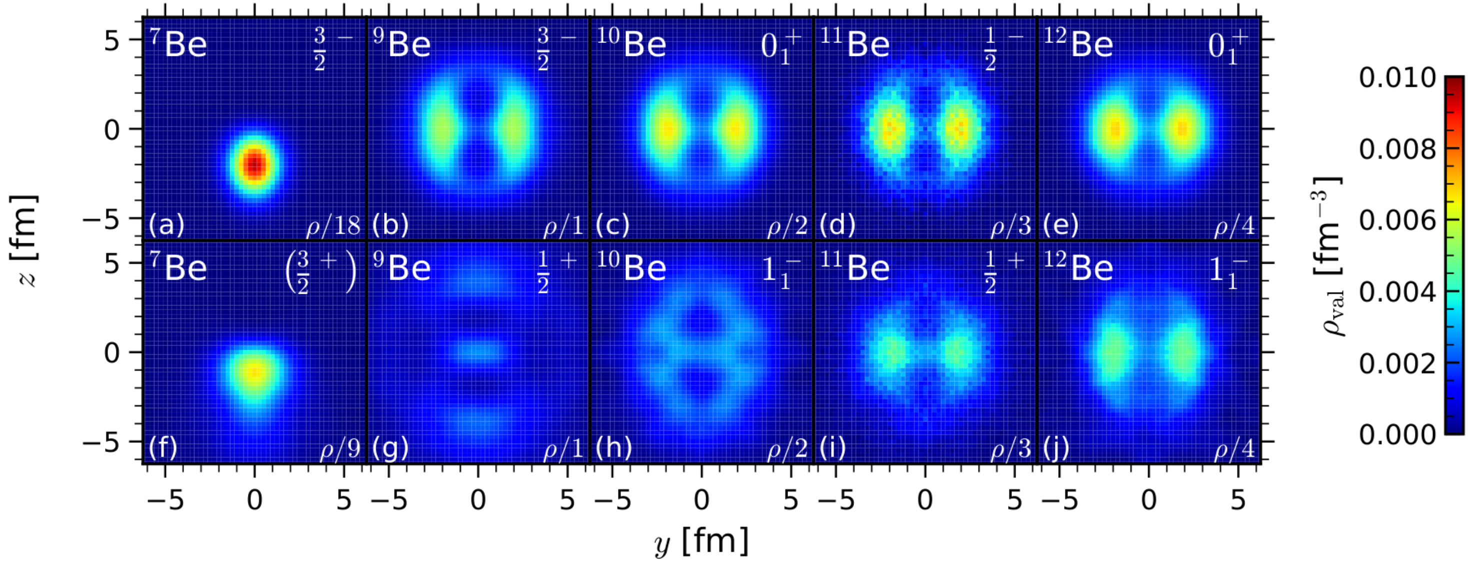








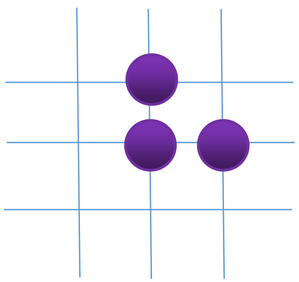
^{7}Be $\frac{3}{2}^-$
^{8}Be 0_1^+
^{9}Be $\frac{3}{2}^-$
^{10}Be 0_1^+
^{11}Be $\frac{1}{2}^-$
^{12}Be 0_1^+
^{7}Be $(\frac{3}{2}^+)$
^{8}Be 2_1^-
^{9}Be $\frac{1}{2}^+$
^{10}Be 1_1^-
^{11}Be $\frac{1}{2}^+$
^{12}Be 1_1^-
(a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k) (l)
z [fm]
y [fm]
$\rho_{\rm tot}$ [fm^{-3}]
0.00 0.05 0.10 0.15 0.20



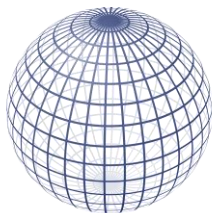
^{7}Be $\frac{3}{2}^-$
^{9}Be $\frac{3}{2}^-$
^{10}Be 0_1^+
^{11}Be $\frac{1}{2}^-$
^{12}Be 0_1^+
^{7}Be $(\frac{3}{2}^+)$
^{9}Be $\frac{1}{2}^+$
^{10}Be 1_1^-
^{11}Be $\frac{1}{2}^+$
^{12}Be 1_1^-
(a) $\rho/18$ (b) $\rho/1$ (c) $\rho/2$ (d) $\rho/3$ (e) $\rho/4$
(f) $\rho/9$ (g) $\rho/1$ (h) $\rho/2$ (i) $\rho/3$ (j) $\rho/4$
z [fm]
y [fm]
$\rho_{\rm val}$ [fm^{-3}]
0.000 0.002 0.004 0.006 0.008 0.010

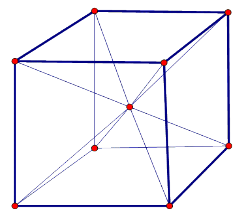
Calculation of the Hoyle State by NLEFT

- Hoyle state

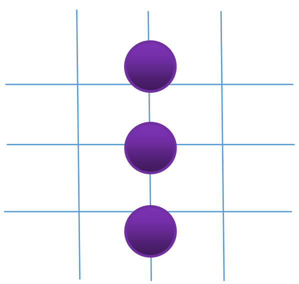


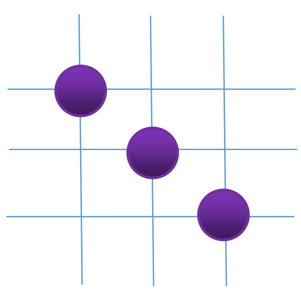
Angular momentum projection: SO(3) group reducted to cubic group O



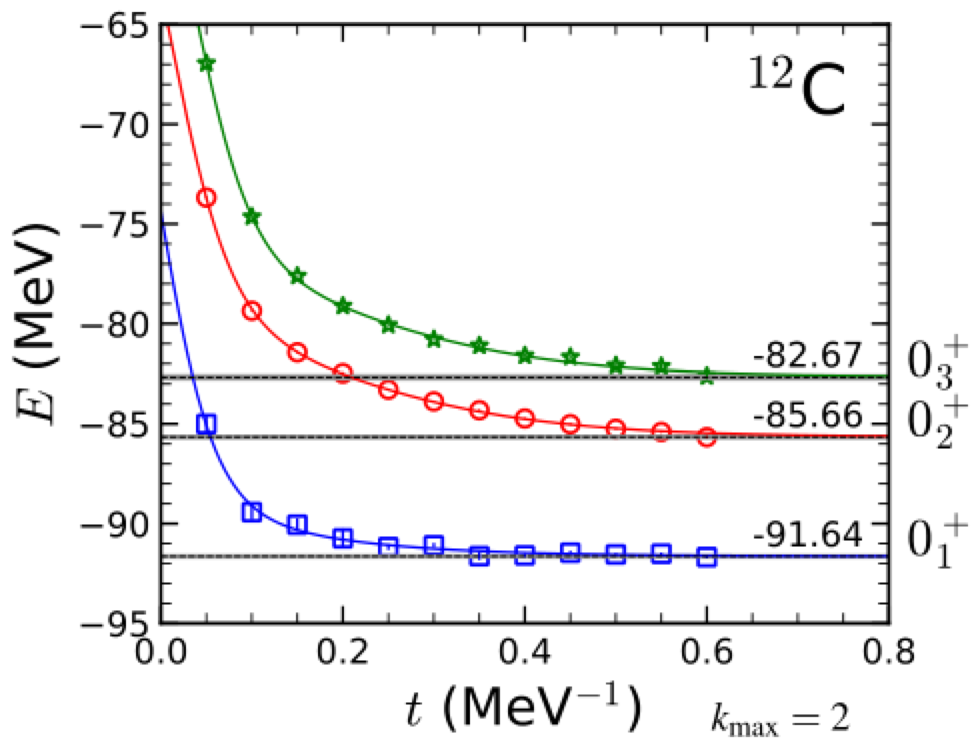


J	irrepresentation
0	A_1
1	T_1
2	$E + T_2$
3	$A_2 + T_1 + T_2$
4	$A_1 + E + T_1 + T_2$





$$\phi(\mathbf{r}) = \exp\left(-(\mathbf{r}-\mathbf{r}_0)^2/2w^2\right)$$

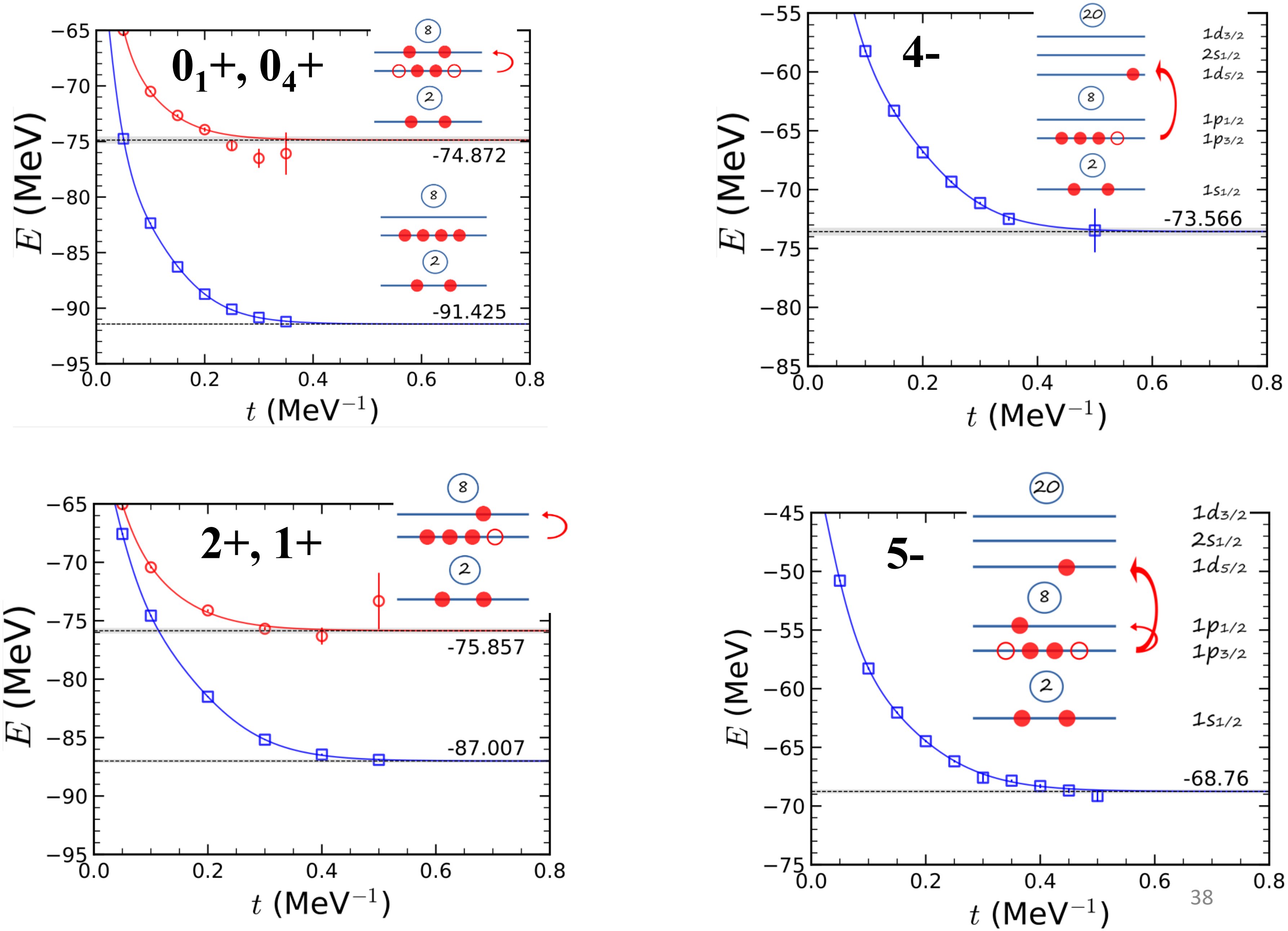


$$E_i(t) = \frac{E_i + \sum_{k=1}^{k_{\max}} (E_i + \Delta E_{i,k}) c_{i,k} e^{-\Delta E_{i,k} t}}{1 + \sum_{k=1}^{k_{\max}} c_{i,k} e^{-\Delta E_{i,k} t}}$$

T. A. Lähde et al., JPG 42 (2015) 034012

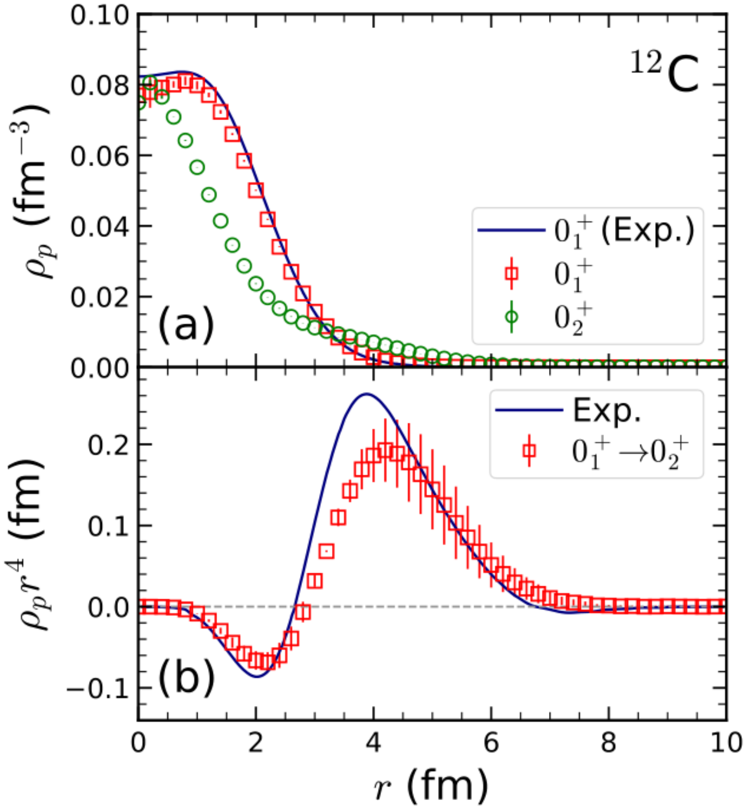
Web figures from: https://en.wikipedia.org/wiki/Sphere
https://math.ucr.edu/home/baez/icosidodecahedron/7.html

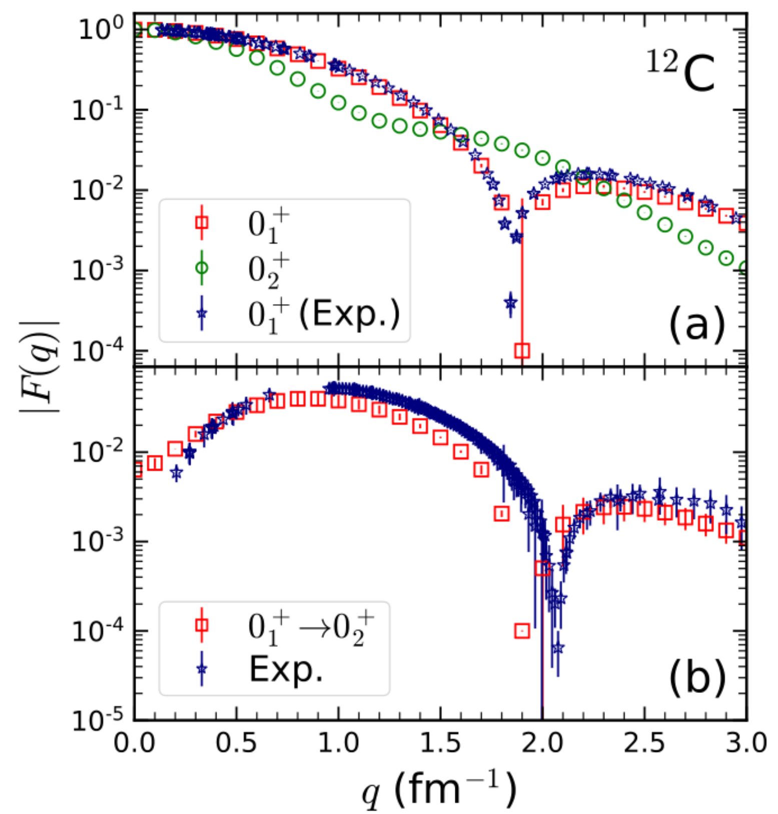
Shell-Model States Used as Initial Wave



Density Profiles

- Charge density distributions (left) and form factors (right) of ground state, Hoyle state, and transitions between them.



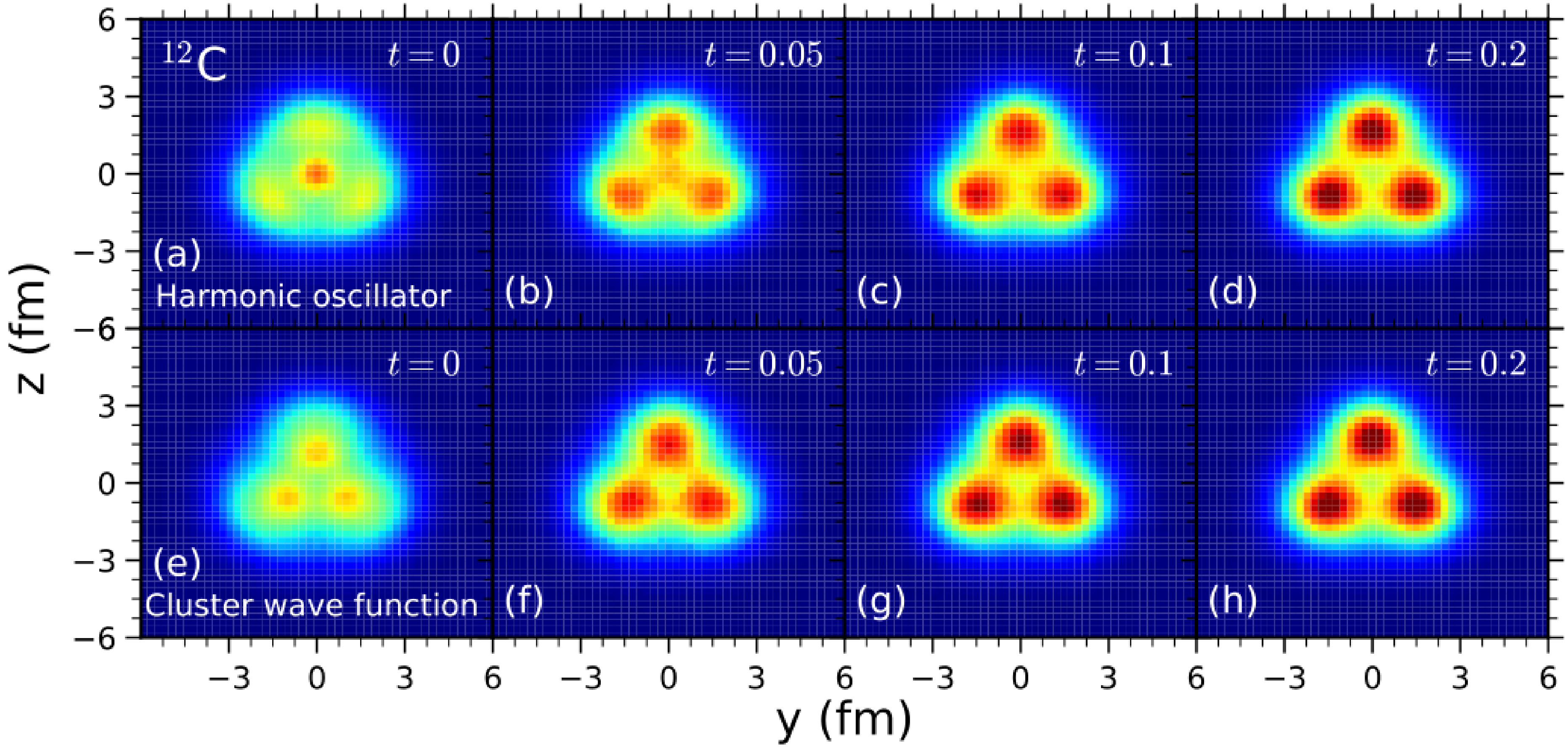


Exp. M. Chernykh et al., PRL 105, 022501 (2010)
I. Sick and J. S. Mccarthy, NPA 150, 631 (1970)
P. Strehl, Z. Phys. 234 (1970) 416; H. Crannell et al., NPA 758, 399 (2005)

$$F(q) = \frac{4\pi}{Z} \int dr r^2 \rho_p(r) j_0(qr)$$

Cluster Formation

- Density distribution of ^{12}C ground state using (a-d) harmonic oscillator or (e-h) cluster wave function as initial states, with Euclidean projection time ranging from t = 0 to 0.2 MeV^{-1}.



confirms the finding in Ref. E. Epelbaum et al., PRL 109, 252501 (2012)