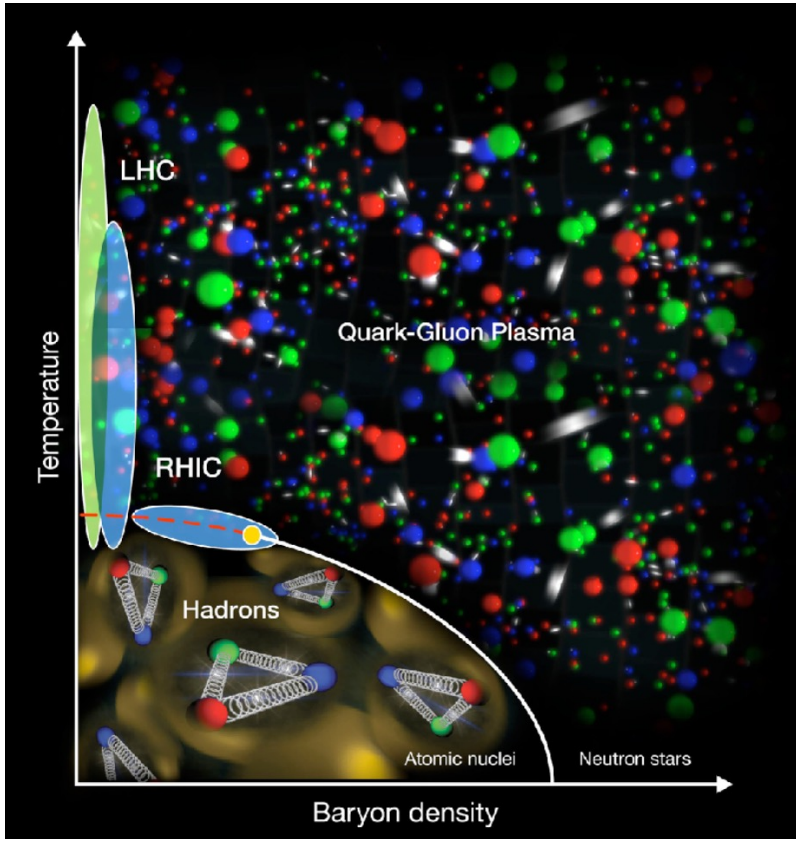
Extract the QCD speed of sound in the presence of quantum fluctuations

Yu-Shan Mu
Fudan University

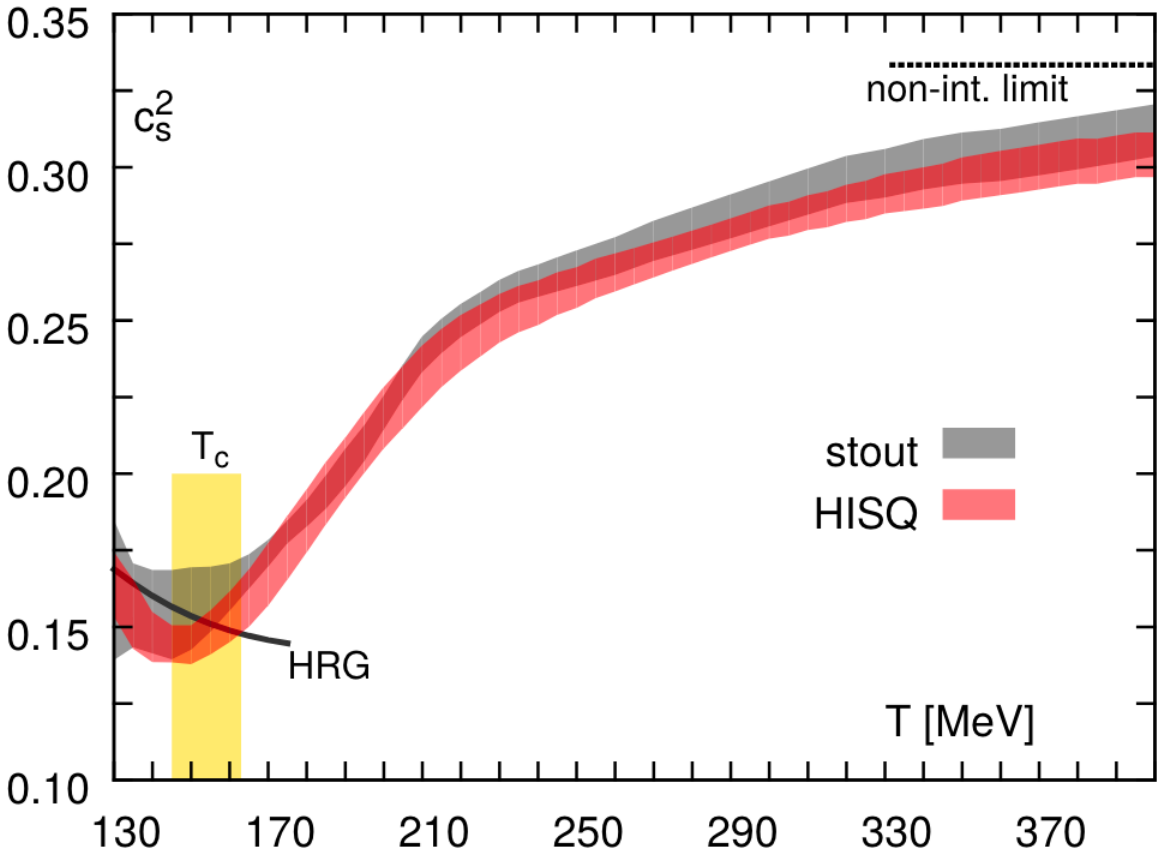
with Jing-An Sun, Li Yan and Xu-Guang Huang

Based on 2501.02777

Relativistic heavy-ion collisions

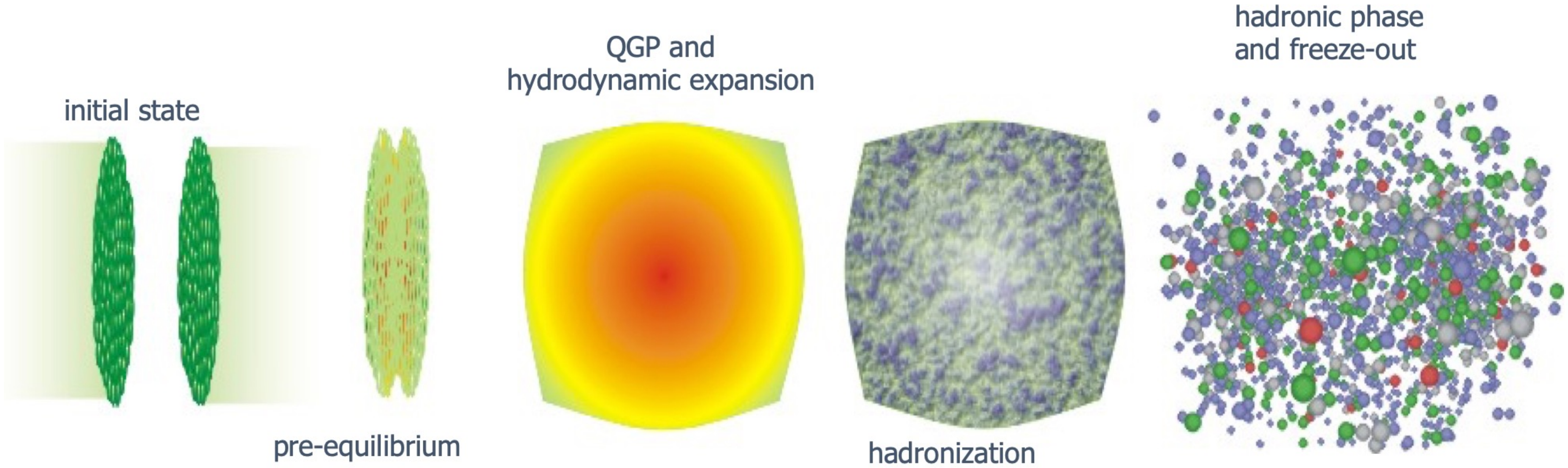


μ_B=0



A direct measurement of QCD equation of state is challenging in heavy-ion study.

Standard model of heavy-ion collisions



- Initial conditions : TRENTo, 3D-Glauber, IP-Glasma…
- Viscous hydrodynamics: MUSIC, Trajectum, CLVis…
- Hadron cascade afterburner: UrQMD, SMASH…

The QCD speed of sound in heavy-ion collisions

- Thermodynamics tells :

$$dP = sdT, \quad de = Tds \qquad \text{constant volume}$$

$$c_s^2 \equiv \frac{\partial P}{\partial e}\bigg|_{\text{Adiabatic}} = \frac{d\ln T}{d\ln s}\bigg|_{\text{Adiabatic}} = \frac{d\ln T}{d\ln S}\bigg|_{\text{Adiabatic}}$$

- In heavy-ion collisions:

$$S \propto N_{ch} \quad \rightarrow \quad d\ln S = d\ln N_{ch}$$

$$T \propto \langle p_T \rangle \qquad d\ln T = d\ln \langle p_T \rangle$$

- [F. Gardim et al , 1908.09728, Nature Physics]

$$c_s^2 = \frac{d\ln\langle p_T\rangle}{d\ln N_{ch}}$$

Effective temperature and effective volume

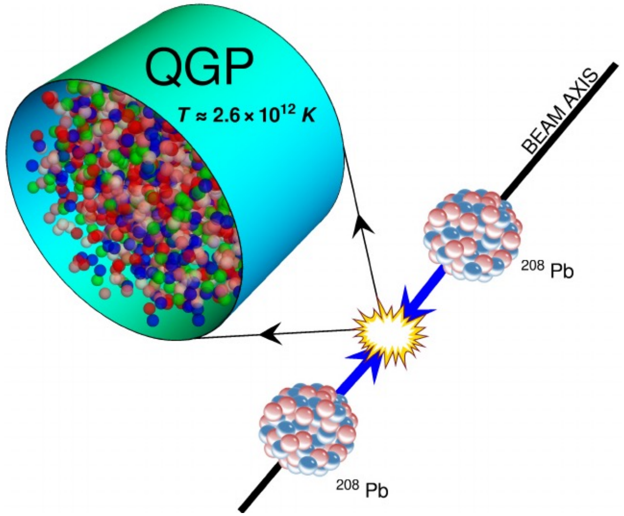
- A uniform fluid at rest with T_{eff} and V_{eff} :

$$E = \int_{\mathrm{f.o.}} T^{0\mu} \mathrm{d}\sigma_\mu = \epsilon(T_{\mathrm{eff}}) V_{\mathrm{eff}}$$

$$S = \int_{\mathrm{f.o.}} s u^\mu \mathrm{d}\sigma_\mu = s(T_{\mathrm{eff}}) V_{\mathrm{eff}}$$

[F. Gardim et al , 1908.09728, Nature Physics]

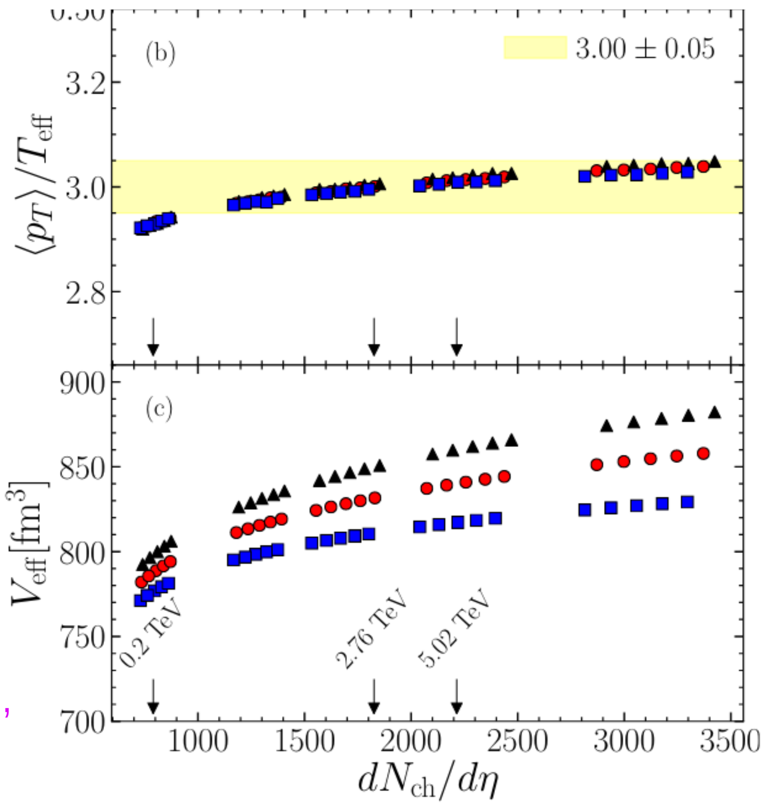
- T_{eff} and s (T_{eff}) are related by the equation of state of the fluid.
- T_{eff} is smaller than the initial temperature and larger than the freeze-out temperature.



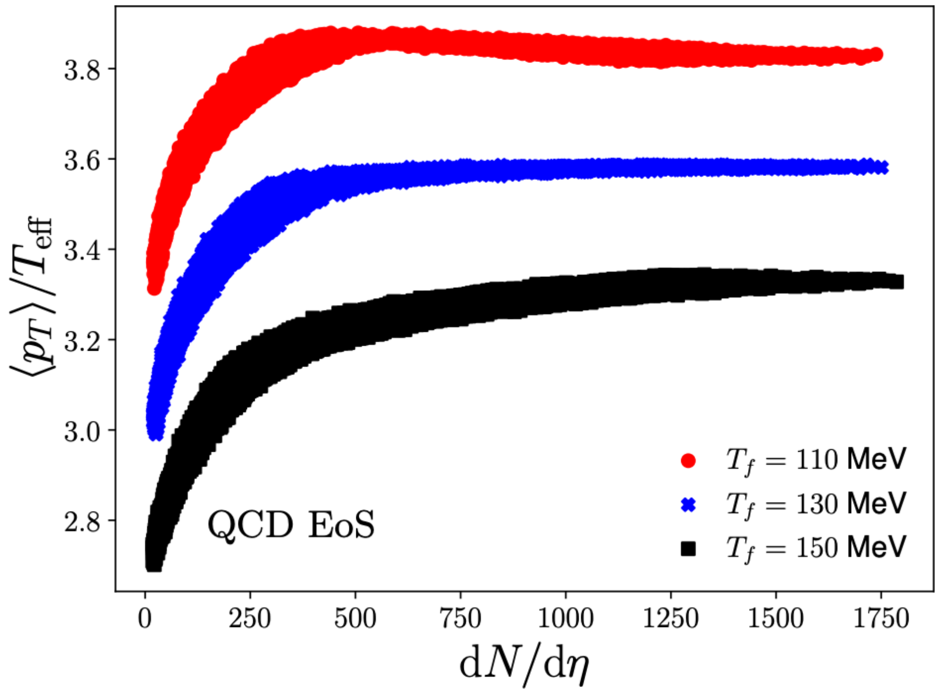
Effective temperature

- In large systems : $\langle p_T \rangle \approx 3\, T_{\text{eff}}$
- In small systems like p-Pb: $\langle p_T \rangle \approx 2.22 \sim 2.8\, T_{\text{eff}}$ for 5.02 TeV

  $\langle p_T \rangle \approx 2.16 \sim 2.58\, T_{\text{eff}}$ for 8.16 TeV



[F. Gardim et al , 2403.06052]

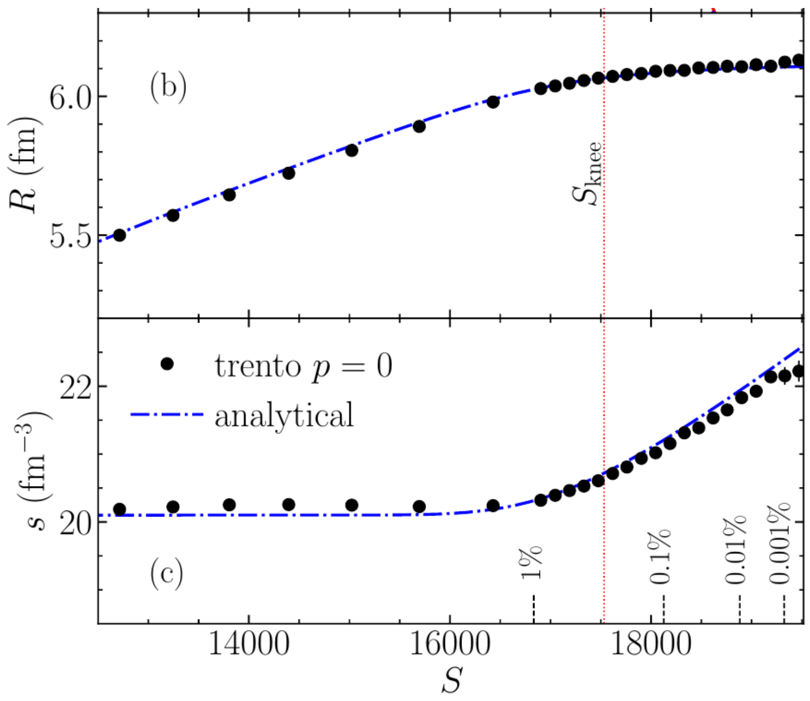


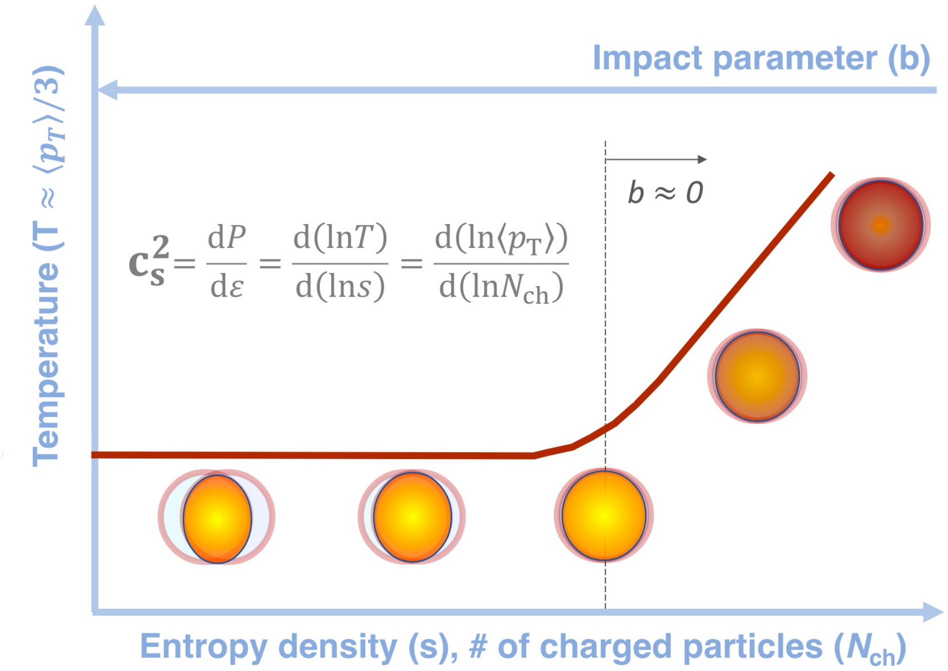
[L. Gavasino et al , 2503.20765]

Ultra-central nucleus-nucleus collisions (UCC)

- Large systems

[F. Gardim et al , 1909.11609]

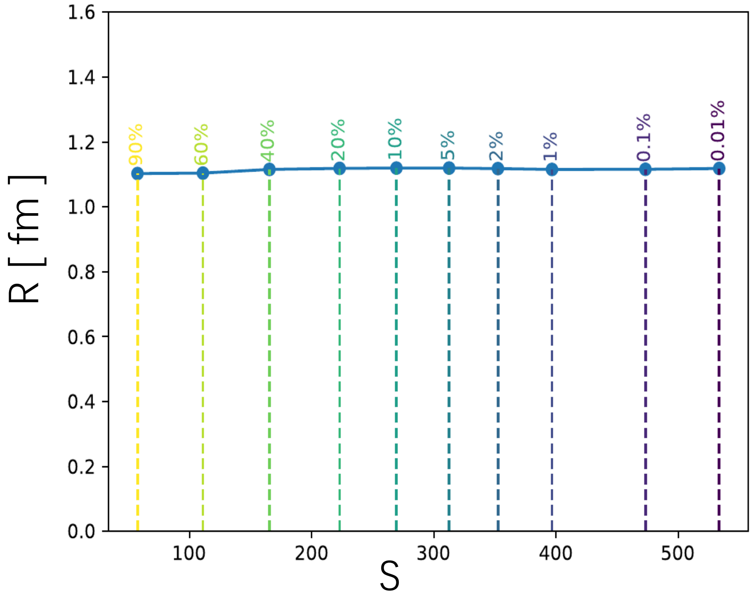


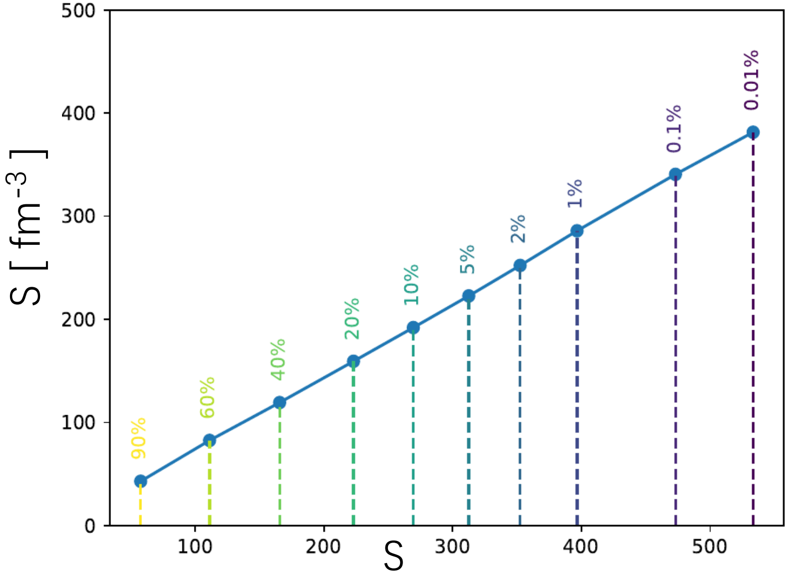


- S_{knee}: the mean value of the entropy at b = 0.
- b ≈ 0 → The volume is fixed.
- The differences in N_{ch} and T are driven by fluctuations.
- The slope is associated with the speed of sound.

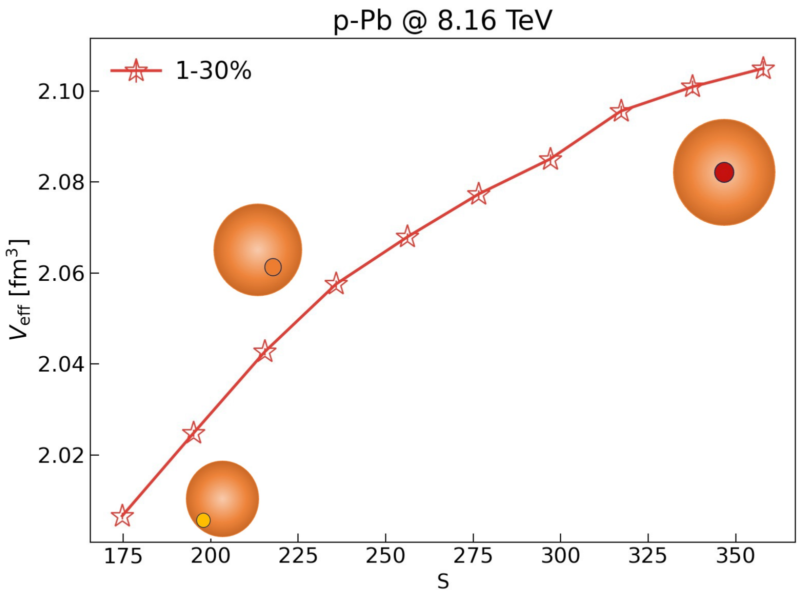
Ultra-central nucleus-nucleus collisions (UCC)

- Small systems





TRENTo model



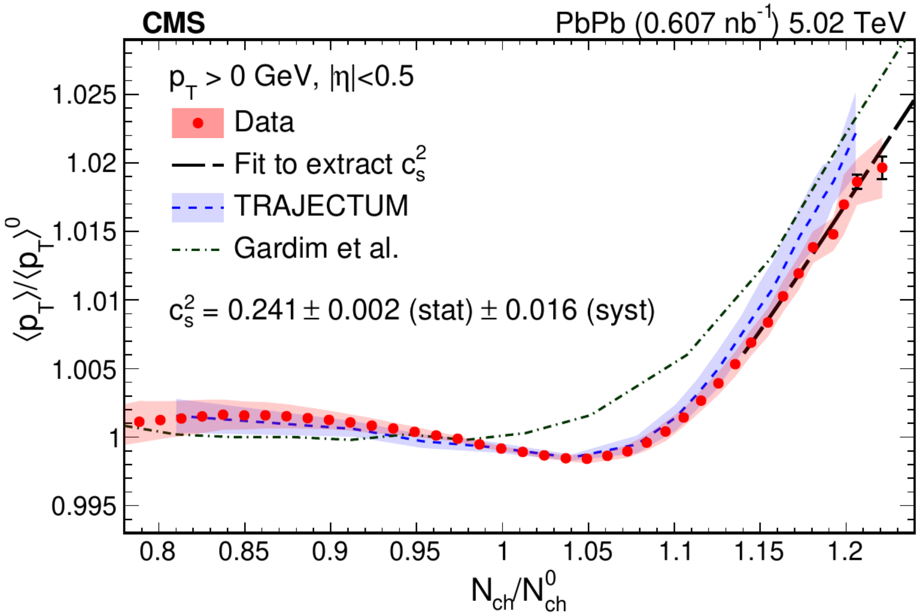
hydro model

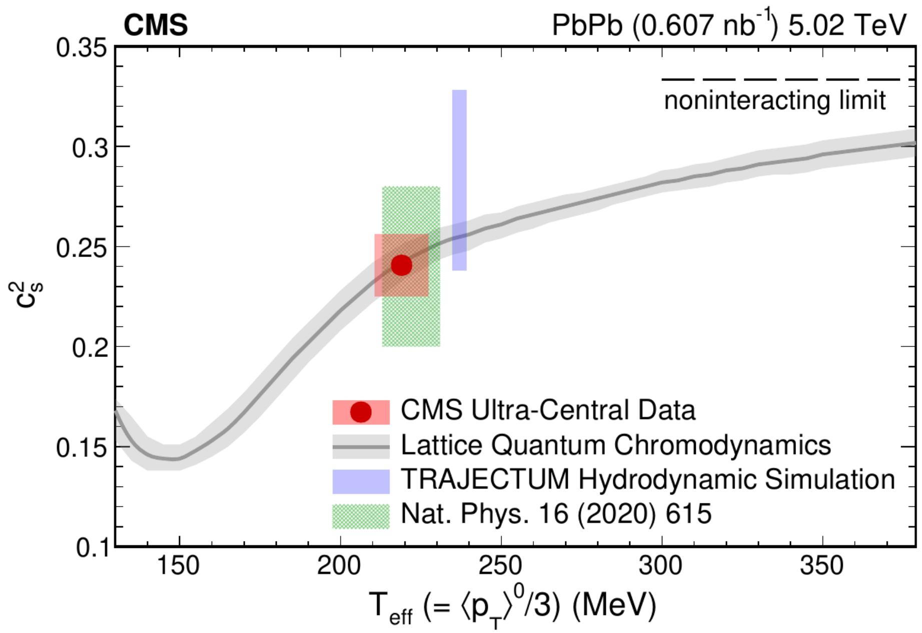
- Initial geometric size which is determined by the proton size is almost fixed in all centralities.
- However, effective volume only saturates in ultra-centra events.

Extract the speed of sound in experiments

$$c_s^2 = \frac{d\ln\langle p_t\rangle}{d\ln N_{\rm ch}} \rightarrow \frac{\Delta_p}{\langle p_T\rangle_0} = c_s^2 \frac{\Delta_N}{N_0} \quad \text{with} \quad \begin{cases} \Delta_p \equiv \langle p_T\rangle - \langle p_T\rangle_0 \\ \Delta_N \equiv N_{ch} - N_0 \end{cases}$$

where $\langle p_T\rangle_0$ and N_0 are the averaged values over the ultra-central events

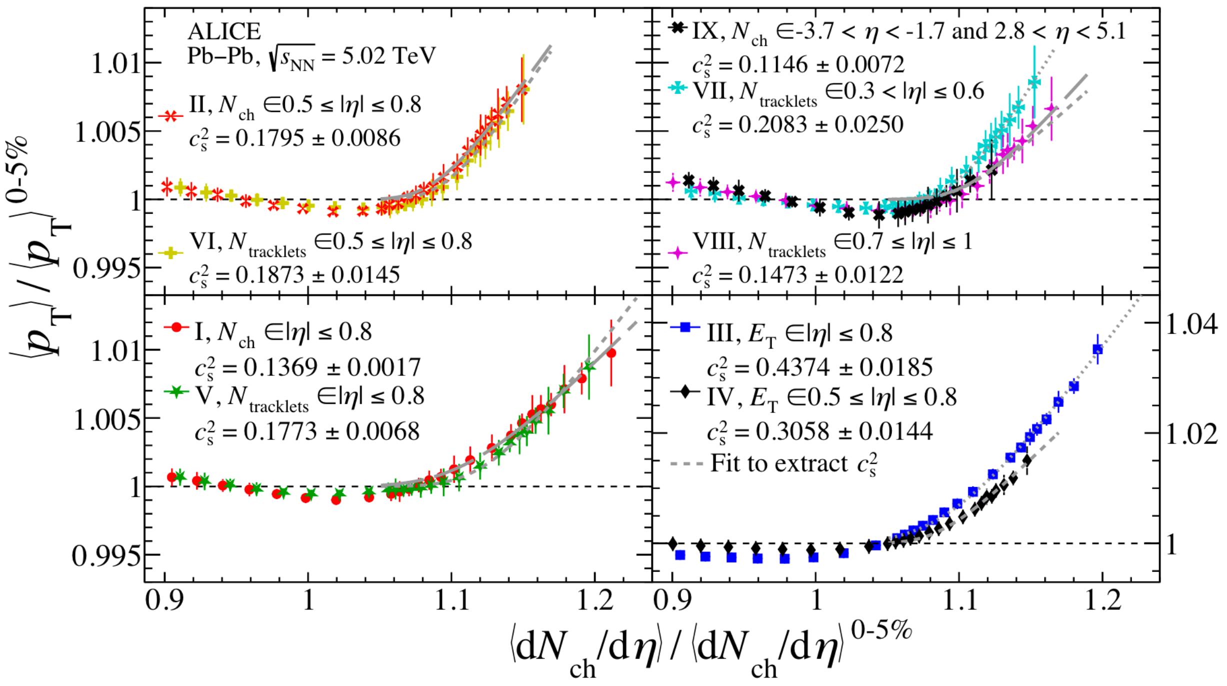




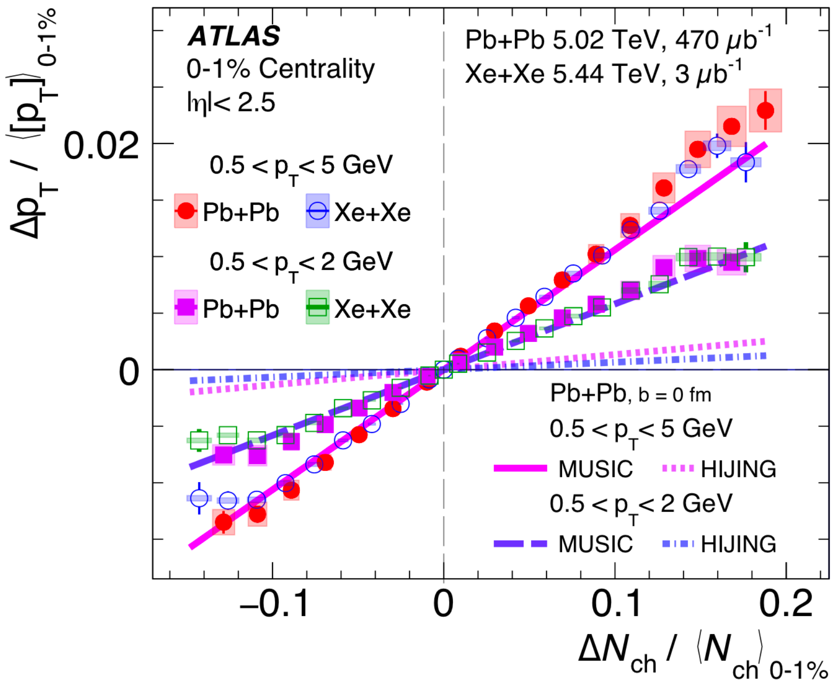
[CMS collaboration, 2401.06896]

Extract the speed of sound in experiments

[ALICE collaboration, 2506.10394]



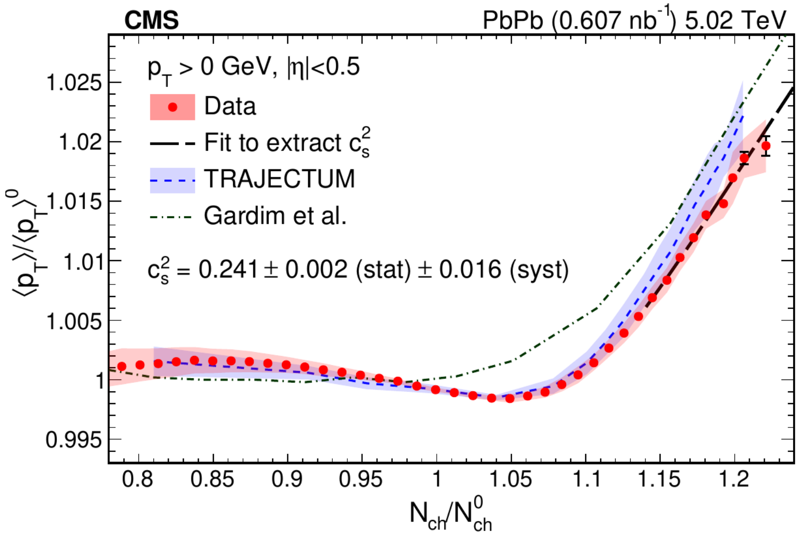
[ATLAS collaboration, 2407.068413]



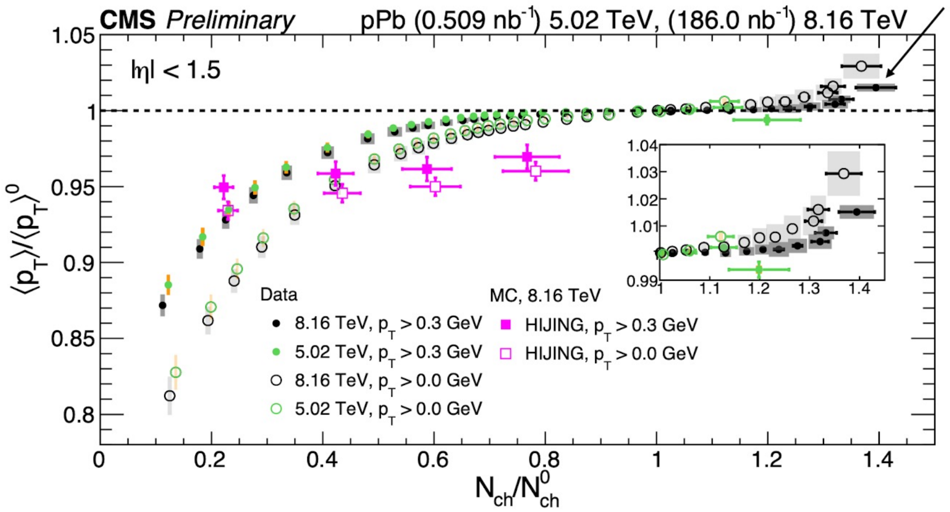
- Extracted value has a dependence of kinematic cut;
- ALICE extracted 9 different slopes with 9 different centrality estimators;
- E_T-based estimators give larger value;
- The slope extracted by ATLAS depends on the pT-range of the particles;

The speed of sound of small systems in experiments

- Multiplicity dependence at fixed energy



UCC pPb: ~10^{-8} in $E_{T,sum}^{HF}$ centrality



The reference $\langle p_T\rangle^0$, N_{ch}^0 chosen from the **0-0.01% centrality**

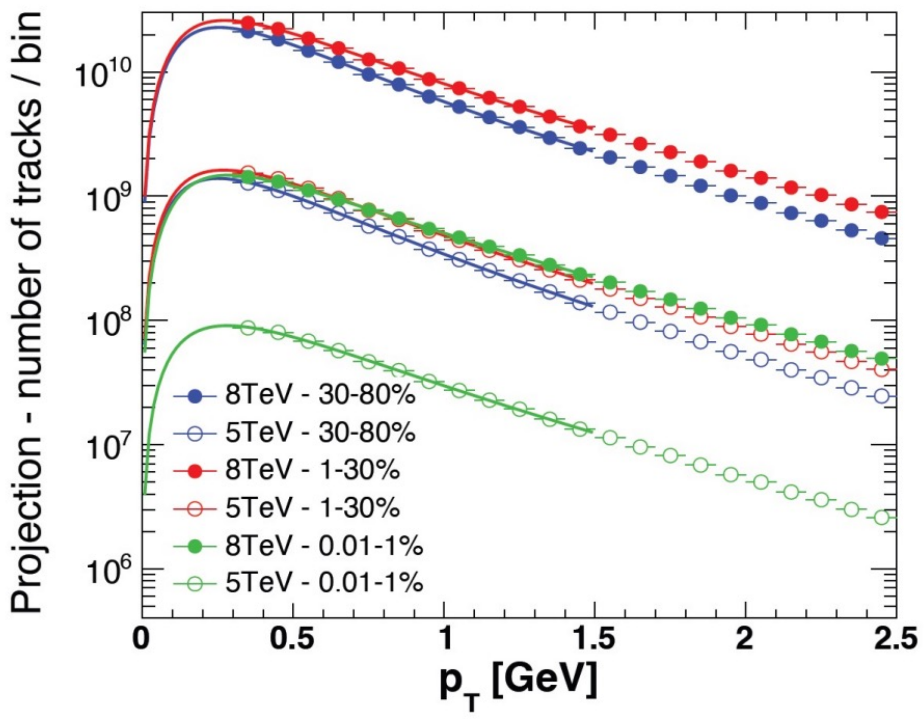
[CMS collaboration, QM2025]

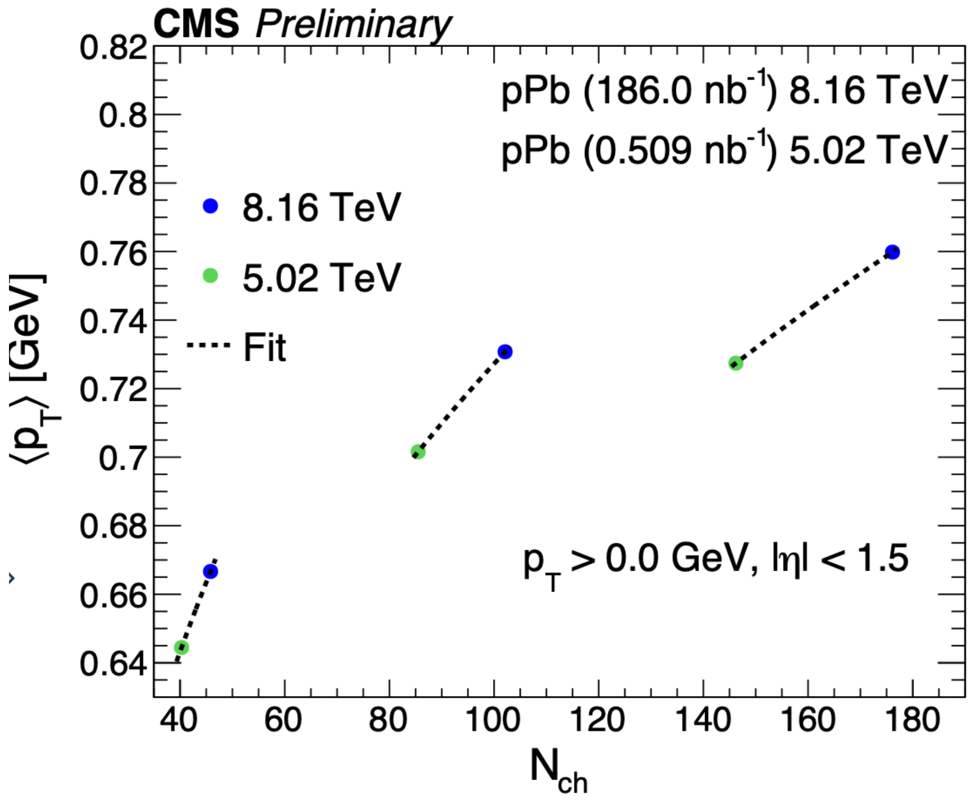
- For both 5.02 and 8.16 TeV, $\langle p_T\rangle/\langle p_T\rangle_0$ initially increases with N_{ch}/N_{ch}^0 at low multiplicities before saturating at higher multiplicities.
- For the 8.16 TeV dataset, an indication of a rise in $\langle p_T\rangle/\langle p_T\rangle_0$ is observed at the highest multiplicities.
- The HIJING model fails to describe the observed trend in the data both qualitatively and quantitatively.

The speed of sound of small systems in experiments

- Energy dependence at fixed centrality

[CMS collaboration, QM2025]

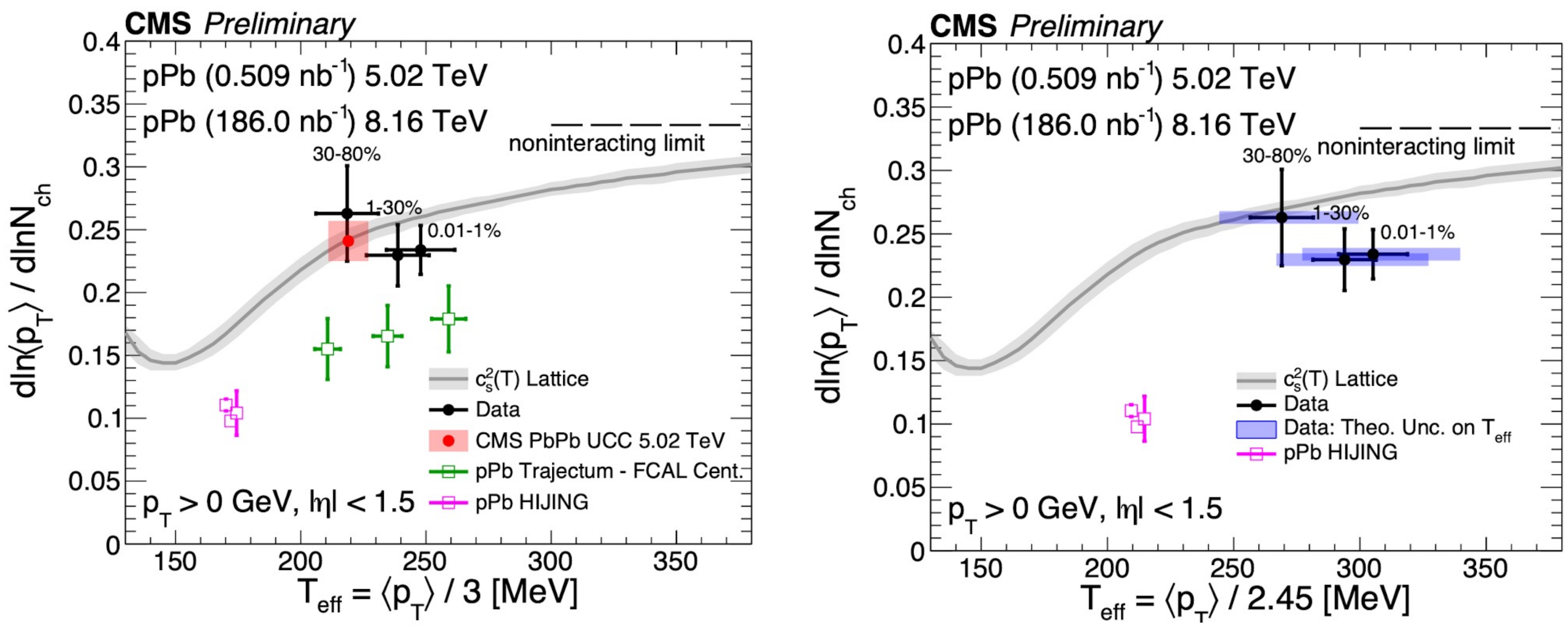




Fit with $\langle p_T \rangle \sim (N_{ch})^{c_s^2}$

The speed of sound of small systems in experiments

- Energy dependence at fixed centrality



[CMS collaboration, QM2025]

- In the boost invariant scenario, p-Pb results exhibit good agreement with lattice QCD prediction.
- In the non-boost invariant scenario, the agreement between p-Pb data and lattice QCD is worse.
- Results from Trjectum model are found to be consistently below the data.
- HIJING fails to describe data in both scenarios.

Two sets of model studying

- Thermal model (Event-by-event hybrid hydro model)

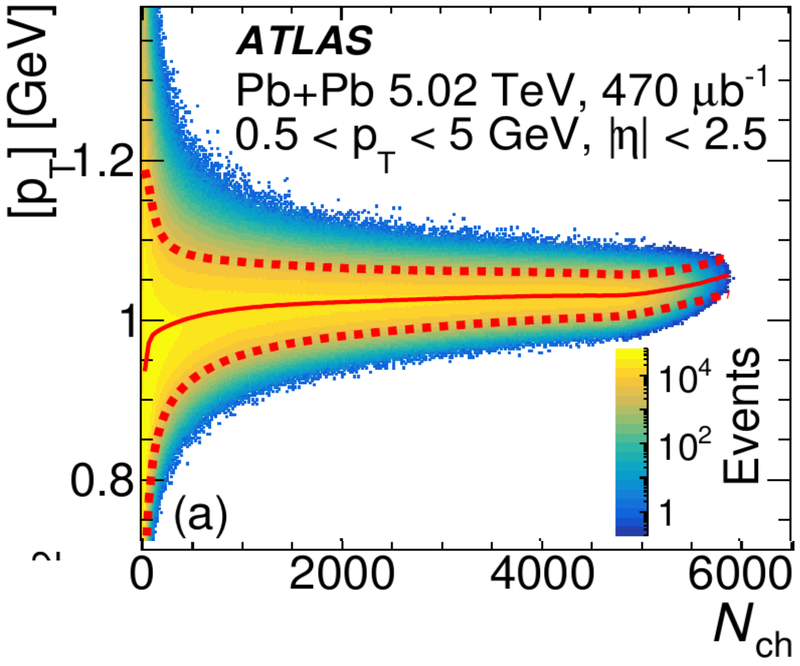
TRENTo + (MUSIC + LQCD) + UrQMD

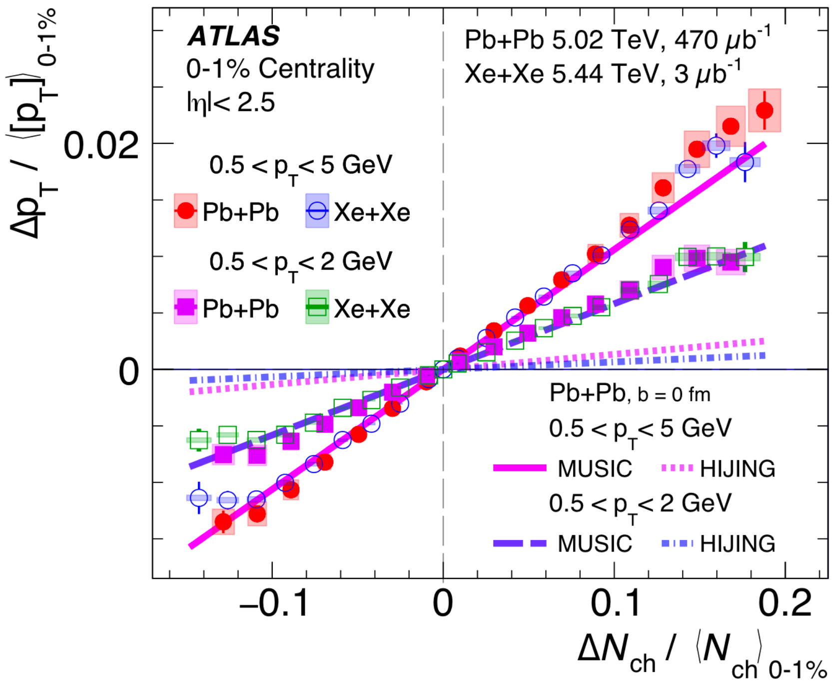
- TRENTo: Initial condition (main source of quantum fluc.)
- (MUSIC + LQCD): Viscous hydro (thermalized QGP)
- UrQMD: non-thermal scatterings

- Non-thermal model

HIJING
Pythia

Problems in linear fit

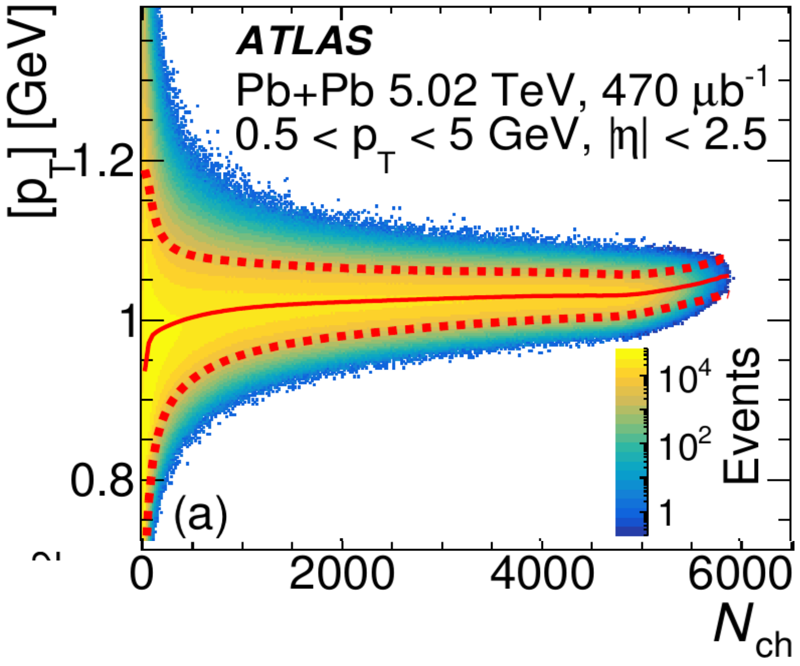


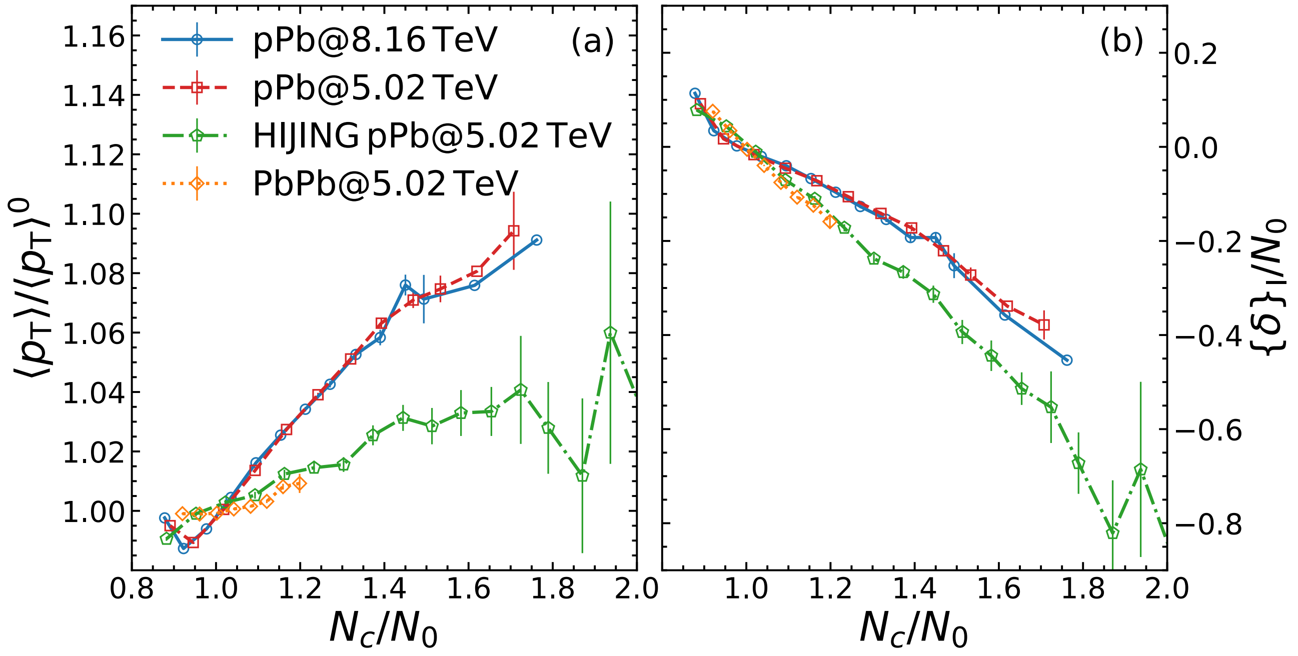


[ATLAS collaboration, PRL 133 (2024) 25, 252301]

- $\frac{\{\Delta_\mathrm{p}\}_I}{\langle \mathrm{p_T} \rangle_0} = \mathrm{c}_s^2 \frac{\{\Delta_\mathrm{N}\}_I}{\mathrm{N}_0}$, where $\{\ \}_I$ denotes event average in the sub-bin I.
- $\{\delta\}_I = -\alpha\{\Delta_\mathrm{N}\}_I$ which leads to $c_s^2 \to c_s^2 - \alpha$
- Quantum fluctuations in the realistic collisions can suppress the slope.

Problems in linear fit

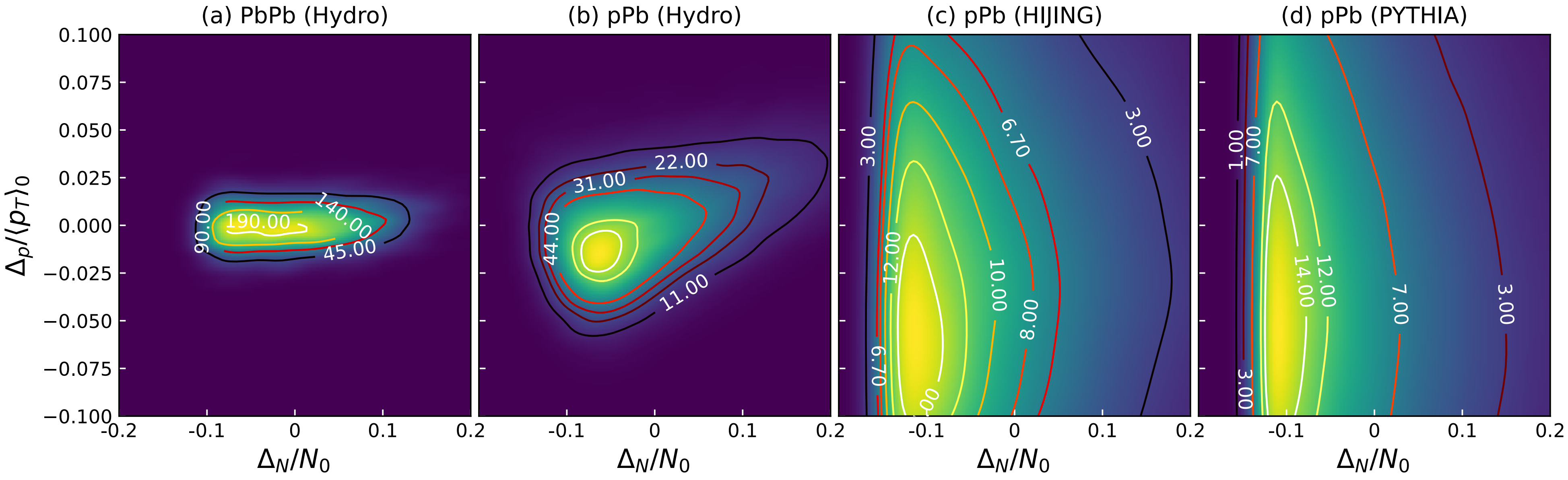




[ATLAS collaboration, PRL 133 (2024) 25, 252301]

- $\frac{\{\Delta_\mathrm{p}\}_I}{\langle \mathrm{p_T}\rangle_0} = \mathrm{c}_s^2 \frac{\{\Delta_\mathrm{N}\}_I + \{\delta\}_I}{\mathrm{N}_0}$, where $\{\ \}_I$ denotes event average in the sub-bin I.
- $\{\delta\}_I = -\alpha\{\Delta_\mathrm{N}\}_I$ which leads to $c_s^2 \rightarrow c_s^2 - \alpha$
- Quantum fluctuations in the realistic collisions can suppress the slope.

2D joint probability distribution $P(\Delta_N, \Delta_p)$



- Presence of quantum fluctuations lead to 2D distribution of Δ_N and Δ_p.
- The tilted tip implies a positive correlation between Δ_N and Δ_p.
- Larger width implies stronger fluctuations.

Extracting the speed of sound in the presence of quantum fluc.

- In a thermalized system,

$$\frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0} \longrightarrow \delta = \frac{N_0}{c_s^2 \langle p_T \rangle_0} \Delta_p - \Delta_N$$

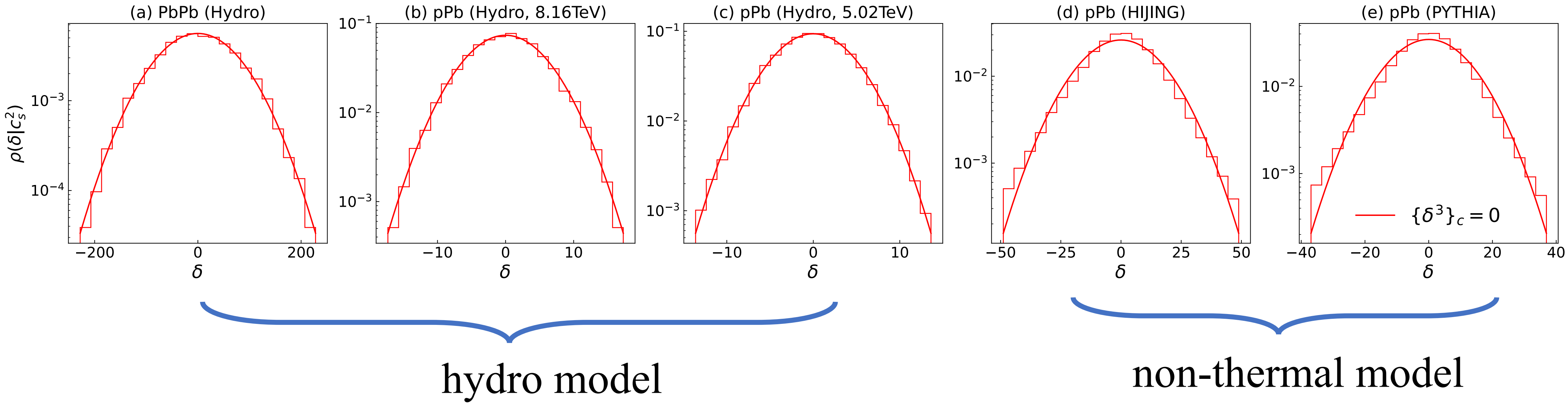
- δ is independent of thermodynamic response.
- Distribution of δ is Gaussian according to Central Limit Theorem (CLT).
- Zero skewness condition: $\{\delta^3\}_c = 0$

$$(c_s^2)^3 \underline{\frac{\{\Delta_N^3\}}{N_0^3}} - 3(c_s^2)^2 \underline{\frac{\{\Delta_N^2 \Delta_p\}}{N_0^2 \langle p_T \rangle_0}} + 3c_s^2 \underline{\frac{\{\Delta_N \Delta_p^2\}}{N_0 \langle p_T \rangle_0^2}} - \underline{\frac{\{\Delta_p^3\}}{\langle p_T \rangle_0^3}} = 0$$

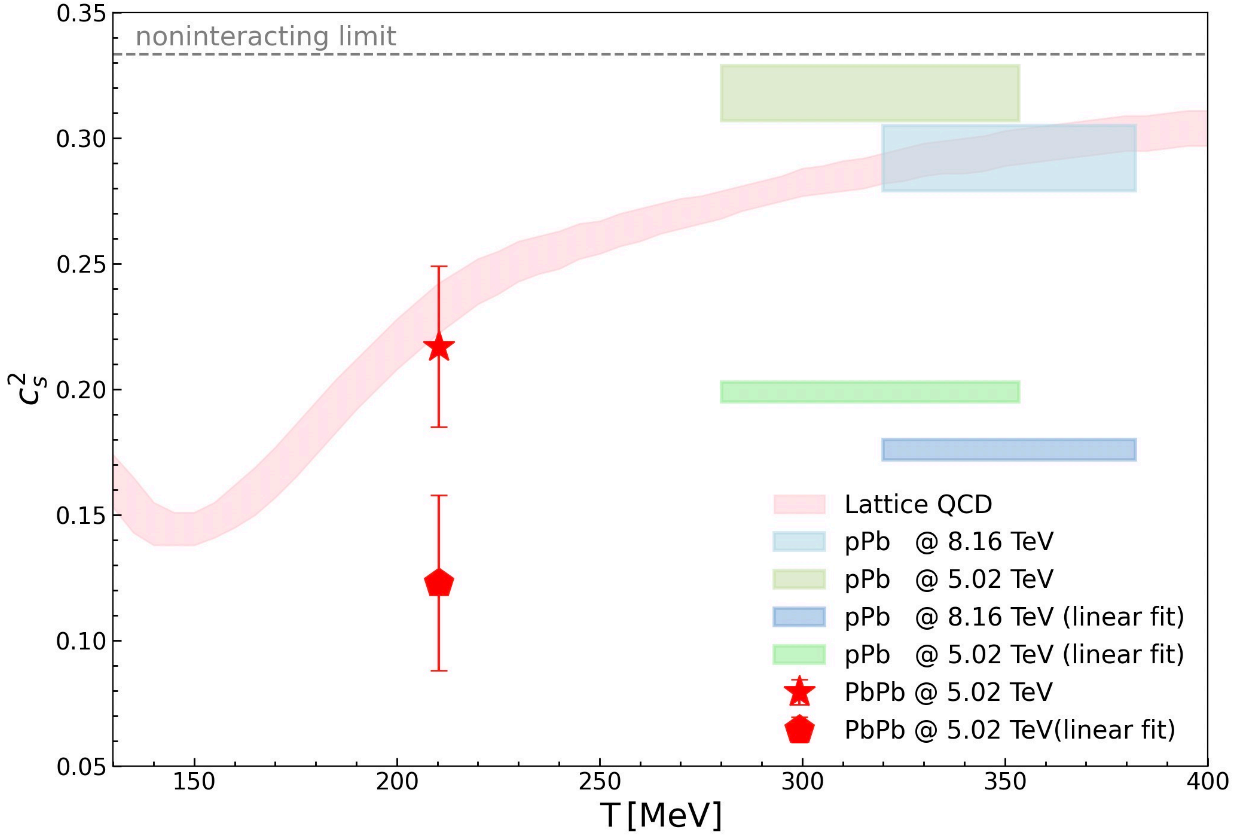
mixed skewness of transverse momentum and charged multiplicity which can be measured in experiments

Verify Gaussianity condition of quantum fluctuations

- Solve the probability distribution of δ with respect to c_s^2 and compare with Gaussian distribution.
- From hydro simulations, δ follow Gaussian distributions.
- From non-thermal models, fluctuations of δ exhibits non-Gaussianity with heavy tails.



Extracted speed of sound from different methods



c_s^2	sub-bin slope	$\{\delta^3\}_c = 0$	$\{\delta^5\}_c = 0$	LEOS
PbPb (Hydro, 5.02TeV)	0.123 ± 0.035	0.217 ± 0.032	0.216 ± 0.041	0.222-0.242
pPb (Hydro, 8.16 TeV)	0.176 ± 0.004	0.292 ± 0.013	0.287 ± 0.012	0.282-0.309
pPb (Hydro, 5.02 TeV)	0.197 ± 0.004	0.318 ± 0.011	0.313 ± 0.008	0.269-0.304

Probe of thermalization

- δ behaves differently in thermalized and non-thermalized systems.

Thermalized system: $$\frac{\Delta_p}{\langle p_T \rangle_0} = c_s^2 \frac{\Delta_N + \delta}{N_0}$$ → thermal response + quantum noise

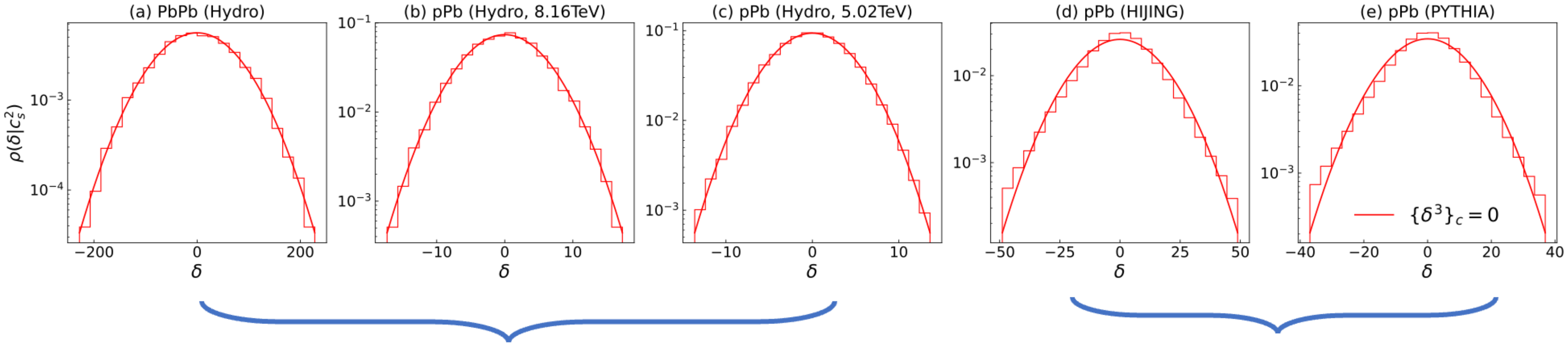
- ➢ The distribution of δ is Gaussian;
- ➢ The extracted speed of sound is physical.

Non-thermalized system: $$\frac{\Delta_p}{\langle p_T \rangle_0} = \kappa \frac{\Delta_N + \delta}{N_0}$$ → quantum response + quantum noise

- ➢ The distribution of δ is non-Gaussian;
- ➢ The extracted value is nonphysical.

Probe of thermalization

- δ behaves differently in thermalized and non-thermalized systems.



hydro model

non-thermal model

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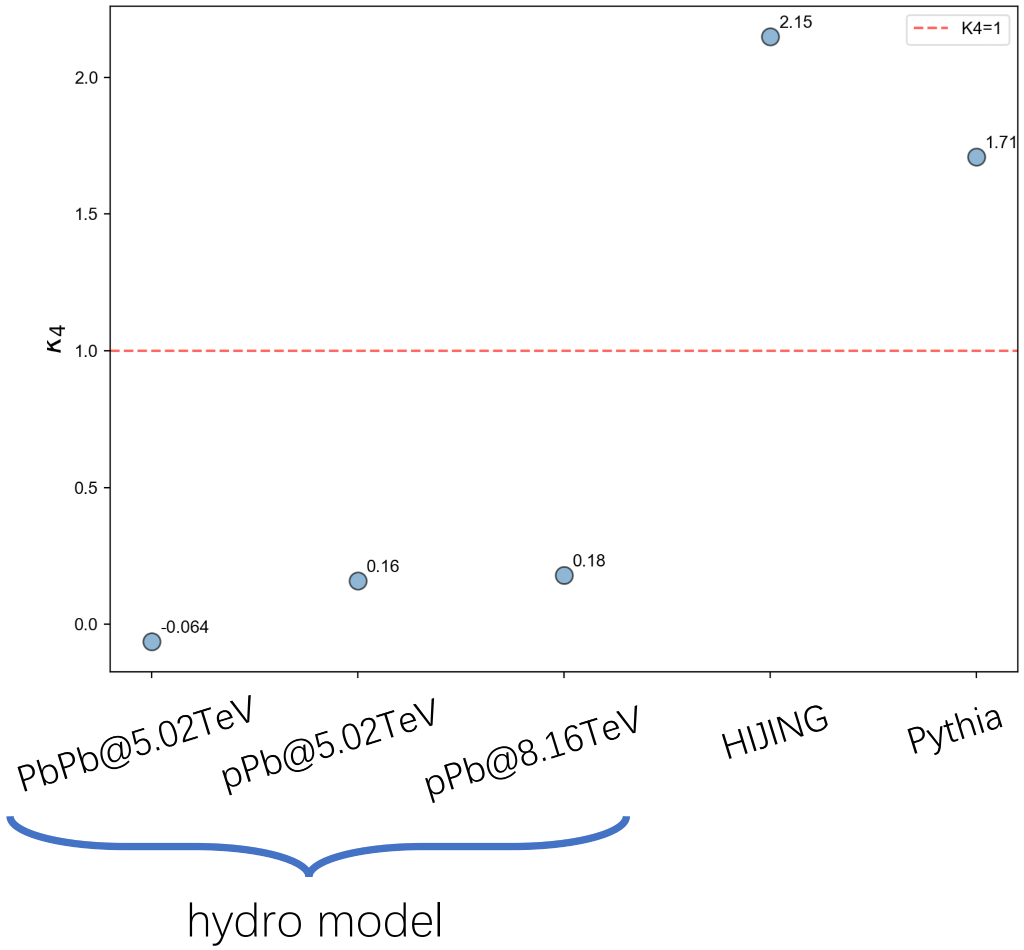
pPb (PYTHIA, 5.02 TeV)	-0.032 ± 0.002	1.178 ± 0.006	1.352 ± 0.019	0.227-0.278
pPb (HIJING, 5.02 TeV)	0.079 ± 0.003	1.104 ± 0.019	1.171 ± 0.053	0.206-0.271

Probe of thermalization

- Deviations from thermalization can be quantified by the standardized kurtosis



$$\kappa_4 = \frac{\{\delta^4\}}{\{\delta^2\}^2} - 3$$

- 0: absolute thermalization
- [0,1] : partial thermalization
- >> 1 : non-thermal

Summary

- The QCD speed of sound can be extracted even in the presence of quantum fluctuations;
- Thermalization can be probed by examining the Gaussianity of quantum fluctuation δ and the physical validity of the extracted speed of sound, which can be quantified by the standardized kurtosis of δ.