Shear and Bulk Viscosities of Gluon Plasma across the Transition Temperature from Lattice QCD

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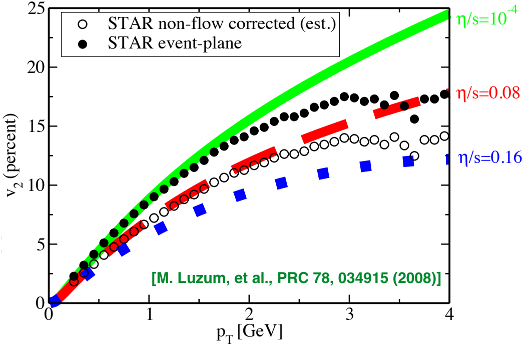
Precision Frontier of QCD Matter
Sep 1 –12, Wuhan

Introduction



• inputs for tranport/hydro models

- η/s quantifies the dissipation processes in the hydrodynamics.

  G. Denicol et al., PRC 80, 064901 (2008)

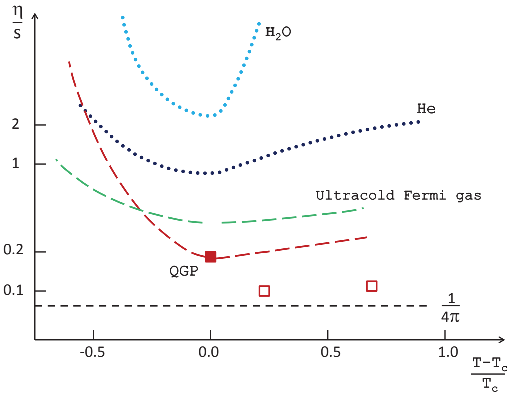
- Small η/s is suggested by phenomenological interpretation of experimental data.

  K. H. Ackermann et al., (STAR), PRL 86, 402 (2001)

- Extracting η/s from experiments needs accurate inputs: initial condition, EoS ...

  U. Heinz et al., Annu. Rev. Nucl. Part. Sci. 63, 123 (2013)

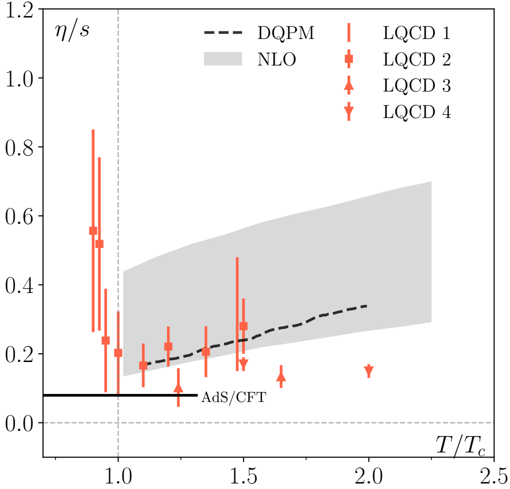
Introduction



S.Cremonini et al., JHEP 1208 (2012) 167

- η/s of QGP is sensitive to the phase transitions.
- Determinations of viscosities require theoretical inputs.

Determinations of viscosities from theory



- NLO weak-coupling calculation
  J. Ghiglieri et al., JHEP 03, 179 (2018)
- Dynamical Quasi-Particle Model (DQPM)
  R. Marty et al., PRC 88, 045204 (2013)
- ...
- Lattice QCD (all quenched):
  multi-level: 2: N. Astrakhantsev et al., JHEP 04, 101 (2017)
  3: H. B. Meyer, PRD 76, 101701 (2007)
  4: S. Borsanyi et al., PRD 98, 014512 (2018)
  gradient flow (extendable to full QCD):
  1: H. T. Shu et al., PRD 108, 014503 (2023)

1.5 $T_c \Rightarrow$ **this work:** $0.76 \leq T/T_c \leq 2.25$

Theoretical framework

$$G_{\text{shear}}(\tau,\tau_{\text{F}}) = \frac{1}{10}\int \mathrm{d}^3x \left\langle \pi_{ij}(0,\vec{0},\tau_{\text{F}})\,\pi_{ij}(\tau,\vec{x},\tau_{\text{F}})\right\rangle$$

Renormalization

$a \to 0, \tau_{\text{F}} \to 0$

$$G(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\frac{\cosh[\omega(1/2T-\tau)]}{\sinh(\omega/2T)}\rho(\omega,T)$$

Inverse problem

Spectra reconstruction $\rho(\omega,T)$

Kubo formula

$$\eta(T) = \lim_{\omega\to 0}\frac{\rho_{\text{shear}}(\omega,T)}{\omega}$$

Key Points:

- Lattice computation of EMT correlators.
- Spectra reconstruction from the correlators.

Challenges:

- Severe UV fluctuations in the correlators.
- Theoretical uncertainties in the spectra reconstruction.

Theoretical framework

$$G_{\text{bulk}}(\tau,\tau_{\text{F}}) = \int \mathrm{d}^3x \left\langle T_{\mu\mu}(0,\vec{0},\tau_{\text{F}})\, T_{\mu\mu}(\tau,\vec{x},\tau_{\text{F}}) \right\rangle$$

Renormalization

$a \to 0, \tau_{\text{F}} \to 0$

$$G(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{\cosh[\omega(1/2T-\tau)]}{\sinh(\omega/2T)} \rho(\omega,T)$$

Inverse problem

Spectra reconstruction $\rho(\omega,T)$

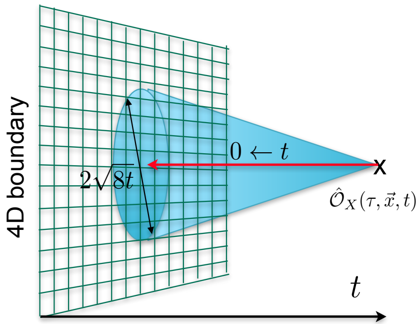
Kubo formula

$$\zeta(T) = \frac{1}{9} \lim_{\omega\to 0} \frac{\rho_{\text{bulk}}(\omega,T)}{\omega}$$

Key Differences:

- Remove the disconnected contributions.
- Smaller cutoff effects for the renormalization constant.
- Same analysis procedure.

Noise reduction technique: gradient flow



Flow equation :

$$\frac{\mathrm{d}B_\mu(x,\tau_\mathrm{F})}{\mathrm{d}\tau_\mathrm{F}} = D_\nu G_{\nu\mu}(x,\tau_\mathrm{F})$$
$$B_\nu(x,\tau_\mathrm{F}=0) = A_\nu(x)$$

LO solution:

$$B_\nu(x,\tau_F) \propto \exp\left(\frac{-(x-y)^2}{\sqrt{8\tau_F}^2/2}\right) B_\nu(y)$$

M. Lüscher, JHEP 08, 071 (2010)

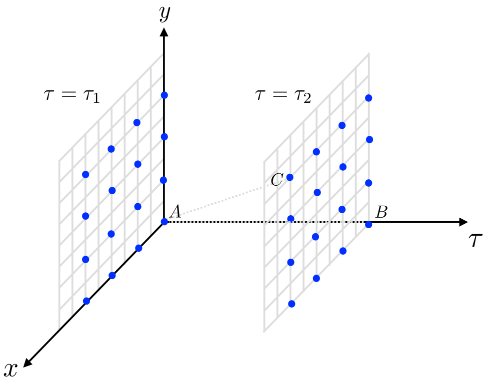
Smearing radius: $\sqrt{8\tau_\mathrm{F}}$.

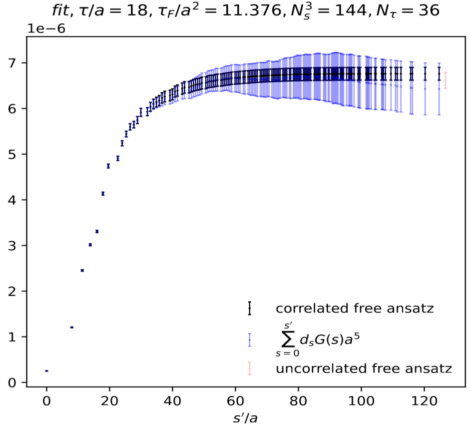
Advantage:

- The UV fluctuations strongly suppressed.
- Well-defined renormalization framework for EMT:

  $$T_{\mu\nu}(\tau_F,x) = c_1(\tau_F)U_{\mu\nu}(\tau_F,x) + 4c_2(\tau_F)\delta_{\mu\nu}E(\tau_F,x)$$
- **O**perator **P**roduct **E**xpansion of $G(\tau,\tau_\mathrm{F})$ in τ_F/τ^2.

Noise reduction technique: blocking fit





$$G(\tau) = \frac{a^3}{V} \sum_{v_1} \Big[\sum_{\vec{m} \in v_1} \mathcal{O}(\tau_1, \vec{m}) \Big] \sum_{v_2} \Big[\sum_{\vec{n} \in v_2} \mathcal{O}(\tau_2, \vec{n}) \Big]$$

3-7 SNR improvement: save computation cost.

L. Altenkort et al. PRD 105, 094505 (2022)

Lattice setup

- **Pure SU(3) Yang-Mills gauge theory:**

Nuclear Science Computing Center at CCNU

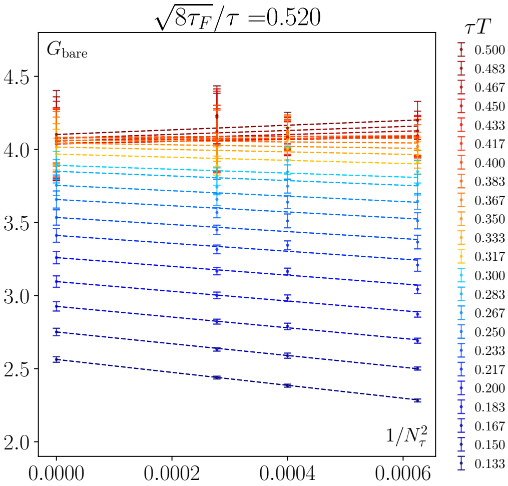
T/T_c	0.76			0.9			1.125			1.267			1.5			1.9			2.25		
N_σ	96	120	144	96	120	144	96	120	144	96	120	144	96	120	144	96	120	144	96	120	144
N_τ	40	50	60	40	50	60	32	40	48	24	30	36	24	30	36	16	20	24	16	20	24
#Conf.	5000			5000			5000			5000			5000			5000			5000		

- **Lattice spacing**:

β	7.0606	7.2005	7.2456	7.3874	7.3986	7.5416
a (fm)	0.02068	0.01746	0.01654	0.01397	0.01379	0.01164

H.-T. Ding, H.-T. Shu and CZ, work in progress

Continuum extrapolation



$a \to 0$ at $0.9T_c$ in the shear channel.

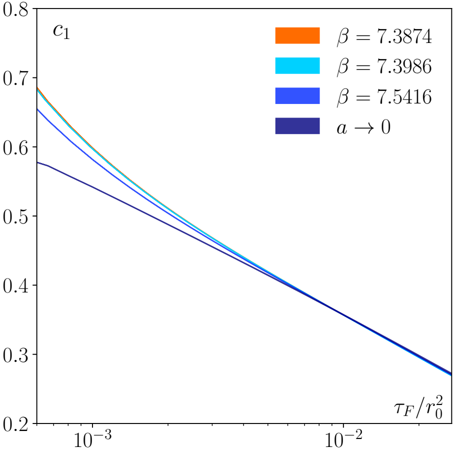
The joint fit Ansatz:

$$G_{\text{bare}}(N_\tau) = G_{\text{bare}}^{\tau T}(a=0) + \left(b + m_1 \cdot \tau T + \frac{m_2}{\tau T}\right)/N_\tau^2$$

$$G_{\text{bare}} = \frac{G^{\text{t.l.}}(\tau T, \tau_F)}{G^{\text{norm}}(\tau T)}$$

The Ansatz describes the data well.

Renormalization

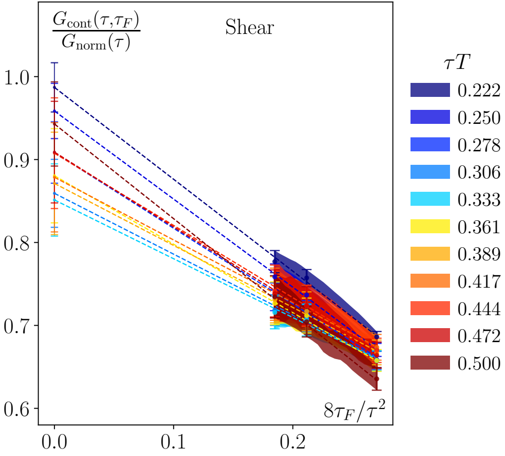


c_1 & c_2: renormalization constants matching Gradient Flow scheme to $\overline{\mathsf{MS}}$ scheme.

$$T_{\mu\nu}(\tau_F, x) = c_1(\tau_F)\, U_{\mu\nu}(\tau_F, x) + 4c_2(\tau_F)\, \delta_{\mu\nu} E(\tau_F, x)$$

$$c_1(\tau_F) = \frac{1}{g^2_{\overline{\mathsf{MS}}}(\mu)} \sum_{n=0}^{2} k_1^{(n)}(L(\mu, \tau_F)) \left[\frac{g^2_{\overline{\mathsf{MS}}}(\mu)}{(4\pi)^2}\right]^n$$

Flow time extrapolation



$\tau_F \to 0$ at $1.5\,T_c$.

$\tau_F \to 0$ extrapolation Ansatz:

$$G(\tau_F/\tau^2, \tau T) = G^{\tau T}_{\tau_F=0} + \left(b + m_1 \cdot \tau T + \frac{m_2}{\tau T}\right) \cdot \tau_F/\tau^2$$

Flow time window:

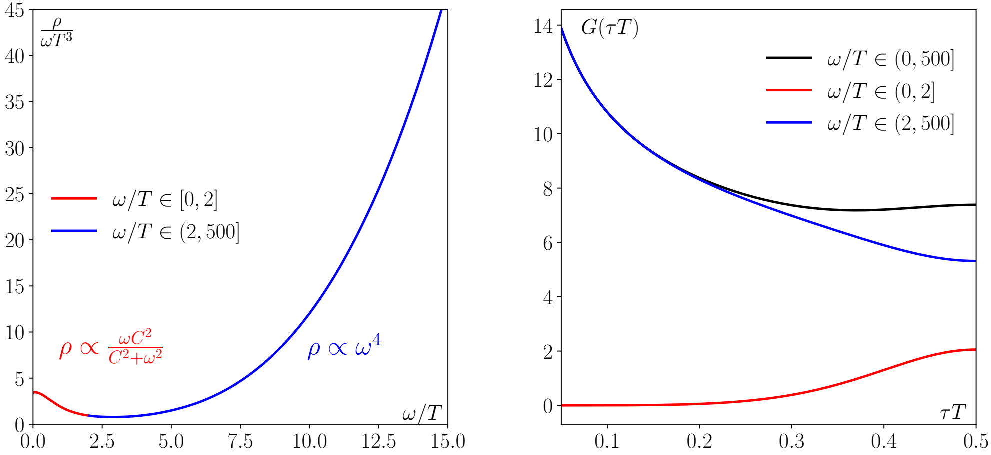
$$\sqrt{8\tau_F}/\tau \in [0.43,\ 0.52]$$

L. Altenkort et al. PRD 103, 114513 (2021)

Avoid over smearing:

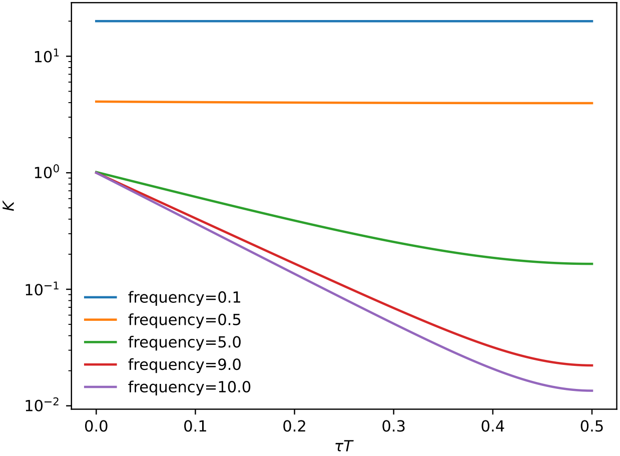
$$\sqrt{8\tau_F} \geq \sqrt{2}a$$

Illustration of sensitivity of correlators to the transport peak



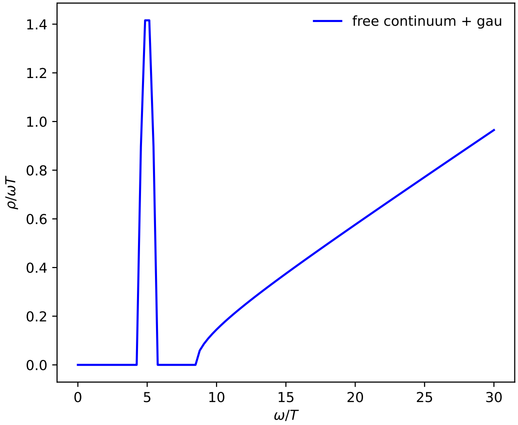
$G(\tau T)$ at $\tau T \sim 0.5$ are more sensitive to the transport peak.

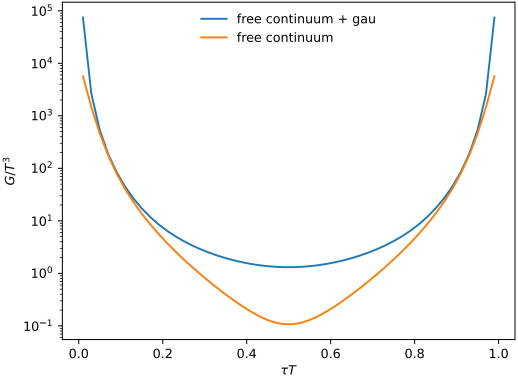
Role of the kernel function



Kernel function $\frac{\cosh\left(\frac{\omega}{T}(\tau T - 0.5)\right)}{\sinh(\omega/2T)}$ at different frequency.

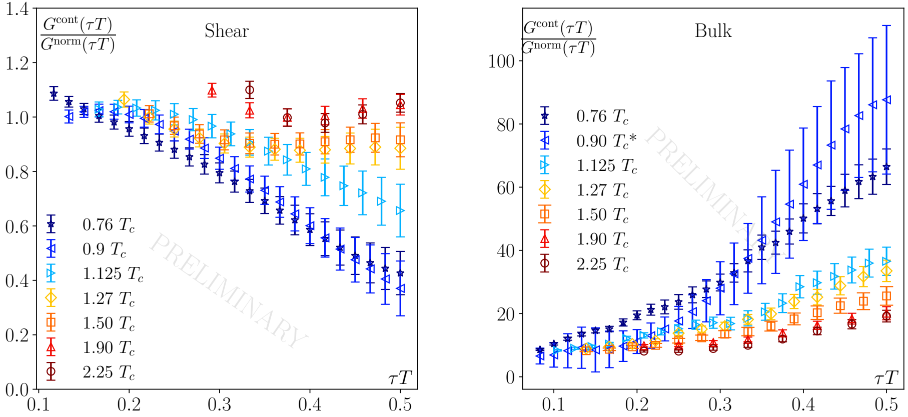
Illustration of the shape of the spectral function





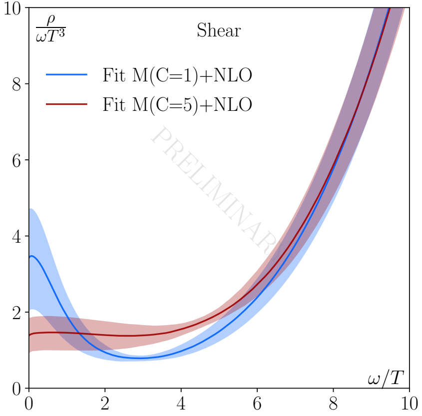
Upper bound of ω/T is 1300.

Normalized correlators in the continuum limit



Clear temperature dependencies for both channels.
Negative slope for low temperatures in the shear channel.

Reconstructed spectral functions in the shear channel at 1.5 T_c



Spectral function at 1.5 T_c.

$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \frac{\cosh[\omega(1/2T - \tau)]}{\sinh(\omega/2T)} \rho(\omega, T)$$

$$\frac{\rho(\omega)}{\omega T^3} = \frac{A}{T^3} \frac{C^2}{C^2 + (\omega/T)^2} + B \frac{\rho_{\text{pert}}(\omega)}{\omega T^3}$$

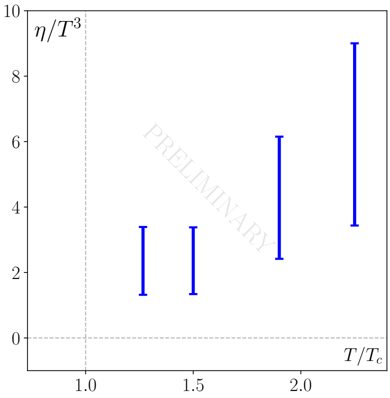
$$\rho_{\text{pert}}(\omega) \propto (\omega/T)^4$$

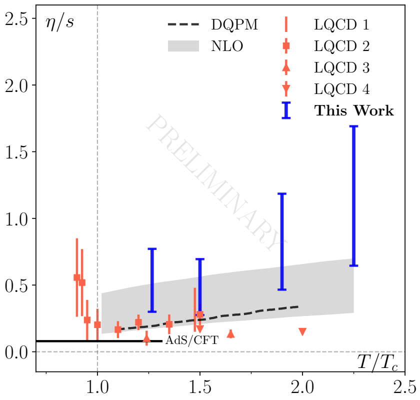
Y. Zhu et al. JHEP 03, 002 (2013) (shear)

M. Laine et al., JHEP 09, 084(2011 (bulk)

$C = 1$, sharp peak, long-lived excitation.
$C = 5$, broad peak, short-lived excitation.

Temperature dependencies of shear viscosity





DQPM: PRC 88, 045204 (2013).
NLO: JHEP 03, 179 (2018).
Gradient Flow:

LQCD1: PRD 108, 014503 (2023).

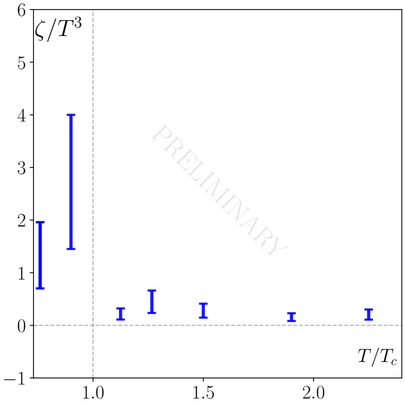
Multi-Level:
LQCD2: JHEP 04, 101 (2017).
LQCD3: PRD 76, 101701 (2007).

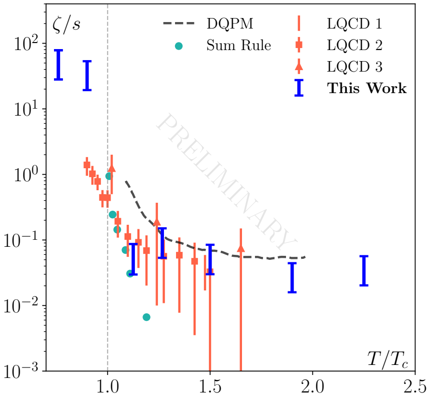
LQCD4: PRD 98, 014512 (2018).

Mild increase with temperature in η/s at $T \gtrsim 1.27\,T_c$.

η/s agrees with LQCD1 & NLO at $T \gtrsim 1.27\,T_c$.

Temperature dependencies of bulk viscosity





DQPM: PRC 88, 045204 (2013)
Sum Rule: JHEP 09, 093 (2008)
LQCD1 (GF): PRD 108, 014503 (2023). $a = 0.0117$ fm
LQCD2 (ML): PRD 98, 054515 (2018). $a \geq 0.0253$ fm
LQCD3 (ML): PRL 100, 162001 (2008). $a \geq 0.0475$ fm

Smaller values of ζ/s at $T > T_c$.

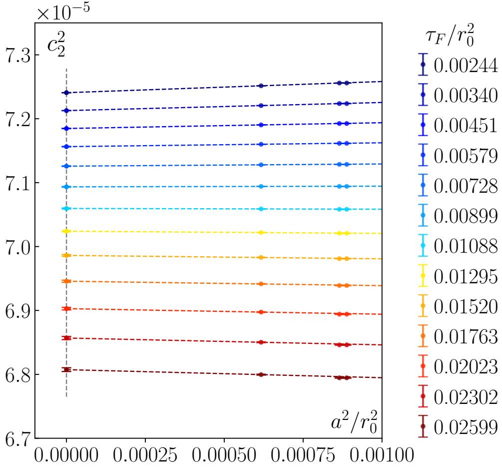
ζ/s agrees with LQCD1 & LQCD2 at $T > T_c$.

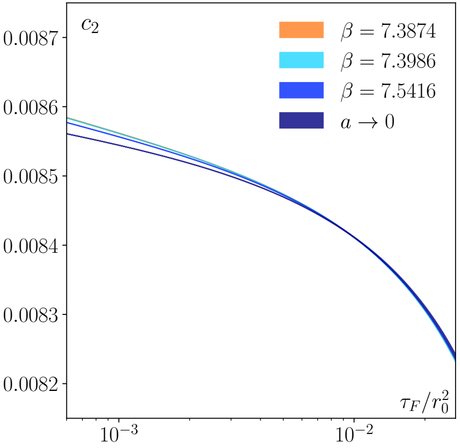
Summary

- Large and fine lattices are generated to extract the viscosities.
- High-precision EMT correlators are obtained via the gradient flow and blocking method.
- Temperature dependencies of η/s and ζ/s are investigated in SU(3) across the phase transition region.
- η/s increases mildly with temperature at $T \gtrsim 1.27T_c$.
- $\zeta/s(T < T_c) \gg \zeta/s(T > T_c)$, and ζ/s is most flat at $T \gtrsim 1.13T_c$.

- η/s at $T < 1.27T_c$ is progressing.
- The full QCD investigation (including dynamical quarks) is progressing.

Back up: renormalization

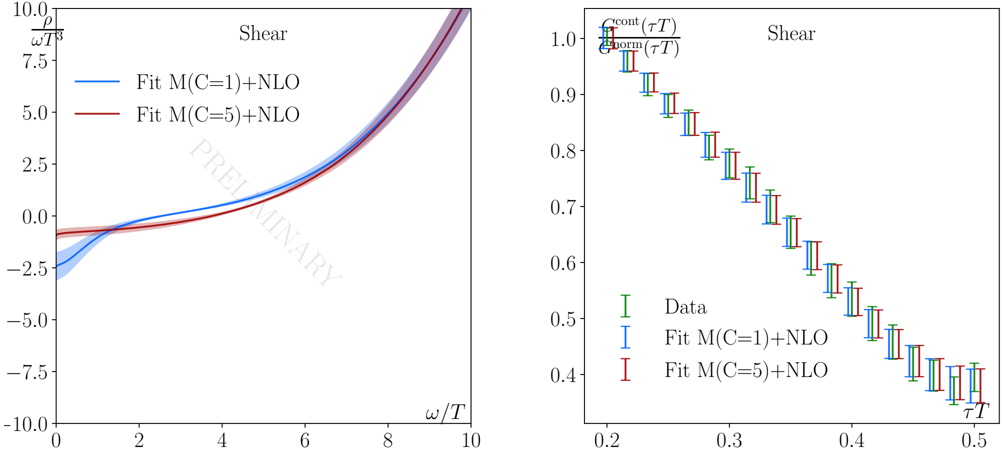




$$c_2^2(a^2/r_0^2) = c_2(a=0) + b\,a^2/r_0^2.$$

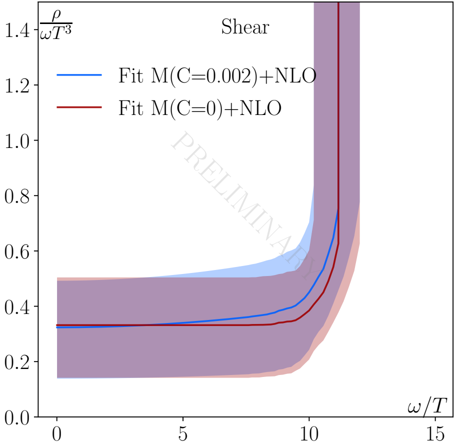
Lattice spacing effects are small.

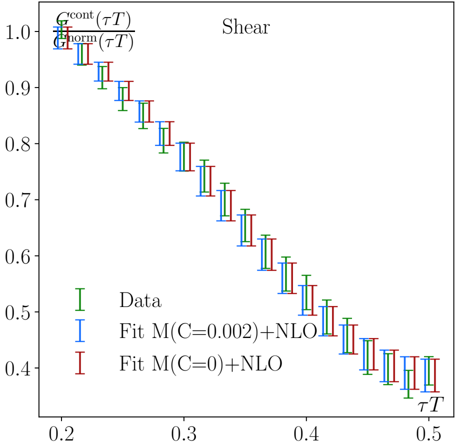
Spectral fit at 0.76Tc



Shear spectral function at 0.76Tc with model: x

Spectral fit at 0.76Tc

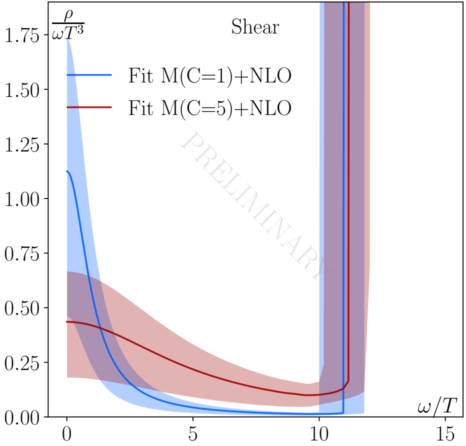


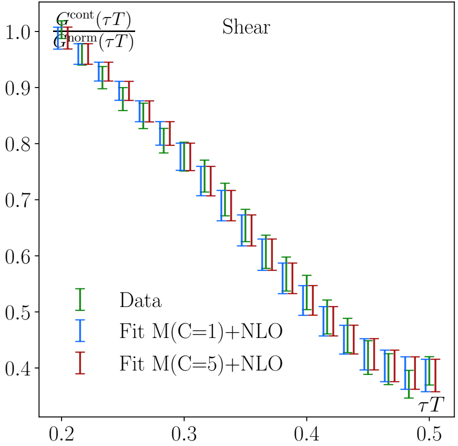


Shear spectral function at 0.76Tc with model[JHEP 04, 101 (2017)]:

$$\rho(\omega) = BT^3\omega\left(1 + C\omega^2\right)\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0).$$

Spectral fit at 0.76Tc





Shear spectral function at 0.76Tc with the transport peak model at small frequency.

Outlook: the possible shape of the spectral function below Tc