A precise determination of top quark electro-weak couplings at the FCC-ee

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Outline

- INTRODUCTION
- THEORETICAL FRAMEWORK
- OPTIMAL-OBSERVABLE STATISTICAL ANALYSIS
- SENSITIVITY TO TOP EW COUPLINGS
- RESULTS AND DISCUSSION
- SUMMARY AND OUTLOOK

INTRODUCTION

It can be generated by the existence of a new strong sector, inspired by QCD that may manifest itself at energies of around 1 TeV.
Standard Model fields would couple to the new sector with a strength that is proportional to their mass.

- **Motivation:**

To understand the nature of **electro-weak symmetry breaking,** the t quark is expected to be a window to any new physics(which will modify the electro-weak $t\bar{t}\gamma/t\bar{t}Z^0$vertex) at the TeV energy scale.

- **$t\bar{t}\gamma$/ $t\bar{t}Z$ vertex(Born level):**

In ILC there is no concurrent QCD production of t quark pairs whcih means a greatly clean measurement

$$\Gamma_\mu^{t\bar{t}X}(k^2,q,\bar{q}) = -ie\left\{\gamma_\mu\left(F_{1V}^X(k^2)+\gamma_5 F_{1A}^X(k^2)\right)+\frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})^\mu\left(iF_{2V}^X(k^2)+\gamma_5 F_{2A}^X(k^2)\right)\right\},$$

Within the Standard Model the Fi have the following values:

$\sin\theta_w$**~0.48** and $\cos\theta_w$ are the sine and the cosine of the Weinberg angle θ_w.

$$F_{1V}^\gamma = -\frac{2}{3}\,, F_{1V}^Z = \frac{1}{4\sin\theta_W\cos\theta_W}\left(1-\frac{8}{3}\sin^2\theta_W\right)\,,$$

$$F_{1A}^\gamma = 0\,, F_{1A}^Z = \frac{1}{4\sin\theta_W\cos\theta_W}\,.$$

Applying A, B, C, D parameterization

$$A_v+\delta A_v = -2i\sin\theta_W\left(F_{1V}^X+F_{2V}^X\right)\,, B_v+\delta B_v = -2i\sin\theta_W F_{1A}^X\,,$$

$$\delta C_v = -2i\sin\theta_W F_{2V}^X\,, \delta D_v = -2\sin\theta_W F_{2A}^X\,.$$

$$A_\gamma = \frac{4}{3}\sin\theta_W,\quad B_\gamma = 0,\quad A_Z = \frac{1}{2\cos\theta_W}\left(1-\frac{8}{3}\sin^2\theta_W\right),\quad B_Z = \frac{1}{2\cos\theta_W}$$

:

$$\Gamma_{t\bar{t}v}^\mu = \frac{g}{2}\left[\gamma^\mu\left\{(A_v+\delta A_v)-\gamma_5(B_v+\delta B_v)\right\}+\frac{(p_t-p_{\bar{t}})^\mu}{2m_t}(\delta C_v-\delta D_v\gamma_5)\right],$$

INTRODUCTION

- **Meaning:**
  - The **Yukawa coupling** is the interaction strength between the top quark and the Higgs boson, and the mass of the top quark is related to this coupling. The production threshold scan allows for precise measurements of these properties.
  - The **precision of top-quark mass**, along with the properties of Z/W is used in a **global electroweak fit which** helps **constraint weakly-coupled new physics** up to a scale of **100 TeV**.
  - The new physics may be manifested by a significant deviation in the top quark electrically weak coupling, which is manifested as a significant difference from the values predicted by the SM.

- **FCC-ee**
  - FCC-ee is known as the first step of the Future Circular Colliders (FCC) physics programme would exploit a high-luminosity e^+e^-collider.
  - **Centre-of-mass energies:** from below the Z pole to the $\mathbf{t\bar{t}}$ threshold and beyond (91.2-346+GeV)
  - **Luminosity:** 3.6ab^{-1} over a period of five years.1ab^{-1} ought to be kept for threshold measurements.
  - **Non-Polarised** electron and positron beams.This means that there are half of the left-hand particles and half of the right-hand particles in the beam.(Degree of polarization P =0)

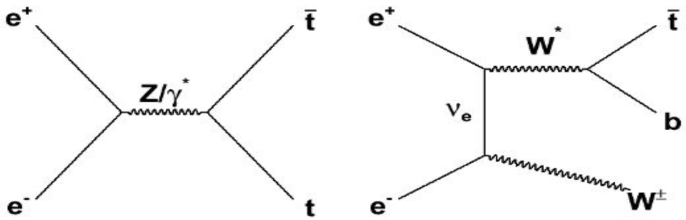
THEORETICAL FRAMEWORK

- **Signal process:** $e^+e^- \rightarrow t\bar{t} \rightarrow l^{\pm}\nu b\bar{b}q'\bar{q}$ ('lepton+jets' final state)
- **Several other SM processes give rise to the same final state.:**
  - **Ignored source(0.1%):** Single-top production $e^+e^- \rightarrow WW^* \rightarrow Wt\bar{b} \rightarrow l^{\pm}\nu b\bar{b}q'\bar{q}$

    At a more fundamental level, interference between between single-top production and top quark pair production renders the separation physically meaningless.
  - **Other source:** $Z^0WW, Z^0\ Z^0, Z^0\ Z^0\ Z^0, WW, q\bar{q}$

    These can be distinguished rather efficiently from top quark pair production.

* Therefore, background is not considered in this article.



(a) $t\bar{t}$ pair production (b) Single top production

THEORETICAL FRAMEWORK

- **Form Factors:**
  - The coupling F_{2A}^{γ} is related to the dipole moment $d = (\frac{e}{2mt})F_{2A}(0)$ that violates the combined Charge and Parity symmetry CP. Here it is fixed to 0 because no significant damage has been observed so far.
  - The coupling F_{2V}^{γ} is related via $F_{2V}^{\gamma} = \frac{Q_{t}(g-2)}{2}$, the anomalous magnetic moment (g − 2) and electrical charge of the t quark Q_t
  - There are six CP conserving form factors defined for the Z and the photon, F_{1V} , F_{1A}, F_{2V} but because close to the tt threshold the observables depend always on the sum F_{1V} + F_{2V} . Therefore a full disentangling of the form factors will be imprecise for energies below about 1 TeV
  - The form factor F_{1A}^{γ} is also kept to its standard model value(0), as a non-zero value would lead to gauge-invariance violation.
  - Hence, in the present study either the three form factors $F_{1V,A}^{Z}$, F_{1V}^{γ} are varied simultaneously, while the two F_{2V}^{X} are kept at their Standard Model values or vice versa.(This part is same as ILC study:arXiv:1307.8102v1 [hep-ex] 30 Jul 2013)

THEORETICAL FRAMEWORK

Observables and Form Factors

- The tree-level angular and energy distributions of the lepton

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}\cos\theta} = \frac{3\pi\beta\alpha^2(s)}{2s} B_\ell S_\ell(x, \cos\theta),$$

  where β is the top velocity, s is the center-of-mass energy squared, α(s) is the QED running coupling constant, and B` is the fraction of $\mathbf{t\bar{t}}$ events with at least one top quark decaying to either eνb or μνb (about 44%).

- As the non-standard form factors(only real part) $\delta(A, B, C, D)_v \equiv \delta_i$ are supposedly small, only the terms linear in δ_i are kept:

$$S(x, \theta) = S^0(x, \theta) + \sum_{i=1}^{8} \delta_i f_i(x, \cos\theta),$$

- x and θ are the lepton (reduced) energy and polar angle, respectively, and S^0 is the standard-model contribution.

$$x = \frac{2E_\ell}{m_t}\sqrt{\frac{1-\beta}{1+\beta}},$$

THEORETICAL FRAMEWORK

- **Expanded form：**

$$S_f^{(*)}(x_f,\theta_f) = S_f^{(0,*)}(x_f,\theta_f) + \sum_{v=\gamma,Z}\Big[\mathrm{Re}(\delta A_v)\mathcal{F}_{Av}^{f(*)}(x_f,\theta_f) + \mathrm{Re}(\delta B_v)\mathcal{F}_{Bv}^{f(*)}(x_f,\theta_f) + \mathrm{Re}(\delta C_v)\mathcal{F}_{Cv}^{f(*)}(x_f,\theta_f) + \mathrm{Re}(\delta D_v)\mathcal{F}_{Dv}^{f(*)}(x_f,\theta_f)\Big]$$

~~$+\mathrm{Re}(f_2^R)\mathcal{F}_{2R}^{f(*)}(x_f,\theta_f)$~~ the determination of this crucially on the chosen beam polarization.

$$S_f^{(0,*)}(x_f,\theta_f) = \Theta_0^{f(0,*)}(x_f) + \cos\theta_f\,\Theta_1^{f(0,*)}(x_f) + \cos^2\theta_f\,\Theta_2^{f(0,*)}(x_f) \qquad (4.6)$$

$$\Theta_0^{f(0,*)}(x) = \frac{1}{2}\Big[(3-\beta^2)D_V^{(0,*)} - (1-3\beta^2)D_A^{(0,*)} - 2\alpha_0^f(1-\beta^2)\mathrm{Re}(D_{VA}^{(0,*)})\Big]f^f(x) + 2\alpha_0^f\mathrm{Re}(D_{VA}^{(0,*)})\,g^f(x) + \frac{1}{2}\Big[D_V^{(0,*)} + D_A^{(0,*)} + 2\alpha_0^f\,\mathrm{Re}(D_{VA}^{(0,*)})\Big]\Big[2h_1^f(x) - h_2^f(x)\Big], \qquad (4.7)$$

$$\Theta_1^{f(0,*)}(x) = 2\Big[2\mathrm{Re}(E_{VA}^{(0,*)}) + \alpha_0^f(1-\beta^2)E_A^{(0,*)}\Big]f^f(x) + 2\alpha_0^f\,(E_V^{(0,*)} + E_A^{(0,*)})\,g^f(x) - 2\Big[2\mathrm{Re}(E_{VA}^{(0,*)}) + \alpha_0^f(E_V^{(0,*)} + E_A^{(0,*)})\Big]h_1^f(x), \qquad (4.8)$$

$$\Theta_2^{f(0,*)}(x) = \frac{1}{2}\Big[(3-\beta^2)(D_V^{(0,*)} + D_A^{(0,*)}) + 6\alpha_0^f(1-\beta^2)\mathrm{Re}(D_{VA}^{(0,*)})\Big]f^f(x) + 2\alpha_0^f\mathrm{Re}(D_{VA}^{(0,*)})\,g^f(x) - \frac{3}{2}\Big[D_V^{(0,*)} + D_A^{(0,*)} + 2\alpha_0^f\mathrm{Re}(D_{VA}^{(0,*)})\Big]\Big[2h_1^f(x) - h_2^f(x)\Big]. \qquad (4.9)$$

$$\mathcal{F}_{Av}^{f(*)}(x,\theta) = \Big[\frac{1}{2}(3-\beta^2)C(D_V{:}A_v)f^f(x) + 2\alpha_0^f C(D_{VA}{:}A_v)g^f(x)\Big](1+\cos^2\theta) - \Big[\alpha_0^f(1-\beta^2)C(D_{VA}{:}A_v)f^f(x) - \frac{1}{2}\{C(D_V{:}A_v) + 2\alpha_0^f C(D_{VA}{:}A_v)\}\{2h_1^f(x) - h_2^f(x)\}\Big](1-3\cos^2\theta) + 2\Big[\alpha_0^f C(E_V{:}A_v)\{g^f(x) - h_1^f(x)\} + 2C(E_{VA}{:}A_v)\{f^f(x) - h_1^f(x)\}\Big]\cos\theta, \qquad (A.2)$$

$$\mathcal{F}_{Bv}^{f(*)}(x,\theta) = \frac{1}{2}\beta^2 C(D_A{:}B_v)f^f(x)(3-\cos^2\theta) + 2\alpha_0^f C(D_{VA}{:}B_v)g^f(x)(1+\cos^2\theta) - \frac{1}{2}\Big[\{C(D_A{:}B_v) + 2\alpha_0^f(1-\beta^2)C(D_{VA}{:}A_v)\}f^f(x) - \{C(D_A{:}B_v) + 2\alpha_0^f C(D_{VA}{:}B_v)\}\{2h_1^f(x) - h_2^f(x)\}\Big](1-3\cos^2\theta) + 2\Big[\{\alpha_0^f(1-\beta^2)C(E_A{:}B_v) + 2C(E_{VA}{:}B_v)\}f^f(x) + \alpha_0^f C(E_A{:}B_v)g^f(x) - \{\alpha_0^f C(E_A{:}B_v) + 2C(E_{VA}{:}B_v)\}h_1^f(x)\Big]\cos\theta, \qquad (A.3)$$

$$\mathcal{F}_{Cv}^{f(*)}(x,\theta) = -\beta^2 C(G_1{:}C_v)f^f(x)(1+\cos^2\theta) + 2\alpha_0^f C(G_2{:}C_v)\Big[f^f(x) + g^f(x) - h_1^f(x)\Big]\cos\theta$$

$$-\{2C(G_1{:}C_v) + 3\alpha_0^f C(G_3{:}C_v)\}h_1^f(x) + \{C(G_1{:}C_v) + \alpha_0^f C(G_3{:}C_v)\}h_2^f(x)\Big](1-3\cos^2\theta) \qquad (A.4)$$

$$\mathcal{F}_{Dv}^{f(*)}(x,\theta) = \alpha_0^f C(F_1{:}D_v)\Big[f^f(x) - h_1^f(x)\Big](1-3\cos^2\theta) - \alpha_0^f C(F_1{:}D_v)g^f(x)(1+\cos^2\theta) - 2\alpha_0^f C(F_4{:}D_v)\Big[f^f(x) + g^f(x) - h_1^f(x)\Big]\cos\theta, \qquad (A.5)$$

$$f^\ell(x) = \frac{3(1+\beta)}{2\beta W}[\omega^2]_{\omega_-}^{\omega_+}\ \Big(\equiv \frac{3(1+\beta)}{2\beta W}(\omega_+^2 - \omega_-^2)\Big),$$
$$g^\ell(x) = f^\ell(x) + \frac{3(1+\beta)^2}{\beta W}x\Big[\omega + \ln|1-\omega|\Big]_{\omega_-}^{\omega_+},$$
$$h_1^\ell(x) = \frac{1-\beta^2}{2\beta W}\frac{1}{x}\Big[\omega^2(3-2\omega)\Big]_{\omega_-}^{\omega_+},$$
$$h_2^\ell(x) = \frac{1}{4\beta W}(1+\beta)(1-\beta)^2\frac{1}{x^2}\Big[\omega^2(6-8\omega+3\omega^2)\Big]_{\omega_-}^{\omega_+},$$
$$\delta f^\ell(x) = -\frac{3(1+\beta)}{\beta W}\sqrt{r}\Big[2\omega + 2\ln|1-\omega| + \frac{3\omega^2}{1+2r}\Big]_{\omega_-}^{\omega_+},$$
$$\delta g^\ell(x) = \delta f^\ell(x) - \frac{6(1+\beta)^2}{\beta W}\sqrt{r}\,x\Big[\ln|1-\omega| + \frac{1}{1-\omega} + \frac{3}{1+2r}(\omega + \ln|1-\omega|)\Big]_{\omega_-}^{\omega_+},$$
$$\delta h_1^\ell(x) = \frac{3(1-\beta^2)}{\beta W}\frac{\sqrt{r}}{x}\Big[\omega^2\Big(1 - \frac{3-2\omega}{1+2r}\Big)\Big]_{\omega_-}^{\omega_+},$$
$$\delta h_2^\ell(x) = \frac{1}{2\beta W}(1+\beta)(1-\beta)^2 \times \frac{\sqrt{r}}{x^2}\Big[2\omega^2(3-2\omega) - \frac{3\omega^2}{1+2r}(6-8\omega+3\omega^2)\Big]_{\omega_-}^{\omega_+}, \qquad (A.10)$$

$$C(D_V{:}A_\gamma) = 2C[\mathcal{P}_\otimes A_\gamma - (\mathcal{P}_\oplus + v_e\mathcal{P}_\otimes)d'A_Z],$$
$$C(E_V{:}A_\gamma) = -2C[\mathcal{P}_\oplus A_\gamma - (\mathcal{P}_\otimes + v_e\mathcal{P}_\oplus)d'A_Z],$$
$$C(D_{VA}{:}A_\gamma) = -C(\mathcal{P}_\oplus + v_e\mathcal{P}_\otimes)d'B_Z,$$
$$C(E_{VA}{:}A_\gamma) = C(\mathcal{P}_\otimes + v_e\mathcal{P}_\oplus)d'B_Z,$$
$$C(D_V{:}A_Z) = -2C[(\mathcal{P}_\oplus + v_e\mathcal{P}_\otimes)d'A_\gamma - \{2v_e\mathcal{P}_\oplus + (1+v_e^2)\mathcal{P}_\otimes\}d'^2A_Z],$$
$$C(E_V{:}A_Z) = 2C[(\mathcal{P}_\otimes + v_e\mathcal{P}_\oplus)d'A_\gamma - \{2v_e\mathcal{P}_\otimes + (1+v_e^2)\mathcal{P}_\oplus\}d'^2A_Z],$$
$$C(D_{VA}{:}A_Z) = C[2v_e\mathcal{P}_\oplus + (1+v_e^2)\mathcal{P}_\otimes]d'^2B_Z,$$
$$C(E_{VA}{:}A_Z) = -C[2v_e\mathcal{P}_\otimes + (1+v_e^2)\mathcal{P}_\oplus]d'^2B_Z, \qquad (A.15)$$

OPTIMAL-OBSERVABLE STATISTICAL ANALYSIS

- **Likelihood fit :**
  - There are nine different functions, and eight form factors δ_i to be evaluated from a given sample of $t\bar{t}$ events. In principle, all eight form factors and their uncertainties can therefore be determined simultaneously, under the condition that the nine functions are linearly independent. Experimentalists usually maximize numerically a global likelihood L :

$$L = \frac{\mu^N}{N!} e^{-\mu} \times \prod^N p(k),$$

where N is the total number of $t\bar{t}$ events observed in the data sample, μ is the number of events expected for the integrated luminosity $\mathcal{L}$ of the data sample ($\mu = \sigma_{\rm tot} \times \mathcal{L}$), and

$$p(k) = \frac{1}{\sigma_{\rm tot}} \frac{d^2\sigma}{dx d\cos\theta}(x_k, \cos\theta_k), \text{ with } \sigma_{\rm tot} = \int \frac{d^2\sigma}{dx d\cos\theta} dx d\cos\theta.$$

- **Uncertainties**
  - The covariance matrix obtained from the numerical minimization of the negative loglikelihood is then inverted to get the uncertainties on the form factors $\sigma(\delta_i)$.

$$S(x,\theta) = S^0(x,\theta) + \sum_{i=1}^{8} \delta_i f_i(x, \cos\theta),$$

$$V_{ij} = \mathcal{L} \int d\Omega \frac{f_i \times f_j}{S^0},$$

OPTIMAL-OBSERVABLE STATISTICAL ANALYSIS

- **Uncertainties:**

  - If fix F_{2V}^{X} , F_{2A}^{X}, F_{1A}^{γ} as SM value, there are only the three coefficients F_{1V}^{γ} , F_{1V}^{Z} and F_{1A}^{Z} are allowed to vary.
  - If fix F_{1V}^{γ} , F_{1V}^{Z} , F_{1A}^{Z} F_{2A}^{X}, F_{1A}^{γ} as SM value, there are only the two coefficients F_{2V}^{X} are allowed to vary.

The distribution will be like following by replace A.B,C,D by form factors.

which leads to the following 3 × 3 covariance matrix$V_1 = 4sin^2\theta_W \times L \times$X , with

$$S(x,\theta) = S^0(x,\theta) - 2i\sin\theta_W \delta F_{1V}^{\gamma} f_A^{\gamma} - 2i\sin\theta_W \delta F_{1V}^{Z} f_A^{Z} + -2i\sin\theta_W \delta F_{1A}^{Z} f_B^{Z} \,,$$

$$X_{11} = \int \mathrm{d}\Omega \frac{(f_A^{\gamma})^2}{S^0} \,,\ X_{12} = \int \mathrm{d}\Omega \frac{f_A^{\gamma} \times f_A^{Z}}{S^0} \,,\ X_{13} = \int \mathrm{d}\Omega \frac{f_A^{\gamma} \times f_B^{Z}}{S^0} \,,$$

$$X_{22} = \int \mathrm{d}\Omega \frac{(f_A^{Z})^2}{S^0} \,,\quad X_{23} = \int \mathrm{d}\Omega \frac{f_A^{Z} \times f_B^{Z}}{S^0} \,,$$

$$X_{33} = \int \mathrm{d}\Omega \frac{(f_B^{Z})^2}{S^0} \,.$$

which leads to the following 2 × 2 covariance matrix$V_2 = 4sin^2\theta_W \times L \times$X , with

$$S(x,\theta) = S^0(x,\theta) - 2i\sin\theta_W \delta F_{2V}^{\gamma} (f_A^{\gamma} + f_C^{\gamma}) - 2i\sin\theta_W \delta F_{2V}^{Z} (f_A^{Z} + f_C^{Z}) \,,$$

$$Y_{11} = \int \mathrm{d}\Omega \frac{(f_A^{\gamma} + f_C^{\gamma})^2}{S^0} \,,\ Y_{12} = \int \mathrm{d}\Omega \frac{(f_A^{\gamma} + f_C^{\gamma}) \times (f_A^{Z} + f_C^{Z})}{S^0} \,,$$

$$Y_{22} = \int \mathrm{d}\Omega \frac{(f_A^{Z} + f_C^{Z})^2}{S^0} \,.$$

SENSITIVITY TO TOP EW COUPLINGS

- **Assumptions:**
  - A perfect event reconstruction
  - An event selection efficiency of 100%
  - A 4π detector acceptance
  - The absence of background processes
- **Event reconstruction:**
  - The only reconstructed quantities used for the determination of the covariance matrices are the lepton direction and the lepton energy (or momentum).
  - To perform with 50 bins in **x(lepton reduced energy)** and **cos θ(polar angle),**this is conservatively assuming a lepton energy resolution of 1 GeV and a lepton angular resolution of 20 mrad, figures vastly exceeded by LEP detectors.
- **Event selection and particle identification**

The only reconstructed quantities used for the determination of the covariance matrices are the lepton direction and the lepton energy (or momentum).

  - The isolated lepton momentum:13.5-120GeV; with an identification efficiency of 80%
  - The b-quark jet energies:49-94GeV;with a b-tagging efficiency of 60%

So to emulate these efficiencies,all covariance matrix elements, **are multiplied by 0.6 × 0.8 = 0.48.**

SENSITIVITY TO TOP EW COUPLINGS

- **Detector acceptance:**
  - Considering the polar-angle coverage of a typical detector(10-170degree), the leptons are assumed here to be detected only for $|\cos\theta| < 0.9$, i.e., in a range from 26 to 154 degrees.
  - So the **cosθ integral calculation** of covariance matrix should be from -0.9-0.9.
  - Given the large value of the minimum lepton energy, the integration bounds over x are left untouched.
- **Other experimental uncertainties :**
  - Different from ILC research needs to consider such as those affecting the **measurement of the beam polarization**; **the effects of beamstrahlung**; or **the ambiguous top-quark reconstruction**, since FCC-ee is non-polarized and beamstrahlung effects are negligible and the top-quark is not used; Therefore none of these contribute to uncertainties;.
  - The total event rate, needed for the present study, requires a precise luminosity determination, a measurement that can be controlled to **a fraction of a per mil**, hence neglected here.
- **Theoretical uncertainties**
  - The total event rate indeed requires an accurate prediction of the total cross section for top pair production.
  - A **few per mil** level precision can be expected at smaller centre-of-mass energy as long as it is reasonably above the production threshold.

SENSITIVITY TO TOP EW COUPLINGS

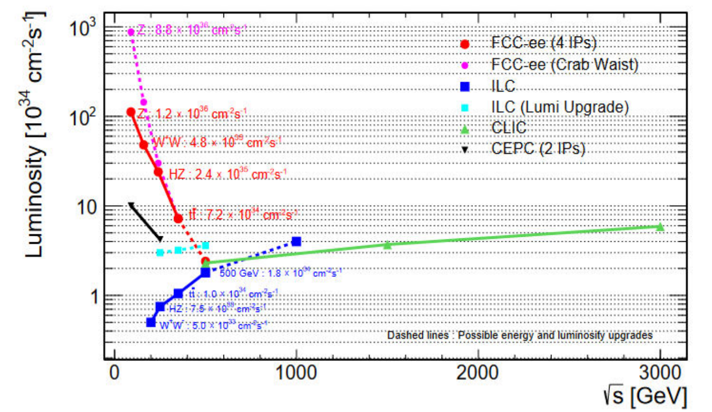
● Integrated luminosity profile :

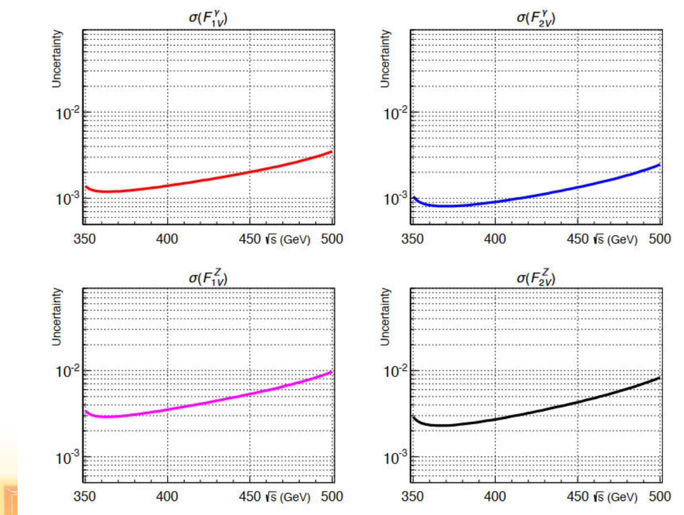
- o The maximum centre-of-mass energy of the FCC-ee is yet unknown. The centre-of-mass energy was therefore varied from 350 to 500 GeV, and the corresponding integrated luminosity was varied linearly with $\sqrt{s}$from 2.6 to 0.5ab^{-1}.

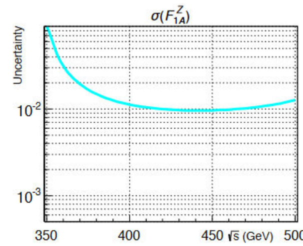
- o The target luminosities at the FCC-ee are displayed in Fig. 2 [11] as a function of the centre-of-mass energy

Above all the expected uncertainties on the top electroweak form factors is from the corrections for the lepton energy and angular resolutions, the event selection efficiency, and the detector acceptance.

Target luminosities of e^+e^-colliders in the world





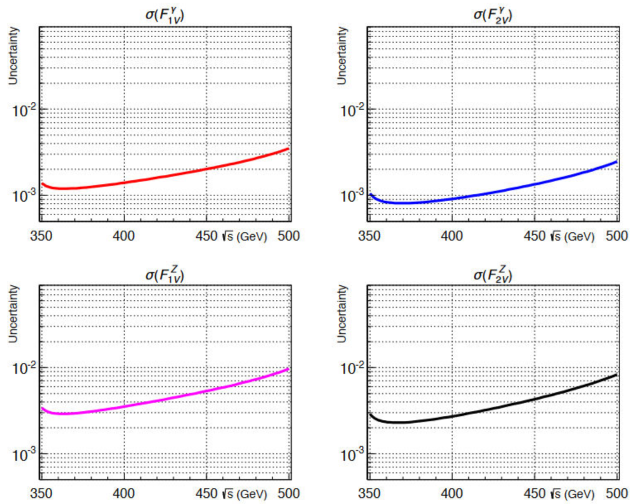


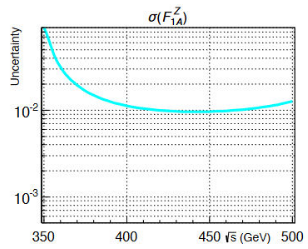
Uncertainties and Centre-of-mass energy

SENSITIVITY TO TOP EW COUPLINGS

Result:

- The first striking observation from the plot of Uncertainties and Centre-of-mass is that an increase of the centre-of-mass energy far beyond the top-pair production threshold is not particularly relevant to improve the precision on the top-quark electroweak couplings.





Uncertainties and Centre-of-mass energy

- For F_{1V}^{γ}, F_{2V}^{γ}, F_{1V}^{Z}, F_{2V}^{Z}, the optimum precision is actually reached with $\sqrt{s}\sim 365GeV$,for F_{1A}^{Z} the precision is also within 50% of optimum.
- The accuracy of these coupling constants in this study reached a high level, which was sufficient to independently determine the values of the ttγ and ttZ couplings **without relying on the initial polarization** .
- The use of the b jets can do sth which is the subject of further studies with more detailed event reconstruction algorithms.

RESULTS AND DISCUSSION

Expected statistical accuracies

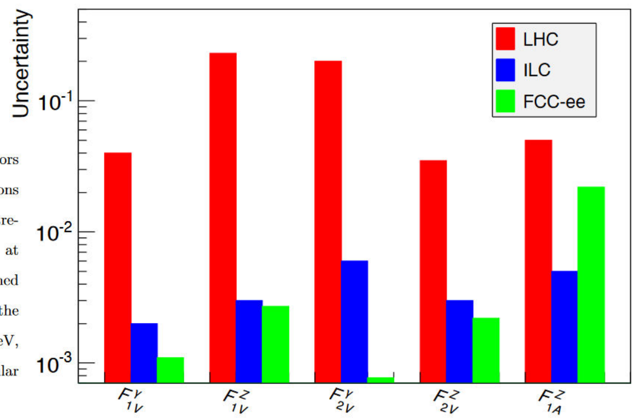
As anticipated, the lack of incoming beam polarization at the FCC-ee is more than compensated by the use of the final state polarization and by a significantly larger integrated luminosity, even with the sole use of the lepton energy and angular distributions, and modest detector performance.

Theory uncertainties

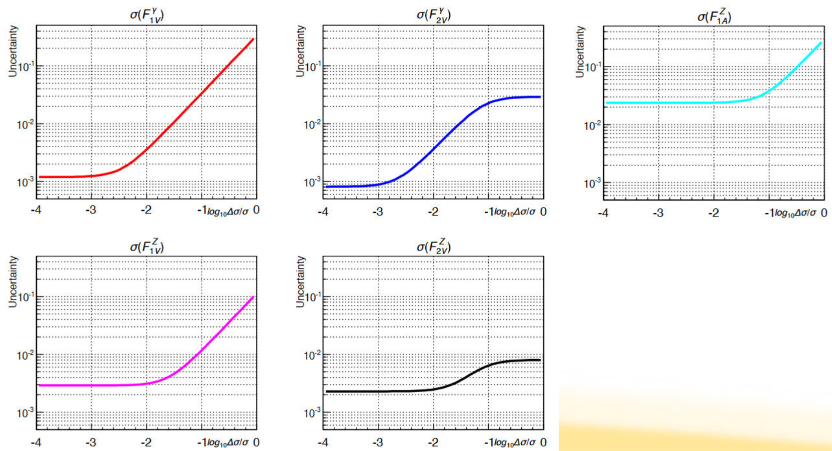
- The dominant systematic error is the theoretical uncertainty on the predicted event rate.
- To evaluate the effects of any value of the assumed cross-section theoretical error, the likelihood in was enhanced with the corresponding Gaussian nuisance factor.

$$L = \frac{\mu^N}{N!} e^{-\mu} \times \prod_{k=1}^{N} p(k),$$

FIG. 4. (Modified from Ref. [3]). Statistical uncertainties on CP-conserving top-quark form factors expected at the ILC (blue) and the LHC (red). The figure was modified to include the projections from the FCC-ee. The results for the LHC assume an integrated luminosity of 300 fb^{-1} and a centre-of-mass energy of 14 TeV. The results for the ILC assume an integrated luminosity of 500 fb^{-1} at $\sqrt{s} = 500$ GeV, and beam polarizations of $\mathcal{P} = \pm 0.8$, $\mathcal{P}' = \mp 0.3$. The ILC projections are obtained from the measurements of the total top-quark pair production cross section, together with the top-quark forward-backward asymmetry. The FCC-ee projections are obtained at $\sqrt{s} = 365$ GeV, with unpolarized beams and with an integrated luminosity of 2.4 ab^{-1}, from the sole lepton angular and energy distributions.



Uncertainty on the form factors at the FCC-ee with 2.4 ab^{-1} at $\sqrt{s} = 365$ GeV, as a function of the relative cross-section theorerical error, varied from 0.01% to 100%. Left column, from top to bottom: F_{1V}^{γ}, F_{1V}^{Z}, and F_{1A}^{Z}. Right column: F_{2V}^{γ} and F_{2V}^{Z}.



RESULTS AND DISCUSSION

Discussion

The precision of two configurations by relaxing the constraints on F^{γ}_{1A}, F^{γ}_{2A} and F^{Z}_{2A} :

- In the first configuration, it turns out that relaxing the constraint on F^{γ}_{1A} does not sizeably change the precision on the other three $F^{X}_{1V,A}$ form factors, as shown in Table:

Precision on	F^{γ}_{1V}	F^{Z}_{1V}	F^{γ}_{1A}	F^{Z}_{1A}
Only three $F^{X}_{1V,A}$	$1.2\,10^{-3}$	$2.9\,10^{-3}$	$0.0\,10^{-2}$	$2.2\,10^{-2}$
All four $F^{X}_{1V,A}$	$1.2\,10^{-3}$	$3.0\,10^{-3}$	$1.3\,10^{-2}$	$2.4\,10^{-2}$
$\sqrt{s} = 500\,\mathrm{GeV}$	$5.5\,10^{-3}$	$1.5\,10^{-2}$	$1.0\,10^{-2}$	$2.2\,10^{-2}$

$2.4ab^{-1}$,$\sqrt{s} = 365GeV$, $F^{X}_{1V,A}$ are fixed to their SM value

- In the second configuration, with vanishing correlation coefficients, the two pairs of form factors(F^{X}_{2A} , F^{X}_{2V}) can therefore be determined independently from each other.
  1. The precisions on F^{γ}_{2V} and F^{Z}_{2A}, expected with $2.4ab^{-1}$ at $\sqrt{s} = 365GeV$ are unchanged, when the constraint on F^{γ}_{2A} and F^{Z}_{2A} is relaxed, and amount to 8.1 10^{-4} and 2.3 10^{-3} respectively.
  2. With 500 fb^{-1} at $\sqrt{s} = 500GeV$, the precisions would be 2.5 10^{-3} and 8.3 10^{-3} respectively

RESULTS AND DISCUSSION

● Discussion

Because of the important correlation between the two distributions $f_D^{\gamma}$and f_D^{Z},the The accuracy of the CP-violating form factors with the sole lepton angle and energy distributions is moderately constraining.

- A relevant precision of 1.7 10^{-2} is reached on the **linear combination(F_{2A}^{γ}+0.17 F_{2A}^{Z})** with 2.4ab^{-1}at $\sqrt{s} = 365GeV$ and it reduced to 0.9 10^{-2} with 500 fb^{-1} at $\sqrt{s} = 500GeV$.

Similarly, when all eight parameters are considered simultaneously, the lepton angle and energy distributions are no longer sufficient to avoid large correlations between form factors.

- There is an optimal-observable analysis of the matrix element squared is carried out with thirteen different observables (**the top quark direction**, **the l^+ and l^- angles and energies, the b and $\bar{b}$ angles and energies, and the invariant masses of the top quarks and W bosons**), with unambiguous identification and reconstruction under the assumption of a perfect detector.
- With these additional variables, the **few degeneracies** between form factors are indeed removed, but the conclusion is identical to that of this paper: the incoming beam polarizations are not essential in the process.

SUMMARY AND OUTLOOK

- **Summary:**
  - The measurements of the angular and energy distributions in semi-leptonic $t\bar{t}$ events ($e^+e^- \rightarrow t\bar{t} \rightarrow l^{\pm}\nu b\bar{b}q'\bar{q}$) at future e^+e^- colliders have a strong potential for a precise determination of the top-quark electroweak couplings.
  - The lack of incoming beam polarization at the FCC-ee is compensated by the polarization of the final state top quarks, and by a significantly larger integrated luminosity.
  - In view of these new estimates, it becomes of particular interest to check their added value to the sensitivity to new physics, especially when combined with the **unequalled precision of the measurements of the Z, the W**, and **the Higgs boson properties**, as well as of **the top-quark mass**, at the FCC-ee.
- **Outlook**
  - Such a Monte Carlo study will also allow a reliable reconstruction of all observables in the event, beyond the **lepton energies and directions,** and is expected to bring sizeable improvements, especially on the few remaining correlations between form factors.