

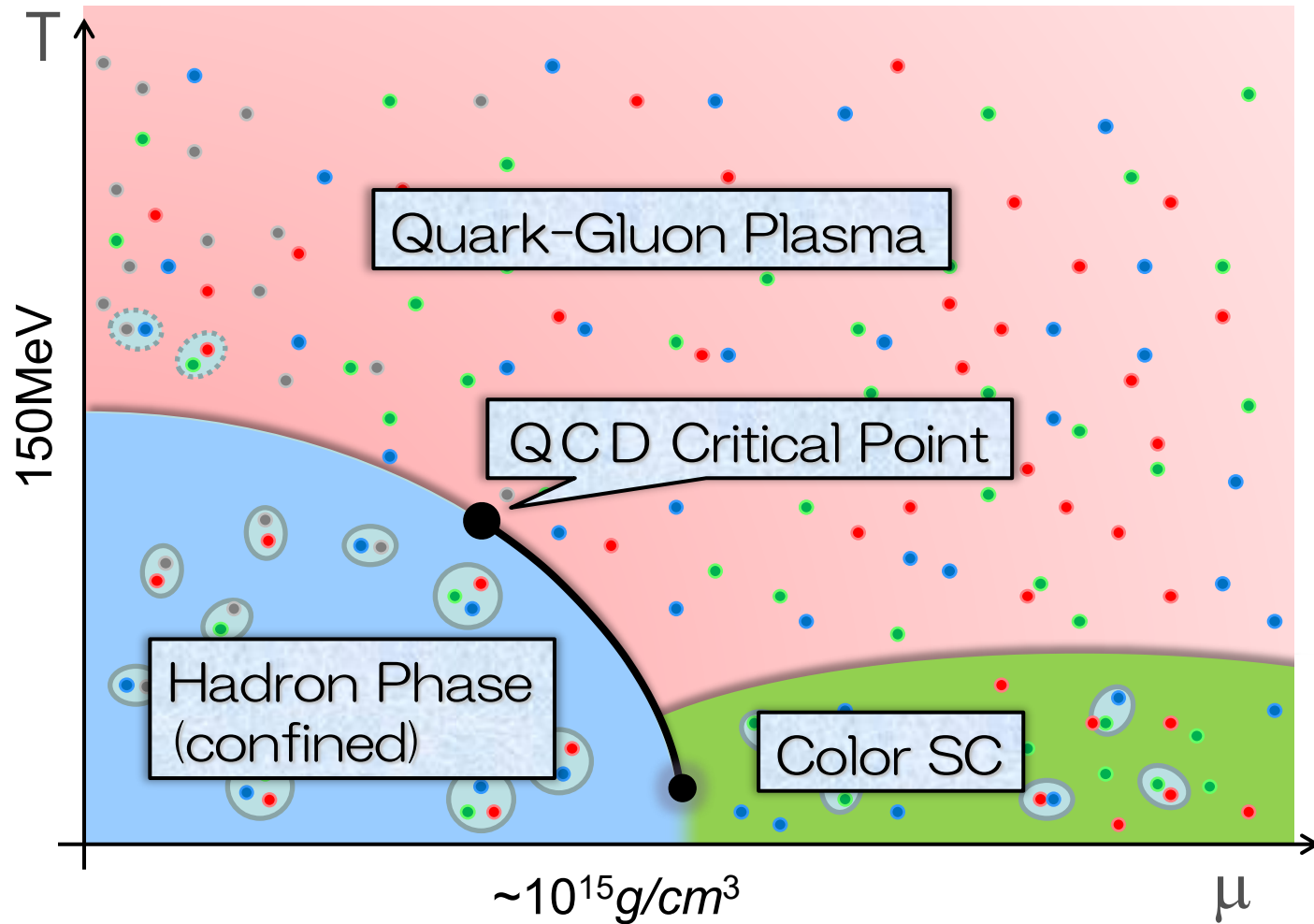
Exploring dense nuclear matter in relativistic heavy-ion collisions

Masakiyo Kitazawa
(YITP, Kyoto)

Taya, Jinno, MK, Nara, PRC C112, L011901 (2025).

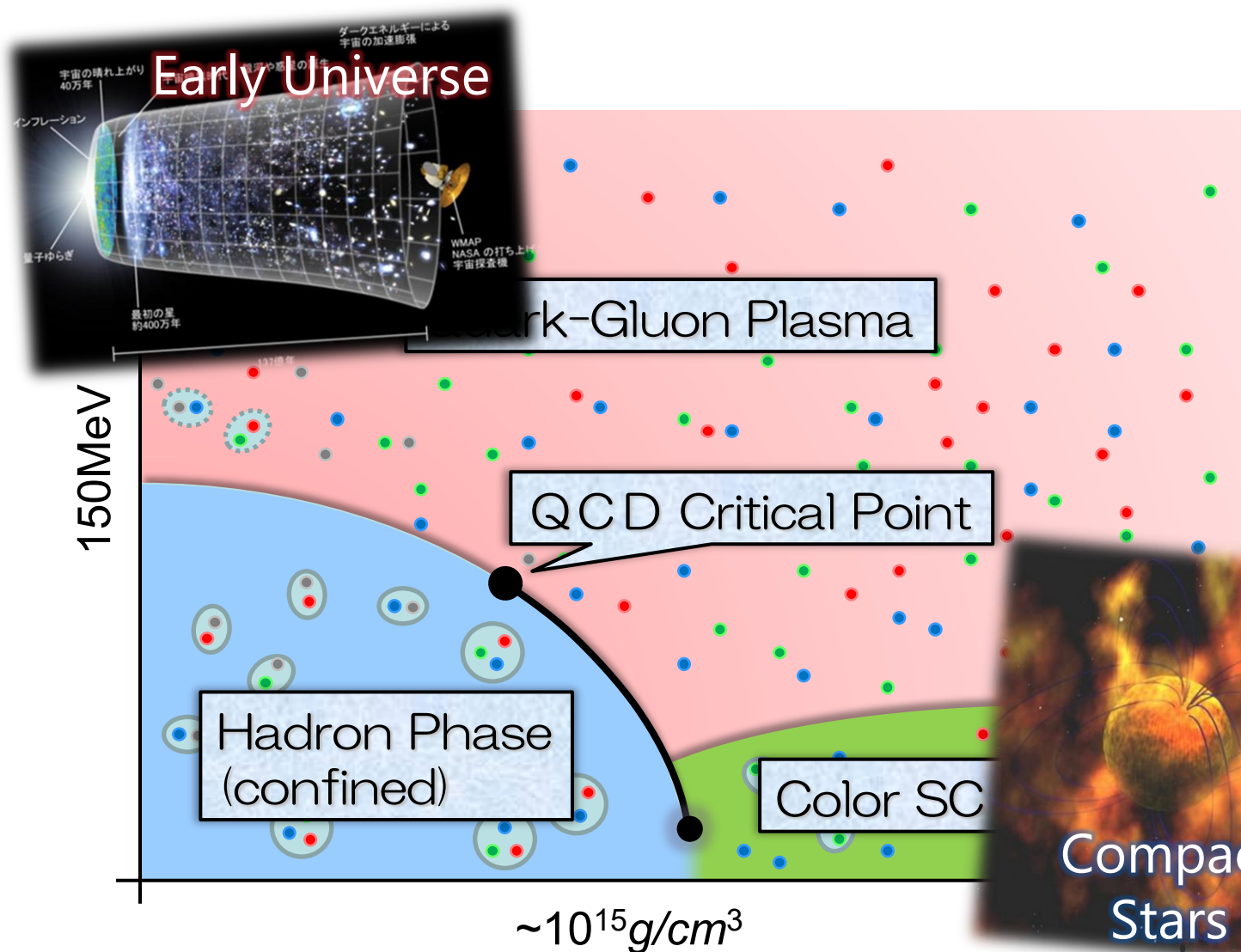
Nishimura, MK, Kunihiro, Ann. Phys. **469**, 169768 (2024); PTEP **2023**, 053D01; PTEP **2022**, 093D02.

QCD Phase Diagram



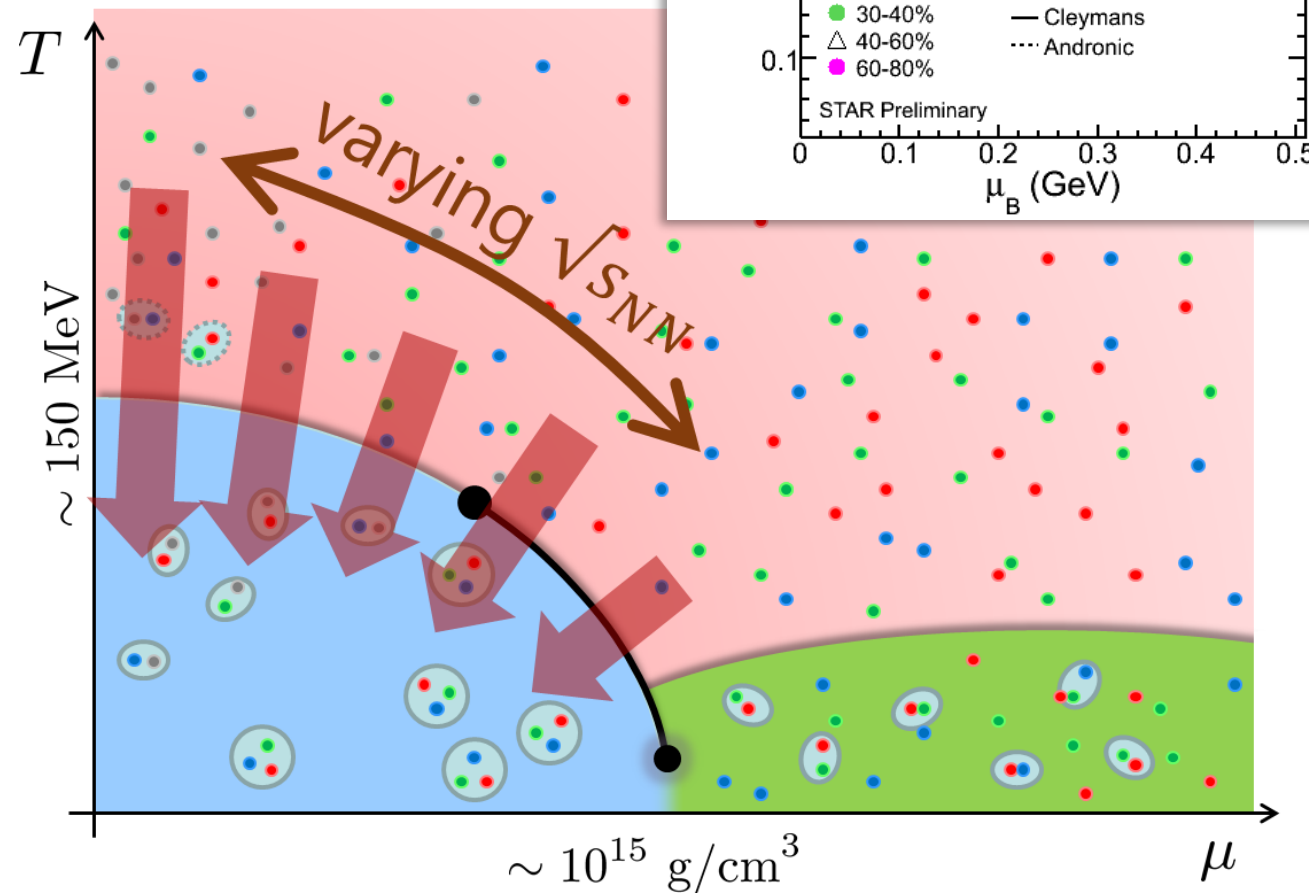
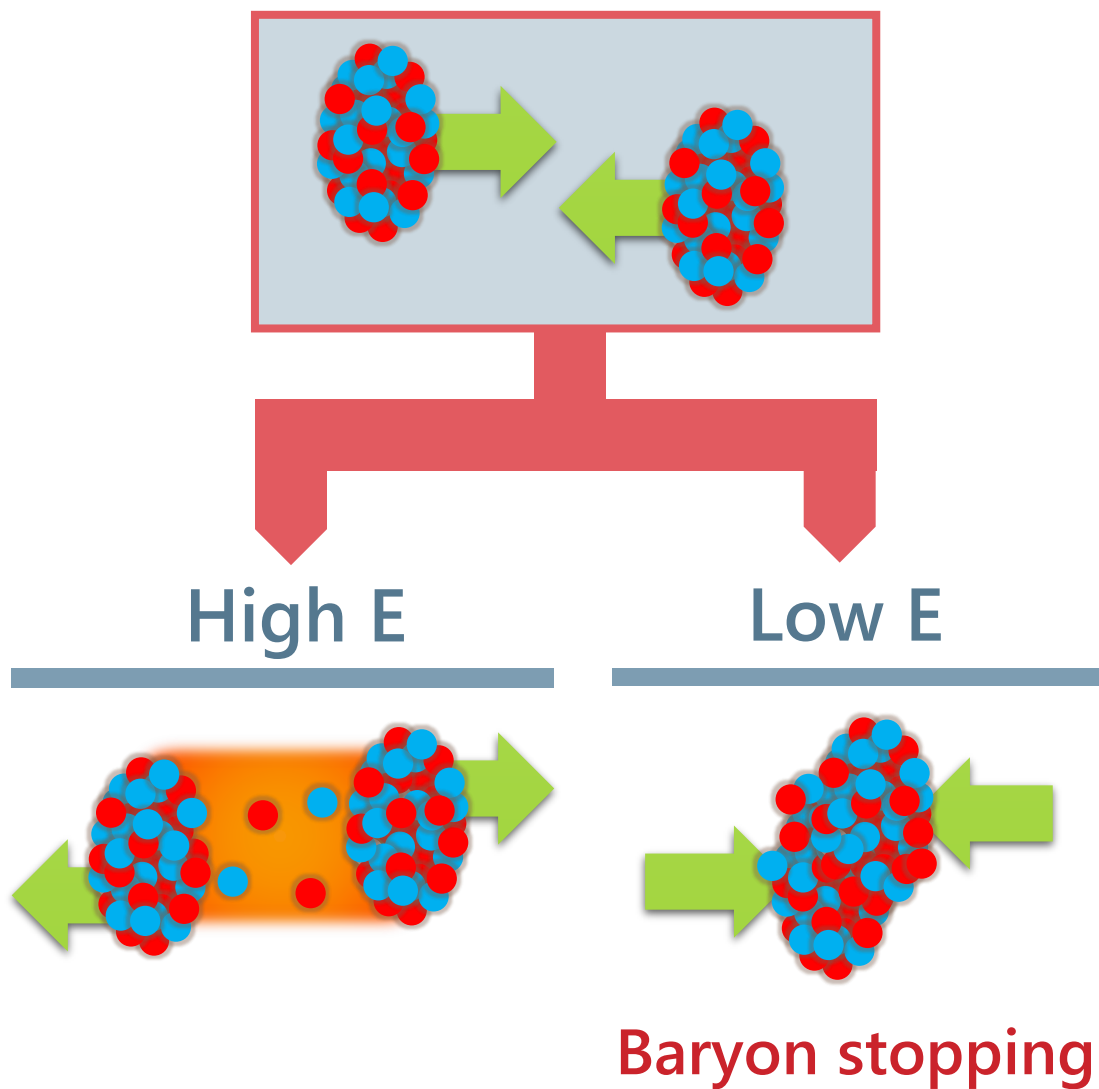
- ❑ Possible first-order transition and QCD critical point in dense region
- ❑ Multiple QCD-CP? [MK+ \('02\)](#)
- ❑ Color superconducting phases in dense and cold quark matter

QCD Phase Diagram

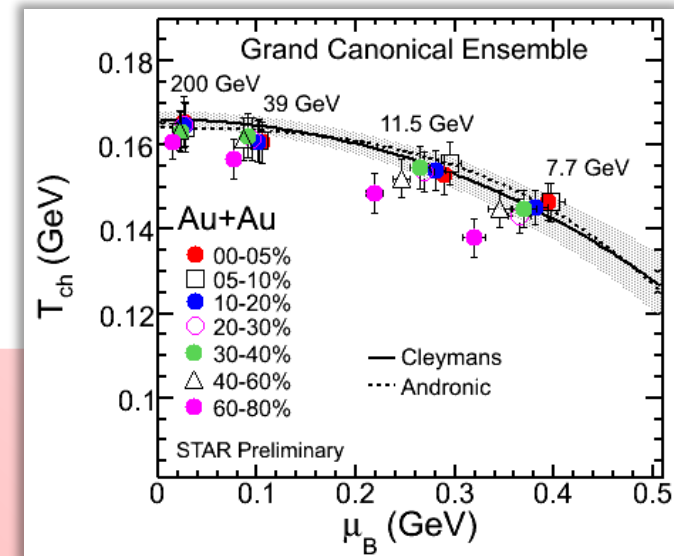


- ❑ Possible first-order transition and QCD critical point in dense region
- ❑ Multiple QCD-CP? [MK+ \('02\)](#)
- ❑ Color superconducting phases in dense and cold quark matter

Beam-Energy Scan



STAR, 2012



HIC across Energy Scales

Various purposes/pictures of the heavy-ion reactions

Low Energy

High Energy

Nuclear reactions

Nuclear liquid-gas tr.

Quark-gluon

breakup reactions

High-density

plasma

Heavy elements

Nuclear matter

Hydro. models

TDHF

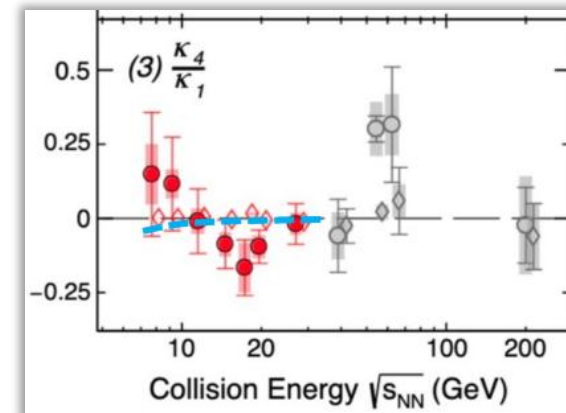
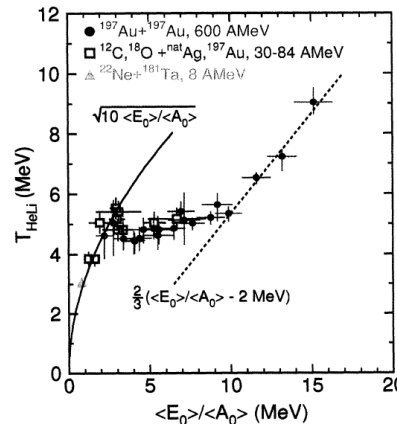
Langevin

BUU

Hadron cascade

AMD

AMPT





見小利則大事不成

Miss the woods for the trees

孔子

(論語、子路13)

Contents

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Taya, Jinno, MK, Nara, PRC C112, L011901 (2025).

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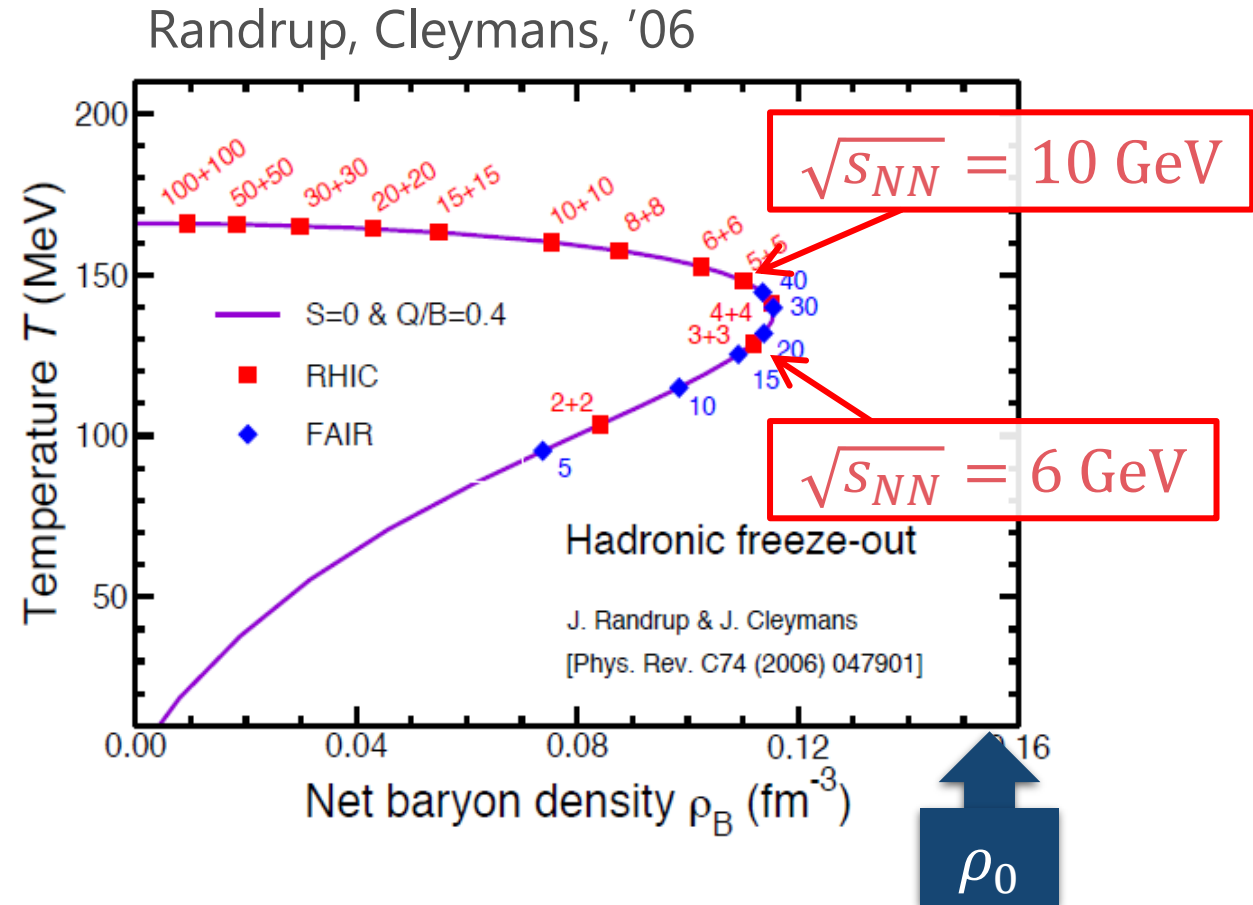
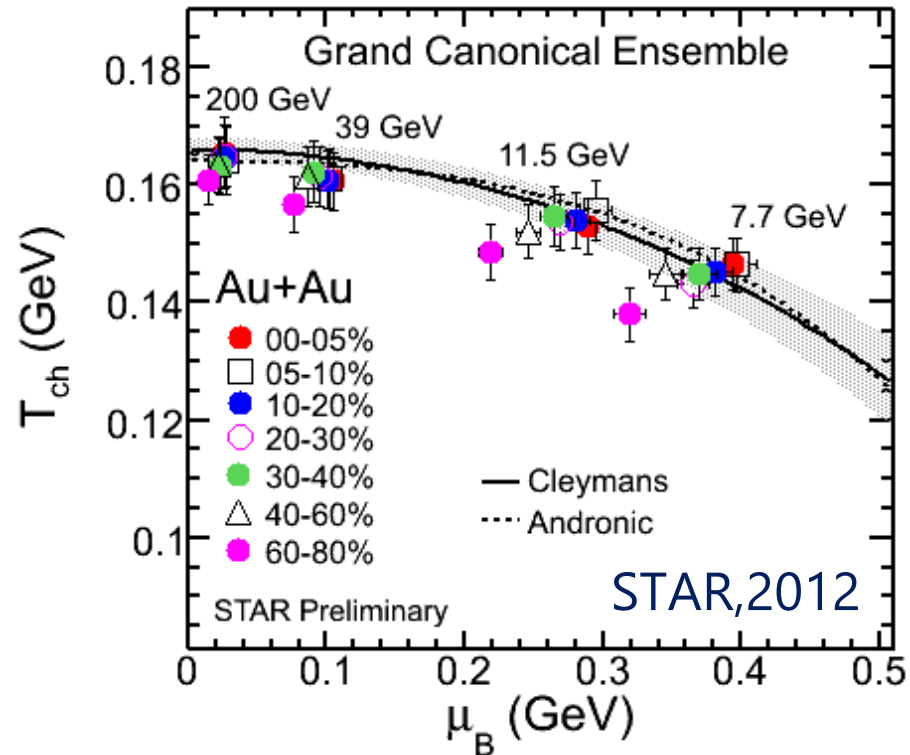
Key Questions

- What is **the optimal collision energy** to explore the baryon-rich matter?
- **How high density** is accessible?

Our Answer

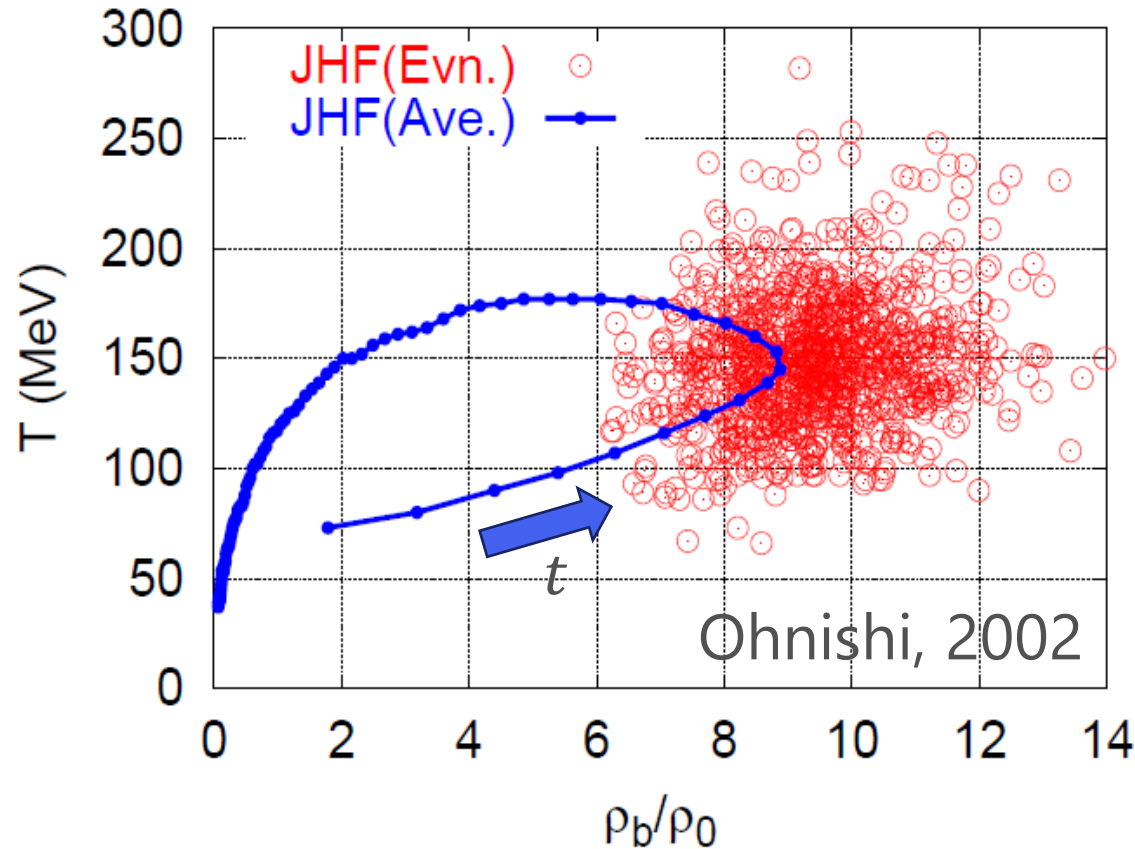
- $\sqrt{s_{NN}} = 3 \text{ GeV}$ is enough to study $\rho = 3\rho_0$.
- $\rho/\rho_0 = 4\sim 5$ may be accessible with $\sqrt{s_{NN}} = 3\sim 6 \text{ GeV}$.

Chemical Freezeout



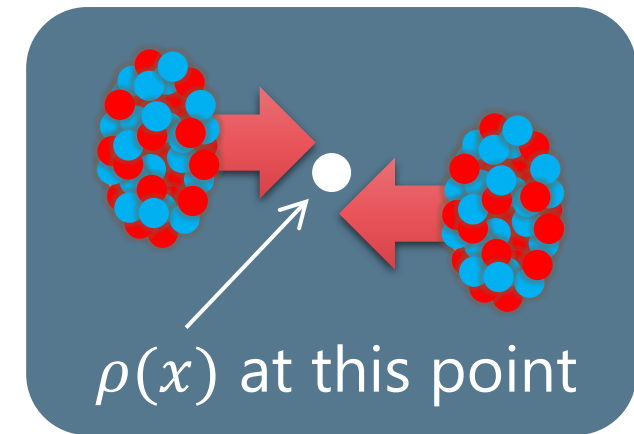
- Highest baryon density **at chemical freezeout** at $\sqrt{s_{NN}} \simeq 6 - 10 \text{ GeV}$?
- Not the highest density in the early stage.
- Density in earlier stage? ➡ Analysis in dynamical models

Baryon Density at Collision Point



Simulation by JAM

$$E/A = 20\text{GeV}, \quad \sqrt{s_{NN}} \simeq 6\text{GeV}$$



- Maximum baryon density exceeds $\rho/\rho_0 \simeq 8$!
- Large event-by-event fluctuations
- How large is the high-density region? How long is the lifetime?

Volume of Dense Region

Taya, Jinno, MK, Nara, PRC ('25).

Volume where the local baryon density is larger than a threshold value ρ_{th}

$$V_3(\rho_{\text{th}}, t) = \int_{\rho(x) > \rho_{\text{th}}} d^3\mathbf{x} \gamma$$

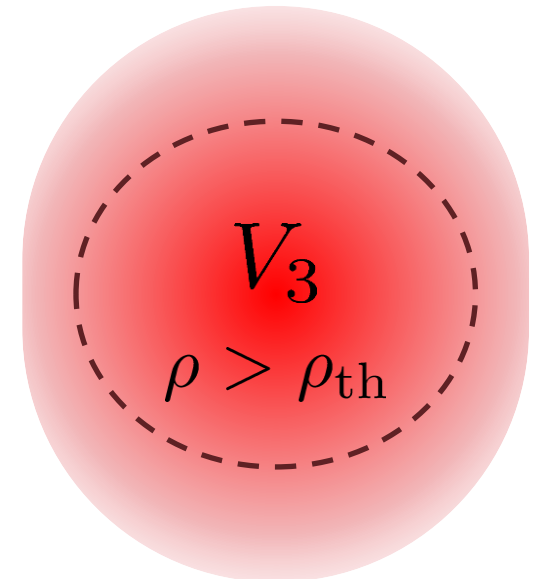
Baryon current $J^\mu(x)$

Baryon density $\rho(x) = \sqrt{J^\mu(x) J_\mu(x)}$

Lorentz factor $\gamma = (1 - (\mathbf{J}/J_0)^2)^{-1/2}$

Note:

- Event-by-event basis / no event average
- Directly calculable in a dynamical model
- We do not care about local thermalization.
 - V_3 is the upper limit of thermalized volume.
 - Even non-thermal, dense region is interesting!



Simulation Setup in JAM

- ❑ Au+Au collision for $2.4 \leq \sqrt{s_{NN}} \leq 20$ GeV
- ❑ Impact parameter $b \leq 3$ fm ~ **top 5% centrality**
- ❑ Momentum-dependent mean field (MF2) Nara, Ohnishi, 2022
 - Setup reproducing $\sqrt{s_{NN}}$ dep. of $dv_1/d\eta$ and v_2

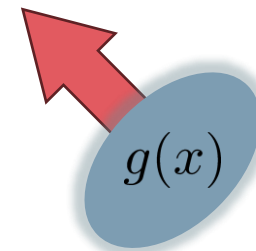
Smeared baryon current

discrete particle distribution → continuous current by smearing

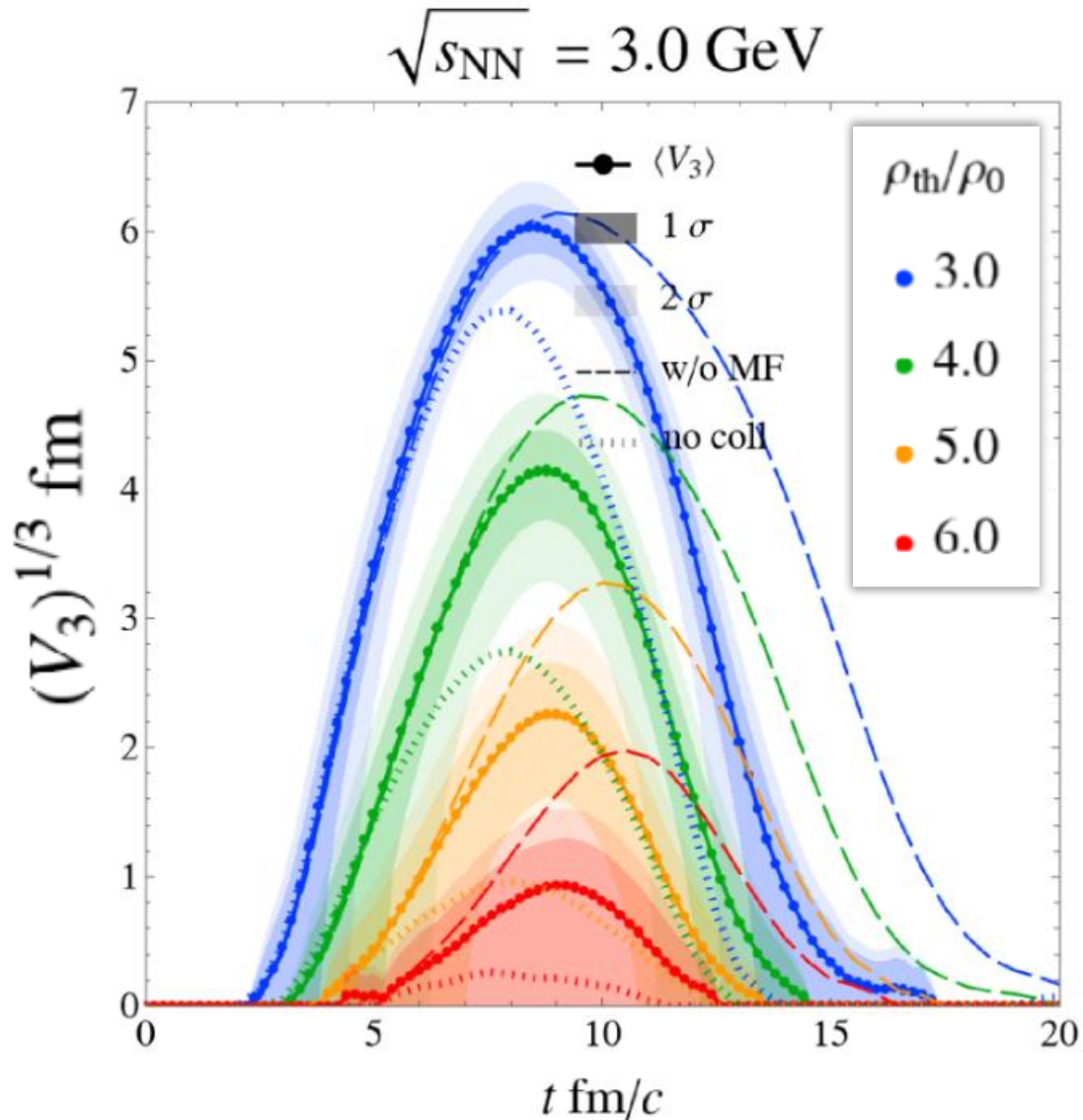
$$J^\mu(x) = \sum_{i \in \text{baryons}} B_i g(x; X_i, P_i) \frac{P_i^\mu}{P_i^0}$$

$$g(x; X, P) := \frac{\gamma}{(\sqrt{2\pi}r)^3} e^{-\frac{|\mathbf{x} - \mathbf{X}|^2 + (\gamma \mathbf{V} \cdot (\mathbf{x} - \mathbf{X}))^2}{2r^2}}$$

$$r = 1 \text{ fm}$$



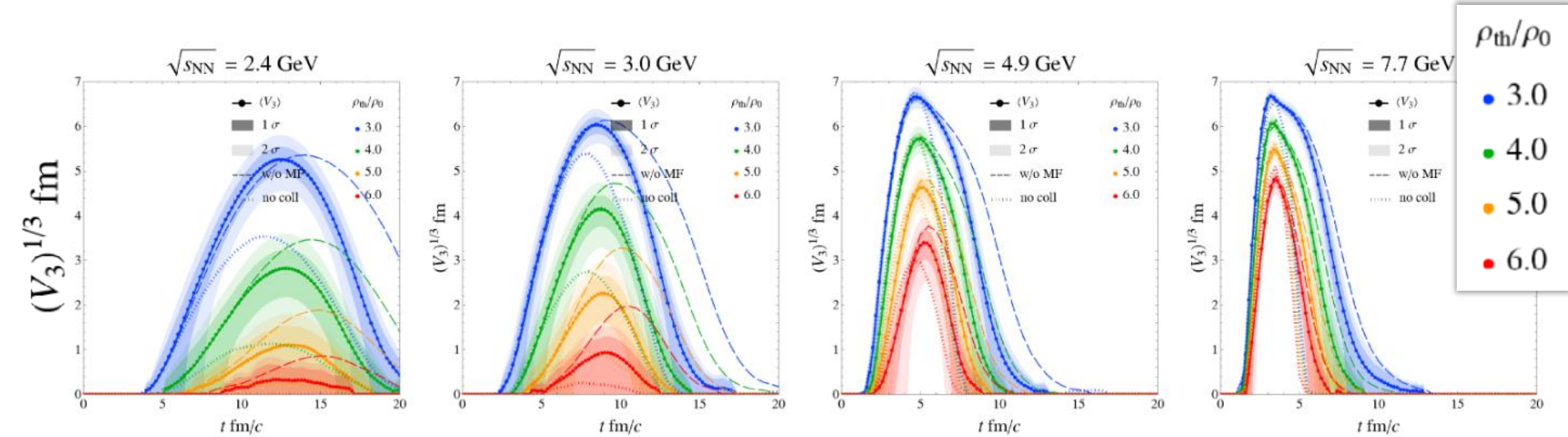
V_3 in JAM



- **solid: JAM+MF** Nara, Ohnishi, 2022
- shaded band: 1σ and 2σ e-v-e fluct.
- dashed: JAM cascade mode
- dotted: no-collision

- Formation of dense region:
 - $V_3(3\rho_0, t) = (6 \text{ fm})^3$
 - $V_3(4\rho_0, t) = (4 \text{ fm})^3$
- Large e-v-e fluctuations
 - separable by event selection?
- Repulsive MF → weaker compression
- Compression owing to interaction

V_3 for various $\sqrt{s_{NN}}$



As $\sqrt{s_{NN}}$ becomes larger,

- $\max V_3(\rho_{th}, t)$ becomes larger.
- The lifetime of dense region becomes shorter.
- E-v-e fluctuations are more suppressed.

Four-Volume / Lifetime

Four Volume

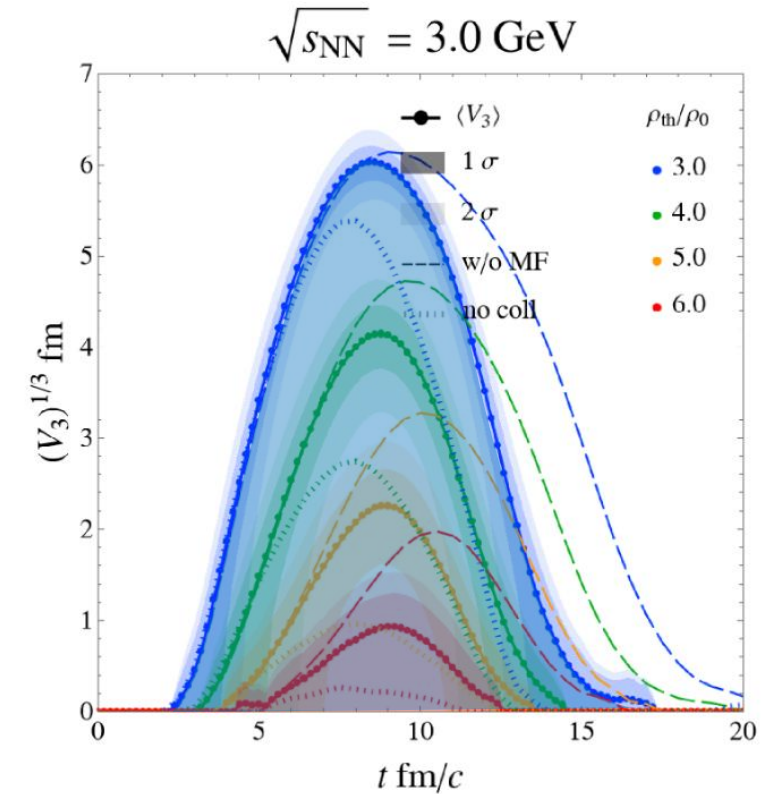
$$V_4(\rho_{\text{th}}) = \int_{-\infty}^{\infty} dt \int_{\rho(x) > \rho_{\text{th}}} d^3x$$

Lifetime

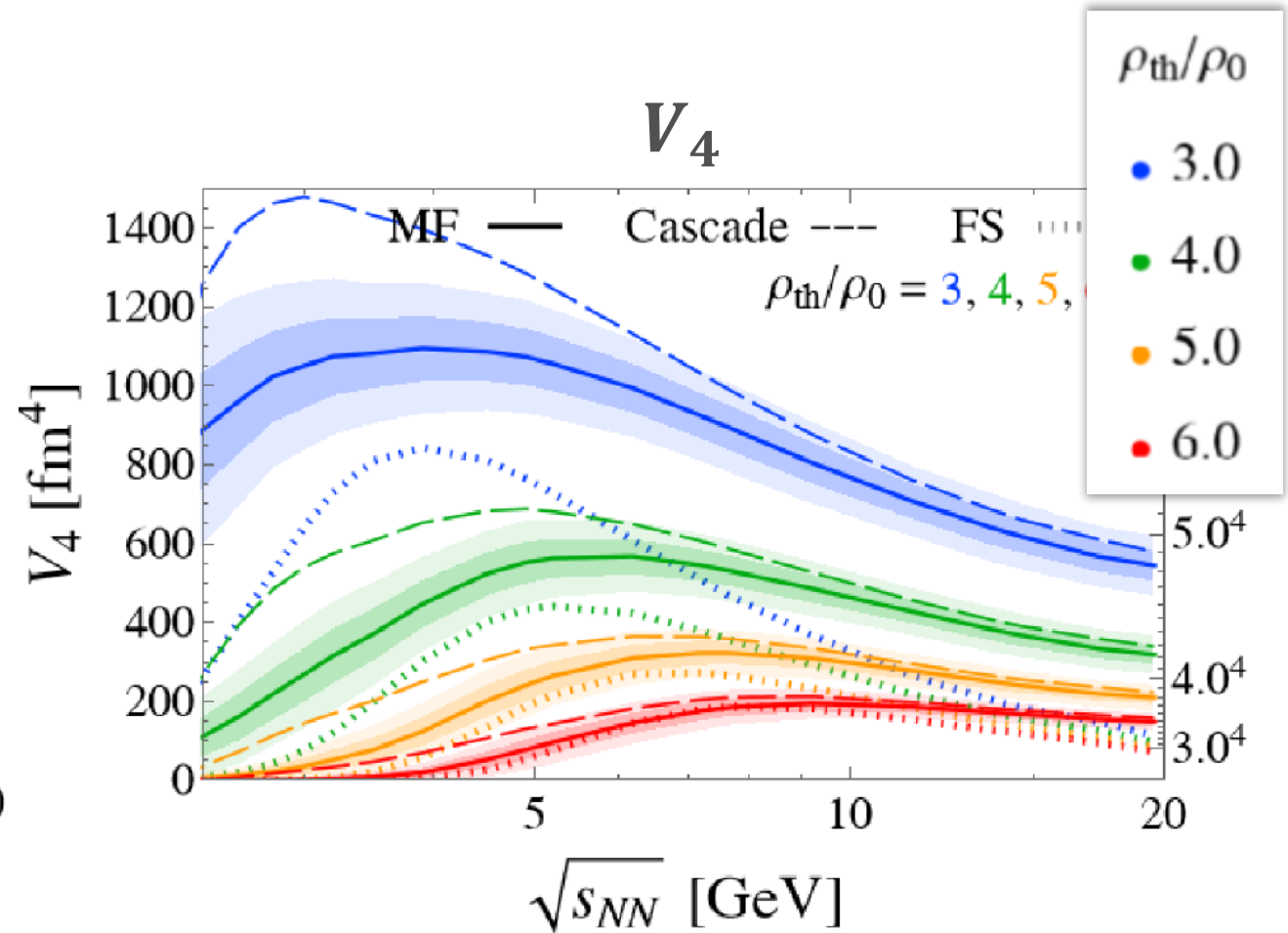
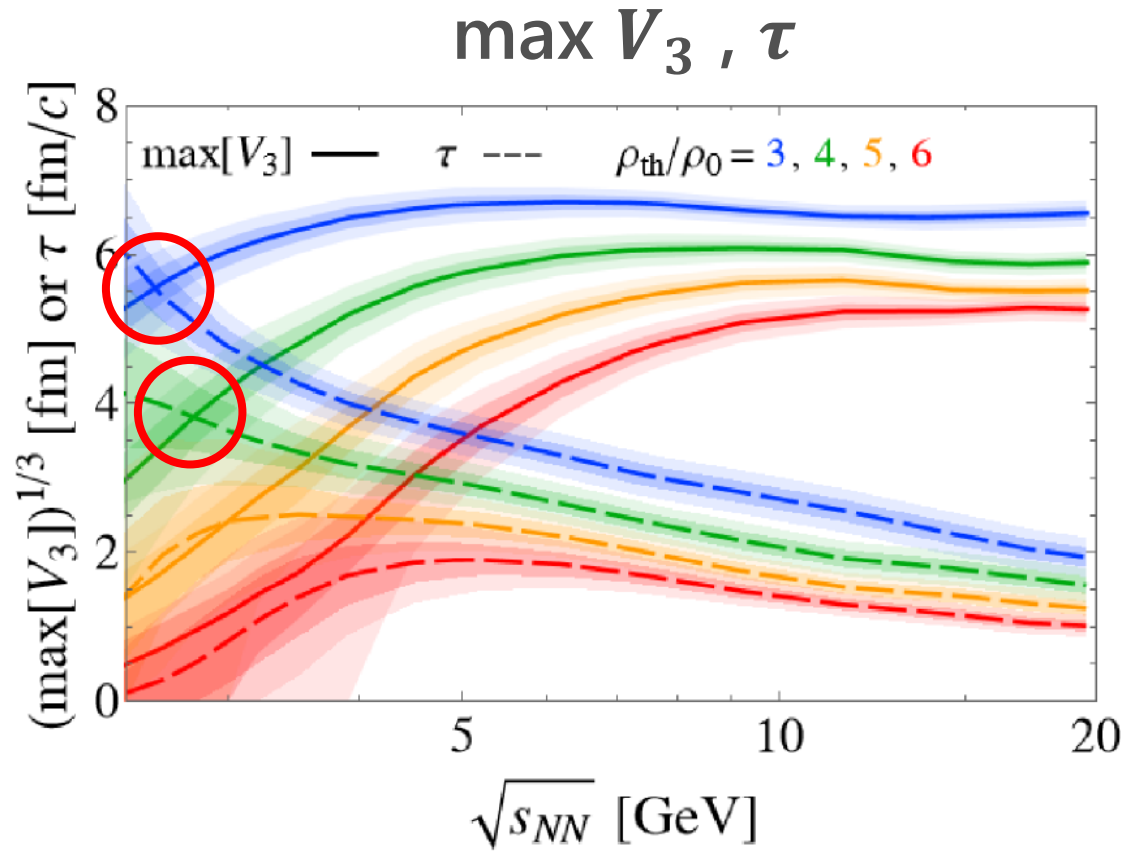
$$\tau(\rho_{\text{th}}) = \frac{V_4(\rho_{\text{th}})}{\max V_3(\rho_{\text{th}}, t)}$$

Note

V_4 may be relevant for the dilepton production rate.



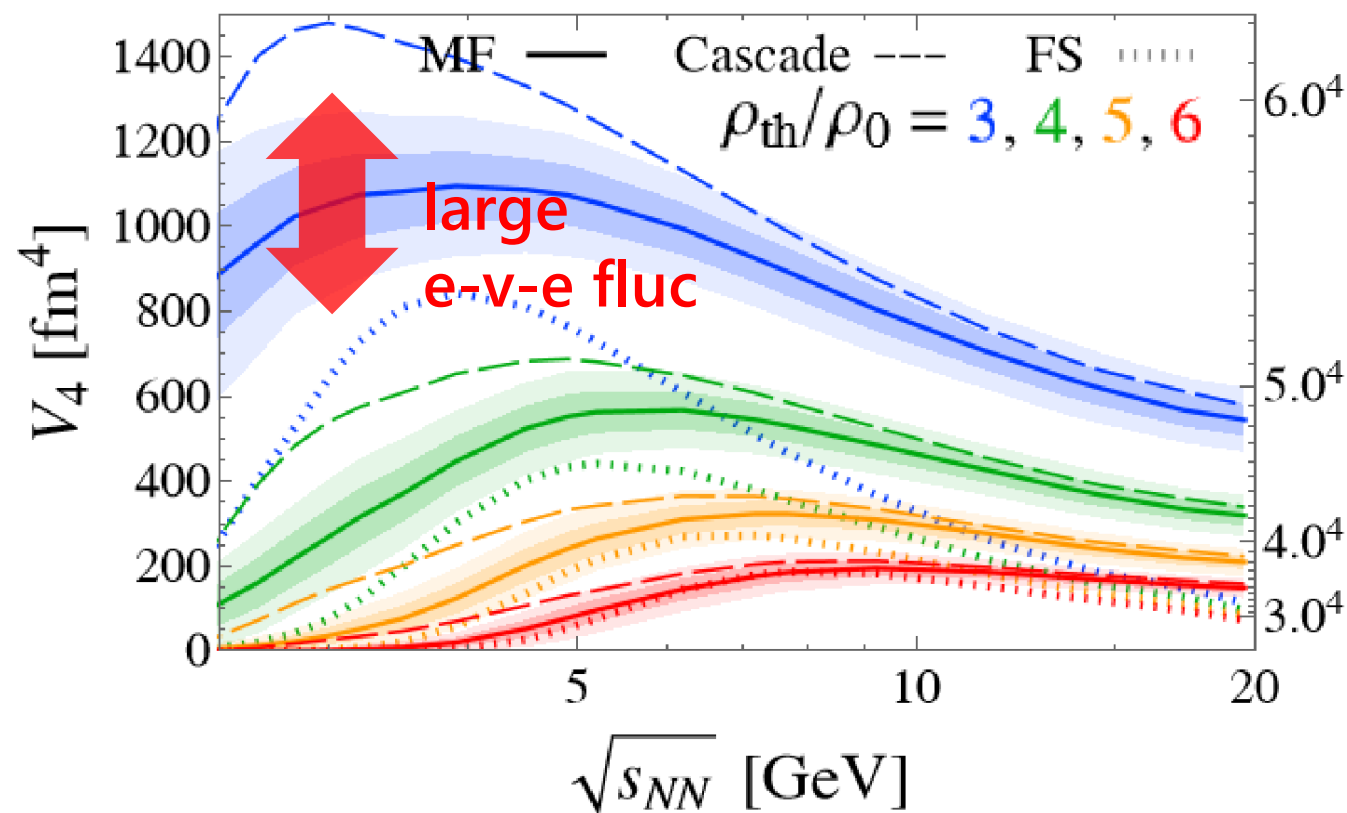
$\sqrt{s_{NN}}$ Dependence



- $\sqrt{s_{NN}} \simeq 3$ GeV would be the best energy to create $\rho = 3 \sim 4\rho_0$ with large V_3 and τ .
- Lower $\sqrt{s_{NN}}$ is suitable to create colder matter.

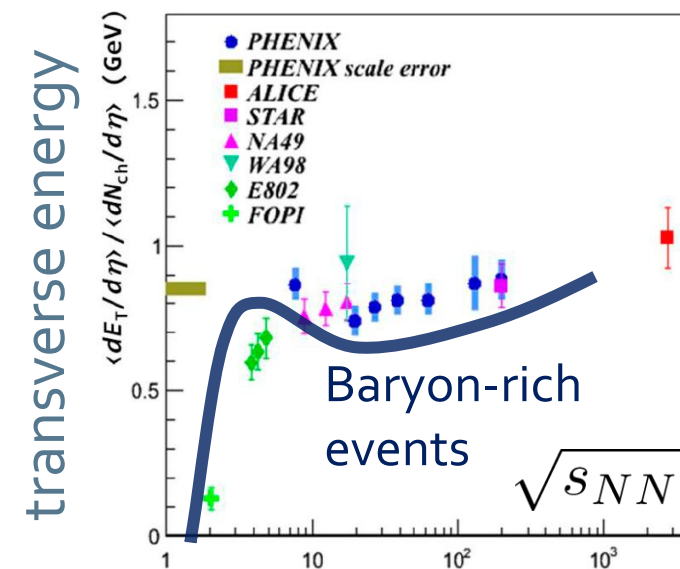
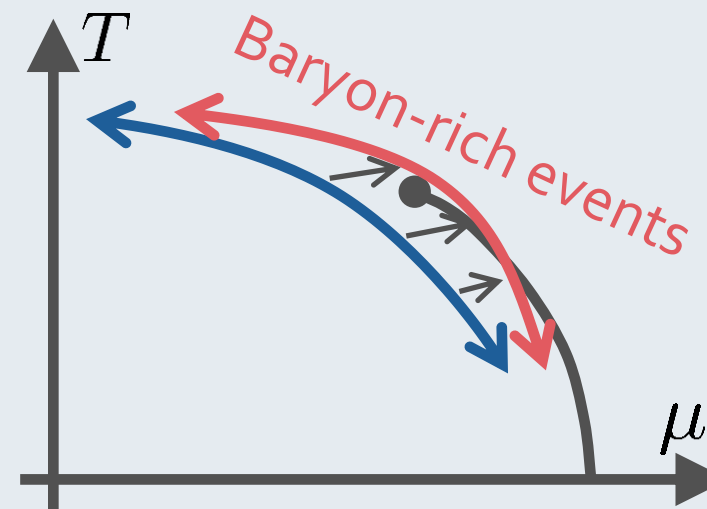
Event Selection

Taya, Jinno, MK, Nara, PRC ('25).

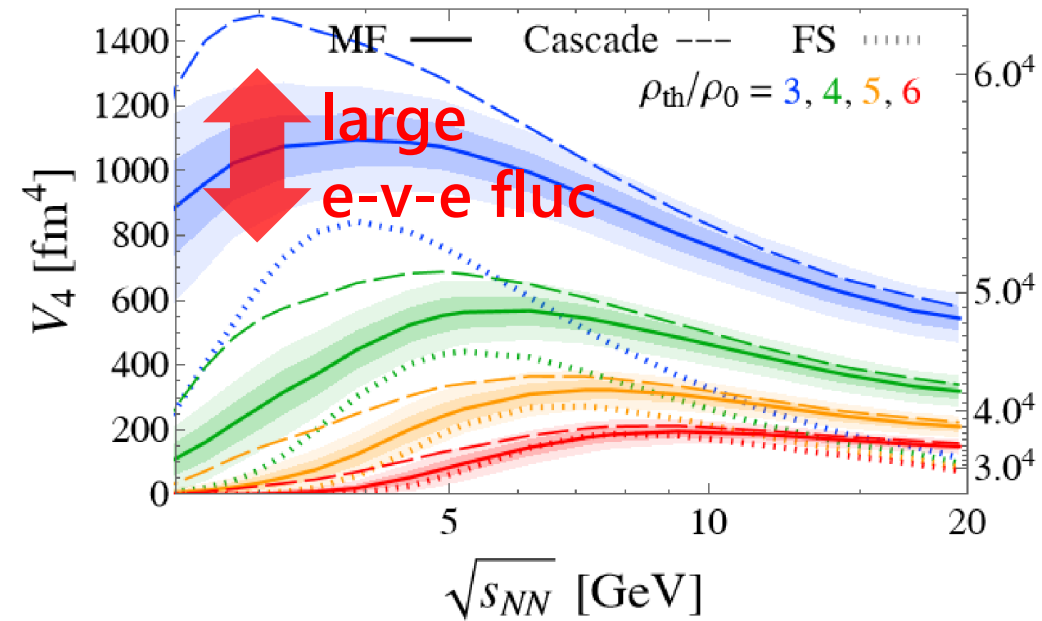
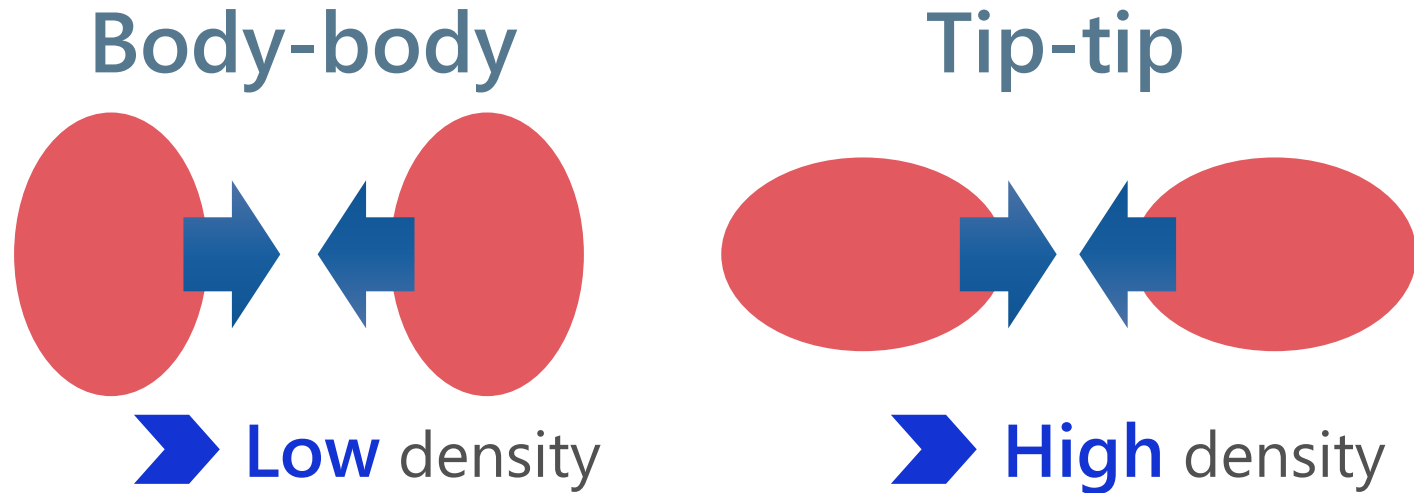


- Event selections via highest baryon/energy density will allow us a detailed study of QCD phase diagram.

Density Scan?



More Density Fluctuations / Denser Matter for Deformed Nuclei?



- Event-by-event density fluctuations will be significantly amplified in the collision of deformed nuclei.
- Tip-tip collisions can create the highest density matter on the Earth.

Short Summary

- $\sqrt{s_{NN}} = 3 \text{ GeV}$ is enough to study $\rho = 3\rho_0$.
- $\rho/\rho_0 = 4\sim 5$ may be accessible with $\sqrt{s_{NN}} = 3\sim 6 \text{ GeV}$.

Future

- Check model independence
 - Analyses in various models
- Experiments at the sweet spot $\sqrt{s_{NN}} = 2.5\sim 6 \text{ GeV}$
 - Future exps. at FAIR, NICA, HIAF, & J-PARC-HI

Contents

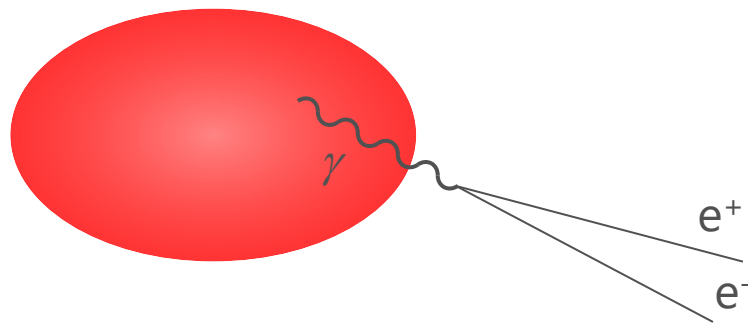
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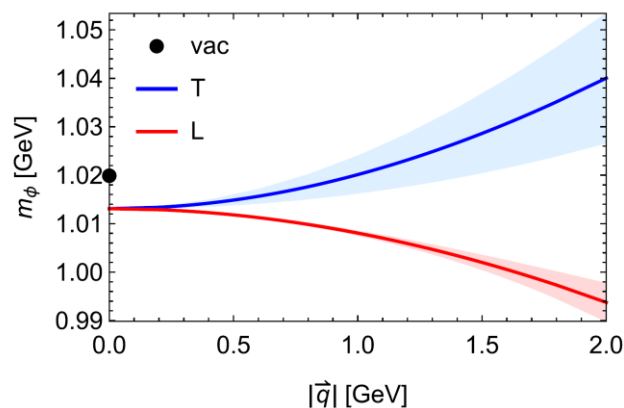
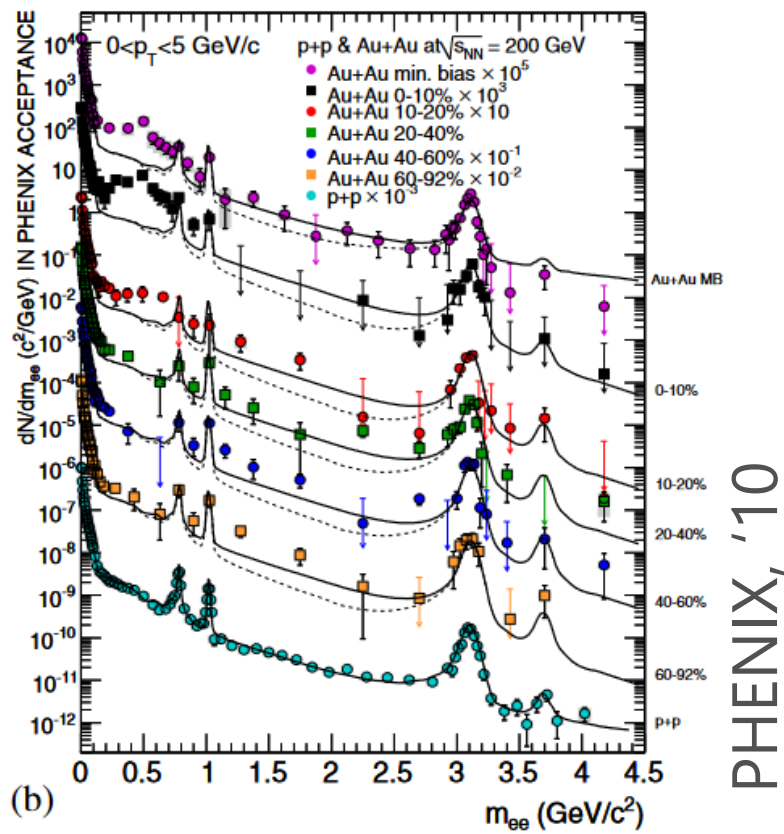
Dilepton Production Rate



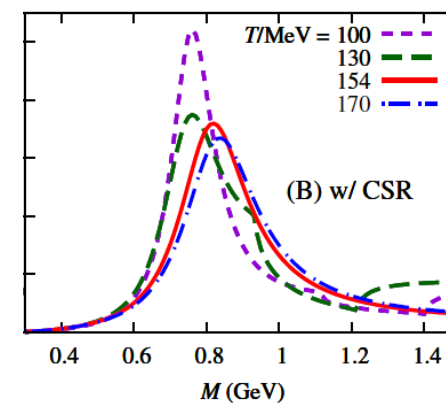
- Generated by the decay of virtual photons
- Carry information of primordial medium

Physics accessible with DPR

- Medium temperature
- Dispersion relations
- Chiral mixing by chiral restoration
- Signal of phase transitions



Kim, Gubler, 2020



Sakai+, 2024

Soft Modes of Second-order Phase Transitions

□ Soft modes

- Divergence of the order-parameter fluctuations at a 2nd-order transition.

- Collective fluctuations become massless there.

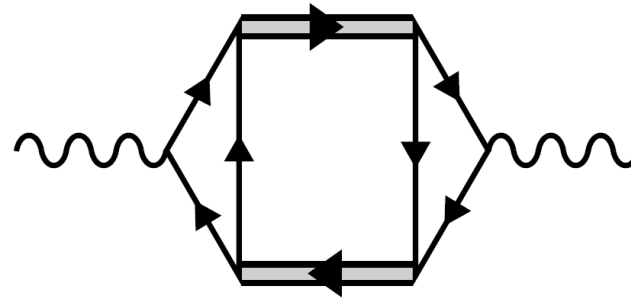
- QCD-CP : density-density fluctuations

- CSC : diquark-pair field



□ Coupling of soft modes with dynamical observables

- Ex.: dilepton production rate



Question: How do the soft modes of QCD-CP/CSC affect DPR?

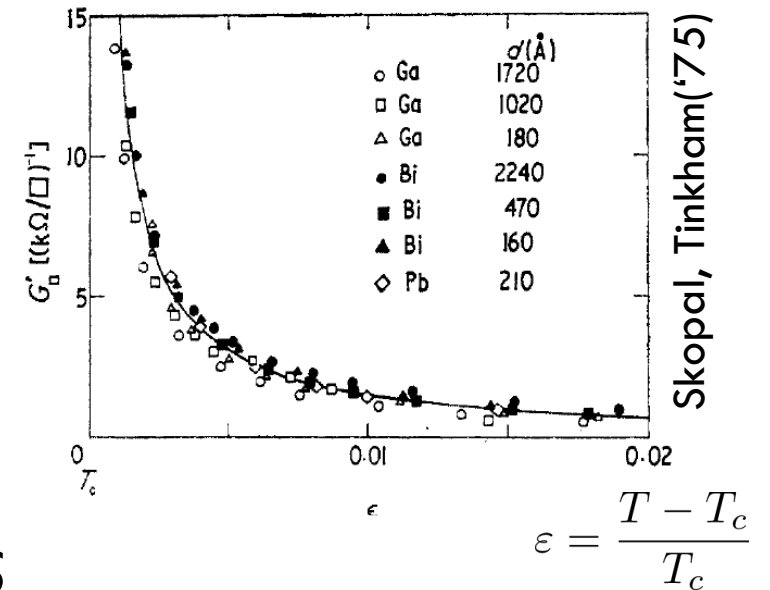
Precursor in Metallic Superconductors

□ Anomalous behavior of observables near but above T_c of SC

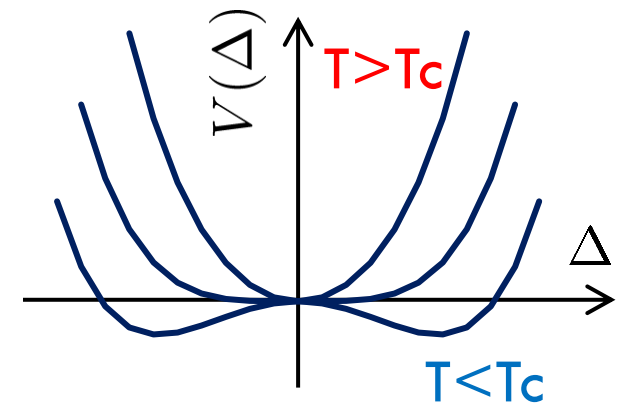
- electric conductivity
- magnetic susceptibility
- pseudogap

- Enhanced pair fluctuations is one of the origins of precursory phenomena.
- More significant phenomena in strongly-coupled systems.

Electric conductivity



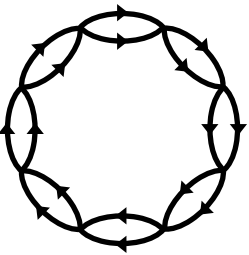
Landau's free energy



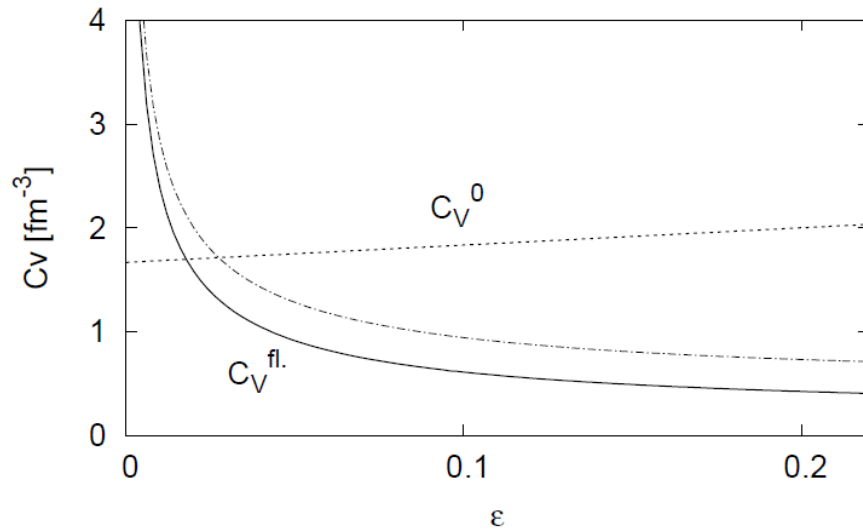
Precursor of Color Superconductivity

MK, Koide, Kunihiro, Nemoto, '03, '05

□ Thermodynamic Potential

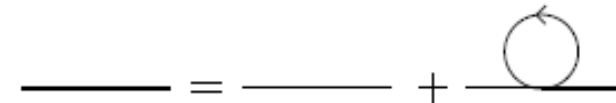
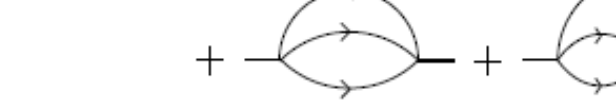
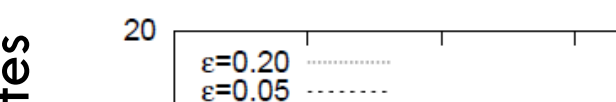
$\Omega =$  $\rightarrow$ Specific heat

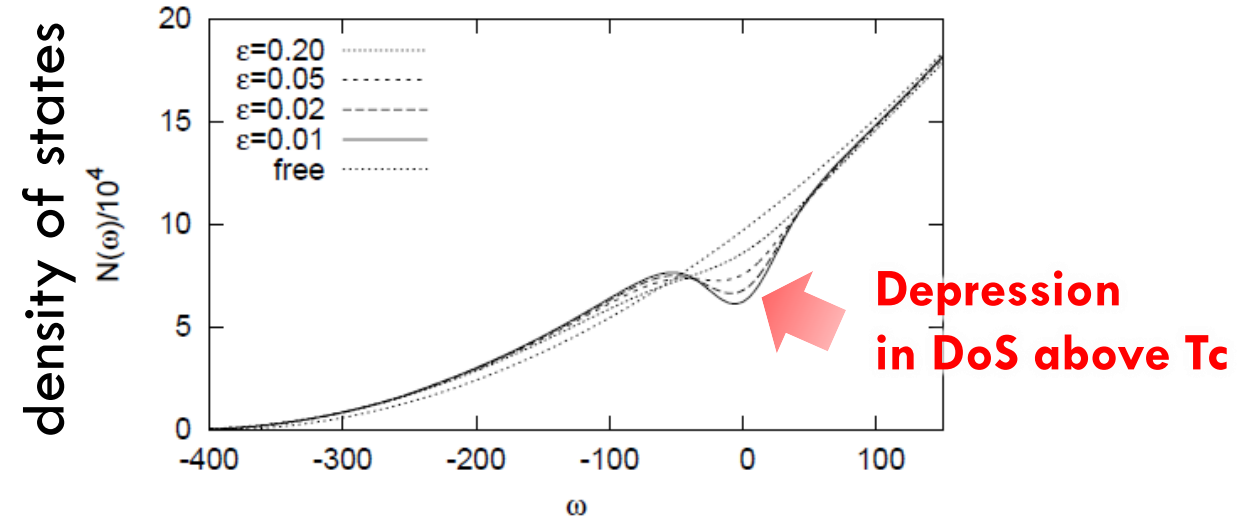
$$c = -T \frac{\partial^2 \Omega}{\partial T^2}$$



$$\varepsilon = \frac{T - T_c}{T_c}$$

□ Pseudogap

 $=$  $+$  $+$  $+$ $\dots$



Model

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S ((\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2)$$

$$\mathcal{L}_C = G_C ((\bar{\psi} i \gamma_5 \tau_A \lambda_A \psi^C)(\text{h.c.}))$$

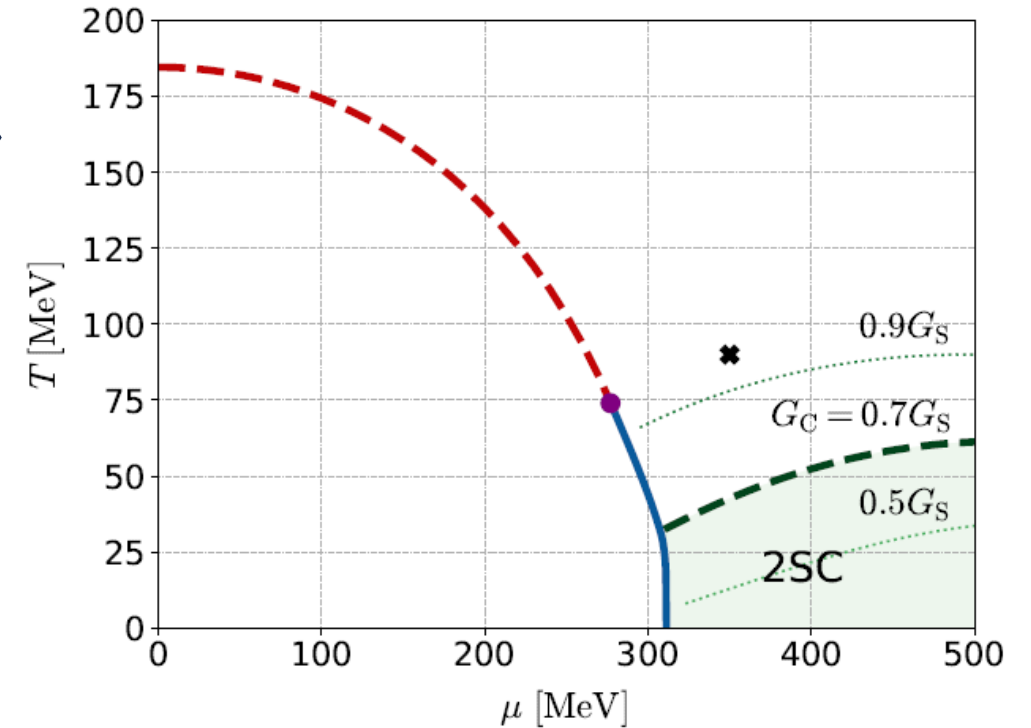
diquark interaction

Parameters

$$G_S = 5.01 \text{ GeV}^{-2}, \quad \Lambda = 650 \text{ MeV}, \quad m_q = 0$$



Phase Diagram in MFA



□ Order of phase transition

□ 2nd in the MFA

□ can be 1st due to gauge fluctuation

Matsuura+('04), Giannakis+('04)

Noronha+('06), Fejos, Yamamoto('19)

Di-quark Fluctuations

□ Diquark Propagator

$$D^R(x) = \langle [\Delta^\dagger(x), \Delta(0)] \rangle \theta(t) = \Rightarrow$$

□ Random Phase Approximation

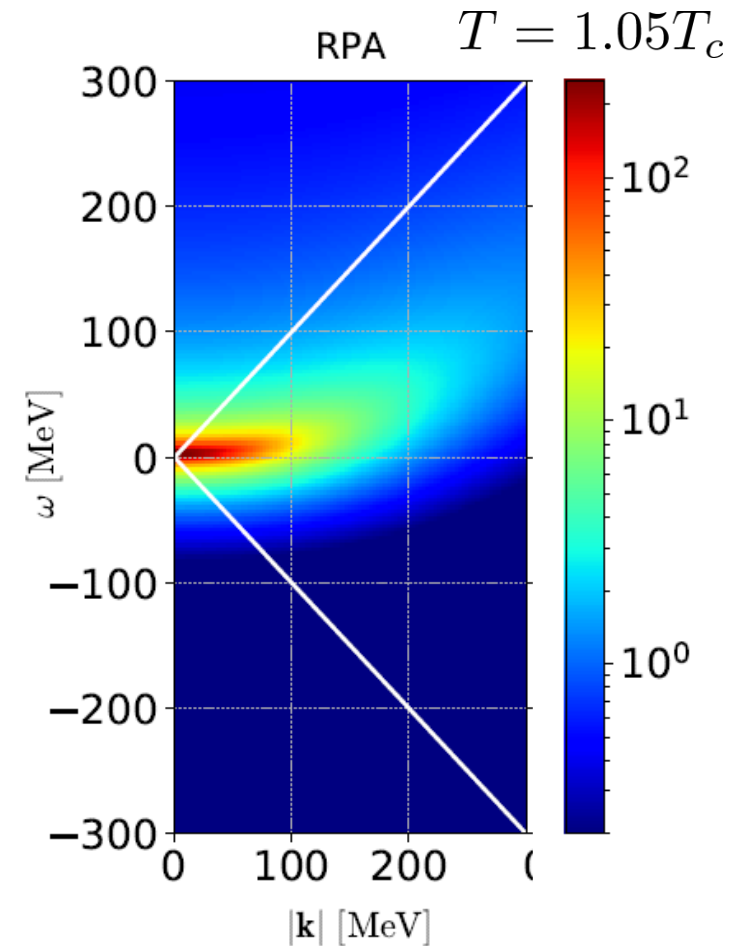
$$\begin{aligned} \Rightarrow &= \text{bubble} + \text{two bubbles} + \dots \\ &= \frac{Q^R(\mathbf{k}, \omega)}{1 + G_C Q^R(\mathbf{k}, \omega)} \\ Q^R(\mathbf{k}, \omega) &= \text{bubble} \end{aligned}$$

- Diquark field becomes massless at $T=T_c$
- Soft mode of CSC transition
- Strength in the space-like region

MK, Koide, Kunihiro, Nemoto, '01,'05

Dynamical Structure Factor

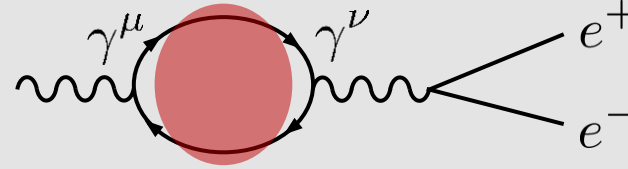
$$S(\mathbf{k}, \omega) = -\frac{1}{\pi} \frac{1}{1 - e^{-\beta\omega}} \text{Im} D^R(\mathbf{k}, \omega)$$



Photon Self-Energy: Precursor of CSC

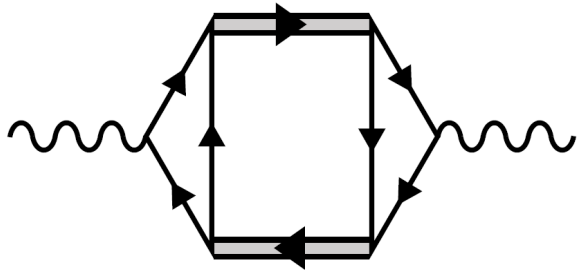
□ Dilepton Production Rate

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^{\beta\omega}-1} \text{Im}\Pi^{R\mu}_{\mu}(k)$$

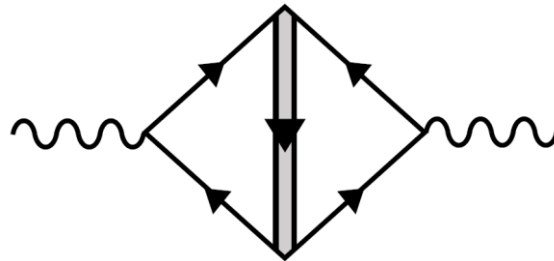


□ Effect of Di-quarks on $\Pi^{\mu\nu}(k)$

Aslamasov-Larkin term



Maki-Thompson term

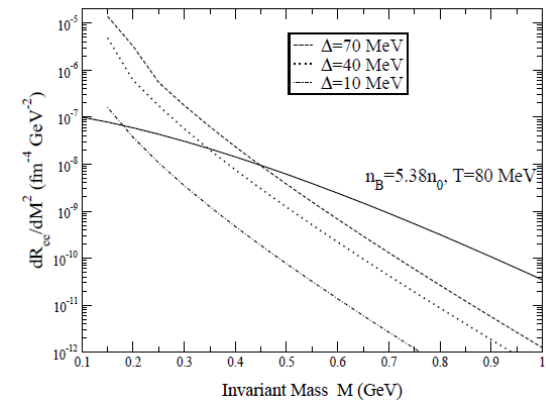
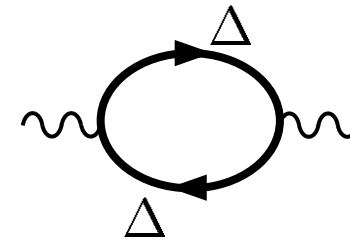


Well-known diagrams in metallic SC
for describing paraconductivity



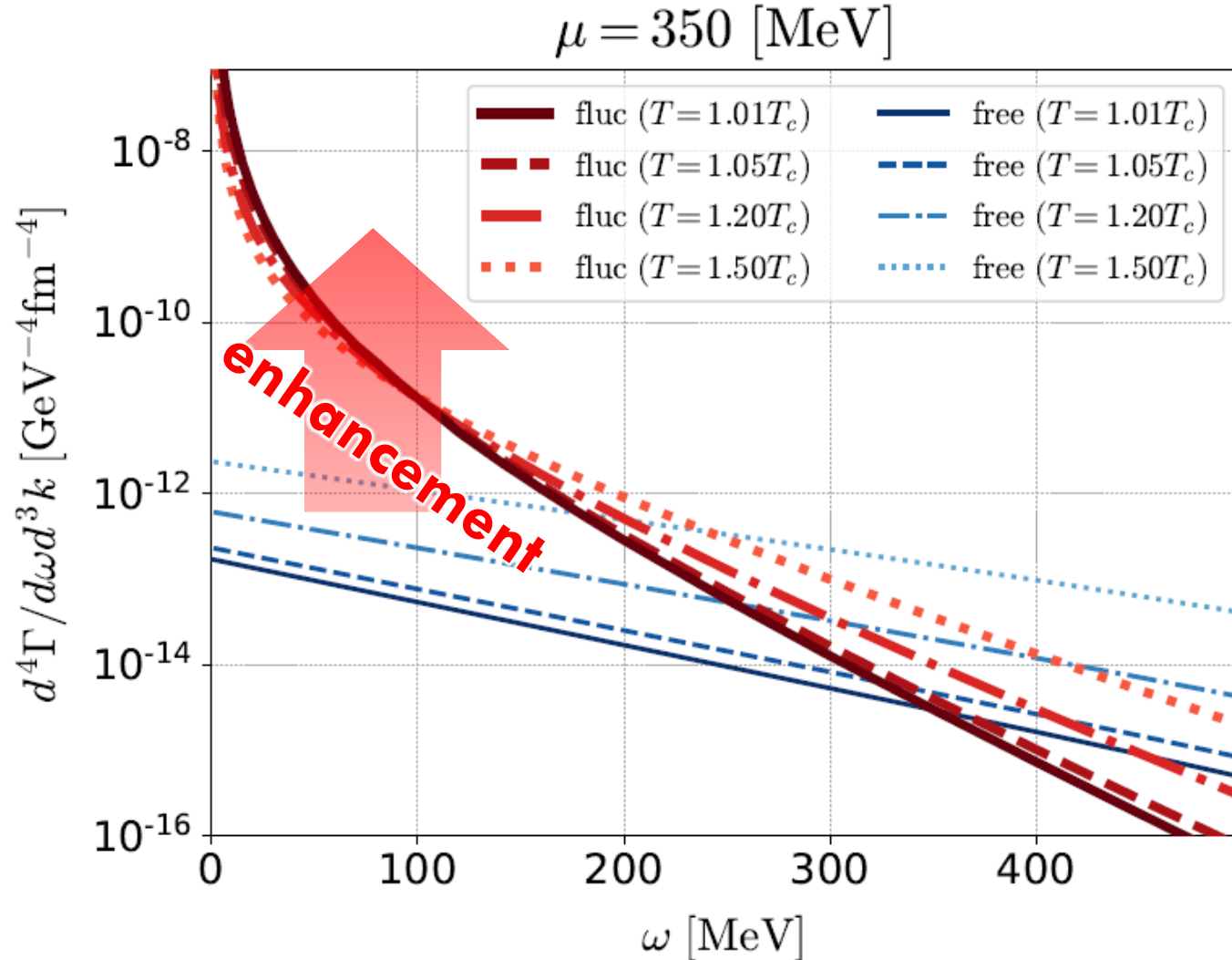
□ DPR from CFL phase

Jaikumar, Rapp, Zahed ('02)



Production Rate at $k = 0$

Nishimura, MK, Kunihiro ('22)



Red: fluctuation contribution

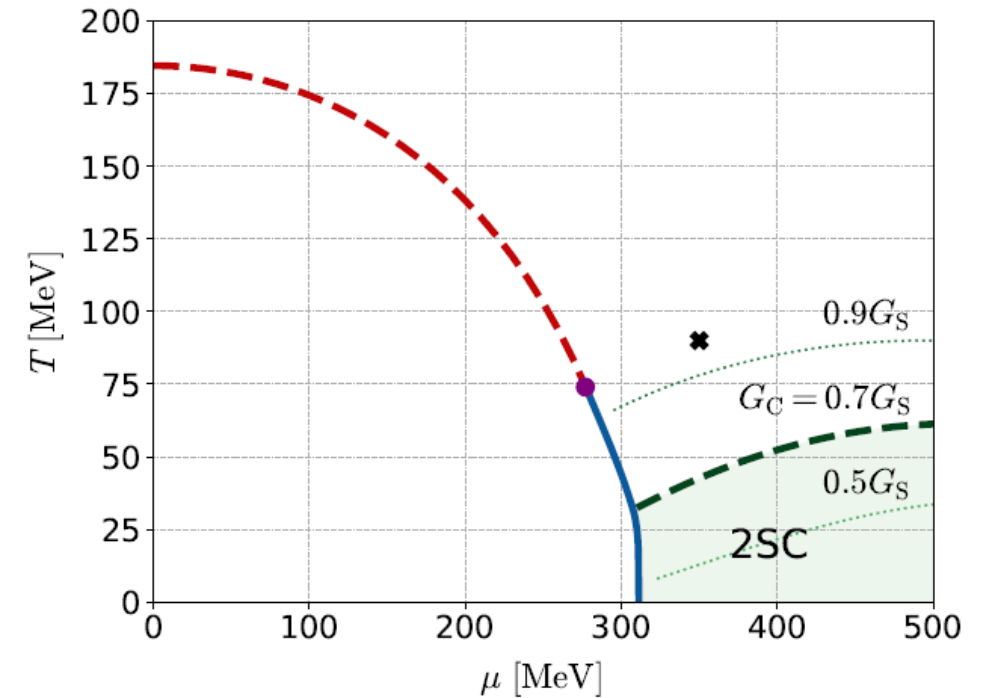
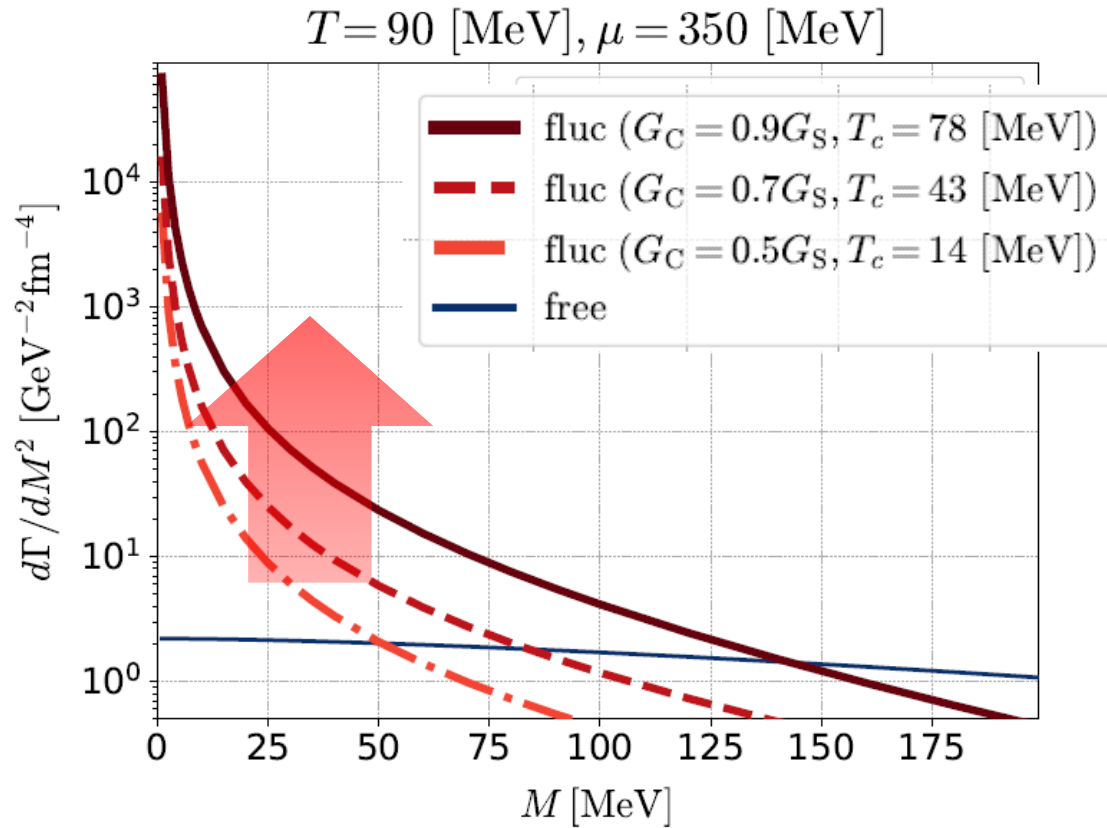
Blue: free quarks

$$G_C = 0.7G_S, T_c \simeq 45 \text{ MeV}$$

- Di-quark fluctuations give rise to large enhancement in the low energy region $\omega < 200$ MeV and $T < 1.5T_c$.
- Anomalous enhancement is not sensitive to T .

Invariant-Mass Spectrum

Nishimura, MK, Kunihiro ('22)



- ❑ Strong enhancement at low invariant mass.
- ❑ Observable in the HIC?

Dileptons from QCD Critical Point

NJL model (2-flavor)

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi + G_S((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2)$$

Parameters

$$G_S = 5.5 \text{ GeV}^{-2}, \quad \Lambda = 631 \text{ MeV}, \quad m_q = 5.5 \text{ MeV}$$

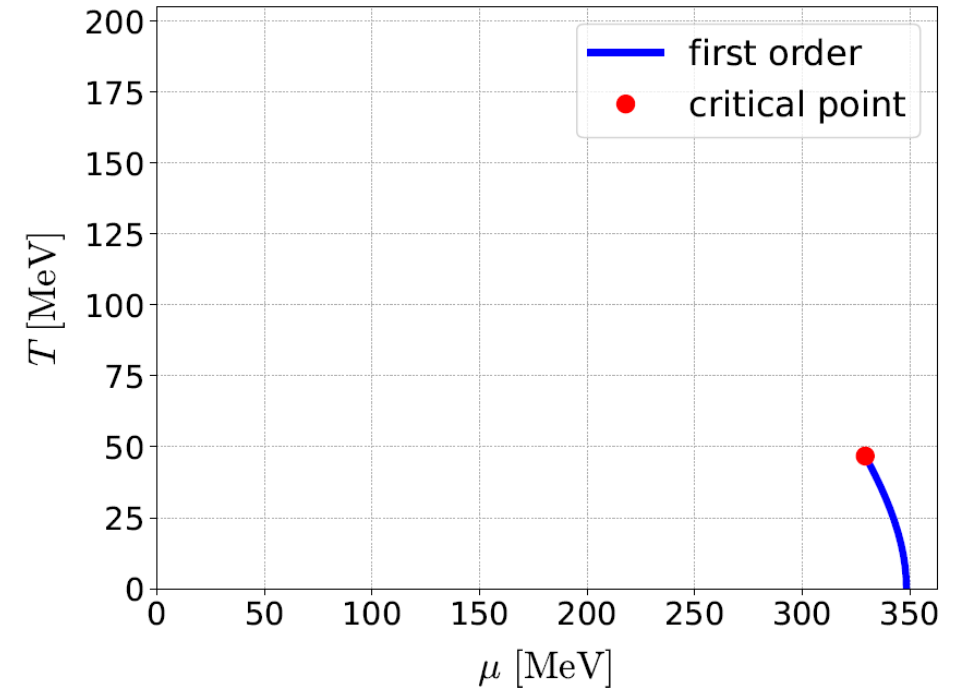
Soft Mode of QCD-CP

= fluctuation of scalar ($\bar{q}q$) channel

$$D^R(x) = \langle [\bar{\psi}\psi(x), \bar{\psi}\psi(0)] \rangle \theta(t) = \Rightarrow \Rightarrow$$

□ Random Phase Approximaion

$$\Rightarrow \Rightarrow = \text{bubble} + \text{two-bubbles} + \dots$$



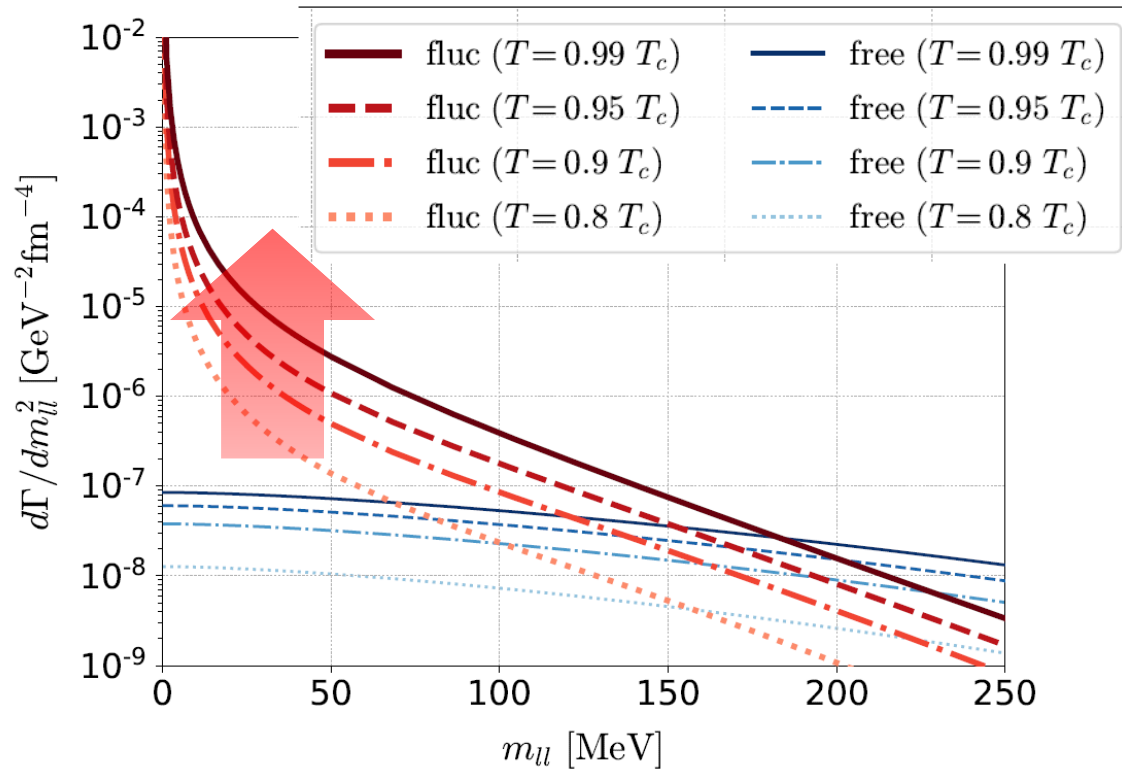
Modification of dilepton production through

A Feynman diagram showing the production of a dilepton pair (two wavy lines) through a quark loop (rectangle) with two red arrows indicating a transition. The diagram is followed by a plus sign and an ellipsis, with the text 'MT, DoS'.

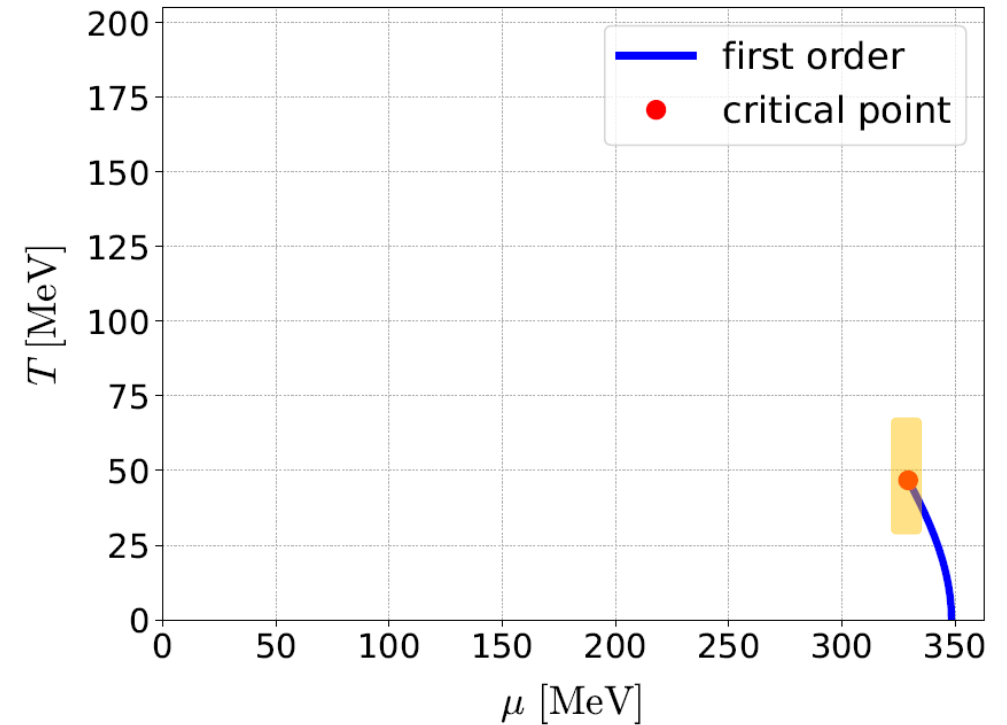
Dilepton production rate near QCD-CP

Nishimura, MK, Kunihiro ('23)

Invariant mass spectrum

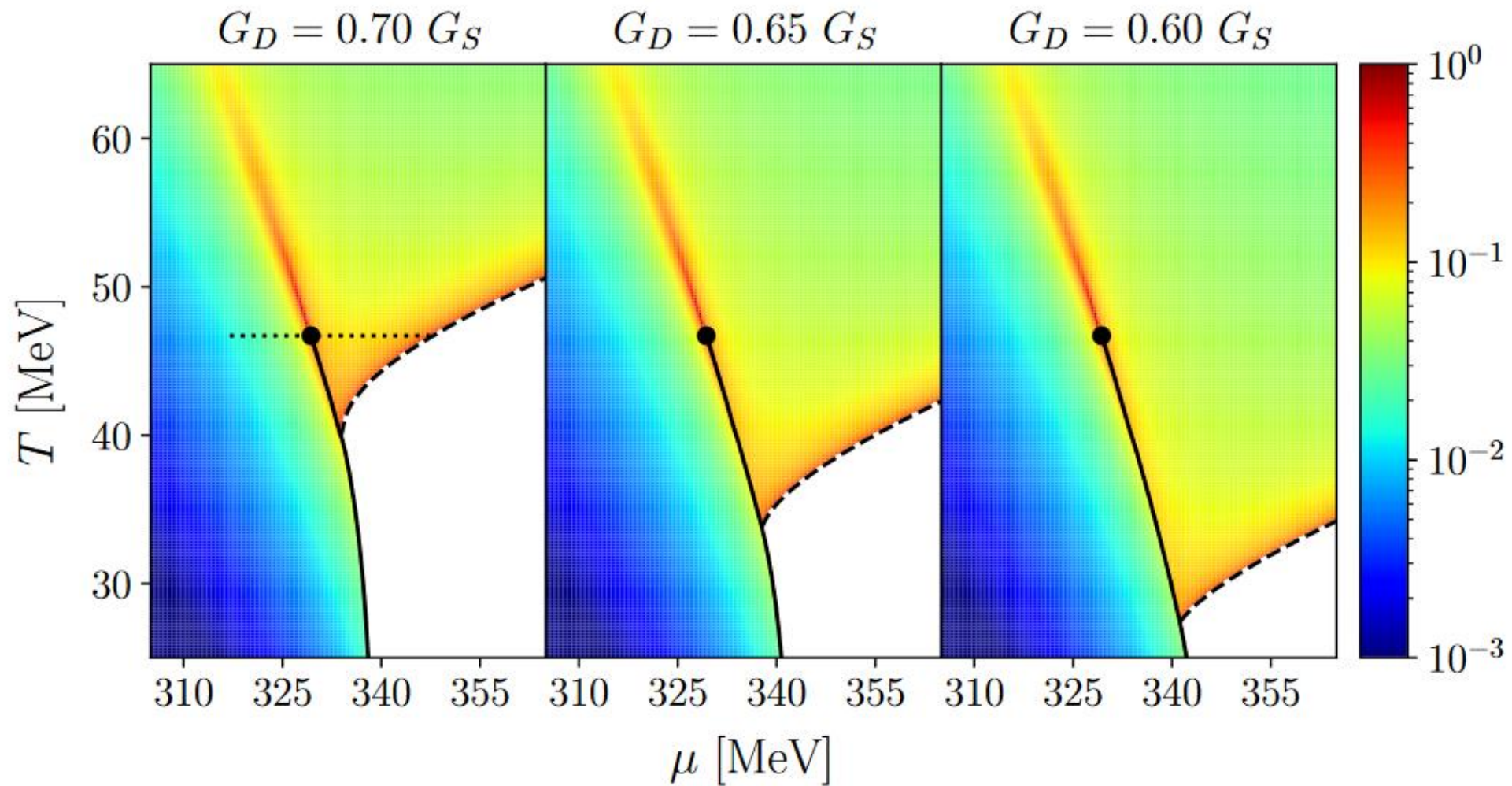


for fixed chem. pot.: $\mu = \mu_c$



- Enhancement at low $M_{\ell\ell}$ region near QCD-CP
- Distinguishment from diquark soft mode may be difficult.

Electric Conductivity on QCD Phase Diagram



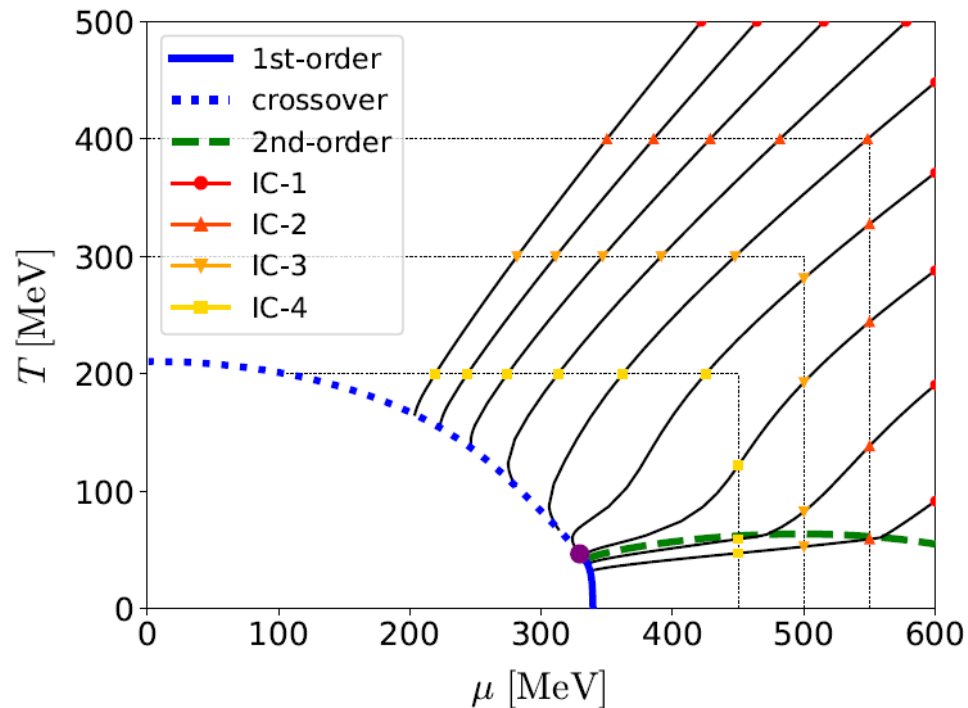
- DPR in the low-energy limit = electric conductivity
- Two “hot spots” on the T - μ plane

Dilepton Yields: Beam-Energy Scan

Dilepton yields

$\simeq$ integrated rate along isentropic lines

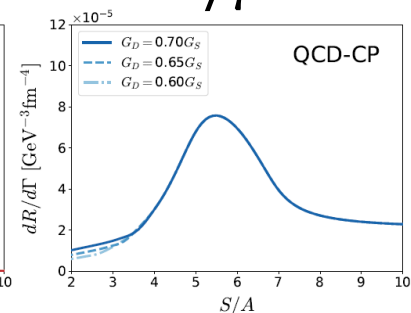
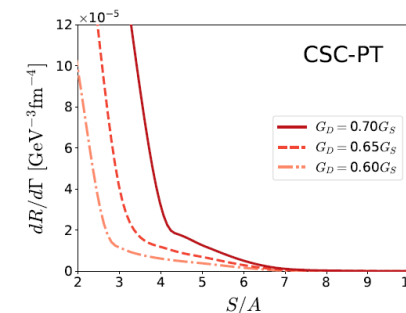
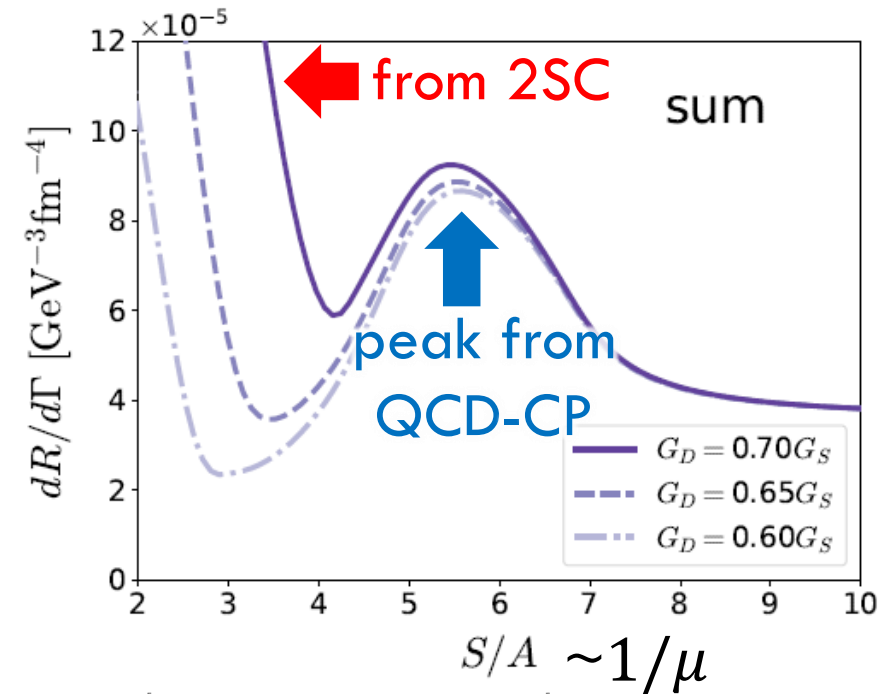
Isentropic lines in NJL model



Effect of 1st-tr on evolution: Savchuk+ 2209.05267

Nishimura, Nara, Steinheimer, Eur.Phys.J.A 60, 2024

Dilepton Yields $50 < M < 100$ MeV

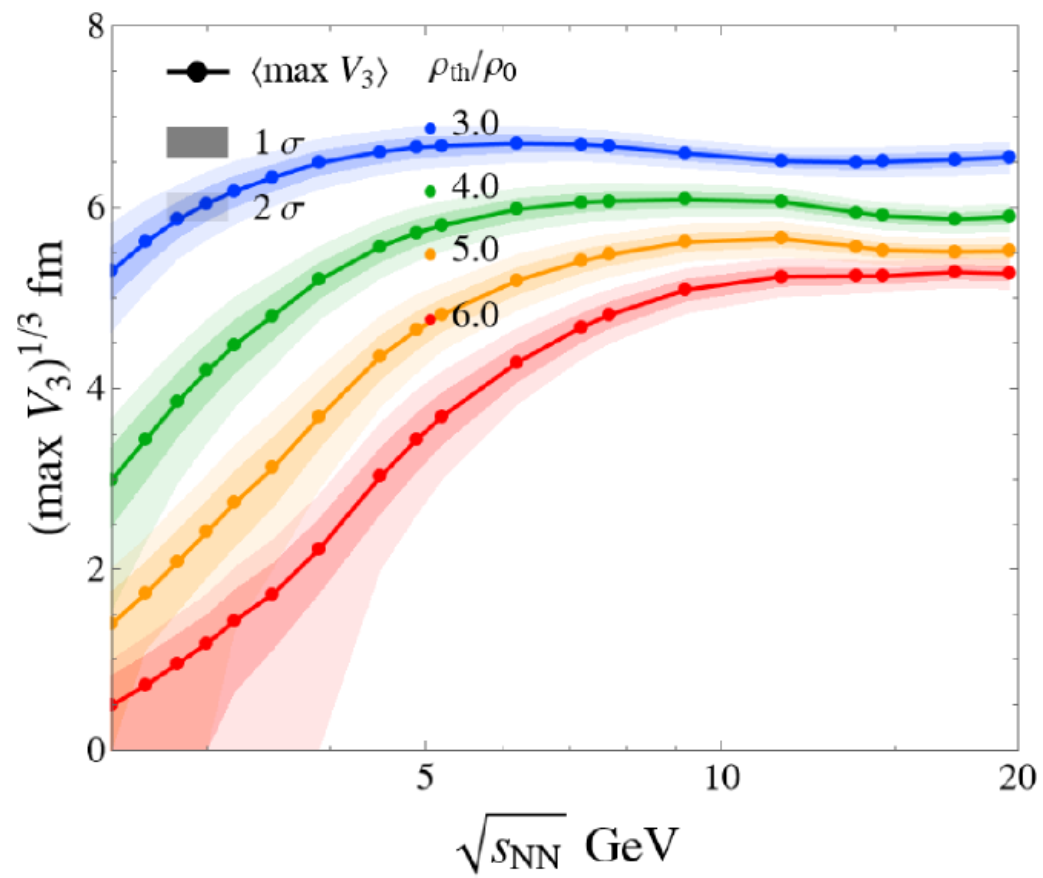


Summary

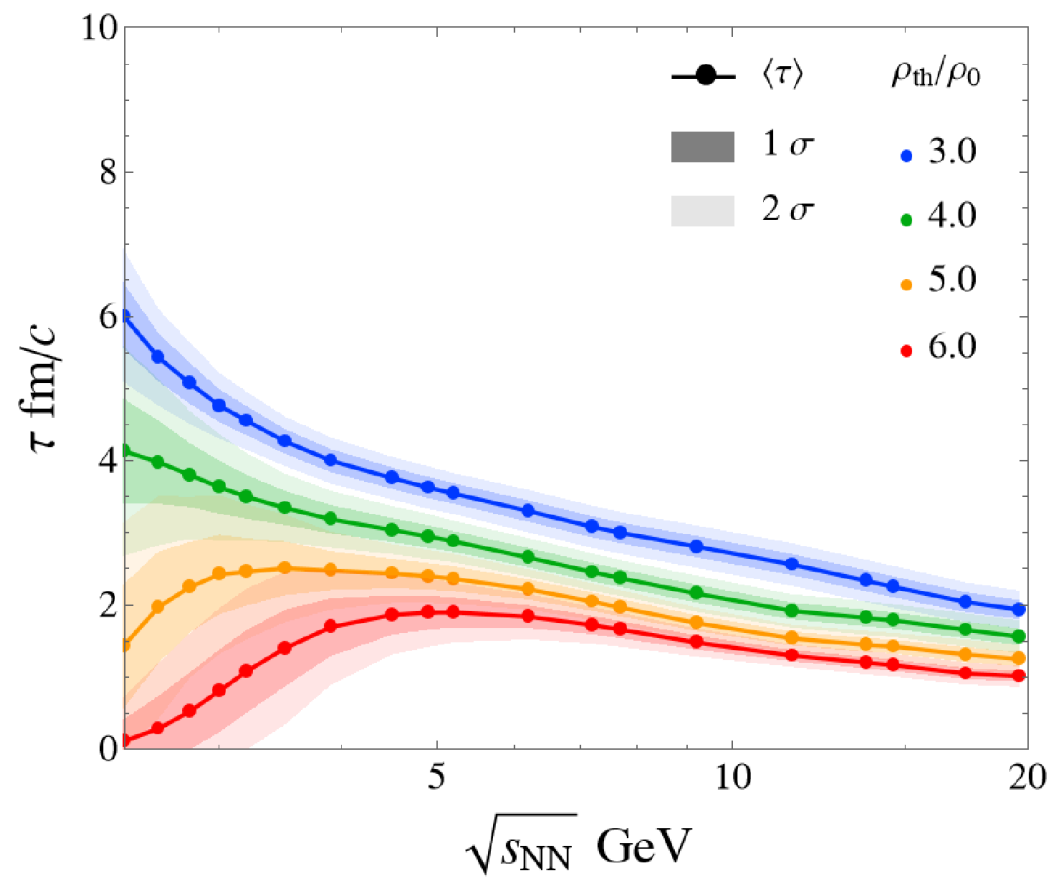
- The beam-energy scan of HIC across energy scales is an interesting subject in nuclear physics.
- At intermediate energies, it will reveal rich structures on QCD phase diagram, such as the QCD critical point and color superconductivity.
- Quantitative analysis of the size and lifetime of the dense region:
 - $\sqrt{s_{NN}} \simeq 3 - 5 \text{ GeV}$ may be an optimal energy to study $\rho = 3 \sim 4\rho_0$.
- Phase transitions in dense quark matter may be detectable through the enhancement of the dilepton production rate at ultra-low-mass-region.

$\sqrt{s_{NN}}$ Dependence

max V_3



Lifetime



ρ_{th}/ρ_0

3.0

4.0

5.0

6.0