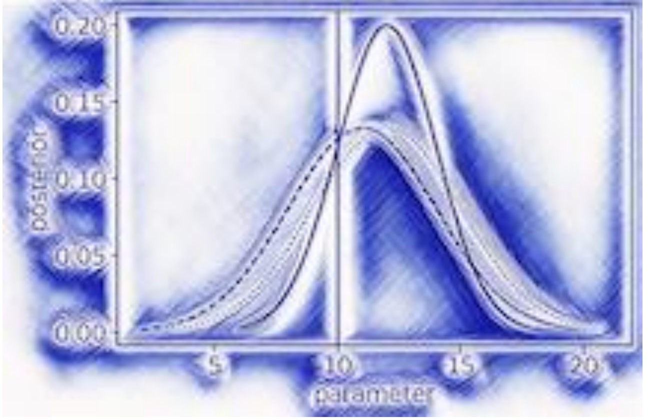
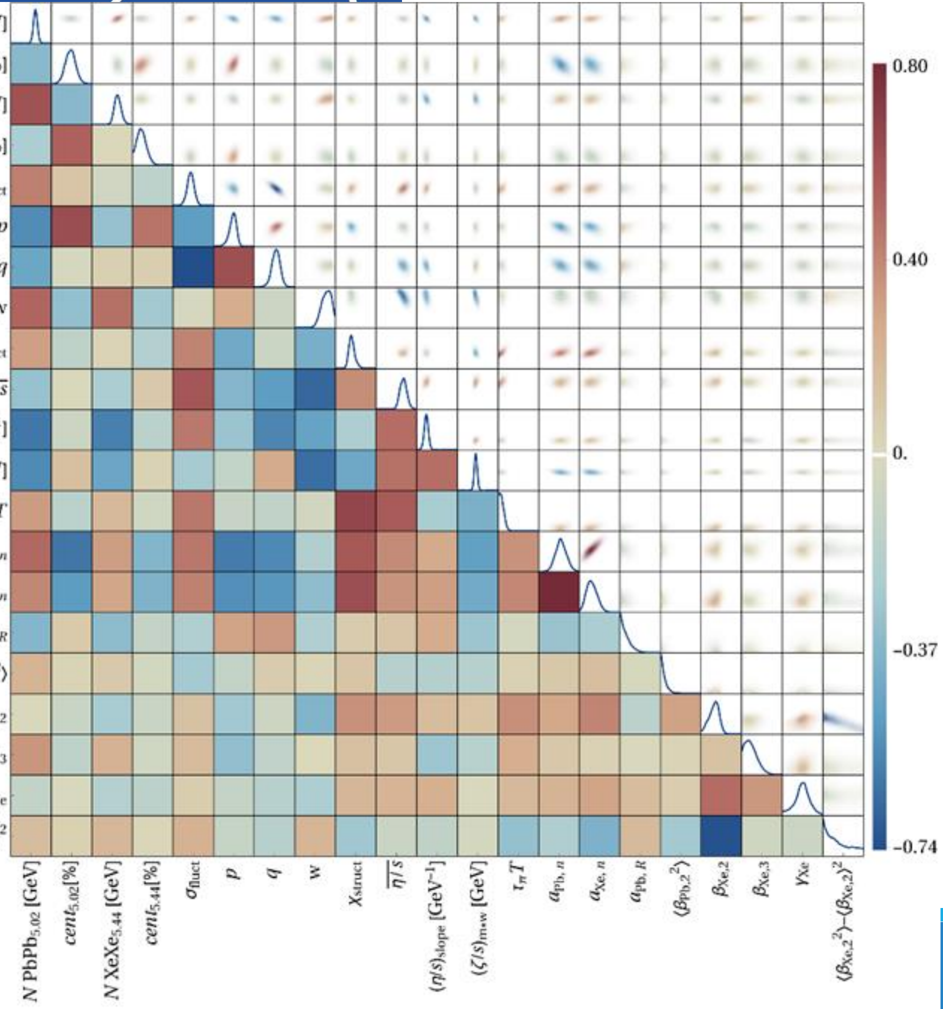
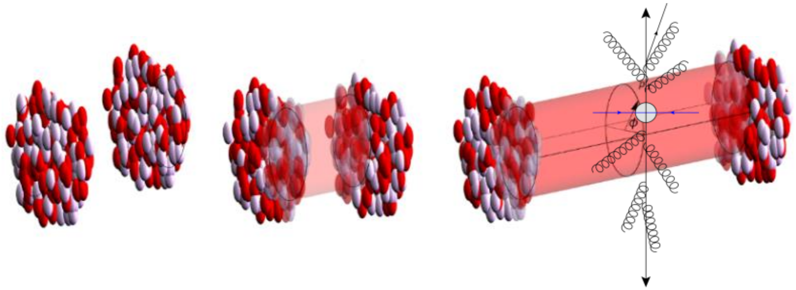




Bayesian inference in heavy ion collisions



Wilke van der Schee

Precision Frontier of QCD Matter: Inference and Uncertainty Quantification

Wuhan, China, 2 September 2025

Bayes theorem is about uncertainties

A mathematical theorem with three basic ingredients:

1. A (physics) model with experimental data
   Parameter posterior not realistic if model is not realistic

2. Prior probability
   Often more relevant than one would wish

3. Uncertainties:
   1. Statistical and emulator uncertainties are relatively easy
   2. Correlated observables already significantly harder
   3. Systematic (model) uncertainty often impossible to estimate
      Becomes more relevant if emulator is more accurate

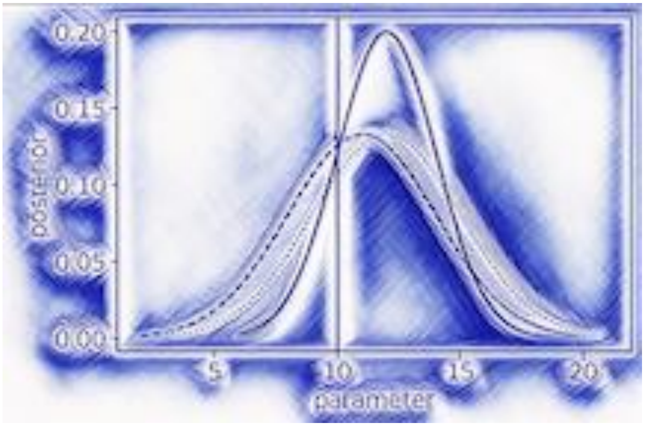
What happens if we have many parameters? Too few parameters? Too many data points? Too few datapoints?

Can we trust our posterior?
Only with a realistic model (for data considered) and realistic uncertainty

Bayes theorem:

$$\mathcal{P}(\boldsymbol{x}|\mathbf{y}_{\text{exp}}) = \frac{e^{-\Delta^2/2}}{\sqrt{(2\pi)^n \det\left(\Sigma(\boldsymbol{x})\right)}} \mathcal{P}(\boldsymbol{x})$$

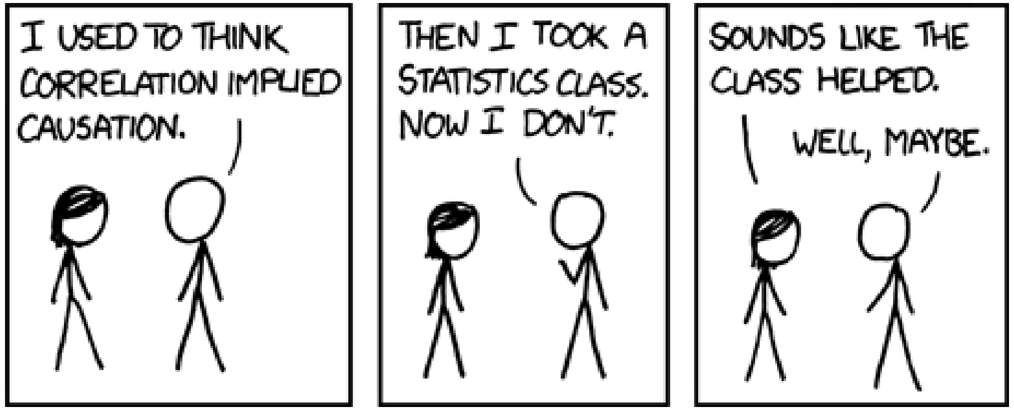
with $\Delta^2 = (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\text{exp}}) \cdot \Sigma(\boldsymbol{x})^{-1} \cdot (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\text{exp}})$



Two messages --- precision era of HIC

Wealth of data poses a significant challenge

1. Necessary to fully utilise experimental knowledge
2. Correlation matrix often important
   (Unknown) systematic uncertainty often important
   Rarely data follows standard χ^2 distribution …



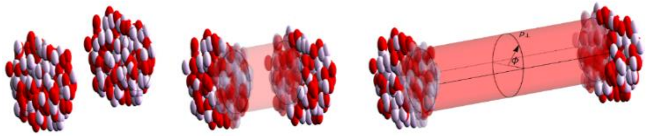
Correlation doesn't imply causation, but it does waggle its eyebrows suggestively and gesture furtively while mouthing 'look over there'.

Often necessary to `interpret' results:

1. Some observables may be more reliable than others
2. Some parameters can be trusted better than others

Wilke van der Schee, CERN/Utrecht

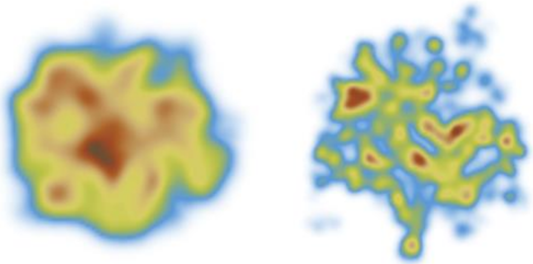
Standard model of heavy ion collisions



(# parameters)

Initial stage (13)

Subnucleonic structure? (8)



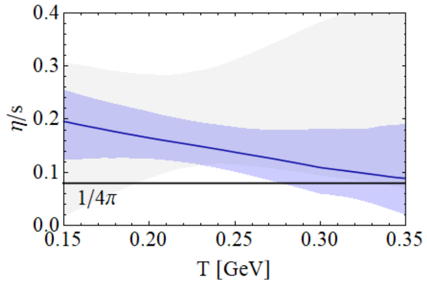
Non-thermal flow? (2)
with hydrodynamised initial stage

Fluctuations? (1)

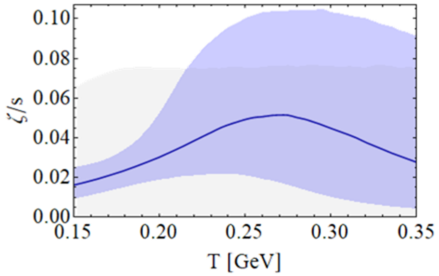
Shape (2)

Viscous hydrodynamics (10)

Shear viscosity (4)



Bulk viscosity (3)

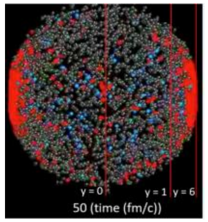


Second order transports: 2

EOS: 1

Cascade of hadrons (2)

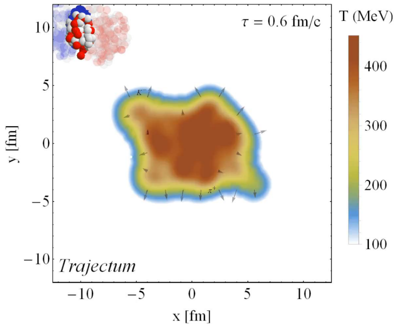
SMASH

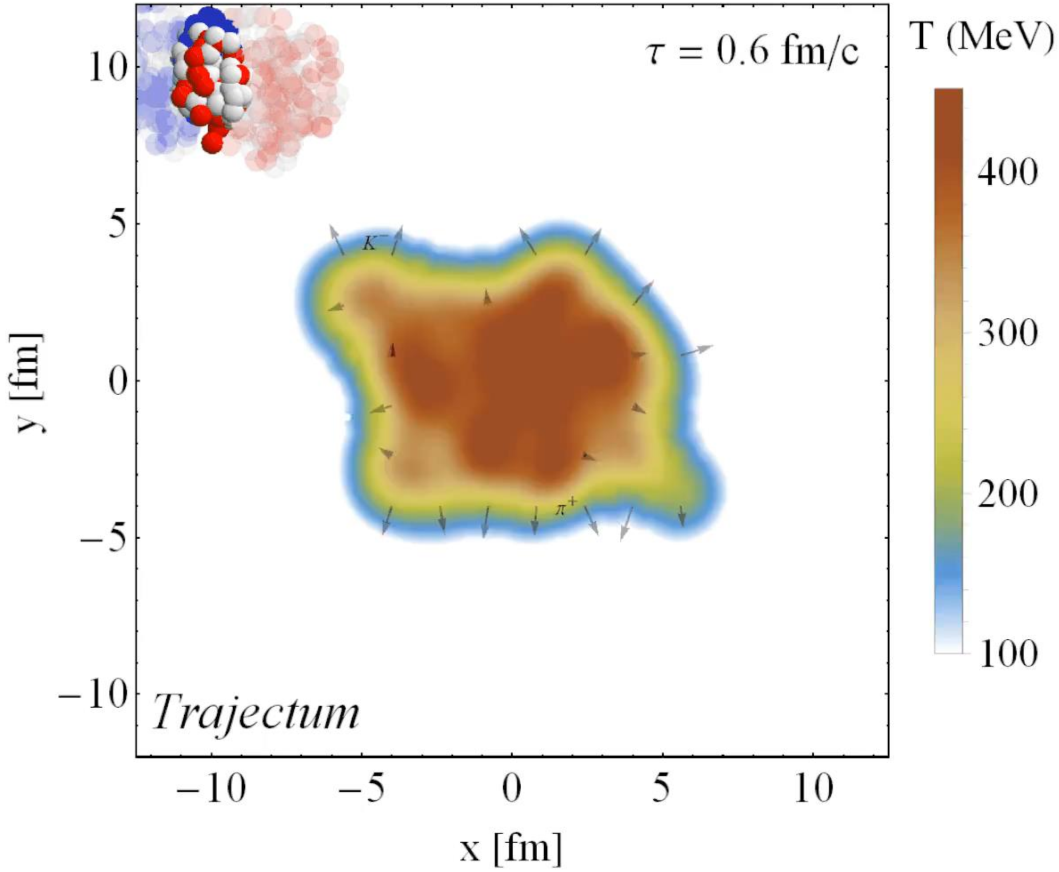


Jonah Bernhard, Scott Moreland and Steffen Bass, Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma (2019)
Govert Nijs, WS, Umut Gursoy and Raimond Snellings, A Bayesian analysis of Heavy Ion Collisions with Trajectum (2020)

A visualisation

1. Depends on parameters (see param file, right)
2. In this case *Trajectum* (many other models available)
3. Compute observables as close to experiment as possible






```
general{
    output=out
    format=smash
    f0500=false
    numevents=1
    seed=7398984.747399307
    debugoutput=true
    numthreads=2
}

entropyacceptanceprobability{
    0:0.0
    24:0.0
    24.5:0.05
    25.5:0.05
    26:0.0
    100:0.0
}

trentosubstructurePbPb{
    dmin=0.63933
    w=0.701919
    sigmann=70.0
    sigmafluct=0.73579
    p=0.14388
    q=1.0
    Eref=0.2
    norm=23.507
    freestreamingreferencetime=1.1708
    freestreamingvelocity=0.62672
    weaktostrong=0.0
    nref=20
    alpha=0
    nc=3.2747
    voverw=0.4892041602706295
}

secondorderhydro{
    numlatticesites=166.0
    latticesize=33.2
}

musclsolverktminmodfastmidpoint{
    cflconstant=0.08
}

LatticeEOStempdepDuke{
    shearhrg=0.0895066
    shearmin=0.0895066
    shearslope=0.43252
    shearcrv=0.231195
    shearrelaxationtime=6.318855
    bulkmax=0.0030138
    bulkT0=0.21471
    bulkwidth=0.10906
    bulkrelaxationtime=0.0687
    deltapipiovertaupi=1.3333333333333
    phi7overpressure=0.128571
    taupipiovertaupi=1.61033
    lambdapiPiovertaupi=1.2
    deltaPiPiovertauPi=0.666666666666
    lambdaPipiovertauPi=1.6
    phi1overpressure=0
    phi3overpressure=0
    phi6overpressure=0
}

cooperfryehadronizer{
    freezeouttemp=153.456
    rapidityrange=0.1
}
```

http://sites.google.com/view/govertnijs/trajectum
http://wilkevanderschee.nl/trajectum

Performing a global analysis

Bayes theorem:

$$\mathcal{P}(\boldsymbol{x}|\mathbf{y}_{\rm exp}) = \frac{e^{-\Delta^2/2}}{\sqrt{(2\pi)^n \det\left(\Sigma(\boldsymbol{x})\right)}}\mathcal{P}(\boldsymbol{x})$$

with $\Delta^2 = (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\rm exp}) \cdot \Sigma(\boldsymbol{x})^{-1} \cdot (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\rm exp})$

Model depends on parameters non-linearly

- Run model on 3000 `design' points
- Use an emulator for any point in parameter space (**GP**)

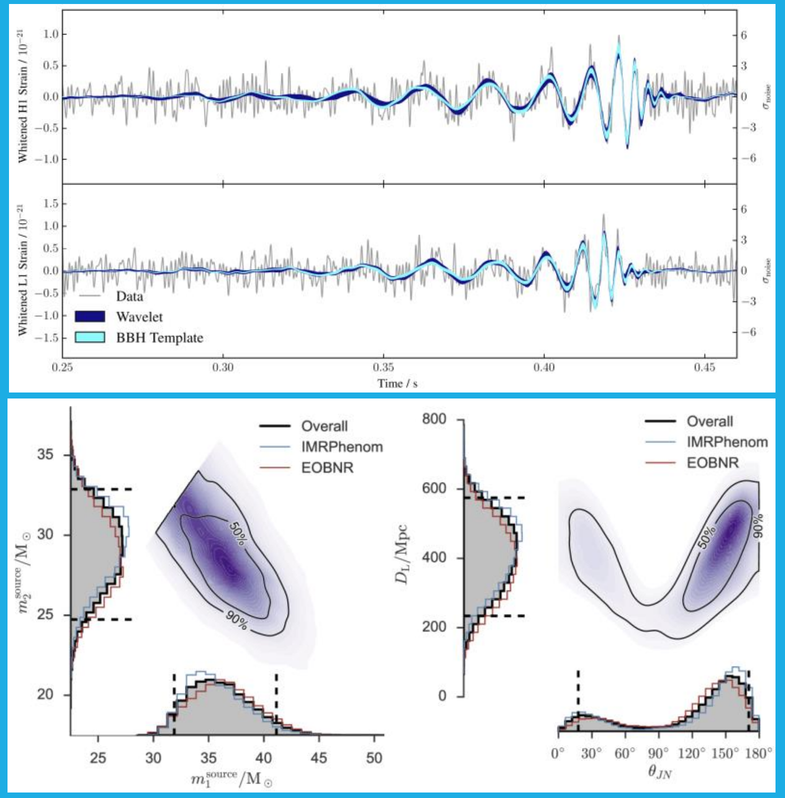
Markov Chain Monte Carlo

- 653 data points
- Obtain posterior probability density of parameters

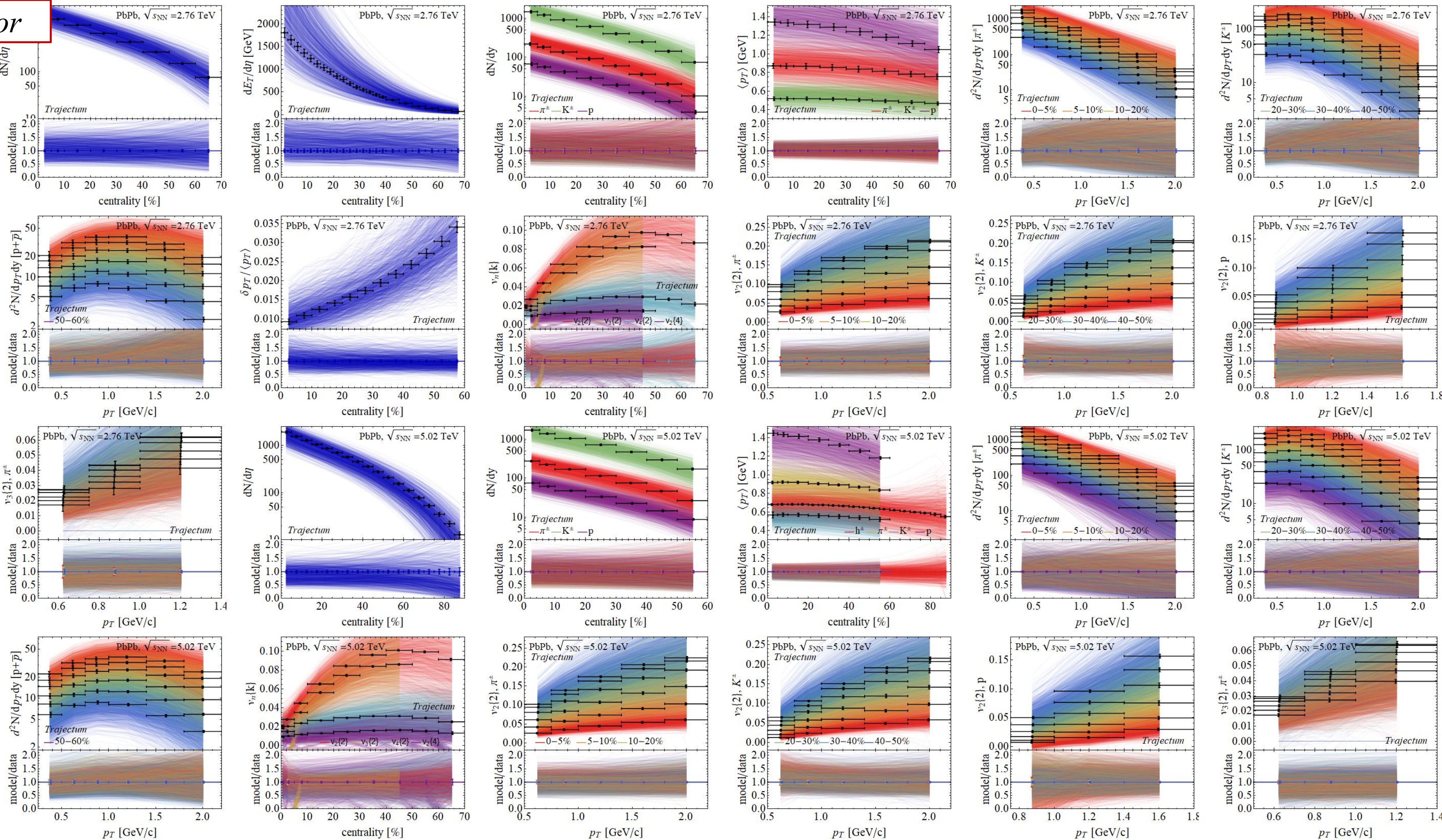
Compare posterior with data

- Can include high statistics run

Same technique: gravitational waves

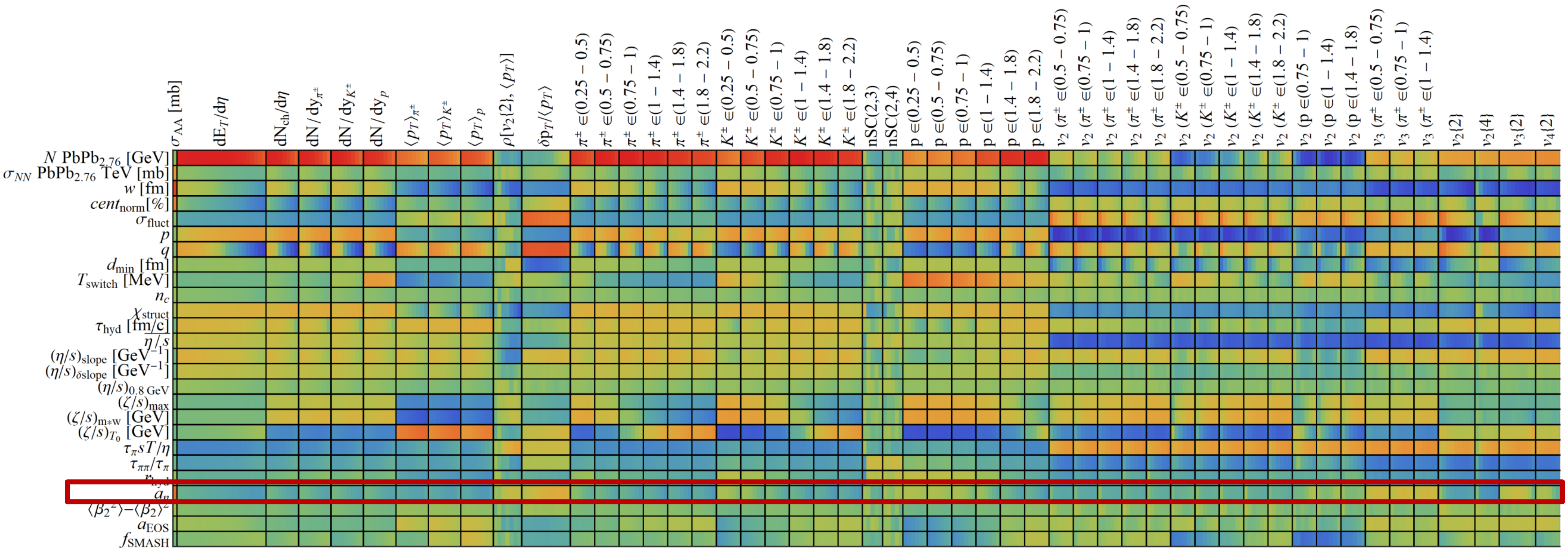


Prior

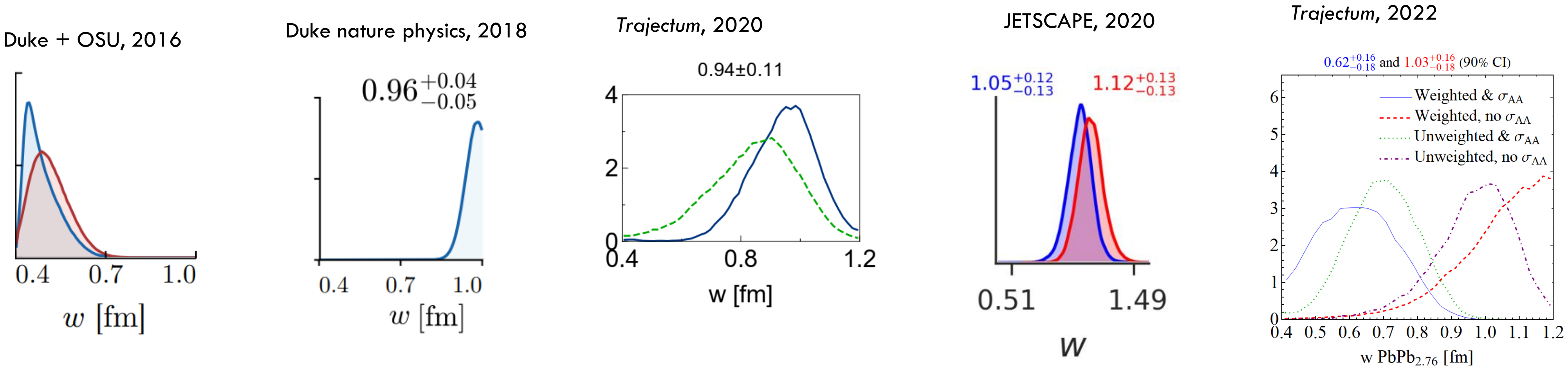


PbPb, $\sqrt{s_{NN}}$ =2.76 TeV
PbPb, $\sqrt{s_{NN}}$ =5.02 TeV
Trajectum
model/data
centrality [%]
p_T [GeV/c]
dN/dη
dE_T/dη [GeV]
dN/dy
⟨p_T⟩ [GeV]
d^2N/d$p_T$dy [$\pi^\pm$]
d^2N/d$p_T$dy [$K^\pm$]
d^2N/d$p_T$dy [p+$\bar{p}$]
δp_T / ⟨p_T⟩
v_n{k}
v_2{2}, $\pi^\pm$
v_2{2}, $K^\pm$
v_2{2}, p
v_3{2}, $\pi^\pm$
$\pi^\pm$ $K^\pm$ p
$h^\pm$ $\pi^\pm$ $K^\pm$ p
0−5% 5−10% 10−20%
20−30% 30−40% 40−50%
50−60%
v_2{2} v_3{2} v_4{2} v_2{4}

Design parameter-observable correlations:



Bayes theorem: three words of caution

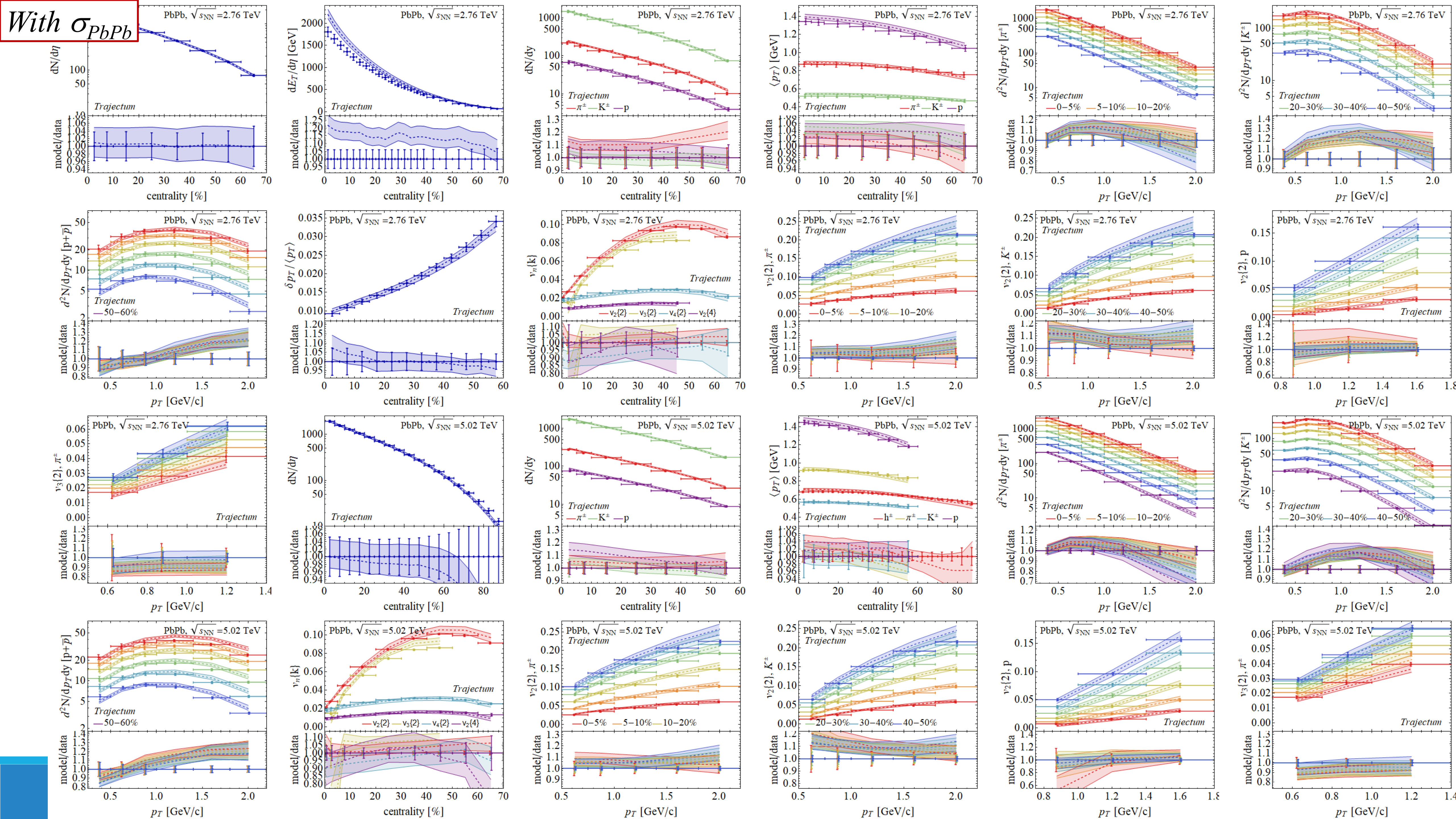


Nucleon width increased in 2018 (very significantly) and decreased (very significantly) in 2022

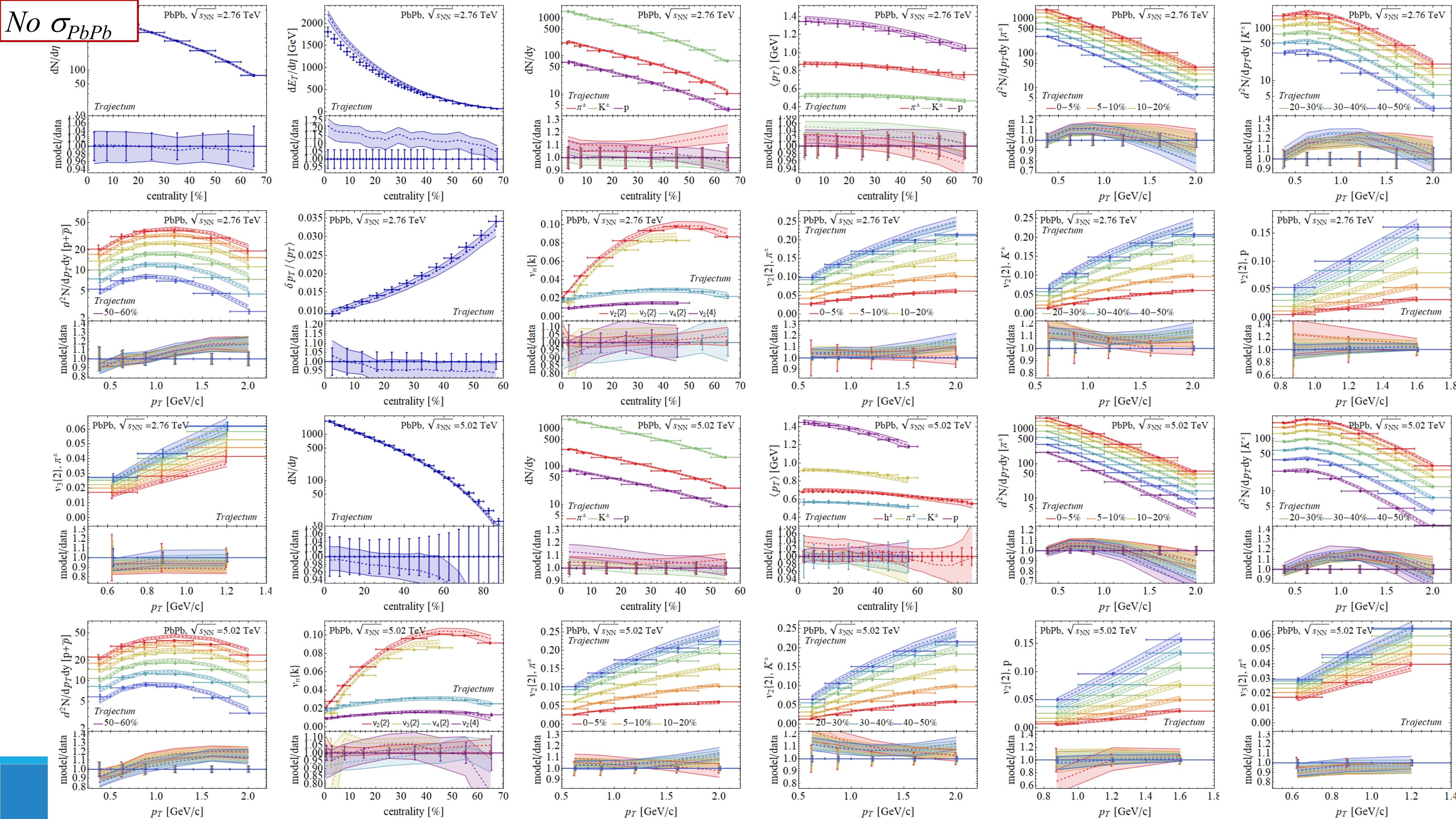
- Turns out that the width was inconsistent with the total hadronic PbPb cross section (measured in 2022)

What happened?

- In 2018: the initial stage model changed
- In 2022: new measurement, but also:
  Many subtle changes in many data points `pulled' to large width



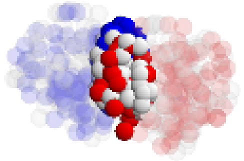
With σ_{PbPb}
PbPb, $\sqrt{s_{NN}}$ =2.76 TeV
PbPb, $\sqrt{s_{NN}}$ =5.02 TeV
Trajectum
dN/dη
$dE_T/d\eta$ [GeV]
dN/dy
$\langle p_T \rangle$ [GeV]
$d^2N/dp_T dy\ [\pi^\pm]$
$d^2N/dp_T dy\ [K^\pm]$
$d^2N/dp_T dy\ [p+\bar{p}]$
$\delta p_T / \langle p_T \rangle$
$v_n\{k\}$
$v_2\{2\}, \pi^\pm$
$v_2\{2\}, K^\pm$
$v_2\{2\}$, p
$v_3\{2\}, \pi^\pm$
model/data
centrality [%]
p_T [GeV/c]
$\pi^\pm$ — $K^\pm$ — p
$h^\pm$ — $\pi^\pm$ — $K^\pm$ — p
0–5% — 5–10% — 10–20%
20–30% — 30–40% — 40–50%
50–60%
$v_2\{2\}$ — $v_3\{2\}$ — $v_4\{2\}$ — $v_2\{4\}$


No σ_{PbPb}
PbPb, $\sqrt{s_{NN}}$ =2.76 TeV
PbPb, $\sqrt{s_{NN}}$ =5.02 TeV
Trajectum
dN/dη
$dE_T/d\eta$ [GeV]
dN/dy
$\langle p_T \rangle$ [GeV]
$d^2N/dp_T dy$ [$\pi^\pm$]
$d^2N/dp_T dy$ [$K^\pm$]
$d^2N/dp_T dy$ [$p+\bar{p}$]
$\delta p_T / \langle p_T \rangle$
$v_n\{k\}$
$v_2\{2\}, \pi^\pm$
$v_2\{2\}, K^\pm$
$v_2\{2\}$, p
$v_3\{2\}, \pi^\pm$
model/data
centrality [%]
p_T [GeV/c]
$\pi^\pm$ — $K^\pm$ — p
$h^\pm$ — $\pi^\pm$ — $K^\pm$ — p
0–5% — 5–10% — 10–20%
20–30% — 30–40% — 40–50%
50–60%
$v_2\{2\}$ — $v_3\{2\}$ — $v_4\{2\}$ — $v_2\{4\}$

The nucleon width and the total PbPb hadronic cross section

What is easier to measure the width than by simply measuring the size?

Fix nucleon-nucleon cross section:

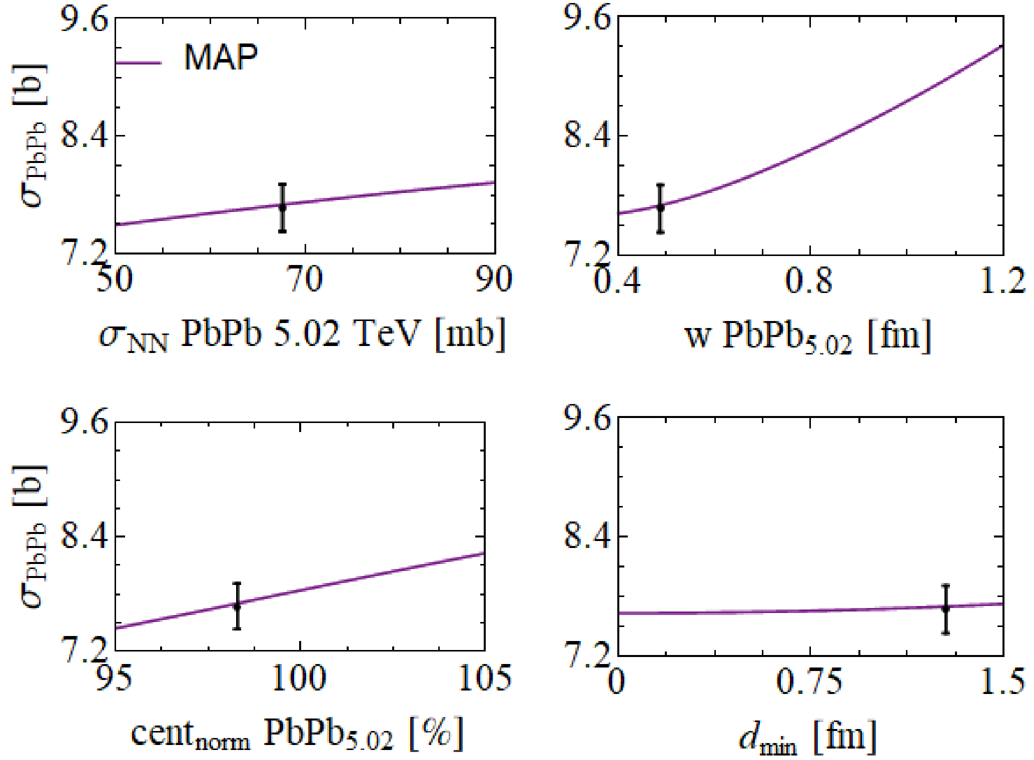


$$P_{\text{coll}} = 1 - \exp\left[-\sigma_{gg} \int dx\, dy \int dz\, \rho_A \int dz\, \rho_B\right]$$

e.g. collision probability tuned to σ_{NN} for Gaussian profile ρ

Theoretically, cross section only depends on

- Nucleon-nucleon cross section
- Nucleon Gaussian width (dominant)
- Centrality normalisation
- Minimum inter-nucleon spacing



Makes the cross section a robust observable

- Basically implying every model needs to get this right
- Basically implying the nucleon width should be small

See also David d'Enterria and Constantin Loizides, Progress in the Glauber Model at Collider Energies (2020)

Why was the width overestimated?

	$\sigma_{PbPb}[b]$	$\sigma_{pPb}[b]$
σ_{AA} & weights	8.03±0.19	2.20±0.06
weights	9.00±0.34	2.50±0.10
σ_{AA}	8.13±0.19	2.23±0.06
neither	8.72±0.29	2.41±0.09
ALICE	7.67±0.24	2.06±0.08

Without cross section width is large, about 1.0 fm

With the cross section width is smaller, about 0.7 fm

- Still tension with cross section: other data pushes width higher

Need to capture `trust' in observables: **weighting**

- Weight unity: cross section + integrated & unidentified
- Weight ½: integrated identified observables
- Weight ¼: p_T-differential identified observables
- Reduced weight: p_T > 1.5 GeV (π+K) and centrality > 50%

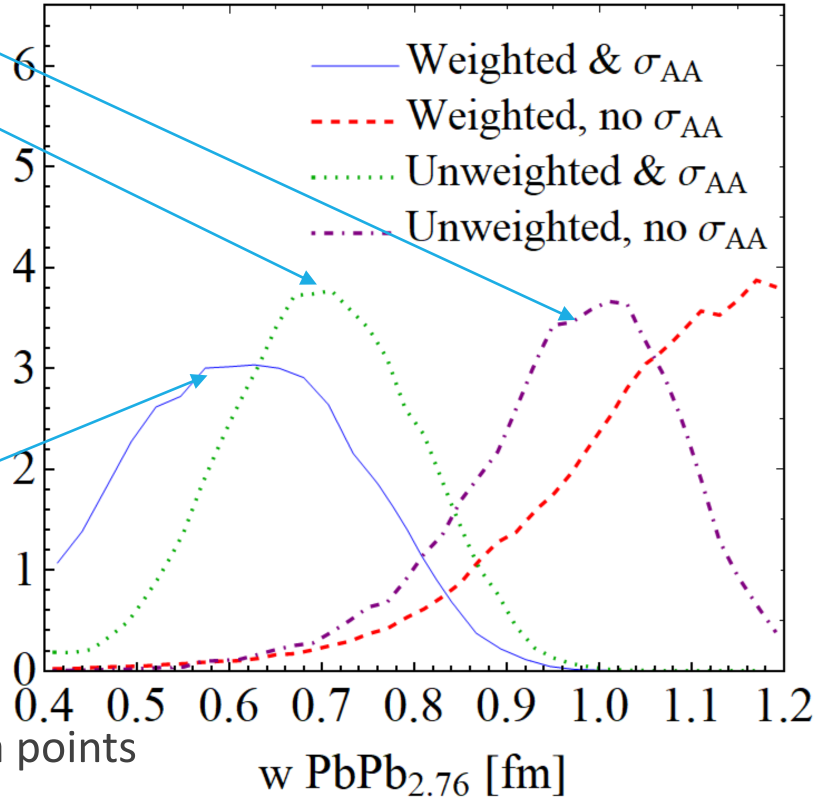
With weighting cross section comes out correctly

- Broader uncertainties: reflect less `trust' due to weighting

Also: description of data not much worse with smaller width

- Important that Bayes factor is an addition of many (correlated!) data points

$0.62^{+0.16}_{-0.18}$ and $1.03^{+0.16}_{-0.18}$ (90% CI)



$$\mathcal{P}(\boldsymbol{x}|\mathbf{y}_{exp}) = \frac{e^{-\Delta^2/2}}{\sqrt{(2\pi)^n \det(\Sigma(\boldsymbol{x}))}}\mathcal{P}(\boldsymbol{x})$$

with $\Delta^2 = (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{exp}) \cdot \Sigma(\boldsymbol{x})^{-1} \cdot (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{exp})$

Four simple examples

Bayes theorem:

$$\mathcal{P}(\boldsymbol{x}|\mathbf{y}_{\text{exp}}) = \frac{e^{-\Delta^2/2}}{\sqrt{(2\pi)^n \det(\Sigma(\boldsymbol{x}))}}\mathcal{P}(\boldsymbol{x})$$

with $\Delta^2 = (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\text{exp}}) \cdot \Sigma(\boldsymbol{x})^{-1} \cdot (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\text{exp}})$

What's the likelihood?

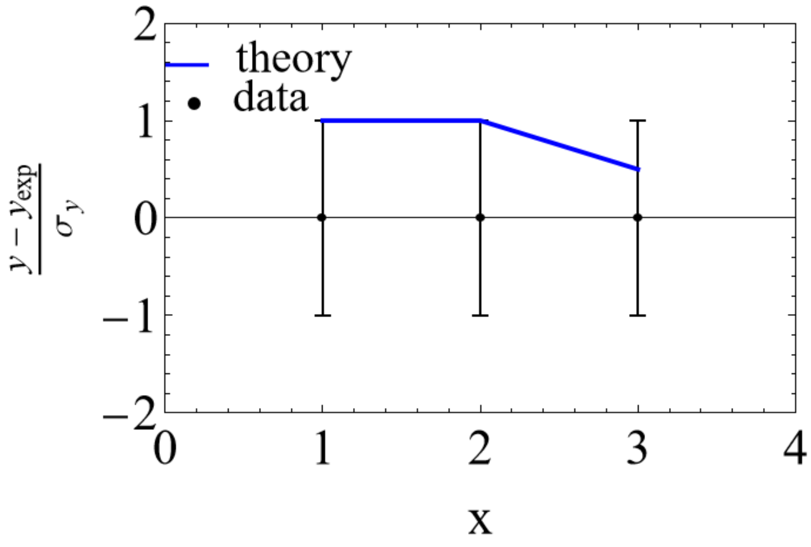
1. For uncorrelated uncertainties?
   - 0.325
2. If 1 & 2 are fully correlated?
   - Ill defined / infinity
3. If 1 & 3 are fully correlated?
   - Ill defined / zero
4. For a realistic covariance matrix?
   - 1.439

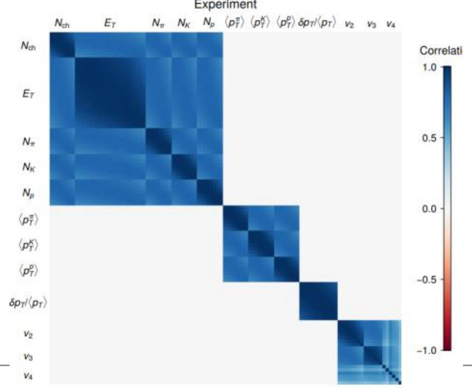
$$\Sigma = \begin{pmatrix} 1 & 0.8 & 0.6 \\ 0.8 & 1 & 0.8 \\ 0.6 & 0.8 & 1 \end{pmatrix};$$

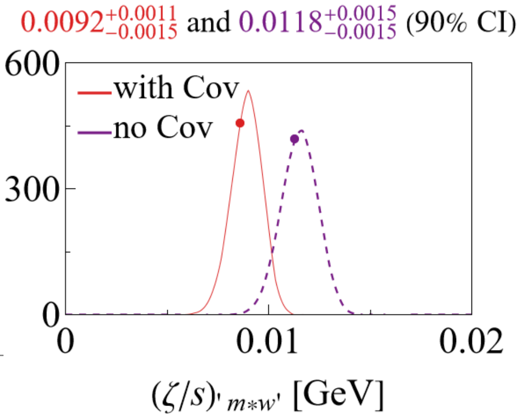
```
δy = {1, 1, 0.5};

Δ2 = δy.Inverse[Σ].δy
likelihood = e^(-Δ2/2) / Sqrt[Det[Σ]]
1.32813

1.43879
```



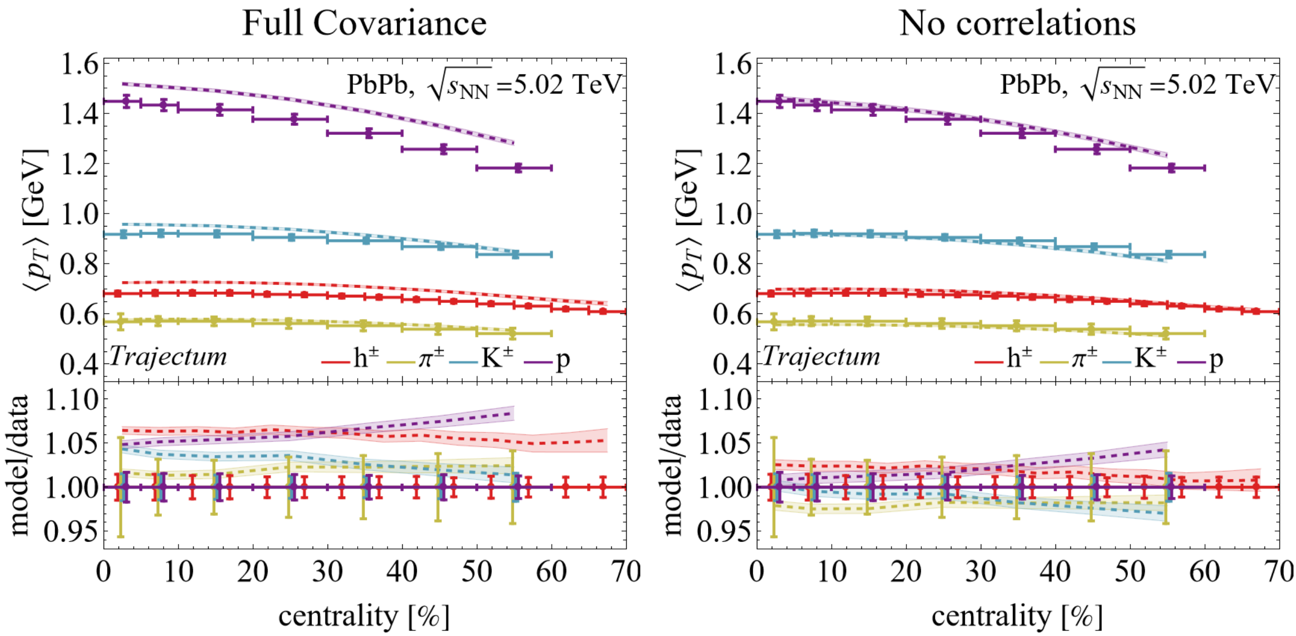




A real example: mean p_T

1. Most analyses likely follow the `Nature Physics' approach*
   - Only correlations between equal observables
     - If subobservables differ: multiply by 80%
   - Covariance between two centralities: $e^{-\frac{1}{2}(c_1-c_2)^2/2}$
2. Likely not very realistic for many observables…
3. For e.g. mean p_T it really matters:

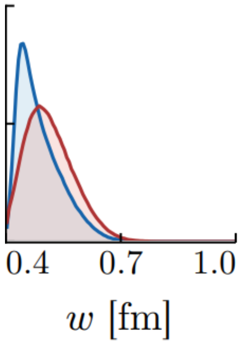
   Changes bulk viscosity beyond uncertainty



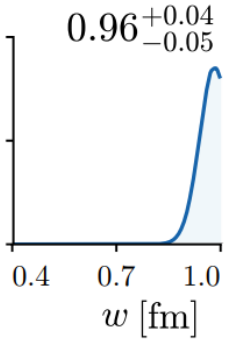
Jonah E. Bernhard, J. Scott Moreland & Steffen A. Bass, Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma (Nature Physics, 2019)
*"for which we assume a reasonable correlation structure,"

Bayes theorem: three words of caution

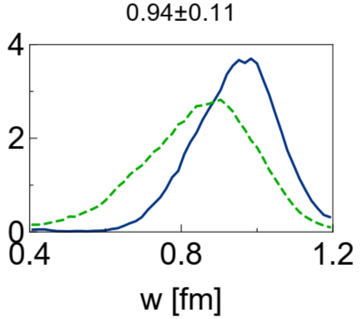
Duke + OSU, 2016



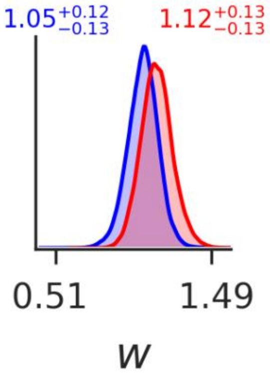
Duke nature physics, 2018



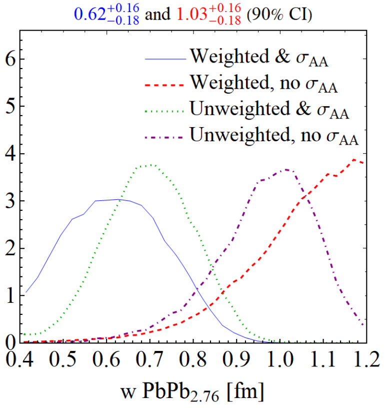
Trajectum, 2020



JETSCAPE, 2020



Trajectum, 2022



First lesson:

- Be careful with many data points that `pull' the Bayes factor
- Are really all *known and unknown* uncertainties included in the covariance matrix? Including correlations?

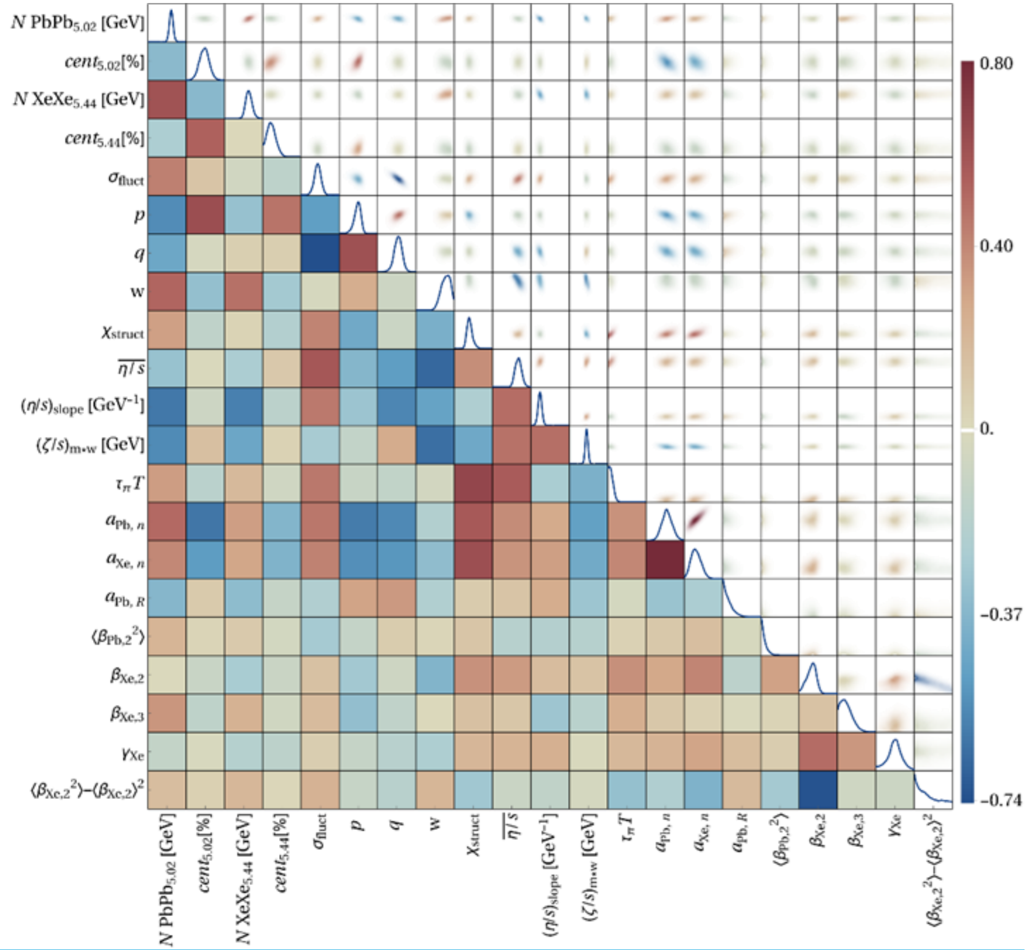
Bayes theorem: three words of caution

Second: does the model capture all the (relevant) physics?

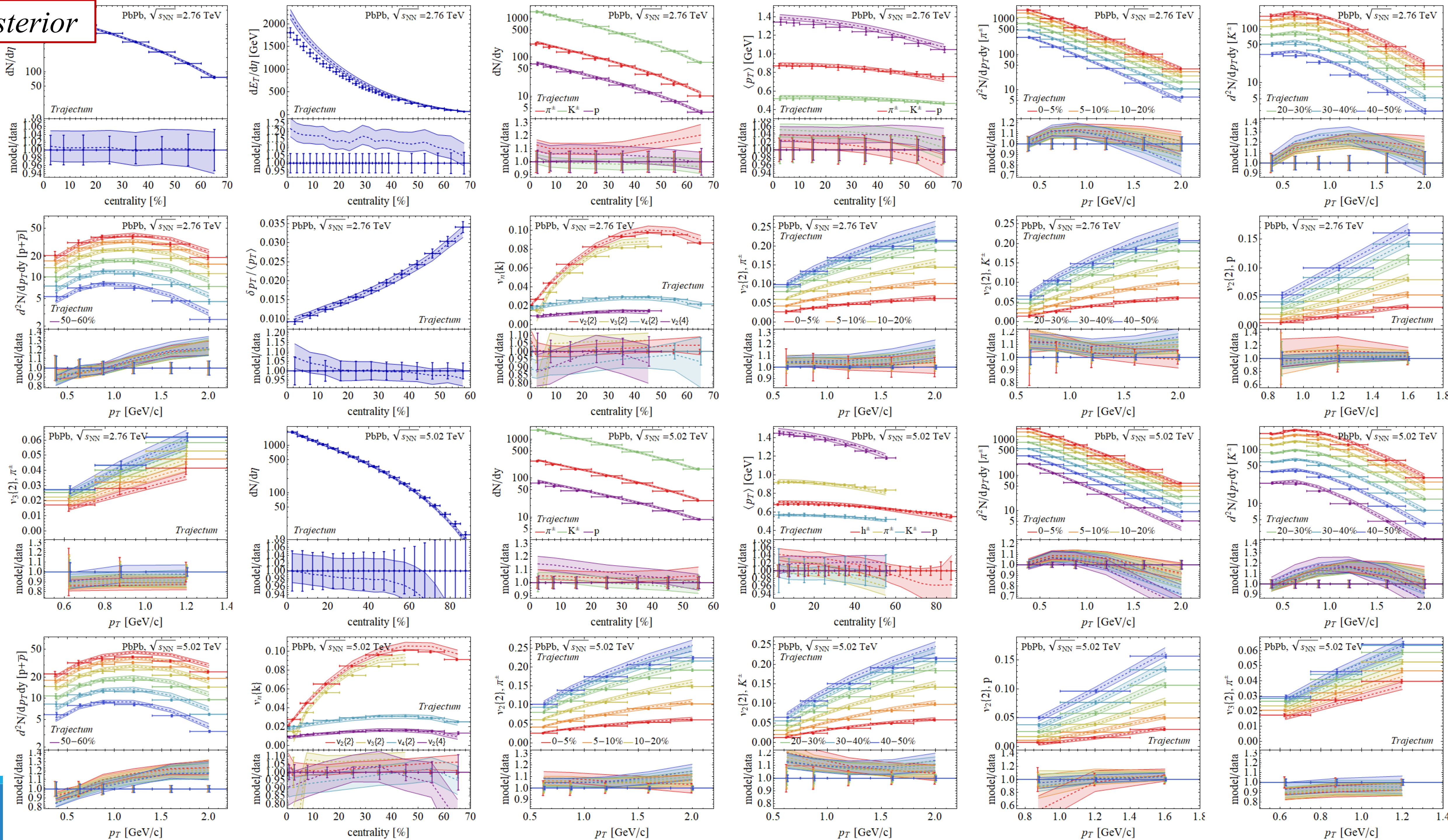
- In practice always compromise between computational cost and completeness of the model

Hints can be seen in correlation matrix

- When `fixing' one parameter this affects all other (correlated!) parameters
- `Posterior' density unreliable if `artificially' restricting a relevant parameter
- Meaning of `relevant' depends on computation

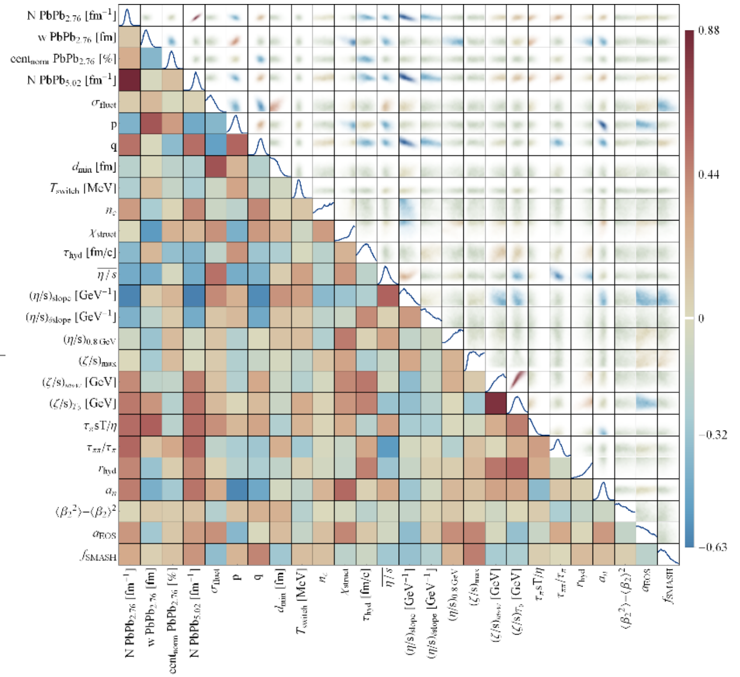


Posterior



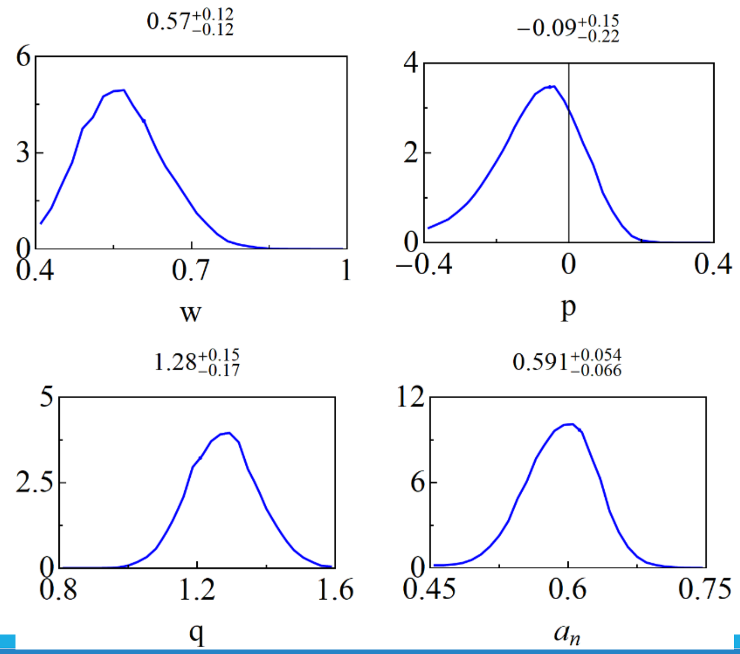
PbPb, $\sqrt{s_{NN}}$ =2.76 TeV
PbPb, $\sqrt{s_{NN}}$ =5.02 TeV
Trajectum
dN/dη
$dE_T/d\eta$ [GeV]
dN/dy
$\langle p_T \rangle$ [GeV]
$d^2N/dp_T dy$ [$\pi^\pm$]
$d^2N/dp_T dy$ [$K^\pm$]
$d^2N/dp_T dy$ [$p+\bar{p}$]
$\delta p_T / \langle p_T \rangle$
$v_n\{k\}$
$v_2\{2\}, \pi^\pm$
$v_2\{2\}, K^\pm$
$v_2\{2\}$, p
$v_3\{2\}, \pi^\pm$
model/data
centrality [%]
p_T [GeV/c]
$\pi^\pm$ — $K^\pm$ — p
$h^\pm$ — $\pi^\pm$ — $K^\pm$ — p
0–5% — 5–10% — 10–20%
20–30% — 30–40% — 40–50%
50–60%
$v_2\{2\}$ — $v_3\{2\}$ — $v_4\{2\}$ — $v_2\{4\}$

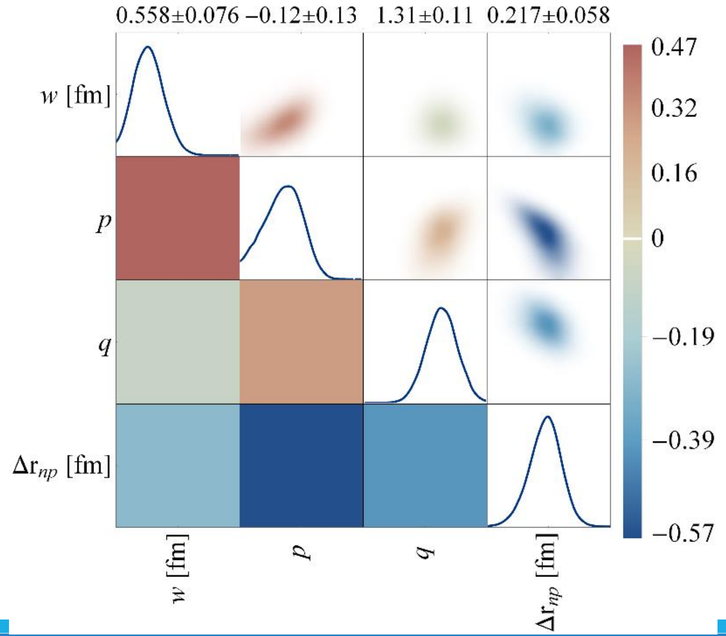
Trento: $\mathcal{E} \propto \left(\frac{1}{2}T_A^p + \frac{1}{2}T_B^p\right)^{q/p} = (T_A T_B)^{q/2}\,|_{p=0}$



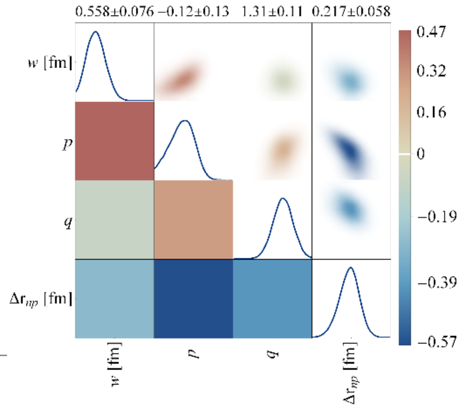
The neutron skin - posterior

1. Three parameters are most sensitive to the neutron skin:
   - The nucleon width and the Trento parameters p and q
   - Small correlation with width (cross section is highly sensitive to w)
   - Very strong anticorrelation with p; centrality dependence is important

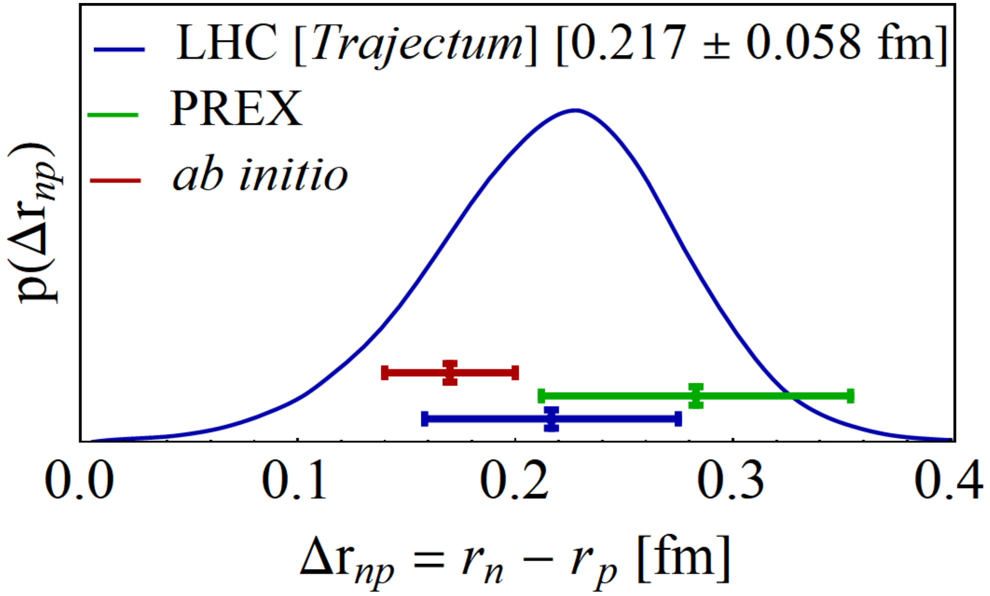


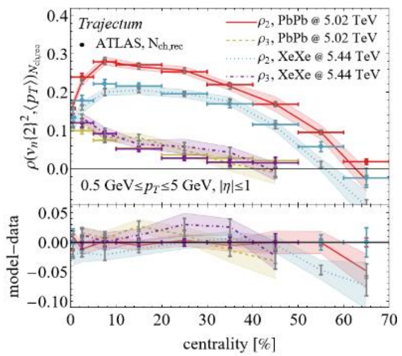


The neutron skin – final result



1. Transform to neutron radius minus proton radius
2. Final result consistent but smaller than PREX II
3. Uncertainty is about 20% smaller than PREX II
4. Cross section is crucially important, but also centrality dependence
   - Important to vary Trento parameters in particular



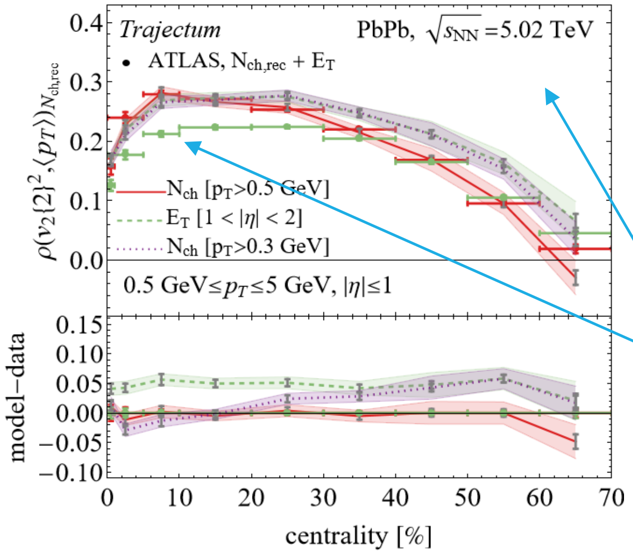


More recent progress: more precision

Slightly technical, message: with more precision a better (model) understanding is needed

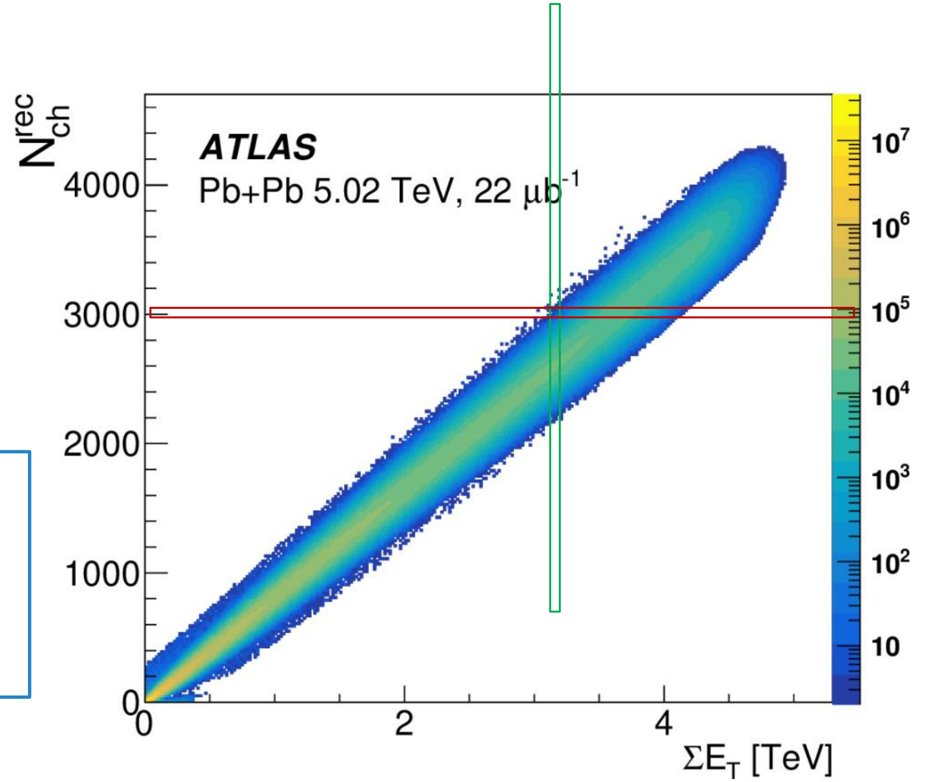
Forward (E_T) versus mid-rapidity (Nch)

- Forward considered `better' (ATLAS, lower spread)
- Mid-rapidity closer to our model (boost invariant)



Somewhat ironic:
ATLAS sees big effect at central collisions
Trajectum has big effect at peripheral collisions

Trajectum: big effect p_T cut on N_{ch} (purple)

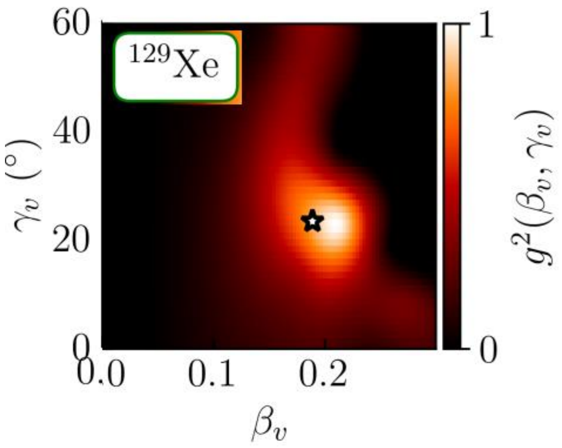


More recent progress: Xenon deformation

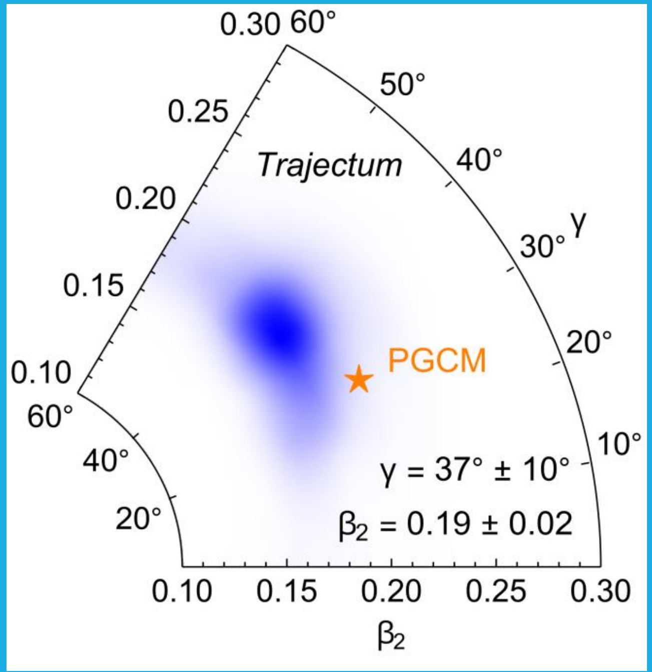
First quantitative measurement of Xenon triaxiality

Matches well with theoretical PGCM estimate

Theory [PGCM]



LHC [*Trajectum*]



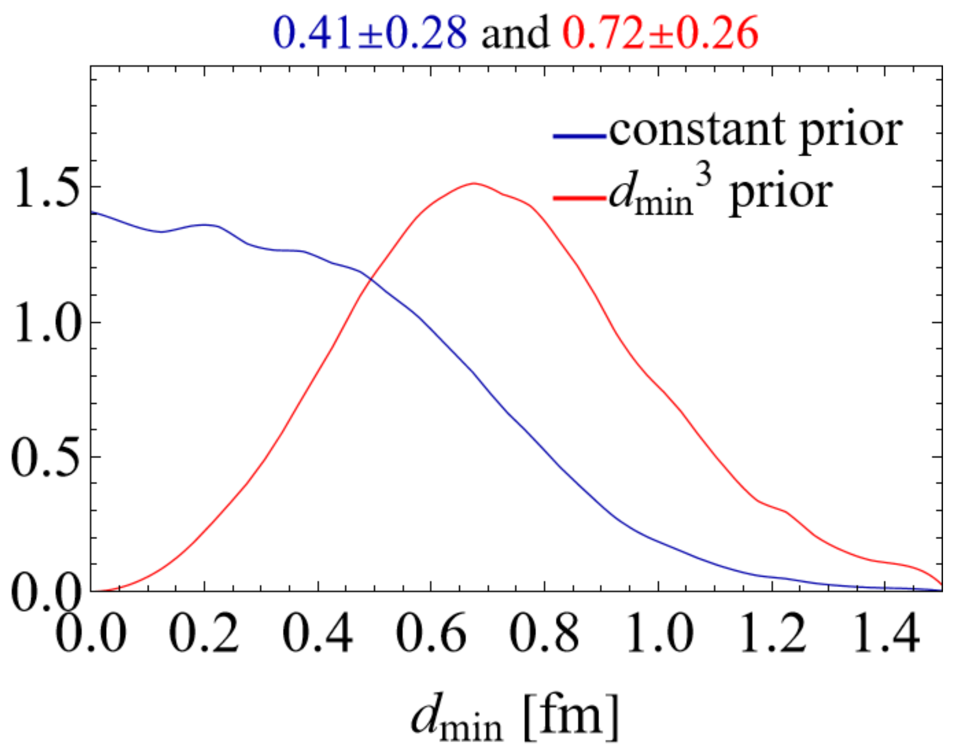
Benjamin Bally, Michael Bender, Giuliano Giacalone and Vittorio Soma, Evidence of the triaxial structure of ^{129}Xe at the Large Hadron Collider (2021)

"Bayesians address the questions everyone is interested in by using assumptions that no one believes. Frequentist use impeccable logic to deal with an issue that is of no interest to anyone."- Louis Lyons

Bayes theorem: three words of caution

Third: the prior is important

- If the data is `strong' the prior doesn't matter
- In practice it often matters…



Bayesian analysis or global analysis

- The only way to `properly' combine all the wealth of data and knowledge we have
- Be careful about the **known and unknown** uncertainties, including correlations

Discussion

1. Bayesian or global analyses
   - Essential tool to assess parameters in view of experimental data
   - Becoming more mainstream: public tools by e.g. JETSCAPE
   - The uncertainties are essential and *sometimes difficult to control*
2. A personal biased overview of an answer to “Bayesian inference”
   - Not covered: Technical implementation, non-flow effects, rapidity gaps, very peripheral or high pT, hadronic afterburner, collisions across beam energies and system sizes (light ions)
3. Heavy ions as a precision science
   - Uncertainties are small: apples-to-apples comparison is essential
   - Many `basic’ observables turn out to be interesting ☺ (no time to cover ultracentral..)

Jonah E. Bernhard, J. Scott Moreland & Steffen A. Bass, Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma (Nature Physics, 2019)

Back up / discussion

$$\mathcal{P}(\boldsymbol{x}|\mathbf{y}_{\mathrm{exp}}) = \frac{e^{-\Delta^2/2}}{\sqrt{(2\pi)^n \det(\Sigma(\boldsymbol{x}))}}\mathcal{P}(\boldsymbol{x})$$

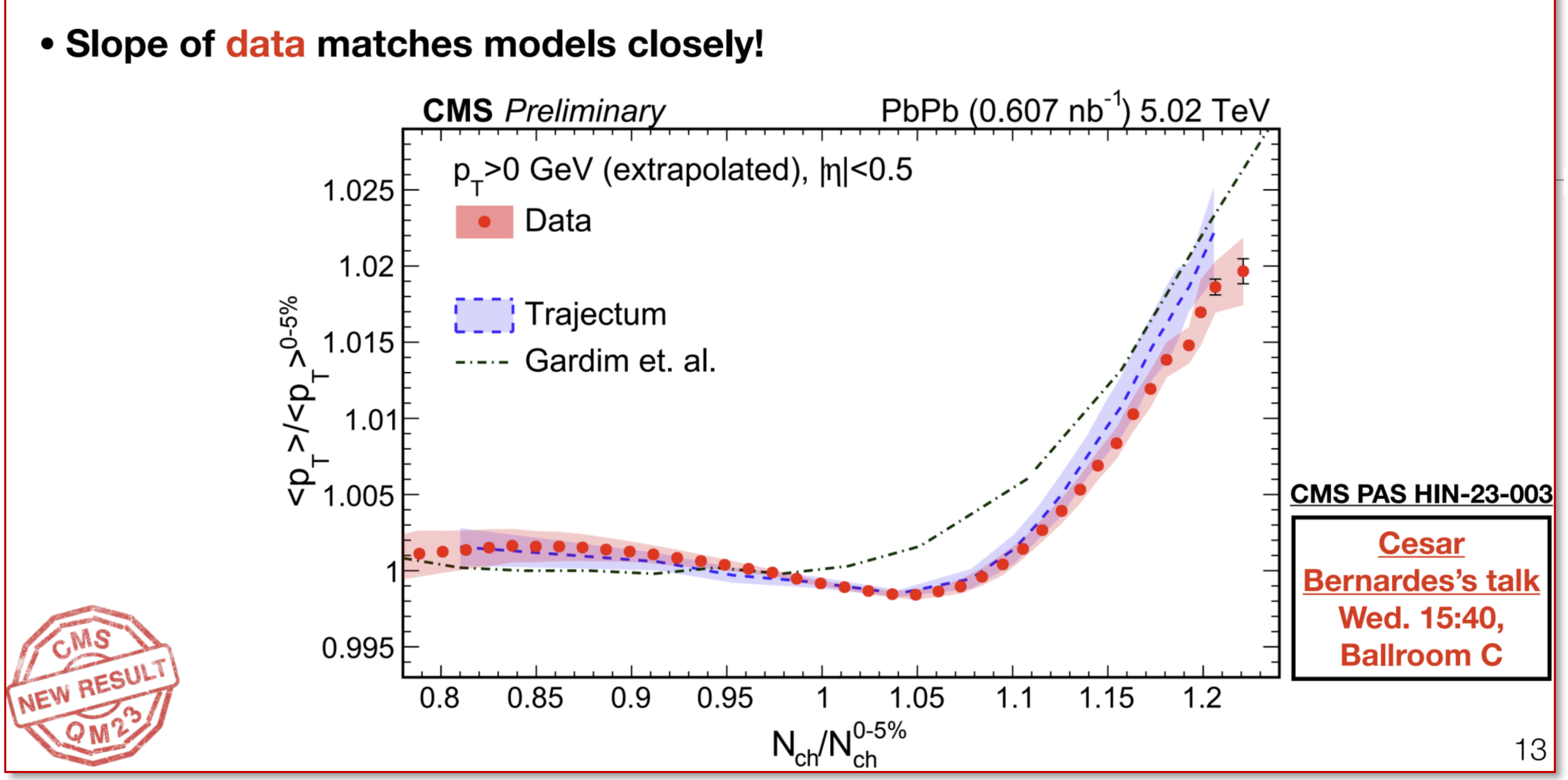
$$\text{with } \Delta^2 = (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\mathrm{exp}}) \cdot \Sigma(\boldsymbol{x})^{-1} \cdot (\mathbf{y}(\boldsymbol{x}) - \mathbf{y}_{\mathrm{exp}})$$

Global analysis, technical practicalities

1. $P(x|y_{exp})$ can be a 25-dimensional function where $y(x)$ is expensive to evaluate
   - Markov Chain Monte Carlo (emcee) can require order 10M evaluations
2. Technical steps to evaluate it
   - Compute $y(x)$ for nicely spreaded sample of points (latin hypercube sampling, order 500 points)
   - PCA on observables, keep order 20 principal components (PCs)
   - Build a Gaussian Process Emulator for those PCs
     - Basically a glorified interpolating function in higher dimensions
     - \`Easier' if observables do not depend much on parameters (large length scale)
     - Validated and includes emulation uncertainty **(often dominant)**
3. Sample posterior of x to obtain systematic model uncertainty due to parameters

Jonah E. Bernhard, J. Scott Moreland & Steffen A. Bass, Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma (Nature Physics, 2019)
Jonah E. Bernhard, Bayesian parameter estimation for relativistic heavy-ion collisions (2018), https://github.com/jbernhard/hic-param-est

Speed of Sound in QGP

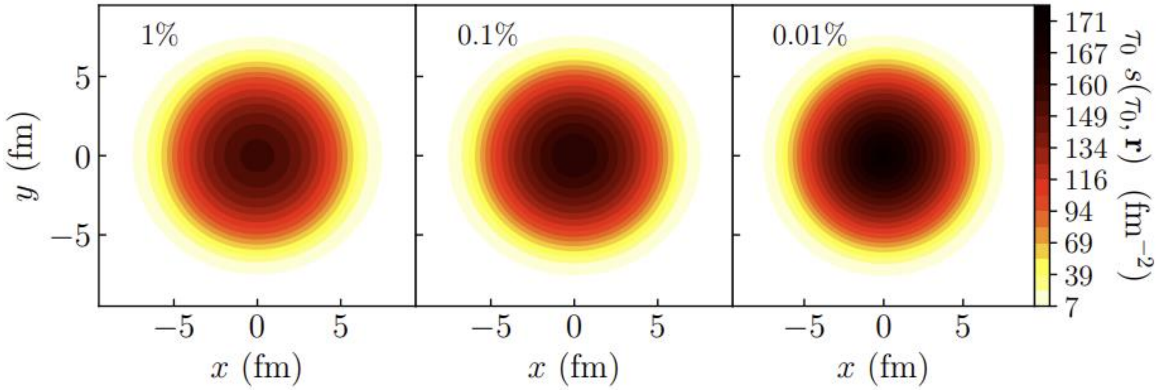


1. A true *prediction* using hydrodynamics (rare these days…)
   - Non-trivial curve; non-monotonicity was unexpected
   - Extremely precise (<0.1%) due to uncertainty *cancellations* in the ratio (theory & experiment (!))

Fernando G. Gardim, Giuliano Giacalone and Jean-Yves Ollitrault, The mean transverse momentum of ultracentral heavy-ion collisions: A new probe of hydrodynamics (2019)
Govert Nijs and WS, Predictions and postdictions for relativistic lead and oxygen collisions with the computational simulation code Trajectum (2021)
Austin Baty, Overview of Recent CMS results, https://indico.cern.ch/event/1139644/contributions/5343932/ (see also **Cesar Bernardes**)

What is happening?

1. (Extremely) ultracentral collisions are not driven by impact parameter
   - Size does not extend beyond radius Lead nucleus
   - (Local) **fluctuations** drive up multiplicity
   - Average **temperature** can still increase

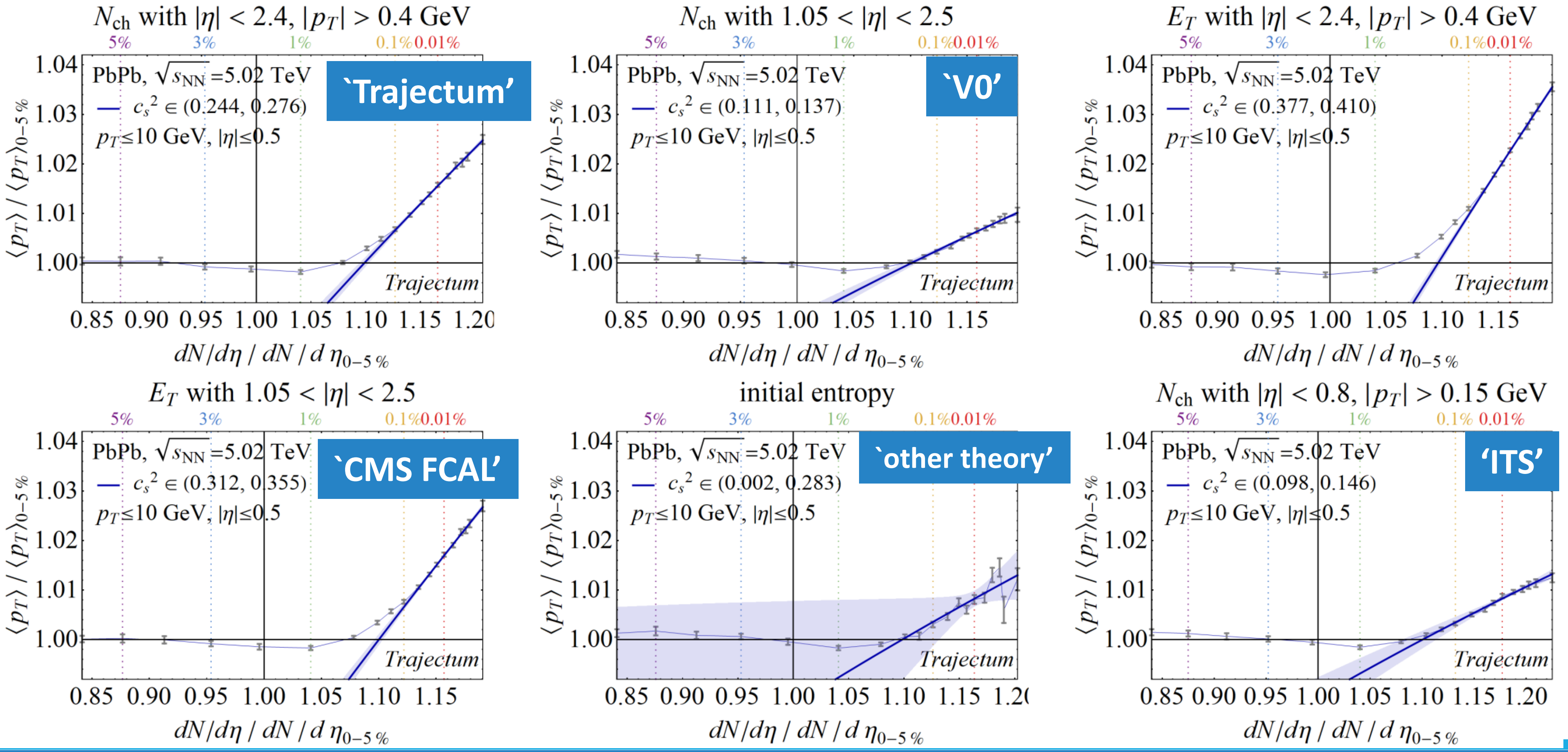


2. A simplified picture:
   - Entropy increase → multiplicity increases
   - Temperature increases → mean pt increases (assuming ideal hydro + constant `work' done in rapidity direction)
   - Slope: sensitive to speed of sound (solely ?!): $c_s^2 = \frac{dP}{d\epsilon} = \frac{d\ln T}{d\ln s} = \frac{d\ln\langle p_t\rangle}{d\ln N_{\rm ch}}$

Fernando G. Gardim, Giuliano Giacalone and Jean-Yves Ollitrault, The mean transverse momentum of ultracentral heavy-ion collisions: A new probe of hydrodynamics (2019)

What then determines the slope?

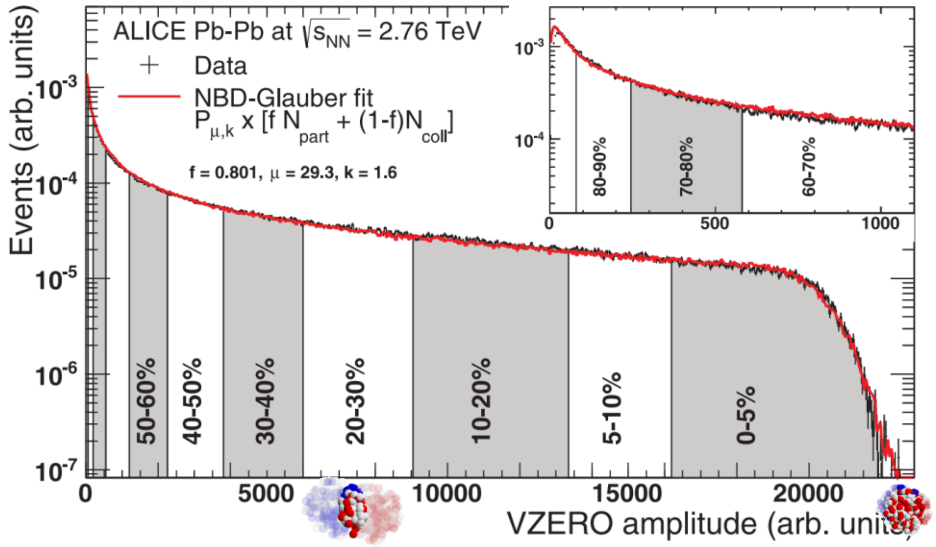
Centrality cuts are very important..;
boost invariant, self-correlation, computationally expensive for |η|>1.5

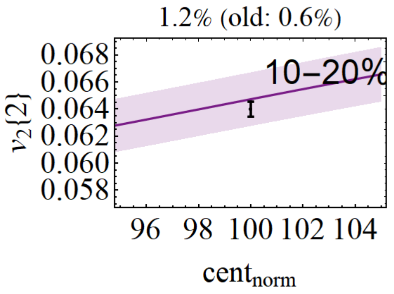


An example analysing data: centrality

An example analysing data: centrality

1. Historically: centrality meant *impact parameter*
   - Impact parameter is however never observable
   - Traditionally V0 signal (multiplicity at forward rapidity) as a proxy
   - Especially with small systems (p-Pb) the definition matters
2. Modern point of view: *centrality is event activity*
   - But where?
   - Also, 90-100% cannot be measured but is modelled
     Do we have the anchor point at 90% under control?
   - If not, all observables shift as a function of centrality (!)
     - `the ultimate' correlation of observables
     - Related to total hadronic PbPb cross section

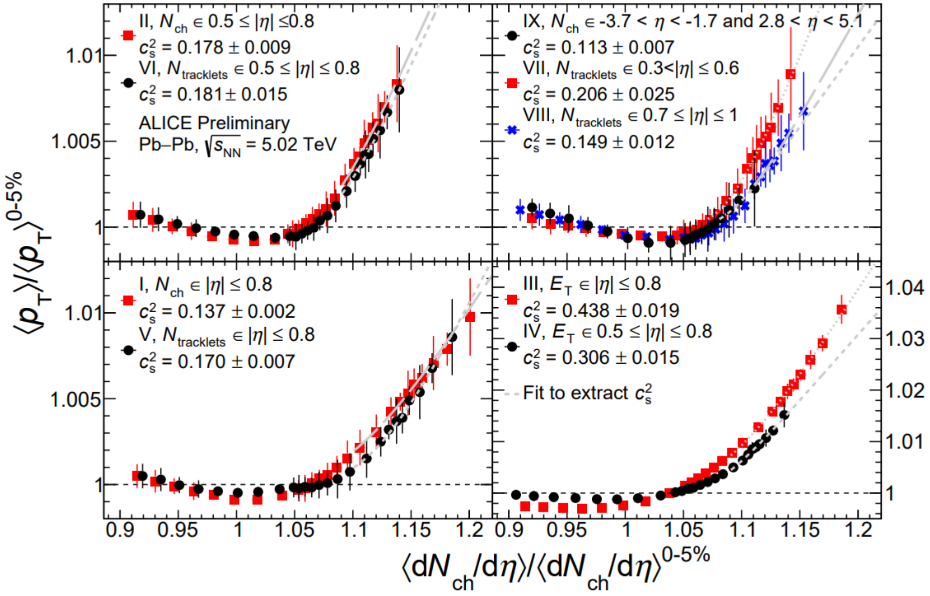




An example analysing data: centrality

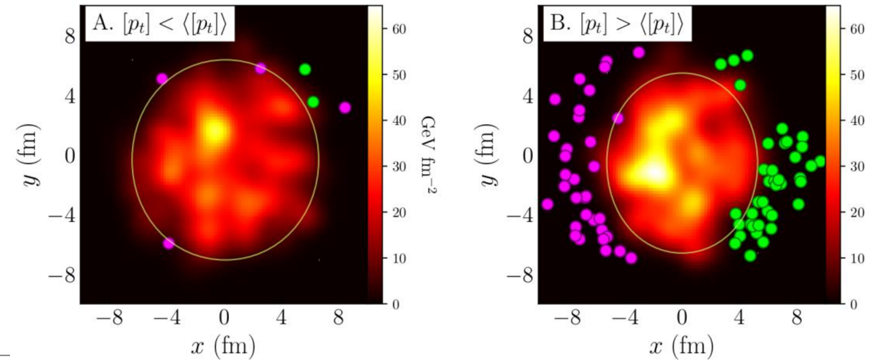
1. An even more modern perspective: apples-to-apples comparison theory/experiment
   - Even relevant in PbPb collisions; for instance in ultracentral:

Observable	Label	Centrality estimation	$\langle p_T \rangle$ and $\langle dN_{ch}/d\eta \rangle$	Minimum $\lvert\Delta\eta\rvert$
N_{ch} in TPC	I	$\lvert\eta\rvert \leq 0.8$	$\lvert\eta\rvert \leq 0.8$	0
	II	$0.5 \leq \lvert\eta\rvert < 0.8$	$\lvert\eta\rvert \leq 0.3$	0.2
E_T in TPC	III	$\lvert\eta\rvert \leq 0.8$	$\lvert\eta\rvert \leq 0.8$	0
	IV	$0.5 \leq \lvert\eta\rvert < 0.8$	$\lvert\eta\rvert \leq 0.3$	0.2
$N_{tracklets}$ in SPD	V	$\lvert\eta\rvert \leq 0.8$	$\lvert\eta\rvert \leq 0.8$	0
	VI	$0.5 \leq \lvert\eta\rvert < 0.8$	$\lvert\eta\rvert \leq 0.3$	0.2
	VII	$0.3 < \lvert\eta\rvert < 0.6$	$\lvert\eta\rvert \leq 0.3$	0
	VIII	$0.7 \leq \lvert\eta\rvert < 1$	$\lvert\eta\rvert \leq 0.3$	0.4
N_{ch} in V0	IX	$-3.7 < \eta < -1.7$ and $2.8 < \eta < 5.1$	$\lvert\eta\rvert \leq 0.8$	0.9



2. Apples-to-apples only possible if theoretical modelling is valid at the centrality definition
   - Can be problematic especially for boost invariant models (forward rapidity poorly described)

Centrality selection: the ρ_2 correlator



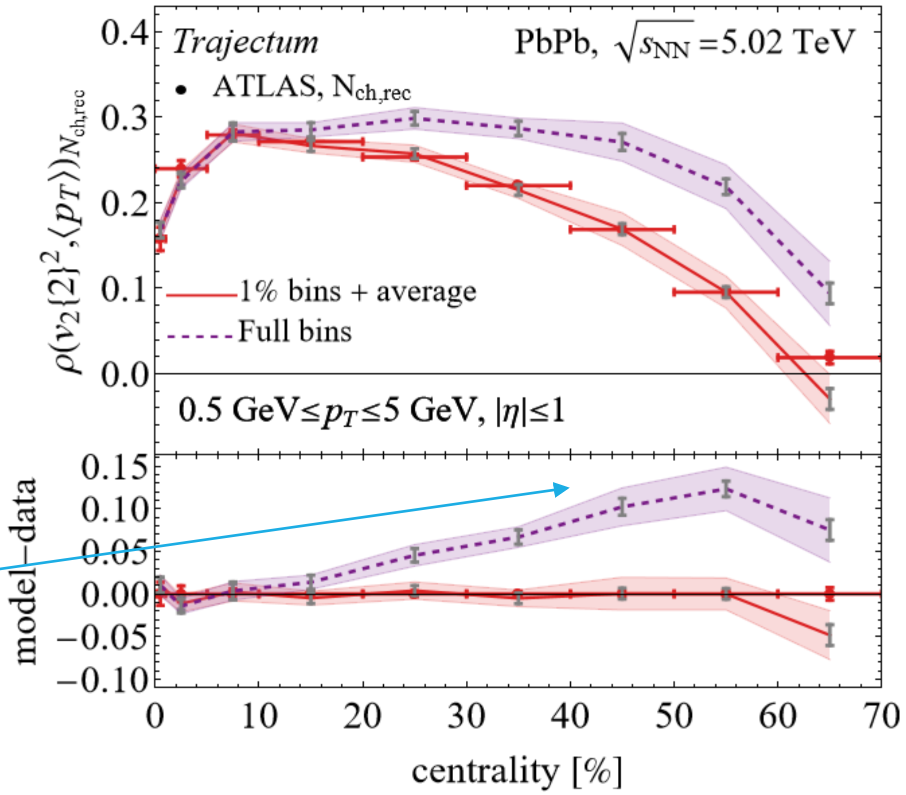
$\rho(v_n\{2\}^2, \langle p_T \rangle)$: correlation anisotropic flow and transverse momentum

- True correlator: zero if flow and momentum are independent
- Intuition: smaller system → larger pT
- Intuition: more elliptic → higher v2{2}
- Intuition: central: smaller system often a bit more elliptical

Centrality is an important detail:

- The correlator is done in small centrality bins (1% for us)
- Obtain larger bins by averaging
- Relatively big effect (up to 0.1, exp uncert. is 0.01)

But how do we define centrality?



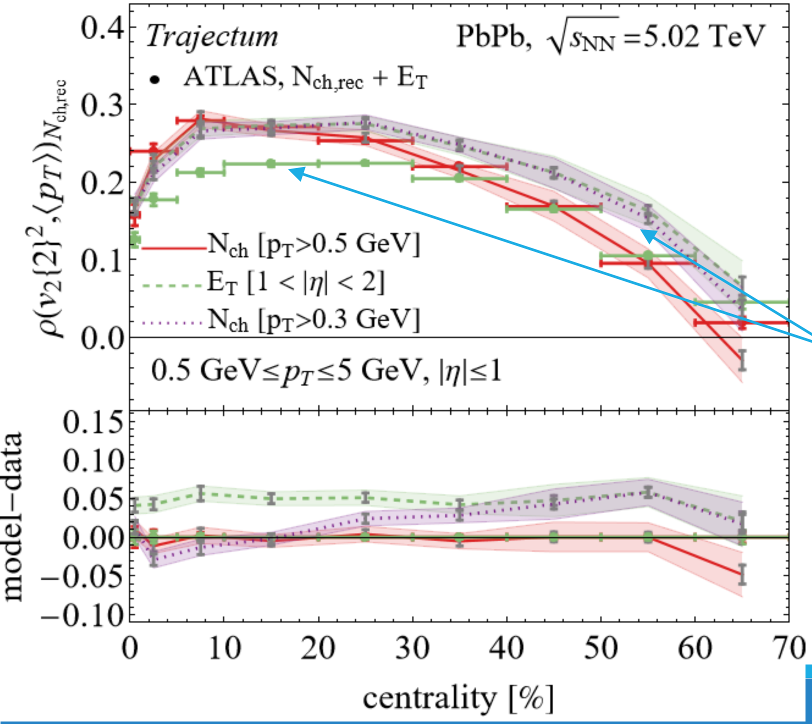
Giuliano Giacalone and Chun Shen, Manipulating strong electromagnetic fields with the average transverse momentum of relativistic nuclear collisions (2021)

Credit to ATLAS: HEPdata has 445 tables (!) that allows to figure these things out

Centrality selection: the ρ_2 correlator

Forward (E_T) versus mid-rapidity (Nch): apples-to-apples

- Forward considered `better' (ATLAS, lower spread)
- Mid-rapidity closer to model (boost invariant)



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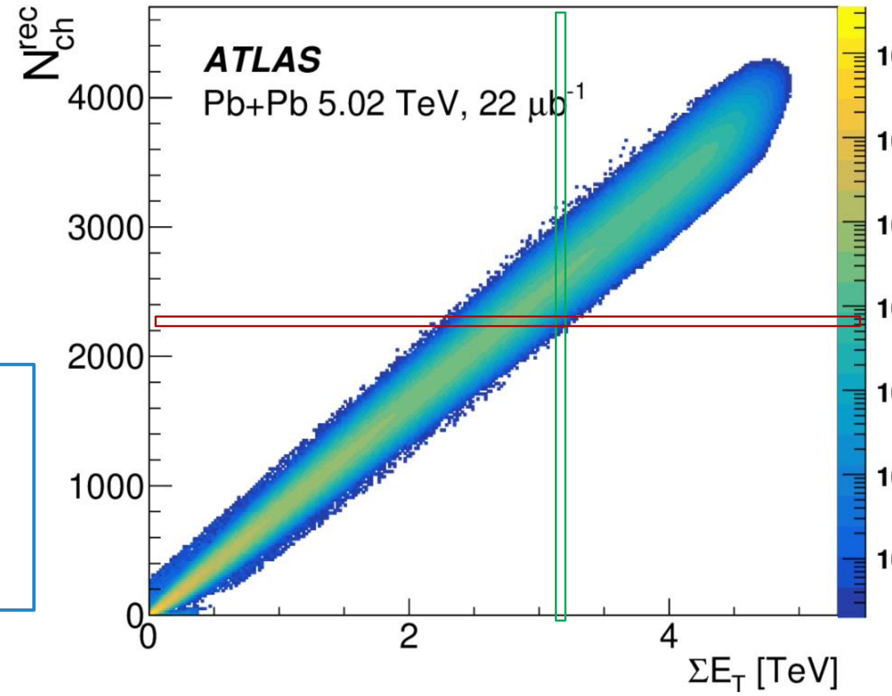

ATLAS, Correlations between flow and transverse momentum in Xe+Xe and Pb+Pb collisions at the LHC with the ATLAS detector: a probe of the heavy-ion initial state and nuclear deformation (2022)