Bell inequality violation of light quarks in dihadron pair production at lepton colliders

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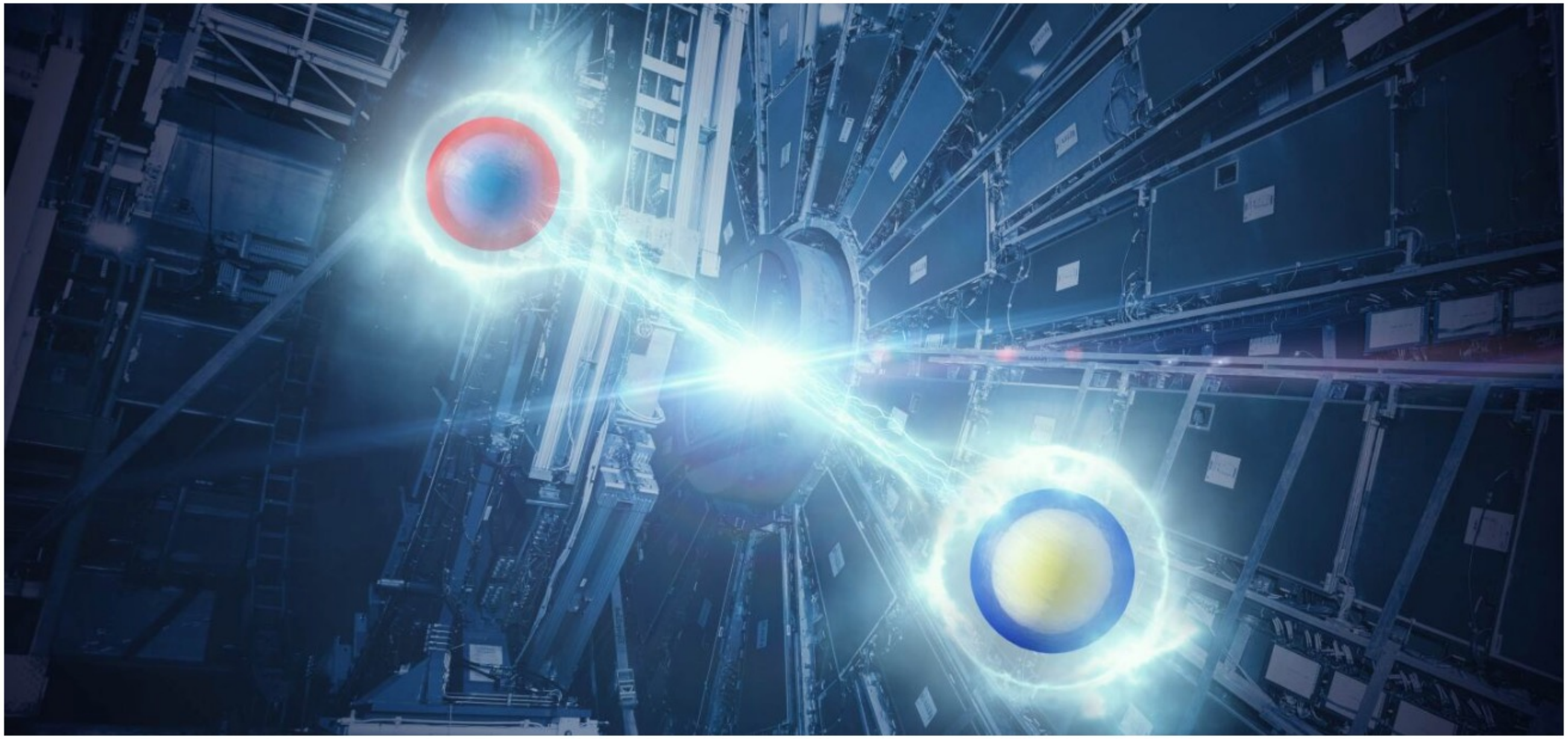
Quantum information at collider

ATLAS and CMS has observed the spin entanglement of top quark pair

The results open up a new perspective on the complex world of quantum physics



Entanglement and Bell inequality

The spin correlation of top quark pair can be described by the general density matrix

$$\rho = \frac{I_2 \otimes I_2 + B_i \sigma_i \otimes I_2 + \bar{B}_i I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

- $B_i, \bar{B}_i$: the polarization of each particle
- C_{ij} : the spin correlation of top quark pair
- Entanglement (non-seperable): concurrence observable

W. K. Wootters, PRL 80 (1998) 2245

$$\mathscr{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

λ_i: eigenvalues of matrix: $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ $\quad \tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$

- Bell's inequality

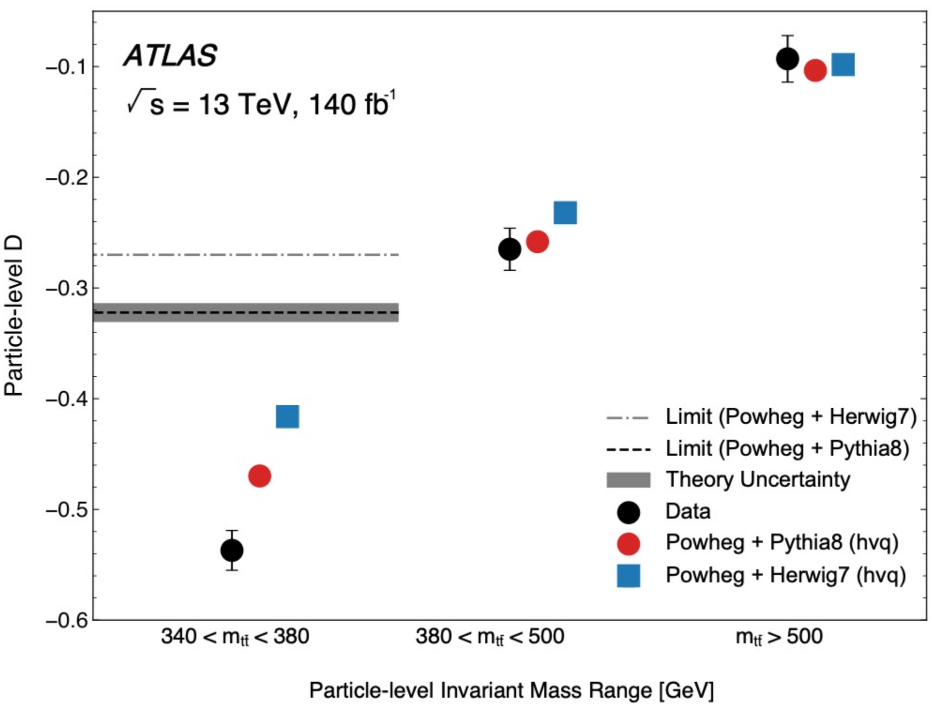
$$\left| \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle \right| \leq 2 \qquad \hat{A} = \vec{a} \cdot \hat{\sigma}, \quad \hat{B} = \vec{b} \cdot \hat{\sigma},$$

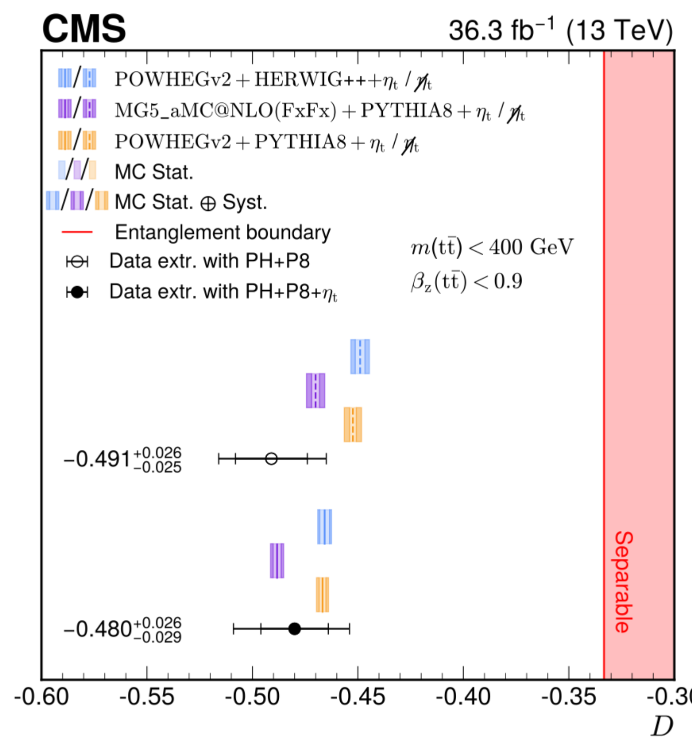
Top quark spin entanglement

Top quark pair: $\mathscr{C}[\rho] = \dfrac{-1-3D}{2}$ $\qquad \dfrac{1}{\sigma}\dfrac{d\sigma}{d\cos\varphi} = \dfrac{1}{2}(1 - D\cos\varphi)$

$$D = \min\left\{\frac{\mathrm{tr}(C)}{3}, \frac{c_1 - c_2 - c_3}{3}, \frac{c_2 - c_1 - c_3}{3}, \frac{c_3 - c_1 - c_2}{3}\right\}, \qquad c_i = \mathrm{eig}(C)$$

Y. Afik, J. de Nova, EPJC 136 (2021) 907





Entangled : $D < -1/3$, separable : $D > -1/3$

Quantum entanglement at colliders

- Top quark pair

Y. Afik, J. R. M. n. de Nova Eur. Phys. J. Plus 136, 907 (2021)
M. Fabbrichesi, R. Floreanini, G. Panizzo, PRL 127, 161801 (2021)
C. Severi. C. D. E. Boschi, F. Maltoni, and M. Sioli, EPJC 82, 285 (2022)
T. Han, M. Low, T. A. Wu, JHEP 07, 192 (2024)
T. Han, M. Low, N. McGinnis, and S. Su, 2412.21158
K. Cheng, T. Han and M. Low, 2410.08303,
…

- Tau lepton pair

M. M. Altakach et al, PRD 107, 093002 (2023)
K. Ehataht et al, PRD 109, 032005 (2024)
Y. Du, X.-G. He, C.-W. Liu and J.-P. Ma, 2409.15418
Y. Zhang et al, 2504.01496
T. Han, M. Low, Y. Su, 2501.04801
…

- Gauge boson pair

A. J. Barr et al, Quantum 7, 1070 (2023)
Q. Bi, Q.-H. Cao, K. Cheng, H. Zhang, PRD 109, 036022 (2024)
R. Ding et al, 2504.09832
…

- Flavor

K. Chen, Z. Xing, R. Zhu, 2407.19242
H. Feng. H. Tang, W. Guo Q. Qin, 2504.15798
….

- Entanglement & NP

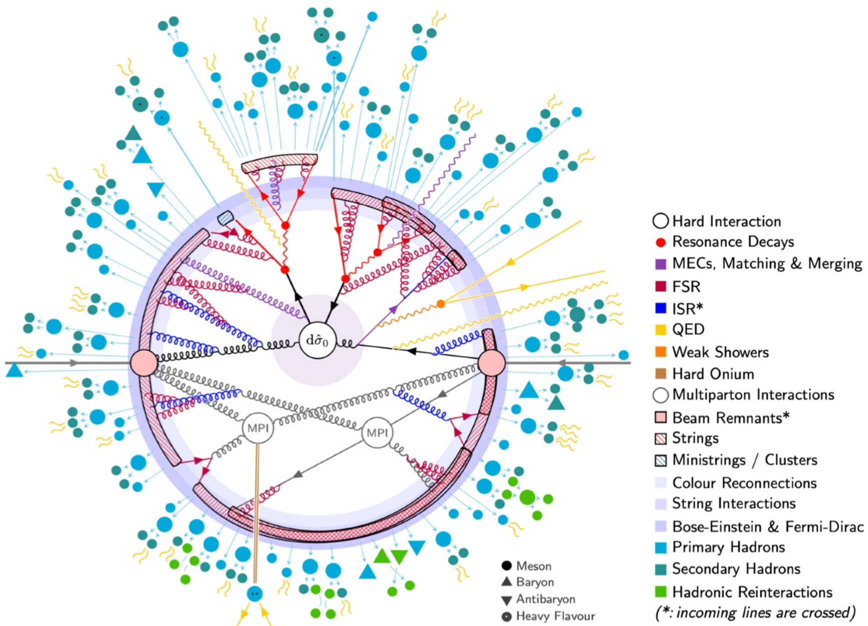
R. Aoude et al, PRD 106 (2022) 055007
M. Fabbrichesi et al, EPJC 83 (2023) 162, JHEP 09 (2023) 195
A. Bernal et al, EPJC 83 (2023) 11, 1050
…

The spin correlation between particles can be measured from its decay products

→ How about the light quarks?

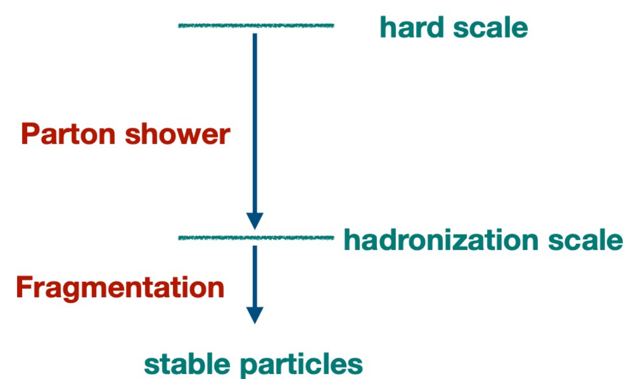
Quantum entanglement of light quarks

- The quark can not be a free particle due to the QCD confinement:



From PYTHIA 8.3

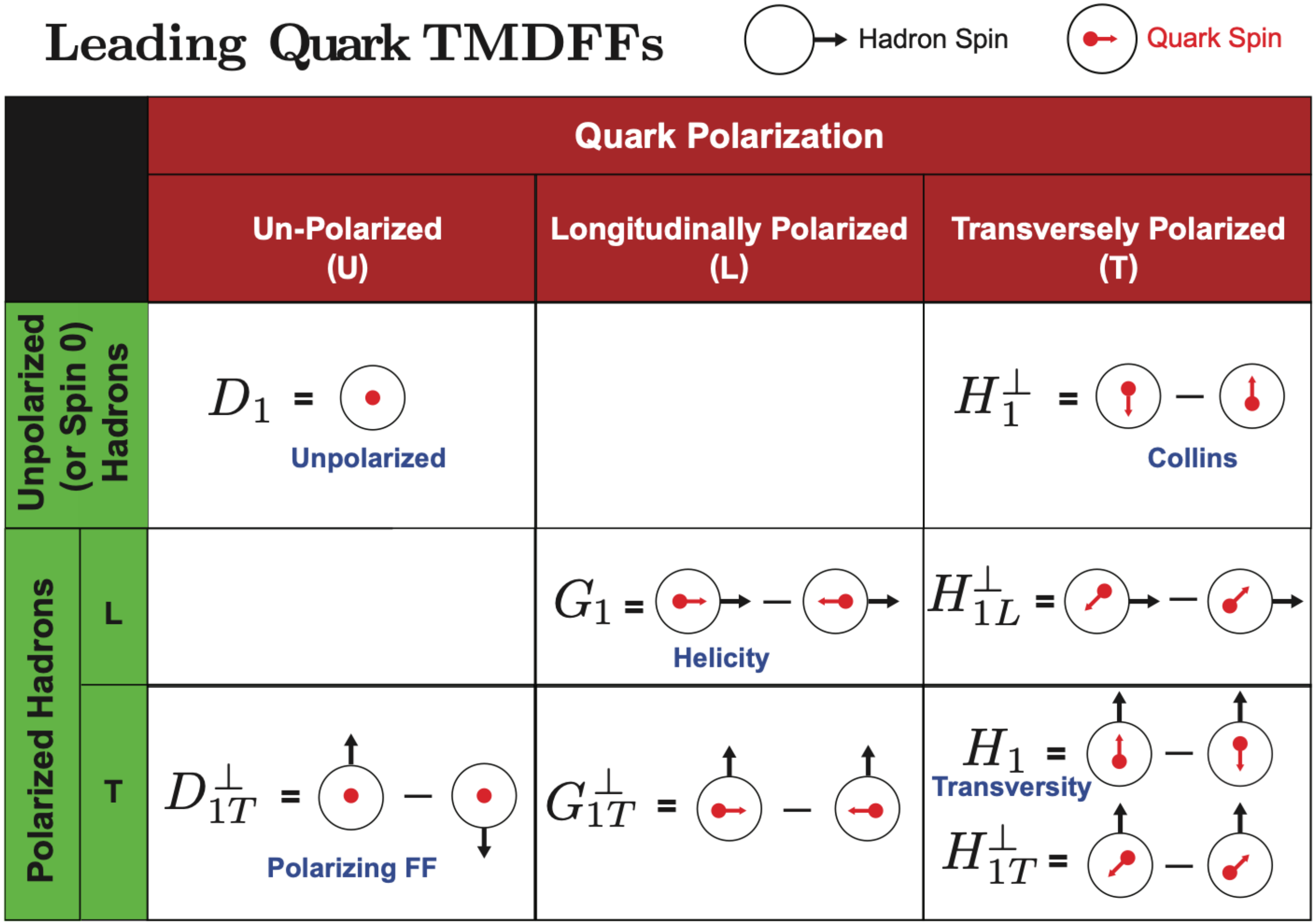
- Hard process in high energy
- Transition from high energy to low energy —parton shower
- Low energy soft regime —fragmentation



- The light quark does not decay but instead fragments into a jet of hadrons after produced from hard scattering

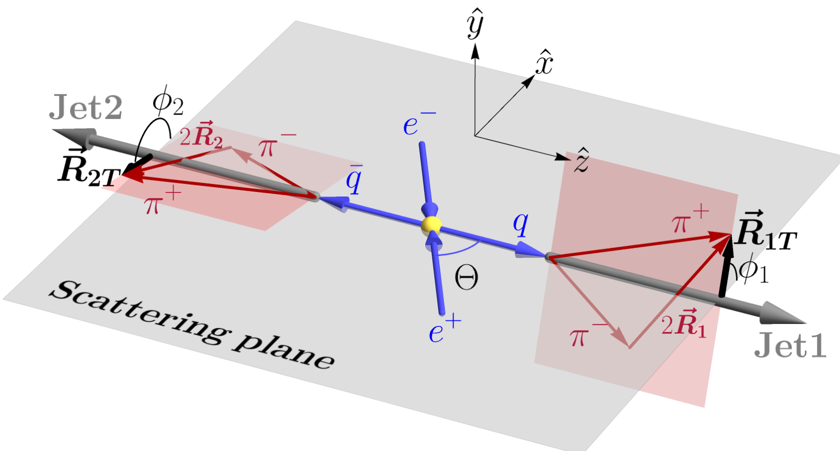
- How to probe the spin information of light quarks?
  The non-perturbative functions: the fragmentation functions

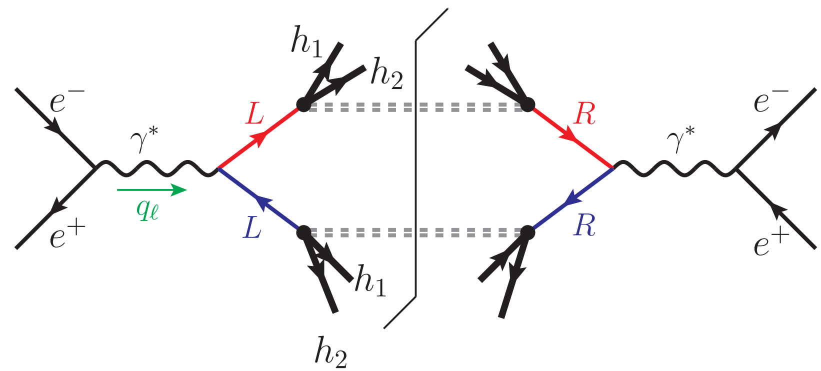
Spin information of light quarks



The spin information of light quarks can transfer to the hadrons: e.g. Collins functions

Dihadron pair production at lepton colldiers





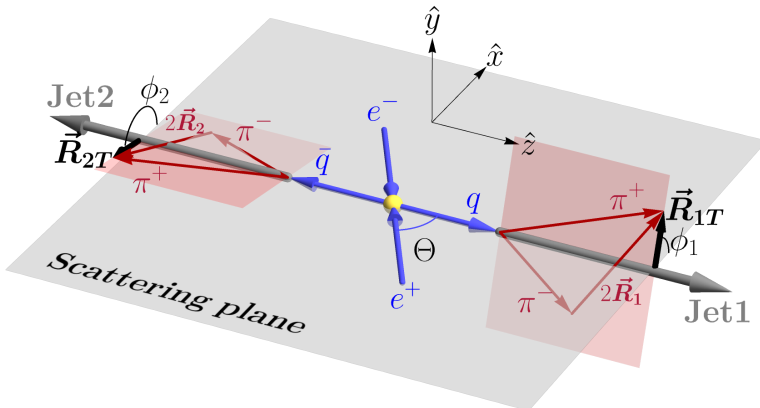
- The transverse spin correlation between light quarks: chiral-odd interference dihadron fragmentations (collinear factorization)

- Light quark pair are 100% correlated in the central scattering region

$$C_{ij} = \mathrm{diag}\left(\frac{\sin^2\Theta}{1+\cos^2\Theta}, -\frac{\sin^2\Theta}{1+\cos^2\Theta}, 1\right)$$

- The maximally entangled Bell state: Bell inequality violation effects

Bell inequality of light quarks



J. C. Collins et al, NPB 420, 565 (1994)

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} = \sigma_{\text{hard}} \sum_{ss'\bar{s}\bar{s}'} \bar{\rho}_{ss',\bar{s}\bar{s}'} \mathcal{D}^{ss'}_{\pi^+\pi^-/q} \mathcal{D}^{\bar{s}\bar{s}'}_{\pi^+\pi^-/\bar{q}}$$

$$\frac{1}{2}\text{Tr}(\mathcal{D}_{\pi^+\pi^-/q}) = D_1^q(z_1, M_1)\,,$$

$$\frac{1}{2}\text{Tr}(\sigma_z \mathcal{D}_{\pi^+\pi^-/q}) = 0,$$

$$\frac{1}{2}\text{Tr}(\sigma_i \mathcal{D}_{\pi^+\pi^-/q}) = -\frac{\varepsilon_T^{ij} R^j_{1,T}}{|\vec{R}_{1,T}|} H_1^{\sphericalangle,q}(z_1, M_1)\,,$$

$$\frac{d\sigma}{dz_1 dz_2 dM_1 dM_2 d\phi_1 d\phi_2} = \sigma_{\text{hard}} \Bigg[\sum_q e_q^2 D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2)$$

Unpolarized diFF

$$+ \frac{1}{2} \sum_q e_q^2 H_1^{\sphericalangle,q}(z_1, M_1) H_1^{\sphericalangle,\bar{q}}(z_2, M_2) \Big(\mathcal{B}_- \cos(\phi_1 + \phi_2) - \mathcal{B}_+ \cos(\phi_1 - \phi_2) \Big) \Bigg]$$

Transverse polarized diFF

$$\mathcal{B}_\pm \equiv C_{xx} \pm C_{yy}$$

$$\mathcal{B}_+ = 0, \quad \mathcal{B}_- = \frac{2\sin^2\Theta}{1 + \cos^2\Theta}.$$

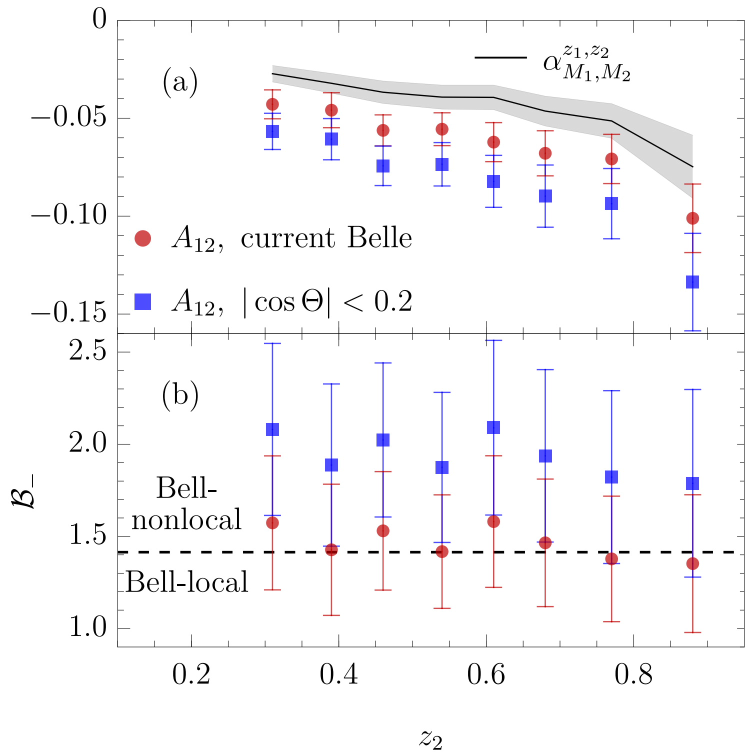
$$\mathcal{B}_- = \frac{2\langle\cos(\phi_1 + \phi_2)\rangle}{\alpha^{z_1,z_2}_{M_1,M_2}} = \frac{A_{12}}{\alpha^{z_1,z_2}_{M_1,M_2}}$$

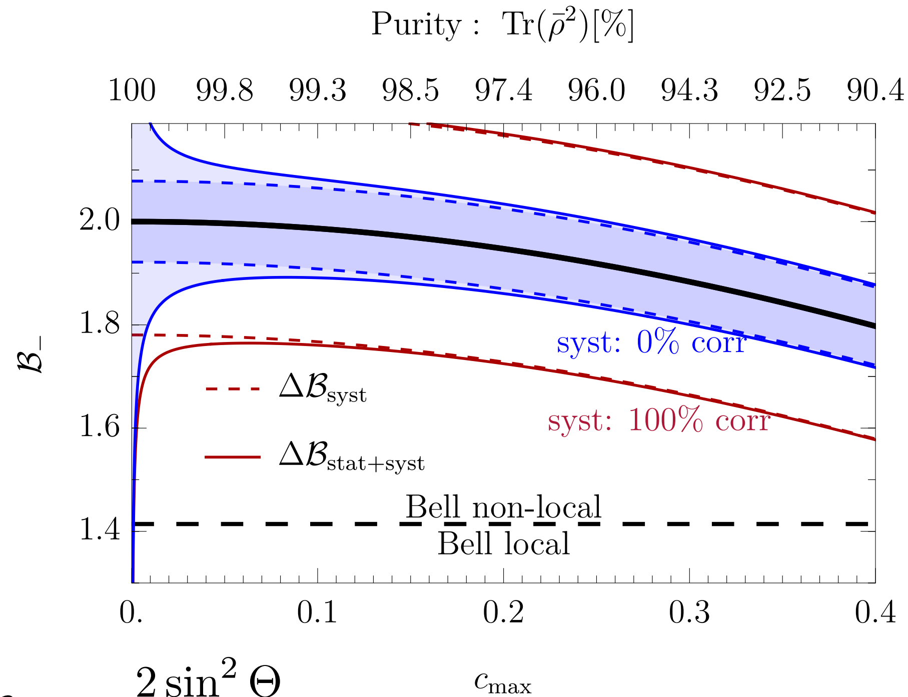
A_{12} → data

$\alpha^{z_1,z_2}_{M_1,M_2}$ → global fitting DiFFs

CHSH type Bell inequality $\quad |\mathcal{B}| > \sqrt{2}$

Dihadron pair production





$$\mathcal{B}_- = \frac{2\sin^2\Theta}{1+\cos^2\Theta}.$$

- Current data exhibiting no significant evidence of Bell inequality violation
- The optimal cuts on scattering angle will significantly improved the results
- The light quark pair would be a highly pure spin Bell state
- Combined results: 2.5 σ for 100% correlated systematic uncertainties and 6.7 σ for the uncorrelated case

Summary

- The era of quantum information study in colliders has just began
- Prior work focused only on massive particles with perturbative decays, leaving massless particles unexplored
- We proposed studying entanglement and Bell inequalities in massless quark pair via the hadron final states by the fragmentation mechanism
- The azimuthal correlations in Belle's $\pi^+\pi^-$ dihadron pair could probe Bell inequality for massless quarks, with $> 5\sigma$ significance when considering uncorrelated systematic uncertainties

Thank you