陕西师范大学 SHAANXI NORMAL UNIVERSITY

A Non-linear Representation of General Scalar Extensions of the SM for HEFT Matching

万霞 **Xia WAN (Shaanxi Normal U)**

Collaborate with 宋华洋 Huayang Song (IBS-CTPU-PTC,Korea)

2412.00355(JHEP),

2503.00707(JHEP)

第30届LHC Mini-Workshop, 洛阳

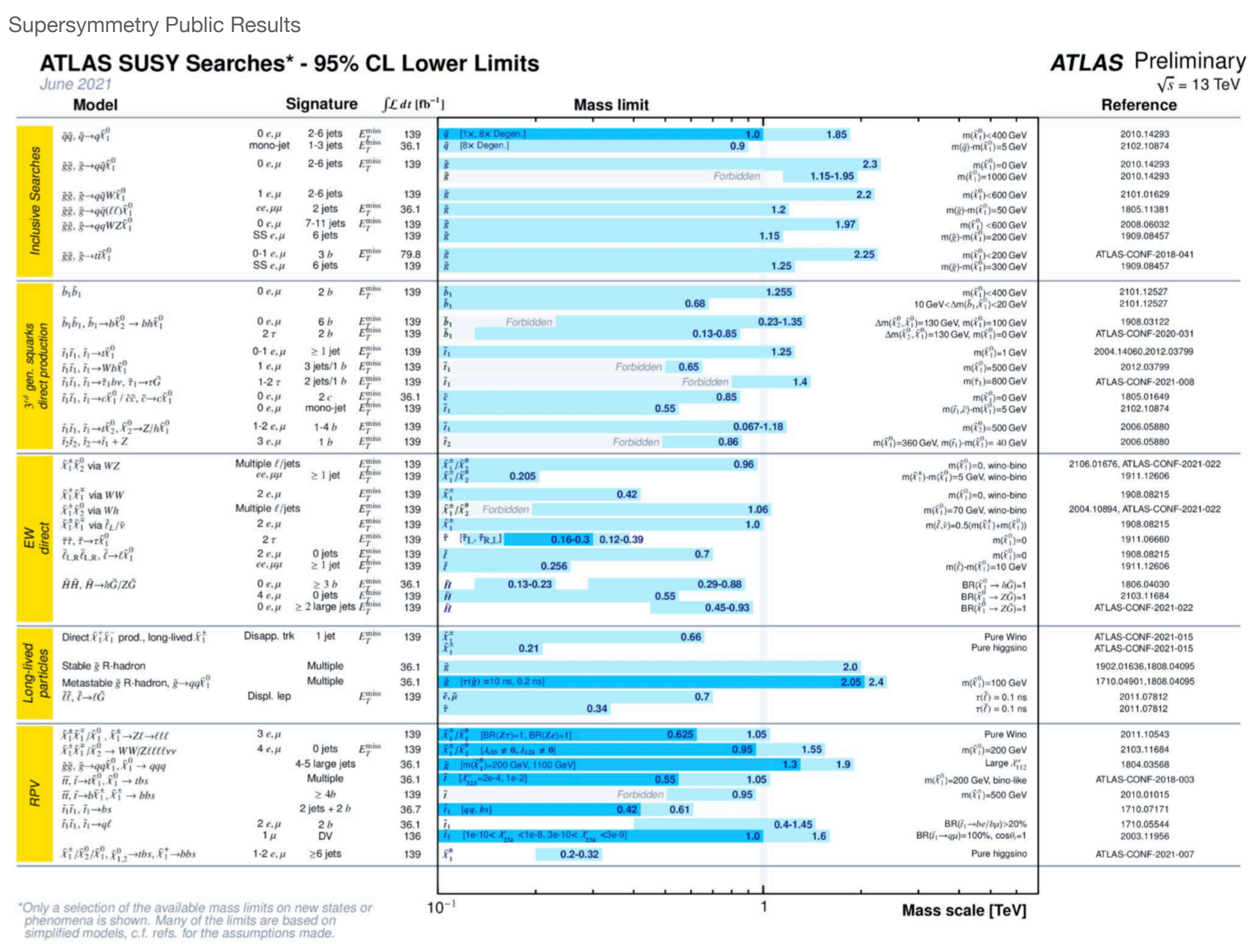
May 24th, 2025.

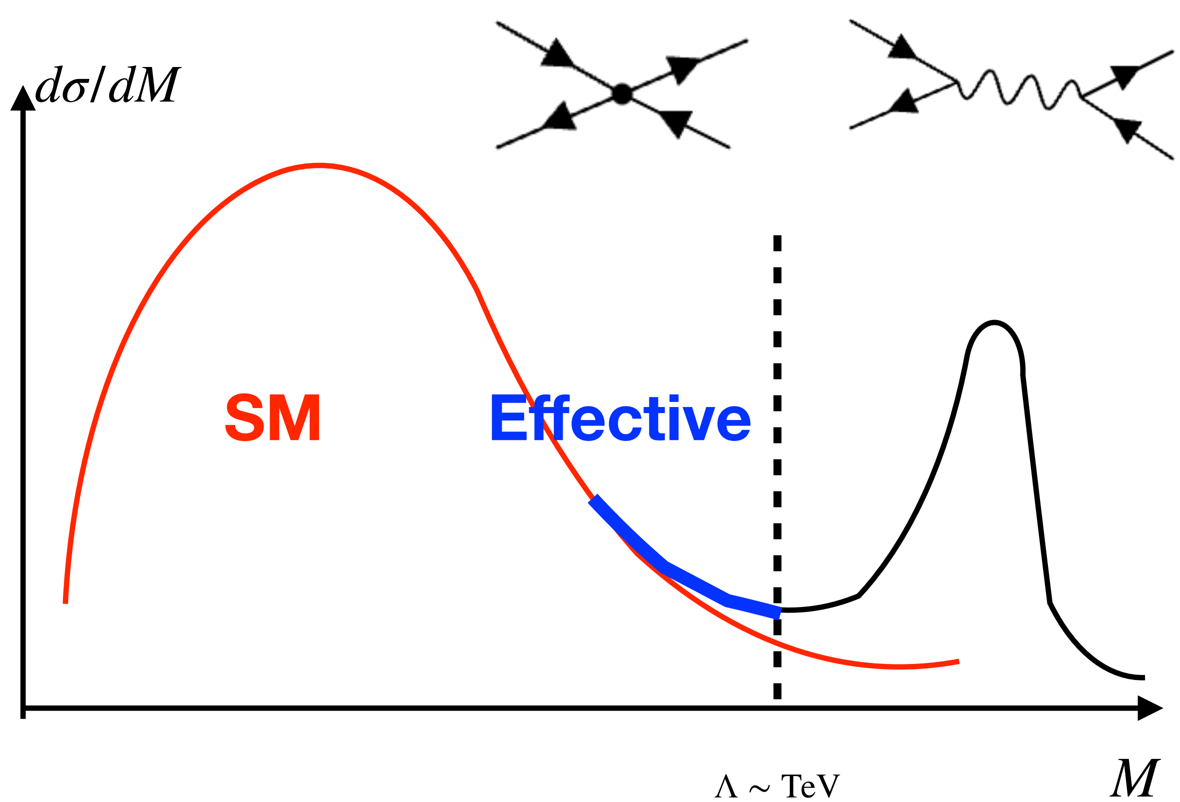
Outline

- Introduction and motivation: EFT, Matching, SMEFT, HEFT.
- The non-linear representation of general scalar extensions for HEFT matching
- Matching HEFT to the Real Higgs Triplet Extension
- Summary and discussion

Why EFT

- Until now, Higgs and **nothing** in direct search of new particles.



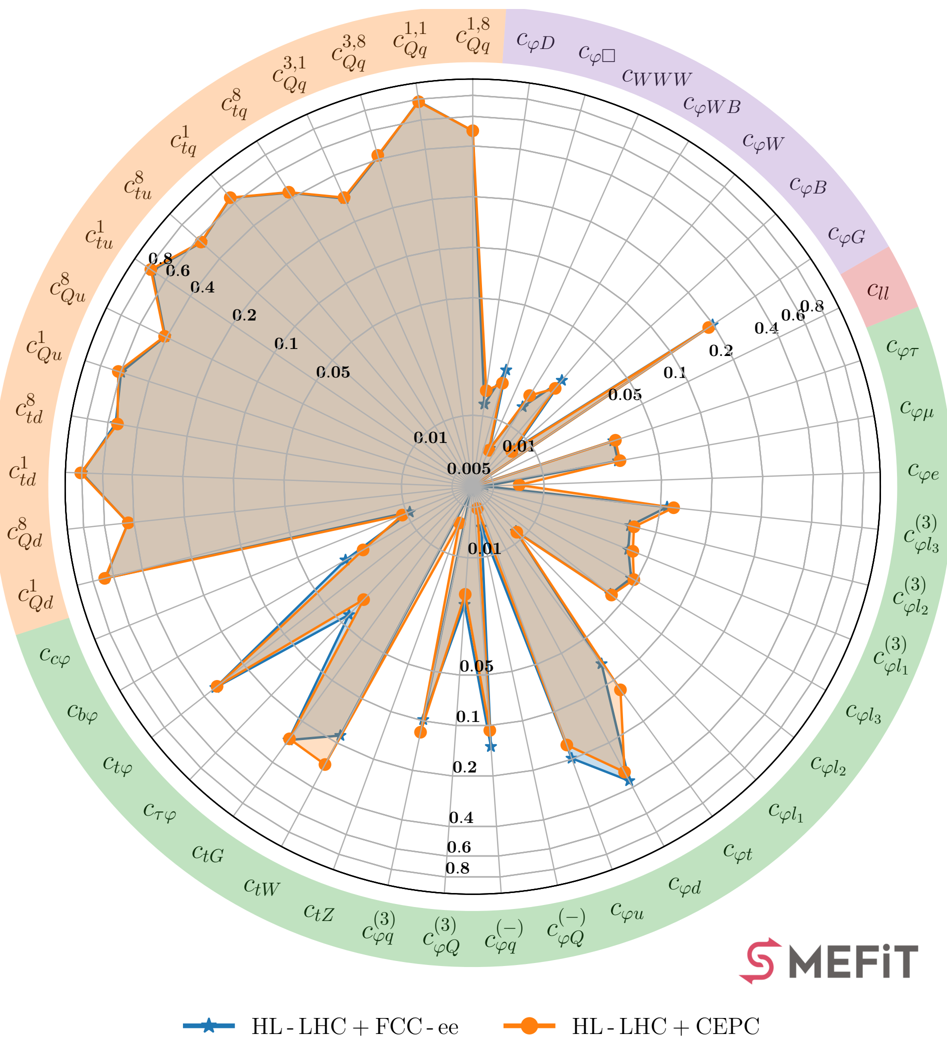


Next strategy of searching for new physics

1. Colliders of higher energy, e.g. muon collider, direct search.
2. Colliders of high luminosity, e.g. CEPC, Indirect effects.
   a. Models (Thousands)
   b. EFT (Model independent, data-driven)

Matching

$$\mathscr{L}_{\text{SMEFT}} \supset \mathscr{L}_{\text{SM}} + \frac{C_H}{\Lambda^2}(H^\dagger H)^3 + \frac{C_{H\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \frac{C_{HD}}{\Lambda^2}(D_\mu H^\dagger H)(H^\dagger D^\mu H) + \cdots$$



From Jaco ter Hoeve, ICHEP 2024

1. Construct a complete basis.
   [H. Sun, M.-L. Xiao, and J.-H. Yu, 2206.07722]
2. Data analysis: simulation and fitting.
3. Matching: relate the Wilson coefficients to the masses and couplings of UV models.

Two EFTs: SMEFT and HEFT

Both are invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry and contains SM fields.

- SMEFT, linear realization of the Higgs and Goldstones, canonical dimension

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ v + h + iG^0 \end{pmatrix}, \quad \mathscr{L}_{\text{SMEFT}} \supset \frac{1}{2} D_\mu H^\dagger D^\mu H + \frac{m^2}{2} H^\dagger H - \lambda (H^\dagger H)^2 + \frac{C_H}{\Lambda^2} (H^\dagger H)^3 + \cdots$$

- HEFT, nonlinear realization, chiral dimension

$$h, U \equiv \exp\left(\frac{i\pi_i \sigma_i}{v_{\text{EW}}}\right), \quad \mathscr{L}_{\text{HEFT}}^{\text{LO}} \supset \frac{1}{2} D_\mu h D^\mu h - V(h) + \frac{v_{\text{EW}}^2}{4} F(h) \text{Tr}(D_\mu U^\dagger D^\mu U) + \cdots$$

$$V(h) = \frac{1}{2} m_h^2 h^2 \left[1 + (1 + \Delta\kappa_3) \frac{h}{v_{\text{EW}}} + \cdots\right], \quad F(h) = 1 + 2(1 + \Delta a) \frac{h}{v_{\text{EW}}} + \cdots$$

[H. Sun, M.-L. Xiao, and J.-H. Yu, 2206.07722]

HEFT is similar to chiral perturbation theory (χ^{PT}) in scalar sector.

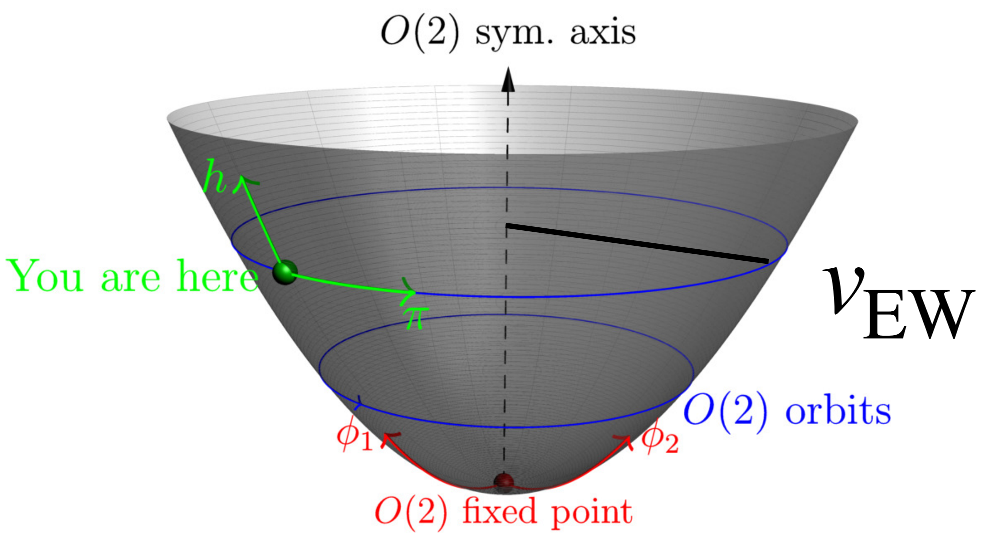
- Goldstones are embedded in the U matrix.
- Power counting use chiral dimension. e.g. p^2, p^4.
- However, Higgs is a **general** scalar, not necessarily composite.

A Geometric Picture

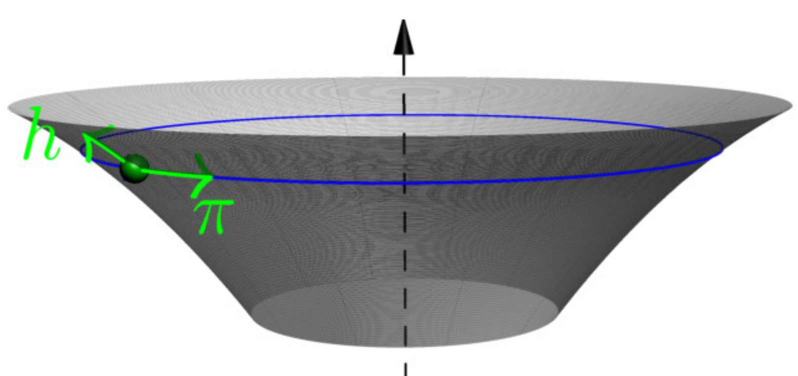
SM doublet

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \longrightarrow \quad \vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \quad \vec{\phi} \to O\vec{\phi}, \text{where} \quad O \in O(4) \supset SU(2) \times U(1)$$

$$\mathcal{L}_{\rm SMEFT} = \frac{1}{2} A(\vec{\phi}\cdot\vec{\phi})(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2} B\left(\vec{\phi}\cdot\vec{\phi}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V\left(\vec{\phi}\cdot\vec{\phi}\right) + \mathcal{O}\left(\partial^4\right)$$

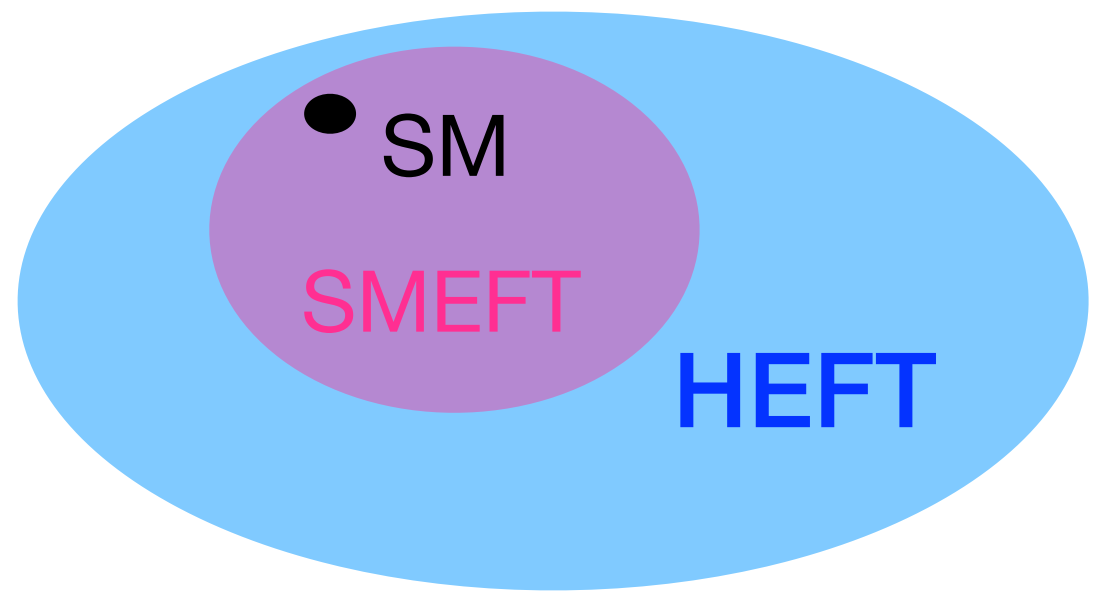


SMEFT/HEFT



from N. Craig, HEFT2021

only HEFT



HEFT encompasses SMEFT

R. Alonso, E. Jenkins, A. Manohar [1511.00724,1605.03602]

Matching a UV Model to HEFT

Through matching we would like to study,

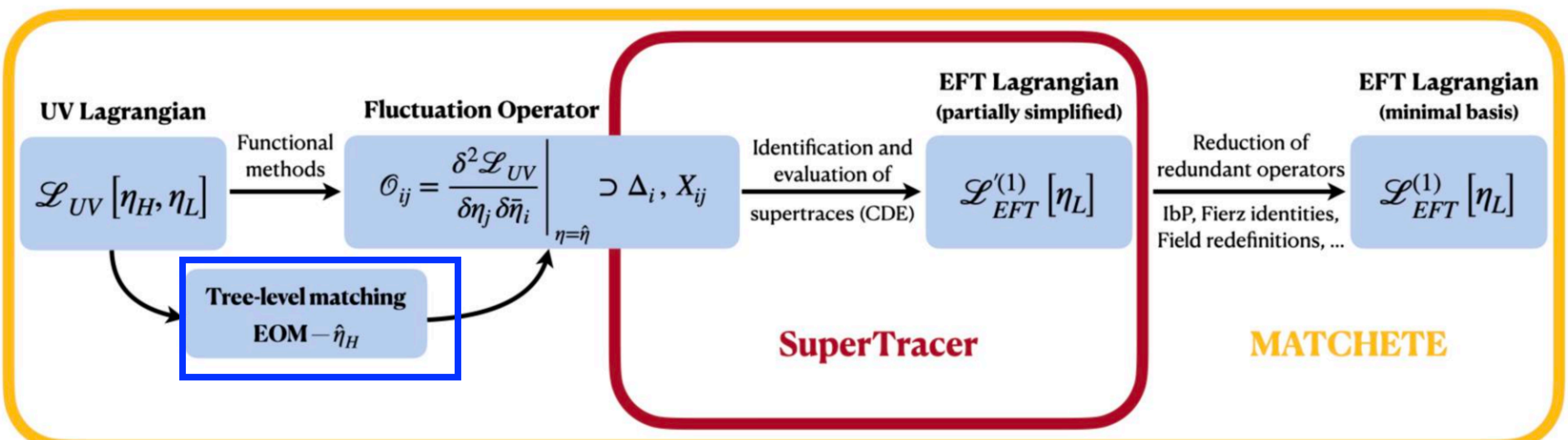
- Could a same UV model match to two EFTs,

A. if possible, which EFT is better?

B. if only HEFT works, why?

- The SMEFT matching is mature at one-loop level (diagrammatic method and functional method). How to make the HEFT matching simply and programmable?

SMEFT matching procedure (Covariant Derivative Expansion)



Real Higgs Triplet Extension of the SM (RHTE)

- A singlet extension, a second doublet extension (2HDM), next is triplet.
  [G. Buchala et al, 1608.03564, 2312.13885], [S. Dawson et al, 2205.01561, 2311.16897],[F. Arco et al, 2307.15693]
- The custodial violation appears at tree level with a non-zero VEV.

The Model: the SM plus a real $SU(2)_L$ triplet with $Y = 0$

Linear form

$$\mathrm{H} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v_\mathrm{H} + h + iG^0\right) \end{pmatrix}, \qquad \Sigma = \frac{1}{2}\Sigma_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1, 2, 3,$$

$$\mathscr{L}_\mathrm{RHTE}(\mathrm{H}, \Sigma) \supset D_\mu \mathrm{H}^\dagger D^\mu \mathrm{H} + \langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle - V(H, \Sigma),$$

$$V(\mathrm{H}, \Sigma) = Y_1^2 \mathrm{H}^\dagger \mathrm{H} + Z_1(\mathrm{H}^\dagger \mathrm{H})^2 + Y_2^2\langle\Sigma^\dagger\Sigma\rangle + Z_2\langle\Sigma^\dagger\Sigma\rangle^2 + Z_3\mathrm{H}^\dagger\mathrm{H}\langle\Sigma^\dagger\Sigma\rangle + 2Y_3\mathrm{H}^\dagger\Sigma\mathrm{H}$$

$Z_i s$ are dimensionless, $Y_i s$ are dimensional

$< \ldots >$ denotes trace

Matching RHTE to SMEFT

$$\mathscr{L}_{\rm RHTE}({\rm H},\Sigma) \supset D_\mu {\rm H}^\dagger D^\mu {\rm H} + \langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle - V(H,\Sigma),$$

$$V({\rm H},\Sigma) = Y_1^2 {\rm H}^\dagger {\rm H} + Z_1 ({\rm H}^\dagger {\rm H})^2 + Y_2^2 \langle \Sigma^\dagger \Sigma \rangle + Z_2 \langle \Sigma^\dagger \Sigma \rangle^2 + Z_3 {\rm H}^\dagger {\rm H} \langle \Sigma^\dagger \Sigma \rangle + 2 Y_3 {\rm H}^\dagger \Sigma {\rm H}$$

mass

$$\mathscr{L}^\Sigma = \frac{1}{2} \vec{\Sigma}^T (-D_\mu D^\mu - Y_2^2 - Z_3\, {\rm H}^\dagger {\rm H}) \vec{\Sigma} + Y_3 \vec{\Sigma} \cdot {\rm H}^\dagger \vec{\sigma} {\rm H} - \frac{1}{4} Z_2 (\vec{\Sigma} \cdot \vec{\Sigma})^2$$

EoM of Σ:

$$(-D_\mu D^\mu - Y_2^2 - Z_3\, {\rm H}^\dagger {\rm H}) \vec{\Sigma}_c = -\, Y_3 {\rm H}^\dagger \vec{\sigma} {\rm H} + Z_2 (\vec{\Sigma}_c \cdot \vec{\Sigma}_c) \vec{\Sigma}_c$$

$$\vec{\Sigma}_c = -\frac{1}{-D_\mu D^\mu - Y_2^2 - Z_3\, {\rm H}^+ {\rm H}} Y_3 {\rm H}^+ \vec{\sigma} {\rm H} + \frac{1}{-D_\mu D^\mu - Y_2^2 - Z_3\, {\rm H}^+ {\rm H}} Z_2 (\vec{\Sigma}_c \cdot \vec{\Sigma}_c) \vec{\Sigma}_c$$

Expansion with $1/Y_2^2$ (if $Y_2^2 \gg v_{\rm EW}^2$)

$$\mathscr{L}_{\rm SMEFT} = \frac{1}{2Y_2^2} Y_3^2 {\rm H}^\dagger \vec{\sigma} H \cdot {\rm H}^\dagger \vec{\sigma} {\rm H} + \frac{1}{2} ({\rm H}^\dagger \vec{\sigma} H)^T \frac{1}{Y_2^2} (-D_\mu D^\mu - Z_3 H^\dagger H) \frac{1}{Y_2^2} {\rm H}^\dagger \vec{\sigma} H + \cdots$$

T. Corbett, A. Helset, A. Martin, M. Trott, [2102.02819]
J. Ellis, K. Mimasu, F. Zamperdri, [2304.06663]

Matching RHTE to HEFT

$$h, U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\rm EW}}\right), \quad \mathscr{L}^{\rm LO}_{\rm HEFT} \supset \frac{1}{2}D_\mu h D^\mu h - V(h) + \frac{v_{\rm EW}^2}{4}F(h){\rm Tr}(D_\mu U^\dagger D^\mu U) + \cdots$$

$$V(h) = \frac{1}{2}m_h^2 h^2\left[1 + (1+\Delta\kappa_3)\frac{h}{v_{\rm EW}} + \cdots\right], \quad F(h) = 1 + 2(1+\Delta a)\frac{h}{v_{\rm EW}} + \cdots$$

RHTE in linear form

$$\mathrm{H} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v_{\rm H} + h + iG^0\right) \end{pmatrix}, \qquad \Sigma = \frac{1}{2}\Sigma_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1, 2, 3,$$

$$\begin{pmatrix} G^+_{\rm EW} \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{pmatrix}\begin{pmatrix} G^+ \\ \Sigma^+ \end{pmatrix} \qquad \begin{pmatrix} h \\ K \end{pmatrix} = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix}\begin{pmatrix} h^0 \\ \Sigma^0 \end{pmatrix}$$

$$\tan\delta = 2v_\Sigma/v_{\rm H}.$$

1. Solve EoMs of $H^\pm, K$.
2. Embed $G^\pm_{\rm EW}, G^0$ into an exponential matrix form. **?! It must be complicated.**

Find a non-linear representation of the UV model for HEFT matching

Non-linear Form

1. $$\mathrm{H} = U \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{\mathrm{H}} + h^0 \end{pmatrix} \quad , \quad U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\mathrm{H}}}\right)$$

$$\Sigma = \frac{1}{2}\Sigma_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1, 2, 3,$$

The mass mixing between Goldstones $\pi^\pm$ and $\Sigma^\pm$ still exists.

Non-linear Form

1. $$\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\mathrm{H}} + h^0 \end{pmatrix} \;,\quad U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\mathrm{H}}}\right)$$

$$\Sigma = \frac{1}{2}\Sigma_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1, 2, 3,$$

The mass mixing between Goldstones $\pi^\pm$ and $\Sigma^\pm$ still exists.

2. $$\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\mathrm{H}} + h^0 \end{pmatrix} \;,\quad U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\mathrm{H}}}\right)$$

$$\Sigma = U\Phi U^\dagger,\ \Phi = \frac{1}{2}\phi_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma - \phi^0 \end{pmatrix}$$

$$2Y_3\mathrm{H}^\dagger\Sigma\mathrm{H}$$

Good news: U disappears in potential, the mass mixing disappears.

Bad news: a term of kinetic mixing appears.

$$\langle D_\mu\Sigma^\dagger D^\mu\Sigma\rangle \supset -v_\Sigma\epsilon_{3jk}D_\mu\phi_j D^\mu\pi_k$$

Non-linear Form

1. $$\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\mathrm{H}} + h^0 \end{pmatrix}, \quad U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\mathrm{H}}}\right)$$ $$\Sigma = \frac{1}{2}\Sigma_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -v_\Sigma - \Sigma^0 \end{pmatrix}, i = 1,2,3,$$

The mass mixing between Goldstones $\pi^\pm$ and $\Sigma^\pm$ still exists.

2. $$\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\mathrm{H}} + h^0 \end{pmatrix}, \quad U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{\mathrm{H}}}\right)$$ $$\Sigma = U\Phi U^\dagger,\ \Phi = \frac{1}{2}\phi_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma - \phi^0 \end{pmatrix}$$

$$2Y_3\mathrm{H}^\dagger\Sigma\mathrm{H}$$

Good news: U disappears in potential, the mass mixing disappears.

Bad news: a term of kinetic mixing appears.

$$\langle D_\mu\Sigma^\dagger D^\mu\Sigma\rangle \supset -v_\Sigma\epsilon_{3jk}D_\mu\phi_j D^\mu\pi_k/v_{\mathrm{EW}}$$

3. $$\mathrm{H} = U\frac{1}{\sqrt{2}}\begin{pmatrix} \chi^\pm \\ v_{\mathrm{H}} + h^0 + i\chi^0 \end{pmatrix}, U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{EW}}\right)$$ $$\Sigma = U\Phi U^\dagger,\ \Phi = \frac{1}{2}\phi_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma - \phi^0 \end{pmatrix}$$

$$\chi^\pm = 2\frac{v_\Sigma}{v_{\mathrm{H}}}\phi^\pm, \chi^0 = 0 \qquad D_\mu H^\dagger D^\mu H \supset v_{\mathrm{H}}\epsilon_{3jk}D_\mu\chi_j D^\mu\pi_k/(2v_{\mathrm{EW}})$$

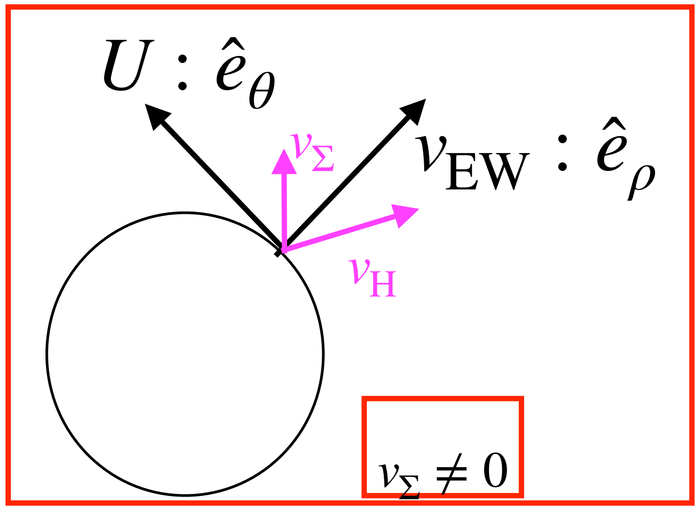
Both mass mixing and kinetic mixing disappear!

The U matrix is separated from heavy states, to "integrate out" heavy states and leave U in HEFT become straightforward.

A Diagrammatic View: Find the Correct U

$$\mathrm{H} = U \begin{pmatrix} \chi^{\pm} \\ \frac{1}{\sqrt{2}}(v_{\mathrm{H}} + h^0 + i\chi^0) \end{pmatrix}, U \equiv \exp\left(\frac{i\pi_i\sigma_i}{v_{EW}}\right) \qquad \chi^{\pm} = 2\frac{v_\Sigma}{v_{\mathrm{H}}}\phi^{\pm}, \chi^0 = 0$$

$$\Sigma = U\Phi U^{\dagger},\ \Phi = \frac{1}{2}\phi_i\sigma_i = \frac{1}{2}\begin{pmatrix} v_\Sigma + \phi^0 & \sqrt{2}\phi^+ \\ \sqrt{2}\phi^- & -v_\Sigma - \phi^0 \end{pmatrix} \qquad (\pi_i s, h^0, \phi^{\pm}, \phi^0)$$



- Use “rotated” scalars.
- Cancel out kinetic mixing.

Does these two rules suitable for a general $SU(2)$ representations?
E.g. a quadruplet, a quintet.

Quadruplet with $Y = 3/2$

$$\begin{pmatrix} \Theta_{111} \\ \sqrt{3}\Theta_{112} \\ \sqrt{3}\Theta_{122} \\ \Theta_{222} \end{pmatrix} \to \begin{pmatrix} \Theta^{3+} \\ \Theta^{++} \\ \boxed{\begin{matrix}\Theta^{+} \\ \Theta^{0}\end{matrix}} \end{pmatrix}$$

$$\mathrm{H}_i = \underline{U_i^j}\mathfrak{h}_j, \quad \mathfrak{h} = \begin{pmatrix} \chi^+ \\ \frac{1}{\sqrt{2}}\left(v_\mathrm{H} + h^0 + i\chi^0\right) \end{pmatrix}$$

$$\Theta_{ijk} = \underline{U_i^l U_j^m U_k^n}\phi_{lmn}$$

$$\langle \phi_{222} \rangle = \langle \phi^{*222} \rangle = v_\Theta/\sqrt{2}, \mathrm{Im}(\phi_{222}) = \eta_4/\sqrt{2}, \phi_{122} = \phi^+/\sqrt{3}$$

$v_H/\sqrt{2}$

$$\mathscr{L}_\mathrm{H}^{\mathrm{mix}} = \langle \mathfrak{h}_2 \rangle \Big((U^\dagger D_\mu U)_1^2 D^\mu \mathfrak{h}^{*1} - (U^\dagger D_\mu U)_2^1 D^\mu \mathfrak{h}_1 + (U^\dagger D_\mu U)_2^2 (D^\mu \mathfrak{h}^{*2} - D^\mu \mathfrak{h}_2) \Big),$$

$$\mathscr{L}_\Theta^{\mathrm{mix}} = 3\langle \phi_{222} \rangle \Big((U^\dagger D_\mu U)_1^2 D^\mu \phi^{*122} - (U^\dagger D_\mu U)_2^1 D^\mu \phi_{122} + (U^\dagger D_\mu U)_2^2 (D^\mu \phi^{*222} - D^\mu \phi_{222}) \Big),$$

$v_\Theta/\sqrt{2}$

$$\chi^+ = -\frac{3v_\Theta}{v_\mathrm{H}}\phi_{122} = -\frac{\sqrt{3}v_\Theta}{v_\mathrm{H}}\phi^+, \quad \chi^0 = -\frac{3v_\Theta}{v_H}\eta_4$$

General Scalar Extensions

$$\Phi_{ijklm\cdots} = U^i_{i_1} U^j_{j_1} U^k_{k_1} U^l_{l_1} U^k_{k_1} U^m_{m_1} \cdots \phi_{i_1 j_1 k_1 l_1 m_1 \cdots}$$

$\underbrace{1\cdots1}_{j-y+1}\underbrace{2\cdots2}_{j+y-1}$ for positive charge

$\underbrace{1\cdots1}_{j-y-1}\underbrace{2\cdots2}_{j+y+1}$ for negative charge

$\underbrace{1\cdots1}_{j-y}\underbrace{2\cdots2}_{j+y}$ Neutral

$$
\begin{aligned}
&(D\Phi^{*i_1i_2i_3i_4i_5\cdots})(D\Phi_{i_1i_2i_3i_4i_5\cdots}) \\
=&(D\phi^{*i_1i_2i_3i_4i_5\cdots})(D\phi_{i_1i_2i_3i_4i_5\cdots}) + (DU^{*i_n}_{k_n} DU^{j_n}_{i_n})\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots} \\
&+ (U^{*i_n}_{k_n} DU^{j_n}_{i_n} D\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots} + DU^{*i_n}_{k_n} U^{j_n}_{i_n}\phi^{*\cdots i_{n-1}k_n i_{n+1}\cdots} D\phi_{\cdots i_{n-1}j_n i_{n+1}\cdots}) \\
&+ (U^{*i_m}_{k_m} DU^{j_m}_{i_m} DU^{*i_n}_{k_n} U^{j_n}_{i_n} + DU^{*i_m}_{k_m} U^{j_m}_{i_m} U^{*i_n}_{k_n} DU^{j_n}_{i_n}) \\
&\quad \phi^{*\cdots i_{m-1}k_m i_{m+1}\cdots i_{n-1}k_n i_{n+1}\cdots}\phi_{\cdots i_{m-1}j_m i_{m+1}\cdots i_{n-1}j_n i_{n+1}\cdots} \qquad (4.12)
\end{aligned}
$$

$$
\begin{aligned}
&\supset \frac{v_\phi}{\sqrt{2}}\left[(j+y)(U^\dagger DU)^2_2 + (j-y)(U^\dagger DU)^1_1\right] D\left(\phi^{0*} - \phi^0\right) \\
&+ v_\phi/\sqrt{2}(U^\dagger DU)^2_1\left[\sqrt{(j+y)(j-y+1)}D\phi^{+*} - \sqrt{(j-y)(j+y+1)}D\phi^-\right] \\
&\quad + v_\phi/\sqrt{2}(U^\dagger DU)^1_2\left[\sqrt{(j-y)(j+y+1)}D\phi^{-*} - \sqrt{(j+y)(j-y+1)}D\phi^+\right]
\end{aligned}
$$

$$\chi^+ = \frac{v_\phi}{v_H}\left(\sqrt{(j-y)(j+y+1)}\phi^{-*} - \sqrt{(j+y)(j-y+1)}\phi^+\right) \quad \chi^0 = -\frac{2yv_\phi}{v_H}\eta^0$$

In this non-linear representation, U and heavy states are separate. As HEFT matching is to “integrate out” heavy states and leave Goldstones in U form, under this representation the matching become straight and simple, further programmable.

HEFT matching of the real Higgs triplet extension (RHTE)

The HEFT (ξ^3)

p^2

Operators	$P(h)/\left[\xi^3/(Y_3 v^3)\right]/(4Z_1 - Z_3)$
$\langle V_\mu V^\mu \rangle$	$-2hv^5 - 10h^2v^4 - 22h^3v^3 - 23h^4v^2 - 11h^5v - 2h^6$
$\langle V_\mu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle$	$2hv^5 + 9h^2v^4 + 16h^3v^3 + 14h^4v^2 + 6h^5v + h^6$
$D_\mu h D^\mu h$	$20h^2v^2 + 24h^3v + 8h^4$
$V(h)$	$h^2v^6(4Z_1 - Z_3) + h^3v^5(16Z_1 - 5Z_3) + \frac{1}{4}h^4v^4(60Z_1 - 41Z_3)$ $-h^5v^3(4Z_1 + 11Z_3 - \frac{1}{2}h^6v^2(24Z_1 + 13Z_3) - 2h^7v(3Z_1 + Z_3)$ $-\frac{1}{4}h^8(4Z_1 + Z_3) + (-4h^2v^4 - 10h^3v^3 - 8h^4v^2 - 2h^5v)Y_3^2/(4Z_1 - Z_3)$

p^4

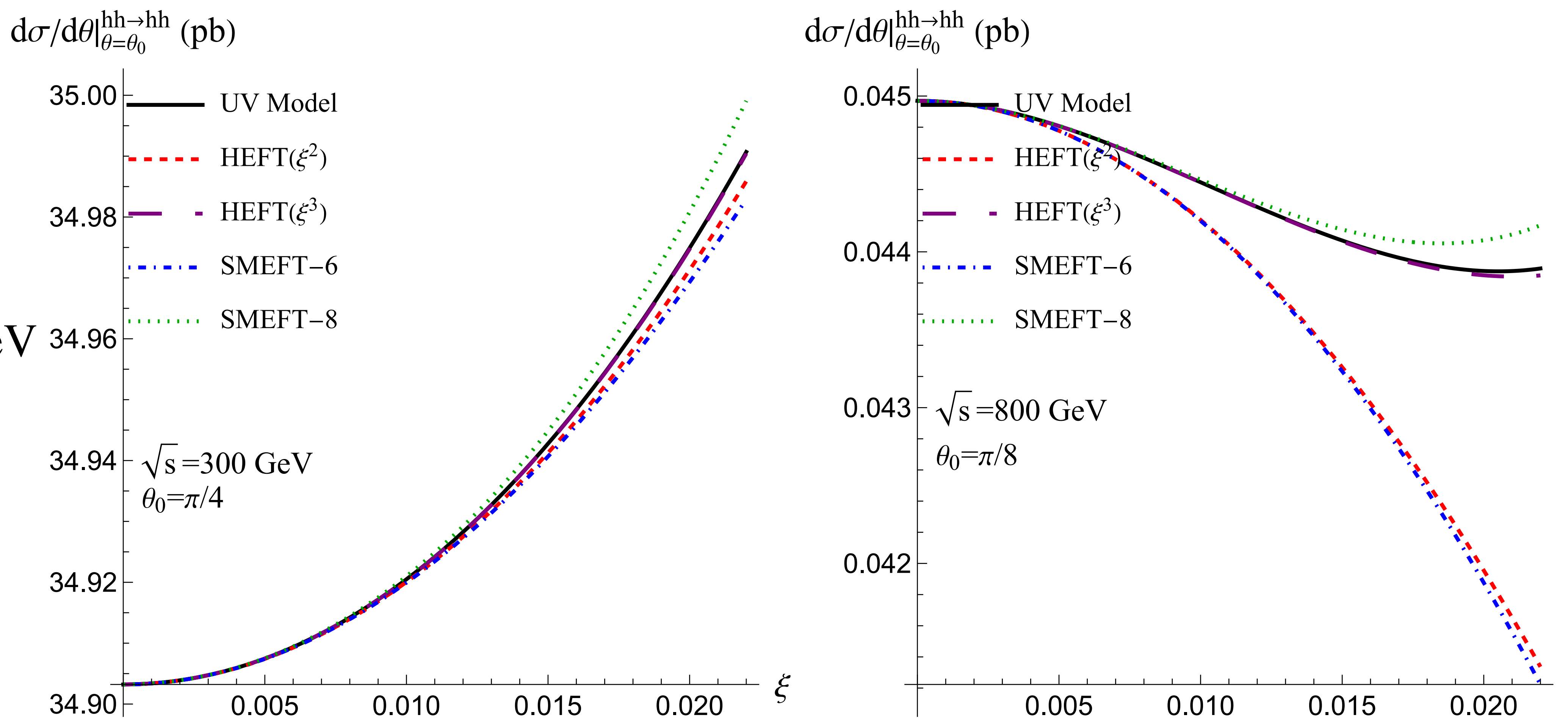
Operator	$P(h)/\left[\xi^3/(Y_3 v^3)\right]$
$\langle V_\mu V^\mu \rangle \langle V_\nu V^\nu \rangle$	$(h+v)^4$
$\langle V_\mu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle \langle V_\nu V^\nu \rangle$	$-2(h+v)^4$
$\langle V_\mu \sigma_3 \rangle \langle V_\nu \sigma_3 \rangle \langle V^\mu V^\nu \rangle$	$2(h+v)^4$
$\langle V_\mu V_\nu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle D^\nu h$	$-4(h+v)^3$
$\langle V_\mu V_\nu \sigma_3 \rangle \langle V^\nu \sigma_3 \rangle D^\mu h$	$4(h+v)^3$
$\langle V_\mu V^\mu \rangle D_\nu h D^\nu h$	$4(h+v)^2$
$\langle V_\mu \sigma_3 \rangle \langle V^\mu \sigma_3 \rangle D_\nu h D^\nu h$	$-4(h+v)^2$
$\langle V_\mu V_\nu \rangle D^\mu h D^\nu h$	$-8(h+v)^2$
$\langle V_\mu \sigma_3 \rangle \langle V_\nu \sigma_3 \rangle D^\mu h D^\nu h$	$4(h+v)^2$
$D_\mu h D^\mu h D_\nu h D^\nu h$	4

Numerical Results (When Y_2^2 is Large)

$$V(\mathrm{H},\Sigma) = Y_1^2 \mathrm{H}^\dagger \mathrm{H} + Z_1(\mathrm{H}^\dagger \mathrm{H})^2 + Y_2^2\langle\Sigma^\dagger\Sigma\rangle + Z_2\langle\Sigma^\dagger\Sigma\rangle^2 + Z_3 \mathrm{H}^\dagger \mathrm{H}\langle\Sigma^\dagger\Sigma\rangle + 2Y_3 \mathrm{H}^\dagger\Sigma \mathrm{H}$$

The bare mass term of Σ

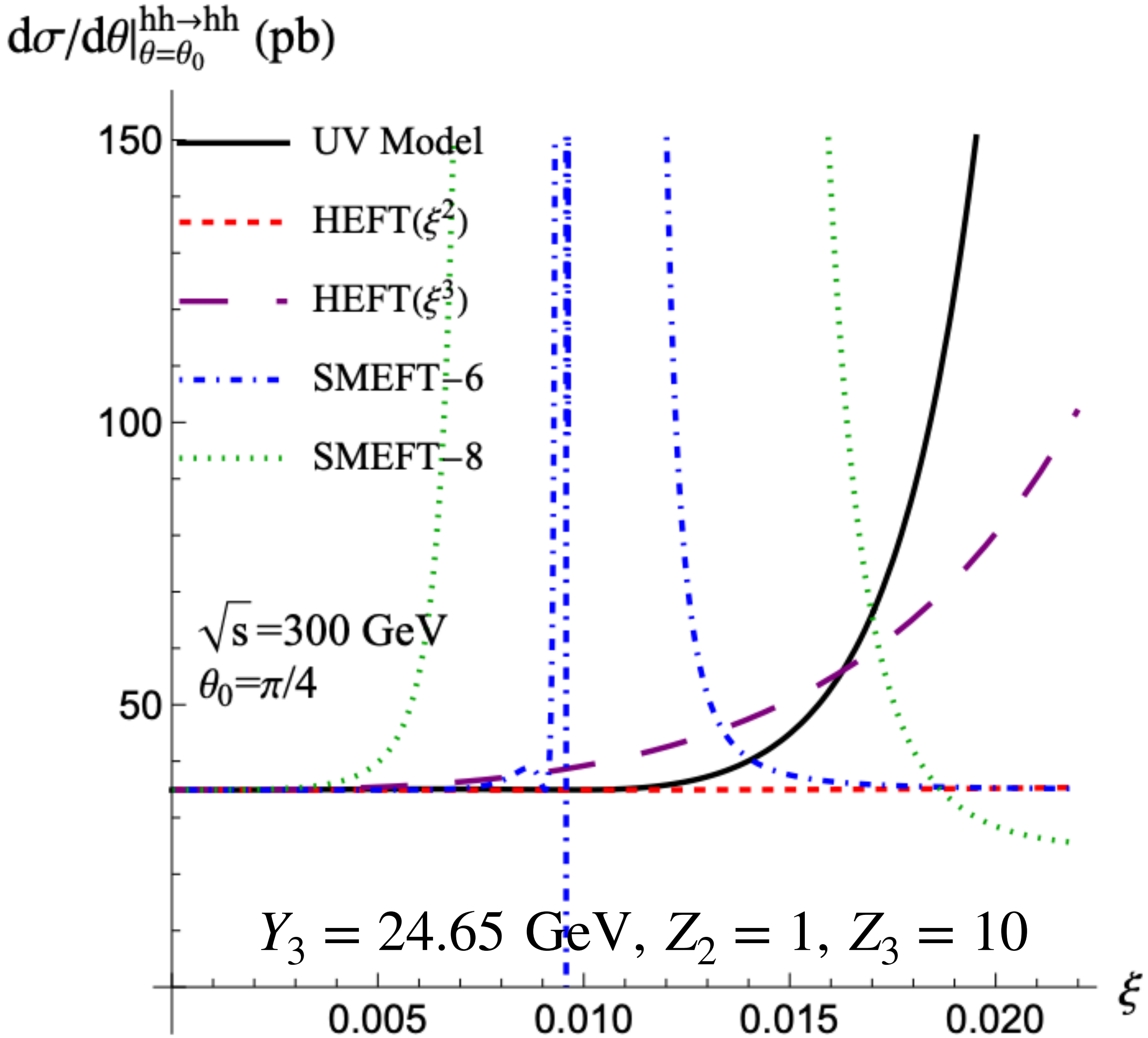
$Y_2 > 2\mathrm{TeV}$



HEFT converges faster, which is same as in $WW \to hh, ZZ \to hh$ process

2504.02580, Yi Liao, Xiao-Dong Ma, Yoshiki Uchida

While $Y_2^2 \sim 0$

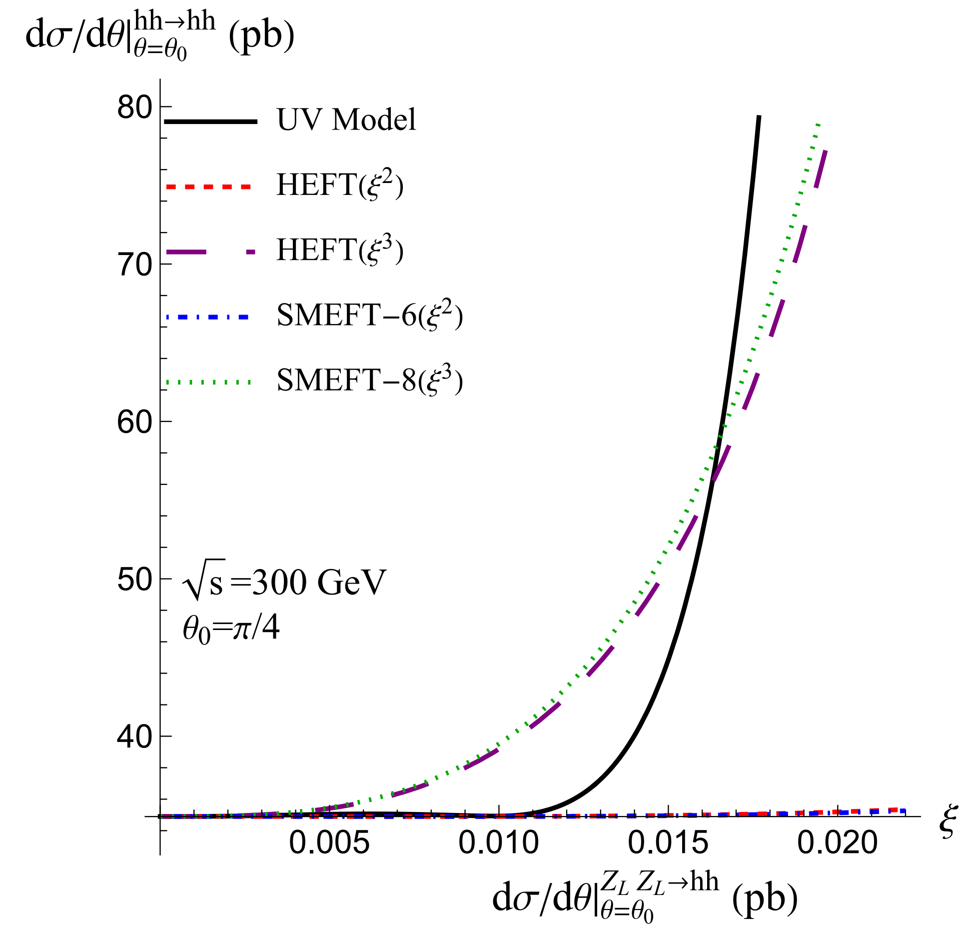


Power counting,

SMEFT: $1/Y_2^2$

HEFT: ξ or $1/m_{\phi^\pm}^2$

After a second expansion with $1/m_{\phi^\pm}^2$



The SMEFT's regular part is consistent with the HEFT's

$$Y_2^2 = -Z_3 v_H^2/2 + m_{\phi^\pm}^2 + \mathcal{O}(\xi)$$

Next Plan

Parameter set 1 $(Z_1, Z_2, Z_3, Y_3, v_{\mathrm{EW}}, \xi)$

$$m^2_{\phi^\pm} = \frac{Y_3 v_H}{2\xi} + 2\xi Y_3 v_H$$

Decoupling case

Parameter set 2 $(m_h, m_{\phi^\pm}, m_K, \sin_\gamma, v_{\mathrm{EW}}, \xi)$

While $m^2_{\phi^\pm}$ approaches infinity, ξ could be kept as a constant, the real model triplet model will not decouple to the SM.

Non-decoupling case

Next Plan

1-loop

$$\mathscr{L}_{\rm RHTE} \supset \frac{1}{2} \begin{pmatrix} K & \phi_1 & \phi_2 \end{pmatrix} \mathscr{X} \begin{pmatrix} K \\ \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\mathcal{X} = \begin{pmatrix} \begin{matrix} -\partial^2 - m_K^2 - 2d_3 h - 2z_3 h^2 \\ -\left(1+3c_\gamma^2\right)\langle V_\mu V^\mu\rangle/2 + c_\gamma^2 V_\mu^3 V_3^\mu \end{matrix} & c_\gamma V_\mu^1 V_3^\mu + 2i(c_\gamma - v_\Sigma s_\gamma/v_H)V_\mu^2 D^\mu & c_\gamma V_\mu^2 V_3^\mu - 2i(c_\gamma - v_\Sigma s_\gamma/v_H)V_\mu^1 D^\mu \\ c_\gamma V_\mu^3 V_1^\mu - 2i(c_\gamma - v_\Sigma s_\gamma/v_H)V_\mu^2 D^\mu & \begin{matrix} -\left(1+4v_\Sigma^2/v_H^2\right)D^2 - m_{\phi^\pm}^2 - d_5 h - z_6 h^2 \\ -2\left(1+v_\Sigma^2/v_H^2\right)\langle V_\mu V^\mu\rangle + V_\mu^1 V_1^\mu \end{matrix} & V_\mu^1 V_2^\mu + 2i(1+2v_\Sigma^2/v_H^2)V_\mu^3 D^\mu \\ c_\gamma V_\mu^3 V_2^\mu + 2i(c_\gamma - v_\Sigma s_\gamma/v_H)V_\mu^1 D^\mu & V_\mu^2 V_1^\mu - 2i(1+2v_\Sigma^2/v_H^2)V_\mu^3 D^\mu & \begin{matrix} -\left(1+4v_\Sigma^2/v_H^2\right)D^2 - m_{\phi^\pm}^2 - d_5 h - z_6 h^2 \\ -2\left(1+v_\Sigma^2/v_H^2\right)\langle V_\mu V^\mu\rangle + V_\mu^2 V_2^\mu \end{matrix} \end{pmatrix} \quad \text{(B.5)}$$

$$D_{ij}^\mu \equiv \partial^\mu \delta_{ij} - g' B^\mu \epsilon_{ij}, \text{ with } i,j \in 1,2 \quad \text{(B.6)}$$

$$V_\mu^i = \langle U^\dagger D_\mu U \sigma_i \rangle \quad \text{(B.7)}$$

Summary

- HEFT encompasses SMEFT. Through matching a UV model to both HEFT and SMEFT, we study their distinction.

- We build a non-linear representation of general scalar extensions of the SM, which is great for HEFT matching in functional method. The key point is that by using “rotated” scalars, we separate the Goldstones’ U matrix and heavy states in the UV model.

- We match the real Higgs triplet extension (RHTE) to both HEFT and SMEFT in the decoupling scenario.

- Further research about non-decoupling effects and 1-loop matching is underway.

Thank you for your attention!