



Resolving the flow puzzle in relativistic heavy ion collisions with detailed nuclear structure

HAO-JIE XU (徐浩浩)

HUZHOU UNIVERSITY(湖州师范学院)

Precision Frontier of QCD Matter: Inference and Uncertainty Quantification

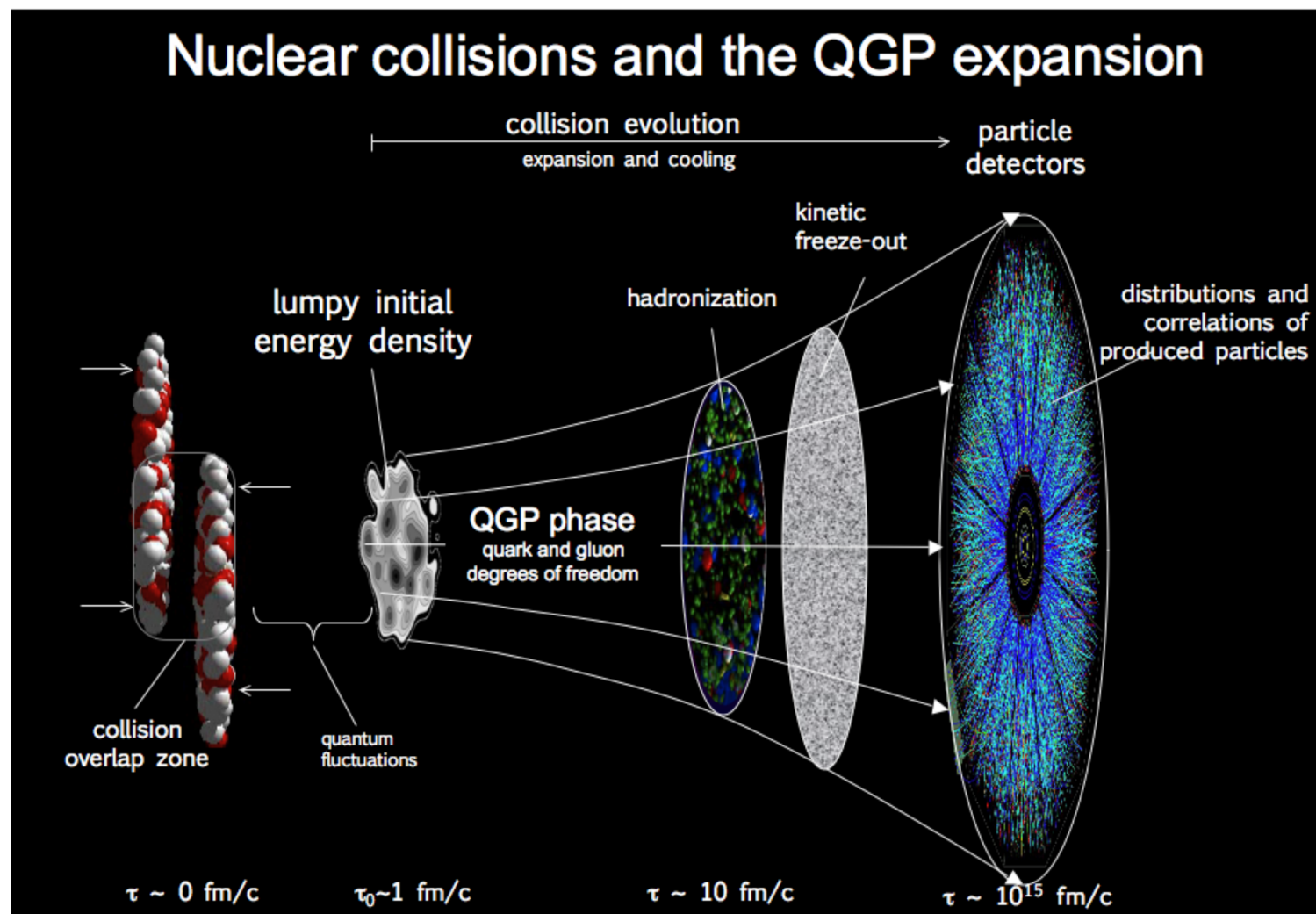
2025.9.1-12, CCNU, Wuhan



Relativistic heavy ion collisions

The
“Little
Bang”

$$\sqrt{s} = 100\text{GeV} \sim \text{TeV}$$

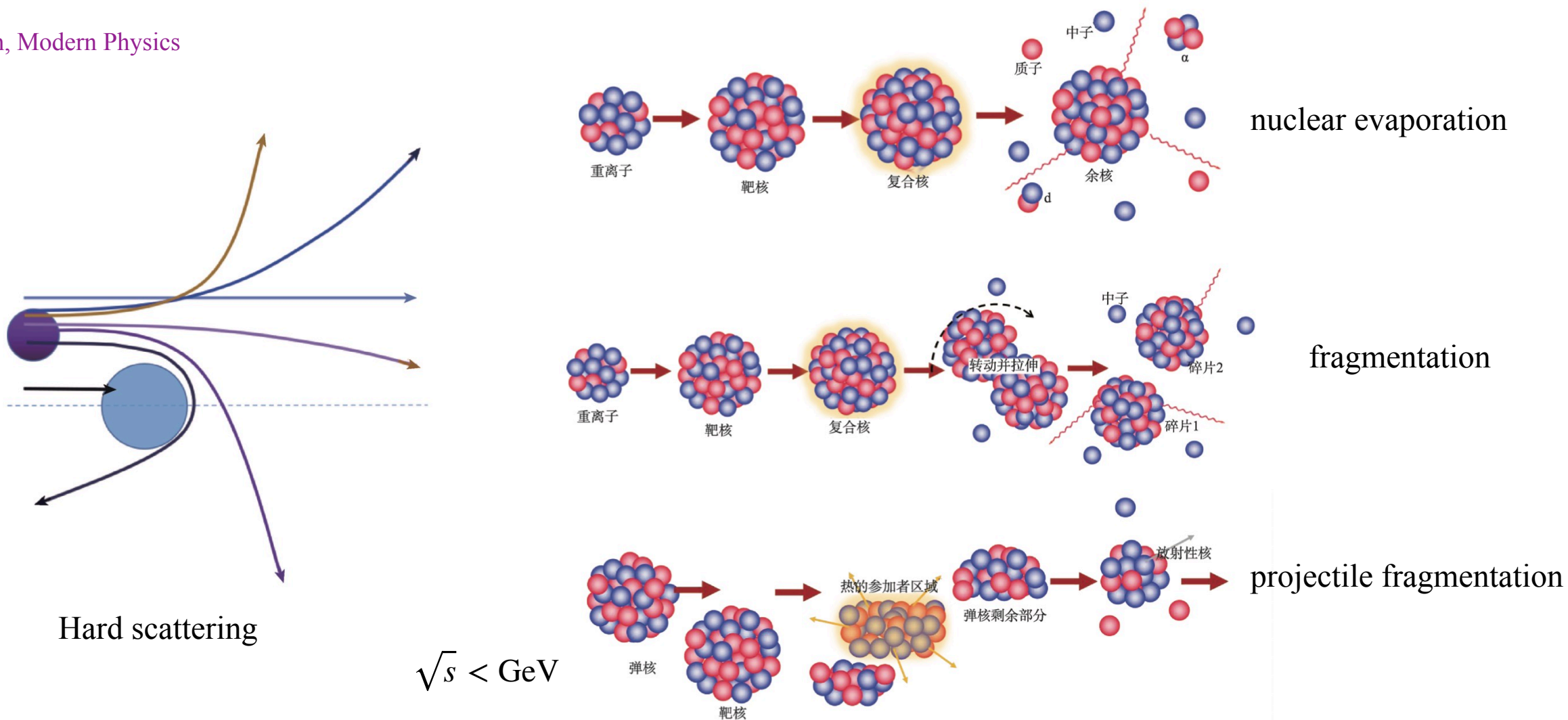


Yoctosecond (10^{-24} s) 幺秒



Nucleus-Nucleus Reactions (Collisions)

G. Jin, Modern Physics

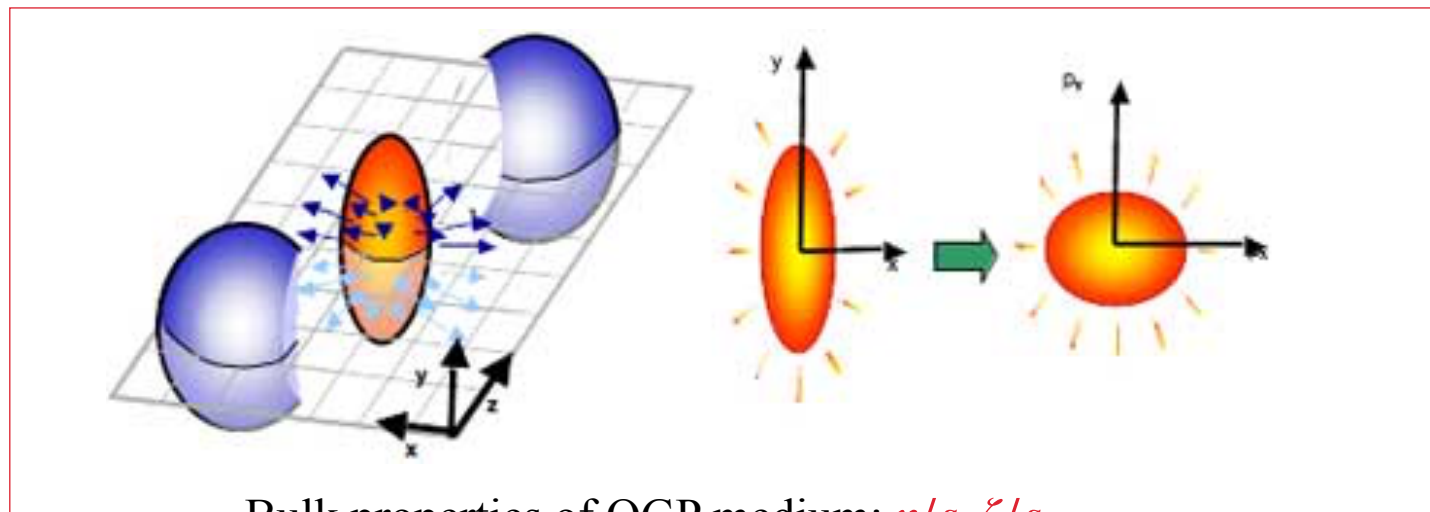




Relativistic Heavy ion collisions and nuclear structure

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - R)/a]}$$

$$R = R_0 [1 + \beta_2 Y_2^0(\theta) + \beta_4 Y_4^0(\theta)]$$

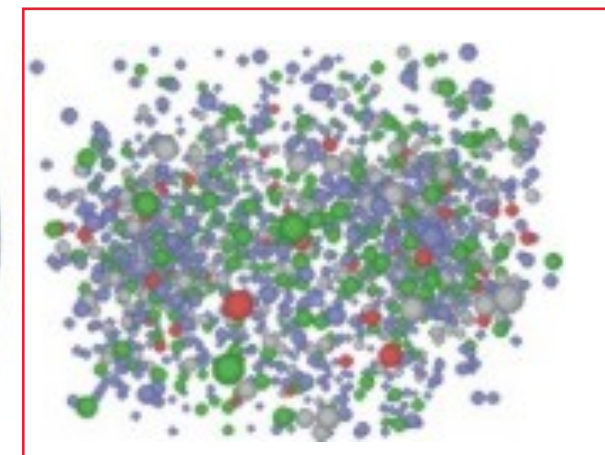
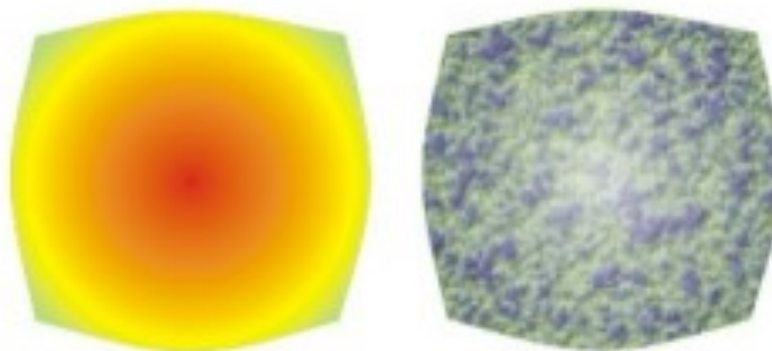
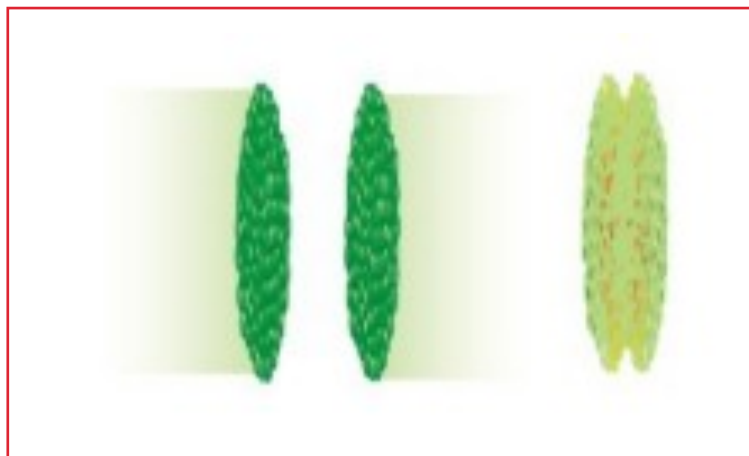


Initial geometry

Bulk properties of QGP medium: $\eta/s, \zeta/s, \dots$

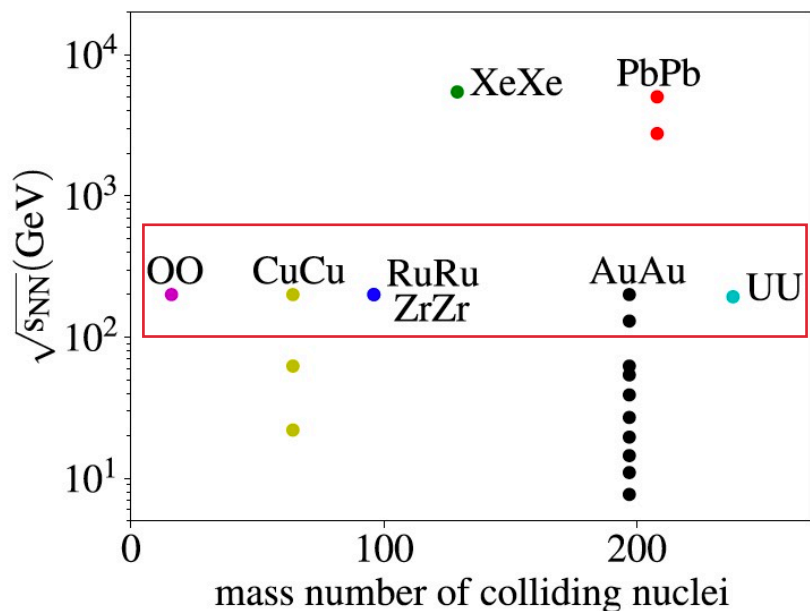
Final observables

Anisotropic flow,
Flow fluctuations
HBT,
....



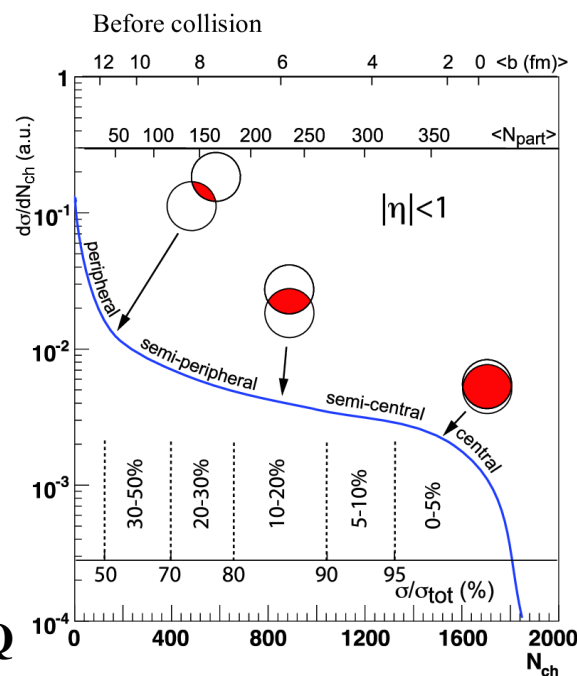
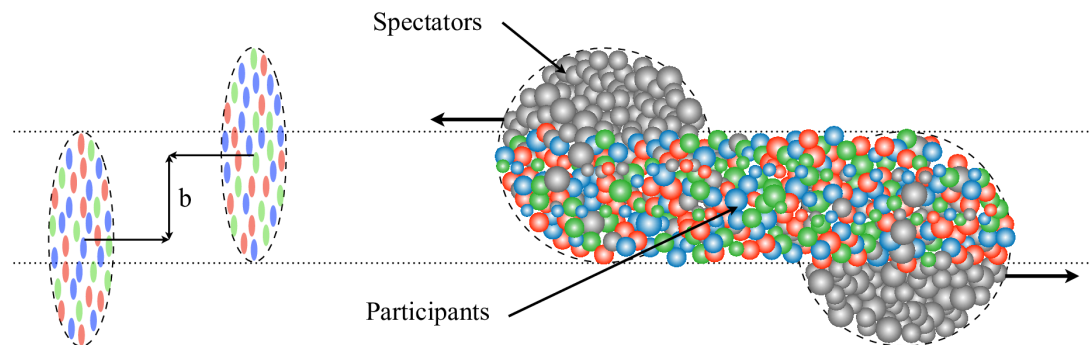


System size dependence and centrality dependence



$$R \sim 1.2A^{1/3}$$

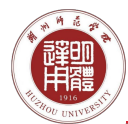
System size dependence



After collision

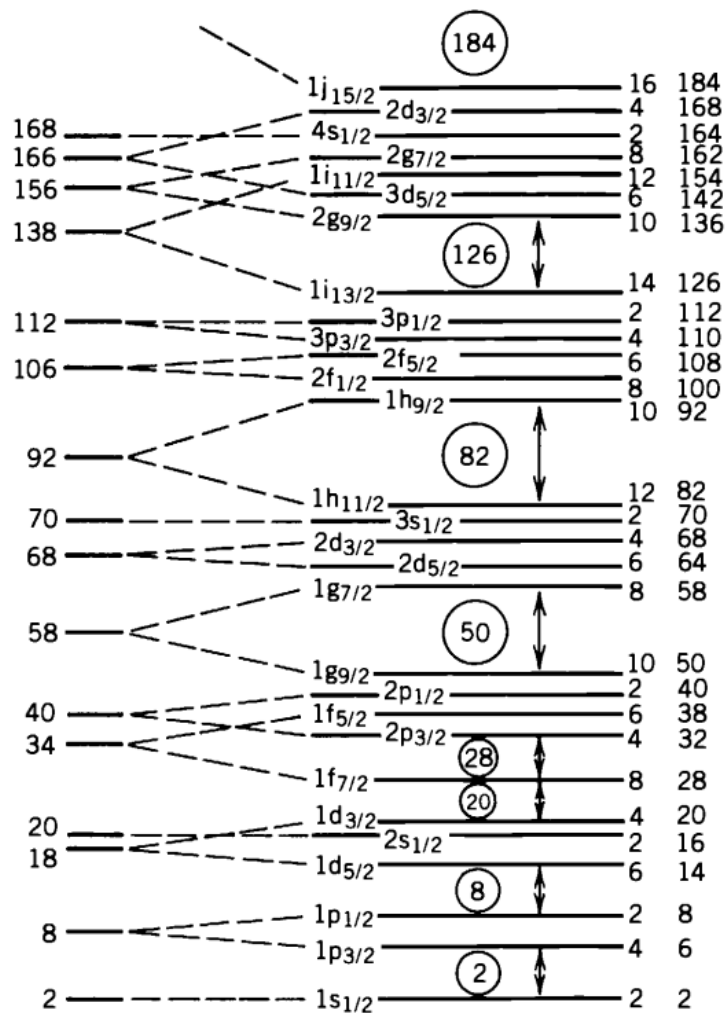
Centrality dependence

b=0 most central collisions

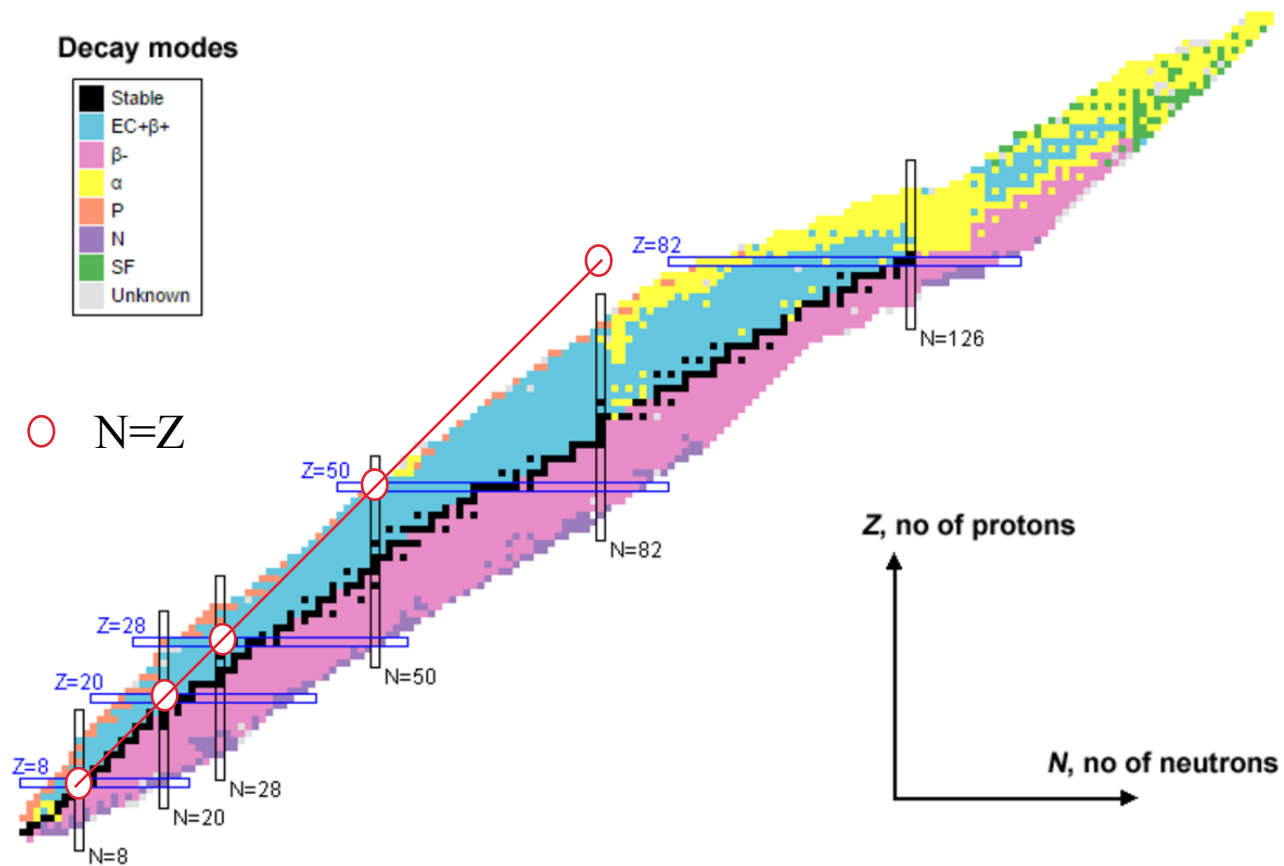


Nuclear structure beyond spherical

Nuclear deformation



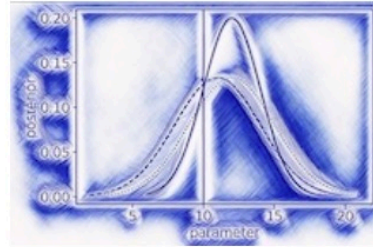
Neutron skin thickness





Intersection of nuclear structure and high-energy nuclear collisions

Chunjian Zhang@ 9:30-10:15, Sep 8



High-energy experimental imaging of nuclear shapes for precise constraints on QGP initial conditions

Chunjian Zhang

Fudan University

September 8, 2025, Wuhan

Precision Frontier of QCD Matter: Inference and Uncertainty Quantification

In this talk, I will discuss some of our own issues in HIC that could be resolved by nuclear structure:

*CME background in isobar collisions * Elliptic flow puzzle in U+U collisions * v_2 - v_3 puzzle in Pb+Pb Collisions

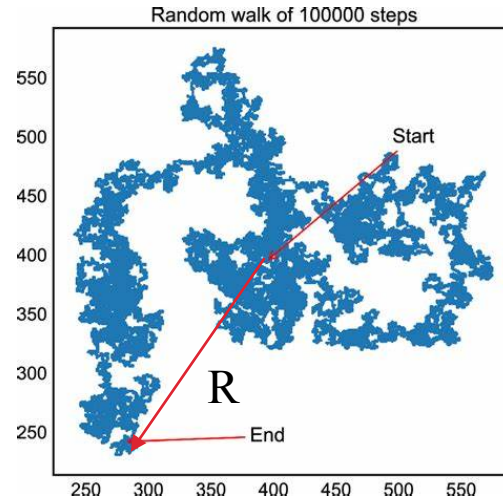


Anisotropic flow in HIC

Q-cumulant method

$$v_n\{2\}^2 = \left\langle \frac{Q_n^2 - M}{M(M-1)} \right\rangle$$

$$Q_n = \sum_{i=1}^M e^{in\varphi_i}$$



Random walk for M steps, the distance between the start point and end point is:

$$R = \sqrt{M}$$

$v_n\{2\} \neq 0 \rightarrow$ nontrivial correlations

- Heavy Ion Collisions boost invariance
2D Fourier series in momentum space

$$\frac{dN}{d\phi} \propto 1 + \sum_n v_n \cos n(\phi - \Phi_n)$$

- Nuclear structure
3D spherical harmonics in coordinate space

$$R(\theta, \phi) = R_0 \left(1 + \sum \beta_{lm} Y_{lm} \right)$$

Collectivity

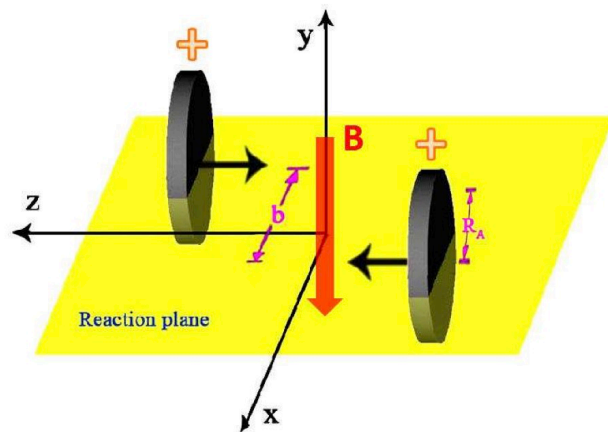
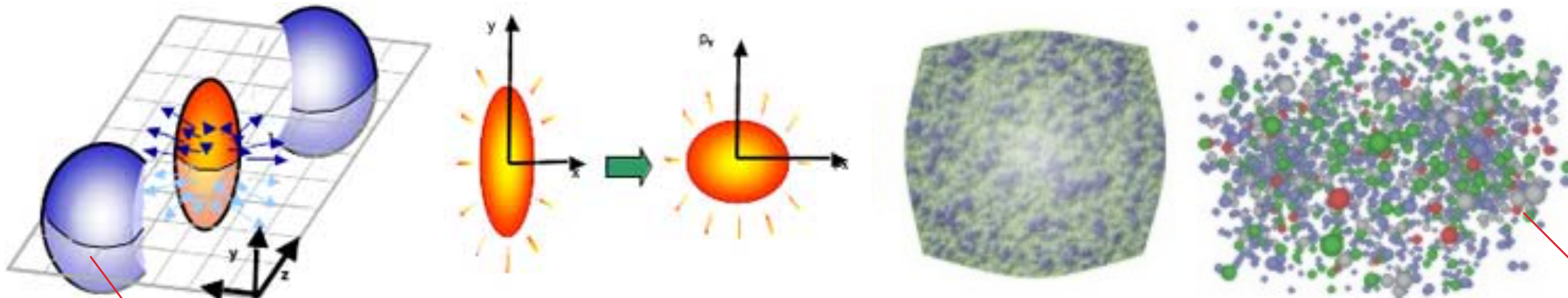
$$\begin{aligned} \frac{dN}{d\phi} \propto & [1 + \underline{v_2} \cos 2(\phi - \Psi_2) \quad \text{probe } \beta_2 \quad \checkmark \\ & + \underline{v_3} \cos 3(\phi - \Psi_3) \quad \text{probe } \beta_3 \quad \checkmark \\ & + \underline{v_4} \cos 4(\phi - \Psi_3) \\ & + \dots] \end{aligned}$$

CME background in isobar collisions (2017-)

HJX, X. Wang, H. Li, J. Zhao, Z. Lin, C. Shen, F. Wang, PRL121, 022301 (2018)
H. Li, **HJX**, J. Zhao, Z. Lin, H. Zhang, X. Wang, C. Shen, F. Wang, PRC98, 054907 (2018)
H. Li, **HJX**, Y. Zhou, X. Wang, J. Zhao, L. Chen, F. Wang, PRL125, 222301(2020)
HJX, H. Li, X. Wang, C. Shen, F. Wang, PLB819, 136453 (2021)
STAR Collaboration, PRC105, 014901(2022)

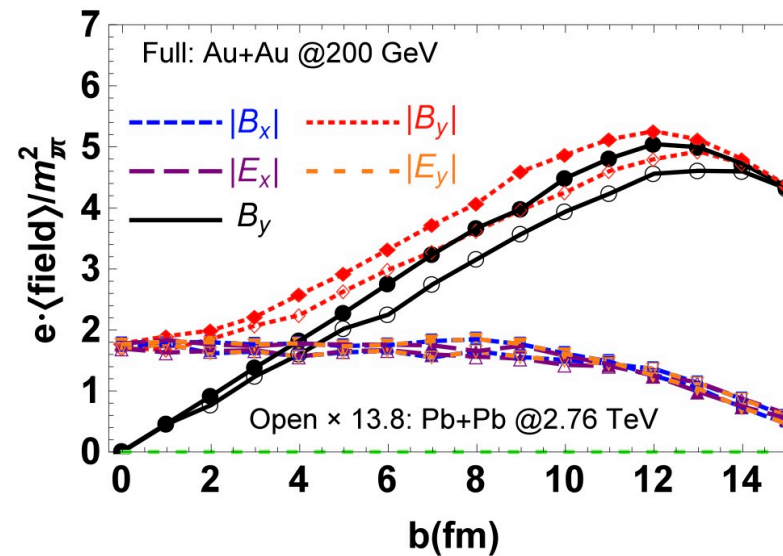
Relativistic heavy ion collisions

Participants



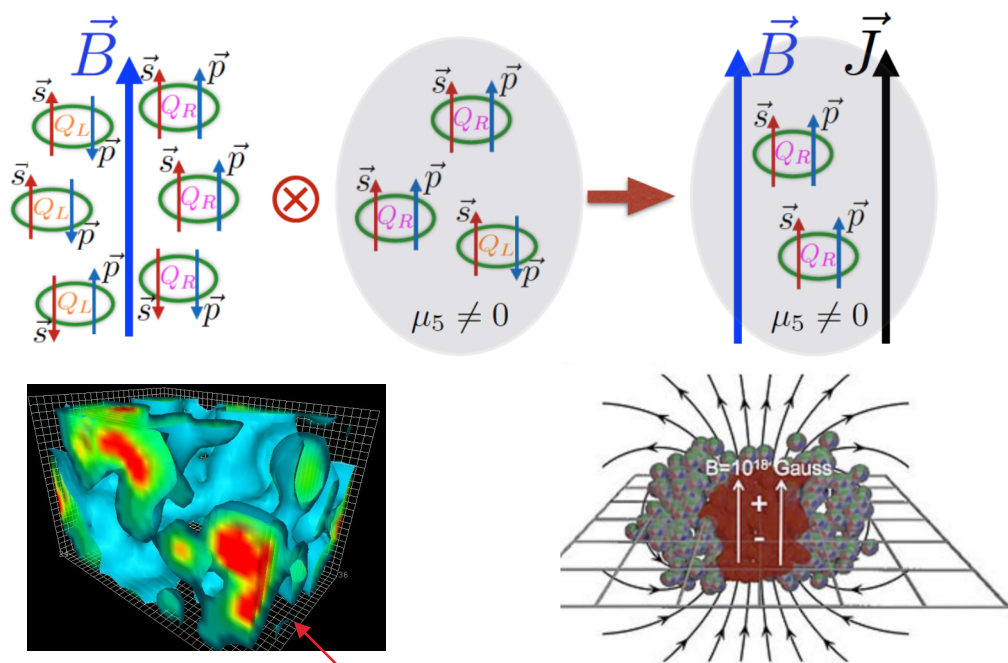
Spectators

Chiral Magnetic Effect



Chiral magnetic effect

Chiral magnetic effect (CME)

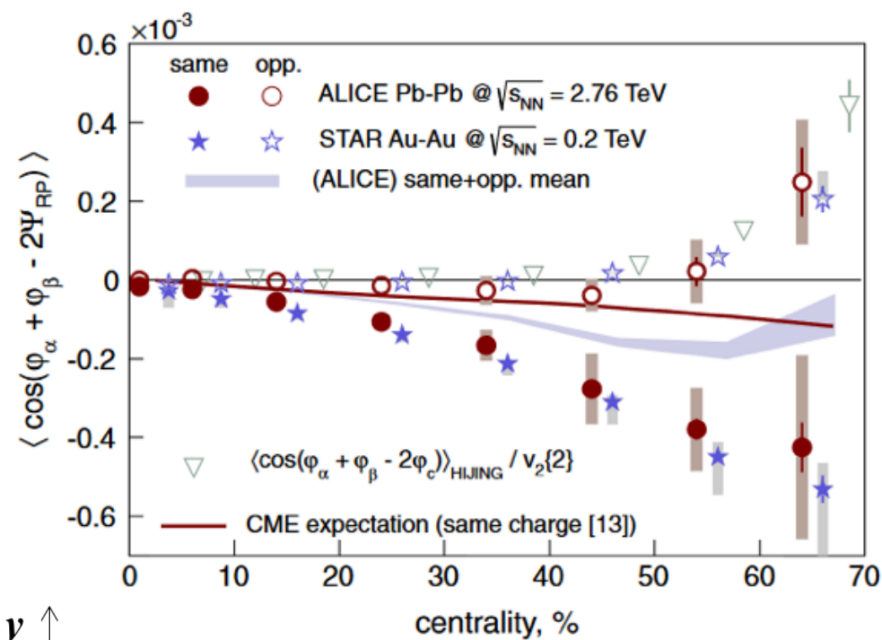


$$\mathbf{J}_{\text{cme}} = \sigma_5 \mathbf{B} = \left(\frac{(Qe)^2}{2\pi^2} \mu_5 \right) \mathbf{B},$$

D. Kharzeev, et al., PPNP88, 1(2016)

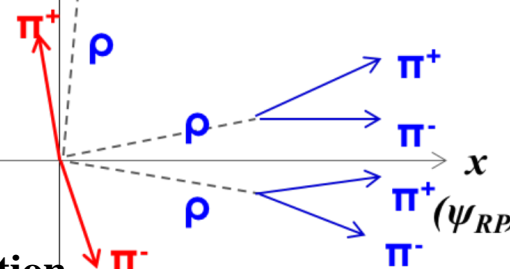
CME signal vs background

STAR, PRL103, 251601 (2009)
ALICE, PRL110, 012301 (2013)



y
(B)

$$\Delta\gamma_{\text{Bkg}} \simeq \langle \cos(\varphi_a + \varphi_b - 2\Psi_{RP}) \rangle v_{2,\text{clust}}$$



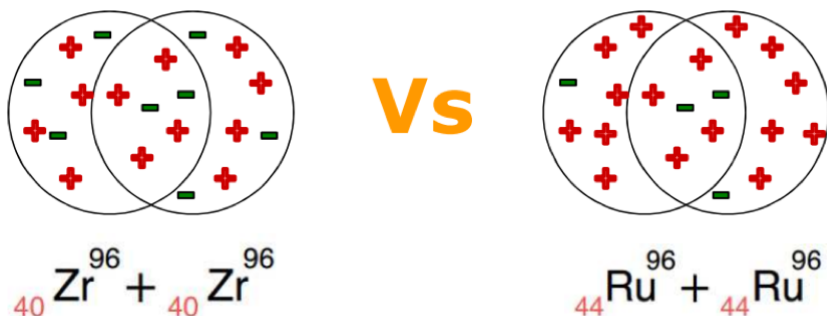
Schlichting, PRC83(2011)
Bzdak, PRC81(2010)
Wang, PRC81(2010)



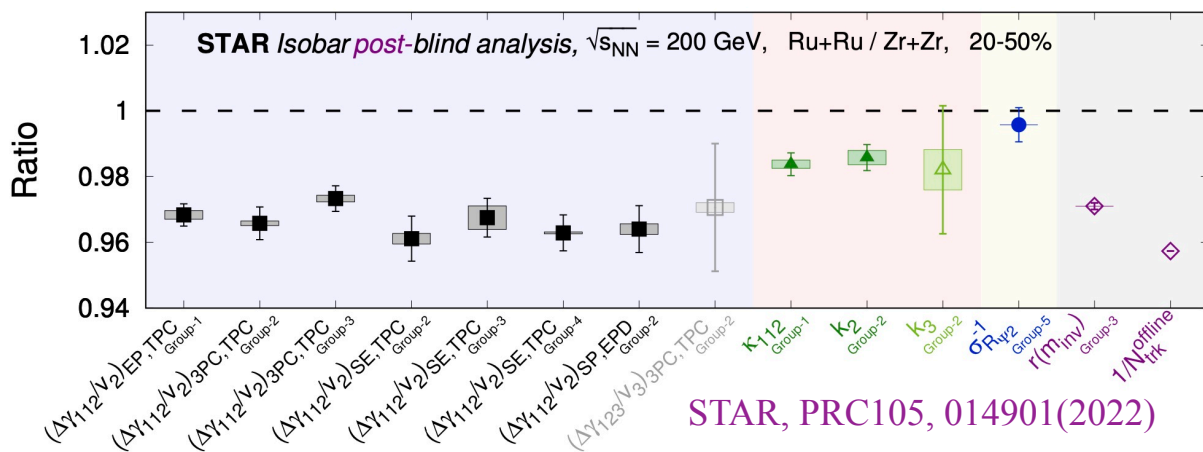
Relativistic isobaric collisions

The isobar collisions were proposed to measure the chiral magnetic effect.

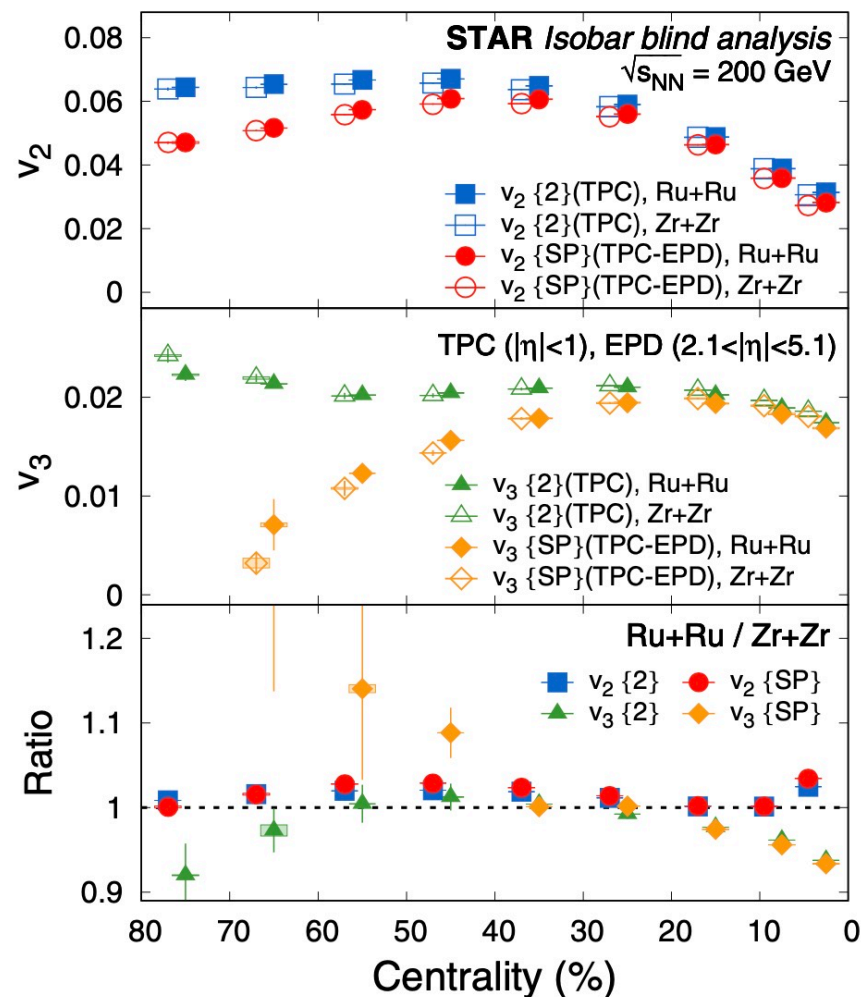
S. Voloshin, PRL105, 172301 (2010)



- Same background
- Different magnetic field => different CME signals



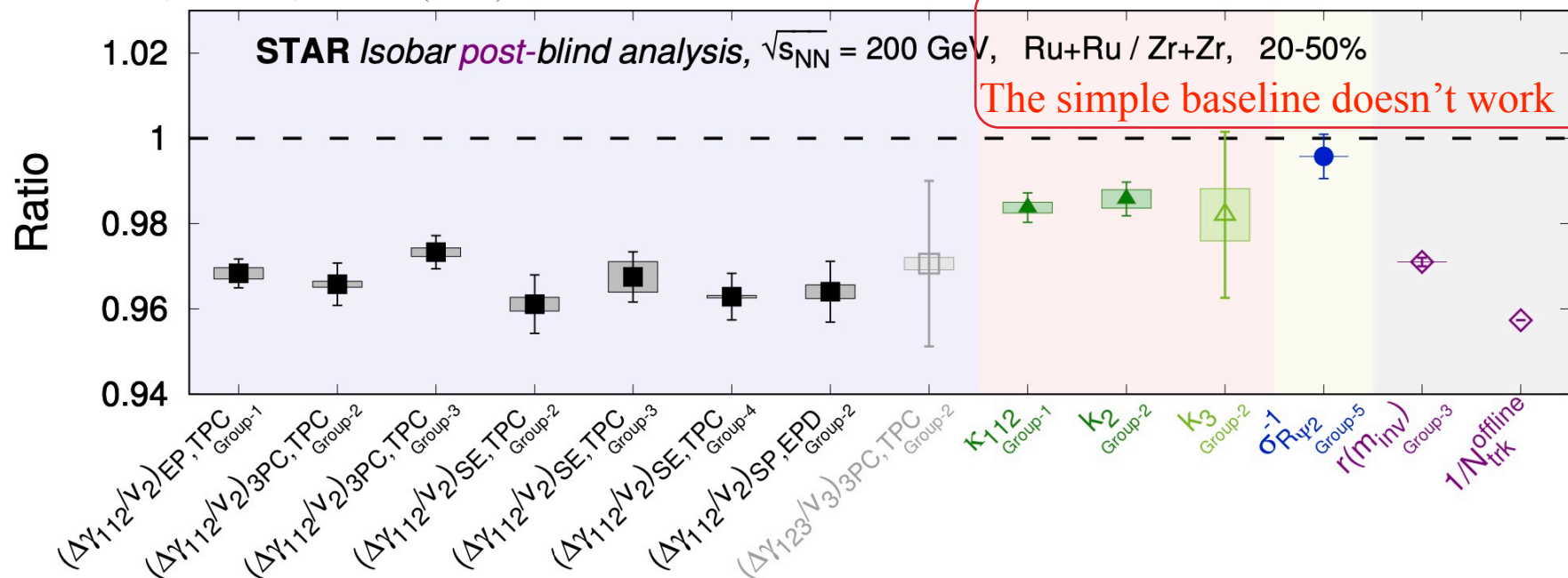
Backgrounds are not identical!!!





Isobar structures are important for the CME search

STAR Collaboration, PRC105, 014901 (2022)



$$\Delta\gamma_{\text{bkg}} = \langle \cos(\varphi_\alpha + \varphi_\beta - 2\Psi_{RP}) \rangle = \frac{N_{\text{cluster}}}{N_\alpha N_\beta} \times \langle \cos(\varphi_\alpha + \varphi_\beta - 2\Psi_{\text{cluster}}) \times v_{2,\text{cluster}} \rangle$$

Multiplicity differences

Flow differences

The **multiplicity and v_2 differences** from isobar structure are crucial for the CME search in the isobar collisions at RHIC



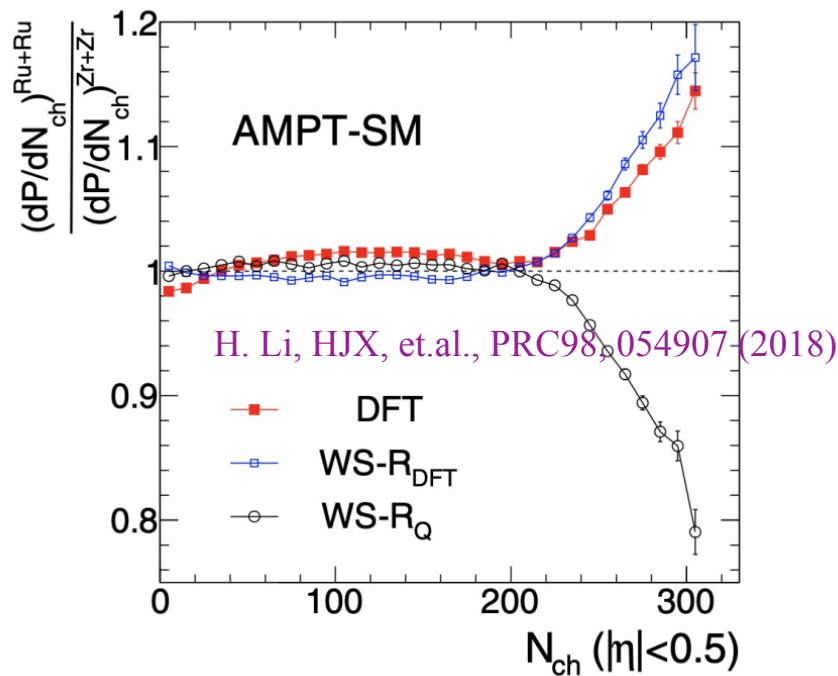
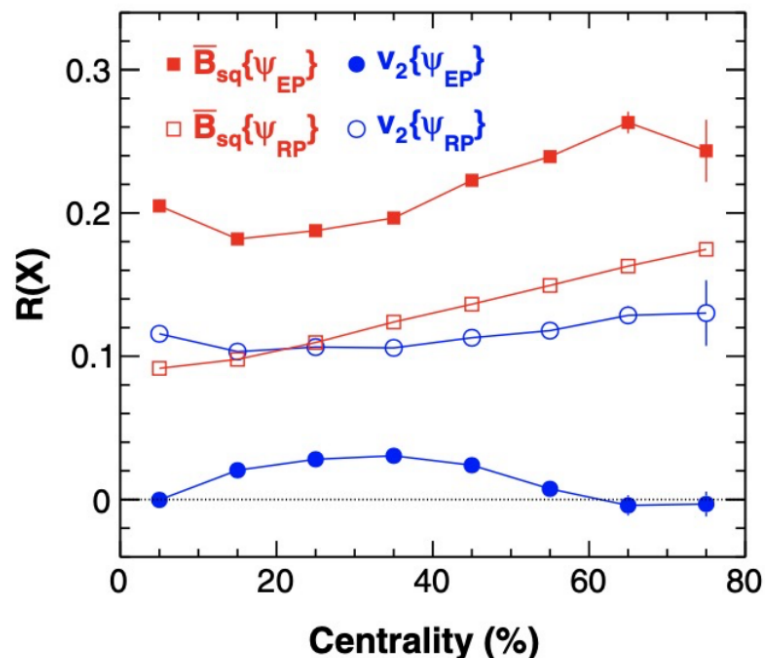
Nuclear densities for HIC models

PHYSICAL REVIEW LETTERS **121**, 022301 (2018)

Instead of WS densities, we use the nuclear densities obtained from density functional theory calculations

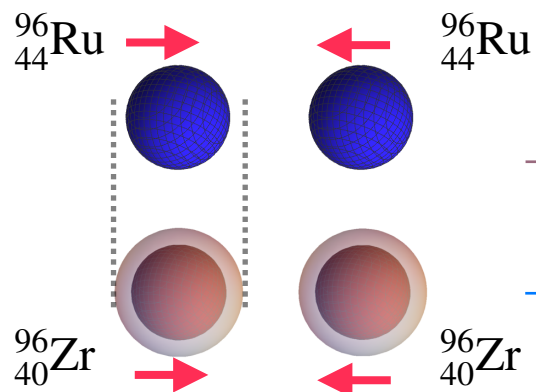
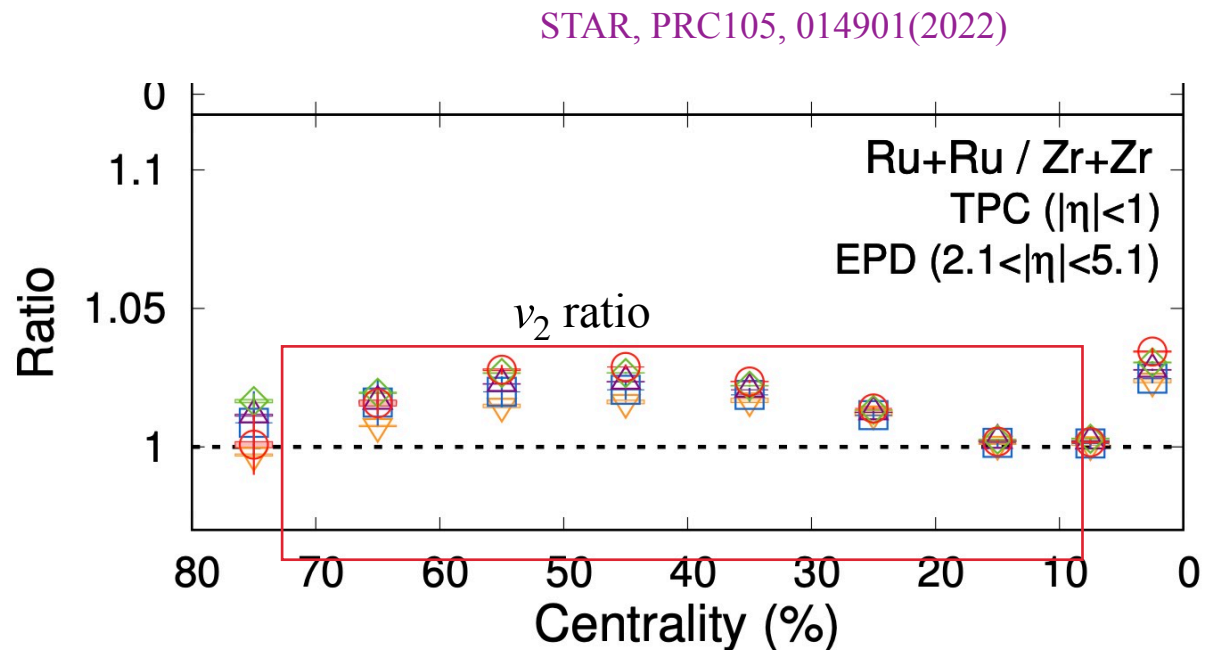
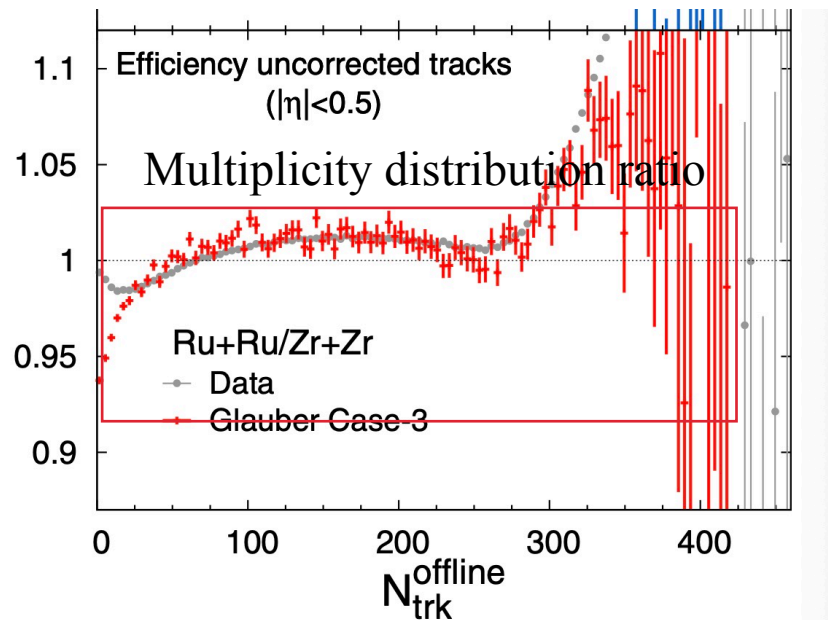
Importance of Isobar Density Distributions on the Chiral Magnetic Effect Search

Hao-jie Xu,¹ Xiaobao Wang,¹ Hanlin Li,² Jie Zhao,³ Zi-Wei Lin,^{4,5} Caiwan Shen,¹ and Fuciang Wang^{1,3,*}





DFT predictions are verified by STAR data



Neutron skin thickness

$$\Delta r_{np} \equiv \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$$

Smaller r , larger density

Larger N_{ch} and $\langle p_T \rangle$

Larger r , smaller density

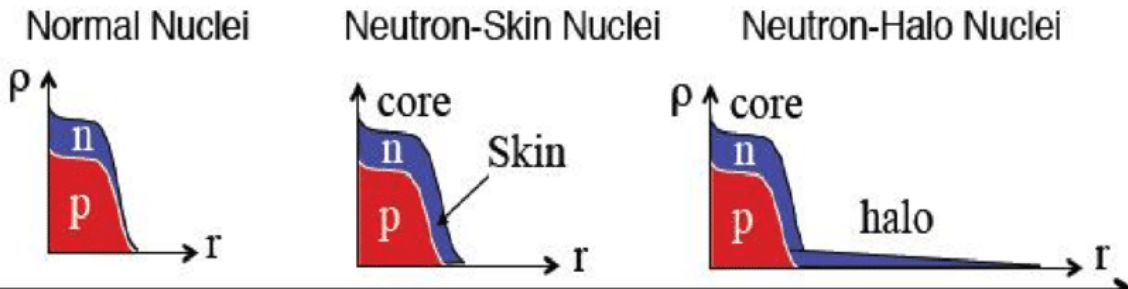
Smaller N_{ch} and $\langle p_T \rangle$



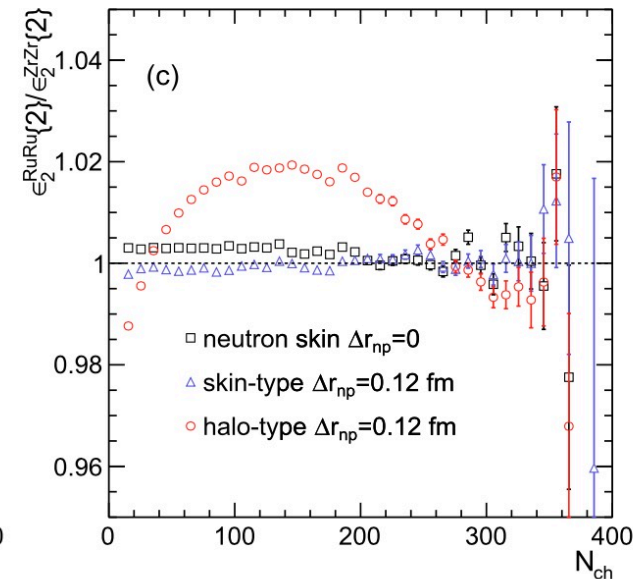
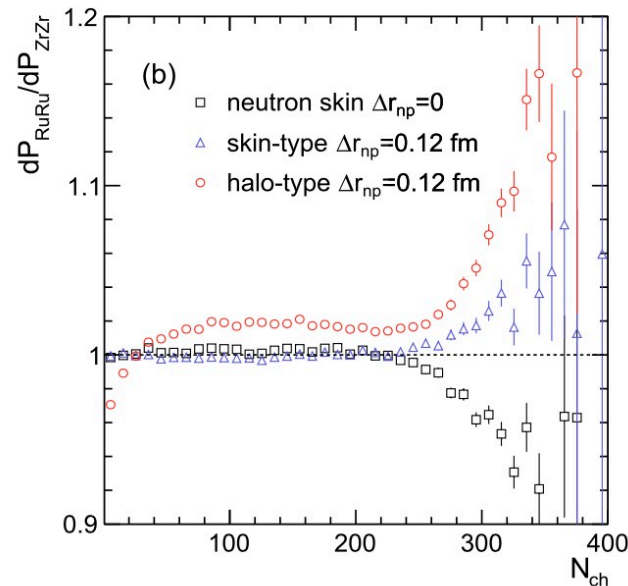
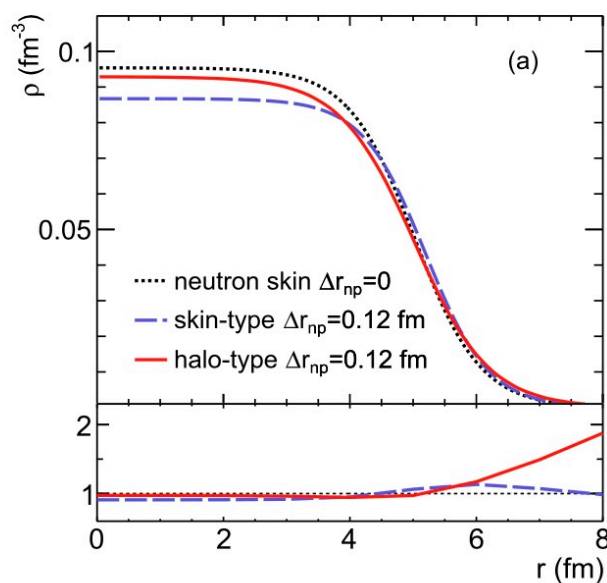
Determine the neutron skin type by STAR data

HJX, et.al., PLB819, 136453 (2021)

● Neutron-skin nuclei and neutron-halo nuclei for Zr



	⁹⁶ Ru		⁹⁶ Zr	
	<i>R</i>	<i>a</i>	<i>R</i>	<i>a</i>
p	5.085	0.523	5.021	0.523
skin-type n	5.085	0.523	5.194	0.523
halo-type n	5.085	0.523	5.021	0.592



The shapes of the Ru+Ru/Zr+Zr ratios of the multiplicity and eccentricity in mid-central collisions can further distinguish between skin-type and halo-type neutron densities.



Probing the neutron skin thickness

PHYSICAL REVIEW LETTERS **125**, 222301 (2020)

Observables sensitive to neutron skin thickness

Probing the Neutron Skin with Ultrarelativistic Isobaric Collisions

Hanlin Li¹, Hao-jie Xu^{2,*}, Ying Zhou,³ Xiaobao Wang,² Jie Zhao,⁴ Lie-Wen Chen,^{3,†} and Fuqiang Wang^{2,4,‡}

More references:

- **HJX**, H. Li, X. Wang, C. Shen, F. Wang, PLB819, 136453 (2021), arXiv:2103.05595
- **HJX**, H. Li, Y. Zhou, X. Wang, J. Zhao, L. Chen, F. Wang, PRC105, L014901 (2022), arXiv:2105.04052
- **HJX**, W. Zhao, H. Li, Y. Zhou, L. Chen, F. Wang, PRC108, L011902 (2023), arXiv:2111.14812
- S. Zhao, **HJX**, Y. Liu, H. Song, PLB840, 137838 (2023), arXiv:2204.02387
- S. Lin, R. Wang, J. Wang, **HJX**, S. Pu, Q. Wang, PRD107, 054004 (2023), arXiv:2210.05106
- J. Wang, **HJX**, F. Wang, Nucl. Sci. Tech. 35, 108(2024), arXiv:2305.17114
- S. Lin, J. Hu, **HJX**, S. Pu, Q. Wang, PRD111, 0774020 (2025) arXiv:2405.16491

Elliptic flow puzzle in U+U collisions (2015-)

W. Ryssens, G. Giacalone, B. Schenke, C. Shen, PRL130, 212302(2023)

HJX, J. Zhao, F. Wang, PRL132, 262301 (2024)



Nuclear deformation

PHYSICAL REVIEW C, VOLUME 61, 021903(R)

Uranium on uranium collisions at relativistic energies

Bao-An Li*

Department of Chemistry and Physics, Arkansas State University, P.O. Box 419, Jonesboro, Arkansas 72467-0419

(Received 12 October 1999; published 12 January 2000)

PHYSICAL REVIEW C, VOLUME 61, 034905

High energy collisions of strongly deformed nuclei: An old idea with a new twist

E. V. Shuryak

Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, New York 11794

(Received 14 July 1999; published 22 February 2000)

PRL **94**, 132301 (2005)

PHYSICAL REVIEW LETTERS

week ending
8 APRIL 2005

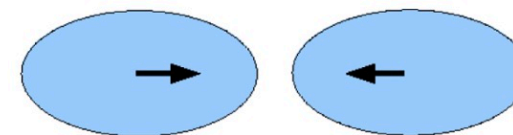
Anisotropic Flow and Jet Quenching in Ultrarelativistic U+U Collisions

Ulrich Heinz and Anthony Kuhlman

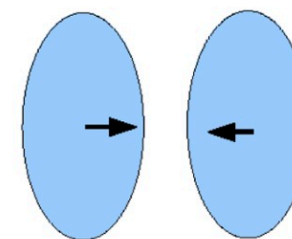
Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA

(Received 16 November 2004; published 6 April 2005)

S. Voloshin, PRL95, 122301 (2010)



(a)



(b)

H. Masui, B. Mohanty, N. Xu, PLB679, 440(2009)

G. Giacalone, PRC99, 024910 (2019)

G. Giacalone, J. Jia, C. Zhang, PRL127, 242301(2021)

J. Jia, PRC105, 014905 (2022)

B. Bally, et.al, PRL128, 082301(2022)

H. Mantysaari, et.al, PRL131, 062301(2023)

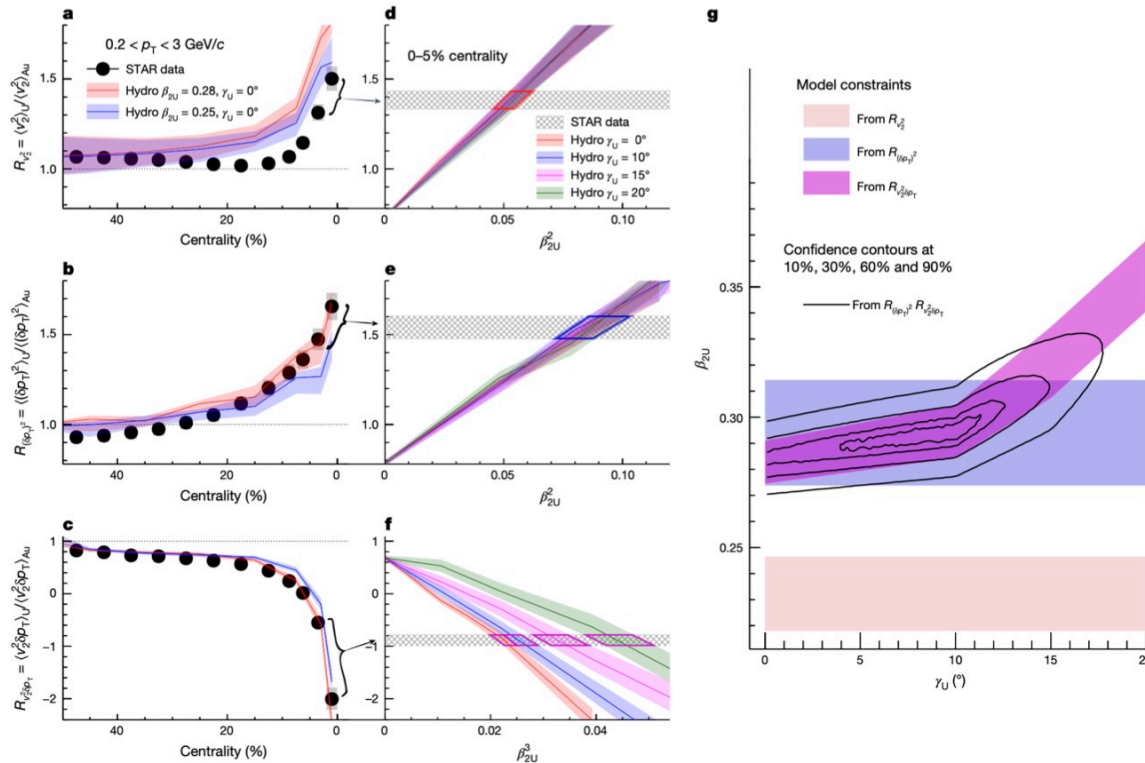
.....



Deformation parameters

Imaging shape of the ground-state ^{238}U : β_2 and γ

C. Zhang QM25



Sufficient precision is achieved from ratios in ultra-central collisions

Relation confirmed from hydro

$$\begin{aligned} \langle v_2^2 \rangle &= a_1 + b_1 \beta_2^2 \\ \langle (\delta p_T)^2 \rangle &= a_2 + b_2 \beta_2^2 \\ \langle v_2^2 \delta p_T \rangle &= a_3 - b_3 \beta_2^3 \cos(3\gamma) \end{aligned}$$

Constraints on β_2 and γ of ^{238}U simultaneously with data-hydro-comparison

$$\beta_{2U} = 0.297 \pm 0.015$$

$$\gamma_U = 8.5^\circ \pm 4.8^\circ$$

STAR, Nature 635, 67-72 (2024)
<https://www.nature.com/articles/s41586-024-08097-2>

A large deformation with a slight deviation from axial symmetry in the nuclear ground-state



April 6-12, 2025, Quark Matter, Frankfurt, Germany

Chunjian Zhang (Fudan University)

11

$R(v_2^2)$ is not used for the above β_2 extractions

$$R(v_2^2) \text{ --- } \beta_2 = 0.234 \pm 0.014$$



Hexadecapole deformation

PHYSICAL REVIEW LETTERS **130**, 212302 (2023)

$$\beta_2^{\text{WS}} \neq \beta_2^*$$

Evidence of Hexadecapole Deformation in Uranium-238 at the Relativistic Heavy Ion Collider

Wouter Ryssens^{1,*}, Giuliano Giacalone², Björn Schenke³, and Chun Shen^{4,5}

¹*Institut d'Astronomie et d'Astrophysique, Université Libre de Bruxelles, Campus de la Plaine CP 226, 1050 Brussels, Belgium*

²*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

³*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

⁴*Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48201, USA*

⁵*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

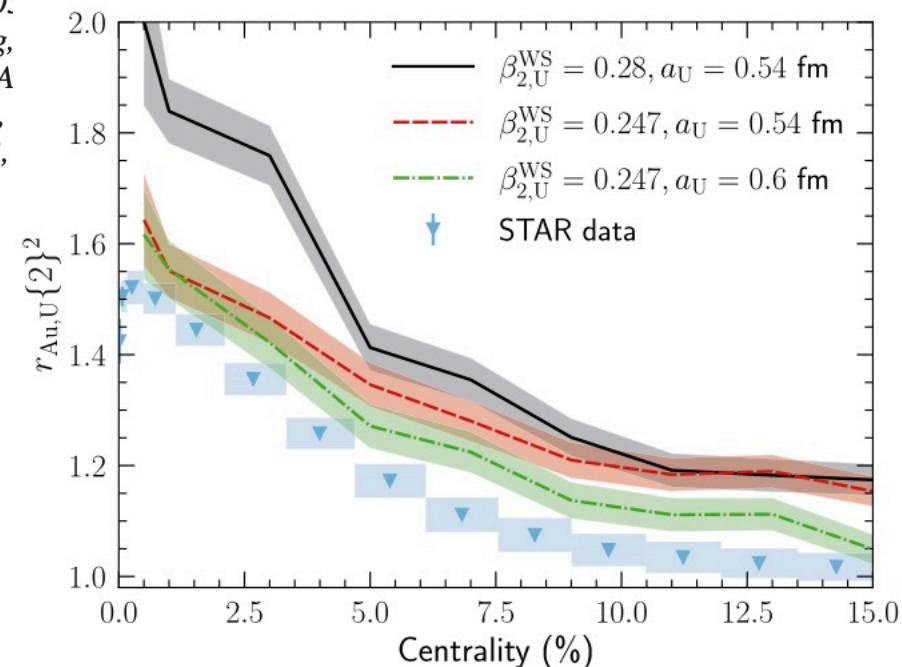
$$\beta_l^* = \frac{4\pi}{(2l+1)ZR_0^l} \sqrt{\frac{B(E_l)}{e^2}}$$

$$B(E2, U^{238}) = 12.09 \pm 0.2 e^2 b^2$$

Liquid drop limit

$$\beta_2^* \propto \left(\beta_2 + \frac{2}{7} \sqrt{\frac{5}{\pi}} \beta_2^2 + \frac{12}{7\sqrt{\pi}} \beta_2 \beta_4 + \dots \right)$$

$$\beta_{2,\text{Au}} = 0.14$$





Deformation of Au

PHYSICAL REVIEW LETTERS **127**, 242301 (2021)

$$\beta_{2,Au} = 0.17$$

Impact of Nuclear Deformation on Relativistic Heavy-Ion Collisions: Assessing Consistency in Nuclear Physics across Energy Scales

Giuliano Giacalone¹, Jianguong Jia^{2,3,*} and Chunjian Zhang²

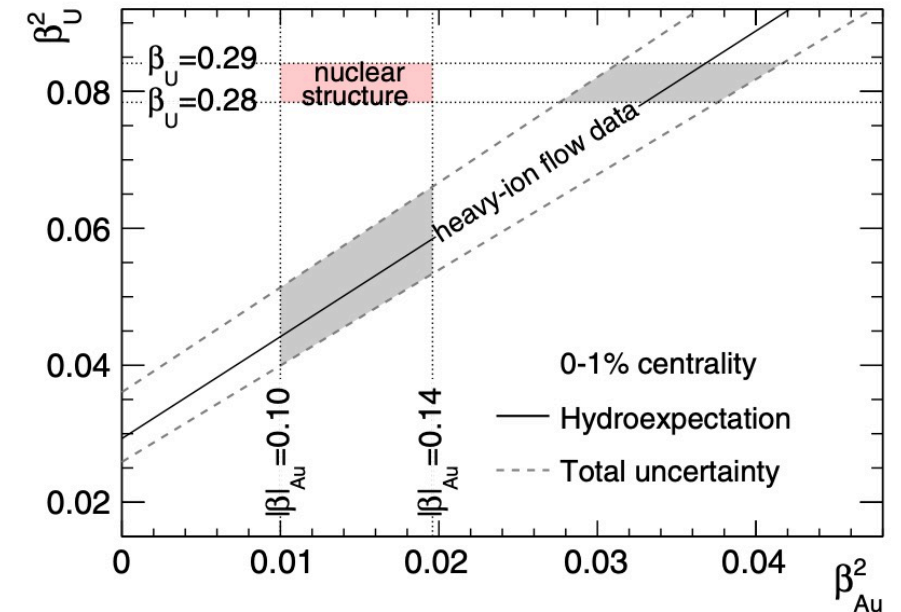
¹*Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

²*Department of Chemistry, Stony Brook University, Stony Brook, New York 11794, USA*

³*Physics Department, Brookhaven National Laboratory, Upton, New York 11976, USA*

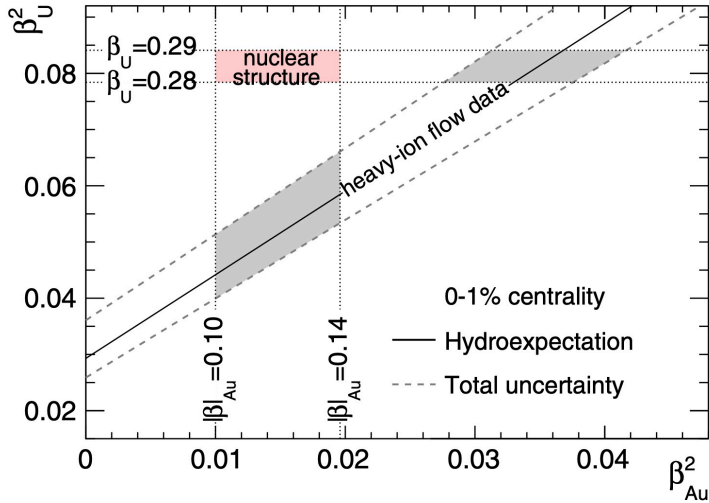
(Received 12 May 2021; revised 18 September 2021; accepted 15 November 2021; published 8 December 2021)

In the hydrodynamic framework of heavy-ion collisions, elliptic flow v_2 is sensitive to the quadrupole deformation β of the colliding ions. This enables one to test whether the established knowledge on the low-energy structure of nuclei is consistent with collider data from high-energy experiments. We derive a formula based on generic scaling laws of hydrodynamics to relate the difference in v_2 measured between collision systems that are close in size to the value of β of the respective species. We validate our formula in simulations of $^{238}\text{U} + ^{238}\text{U}$ and $^{197}\text{Au} + ^{197}\text{Au}$ collisions at top Relativistic Heavy Ion Collider (RHIC) energy, and subsequently apply it to experimental data. Using the deformation of ^{238}U from low-energy experiments, we find that RHIC v_2 data implies $0.16 \lesssim |\beta| \lesssim 0.20$ for ^{197}Au nuclei, i.e., significantly more deformed than reported in the literature, posing an interesting issue in nuclear phenomenology.



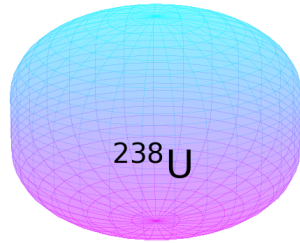


Hexadecapole deformation

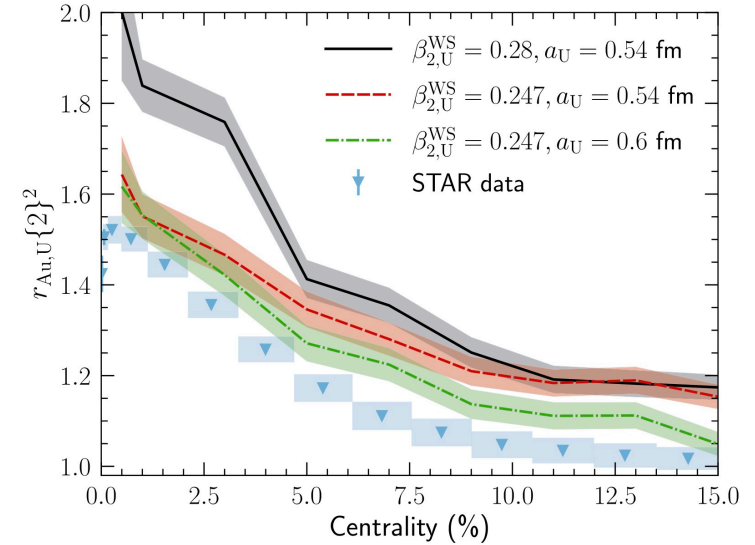
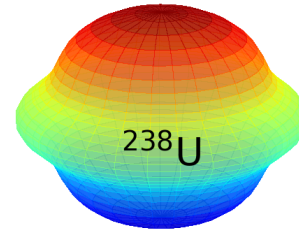


$$\beta_2^* \propto \text{BE}(2)$$

$$\text{BE}(2, \text{U}) = 12.09 \pm 0.02 \text{ e}^2 \text{b}^2$$



or



$$\beta_{2,U} \sim 0.28, \quad \beta_{4,U} \sim 0$$

$$\beta_2^* \propto \left(\beta_2 + \frac{2}{7} \sqrt{\frac{5}{\pi}} \beta_2^2 + \frac{12}{7\sqrt{\pi}} \beta_2 \beta_4 + \dots \right)$$

$$\beta_{2,U} \sim 0.25, \quad \beta_{4,U} \sim 0.1$$

$$\beta_{2,Au} \sim 0.17$$

$$R = R_0 [1 + \beta_2 Y_{20} + \beta_4 Y_{40}]$$

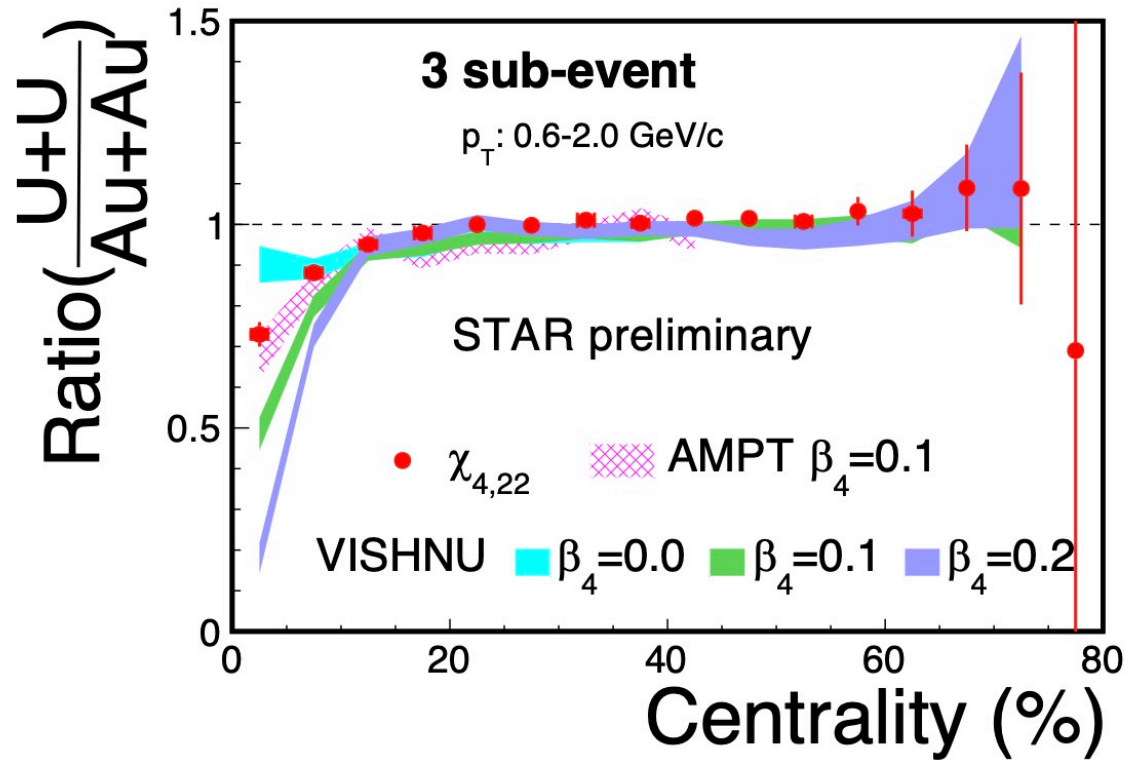
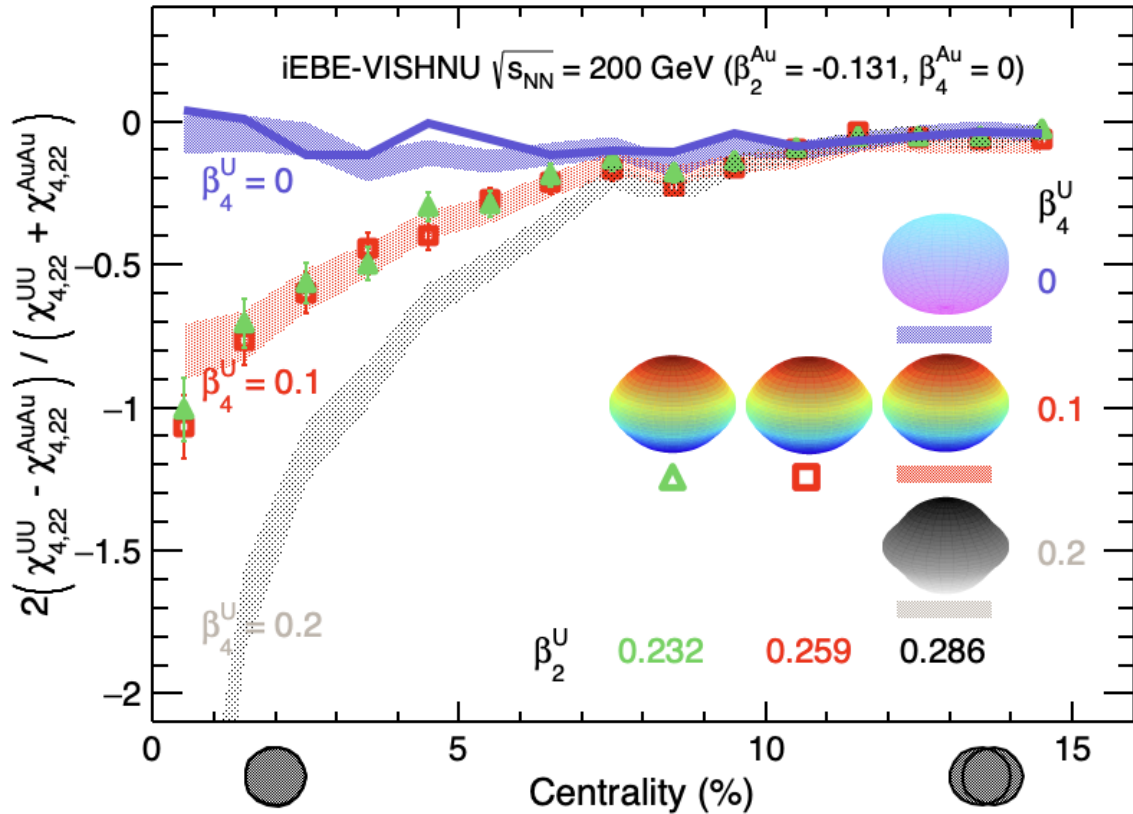
$$\beta_{2,Au} \sim 0.13$$

$\beta_{4,U}$ is poorly known from low-energy nuclear experiment, can it be measured in relativistic heavy ion collisions? **YES!**



Determine the hexadecapole deformation

$$v_4 = v_{4L} + \chi_{4,22} v_2^2$$



HJX, J. Zhao, F. Wang, PRL132, 262301 (2024)

J. Zhao, QM2025

$$\chi_{4,22} \equiv \frac{v_4\{\Phi_2\}}{\langle v_2^4 \rangle^{1/2}} = \frac{ac_2\{3\}}{\langle v_2^4 \rangle}$$



Hexadecapole flow

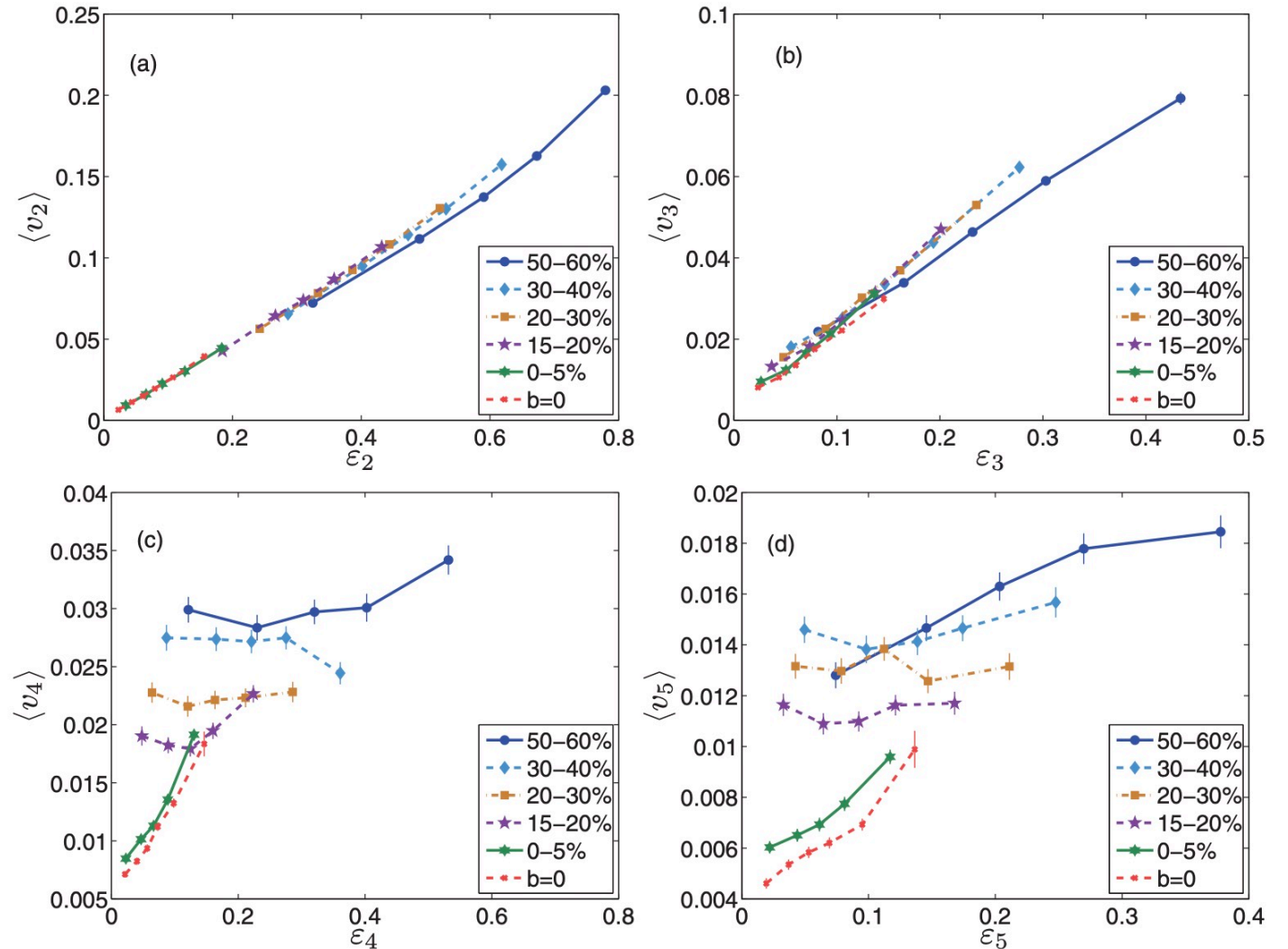
$$v_4 = v_{4L} + \chi_{4,22} v_2^2$$

$$\epsilon_4^2 \propto \beta_4^2 \quad \checkmark$$

$$v_4^2 \propto \epsilon_4^2 \quad \times$$

The hydrodynamic response for v_n ($n \geq 4$) with event-by-event fluctuations is not only non-diagonal but also nonlinear.

Zhi Qiu and Ulrich Heinz, PRC84, 024911(2011)





Linear/nonlinear response coefficients

$$v_n = f(\epsilon_m) \quad \text{from} \quad \partial_\mu T^{\mu\nu} = 0$$

Gubser flow for linear response

$$\frac{v_n}{\epsilon_n} = \frac{9}{64} \frac{\Gamma(3n)}{\Gamma(4n)} \left(\frac{128}{B^3}\right)^n \Gamma^2\left(\frac{n}{2}\right) \frac{n^2(3n+2)^2(n-1)}{2(4n+1)} \quad \text{ideal}$$

$$\text{viscous} + \frac{27K}{256} \frac{\Gamma(3n)}{\Gamma(4n)} \left(\frac{128}{B^3}\right)^n \Gamma^2\left(\frac{n}{2}\right) \frac{n^3(n-1)}{3n-1} \quad \text{charge dependence}$$

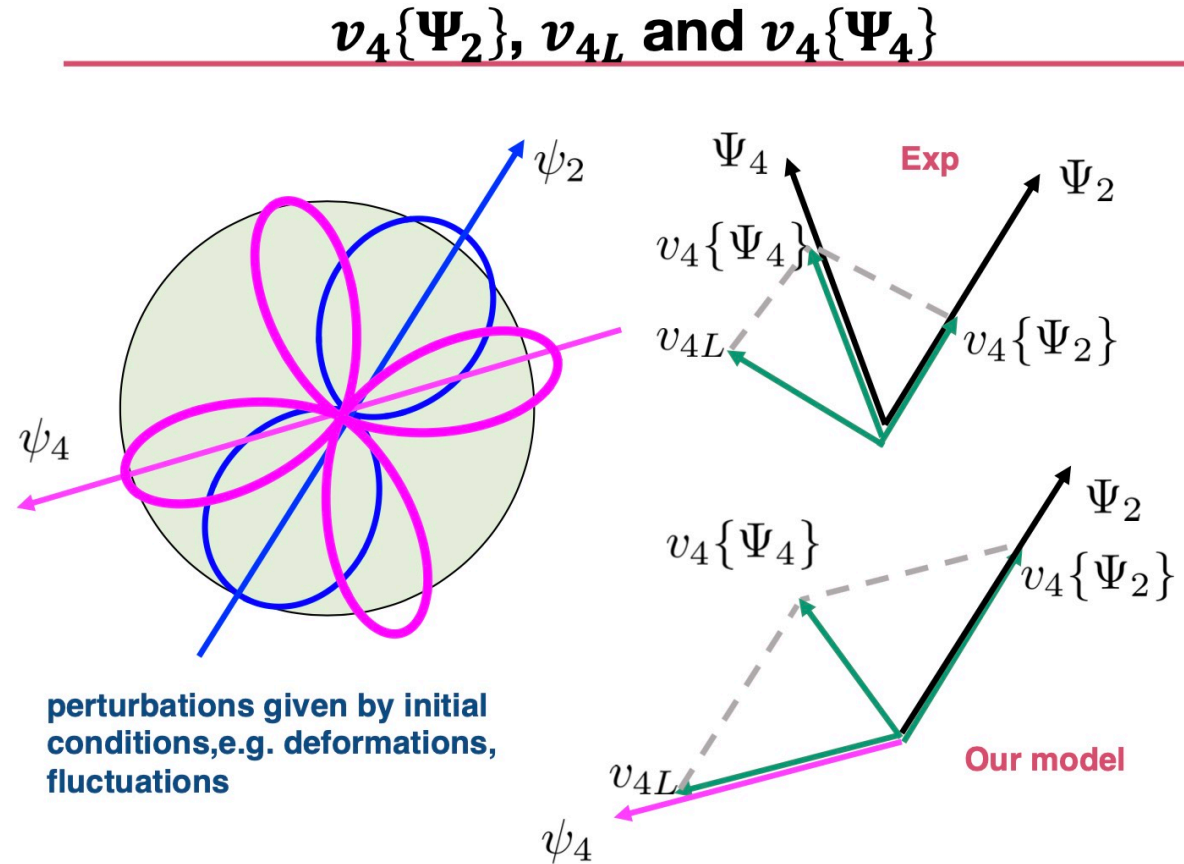
$$\times \left\{ -\frac{1}{4}(3n^2 + 3n + 2) + \frac{\mu_B}{2T} \left(\frac{3n}{2} + 1\right) \left(\pm 1 - \frac{3f'}{4f}\right) \right\}$$

Y. Hatta, et.al, PRD89, 051702 (2014)

Gubser flow for nonlinear response

$$v_4 = A_4 \epsilon_4 + C_4 \epsilon_2^2$$

X. Ren, et.al, in progress

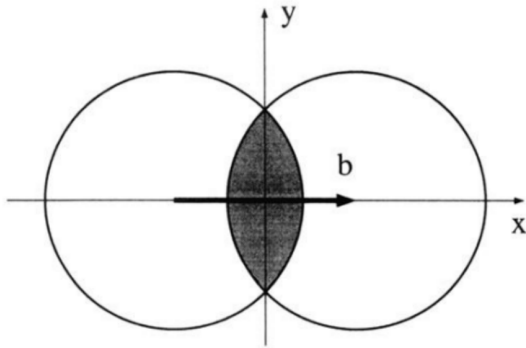


v2-to-v3 puzzle in Pb+Pb collisions (2011-)

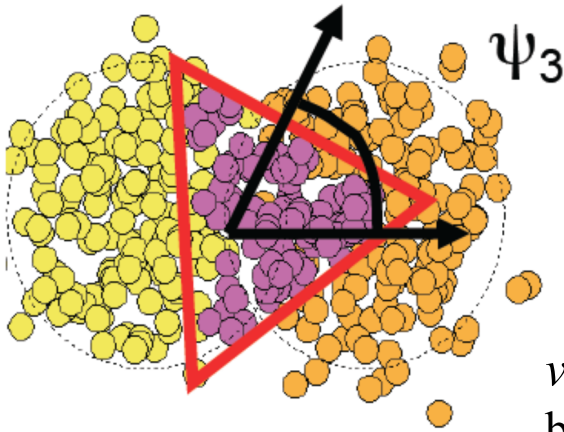
HJX, D. Xu, S. Zhao, W. Zhao, H. Song, F. Wang, "Breathing" octupole Pb nucleus to resolve the elliptical-to-triangular azimuthal anisotropy puzzle in ultracentral relativistic heavy ion collisions, arXiv:2504.19644, PRC in press



Triangular flow



$$v_n = 0 \text{ for } n=2k+1$$



v_3 is dominated by event-by-event fluctuations

literature ▾ find triangular flow and d<2011 and d>2007

Collision geometry fluctuations and triangular flow in heavy-ion collisions #1

B. Alver (MIT), G. Roland (MIT) (Mar, 2010)

Published in: *Phys.Rev.C* 81 (2010) 054905, *Phys.Rev.C* 82 (2010) 039903 (erratum) • e-Print: [1003.0194](#) [nucl-th]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [1,035 citations](#)

Elliptic and triangular flow in event-by-event (3+1)D viscous hydrodynamics #2

Bjorn Schenke (McGill U.), Sangyong Jeon (McGill U.), Charles Gale (McGill U.) (Sep, 2010)

Published in: *Phys.Rev.Lett.* 106 (2011) 042301 • e-Print: [1009.3244](#) [hep-ph]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [823 citations](#)

Triangular flow in hydrodynamics and transport theory #3

Burak Han Alver (MIT, LNS), Clement Gombeaud (Saclay, SPHT), Matthew Luzum (Saclay, SPHT), Jean-Yves Ollitrault (Saclay, SPHT) (Jul, 2010)

Published in: *Phys.Rev.C* 82 (2010) 034913 • e-Print: [1007.5469](#) [nucl-th]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [414 citations](#)

Triangular flow in event-by-event ideal hydrodynamics in Au+Au collisions at $\sqrt{s_{NN}} = 200$ A GeV #4

Hannah Petersen (Duke U.), Guang-You Qin (Duke U.), Steffen A. Bass (Duke U.), Berndt Muller (Duke U.) (Aug, 2010)

Published in: *Phys.Rev.C* 82 (2010) 041901 • e-Print: [1008.0625](#) [nucl-th]

[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [233 citations](#)

The effect of triangular flow on di-hadron azimuthal correlations in relativistic heavy ion collisions #5

Jun Xu (Texas A-M), Che Ming Ko (Texas A-M, Cyclotron Inst. and Texas A-M) (Nov, 2010)

Published in: *Phys.Rev.C* 83 (2011) 021903 • e-Print: [1011.3750](#) [nucl-th]

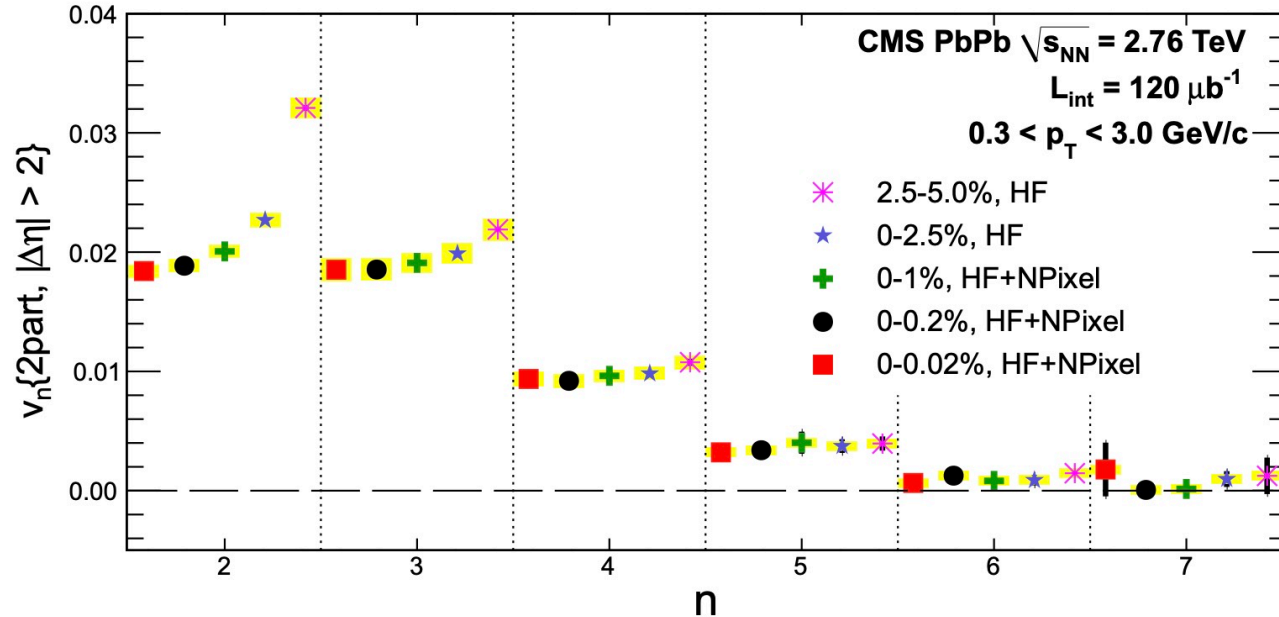
[pdf](#) [DOI](#) [cite](#) [claim](#)

[reference search](#) [72 citations](#)



v2-to-v3 puzzle

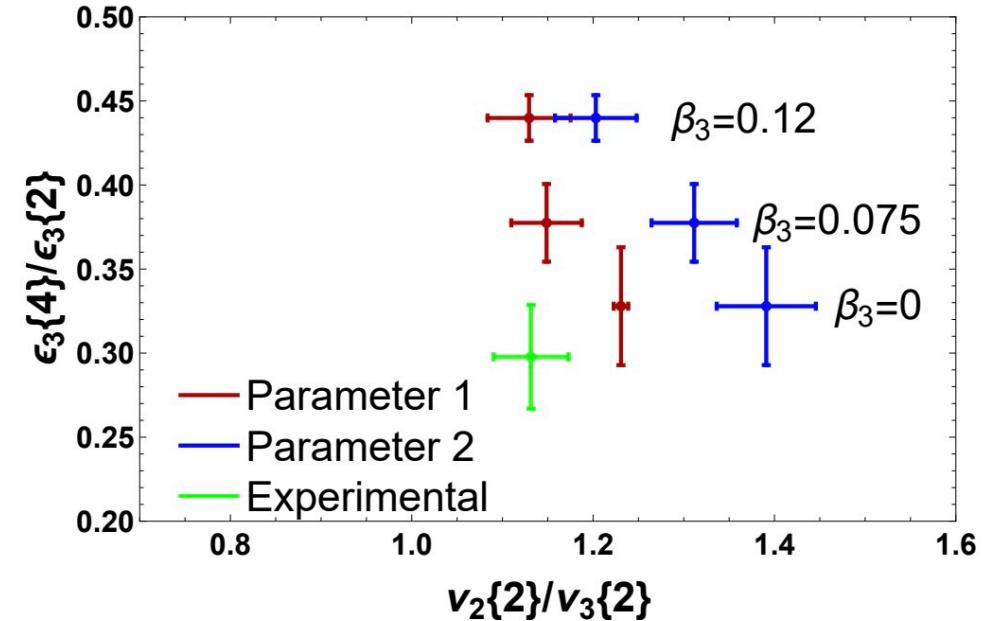
CMS Collaboration, Phys.Rev. C89, 044906 (2014)



$v_2 \simeq v_3$ at most-central collisions, can not be described by hydrodynamic simulations with spherical Pb.

$\epsilon_2 \simeq \epsilon_3$, but

$k_3 (= v_3/\epsilon_3) < k_2 (= v_2/\epsilon_2)$ due to viscous damping effect



With octupole deformation of Pb, “the v2-to-v3 puzzle remains a challenge for hydrodynamic models”.

P. Carson, et.al, Possible octupole deformation of Pb and the ultracentral v2 to v3 puzzle, Phys.Rev. C102, 054905 (2020).



The idea to solve the puzzle

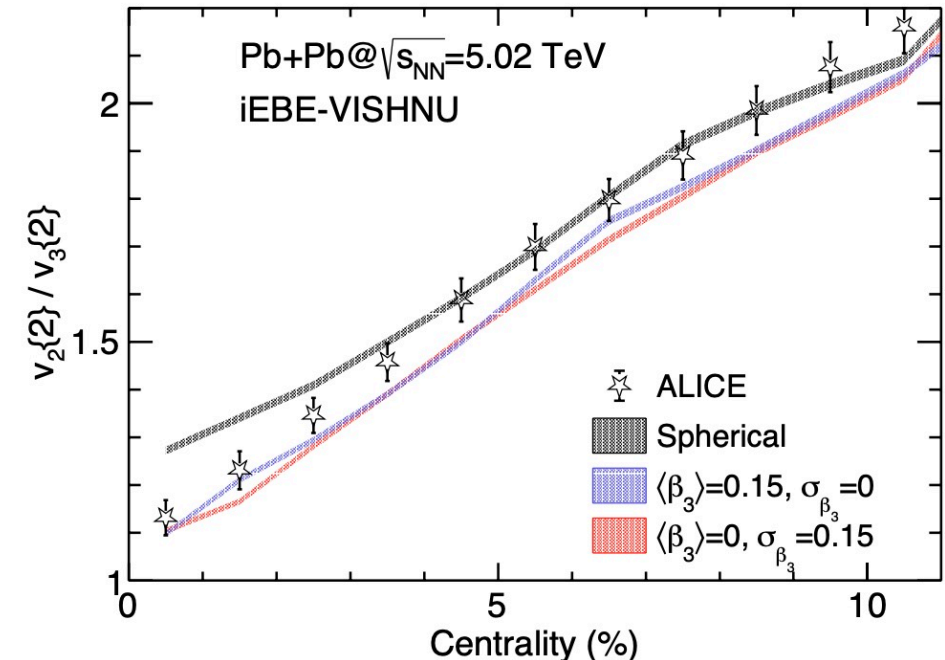
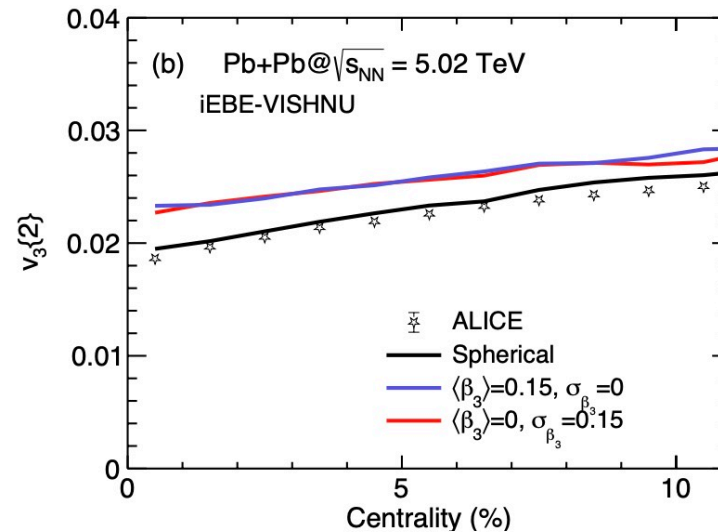
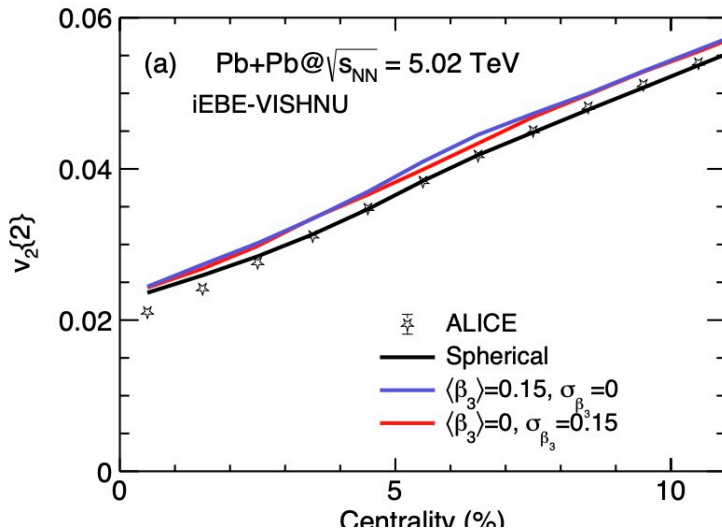
v_3 from two-particle cumulant and four-particle cumulant methods

$$v_3\{2\} \sim v_3 + \sigma \quad v_3\{4\} \sim v_3 - \sigma$$

To reduce $v_3\{4\}/v_3\{2\}$, additional fluctuation are required to enhance σ .
Based on $v_n^2 \propto \beta_n^2$, we introduce octupole shape fluctuation for Pb:

HJX, et.al, arXiv:2504.19644

$$P(\beta_3) \propto \exp \left[-\frac{(\beta_3 - \langle \beta_3 \rangle)^2}{2\sigma_{\beta_3}^2} \right].$$

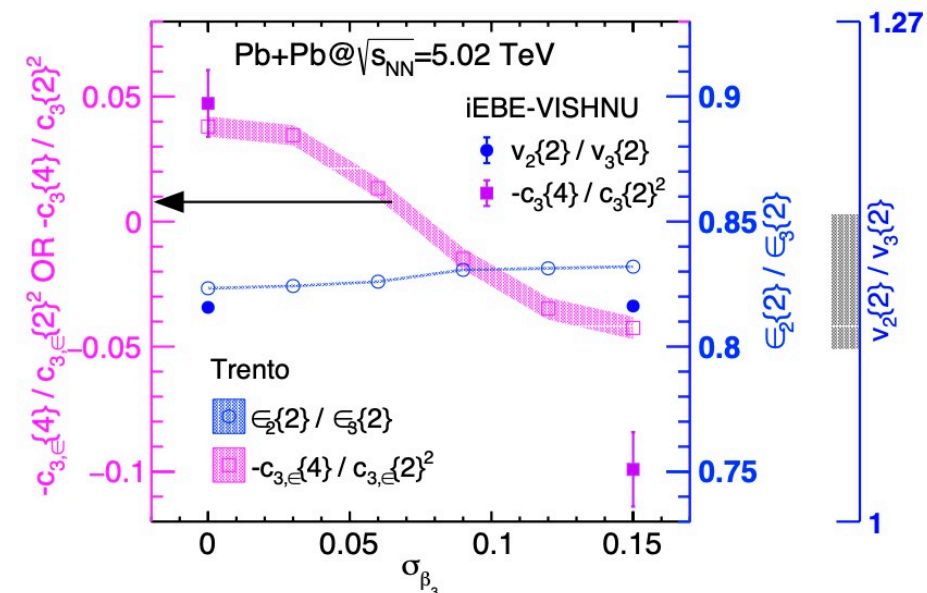




The idea to solve the puzzle

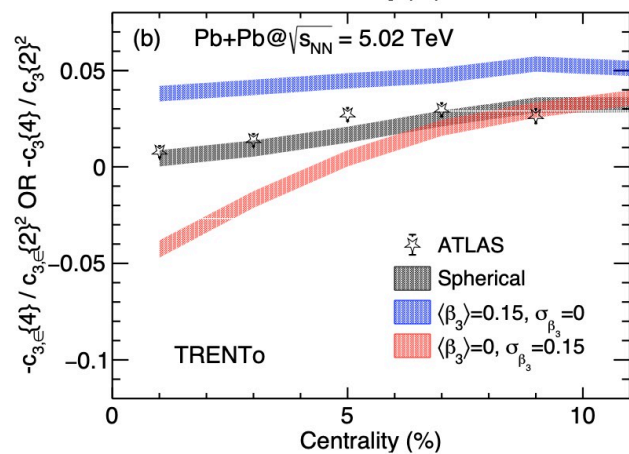
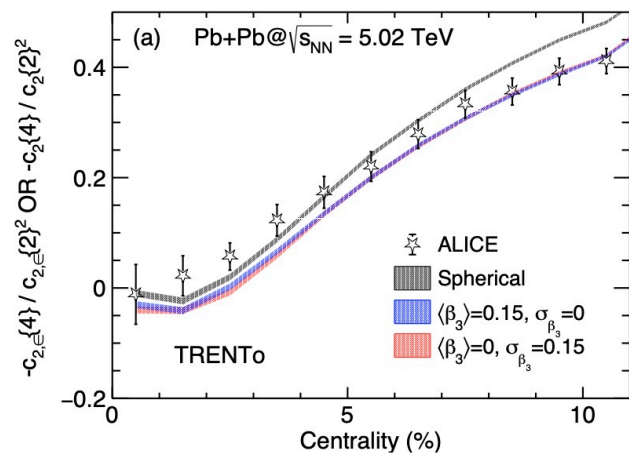
HJX, et.al, arXiv:2504.19644

$$P(\beta_3) \propto \exp \left[-\frac{(\beta_3 - \langle \beta_3 \rangle)^2}{2\sigma_{\beta_3}^2} \right]$$

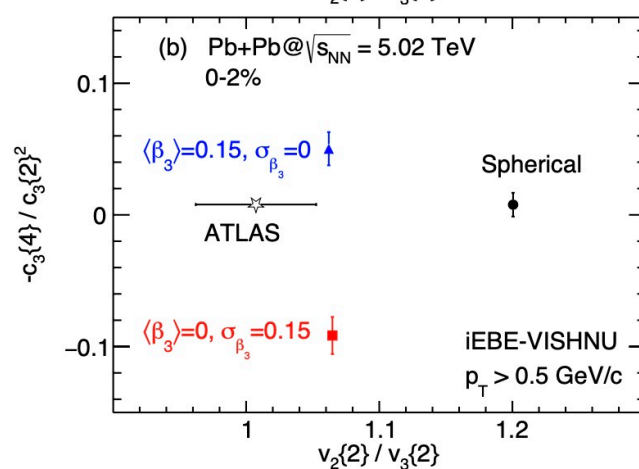
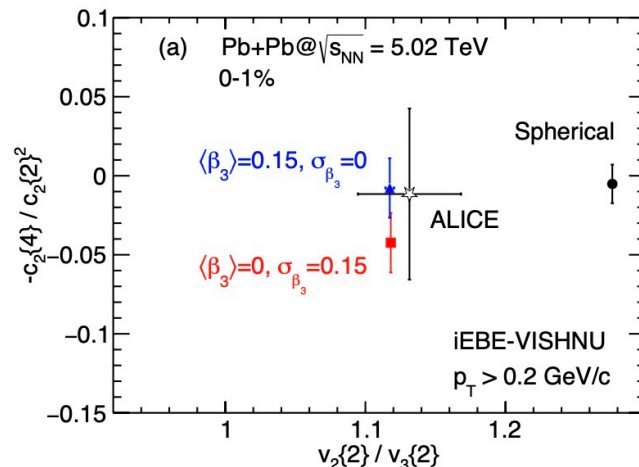


$$\sqrt{\langle \beta_3 \rangle^2 + \sigma_{\beta_3}^2} = 0.15$$

Trento: 0-10%

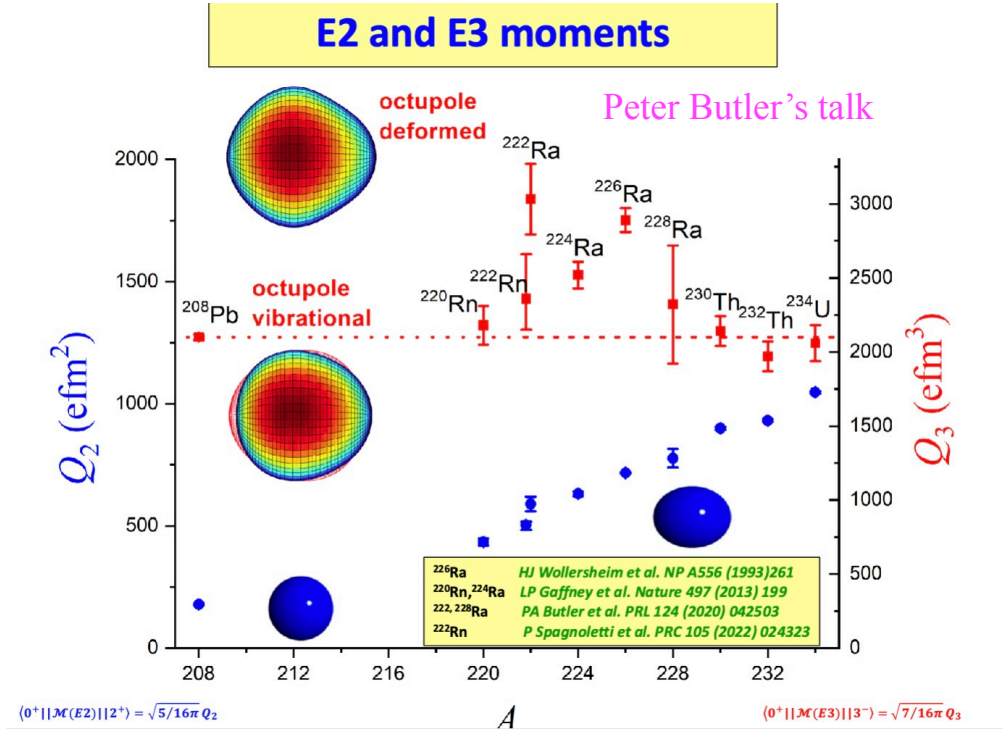
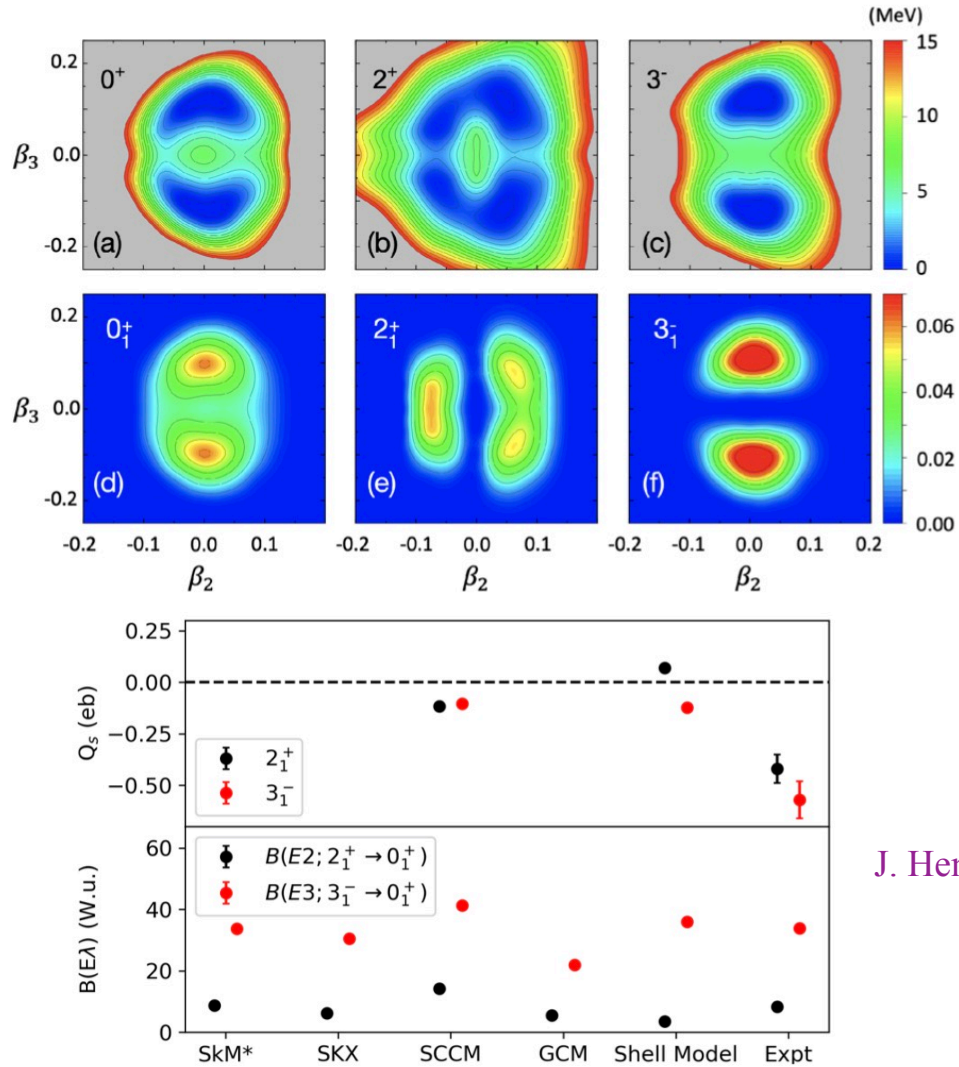


iEBE-VISHNU: 0-2%





Connect to octupole deformation/vibration



J. Henderson, et al, Phys.Rev.Lett., 134, 062502 (2025):

“Even as a cornerstone of the nuclear landscape, ^{208}Pb remains a puzzle for nuclear structure theories.”

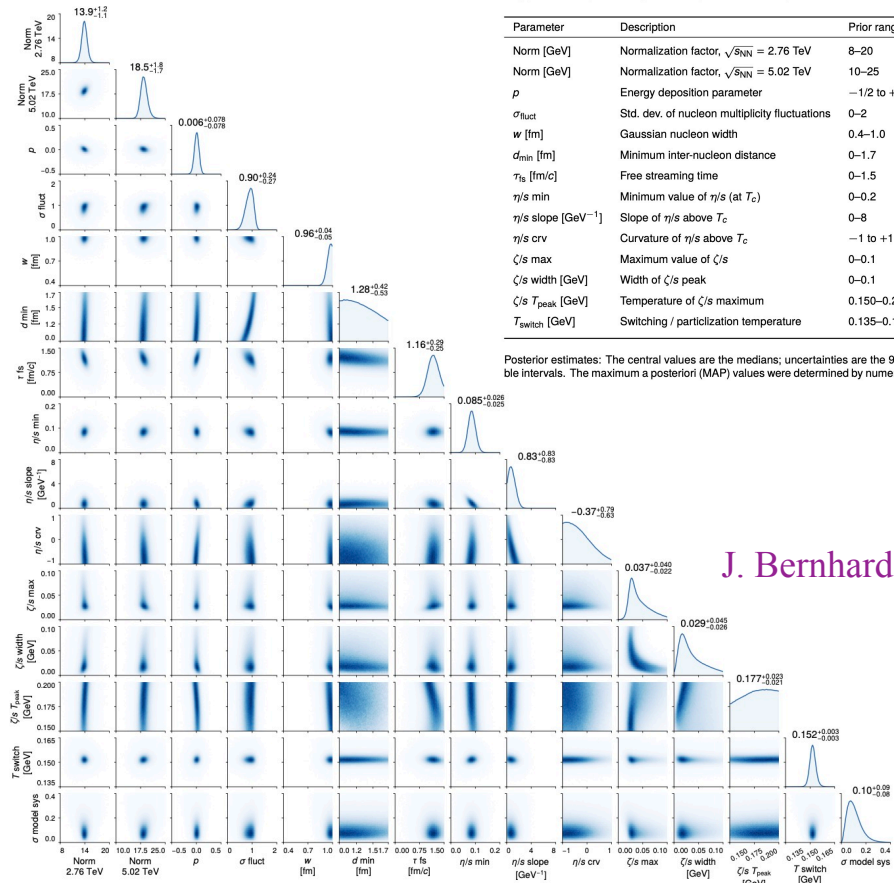


Relativistic hydrodynamic model

Supplementary Table 1 | Summary of estimated parameters

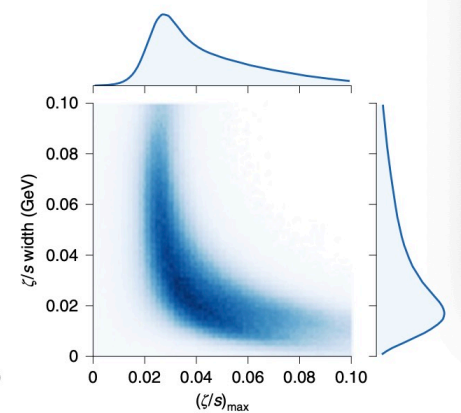
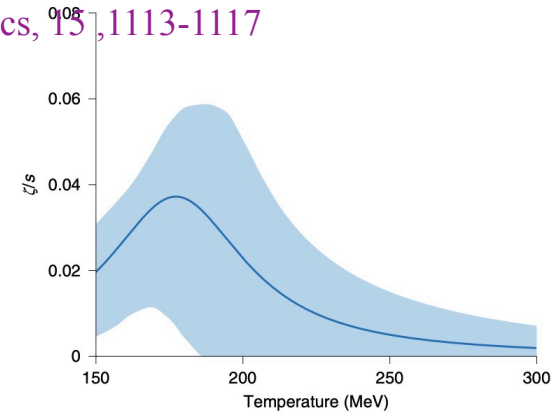
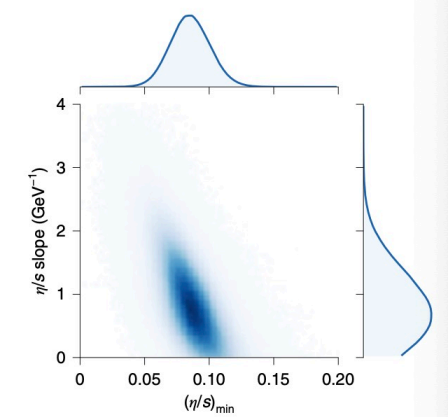
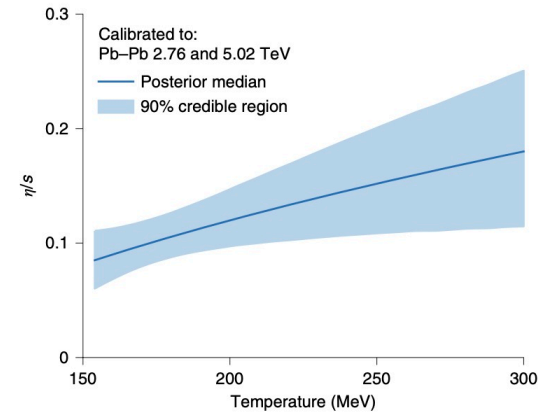
Parameter	Description	Prior range	Posterior estimate	MAP value
Norm [GeV]	Normalization factor, $\sqrt{s_{NN}} = 2.76$ TeV	8–20	$13.9^{+1.2}_{-1.1}$	13.94
Norm [GeV]	Normalization factor, $\sqrt{s_{NN}} = 5.02$ TeV	10–25	$18.5^{+1.8}_{-1.7}$	18.38
p	Energy deposition parameter	-1/2 to +1/2	$0.006^{+0.078}_{-0.078}$	0.007
σ_{fluct}	Std. dev. of nucleon multiplicity fluctuations	0–2	$0.90^{+0.24}_{-0.27}$	0.918
w [fm]	Gaussian nucleon width	0.4–1.0	$0.96^{+0.04}_{-0.05}$	0.956
d_{min} [fm]	Minimum inter-nucleon distance	0–1.7	$1.28^{+0.42}_{-0.53}$	1.27
τ_{fs} [fm/c]	Free streaming time	0–1.5	$1.16^{+0.29}_{-0.25}$	1.16
η/s min	Minimum value of η/s (at T_c)	0–0.2	$0.085^{+0.026}_{-0.025}$	0.081
η/s slope [GeV $^{-1}$]	Slope of η/s above T_c	0–8	$0.83^{+0.83}_{-0.83}$	1.11
η/s crv	Curvature of η/s above T_c	-1 to +1	$-0.37^{+0.79}_{-0.63}$	-0.48
ζ/s max	Maximum value of ζ/s	0–0.1	$0.037^{+0.040}_{-0.022}$	0.052
ζ/s width [GeV]	Width of ζ/s peak	0–0.1	$0.029^{+0.045}_{-0.028}$	0.022
ζ/s T_{peak} [GeV]	Temperature of ζ/s maximum	0.150–0.200	$0.177^{+0.029}_{-0.021}$	0.183
T_{switch} [GeV]	Switching / partitization temperature	0.135–0.165	$0.152^{+0.003}_{-0.003}$	0.151

Posterior estimates: The central values are the medians; uncertainties are the 90% highest posterior density (HPD) credible intervals. The maximum a posteriori (MAP) values were determined by numerically maximizing the posterior probability.



Supplementary Fig. 2 | Posterior distribution for all parameters. Diagonal: Marginal distributions (histograms) for each parameter. Annotated are the posterior medians and 90% highest posterior density (HPD) credible intervals. Off-diagonal: Joint distributions (density histograms) showing correlations between pairs of parameters.

J. Bernhard, et.al, Nature Physics, 13, 1113-1117

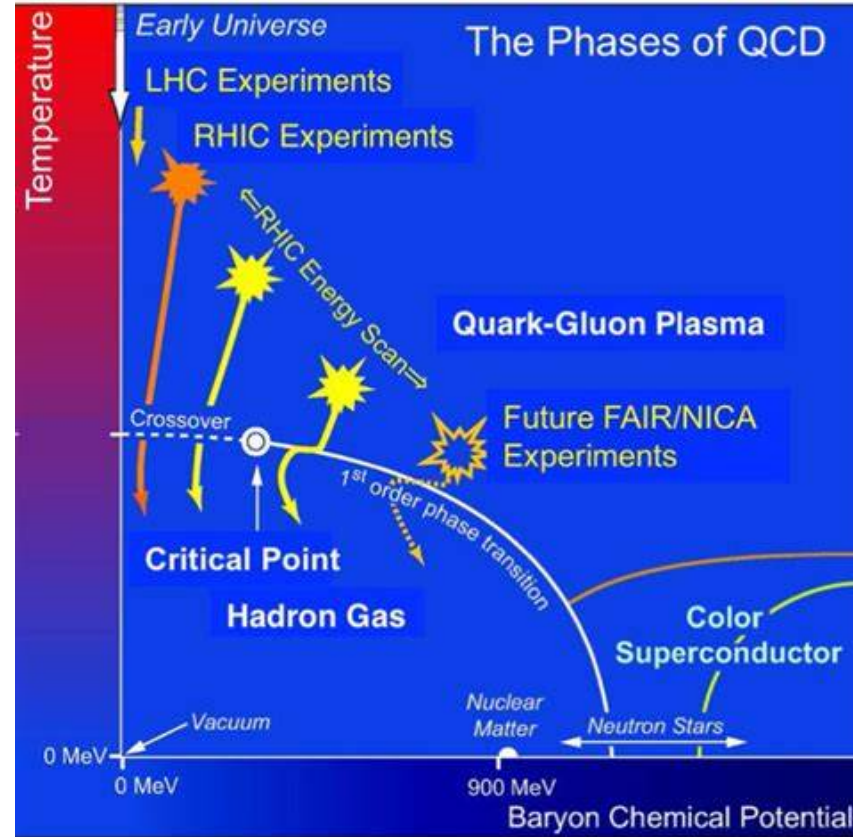


Proceed with extreme caution when extracting anything from the single collision system!



Summary

Precision Frontier of QCD Matter: Inference and Uncertainty Quantification



**Thank you for
your attention!**

Haojie Xu(徐浩浩)

Huzhou University(湖州师范学院)

