A precise determination of top quark electro-weak couplings at the ILC

Huayu Liu
huayuliu9499@163.com

Sun Yat-sen University
14/2/2025

Outline

- Introduction
- Top quark production at the ILC
- Event generation and technical remarks
- Event selection
- Measurement of the forward backward asymmetry
- The slope of the helicity angle distribution
- Precision of Form Factors
- Summary and outlook

Introduction

It can be generated by the existence of a new strong sector, inspired by QCD that may manifest itself at energies of around 1 TeV.
Standard Model fields would couple to the new sector with a strength that is proportional to their mass.

- **Motivation:**

To understand the nature of **electro-weak symmetry breaking,** the t quark is expected to be a window to any new physics(which will modify the electro-weak $t\bar{t}\gamma/t\bar{t}Z^0$vertex) at the TeV energy scale.

- **$t\bar{t}\gamma$/ $t\bar{t}Z$ vertex(Born level):**

In ILC there is no concurrent QCD production of t quark pairs whcih means a greatly clean measurement

$$\Gamma_\mu^{ttX}(k^2, q, \bar{q}) = ie\left\{\gamma_\mu \left(\widetilde{F}_{1V}^X(k^2) + \gamma_5\widetilde{F}_{1A}^X(k^2)\right) + \frac{(q-\bar{q})_\mu}{2m_t}\left(\widetilde{F}_{2V}^X(k^2) + \gamma_5\widetilde{F}_{2A}^X(k^2)\right)\right\}.$$

Within the Standard Model the Fi have the following values:

$$F_{1V}^{\gamma,SM} = -\frac{2}{3},\ F_{1A}^{\gamma,SM} = 0,\ F_{1V}^{Z,SM} = -\frac{1}{4s_w c_w}\left(1 - \frac{8}{3}s_w^2\right),\ F_{1A}^{Z,SM} = \frac{1}{4s_w c_w},$$

s_w and c_w are the sine and the cosine of the Weinberg angle θ_w.

Applying the Gordon identity to the vector and axial vector:

$$\Gamma_\mu^{t\bar{t}X}(k^2, q, \bar{q}) = -ie\left\{\gamma_\mu \left(F_{1V}^X(k^2) + \gamma_5 F_{1A}^X(k^2)\right) + \frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})^\mu \left(iF_{2V}^X(k^2) + \gamma_5 F_{2A}^X(k^2)\right)\right\},$$

Introduction

- **Form Factors:**
  - The coupling F_{2V}^{γ} is related via $F_{2V}^{\gamma} = \frac{Q_t(g-2)}{2}$, the anomalous magnetic moment (g − 2) and electrical charge of the t quark Q_t
  - The coupling F_{2A}^{γ} is related to the dipole moment $d = (\frac{e}{2mt})F_{2A}(0)$ that violates the combined Charge and Parity symmetry CP. Here it is fixed to 0 because no significant damage has been observed so far.
  - There are six CP conserving form factors defined for the Z and the photon, F_{1V} , F_{1A}, F_{2V} but because close to the tt threshold the observables depend always on the sum $F_{1V} + F_{2V}$. Therefore a full disentangling of the form factors will be imprecise for energies below about 1 TeV.
  - Hence, in the present study either the four form factors $F_{1V,A}$ are varied simultaneously, while the two F_{2V} are kept at theirStandard Model values or vice versa.

Introduction

- **International Linear Collider, ILC**
  - ILC Centre-of-mass energies: from Z^0 mass to 1TeV
  - Integrated luminosity:500fb^{-1}
  - Polarised electron and positron beams are used in ILC so t and $\bar{\mathrm{t}}$ oriented toward different angular regions in the detector are enriched in left-handed or righthanded t quark helicity. The Realistic values of beam polarisations at the ILC(**P,P'** is the polarization degree of beam) :
    - P,P'=+0.8,-0.3: 0.8 **left-handed electron** beams and 0.3 **for right-handed** positrons.
    - P,P'=-0.8,+0.3: 0.8 **right-handed electron** beams and 0.3 **for left-handed** positrons.
  - This means that the experiments can independently access the couplings of left- and right-handed chiral parts of the t quark wave function to the Z^0 boson and the photon.
  - For left-handed helicity state t_L,and right-handed helicity state t_R , there are therefore six independent observables:

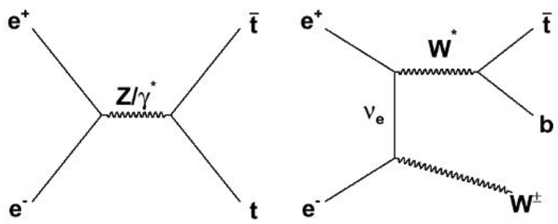
    1. The **cross section**;
    2. The **forward backward asymmetry** A_{FB}^{t};
    3. The **slope of the distribution of the helicity angle**(the angle of the decay lepton in semi-leptonic decays of the $\mathrm{t\bar{t}}$ in the rest frame of the t quark).

Top quark production at the ILC

- **Signal process:** $e^+e^- \to t\tilde{t} \to l^{\pm}\nu b\bar{b}q'\bar{q}$ ('lepton+jets' final state)
- **Several other SM processes give rise to the same final state.:**
  - **Most important source(10%):** Single-top production $e^+e^- \to WW^* \to Wt\bar{b} \to l^{\pm}\nu b\bar{b}q'\bar{q}$

    At a more fundamental level, interference between between single-top production and top quark pair production renders the separation physically meaningless.
  - **Other source:** Z^0WW (5%), $Z^0\ Z^0$, $Z^0\ Z^0\ Z^0$, WW, $q\bar{q}$

    These can be distinguished rather efficiently from top quark pair production.



(a) $t\bar{t}$ pair production (b) Single top production

Cross Section Table

Channel	$\sigma_{unpol.}$ [fb]	$\sigma_{-,+}$ [fb]	$\sigma_{+,-}$ [fb]	$A_{LR}^{SM}\%$
$t\bar{t}$	572	1564	724	36.7
$\mu\mu$	456	969	854	6.3
$\sum_{q=u,d,s,c} q\bar{q}$	2208	6032	2793	36.7
$b\bar{b}$	372	1212	276	62.9
γZ^0	11185	25500	19126	14.2
WW	6603	26000	150	98.8
Z^0Z^0	422	1106	582	31.0
Z^0WW	40	151	8.7	89
$Z^0Z^0Z^0$	1.1	3.2	1.22	45

Top quark production at the ILC

Observables and Form Factors

- The cross section for any process in e^+e^-collisions in case of polarised beams can be written as:

$$\sigma_{\mathcal{P},\mathcal{P}'} = \frac{1}{4}\left[(1-\mathcal{P}\mathcal{P}')(\sigma_{-,+}+\sigma_{+,-}) + (\mathcal{P}-\mathcal{P}')(\sigma_{+,-}-\sigma_{-,+})\right]$$

  - The symbc … eams with electrons and positrons of left-handed, L, or right-handed, R, helicity, respectively.
  - σ_{--} and σ_{++} have been neglected due to helicity conservation at the electron vertex in the high energy limit.
  - The degree of polarisation of the incoming beams is expressed by **P**, for electrons, and **P**′, for positrons.

- The Born level cross section for $t\bar{t}$ quark production for electron beam polarisation I = L, R reads:

$$\sigma_I = 2\mathcal{A}N_c\beta\left[(1+0.5\gamma^{-2})(\mathcal{F}^I_{1V})^2 + (\mathcal{F}^{I'}_{1A})^2 + 3\mathcal{F}^I_{1V}\mathcal{F}^I_{2V}\right],$$

- The forward-backward asymmetry A^t_{FB} can be expressed as:

$$(A^t_{FB})_I = \frac{-3\mathcal{F}^{I'}_{1A}(\mathcal{F}^I_{1V}+\mathcal{F}^I_{2V})}{2\left[(1+0.5\gamma^{-2})(\mathcal{F}^I_{1V})^2 + (\mathcal{F}^{I'}_{1A})^2 + 3\mathcal{F}^I_{1V}\mathcal{F}^I_{2V}\right]},$$

- The fraction of right-handed tops related to the defined helicity angle is given by the following expression:

$$(F_R)_I = \frac{(\mathcal{F}^I_{1V})^2(1+0.5\gamma^{-2}) + (\mathcal{F}^{I'}_{1A})^2 + 2\mathcal{F}^I_{1V}\mathcal{F}^{I'}_{1A} + \mathcal{F}^I_{2V}(3\mathcal{F}^I_{1V}+2\mathcal{F}^{I'}_{1A}) - \beta\mathcal{F}^I_{1V}\Re(\mathcal{F}^I_{2A})}{2\left[(1+0.5\gamma^{-2})(\mathcal{F}^I_{1V})^2 + (\mathcal{F}^{I'}_{1A})^2 + 3\mathcal{F}^I_{1V}\mathcal{F}^I_{2V}\right]}.$$

Here is the form factors in terms of the helicity of the incoming electrons

$$\mathcal{F}^L_{ij} = -F^\gamma_{ij} + \left(\frac{-\frac{1}{2}+s^2_w}{s_w c_w}\right)\left(\frac{s}{s-m^2_Z}\right)F^Z_{ij}$$

$$\mathcal{F}^R_{ij} = -F^\gamma_{ij} + \left(\frac{s^2_w}{s_w c_w}\right)\left(\frac{s}{s-m^2_Z}\right)F^Z_{ij},$$

Event generation and technical remarks

- **Events are generated with WHIZARD 1.95 in the form of:**
  - Six things final state for signal.
  - Two and four final state for background.

The most relevant background contributions are the W W and $b\bar{b}$ final states as well as fully hadronic and fully leptonic decays of $t\bar{t}$ pairs.

The study has been carried out on a fully polarised sample.Because the σ can scales with the polarization and A_{FB}^{t},λ vary only very mildly with the beam polarisation.

- **Other Configuration：**
  - **Using PYTHIA to generate parton showers and subsequent hadronisation.**
  - **Full Simulation of the ILD detector**
  - **Using the version ILD o1 v05 of the ILC software to do reconstruction.**

- **WHIZARD cut:**
  - In case of the signal sample events are flagged for which the difference between the invariant masses of the three fermion systems forming a top and the input t mass to WHIZARD of 174 GeV is smaller than 5Γ. (Here the Γ is the **total decay width** of the t quark.)
  - In this level only about 70% of the generated events can betreated accordingly by PYTHIA.
  - Different will introduce a different systematic uncertainty which is however expected to be reasonable small.

Event selection

$$t\bar{t} \to (bW)(bW) \to (bqq')(b\ell\nu)$$

- **In the SM the fraction of semi-leptonic final states is about 43%.**
- **Such processes can yield additional particles Interfering reconstruction:**
  - $\gamma\gamma \to$ *hadrons* (removed by longitudinally invariant k_t algorithm)
  - electron-positron pairs etc. (removed by a next Durham algorithm)
- **Top quark reconstruction:**
  - The charged lepton allows for the determination of the t quark charge.

    In general leptons(is either the most energetic particle in a jet or has a sizable transverse momentum w.r.t. neighboured jets.) are identified using typical selection criteria :

$$x_T = p_{T,lepton}/M_{jet} > 0.25 \quad \text{and} \quad z = E_{lepton}/E_{jet} > 0.6,$$

    where E_{lepton} is the energy and $p_{T,lepton}$ is the transverse momentum of the lepton within a jet with energy E_{jet} and mass M_{jet} .

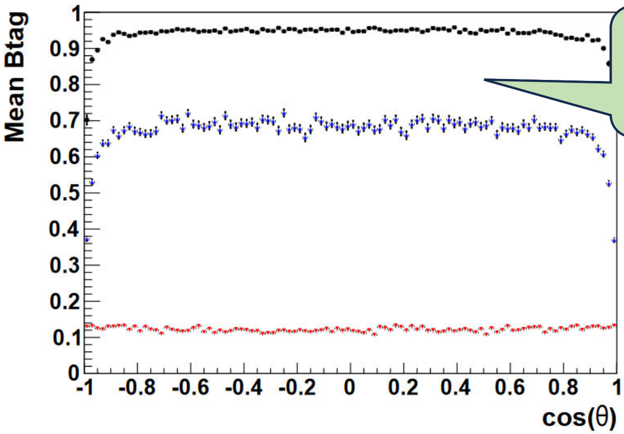
    Efficiency: 85% for electron and muon,70 for τ。
  - The t quark mass is reconstructed from the hadronically decaying W which is combined with one of the b-quark jets.

Event selection

● Top quark reconstruction

- The t quark mass is reconstructed from the hadronically decaying W which is combined with one of the b-quark jets.
- **Cut on jet**: thrust<0.9,total hadronic mass of the final state ∈(180,420) *GeV*.

➢ Two of these must be identified as being produced by the b-quarks of the t quark decay. The b-likeness or b-tag is determined with the LCFIPlus package,



The higher b-tag value is typically 0.92 while the smaller one is still around 0.65 which is greatly more than the one from jets light quarks

The two highest B-tag values (black and blue dots) are associated to B quark jets. the third set of values (red dots) is obtained for jets from light quarks.

➢ Select two of the remaining jets candidates with b-jets to reconstruct the top quark to minimize the following equation:

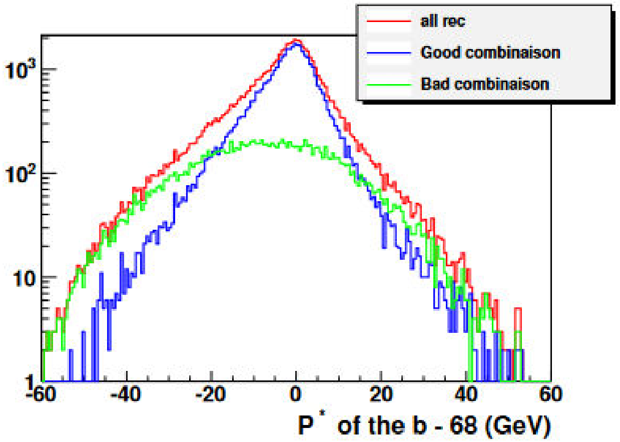
$$d^2 = (\frac{m_{cand} - m_t}{\sigma_{m_{cand}}})^2 + (\frac{E_{cand} - E_{beam}}{\sigma_{E_{cand}}})^2 + (\frac{p_b^* - 68}{\sigma_{p_b^*}})^2 + (\frac{cos\theta_{bW} - 0.23}{\sigma_{cos\theta_{bW}}})^2$$

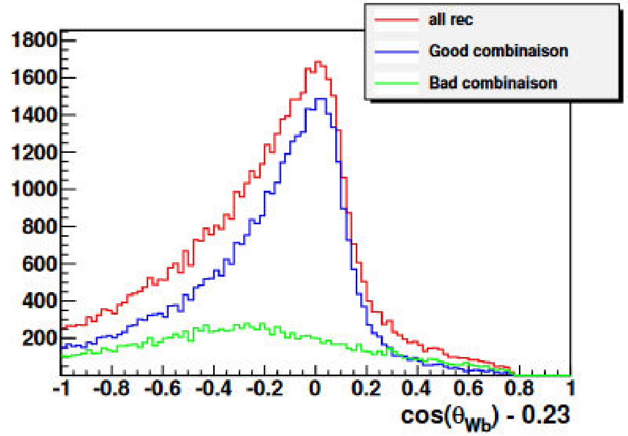
In this equation $m_{cand.}$ and $E_{cand.}$ are invariant mass and energy of the t quark candidate decaying hadronically, respectively, and m_t and E_{beam} are input t mass and the beam energy of 250 GeV. Beyond that it introduces the momentum of the b quark jet in the centre-of-mass frame of the t quark, p_b^*, which has a defined value of 68 GeV, and the angle between the b quark and the W boson.

➢ Further cut : W mass ∈ $(50,250)GeV$,t mass ∈ $(120,270)GeV$

Event selection

- **Top quark reconstruction**
  - **Comparison of combination results:**





  - **Reconstruction efficiency:**
    - **P,P'=+0.8,-0.3:** 54.4% for signal events and 9% for background events.
    - **P,P'=-0.8,+0.3:** 55.9% for signal events and 7.2% for background events.

  This background is predominantly composed by events from non-semileptonic decays of the $t\bar{t}$.
  - The **Reconstruction efficiency** will be worse if considering the generated sample is contaminated by single top events.

Measurement of the forward backward asymme

- **The determination of the A_{FB}^{t}:**

After getting the reconstruction level information, the A_{FB}^{t} can be calculated by the number of events in the hemispheres of the detector and the the polar angle θ of the t quark: $A_{FB}^{t} = \frac{N(\cos\theta > 0) - N(\cos\theta < 0)}{N(\cos\theta > 0) + N(\cos\theta < 0)}$.

  - **For a left-handed polarised electron beam:**

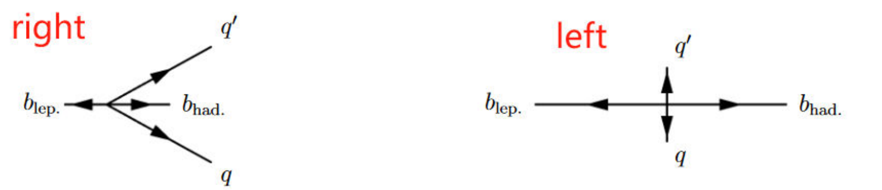
  The W boson decays into two energetic jets. The b quark from the decay of the t quark are comparatively soft. Therefore, the direction of the t quark is essentially reconstructed from the direction of the energetic jets from the W boson decay.

  - **For a left-handed polarised electron beam:**

  The W boson is nearly at rest and the b quarks are very energetic therefore dominate the reconstruction of the polar angle of the t quark.

  - The wrong association of the jet from the b quark decay to the jets from the W boson decay will only affect the left-handed situation.(In fact it will flip the reconstructed polar angle by π giving rise to migrations in the polar angle distribution of the t quark.)

-

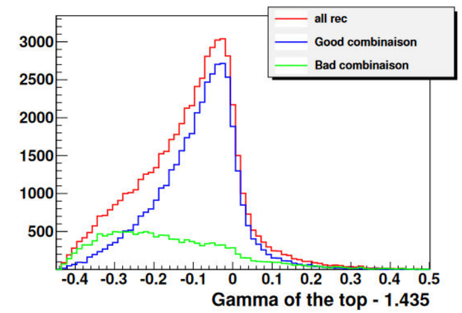


Measurement of the forward backward asymme

● Optimization of left-handed situation:

The combination of b-jet and W is required to ensure that the distribution of θ is as consistent as possible with the truth level.

After the correct association of the of jets from b quarks to that from W bosons which is checked with the MC truth information ,there are such distribution about **the Lorentz factor** $\gamma_t = \boldsymbol{E_t/M_t}$, $\boldsymbol{E_t^*}$, $\boldsymbol{cos}\theta_{bW}$

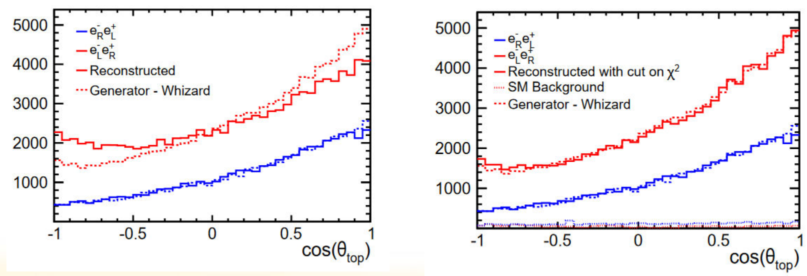


The quality of the reconstructed events is estimated by this:

$$\chi^2 = \left(\frac{\gamma_t - 1.435}{\sigma_{\gamma_t}}\right)^2 + \left(\frac{E_b^* - 68}{\sigma_{E_b^*}}\right)^2 + \left(\frac{cos\theta_{bW} - 0.26}{\sigma_{cos\theta_{bW}}}\right)^2$$

For setting cut χ^2 <15, the reconstructed spectrum agrees very well with the generated one with the reconstruction efficiency 28.5%.

- Reconstructed forward backward asymmetry compared with the prediction by the event generator before and after cut.



Measurement of the forward backward asymme

- Result:

P, P’	SM value	$(A^t_{FB})_{gen}$	$(A^t_{FB})_{rec}$
−1, +1	0. 38	0. 339	0. 326
+1, −1	0. 47	0. 432	0. 42

Slope of the helicity angle distribution

- **Determination of the slope of the helicity angle distribution:**
  - In the rest system of the t quark, the angle of the lepton from the W boson is distributed like:

$$\frac{1}{\Gamma}\frac{d\Gamma}{dcos\theta_{hel}} = \frac{1+\lambda_t \cos\theta_{hel}}{2} = \frac{1}{2} + (2F_R - 1)\frac{\cos\theta_{hel}}{2} \qquad \lambda_t = 1 \text{ for } t_R \quad \lambda_t = -1 \text{ for } t_L$$

Here Γ is cross section, F_R is the fraction of right-handed tops, $\theta_{\mathbf{hel}}$ is the angle of the decay lepton in semi-leptonic decays of the $\mathbf{t\bar{t}}$ in the rest frame of the t quark and the z-axis defined by the direction of motion of the t quark in the laboratory.

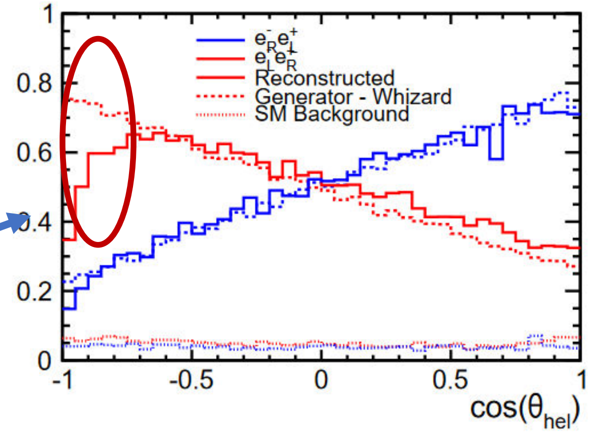
  - This angular distribution is therefore linear and very contrasted between t_L and t_R . In practice there will be a mixture of two kinds of fully polarised samples.

  - If initial state radiation effects (with the photon lost in the beam pipe) are neglected, by means of the energy-momentum of the t quark decaying hadronically, allows for deducing the energy-momentum of the t quark decaying semi-leptonically.

Slope of the helicity angle distribution

Analysis of the helicity angle distribution

- The angular distribution of the decay lepton in the rest frame of the t quark
- The background is small relative to the signal and to a good approximation flat so it can be neglected.
- For the **left-handed polarised electron beam ,**The distribution exhibits a drop in reconstructed events towards $cos\theta_{\mathrm{hel}}$=-1 because :
  - The wrong association of b and W
  - It leads to a wrong top quark decaying hadronically
  - In turn, it leads to wrong four- momentum of the t quark decaying leptonically.
  - In fact it suppresses leptons with small energies and lead to this phenomenon
- The parameter $\lambda_t$can be derived from the slope of the helicity angle distribution that is obtained by a fit to the linear part of the angular distribution in the range:
  - **P,P'=-1,+1:** linear part is [-0.6,0.9].
  - **P,P'=-1,+1:** linear part is [-0.9,0.9].



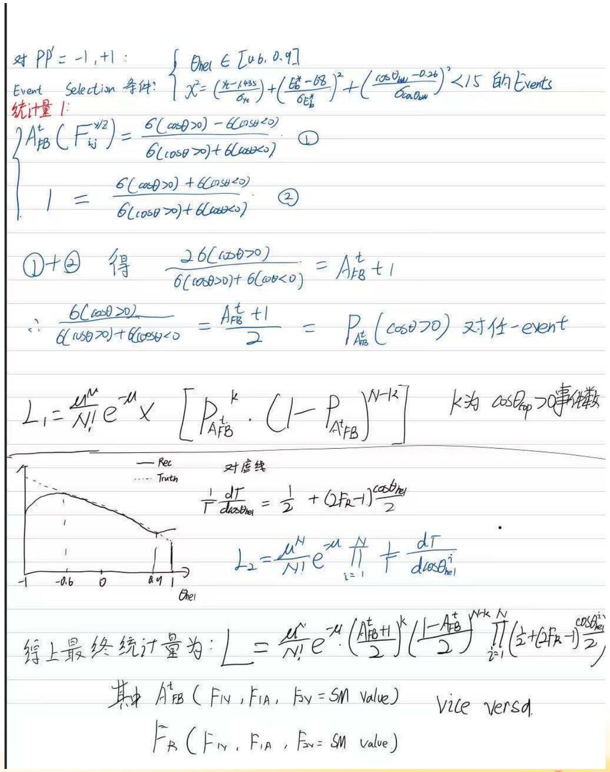
$\mathcal{P},\mathcal{P}'$	$(\lambda_t)_{gen.}$	$(\lambda_t)_{rec.}$	$(\delta\lambda_t)_{stat.}$ for $\mathcal{P},\mathcal{P}'=\mp0.8,\pm0.3$	$(\delta\lambda_t)_{syst.}$
$-1,+1$	-0.484	-0.437	0.011	0.013
$+1,-1$	0.547	0.534	0.013	0.006

$$F_R = \frac{\lambda_t + 1}{2}$$

The values expected in the Standard Model are $(F_R)_L = 0.25$ and $(F_R)_R = 0.76.$

Precision of Form Factors

- **Calculation of Form Factors:**
  - **The results on the reconstruction efficiency A_{FB}^{t} and λ_t presented in the previous sections are transformed into precisions on the form factors .**
  - **Construct the negative Log-likelihood to minimize:**



Result compared with LHC and TESLA TDR

Coupling	SM value	LHC [3] $\mathcal{L} = 300\ \text{fb}^{-1}$	e^+e^- [8] $\mathcal{L} = 300\ \text{fb}^{-1}$ $\mathcal{P},\mathcal{P}' = -0.8, 0$	e^+e^- [ILC DBD] $\mathcal{L} = 500\ \text{fb}^{-1}$ $\mathcal{P},\mathcal{P}' = \pm0.8, \mp0.3$
$\Delta\widetilde{F}_{1V}^{\gamma}$	0.66	$^{+0.043}_{-0.041}$	− −	$^{+0.002}_{-0.002}$
$\Delta\widetilde{F}_{1V}^{Z}$	0.23	$^{+0.240}_{-0.620}$	$^{+0.004}_{-0.004}$	$^{+0.002}_{-0.002}$
$\Delta\widetilde{F}_{1A}^{Z}$	-0.59	$^{+0.052}_{-0.060}$	$^{+0.009}_{-0.013}$	$^{+0.006}_{-0.006}$
$\Delta\widetilde{F}_{2V}^{\gamma}$	0.015	$^{+0.038}_{-0.035}$	$^{+0.004}_{-0.004}$	$^{+0.001}_{-0.001}$
$\Delta\widetilde{F}_{2V}^{Z}$	0.018	$^{+0.270}_{-0.190}$	$^{+0.004}_{-0.004}$	$^{+0.002}_{-0.002}$

Summary and outlook

- This article presents a comprehensive analysis of $\mathbf{t\bar{t}}$ quark production using the semi-leptonic decay channel. Results are given for a centre-of-mass energy of $\sqrt{s} = 500\text{GeV}$ and an integrated luminosity of $500fb^{-1}$ and a beam polarization(P,P'=$\mp 0.8, \pm 0.3$).
- Semi-leptonic events, including those with τ leptons in the final state can be selected with an efficiency of about 55%.
- The cross section of the semi-leptonic channel of tt quark production can therefore be measured to a statistical precision of about 0.5%.
- It was shown that in particular for predominantly left handed polarisation of the initial electron beam the $V-A$ structure leads to migrations, which distort the theoretical expected A_{FB}^{t}. These migrations can be remedied by tightening the selection criteria of the events.There is a a precision of better than 2% for both beam polarisations.
- It is introduced the slope of the helicity angle distribution, which is a new observable for ILC studies. It can be measured to a precision of about 4%.
- These couplings can be measured with high precision at the ILC and always more than one order of magnitude better than it will be possible at the LHC with L=300 fb^{-1}.
- In extension of a future analysis will disentangle the CP violating form factors.

Schedule

- **Plot the distribution.**
- **I have finished a code to find best combinaison to reconstruct the top from recon level data(signal).**
- **I have finished a code to calculate the** A_{FB}^{t} and its form factors expanded form.

$$(A_{FB}^t)_I = \frac{-3\mathcal{F}_{1A}^{I'}(\mathcal{F}_{1V}^I + \mathcal{F}_{2V}^I)}{2\left[(1+0.5\gamma^{-2})(\mathcal{F}_{1V}^I)^2 + (\mathcal{F}_{1A}^{I'})^2 + 3\mathcal{F}_{1V}^I\mathcal{F}_{2V}^I\right]},$$

but I don’t sure it is the right form for CEPC Non-polarized beam(composed by 50% left-hand and 50% right-hand particles)