Quantum Non-locality at Colliders

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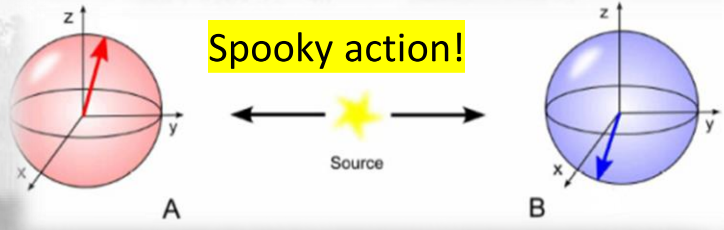
Based on JHEP 10 (2024) 211 & Phys.Rev.D 111 (2025) 3, 036008

◆ Einstein-Podolsky-Rosen——EPR文章（1934），试图证明QM是不完整的。被关联的两个粒子的物理观测量是以某种方式互相关联：测量一个粒子的物理量会影响另一个粒子的观测量，如动量、位置。



Quantum Mechanical Description of physics reality described by wave function is not complete without a "hidden-variable theory"

"Can Quantum Mechanical Description of Physical Reality Be Considered Complete?"

A. Einstein B. Podolski N. Rosen

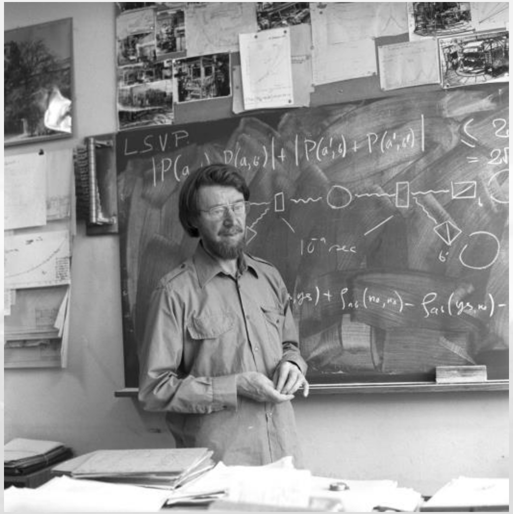
Physical reality must be local! - Podolsky

EPR Paradox

A short history of quantum Entanglement (2)

◆ 贝尔不等式的提出

1964年，物理学家**约翰·贝尔**提出了著名的“贝尔不等式”。贝尔不等式为局域性隐变量理论提供了可检验的预测：如果该理论正确，某些物理量的测量结果应该满足特定的数学关系。



$$|E(a,b)+E(a,b')+E(a',b)-E(a',b')|\leq 2$$

量子力学是非局域性理论！

Observation by LHC experiments

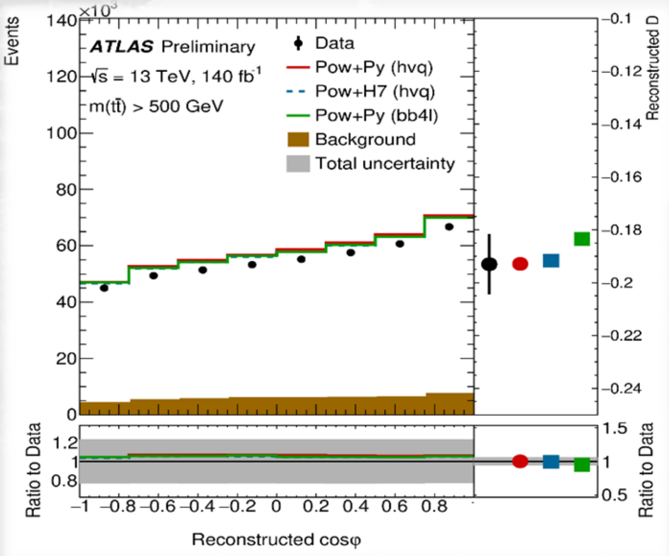
Entanglement marker

$$D = \frac{tr[C]}{3} = -3\,\langle cos\varphi \rangle$$

$$D \le -\frac{1}{3}$$ Entanglement condition

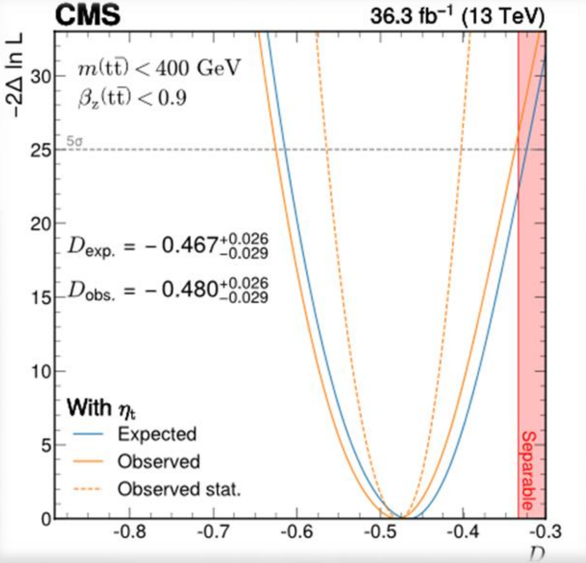
Y. Afik
Eur. Phys. J. Plus **136**, 907 (2021).

ATLAS collaboration



ATLAS Collaboration
Nature **633**, 542 (2024).

CMS collaboration



CMS collaboration.
Rep. Prog. Phys. **87**, 117801 (2024).

Two-qutrit : $VV(ZZ, WW)$

- **Bell-type inequality（CGLMP）**

$$I_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \leq 2$$

$$I_3 = \langle \mathcal{O}_{\text{Bell}} \rangle = \text{Tr}\{\rho\, \mathcal{O}_{\text{Bell}}\} \leq 2$$

A.J. Barr et.al, Prog.Part.Nucl.Phys. 139 (2024) 104134

- **Spin density matrix (SDM)**
  - **Gell-Mann Parameterization**

$$\rho(\lambda_1, \lambda_1', \lambda_2, \lambda_2') = \left(\frac{1}{9}[\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [T^a \otimes \mathbb{1}] + \sum_a g_a [\mathbb{1} \otimes T^a] + \sum_{ab} h_{ab} \left[T^a \otimes T^b\right]\right)_{\lambda_1 \lambda_1' \lambda_2 \lambda_2'}$$

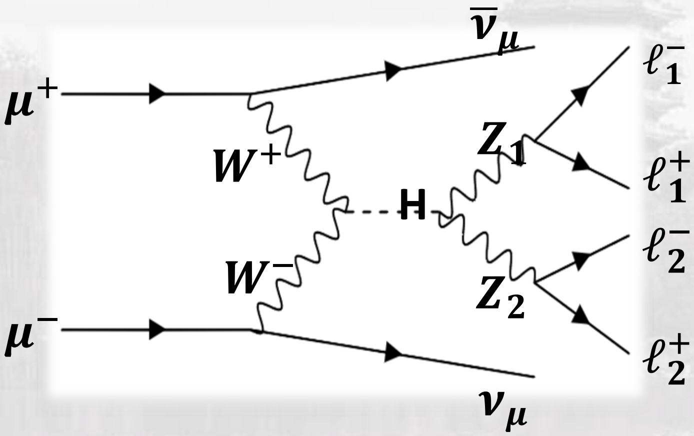
  - **Irreducible Tensor Parameterization**

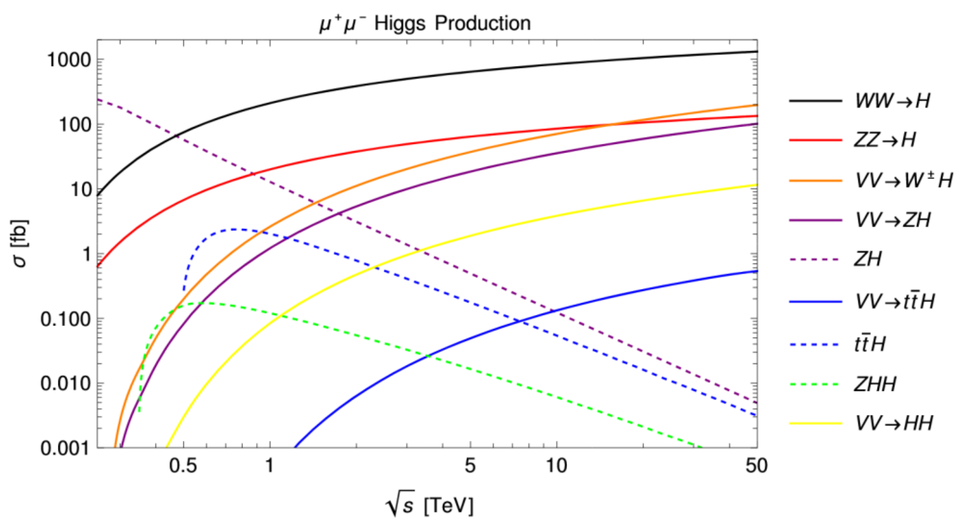
$$\rho = \frac{1}{9}\left[\mathbb{1}_3 \otimes \mathbb{1}_3 + A^1_{LM}\, T^L_M \otimes \mathbb{1}_3 + A^2_{LM}\, \mathbb{1}_3 \otimes T^L_M + C_{L_1 M_1 L_2 M_2}\, T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right]$$

J.A Aguilar et.al PhysRevD.107.016012

$H \to ZZ \to \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$

- Large number of Higgs events from Vector Boson Fusion (VBF)





Muon Collider Forum Report

Quantum State Tomography（量子层析）

- Differential Cross section: $zz \to \ell_1^+\ell_1^-\ell_2^+\ell_2^-$ (parent particle's rest frame)

J.A Aguilar et.al
PhysRevD.107.016012

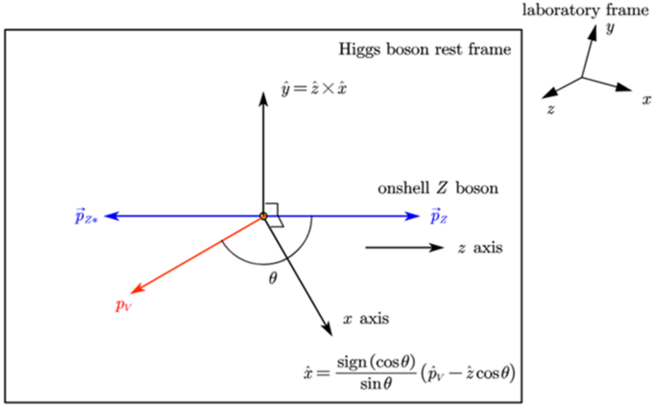
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left\{\rho\left(\Gamma_1 \otimes \Gamma_2\right)^T\right\} \qquad \Gamma \propto \mathcal{M}^\dagger \mathcal{M}$$

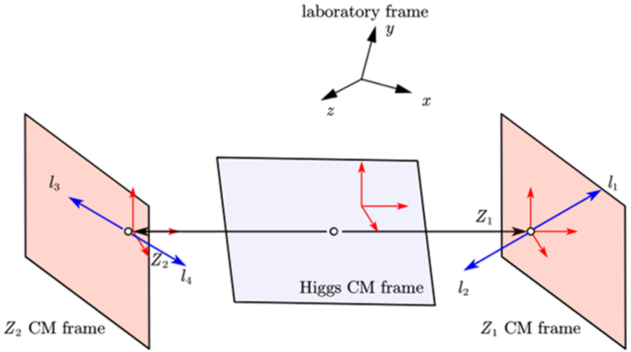
$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + A^1_{LM} B_L Y_L^M(\theta_1,\varphi_1) + A^2_{LM} B_L Y_L^M(\theta_2,\varphi_2)$$
$$+ C_{L_1 M_1 L_2 M_2} B_{L_1} B_{L_2} Y_{L_1}^{M_1}(\theta_1,\varphi_1) Y_{L_2}^{M_2}(\theta_2,\varphi_2)\Big]$$

- Statistical average over events gives the polarization and correlation coefficients

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_L^M(\Omega_j) d\Omega_j = \frac{B_L}{4\pi} A^j_{LM}, \qquad j = 1,2\,.$$
$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_{L_1}^{M_1}(\Omega_1) Y_{L_2}^{M_2}(\Omega_2) d\Omega_1 d\Omega_2 = \frac{B_{L_1} B_{L_2}}{(4\pi)^2} C_{L_1 M_1 L_2 M_2}\,,$$

Reference frame: helicity basis



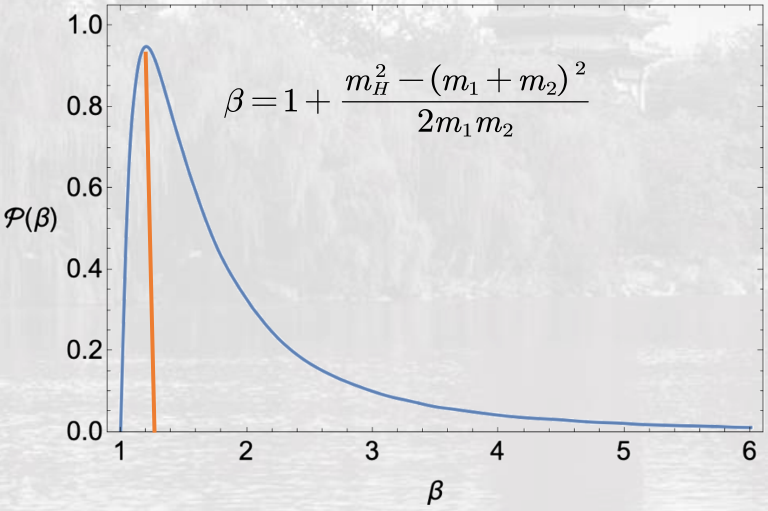


- The $\hat{z}$ axis is taken in the direction of the Z_1 three-momentum in the Higgs boson rest frame.
- The $\hat{x}$ axis is in the production plane and defined as $\hat{x} = \text{sign}(\cos\theta)(\hat{p}_p - \cos\theta\hat{z})/\sin\theta$, with $\hat{p}_p = (0,0,1)$ the direction of one proton in the laboratory frame, $\cos\theta = \hat{z}\cdot\hat{p}_p$. The definition for $\hat{x}$ is the same if we use the direction of the other proton $-\hat{p}_p$.
- The $\hat{y}$ axis is taken such that $\hat{y} = \hat{z} \times \hat{x}$, orthogonal to the production plane.

Quantum State of $H \to ZZ$

$$|\psi_{ZZ}\rangle = \frac{1}{\sqrt{2+\beta^2}}\left(|+-\rangle - \beta|00\rangle + |-+\rangle\right)$$

Only 9 entries are non-zero!



$$\rho = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\beta & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta & 0 & \beta^2 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\beta & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Numerical results

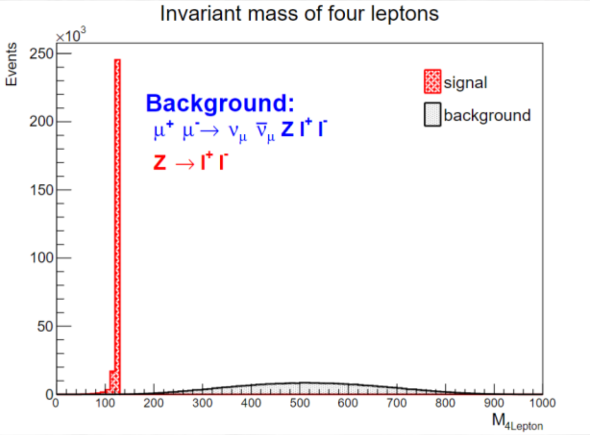
◆ $\mu^+\mu^- \to H \to 4\ell$ Events

Simulation Results from Madgraph

$\sqrt{s_\mu}$[TeV]	σ[fb]	Luminosity	Events
1	1.51×10^{-2}	30 ab^{-1}	455
3	3.56×10^{-2}	30 ab^{-1}	1089
10	6.06×10^{-2}	30 ab^{-1}	1890

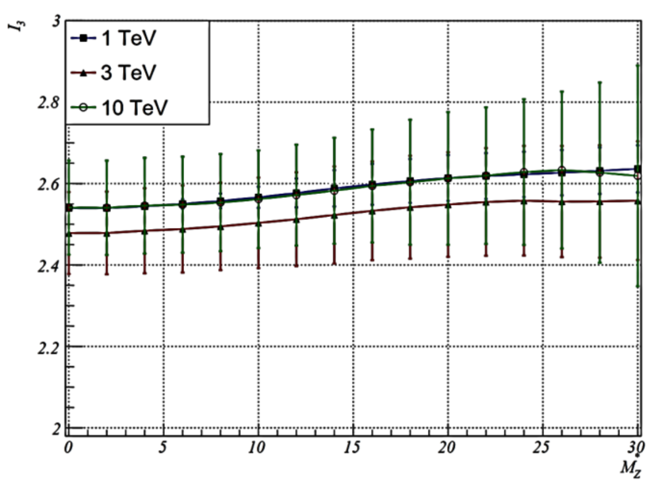
◆ On-shell and off-shell bosons are identified based on the invariant mass of the lepton pairs.

◆ Almost background free.



Numerical Results

◆ $\mu^+\mu^- \to H \to 4\ell$ Events



A very good manifestation of statistical behavior, towards large M_Z, smaller events, thus large uncertainty.

The expectation value of I_3 as function of the off-shell Z mass M_Z^* .

Numerical Results

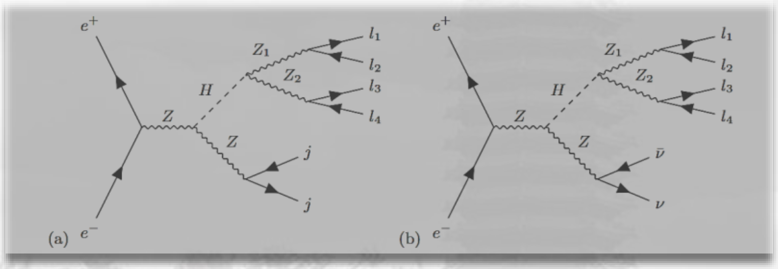
- Entanglement criteria:

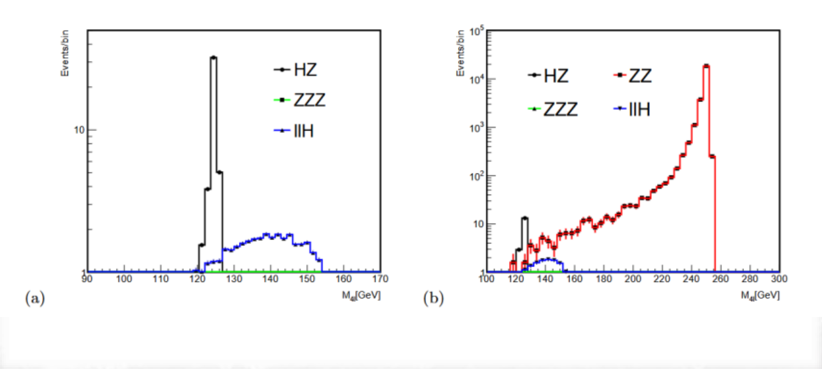
$\sqrt{s} = 1$ TeV			
M_{Z_2} (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.563 ± 0.325	-0.928 ± 0.216	0.527 ± 0.164
10.000	2.596 ± 0.335	-0.943 ± 0.220	0.553 ± 0.179
20.000	2.654 ± 0.373	-0.977 ± 0.248	0.574 ± 0.192
30.000	2.663 ± 0.508	-0.979 ± 0.334	0.589 ± 0.248

$\sqrt{s} = 3$ TeV			
M_{Z_2} (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.467 ± 0.217	-0.871 ± 0.121	0.493 ± 0.377
10.000	2.499 ± 0.225	-0.891 ± 0.135	0.502 ± 0.390
20.000	2.538 ± 0.254	-0.908 ± 0.163	0.536 ± 0.365
30.000	2.543 ± 0.342	-0.890 ± 0.216	0.606 ± 0.423

$\sqrt{s} = 10$ TeV			
M_{Z_2} (GeV)	I_3	$C_{2,1,2,-1}$	$C_{2,2,2,-2}$
0.000	2.539 ± 0.312	-0.930 $\pm$ 0.196	0.466 ± 0.232
10.000	2.569 ± 0.295	-0.946 $\pm$ 0.194	0.482 ± 0.217
20.000	2.616 ± 0.321	-0.969 $\pm$ 0.218	0.514 ± 0.219
30.000	2.644 ± 0.517	-0.943 $\pm$ 0.334	0.527 ± 0.280

Application in CEPC





M_Z^* [GeV]	$\mathcal{I}_3$	C_{212-1}	C_{222-2}
0	$2.823 \pm 0.640 (1.29\sigma)$	$-1.080 \pm 0.420 (2.57\sigma)$	$0.637 \pm 0.559 (1.14\sigma)$
10	$2.913 \pm 0.692 (1.32\sigma)$	$-1.126 \pm 0.451 (2.50\sigma)$	$0.677 \pm 0.598 (1.13\sigma)$
20	$3.092 \pm 0.800 (1.37\sigma)$	$-1.225 \pm 0.514 (2.38\sigma)$	$0.761 \pm 0.734 (1.04\sigma)$
30	$3.048 \pm 1.816 (0.58\sigma)$	$-1.160 \pm 1.192 (0.97\sigma)$	$0.875 \pm 1.338 (0.65\sigma)$

Testing Bell inequalities and probing quantum entanglement at CEPC

Youpeng Wu[1,*], Ruobing Jiang[1,†], Alim Ruzi[1,‡], Yong Ban[1,§], Xueqing Yan[1,2], and Qiang Li[1,∥]

Summary

- Quantum mechanics is non-local
- No local hidden variable theory is compatible with quantum theory
- Higgs decay can provide entangled qubit and qutrit system
- Quantum entanglement is still present at extremely relativistic environment.