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Form Factors for $B_c^* \to \eta_c + l\nu_l$ at NLO in QCD

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Based on
on-going work
with Qin Chang, Ruilin Zhu, Zhen-Jun Xiao, et al.

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Outline

1. Introduction
2. Calculation Procedure
3. Results
4. Summary

B_c^* semileptonic decay and form factors

➢ $c\bar{b}$ meson

- the only meson containing two different heavy flavors
- Bc(1S) discovered in 1998 through $B_c \to J/\psi + l\nu_l$
- Bc*(1S) not yet observed due to low production rate and difficulty in detecting γ of $B_c^* \to B_c + \gamma$
- Bc*(1S) weak decays (e.g. $B_c^* \to \eta_c + l\nu_l$) help search for Bc*(1S)

[CDF, PRL(1998)]
[CMS, PRL(2019)]
[LHCb, PRL(2019)]

➢ Form factors (FFs)

- Building blocks for decay widths and branching fractions
- Lattice QCD data available for $B_c \to J/\psi\,(\eta_c)$ FFs, not for B_c^*
- $B_c^* \to \eta_c$ FFs calculated by light-front quark model (LFQM), Bauer-Stech-Wirbel (BSW) model, QCD sum rules (QCDSR),

[HPQCD, 1611.01987]
[HPQCD, PRD(2020)]
[Q.Chang,L.L.Chen,S.Xu, JPG(2018)]
[P.R,R.Dhir, 1908.00242]
[Y.Yang,et al., CPC(2023)]
[S.Y.Wang,et al., CPC(2024)]

Why NRQCD higher-order calculations

[G.T.Bodwin,E.Braaten,G.P.Lepage,PRD(1995)]

➢ Non-Relativistic Quantum Chromodynamics (NRQCD)

- Form factor = $\boxed{\text{short-distance coefficient}}$ ✖ $\boxed{\text{wavefunction at origin}}$

  perturbative expansion in α_s nonperturbative

- To test perturbative expansion convergence and renormalization scale dependence in NRQCD
- To obtain more precise theoretical predictions and test the Standard Model
- To study new physics by calculating (axial-)tensor form factors
- 2007-2022, $B_c \to J/\psi\,(\eta_c)$ (axial-)vector and (axial-)tensor FFs at next-to-leading order (NLO) [G. Bell, 0705.3133] [C.F.Qiao,P.Sun, JHEP(2012)] [C.F.Qiao,R.Zhu, PRD(2013)] [W.Tao,Z.J.Xiao,R.Zhu, PRD(2022)]
- 2024-2025, $B_c^* \to J/\psi$ (axial-)vector and (axial-)tensor FFs at NLO

[Y.Geng,M.Cao,R.Zhu, PRD(2024)]
[Q.Chang,et al., PRD(2025)]

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Definitions & Feynman diagrams for FFs

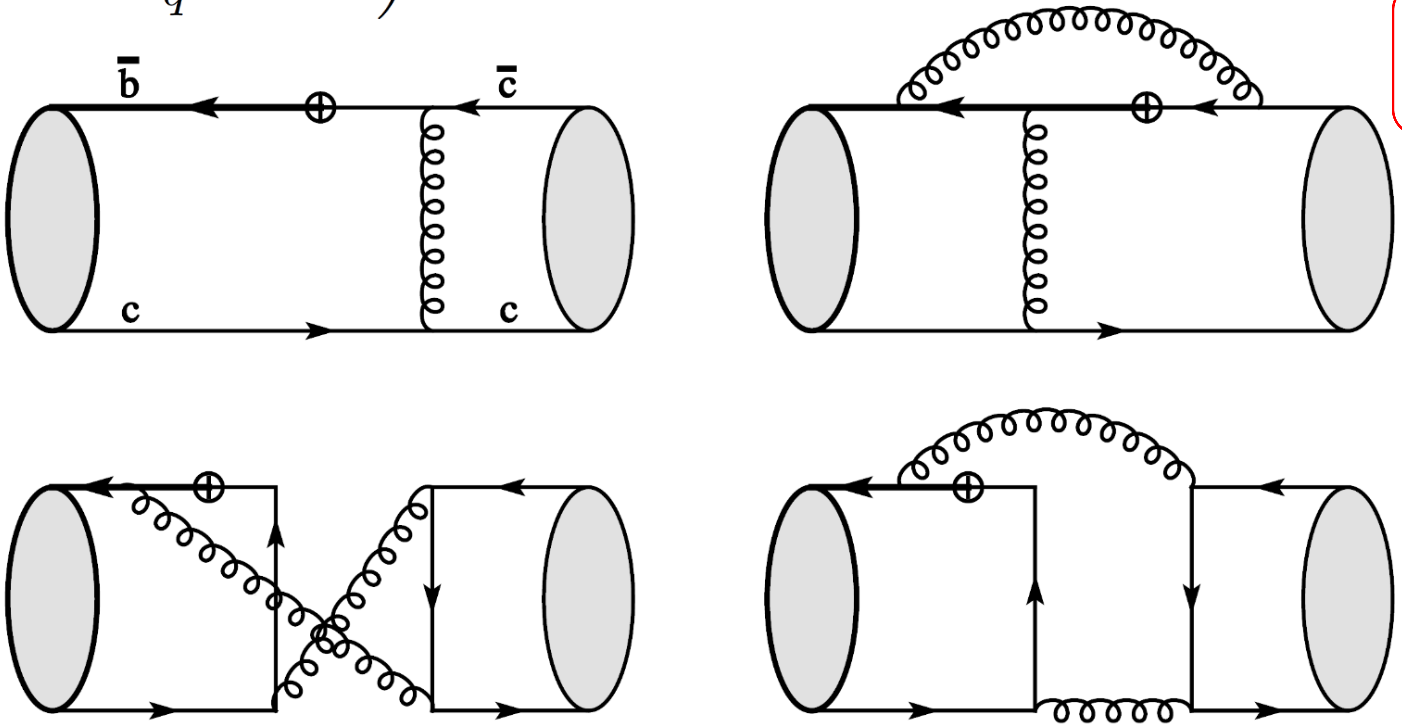
$$\langle \eta_c(p') | \bar{b}\gamma_\mu c | B_c^*(\epsilon, p) \rangle = \frac{i\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu P^\alpha q^\beta}{m_{B_c^*} + m_{\eta_c}} V,$$

$$\langle \eta_c(p') | \bar{b}\gamma_\mu\gamma_5 c | B_c^*(\epsilon, p) \rangle = \frac{2m_{B_c^*}\epsilon \cdot q q_\mu}{q^2} A_0 + \left(\epsilon_\mu - \frac{\epsilon \cdot q}{q^2} q_\mu\right)(m_{B_c^*} + m_{\eta_c}) A_1 + \epsilon \cdot q \left(\frac{P_\mu}{m_{B_c^*} + m_{\eta_c}} - \frac{m_{B_c^*} - m_{\eta_c}}{q^2} q_\mu\right) A_2,$$

$$\langle \eta_c(p') | \bar{b}\sigma_{\mu\nu} q^\nu c | B_c^*(\epsilon, p) \rangle = -\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu P^\alpha q^\beta T,$$

$$\langle \eta_c(p') | \bar{b}\sigma_{\mu\nu}\gamma_5 q^\nu c | B_c^*(\epsilon, p) \rangle = i\left(\epsilon_\mu - \frac{\epsilon \cdot q}{q^2} q_\mu\right)(m_{B_c^*}^2 - m_{\eta_c}^2) T_1' - i\epsilon \cdot q \left(P_\mu - \frac{m_{B_c^*}^2 - m_{\eta_c}^2}{q^2} q_\mu\right) T_2',$$

$q = p - p'$:
transfer momentum



Express amplitudes as $A_0, B_0, C_{0,1}, D_0$ & calculate them

amplitudes

Consisting of scalar products of momenta

FeynCalc TID

$A_0, B_0, C_{0,1}, D_0, E_0$

IBP reduction

Package-X

[K.G.Chetyrkin,F.V.Tkachov, NPB(1981)]
[H.H.Patel, CPC(2017)]

Li_2, Log, Sqrt

Series: expanding in small m_c and taking the leading-order terms

hierarchical heavy quark limit

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Analytical results for $B_c^* \to \eta_c$ form factors

$\Psi_{B_c^*(\eta_c)}(0)$: $B_c^*(\eta_c)$ wavefunction at origin

- LO

$$V=\frac{16\sqrt{2}\pi\alpha_s C_F s^2(1+3x)(1+x)^{\frac{3}{2}}\Psi_{B_c^*}(0)\Psi_{\eta_c}(0)}{m_b^3 x^{\frac{3}{2}}(1+s(x-2)x)^2},\quad V=\frac{4s^2(1+3x)^2(x^2-1)T_1'}{1-s(6+4x-8x^2)-s^2x(12+10x-8x^2-15x^3)},$$

$$V=\frac{1+3x}{1+x}A_0=\frac{2s(1+3x)^2}{3+sx(6+7x)}A_1=2A_2,\qquad T_2'=T=\frac{1+s(4+6x+5x^2)}{4s(1+x)^2}A_0,$$

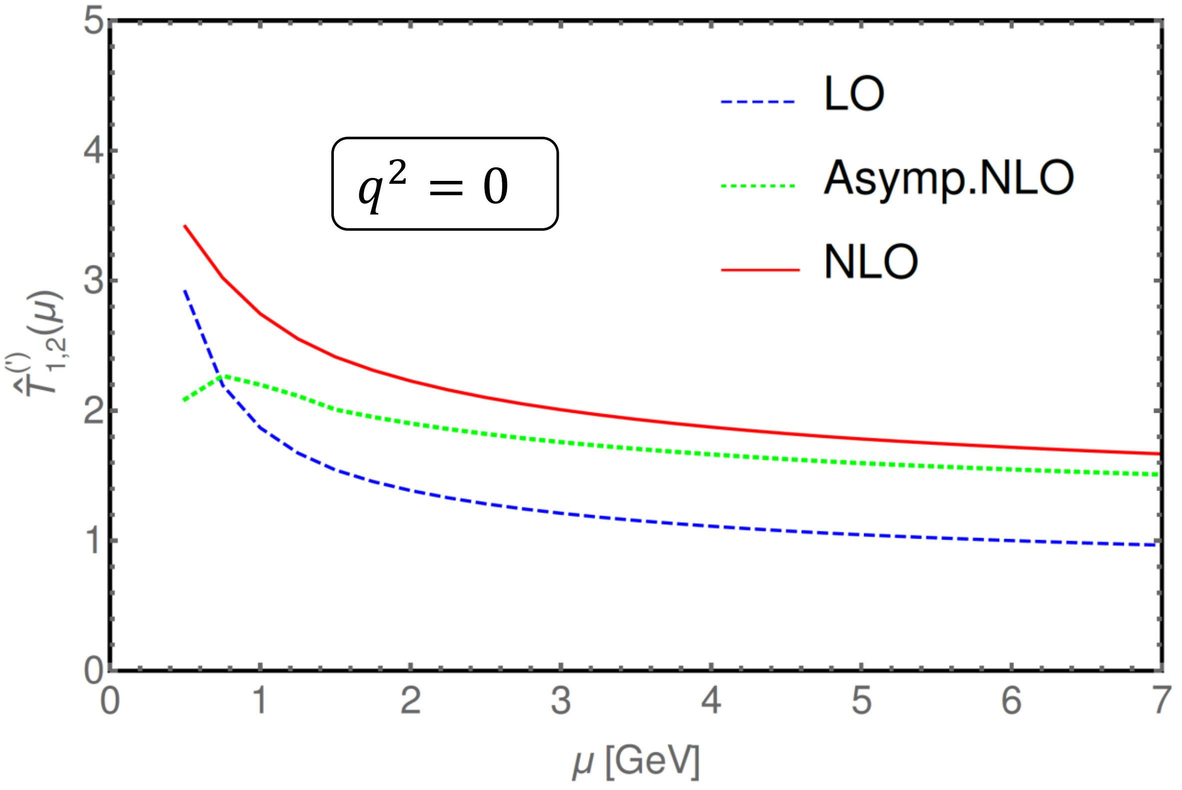
- Asymptotic NLO

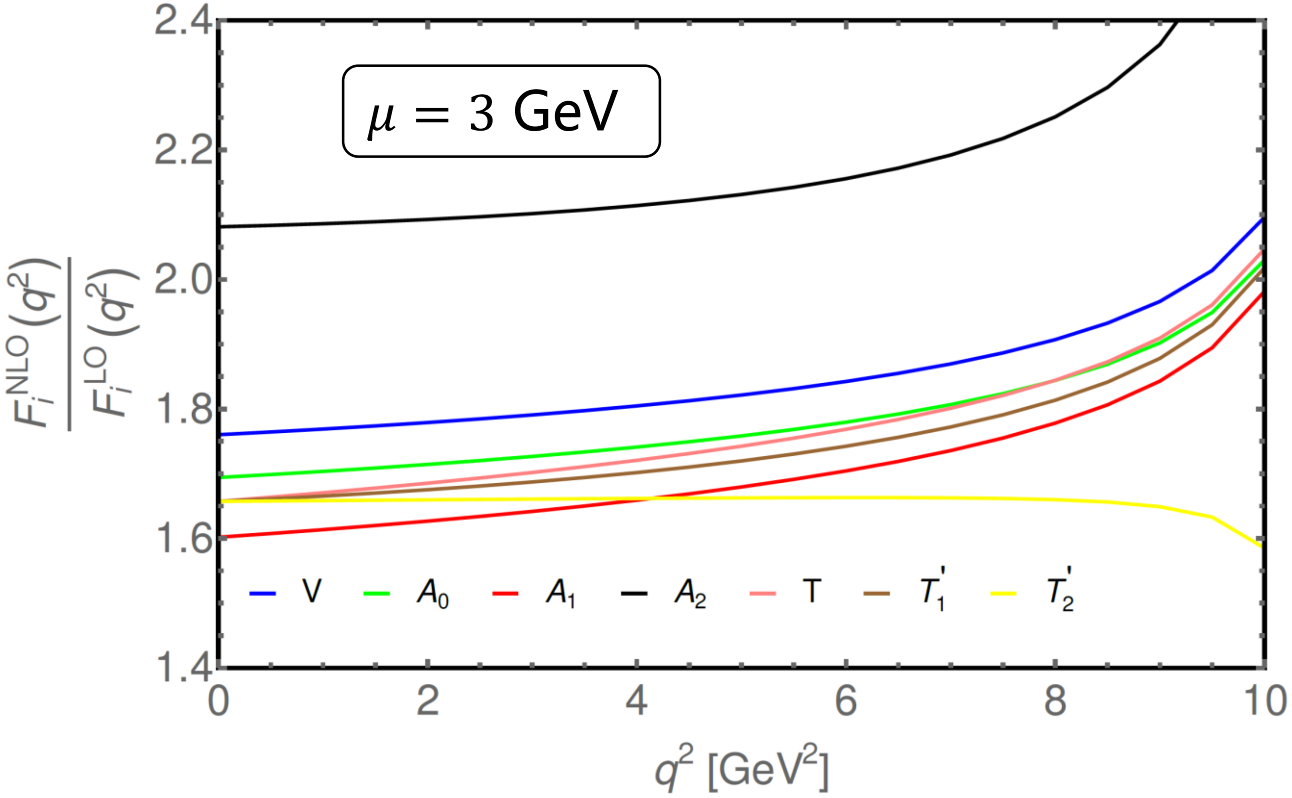
$$x=\frac{m_c}{m_b},\quad y=\frac{\mu}{m_b},\quad s=\frac{1}{1-\frac{q^2}{m_b^2}}$$

$$\begin{aligned}\frac{T_1'^{\mathrm{NLO}}}{T_1'^{\mathrm{LO}}}=&1+\frac{\alpha_s}{4\pi}\left\{\left(\frac{11C_A}{3}-\frac{2n_f}{3}\right)\ln\frac{2sy^2}{x}-\frac{10n_f}{9}+\left(-\frac{2\ln x}{3}+\frac{2\ln s}{3}+\frac{2\ln 2}{3}+\frac{10}{9}\right)n_b+\frac{2s\ln x}{1-6s}\right.\\
&+\frac{2s\ln s}{1-6s}+\frac{(4s+2)\ln 2}{1-6s}+\frac{\pi^2}{18s-3}+C_A\left[\frac{2(s-1)(4s-1)\mathrm{Li}_2(1-2s)}{6s-1}+\frac{(4s^2-6s+2)\,\mathrm{Li}_2(1-s)}{1-6s}\right.\\
&+\frac{s\ln^2 x}{1-6s}+\left(\frac{2s}{1-6s}+\frac{2s\ln s}{1-6s}+\frac{(2s+2)\ln 2}{1-6s}\right)\ln x+\frac{s(2s-3)\ln^2 s}{6s-1}+\left(\frac{2s}{1-6s}+\frac{4(2s-3)s\ln 2}{6s-1}\right)\ln s\\
&\left.+\frac{2s(2s-3)\ln^2 2}{6s-1}+\frac{6s\ln 2}{1-6s}+\frac{438s+3\pi^2(s(2s-5)+1)-85}{54s-9}\right]+C_F\left[-\frac{4(s-1)(4s-1)\mathrm{Li}_2(1-2s)}{6s-1}\right.\\
&+\frac{(8s^2+2)\,\mathrm{Li}_2(1-s)}{6s-1}+\frac{(3s+1)\ln^2 x}{6s-1}+\left(\frac{6-35s}{6s-1}+\left(\frac{3}{6s-1}+1\right)\ln s+\frac{(20s+2)\ln 2}{6s-1}\right)\ln x\\
&+\frac{(13-4s)s\ln^2 s}{6s-1}+\left(\frac{(27-34s)s-7}{4s(3s-2)+1}+\frac{(8(5-2s)s-2)\ln 2}{6s-1}\right)\ln s+\frac{(4s(2s-5)+1)\ln^2 2}{1-6s}\\
&\left.\left.+\frac{(16s(2s-1)-2)\ln 2}{4s(3s-2)+1}+\frac{-309s+\pi^2((19-4s)s+2)+51}{18s-3}\right]\right\},\end{aligned}$$

3.2

μ & q^2 dependence of form factors





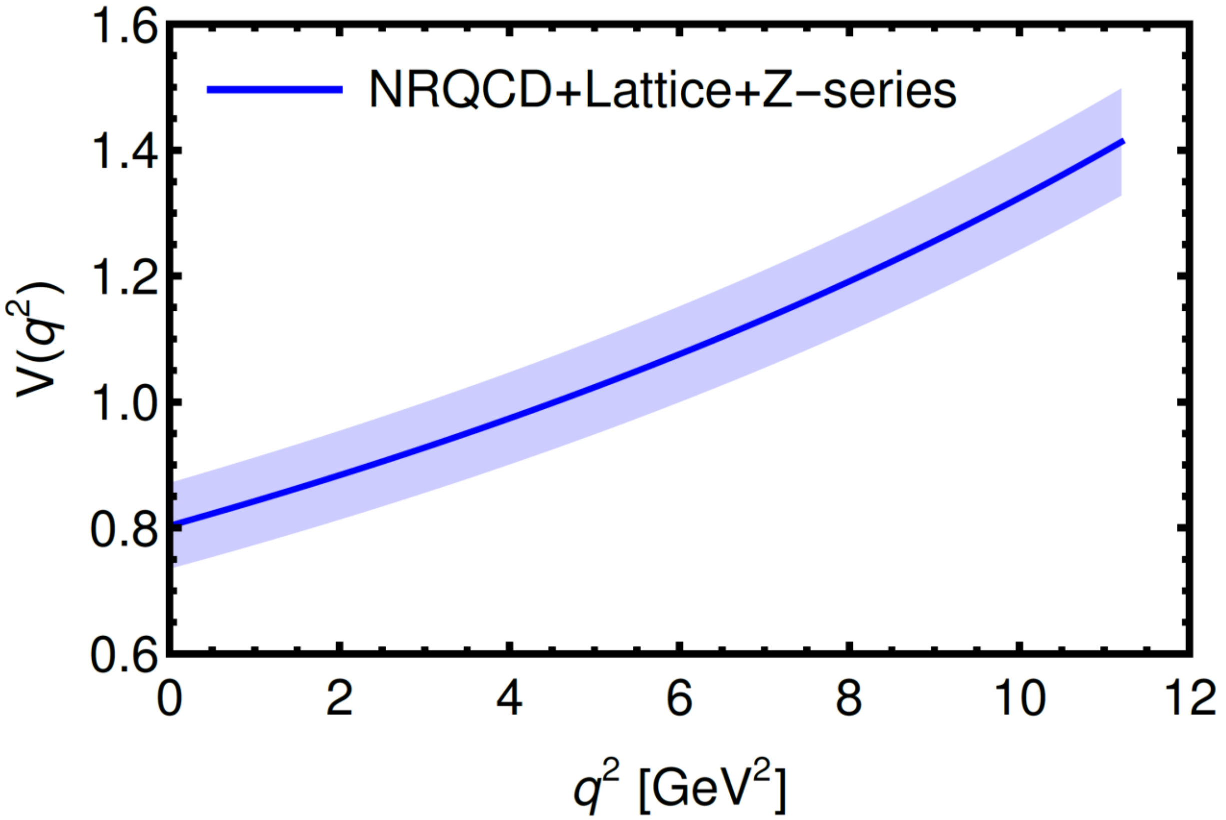
- NLO corrections reduce the renormalization scale dependence
- NLO corrections are sizable but convergent in low q^2 region
- The convergence breaks down at high q^2

3.3 NRQCD+Lattice+Z-series predictions in full q^2 range

$$F_{i,\mathrm{NRQCD+lattice}}^{B_c^*\to\eta_c}(q^2) \approx \frac{1}{2}\sum_{j=1}^{2}\frac{F_{i,\mathrm{NRQCD}}^{B_c^*\to\eta_c}(q^2)}{F_{j,\mathrm{NRQCD}}^{B_c\to\eta_c}(q^2)}F_{j,\mathrm{lattice}}^{B_c\to\eta_c}(q^2),$$

$$F_i(q^2) = \frac{1}{1-\frac{q^2}{m_R^2}}\sum_{n=0}^{N}\alpha_{i,n}\, z^n\left(q^2\right),$$

	NRQCD+Lattice	LFQM		BSW
		[5]	[4]	[6, 7]
$V(0)$	$0.8062^{-0.0082}_{+0.0121} \pm 0.0682$	0.91	$0.88^{+0.01+0.04}_{-0.01-0.06}$	$0.753^{+0.107}_{-0.129}$
$A_0(0)$	$0.5243^{-0.0008}_{+0.0012} \pm 0.0443$	0.66	$0.56^{+0.01+0.01}_{-0.02-0.02}$	0.526
$A_1(0)$	$0.5410^{+0.0064}_{-0.0095} \pm 0.0457$	0.69	$0.59^{+0.01+0.02}_{-0.02-0.03}$	$0.561^{+0.080}_{-0.096}$
$A_2(0)$	$0.4766^{-0.0211}_{+0.0314} \pm 0.0403$	0.59	$0.45^{+0.02+0.01}_{-0.02-0.02}$	$0.495^{+0.035}_{-0.062}$
$T_{1,2}^{(\prime)}(0)$	$0.5475^{+0.0020}_{-0.0030} \pm 0.0463$	–	–	–



[HPQCD, 1611.01987]
[Q.Chang,L.L.Chen,S.Xu, JPG(2018)]
[P.R,R.Dhir, 1908.00242]
[Y.Yang,et al., CPC(2023)]
[S.Y.Wang,et al., CPC(2024)]

[C.G.Boyd,B.Grinstein,R.F.Lebed, PRD(1997)]
[D.Leljak,B.Melic,M.Patra, JHEP(2019)]

3.4

Decay Widths & Branching Fractions

$$\frac{d\Gamma_L}{dq^2}=\left(\frac{q^2-m_l^2}{q^2}\right)^2\frac{\sqrt{\lambda(q^2)}G_F^2|V_{cb}|^2}{384m_{B_c^*}^3\pi^3q^2}\Bigg\{3m_l^2\lambda(q^2)A_0^2(q^2)$$
$$+\Bigg|(m_{B_c^*}^2-m_{\eta_c}^2-q^2)(m_{B_c^*}+m_{\eta_c})A_1(q^2)$$
$$-\frac{\lambda(q^2)}{m_{B_c^*}+m_{\eta_c}}A_2(q^2)\Bigg|^2\frac{m_l^2+2q^2}{4m_{\eta_c}^2}\Bigg\},$$

$$\frac{d\Gamma_\pm}{dq^2}=\left(\frac{q^2-m_l^2}{q^2}\right)^2\frac{\sqrt{\lambda(q^2)}G_F^2|V_{cb}|^2}{384m_{B_c^*}^3\pi^3}\Bigg\{(m_l^2+2q^2)\lambda(q^2)$$
$$\times\left|\frac{V(q^2)}{m_{B_c^*}+m_{\eta_c}}\mp\frac{(m_{B_c^*}+m_{\eta_c})A_1(q^2)}{\sqrt{\lambda(q^2)}}\right|^2\Bigg\},$$

$$\frac{d\Gamma_T}{dq^2}=\frac{d\Gamma_+}{dq^2}+\frac{d\Gamma_-}{dq^2},\quad\frac{d\Gamma}{dq^2}=\frac{d\Gamma_L}{dq^2}+\frac{d\Gamma_T}{dq^2},$$

$$\Gamma(B_c^*\to\eta_c l\nu_l)=\int_{m_l^2}^{(m_{B_c^*}-m_{\eta_c})^2}\frac{d\Gamma}{dq^2}dq^2.$$

[S.Y.Wang,et al., CPC(2024)]
[T.Wang,et al., JPG(2018)]
[Z.G.Wang, CTP(2014)]

	This work	LFQM [4]	BS [21]	QCDSR [8]
$\frac{10^{14}}{\mathrm{GeV}}\Gamma(B_c^*\to\eta_c e\nu_e)$	$2.541^{+0.494}_{-0.446}$	–	$0.966^{+0.094}_{-0.084}$	$0.686^{+0.225}_{-0.195}$
$\frac{10^{14}}{\mathrm{GeV}}\Gamma(B_c^*\to\eta_c \mu\nu_\mu)$	$2.529^{+0.489}_{-0.442}$	–	$0.963^{+0.094}_{-0.084}$	$0.684^{+0.224}_{-0.195}$
$\frac{10^{14}}{\mathrm{GeV}}\Gamma(B_c^*\to\eta_c \tau\nu_\tau)$	$0.688^{+0.104}_{-0.096}$	–	$0.290^{+0.029}_{-0.026}$	$0.215^{+0.075}_{-0.065}$
$10^7\mathcal{B}(B_c^*\to\eta_c e\nu_e)$	$4.235^{+0.824}_{-0.743}$	$4.48^{+1.14}_{-0.76}$	$4.20^{+0.41}_{-0.37}$	–
$10^7\mathcal{B}(B_c^*\to\eta_c \mu\nu_\mu)$	$4.214^{+0.816}_{-0.736}$	$4.45^{+1.14}_{-0.75}$	$4.19^{+0.41}_{-0.37}$	–
$10^7\mathcal{B}(B_c^*\to\eta_c \tau\nu_\tau)$	$1.147^{+0.173}_{-0.161}$	$1.03^{+0.26}_{-0.17}$	$1.26^{+0.13}_{-0.11}$	–
$\mathcal{R}_{\eta_c}$	$0.271^{+0.107}_{-0.076}$	$0.229^{+0.059}_{-0.059}$	0.300	–

Summary

- ✓ Obtain complete and asymptotic analytical results for NLO QCD corrections to $B_c^* \to \eta_c$ (axial-)vector and (axial-)tensor form factors
- ✓ Find NLO corrections reduce renormalization scale dependence, and are sizable but convergent in low q^2 region
- ✓ Provide NRQCD+Lattice+Z-series predictions for $B_c^* \to \eta_c$ form factors across full physical q^2 range
- ✓ Calculate decay widths and branching fractions for $B_c^* \to \eta_c + l\nu_l$

Thank you!