Calibration of rapidity dependent mulitplicities using a three-dimensional initial state model

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Precision Frontier of QCD Matter: Inference and Uncertainty Quantification
Workshop C3NT

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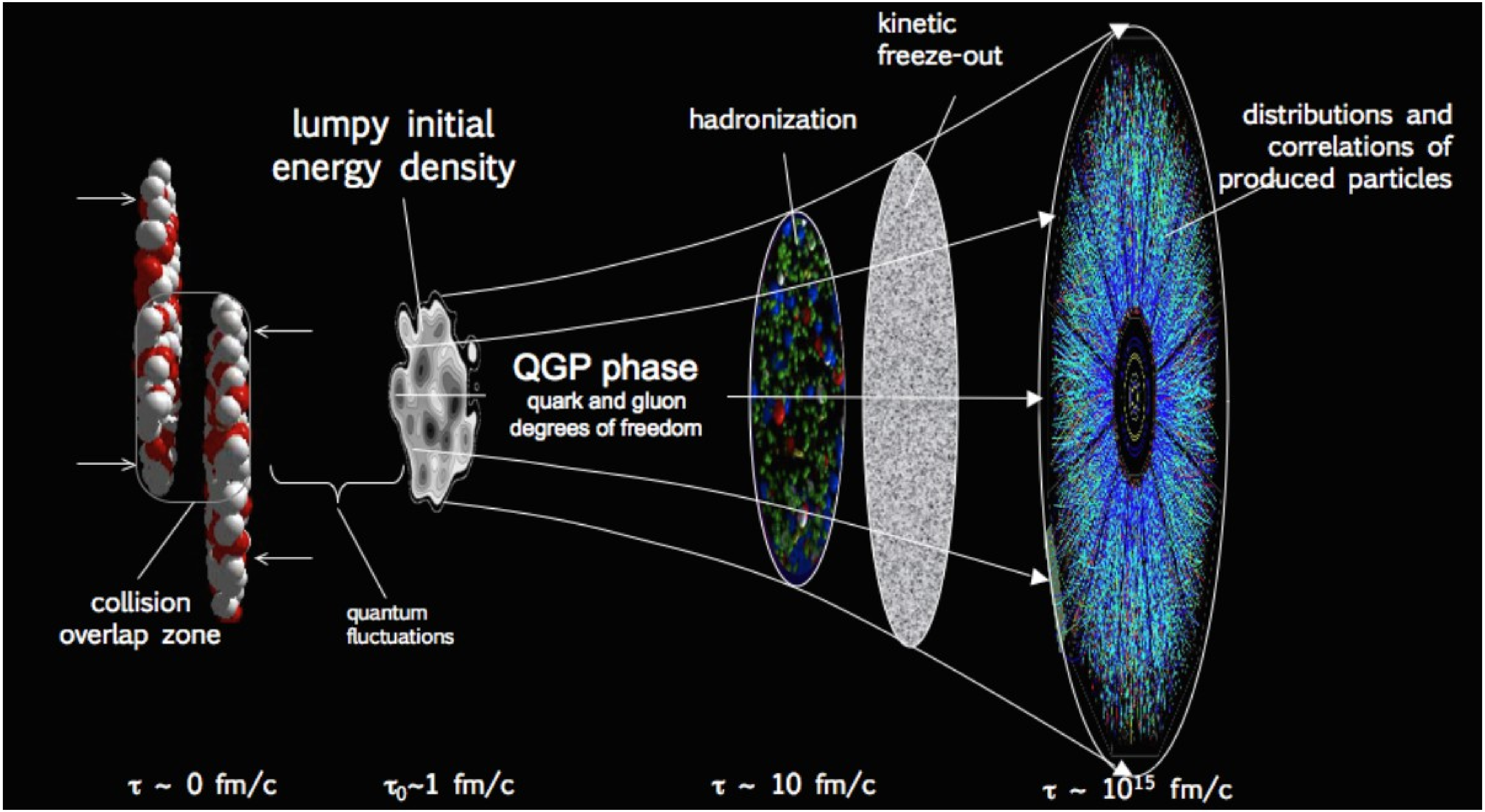
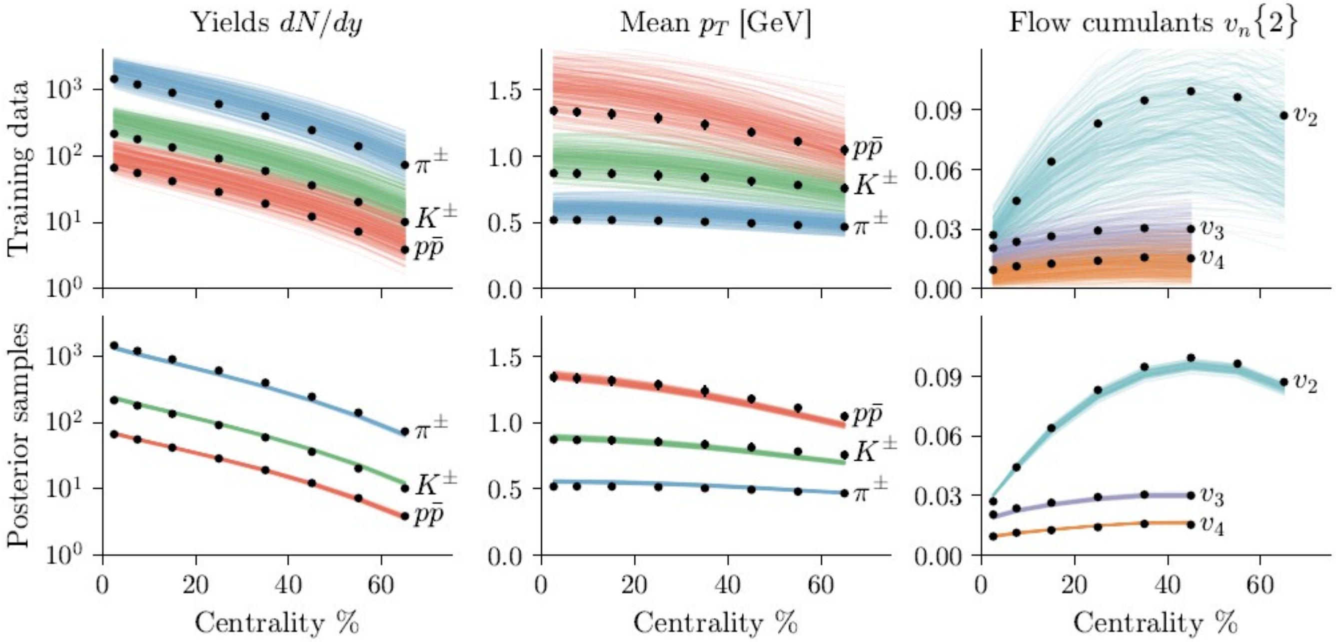
Heavy ion collision - overview

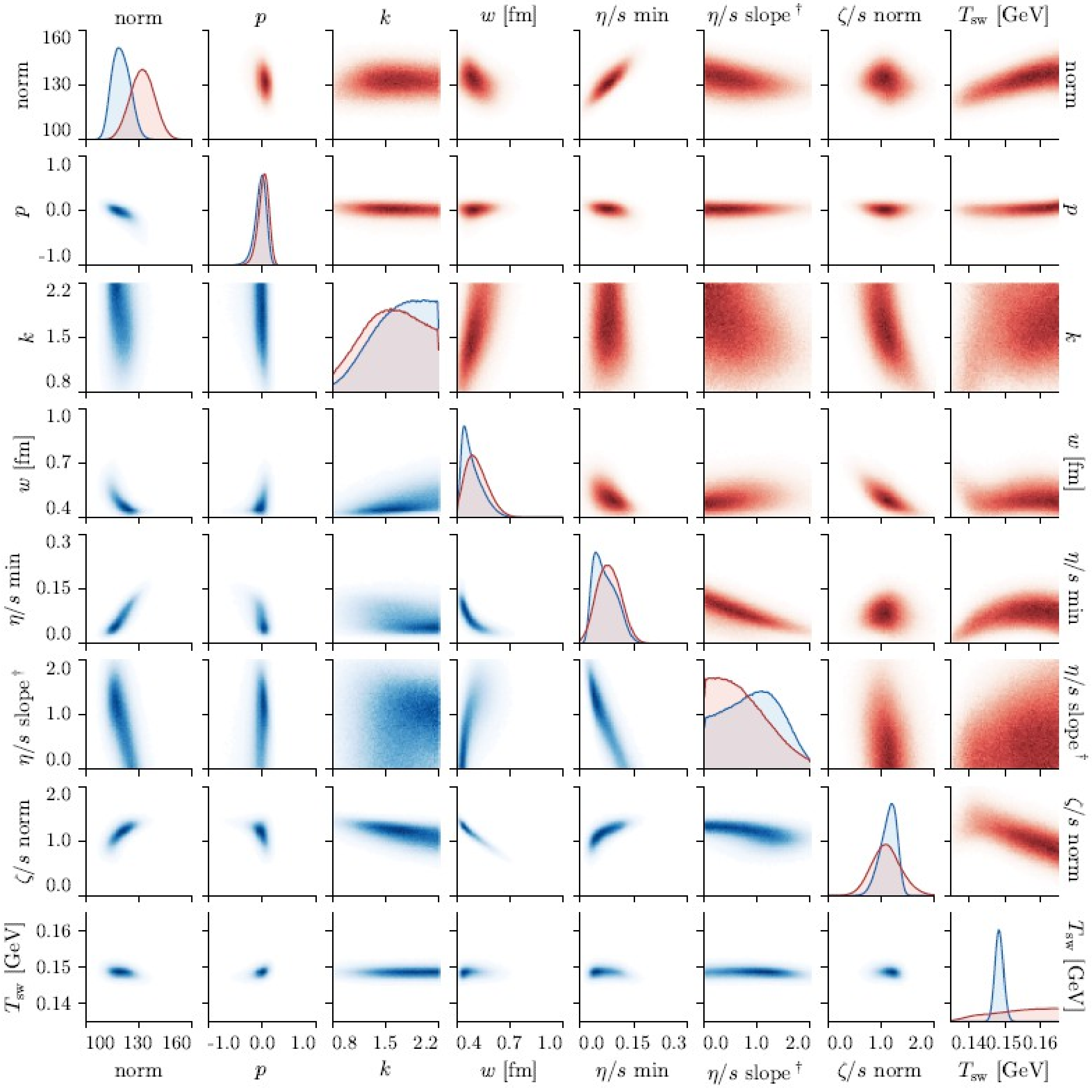


Heavy ion collision – overview

- Models include many parameters for initial state, fluid evolution, freeze out etc
- → Constrained through bayesian calibration
- Constraint on parameters depends on data set
- Some parameters can be linked to QCD/first principle calculations



Heavy ion collisions - overview



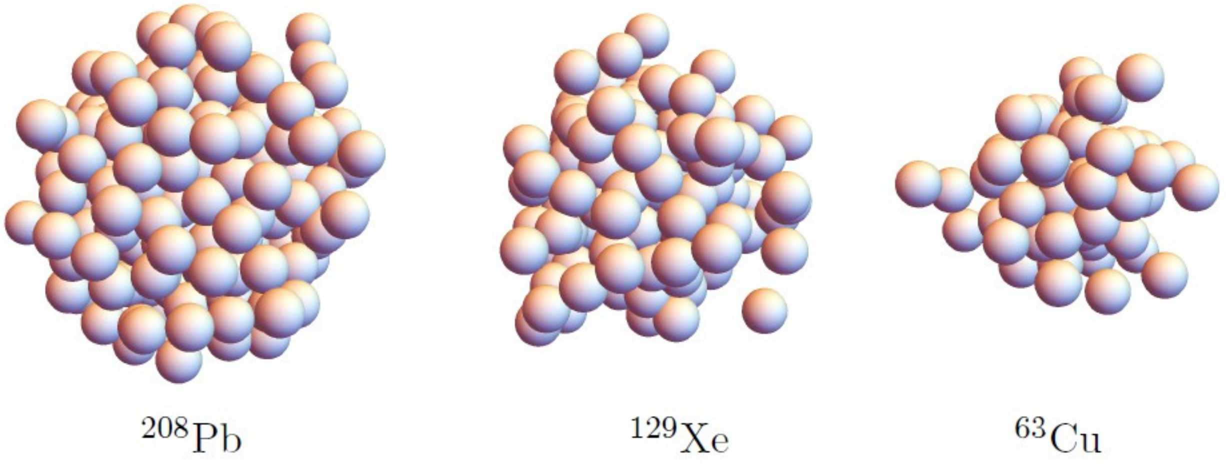
- Different constraints based on identified particles (blue) or charged particles (red)
- Most parameters from initial state

  → Hard to calculate from first principles

Initial conditions – Trento (2D)

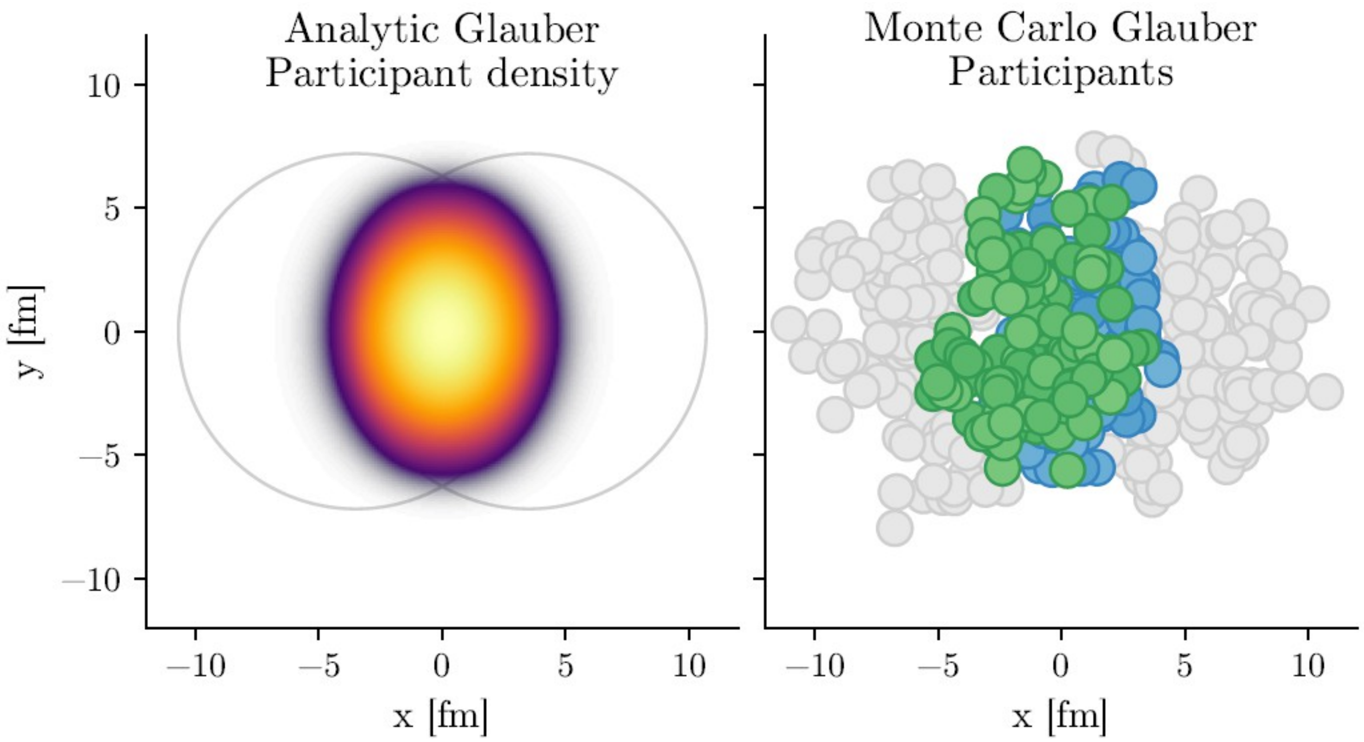
- Widely used model for generating initial conditions
- Based on Glauber approach:

1. Sample nucleon positions based on density



Initial conditions – Trento (2D)

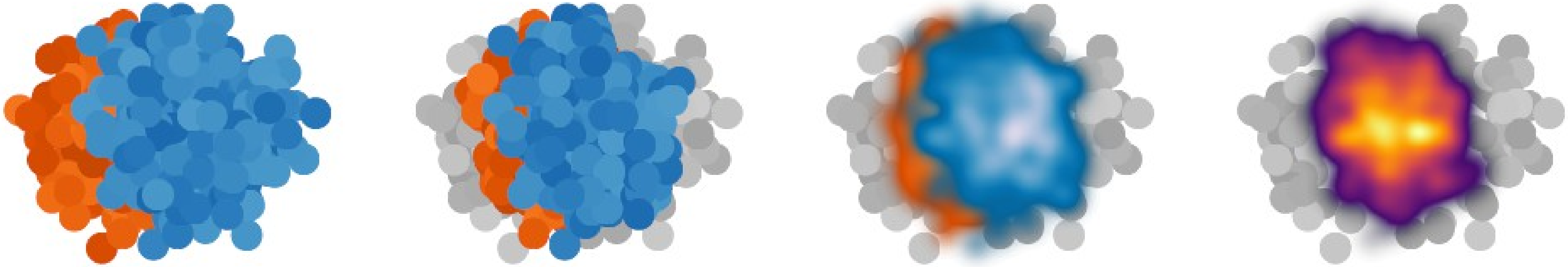
2. Determine collisions based on pairwise probability (distance & cross section)



Initial conditions – Trento (2D)

2. Deposit Gaussian with fluctuating norm at each collision site

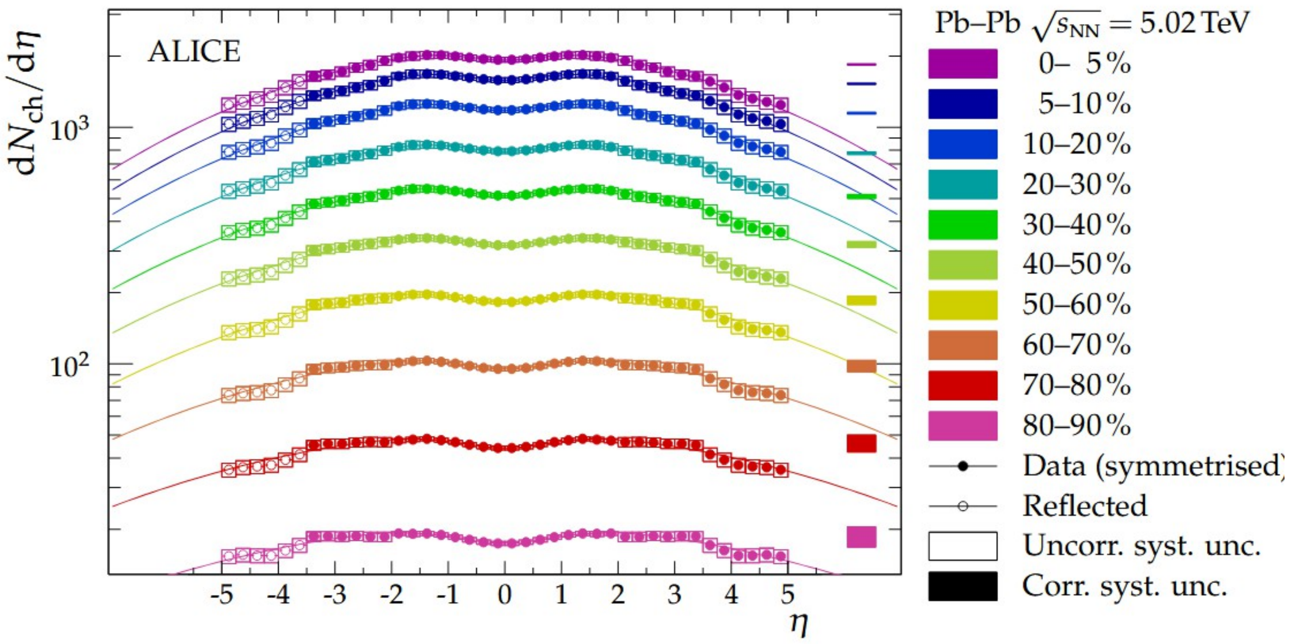
3. Combine into one profile using generalized norm

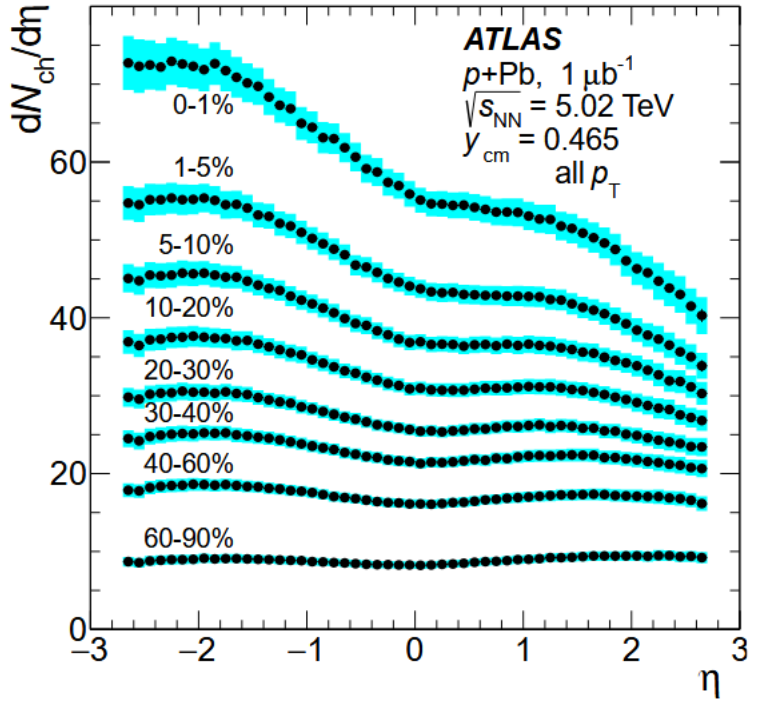


1. Nucleon positions 2. Nucleon participants 3. Participant thicknesses 4. Reduced thickness

Initial conditions – Trento (2D)

Good model for mid rapidity for symmetric collisions, but also interesting physics in asymmetric collisions & at larger rapidities

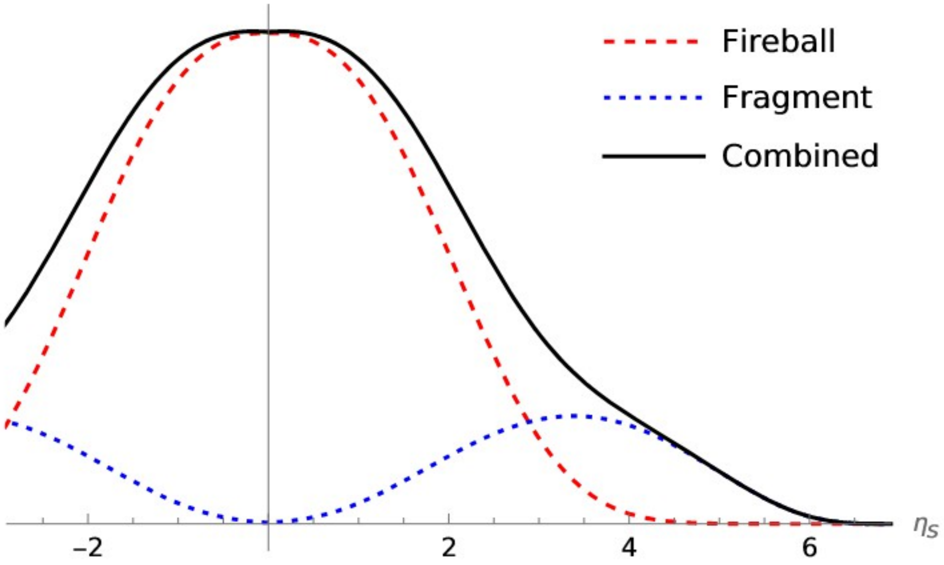




→ Extension in 3D required!

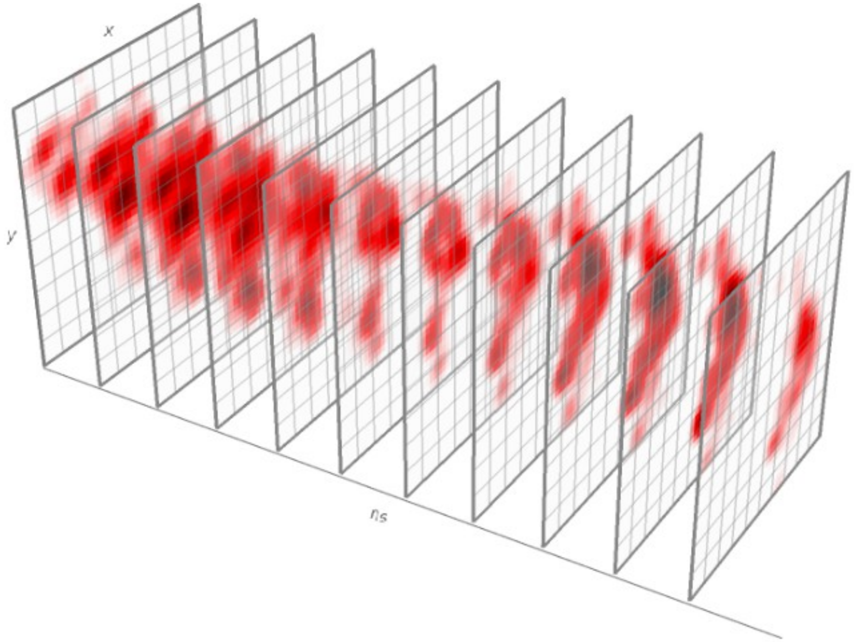
Trento 3D

- Trento 3D is extension of Trento to account for longitudinal extension
- Idea: Combine fireball at midrapidity with fragmentation for larger/smaller rapidities
  → New parameters related to shape & size of fireball etc



Trento 3D

Profiles evaluated on 2d x-y grids for constant values of rapidity



Trento 3D calibration

- Use Trento 3D for calibration on rapidity-dependent data
- Full simulation of 3+1D hydro with afterburner etc very expensive
  → Use 1+1D hydro evolution together with transverse Gaussianization

$$\varepsilon_{\mathrm{G}}(\tau, r, \phi, \eta_s) = \varepsilon_{\mathrm{LH}}(\tau, \eta_s) \frac{1}{2\pi\sigma_{\mathrm{G}}^2} e^{-r^2/2\sigma_{\mathrm{G}}^2}$$

1+1D hydro evolution

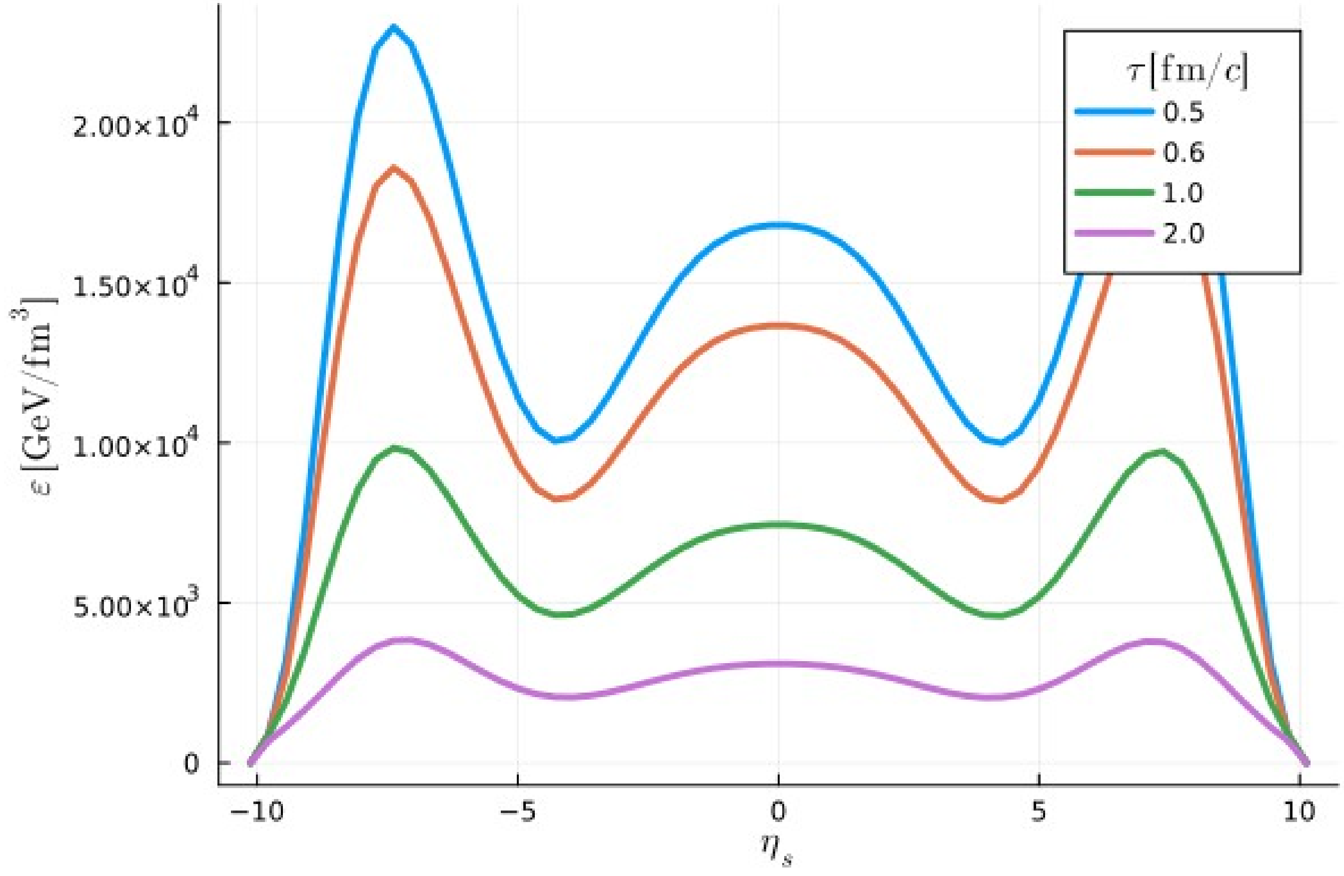
- Fluid dynamic evolution constrained to longitudinal direction
  → 2D metric & energy-momentum tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -\tau^2 \end{pmatrix} \qquad T^{\mu\nu} = \begin{pmatrix} \varepsilon_{\mathrm{LH}} & \frac{1}{\tau} p_{z,\mathrm{LH}} \\ \frac{1}{\tau} p_{z,\mathrm{LH}} & \frac{c_s^2}{\tau^2} \varepsilon_{\mathrm{LH}} \end{pmatrix}$$

- Use simplified equation of state $P = c_s^2\, \varepsilon_{\mathrm{LH}}$ with c_s^2=1/3

1+1D hydro

Equations can be solved quickly through Greens functions



Transverse Gaussianization

- Employ Cooper-Frye particle production on 2+1D hypersurface
  → transverse Gaussianization
- Transverse energy density given by Gaussian

$$\varepsilon_{\mathrm{G}}(\tau, r, \phi, \eta_s) = \varepsilon_{\mathrm{LH}}(\tau, \eta_s)\frac{1}{2\pi\sigma_{\mathrm{G}}^2}e^{-r^2/2\sigma_{\mathrm{G}}^2}$$

- Radius at freeze-out given by

$$r_{\mathrm{FO}}(\tau, \eta_s) = \sqrt{2\sigma_{\mathrm{G}}^2 \ln\left\{\frac{\varepsilon_{\mathrm{LH}}(\tau, \eta_s)}{2\pi\sigma_{\mathrm{G}}^2\varepsilon_{\mathrm{FO}}}\right\}}$$

Calibration setup

Use Trento 3D + 1+1D hydro for calibration in 3 steps:

1) Closure test with Trento 3D & 1+1D hydro
2) Cross-model validation vs Trento 3D & MUSIC
3) Calibration on experimental data

Parameter	**Symbol**	**Range**
Form width [fm]	u	0.35 – 1.0
Nucleon width [fm]	w	0.35 – 1.0
Constituent number	n_c	2.0 – 20.0
Structure	χ	0.2 – 0.9
Transverse mom. scale [GeV]	$k_{\mathrm{T,min}}$	0.2 – 0.9
Shape parameter	α	3.0 – 5.0
Shape parameter	β	−0.5 – 1.5
Fireball norm. [GeV] ($\sqrt{s_{\mathrm{NN}}} = 200$ GeV)	N_{200}	1.0 – 15.0
Fireball norm. [GeV] ($\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV)	N_{5020}	15.0 – 30.0
Fluctuation	k	0.1 – 0.6
Flatness	f	1.0 – 2.5
Hydrodyn. time [fm/c] (Pb–Pb 5.02 TeV)	$\tau_{0,\mathrm{Pb}}$	0.1 – 1.5
Hydrodyn. time [fm/c] (p–Pb 5.02 TeV)	$\tau_{0,p}$	0.1 – 1.5
Hydrodyn. time [fm/c] (Au–Au 200 GeV)	$\tau_{0,\mathrm{Au}}$	0.1 – 1.5
Hydrodyn. time [fm/c] (d–Au 200 GeV)	$\tau_{0,d}$	0.1 – 1.5
Overall scale	N_{scale}	0.8 – 2.0

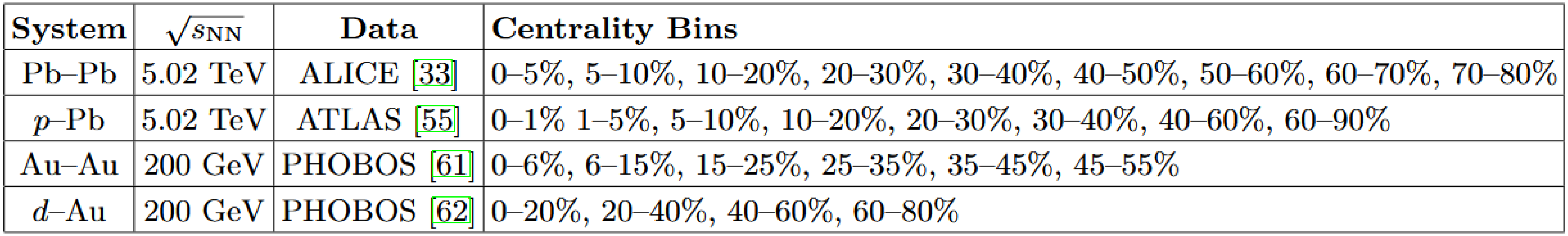
Calibration - data

Charged particle multiplicity as function of rapidity for four different collisions systems:

- PbPb 5.02 TeV
- pPb 5.02 TeV
- AuAu 200 GeV
- dAu 200 GeV

→ Two symmetric and two asymmetric systems

Different centrality bins in systems:



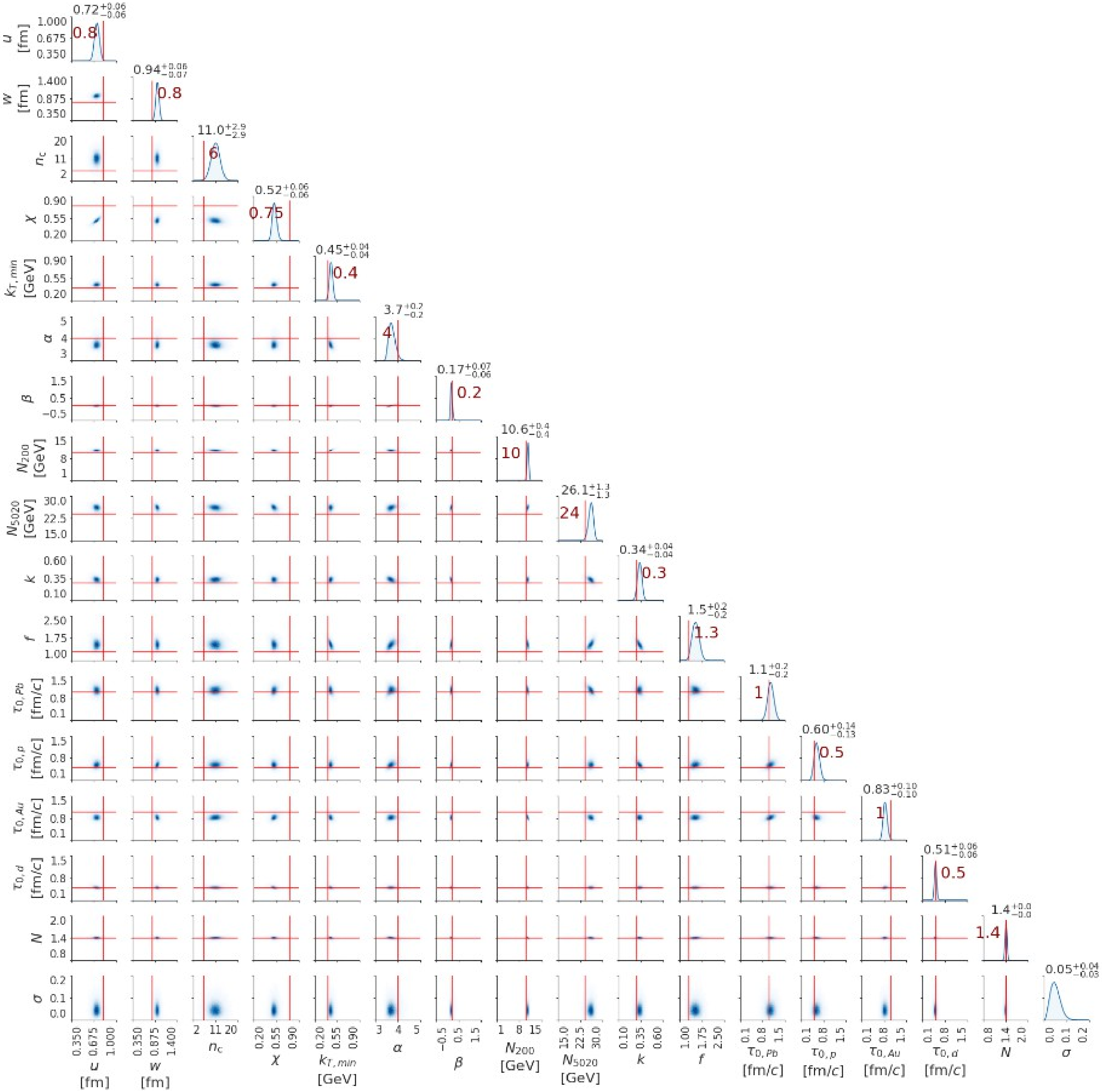
System	$\sqrt{s_{\mathrm{NN}}}$	**Data**	**Centrality Bins**
Pb–Pb	5.02 TeV	ALICE [33]	0–5%, 5–10%, 10–20%, 20–30%, 30–40%, 40–50%, 50–60%, 60–70%, 70–80%
p–Pb	5.02 TeV	ATLAS [55]	0–1% 1–5%, 5–10%, 10–20%, 20–30%, 30–40%, 40–60%, 60–90%
Au–Au	200 GeV	PHOBOS [61]	0–6%, 6–15%, 15–25%, 25–35%, 35–45%, 45–55%
d–Au	200 GeV	PHOBOS [62]	0–20%, 20–40%, 40–60%, 60–80%

Calibration - design

- Calibration uses 1000 design points
- 5000 events per system and design point
- Events are sorted and averaged in centrality classes
- Reduction to 1D profiles
- 1+1D hydro + Cooper-Frye freeze-out

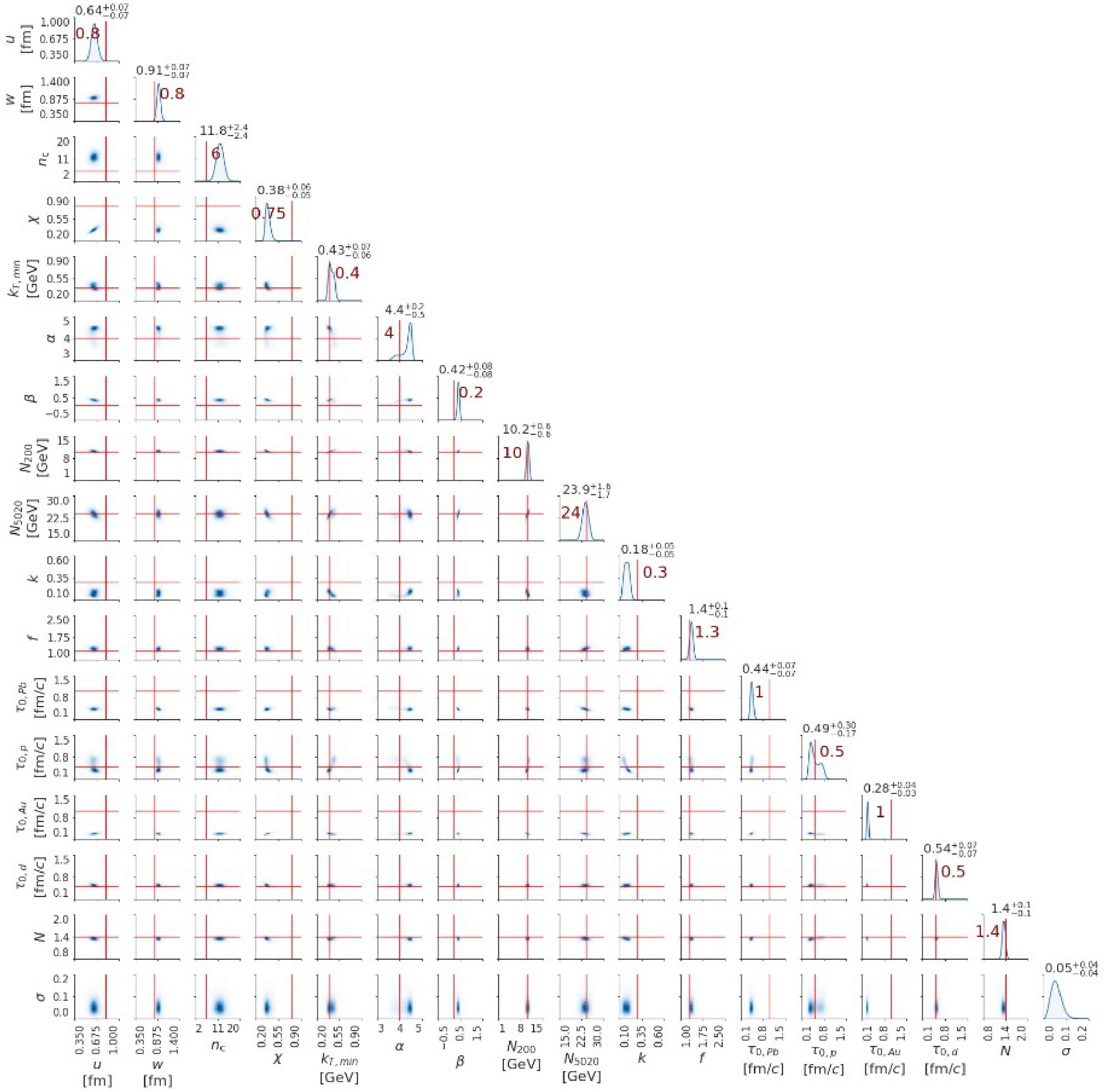
Closure test - results

Closure posteriors in agreement with true values



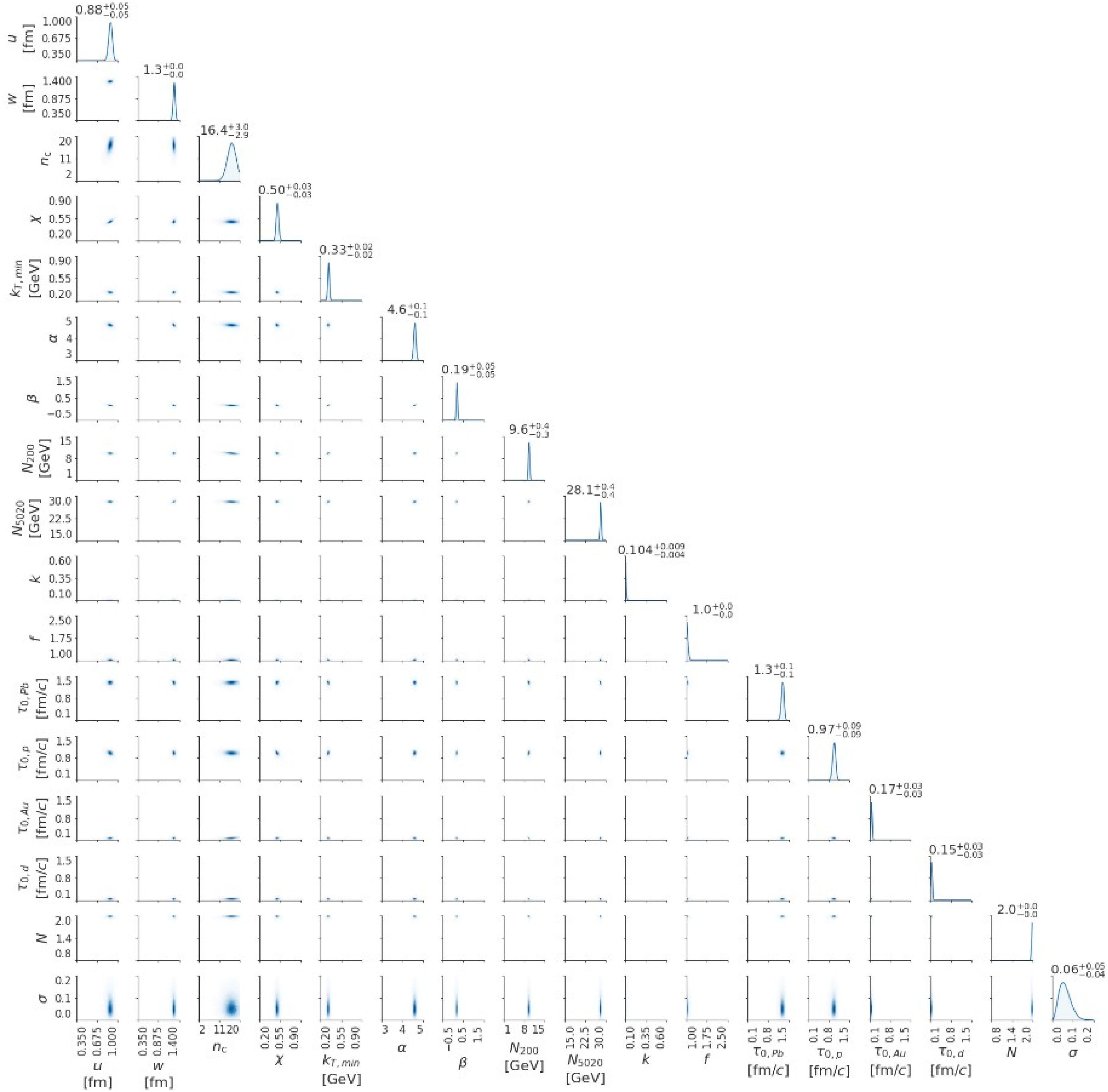
Cross validation - results

Some discrepancy between model and data & very narrow distributions



Calibration - results

Posteriors very narrow (almost delta functions) → Hint for some issue of the calibration

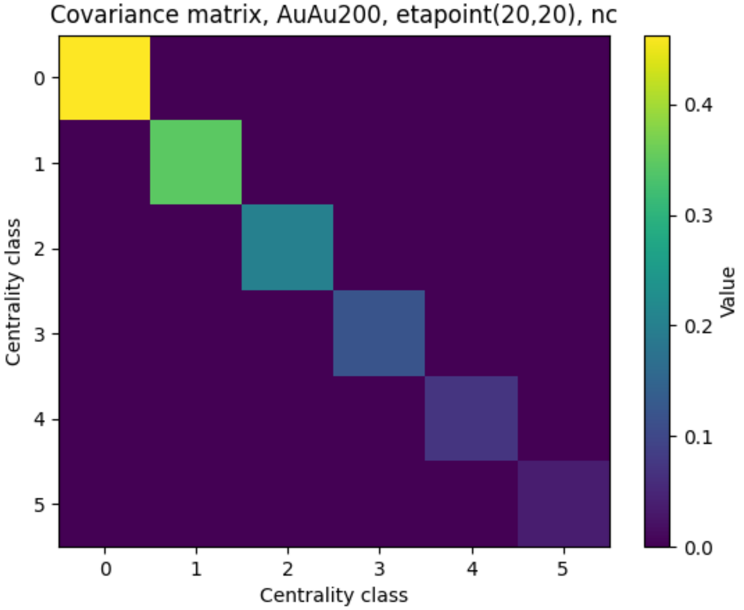


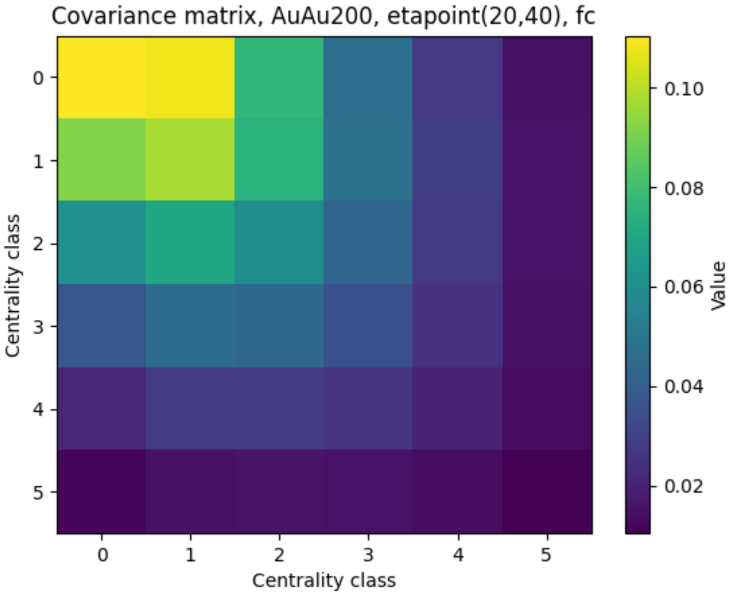
Possible issues

- Treatment of uncertainties/setup of covariance matrix
- Choice of emulator
- Treatment of data
- Choice of centrality range

Covariance matrix

- Initial setup of covariance matrix purely diagonal → No correlation between different observables
- N_ch should be strongly correlated across different rapidities and centralities

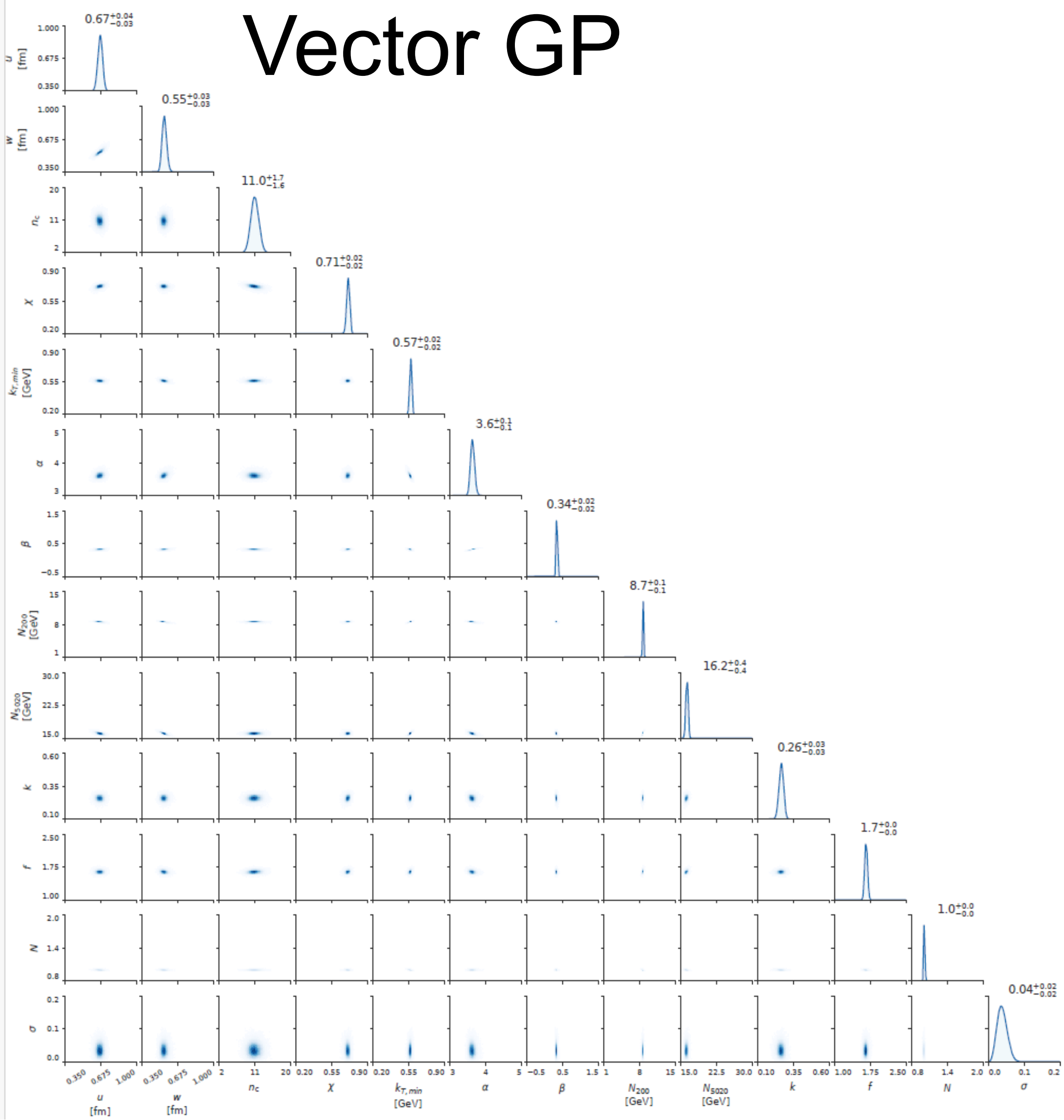

Covariance matrix, AuAu200, etapoint(20,20), nc


Covariance matrix, AuAu200, etapoint(20,40), fc

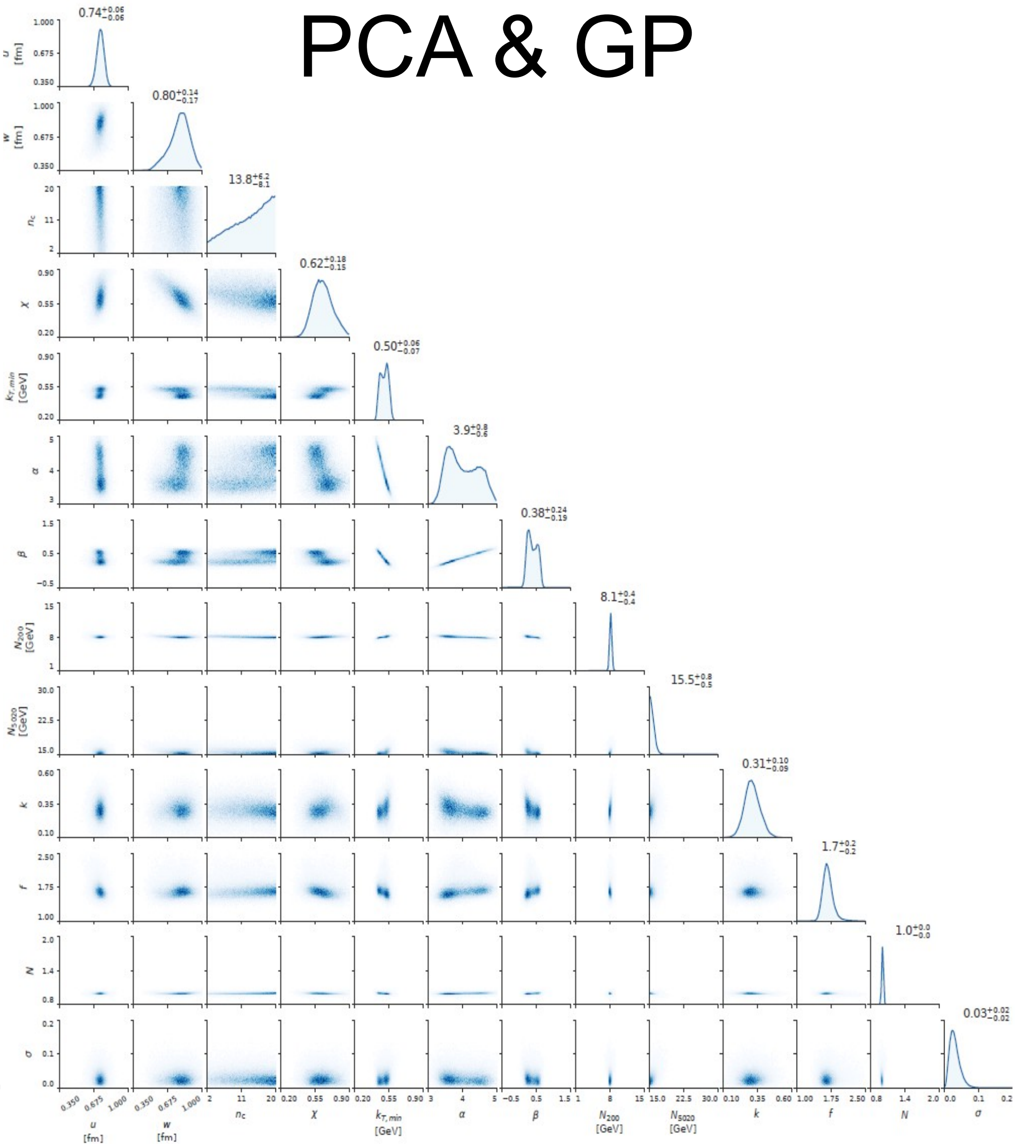
Choice of emulator

Two different approaches:

Vector GP



PCA & GP

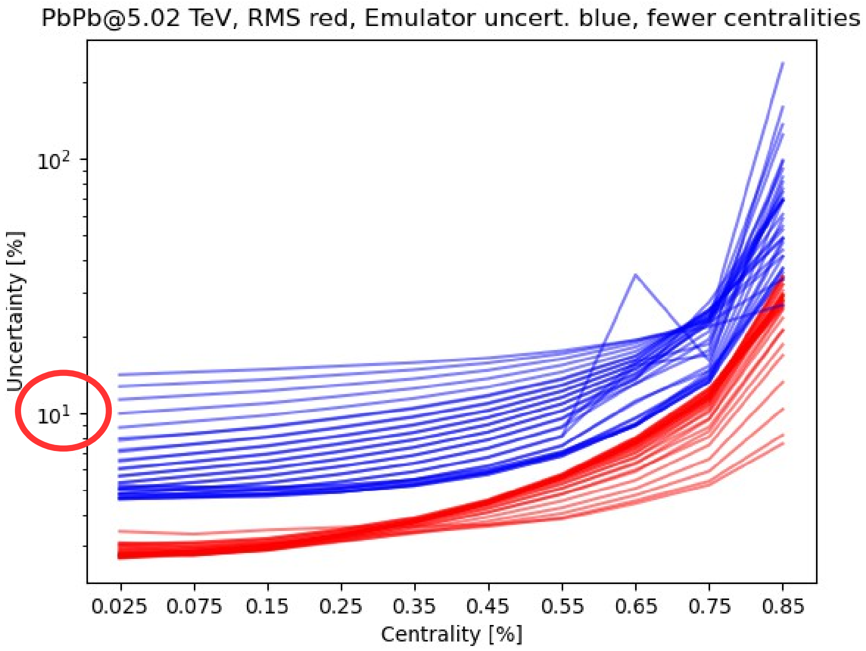


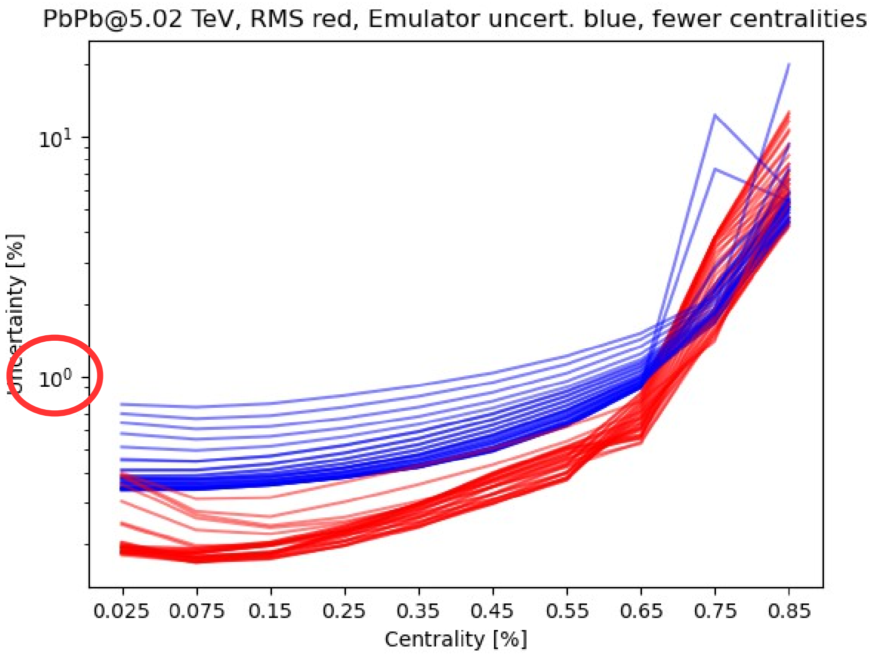
VectorGP seems to overfit

Treatment of data

- Multiplicities very different across different systems
- Use transformation to have data & uncertainties at similar scales

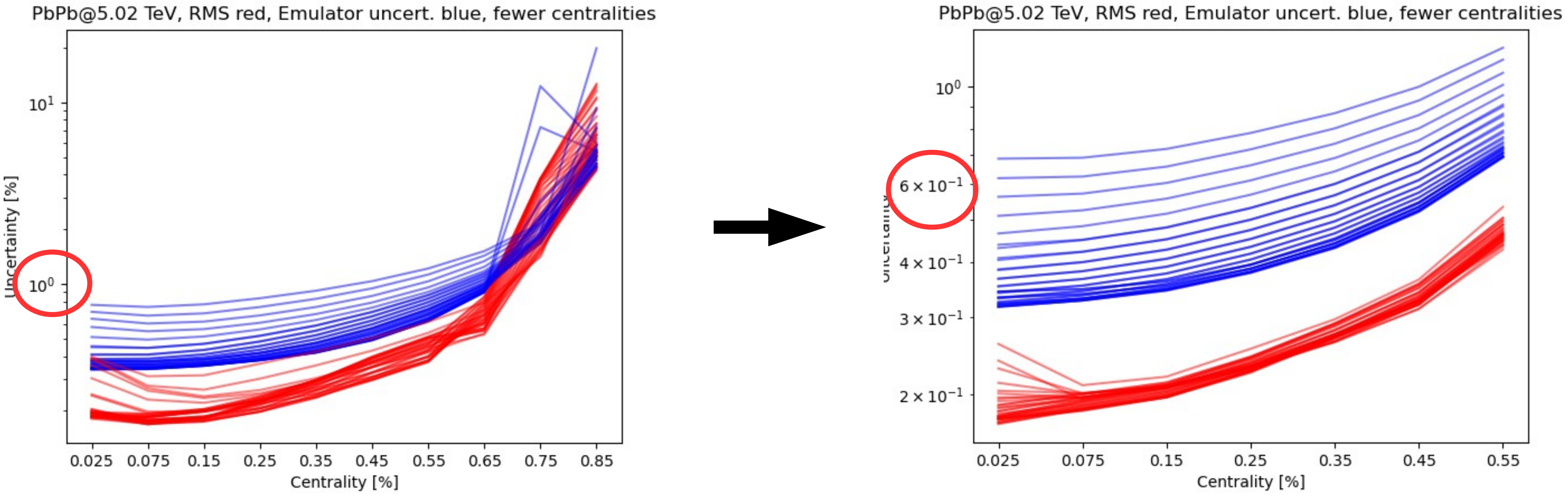
$$N^* = \log(N+1)$$





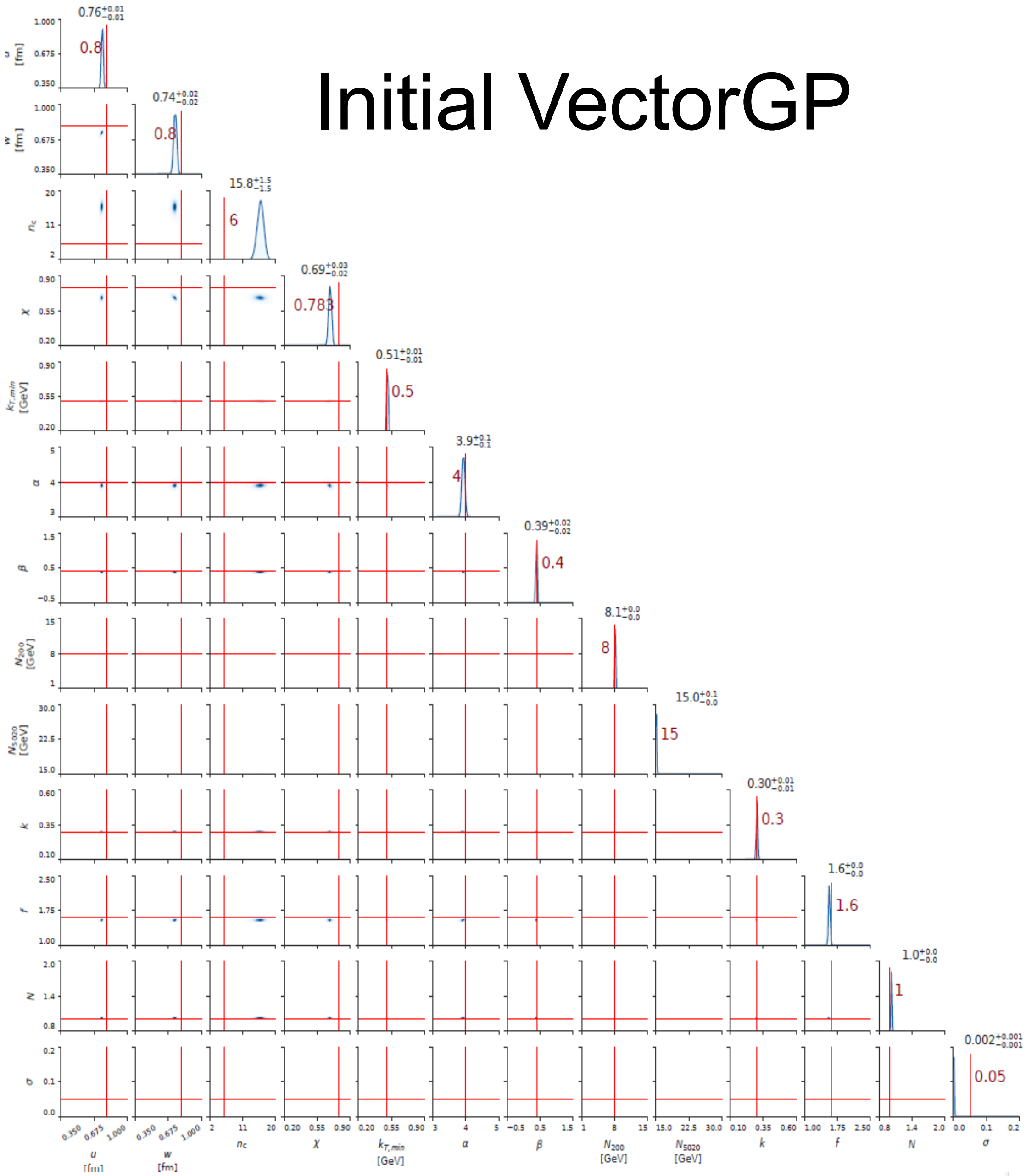
Choice of centrality range

- Hydro expected to work better for more central collisions & larger systems
- Emulator uncertainty grows with centrality
  → Only use data up to 60% centrality

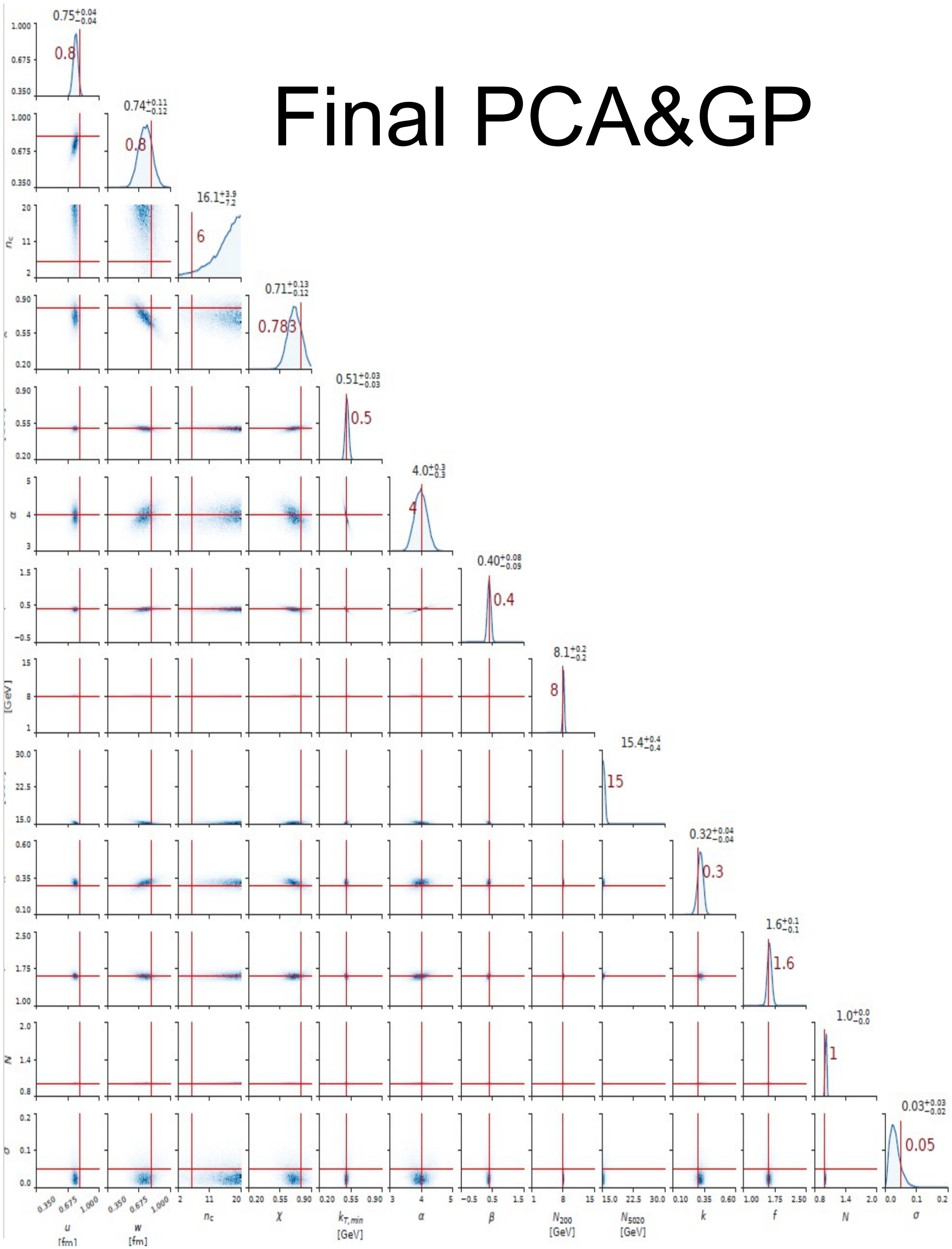


Final closure

Initial VectorGP



Final PCA&GP



Product of incremental improvements

Conclusion & outlook

- First steps toward 3D calibration using Trento3D

  → Examine interesting effects at large rapidities and in asymmetric collision systems

  Next steps:

  - Redo closure, cross-validation & calibration with incremental improvements added
  - Use more realistic fluid dynamics & hadronization models and include more observables

    → Stay tuned for updated 1+1D calibration (soon) and full 3+1D calibration (not so soon)