UNIVERSITÀ DEGLI STUDI
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Istituto Nazionale di Fisica Nucleare

Electroweak Challenges at future colliders

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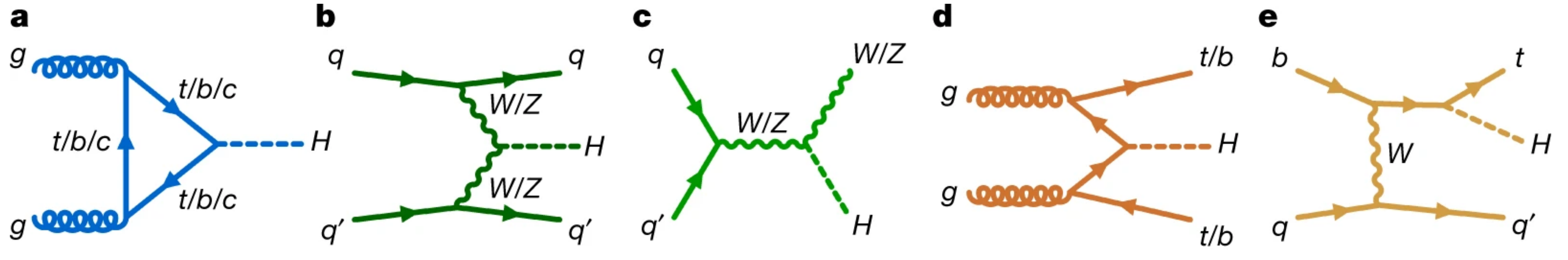
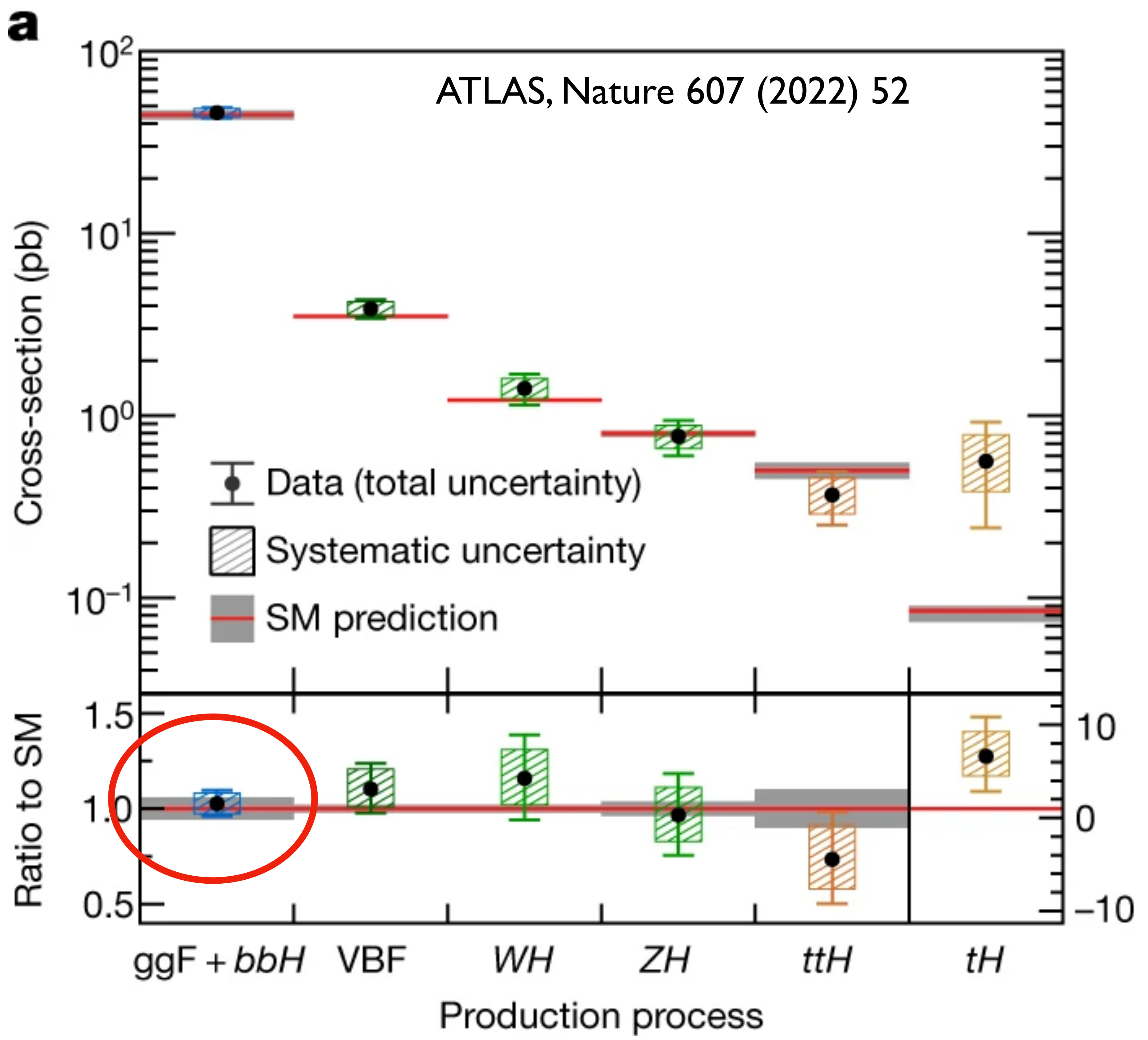
CEPC2025, Guangzhou, November 6th 2025

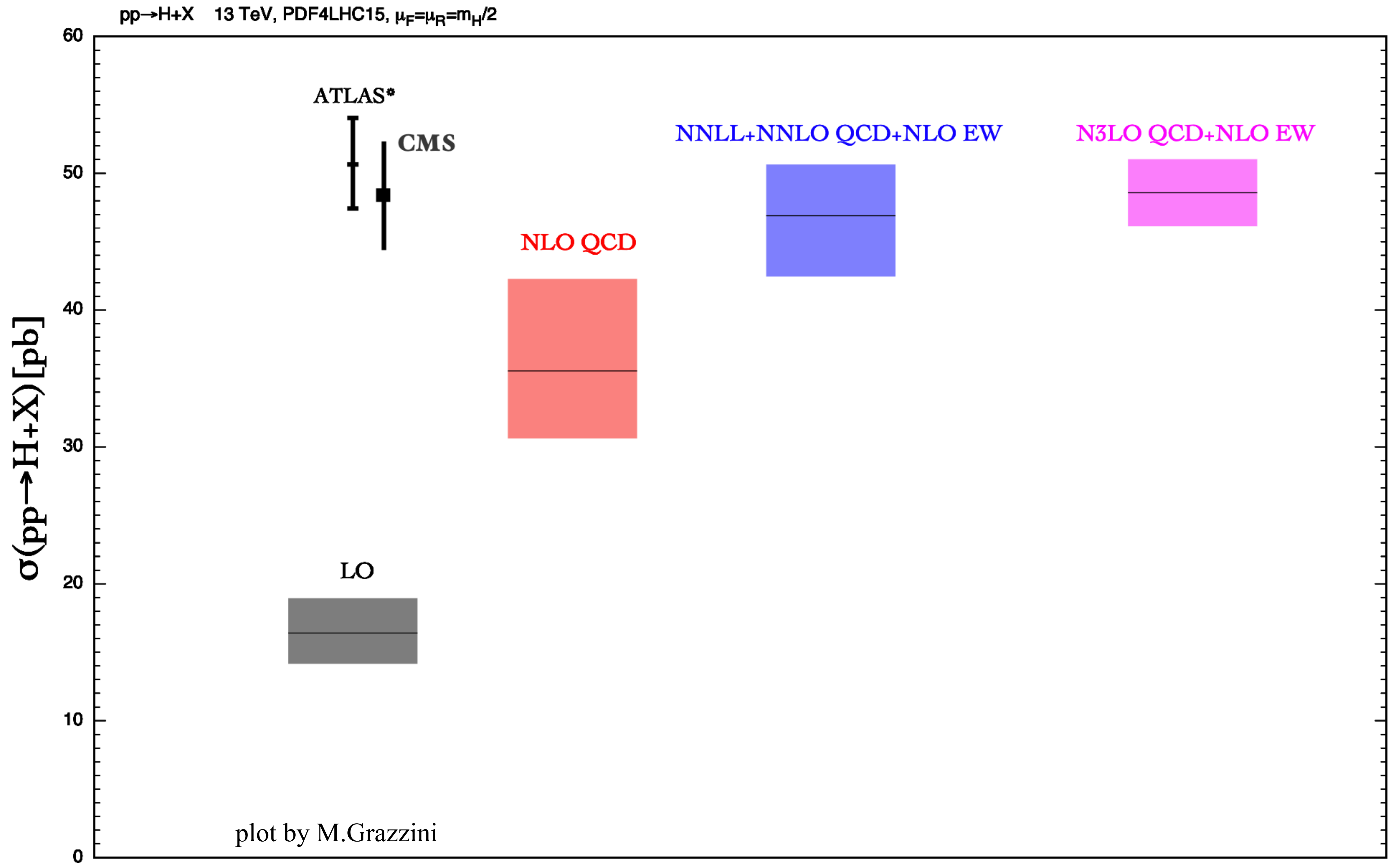
When Precision is a crucial tool: deciphering the nature of the Higgs boson

Is the scalar resonance observed at the LHC the Standard Model Higgs boson ?



Quantum corrections up to third order needed for a significant (accurate and precise) comparison with the Higgs production cross sections





Motivations and open questions

- Open questions (dark matter/energy, matter-antimatter asymmetry, EW phase transition) investigated at high-energy colliders:

  the EW sector of the SM offers many opportunities as a portal towards new physics

  (Higgs and gauge boson properties, new gauge bosons, CKM phases)

- Scrutiny of physical observables:

  1) can we get a consistent description (i.e. SM cross sections) of scattering processes from 10 GeV to 10 TeV ?

- Extraction of the lagrangian parameters from the data and comparison with their theoretical prediction:

  2) do we have control over the theoretical systematic of the fitting procedures ?

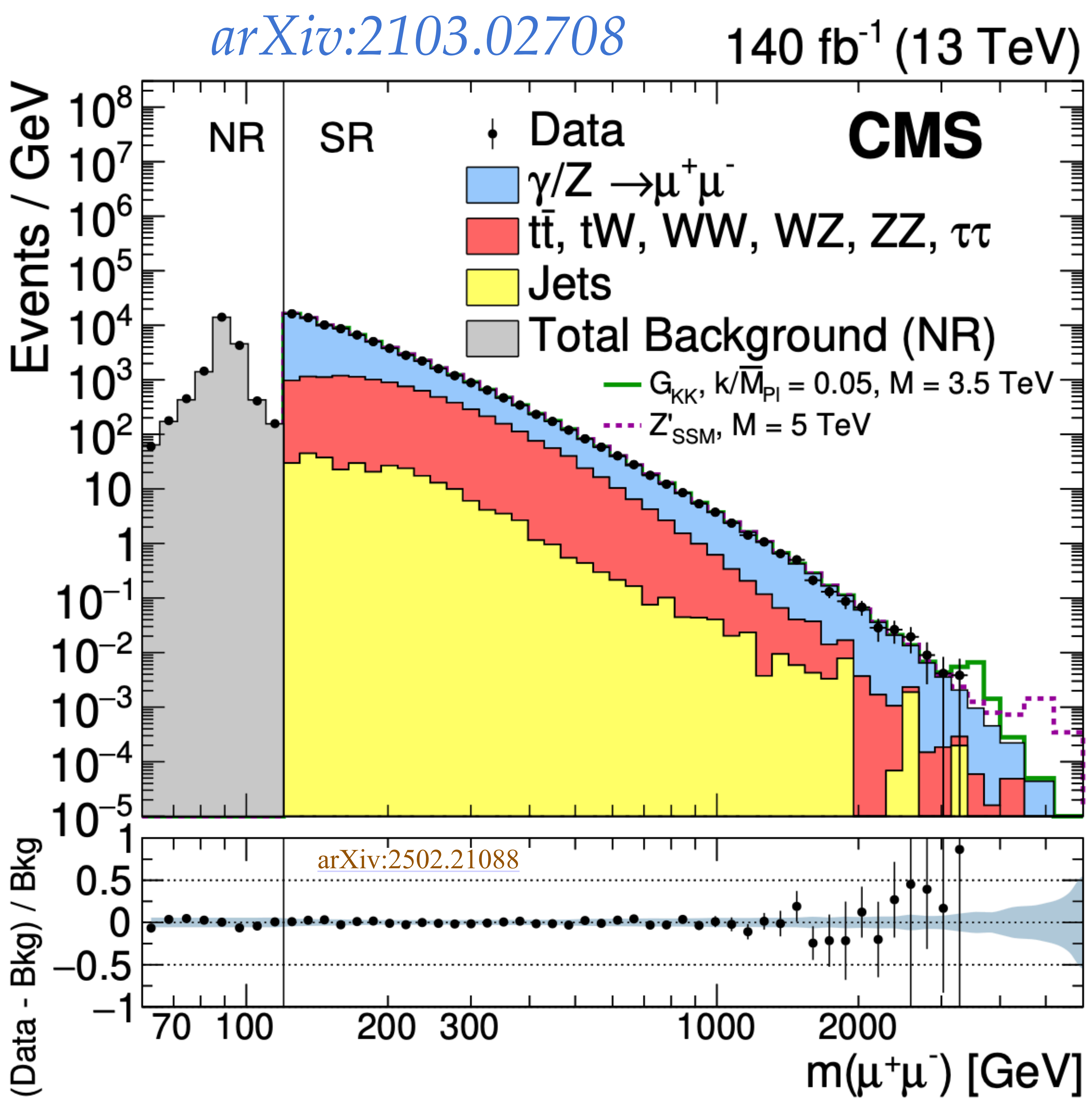
  3) are the current theoretical predictions sufficiently precise?

- Extending the SM:

  4) are we able to prepare a statistical test of the SM and of its extension, both with comparable theoretical accuracy ?

Scrutiny of physical observables

Cross sections: status at the LHC and challenges at the HL-LHC



statistical error only

bin range (GeV)	% error 140 fb^{-1}	% error 3 ab^{-1}
91-92	0.03	6 10^{-3}
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

- sub-percent precision in the TeV region → sensitivity to quantum effects, possibly BSM
- sub-permille precision in the 120-400 GeV interval → stringent test of the SM energy dependence

Cross sections: challenges at the CEPC / FCC-ee

- muon-pair production cross sections, at FCC-ee, with design luminosities, at different cms energies

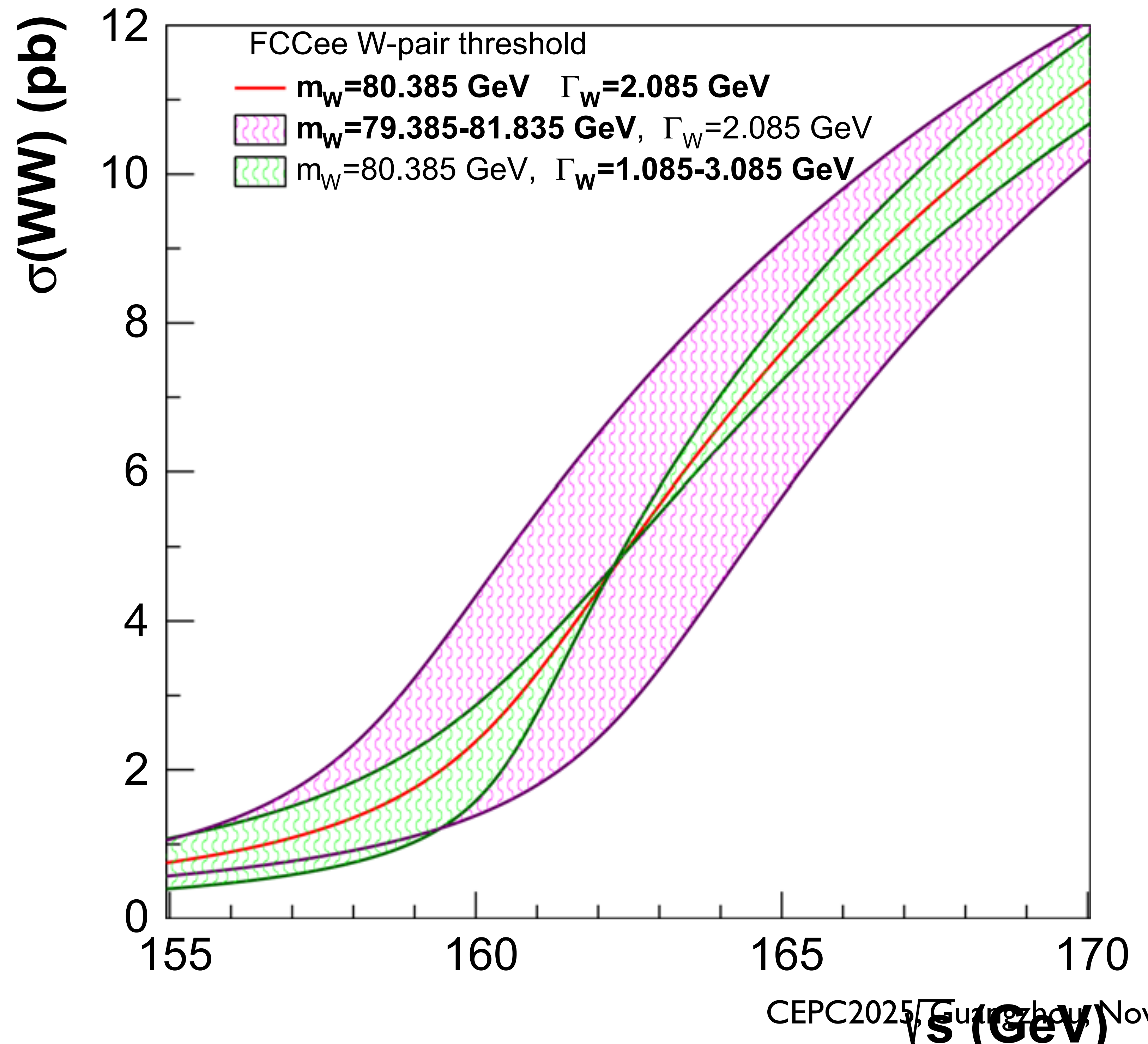
sqrt(S) (GeV)	luminosity (ab^{-1})	σ (fb)	% stat error
91	150	2.17595 10^6	0.0002
240	5	1870.84 ± 0.612	0.03
365	1,5	787.74 ± 0.725	0.09

$\sigma(e^+e^- \to \mu^+\mu^- + X)$ arXiv:2206.08326

- a reduction by a factor O(15-20) for the error on $A_{FB}(m_Z^2)$ → $\Delta \sin^2\theta^{\ell}_{eff} \sim (0.6 \cdot 10^{-5})_{exp}$

- 4-fermion production:

$$12\ \text{ab}^{-1}\ \text{at}\ \sqrt{s} = 157.5, 162.5\ \text{GeV} \quad \to \quad \Delta m_W \sim (0.5\ \text{MeV})_{stat}$$



Size of the main subsets of electroweak corrections

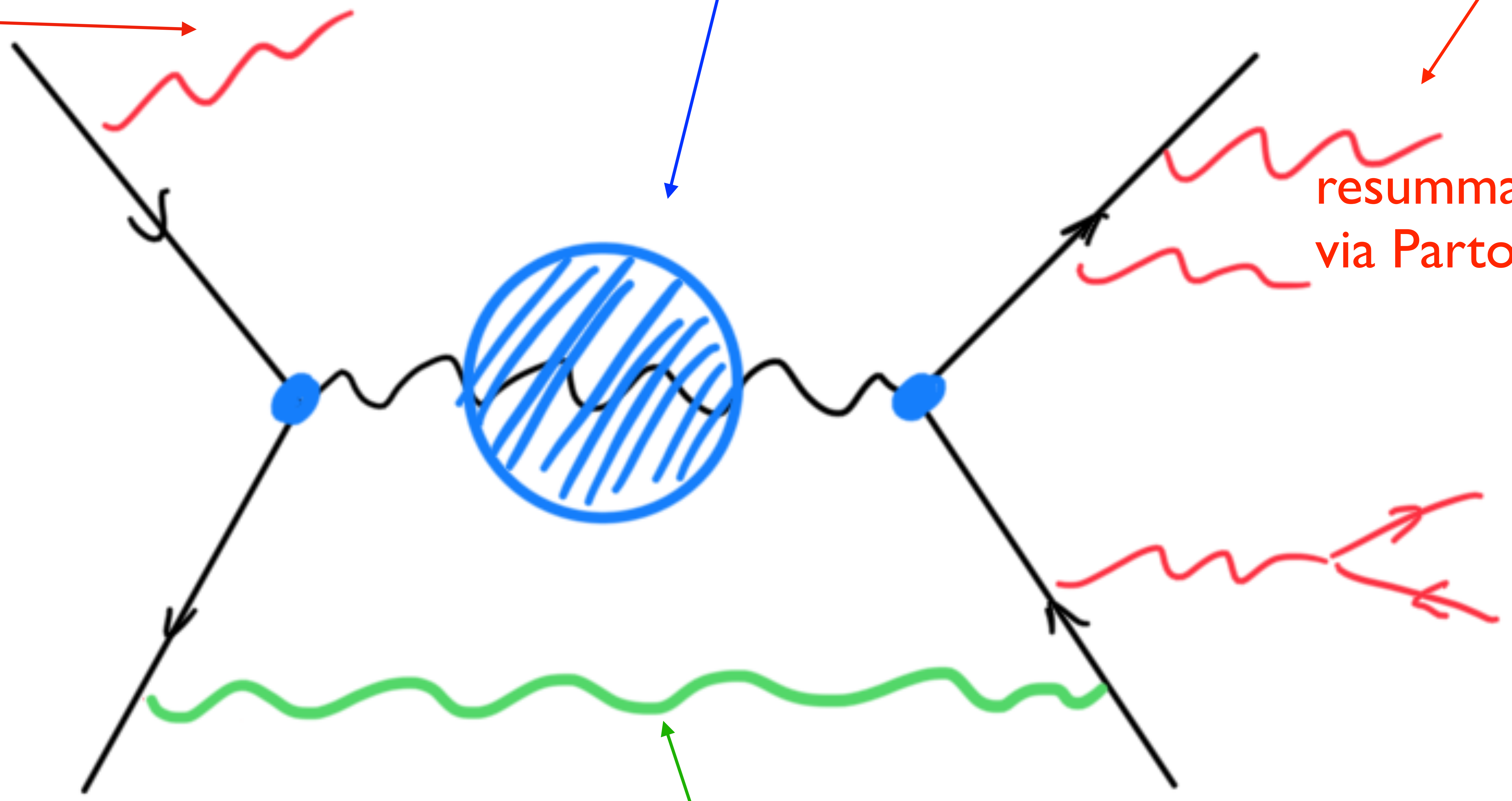
QED radiation
couplings redefinition
EW Sudakov logarithms

complete 3-loop EW form factors
at the Z resonance
+ leading 4-loop terms

$\rho \sim 1.01$

$$\left(\alpha \log \frac{m_Z^2}{m_\mu^2}\right)^n \sim (0.098)^n$$

$$\left(\alpha \log \frac{m_Z^2}{m_e^2}\right)^n \sim (0.176)^n$$



resummation of the soft/collinear logs
via Parton Shower + matching ?

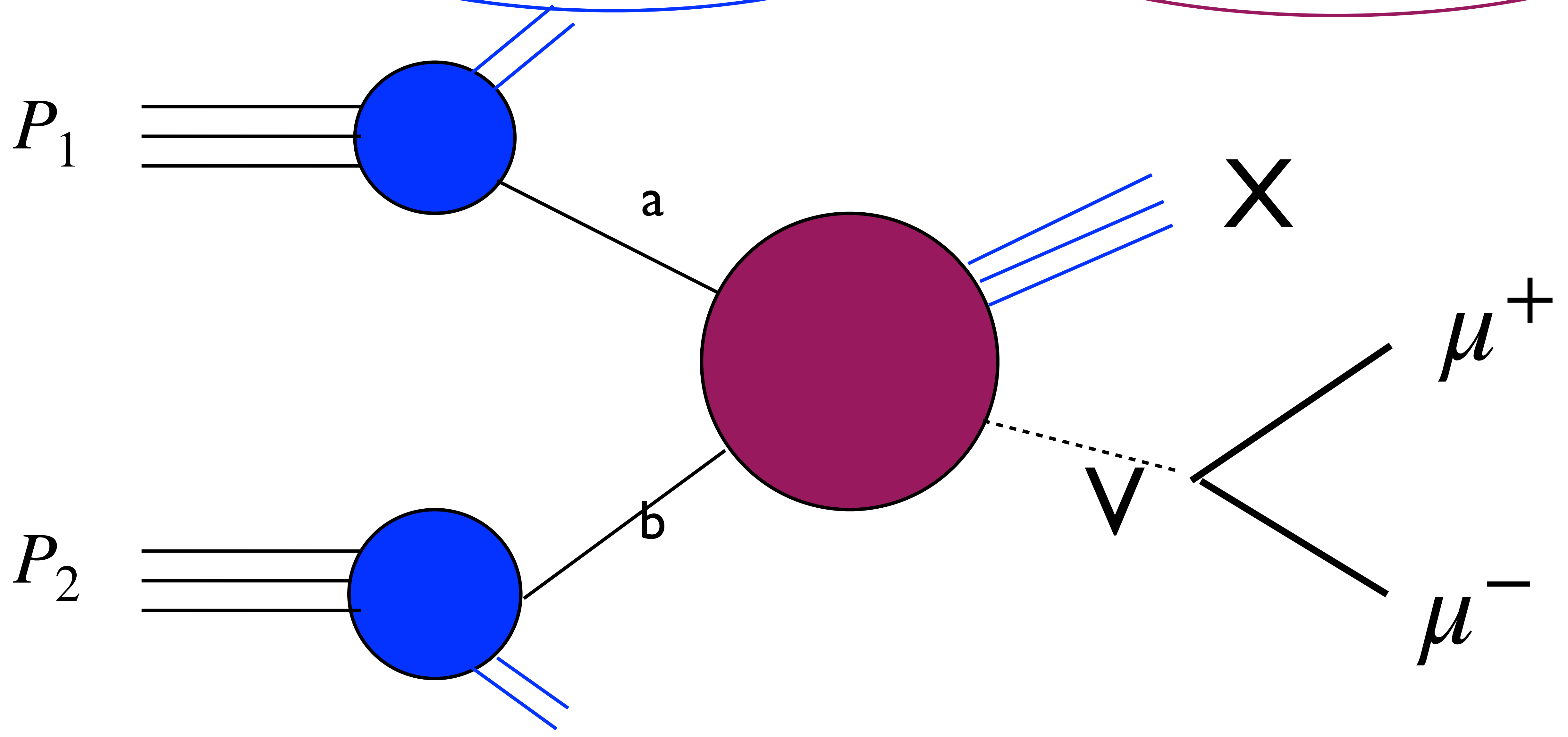
resummation of the initial state
collinear log solving the
DGLAP equations ?

$$\alpha \log^2 \left(\frac{(1000\ \mathrm{GeV})^2}{m_W^2}\right) \sim 0.185$$

complete NNLO-EW corrections
to the differential cross sections

Factorisation theorems and the cross section in the partonic formalism

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



Particles $P_{1,2}$ can be protons (→ Drell-Yan @ LHC) or leptons (→ CEPC / FCC-ee, muon collider)

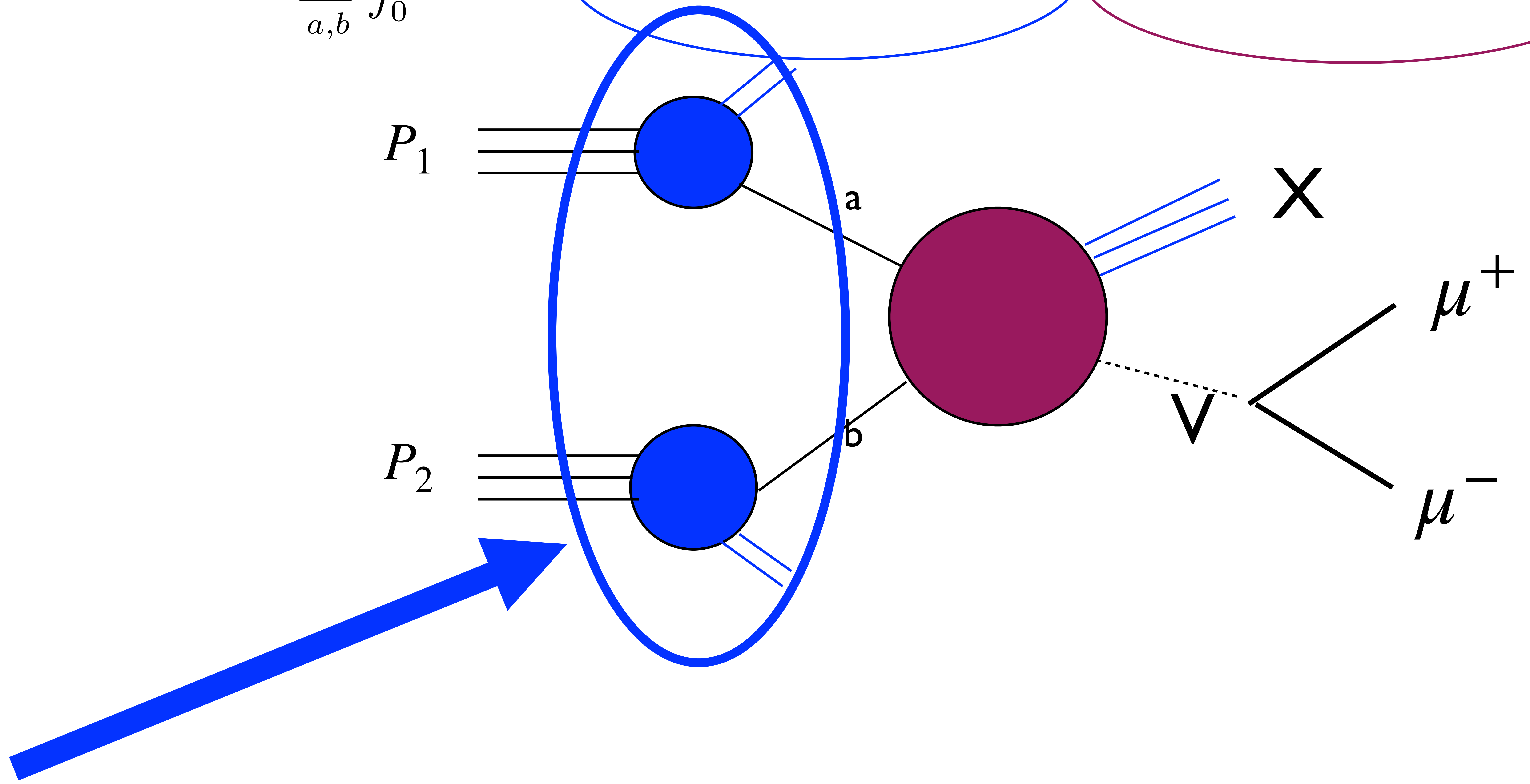
The partonic content of the scattering particles can be expressed in terms of PDFs both for protons and leptons

The partonic scattering can be computed in perturbation theory, in the full QCD+EW theory,

Factorisation theorems guarantee the validity of the above picture up to power correction effects

Factorisation theorems and the cross section in the partonic formalism

$$\sigma(P_1,P_2;m_V) = \sum_{a,b}\int_0^1 dx_1 dx_2\, f_{h_1,a}(x_1,M_F) f_{h_2,b}(x_2,M_F)\, \hat\sigma_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



Leading QED contributions can be resummed to all perturbative orders by solving the DGLAP equations

The DGLAP formalism is clearly mature (from QCD up to N3LO) and available also for e^+e^- collisions

Initial state QED radiation in the PDF language

$$d\sigma_{kl}(p_k,p_l) = \sum_{ij}\int dz_+ dz_-\, \Gamma_{i/k}(z_+,\mu,m)\,\Gamma_{j/l}(z_-,\mu,m)\, d\hat\sigma_{ij}(z_+p_k, z_-p_l,\mu)$$

$$\Gamma^{\rm LL}_{e^-}(z,\mu^2) = \frac{\exp\left[(3/4-\gamma_E)\eta\right]}{\Gamma(1+\eta)}\eta(1-z)^{-1+\eta} - \frac{1}{2}\eta(1+z) + \mathcal{O}(\alpha^2),$$

all-orders large z bulk

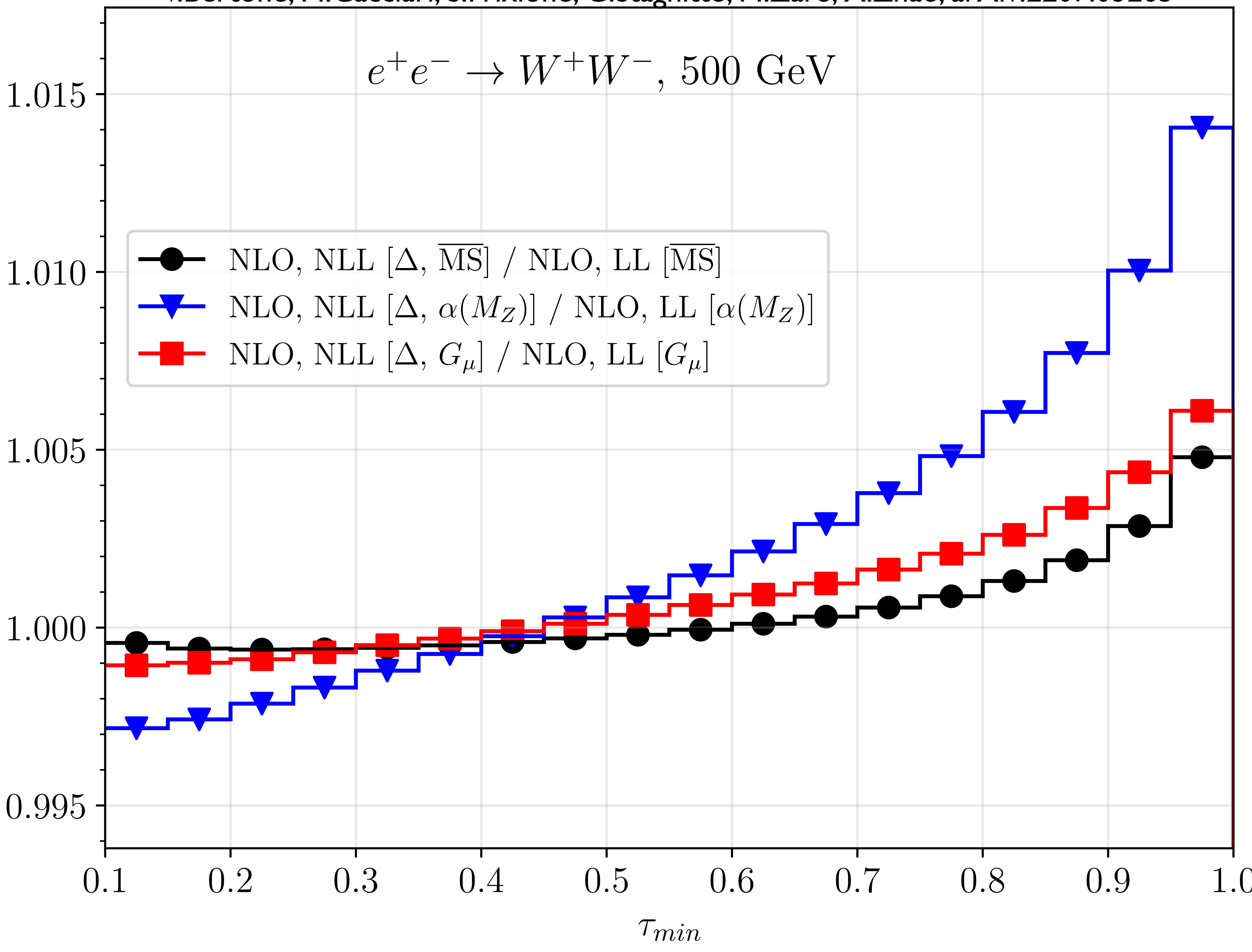
fixed-order all z correction

Gribov, Lipatov, (1972)

Skrzypek, Jadach (1990);
Cacciari, Deandrea, Montagna, Nicrosini (1992)

$\Gamma^{\rm NLL}_{e^-}(z,\mu^2)$ presented in
Frixione et al., arXiv:1909.03886, 1911.12040, 2105.06688, 2207.03265,

V.Bertone, M.Cacciari, S.Frixione, G.Stagnitto, M.Zaro, X.Zhao, arXiv:2207.03265

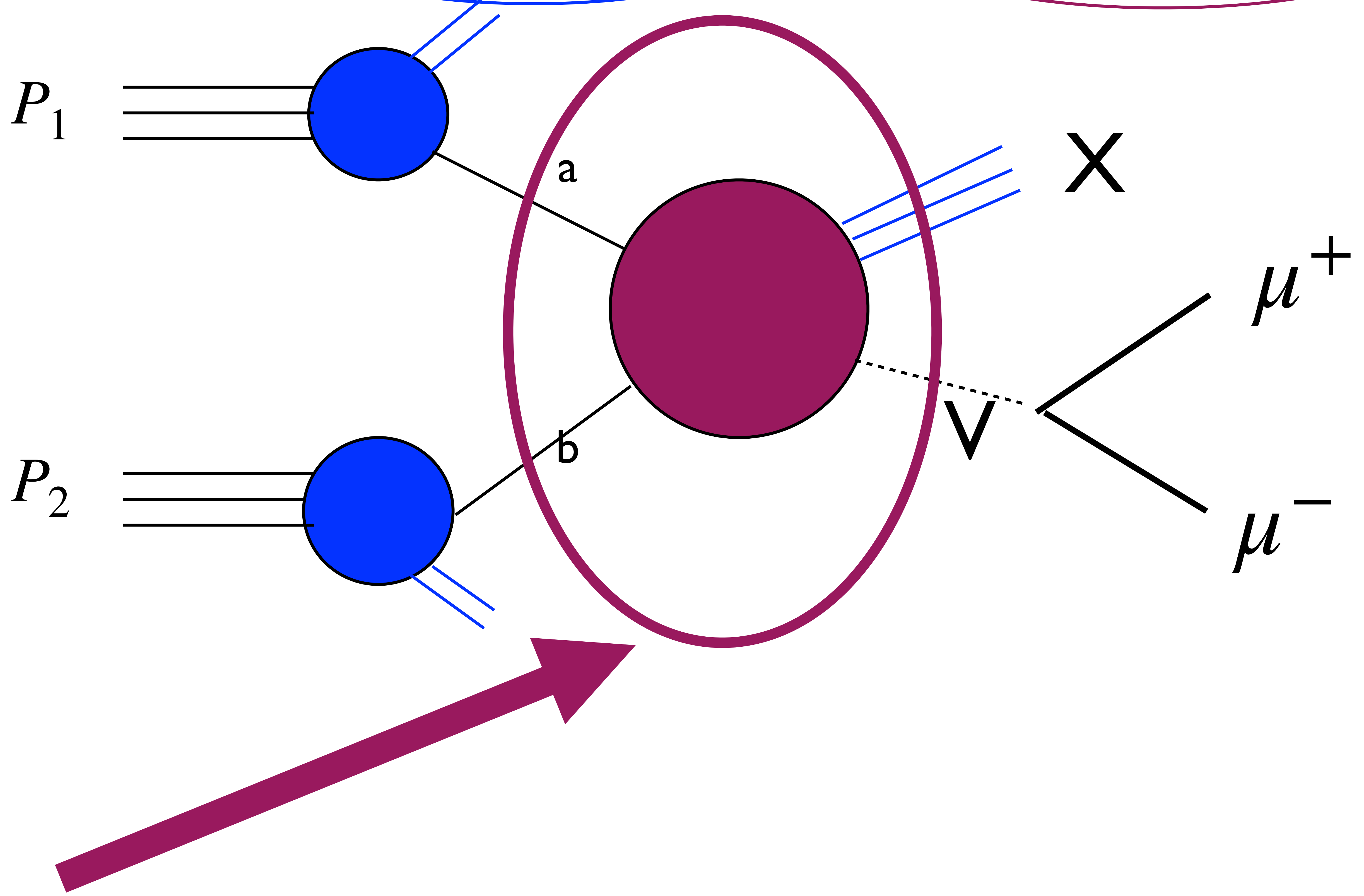


Sizeable initial state logarithms, beyond those included at fixed NLO

The lepton PDF are needed at N3LL to match the precision needs of CEPC / FCC-ee

Factorisation theorems and the cross section in the partonic formalism

$$\sigma(P_1,P_2;m_V) = \sum_{a,b}\int_0^1 dx_1 dx_2\, f_{h_1,a}(x_1,M_F) f_{h_2,b}(x_2,M_F)\, \hat\sigma_{ab}(x_1P_1,x_2P_2,\alpha_s(\mu),M_F)$$



Significant progress in the last 5 years in the evaluation of the partonic cross sections (N3LO QCD and NNLO QCD-EW)

but

NNLO-EW corrections are still missing !!!

Recent progress in the prediction of the lepton-pair production cross sections

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} + \alpha_s\, \sigma^{(1,0)} + \alpha\, \sigma^{(0,1)} + \alpha_s^2\, \sigma^{(2,0)} + \alpha\, \alpha_s\, \sigma^{(1,1)} + \alpha^2\, \sigma^{(0,2)} + \alpha_s^3\, \sigma^{(3,0)} + \ldots$$

Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

Altarelli, Ellis, Martinelli (1979)

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

C.Duhr, B.Mistlberger, arXiv:2111.10379

still missing
Sudakov high-energy approximations

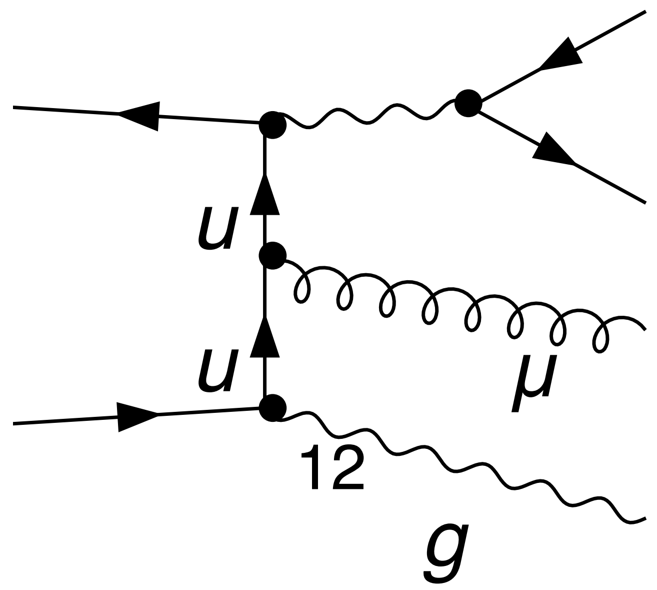
Neutral Current

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, (2021)
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2022)
F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller, A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, (2022)

Charged-current 2-loop amplitude — T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, (2024)

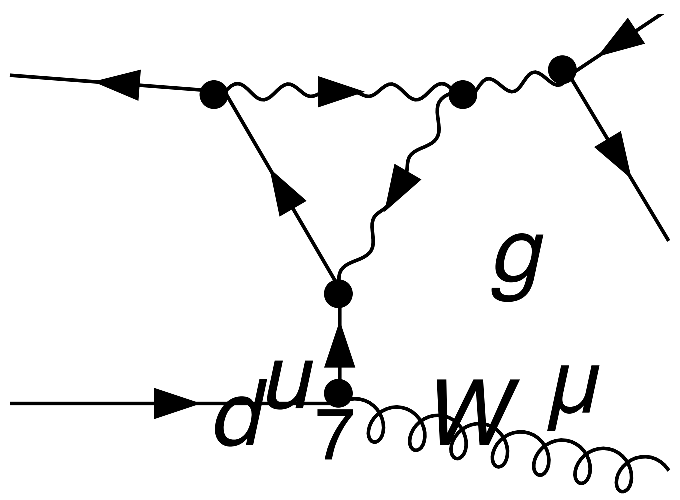
Neutral-current NNLO phenomonology — T.Armadillo, R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, AV, arXiv:2412.16095

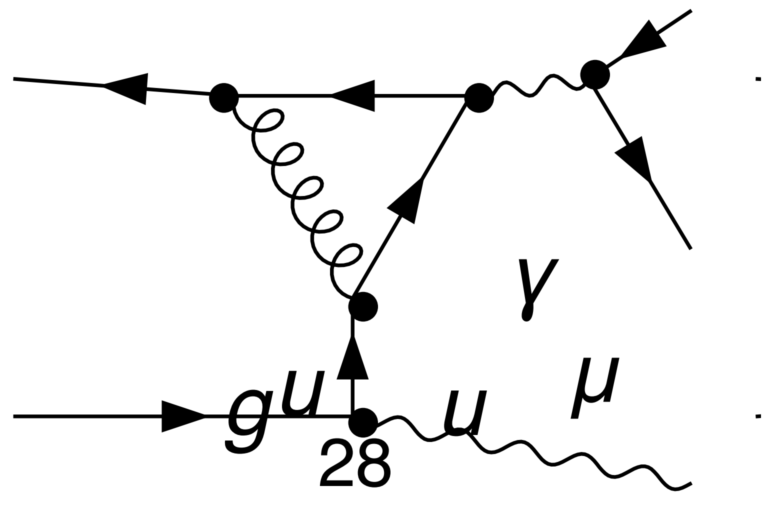
Different kinds of contributions at second order and corresponding problems



double-real contributions

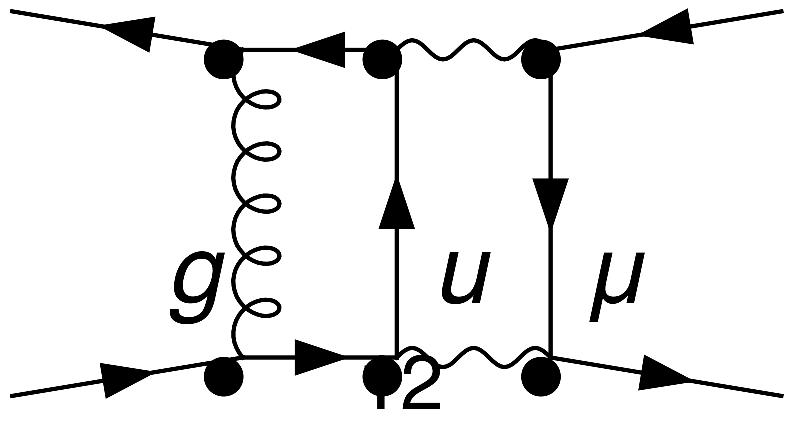
amplitudes are easily generated with OpenLoops
IR subtraction
challenging numerical convergence when aiming at 0.1% precision

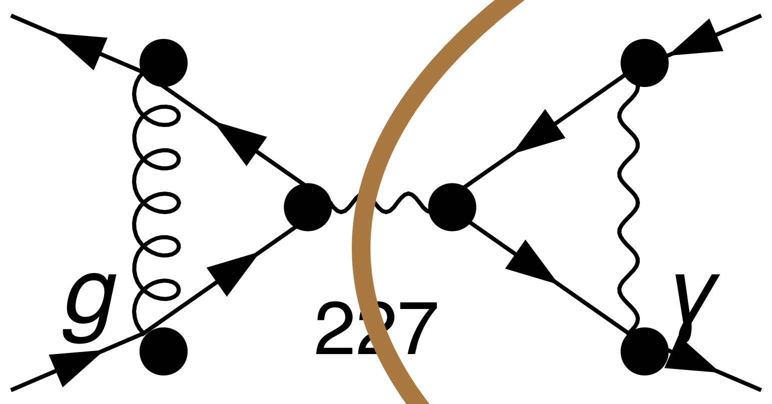




real-virtual contributions

amplitudes are easily generated with OpenLoops or Recola
1-loop UV renormalisation and IR subtraction
challenging numerical convergence when aiming at 0.1% precision





double-virtual contributions

generation of the amplitudes
γ_5 treatment
2-loop UV renormalization
solution and evaluation of the Master Integrals
subtraction of the IR divergences
numerical evaluation of the squared matrix element

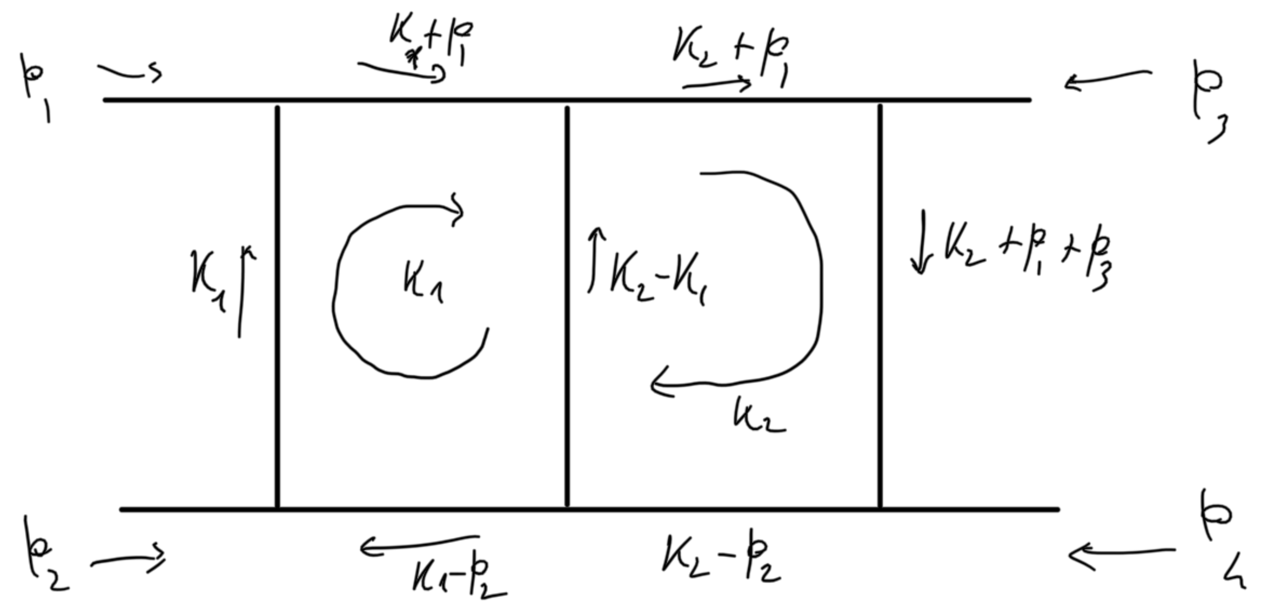
Theoretical challenges in the double-virtual contributions

topic	open issues
generation of the amplitudes	$\mathcal{O}(10^4)$ Feynman diagrams → amplitude size in the GB range
γ_5 treatment	chiral couplings in dimensional regularisation → consistent prescriptions CPU intensive
2-loop UV renormalization	unstable particles (meaning of mass, gauge invariance, unitarity)
evaluation of the Feynman Integrals	increasing number of energy scales → closed form solution not available
subtraction of the IR divergences	separation of IR-finite remainder from IR-enhanced (→ resummation) terms
numerical evaluation	stability (large cancellations → precision loss) and speed

Master Integrals evaluation

The Feynman Integrals $\mathcal{I}_i$ are one of the major challenges in the evaluation of the virtual corrections

$$\mathcal{I}(p_i \cdot p_j; \vec{m}) = \int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{1}{[k_1^2 - m_0^2]^{\alpha_0} \, [(k_1+p_1)^2 - m_1^2]^{\alpha_1} \, \ldots \, [(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j} \, \ldots \, [(k_2+p_l)^2 - m_l^2]^{\alpha_l}}$$



The complexity of the solution grows with the number of energy scales (masses and invariants) upon which it depends

2 fermion production: 7 masses + 2 invariants, 4 fermion production: 9 masses + 5 invariants

Solution strategies

Numerical — Monte Carlo integration, Sector decomposition (Fiesta, PySecDec)

→ limited numerical precision, possible issues with rich threshold structures

Analytical in closed form — Generalised Polylogarithms, Symbols, Elliptic multiple Polylogarithms,…

→ mostly developed for QCD massless amplitudes

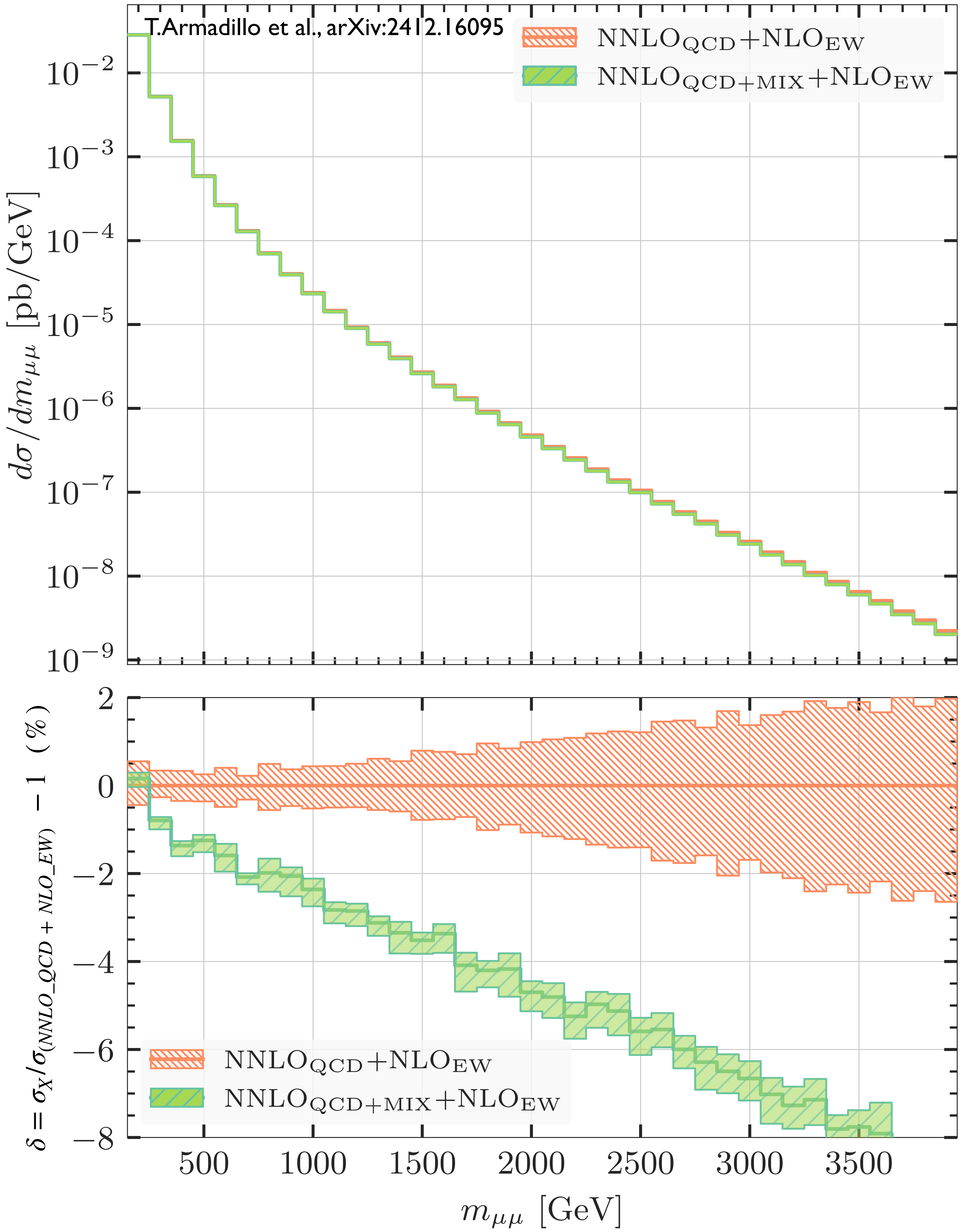
Semi-analytical via series expansions — Solution of the MIs differential equations via series expansion (SeaSyde, AMFlow)

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

→ no closed form, but arbitrary precision is available

→ solution of a generic integral is available with complex internal masses!

Progress in the Standard Model predictions



The NNLO-QCD + NLO-EW predictions were a standard in Drell-Yan studies

The theoretical uncertainty estimates in the few TeV region were encouraging

but

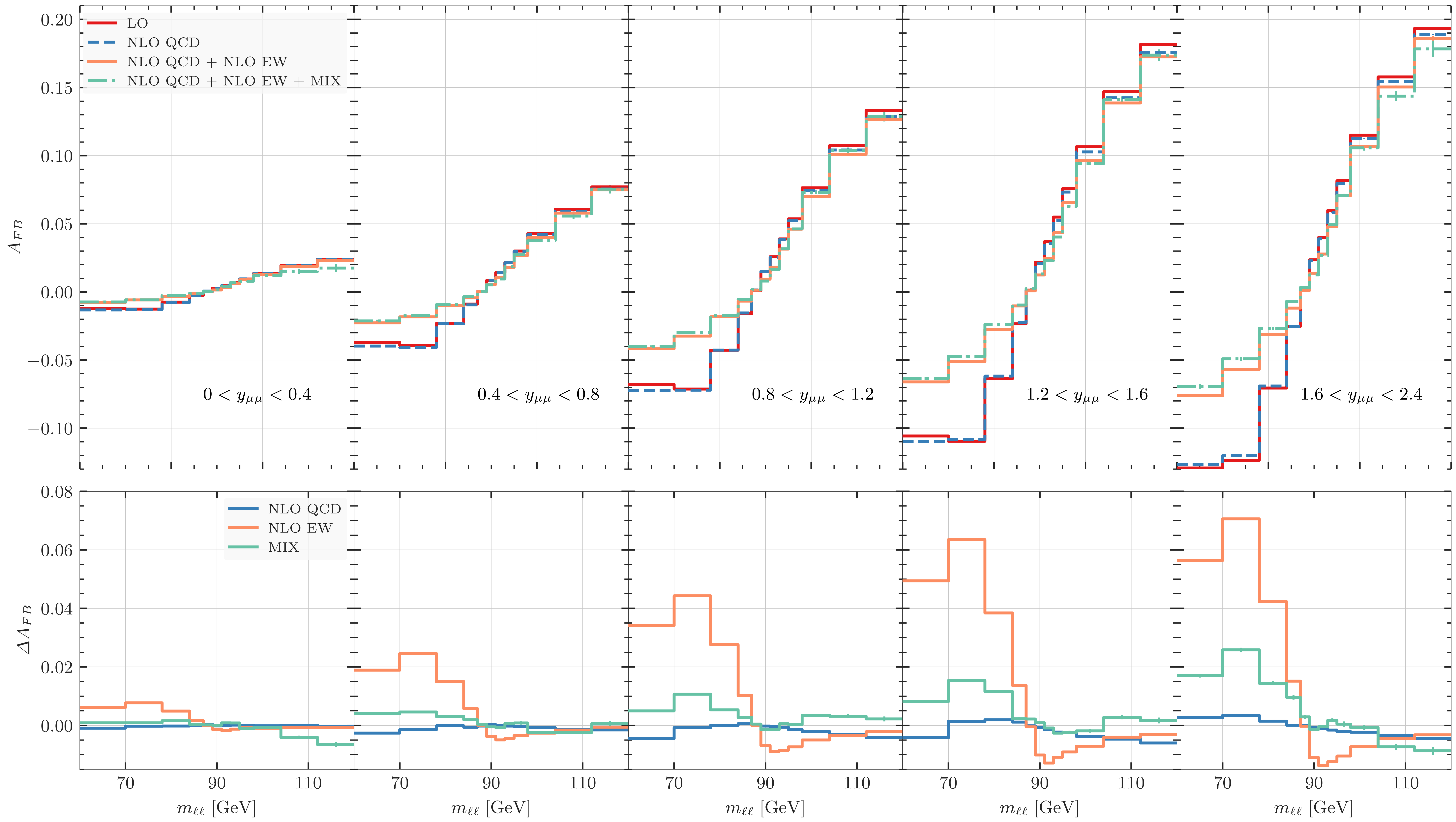
the NNLO QCD-EW corrections combine two large effects,
with a new large shift of the predictions,
well outside the previous uncertainty band

(still missing) NNLO-EW corrections introduce additional new structures
featuring "large" effects,
in the same region where a tiny BSM signal might be present
→ needed to improve the accuracy of the SM prediction

The same comments apply in the CEPC / FCC-ee case:
in the [91,365] GeV range smaller quantum corrections, but
higher experimental precision
→ NNLO-EW corrections will be needed

Progress in the Standard Model predictions

T.Armadillo et al., arXiv:2412.16095



NNLO QCD-EW corrections distort the forward-backward asymmetry $A_{FB}(m_{\ell\ell})$, much more than the pure QCD effects with a large potential impact on the weak mixing angle determination at the LHC

These results might help to revise and improve the study of $A^b_{FB}(m_Z^2)$ in e^+e^- collisions

Testing the Standard Model with its fundamental parameters

Tests of the Standard Model: theoretical predictions vs experimental determinations

The SM lagrangian depends on a fixed number of inputs $\mathscr{L}_{SM} = \mathscr{L}_{SM}(\alpha, G_\mu, m_Z, m_H; m_f, CKM)$

any other quantity can be predicted with a perturbative calculation, e.g. $(m_W^{th}, \sin^2\theta_{eff}^{\ell,th})$

Higher-order corrections are needed for a precise prediction

vs

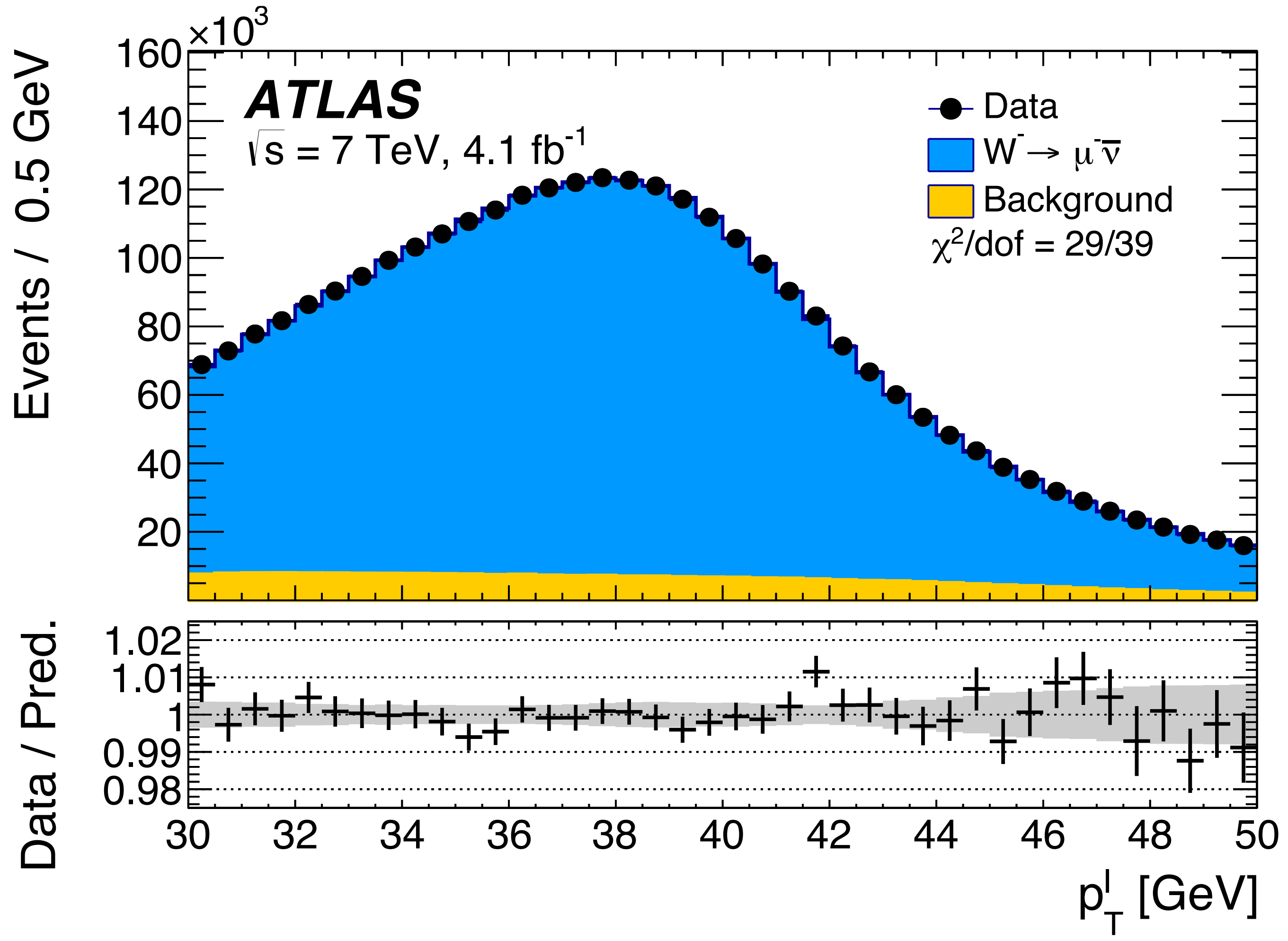
The kinematical distributions of the final state particles feature specific patterns (resonances, asymmetries)

The interpretation of these structures requires a model (e.g. the SM)

Theoretical predictions of the distributions can be fit to the data, keeping masses / couplings as a free parameter

The templates are computed with $\mathscr{L}_{SM} = \mathscr{L}_{SM}(G_\mu, m_W, m_Z, m_H; m_f, CKM)$ or

$$\mathscr{L}_{SM} = \mathscr{L}_{SM}(G_\mu, \sin^2\theta_{eff}^{\ell}, m_Z, m_H; m_f, CKM)$$



The best fit value is the experimental determination of $(m_W^{exp}, \sin^2\theta_{eff}^{\ell,exp})$

Higher-order corrections are needed to minimise the theoretical systematics

High-precision theoretical prediction of EW parameters: current status

Full 2-loop EW and leading 3- and 4-loop results included

on-shell scheme $m_W^{os} = 80.353 \pm 0.004$ GeV (Freitas, Hollik, Walter, Weiglein)

MSbar scheme. $m_W^{\overline{MS}} = 80.351 \pm 0.003$ GeV (Degrassi, Gambino, Giardino)

$$\frac{\Delta m_W^{th}}{m_W} \sim 1 \cdot 10^{-4}$$

parametric uncertainties $\delta m_W^{par} = \pm 0.005$ GeV due to the $(\alpha, G_\mu, m_Z, m_H, m_t)$ values

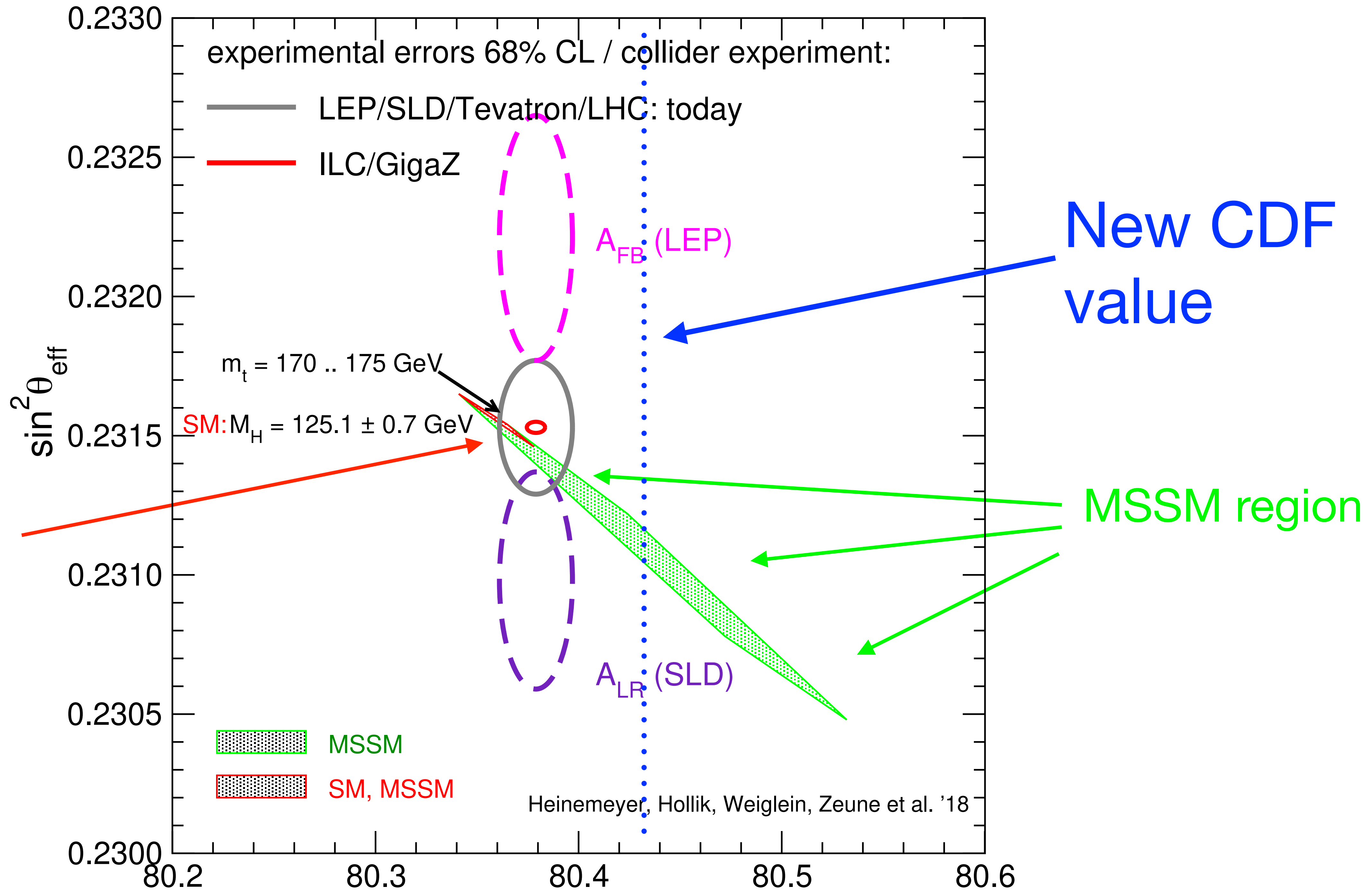
I.Dubovyk, A.Freitas, J.Gluza, T.Riemann, J.Usovitsch, arXiv:1906.08815

$$\sin^2\theta_{\rm eff}^f = s_0 + d_1 L_H + d_2 L_H^2 + d_3 L_H^4 + d_4 \Delta_\alpha + d_5 \Delta_t + d_6 \Delta_t^2 + d_7 \Delta_t L_H + d_8 \Delta_{\alpha_s} + d_9 \Delta_{\alpha_s} \Delta_t + d_{10} \Delta_Z$$

$$\frac{\Delta(\sin^2\theta_{eff}^\ell)^{th}}{\sin^2\theta_{eff}^\ell} \sim 3 \cdot 10^{-4}$$

not adequate for the CEPC / FCC-ee requirements

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]



At the CEPC / FCC-ee we will have (statistical errors)

$$\frac{\Delta m_W^{th}}{m_W} \sim 1 \cdot 10^{-5} \qquad \frac{\Delta(\sin^2\theta_{eff}^\ell)^{th}}{\sin^2\theta_{eff}^\ell} \sim 2 \cdot 10^{-5}$$

We need the complete 3-loop EW+QCD and leading 4-loop corrections to

- the muon decay amplitude
- the Z form factors

High-precision experimental determination of $\sin^2\theta^{\ell}_{eff}$: current status

Given the LEP/SLD precision (17M of Z bosons collected) and the computational power available in the '90s

the deconvolution of QED and QCD from the data,

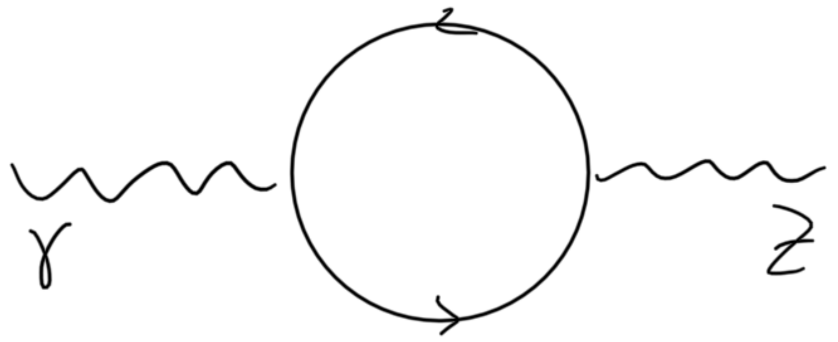
the subtraction of non-factorisable effects from the data,

the decomposition of the Z resonance in pseudoobservables

were an ingenious procedure to evaluate the Z EW form factor at $q^2 = m_Z^2$ → the g_V/g_A ratio expressed by $\sin^2\theta^{\ell}_{eff}$

$$A_{FB}^{exp,deconv}(m_Z^2) - \mathscr{A}_{nonfact} = \frac{3}{4}\mathscr{A}_e\mathscr{A}_f = 3\,\frac{g_V^e/g_A^e}{1+(g_V^e/g_A^e)^2}\;\frac{g_V^f/g_A^f}{1+(g_V^f/g_A^f)^2}$$

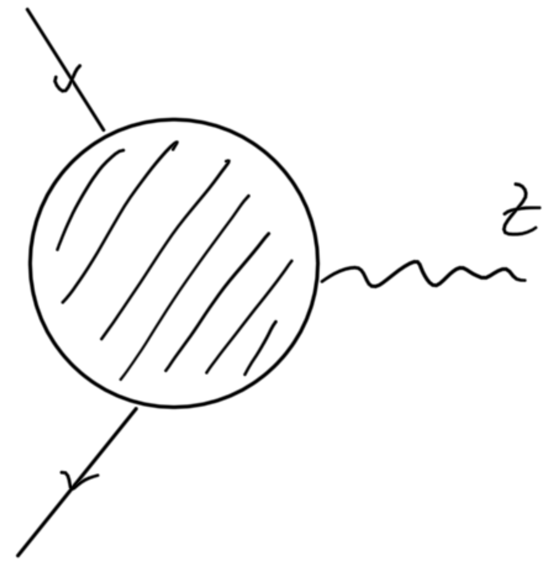
$$= \frac{3}{4}\,\frac{1-4|Q_e|\sin^2\theta^e_{eff}}{1-4|Q_e|\sin^2\theta^e_{eff}+8|Q_e|^2\sin^4\theta^e_{eff}}\;\frac{1-4|Q_f|\sin^2\theta^f_{eff}}{1-4|Q_f|\sin^2\theta^f_{eff}+8|Q_e|^2\sin^4\theta^f_{eff}}$$



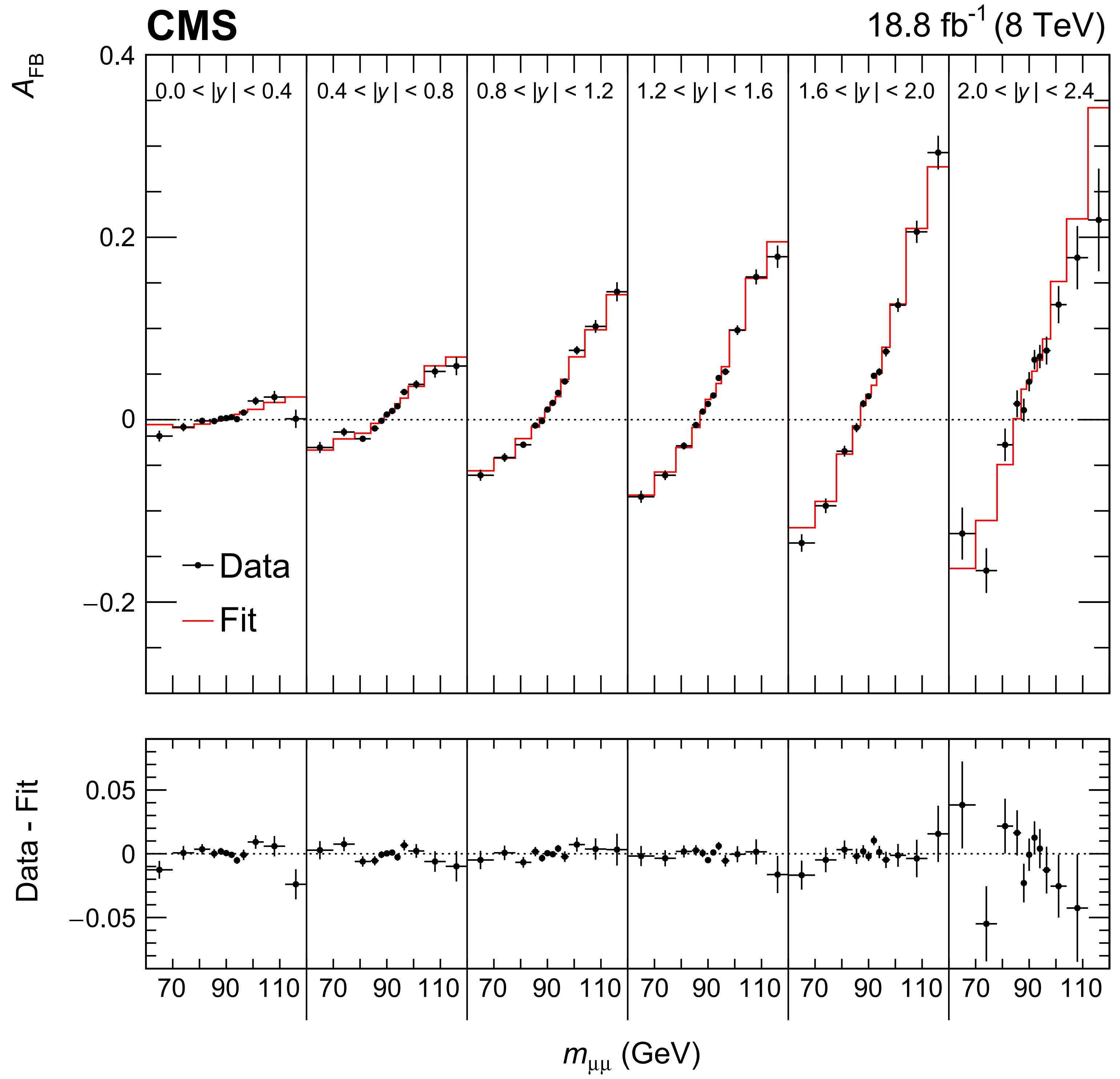
The mixing angle is "effective" because it reabsorbs in a tree-level parameter

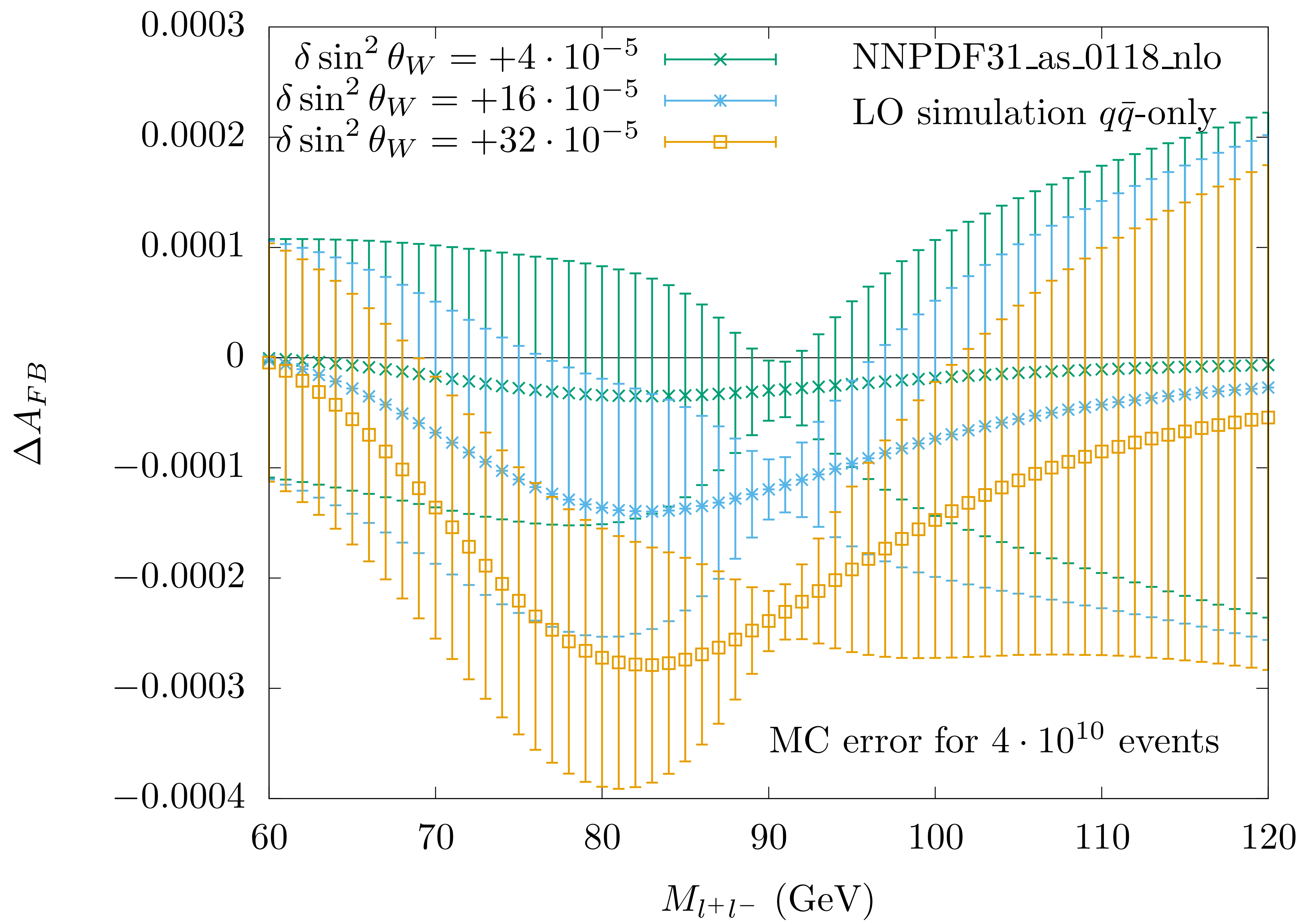
all the SM (+BSM?) quantum corrections (self-energies and vertex → flavour dependent)

at the Z pole we can define a gauge invariant form factor



High-precision experimental determination of $\sin^2\theta^{\ell}_{eff}$: current status





At hadron colliders, $A_{FB}(m_{\ell\ell})$ is a PDF-weighed combination of elementary asymmetries

PDF uncertainties mimic the dependence of $A_{FB}(m_{\ell\ell})$ on $\sin^2\theta^{\ell}_{eff}$

→ the whole distribution is needed to reduce the PDF uncertainty

a template fit procedure is needed to interpret the data at different invariant masses M.Chiesa, F.Piccinini, AV, arXiv:1906.11569

the $(G_\mu, \sin^2\theta^{\ell}_{eff}, m_Z)$ input scheme i) allows the fit

ii) reduces the perturbative theoretical systematic errors in the fit

High-precision experimental determination of $\sin^2\theta^{\ell}_{eff}$: prospects at CEPC / FCC-ee

- Are all the LEP/SLD assumptions still valid at the precision level offered by CEPC / FCC-ee ?
- Is there a bias in the pseudoobservables formulation ?

  How can we deconvolute radiation beyond LL ? → we need quantitative answers

  How can we subtract non-factorizable corrections in a gauge invariant way?

- Shall we exploit the incredible precision of the observables at the CEPC / FCC-ee with a direct template fit approach like it is regularly done at the LHC ?

  → high precision cross sections will be available (with a non negligible effort!)

  → theory error propagation is transparent in a direct template fit approach of the observables

- The LEP/SLD approach is in fact a BSM analysis, for the effective angle is an extra coupling ready to parameterise BSM effects

  → shall we attempt a more systematic approach (e.g. in terms of SMEFT, if we are expecting new heavy physics) ?

  → we need to develop the SMEFT renormalization program

Towards BSM searches

The weak mixing angle(s)

- in the classical SM lagrangian the weak mixing angle expresses the amount of mixing between $SU(2)_L$ and $U(1)_Y$

$$\tan\theta_W = \frac{g'}{g}$$

- upon renormalisation, various definitions are possible, with sensitivity to different subsets of quantum corrections

  - on-shell definition: $$\sin^2\theta_{OS} = 1 - \frac{m_W^2}{m_Z^2}$$ definition valid to all orders

    Sirlin, 1980

  - the effective leptonic weak mixing angle

$$\mathcal{M}^{eff}_{Zl^+l^-} = \bar{u}_l \gamma_\alpha \left[\mathcal{G}_v^f(m_Z^2) - \mathcal{G}_a^f(m_Z^2)\,\gamma_5 \right] v_l\, \varepsilon_Z^\alpha \qquad 4\,|Q_f|\sin^2\theta^f_{eff} = 1 - \frac{\mathcal{G}_v^f}{\mathcal{G}_a^f}$$

  - MSbar definition: $$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8m_{W,0}^2} \longrightarrow \hat{s}^2\hat{c}^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2(1-\Delta\hat{r})} \qquad \hat{s}^2 \equiv \sin^2\hat{\theta}(\mu_R = m_Z)$$

    Marciano, Sirlin, 1980; Degrassi, Sirlin, 1991

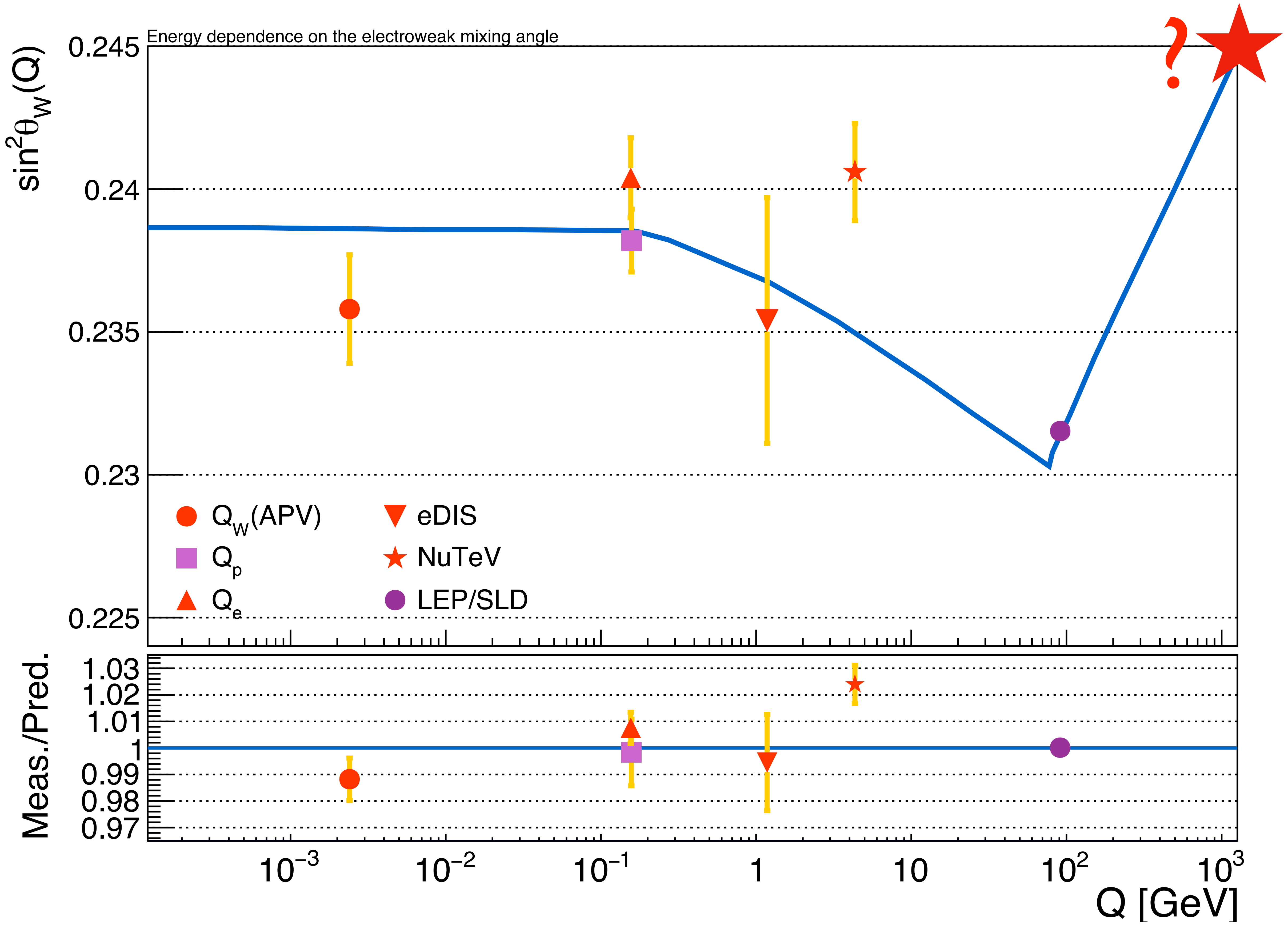
The $\overline{MS}$ weak mixing angle $\sin^2\hat{\theta}(\mu_R)$ as a probe for BSM searches

- To claim a discrepancy from the SM, it is necessary to define with high precision what is the SM !
- Test of the SM: ▻ distribution of physical observables → probe of a range of energies

  ▻ value of fundamental parameters → defined at specific energy scales

- The study of running couplings, e.g. the $\overline{MS}$ weak mixing angle $\sin^2\hat{\theta}(\mu_R)$, opens the scope of the SM tests:

  the same parameter (same formal definition) can be studied from 1 MeV up to few TeV as a function of μ_R



Erler, J., Schott, M., Progr. in Part. and Nucl. Phys. 106 (2019) 68-119

The determination of $\sin^2\hat{\theta}(m_{\ell\ell})$ at different scales

tests the SM prediction

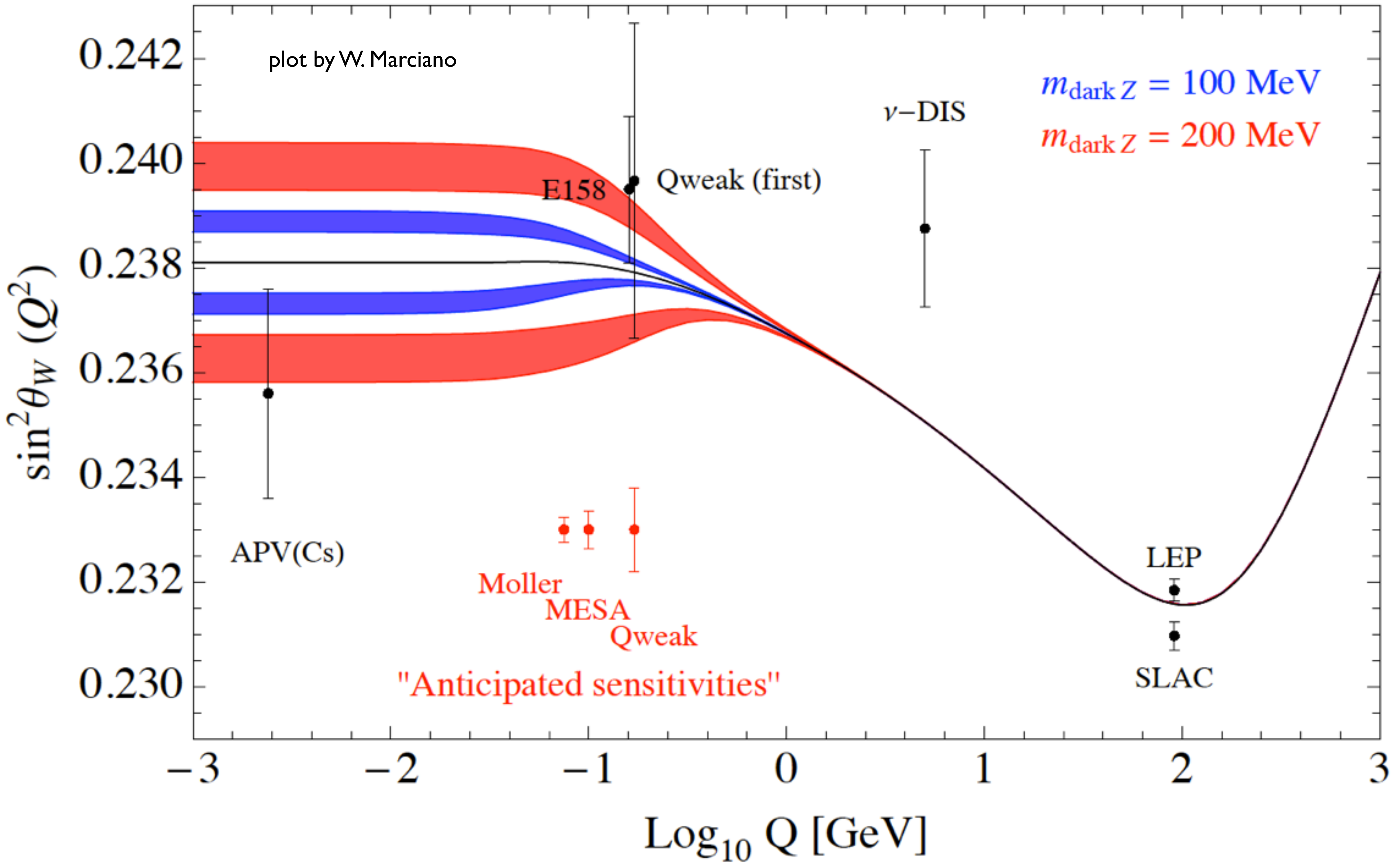
(fixed by one boundary condition e.g. at m_Z)

A deviation from the SM might be due to:

- missing SM higher orders
- BSM contributions which modify the running
- BSM constributions entering at tree-level in the xsecs

Light new physics and $\sin^2\hat{\theta}(\mu_R)$

New dark parity-violating bosons



A new dark bosons, mixing with the SM Z boson, may modify the strength of the parity-violating couplings

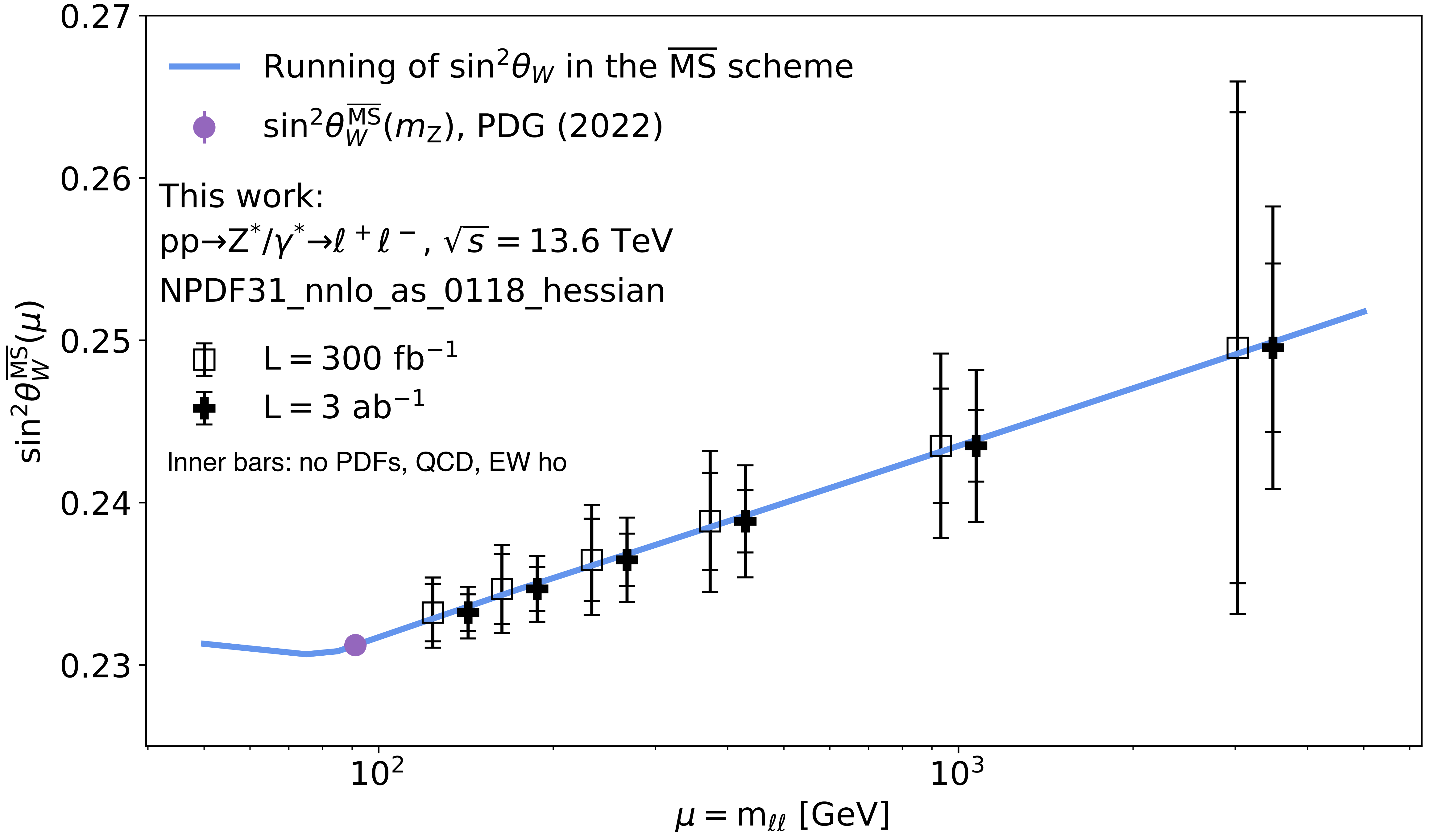
The effects can be completely absent at the Z resonance, where the SM amplitude is purely imaginary.

The presence of the extra boson modifies the asymmetry and, in turn, mimics a change in the running of $\sin^2\hat{\theta}(\mu_R)$, with a modulation due to the assumed boson mass and couplings

The sensitivity to this kind of interaction is quite unique to the low-energy electron-scattering experiments

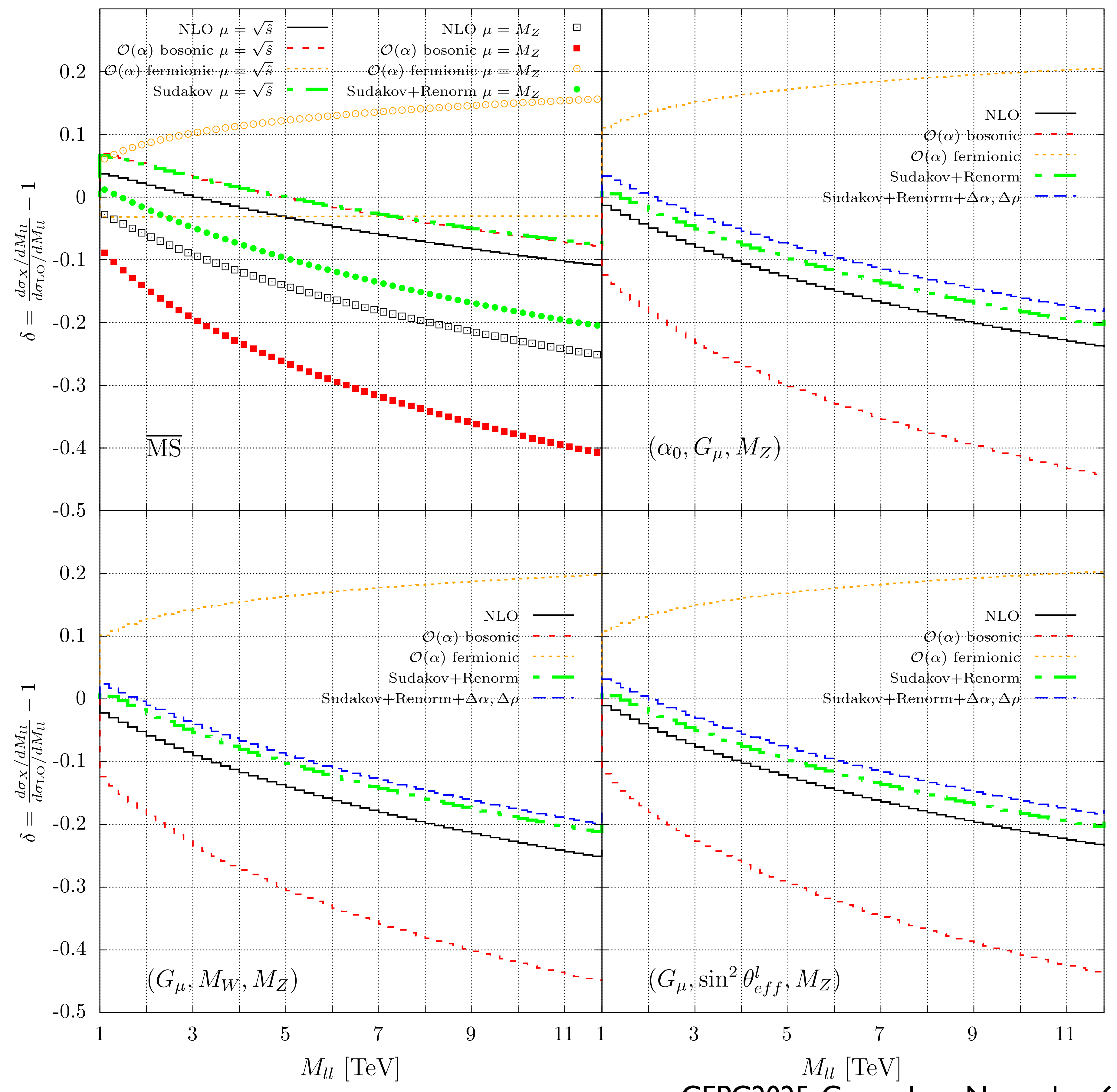
Determination of the running of $\sin^2\hat{\theta}(\mu_R^2)$ in the TeV region

The sensitivity to determine the running of $\sin^2\hat{\theta}(\mu_R^2)$ at the LHC and HL-LHC has been demonstrated in arXiv: 2302.10782 using POWHEG with NLO-EW corrections in the ($\hat{\alpha}(\mu_R), \sin^2\hat{\theta}(\mu_R), m_Z$) input scheme



$\sin^2\hat{\theta}(\mu_R^2 = m_{\ell\ell}^2)$ can be fit to the data, bin by bin in the invariant mass distribution

M.Chiesa, C.Del Pio, F.Piccinini, arXiv:2402.14659



the slope of the invariant mass distribution depends on several competing factors (QCD, EW Sudakov logs, $\hat{\alpha}(\mu_R)$ running)

→ spread ranging from 1% (at 1 TeV) to 10% (at 12 TeV)

Missing SM higher orders could be misinterpreted as a BSM signal

→ NNLO-EW crucial to reduce this risk

Actions to face these challenges

TDLI Institute - Shanghai, October 13-16 2025

School on Precision Higgs Factory Physics

in the framework of the ongoing project based at TDLI

Precision Calculation for Electron-Positron Colliders

CERN, Geneva, June 22-26 2026

Gearing up for Future Colliders -- School on High-Precision Calculations for Future Lepton Colliders

followed by

Gearing up for Future Colliders -- Workshop on Radiative Corrections at Future Lepton Colliders

Conclusions

Can we exploit the precision offered by future colliders ?

Significant qualitative progress in the last 5 years to improve the SM predictions:

- methodological advances (2-loop Master Integrals, DGLAP for the leptons, γ_5)
- conceptual advances (new renormalization schemes, studies about the mass definition for unstable particles)
- phenomenological results (first NNLO QCD-EW cross sections)

These first steps require a huge amount of work to become standardised procedures

- engineering of the software
- optimisation
- new mathematical advances

The HL-LHC phase offers an excellent testing ground of all these ideas

The precision level of the cross sections at CEPC / FCC-ee requires an extremely robust interpretation framework

- BSM signals should not be confused with the SM quantum corrections

Thank you

Back-up

Phenomenology challenges: high-precision theoretical prediction of EW parameters

Quantity	FCC-ee	Current intrinsic error		Projected intrinsic error
M_W [MeV]	0.5–1 ‡	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ $[10^{-5}]$	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.4	$(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.15
R_b $[10^{-5}]$	6	11	$(\alpha^3, \alpha^2\alpha_s)$	5
R_l $[10^{-3}]$	1	6	$(\alpha^3, \alpha^2\alpha_s)$	1.5

arXiv:1906.05379

Quantity	FCC-ee	future parametric unc.	Main source
M_W [MeV]	$0.5 - 1$	1 (0.6)	$\delta(\Delta\alpha)$
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ $[10^{-5}]$	0.6	2 (1)	$\delta(\Delta\alpha)$
Γ_Z [MeV]	0.1	0.1 (0.06)	$\delta\alpha_s$
R_b $[10^{-5}]$	6	< 1	$\delta\alpha_s$
R_ℓ $[10^{-3}]$	1	1.3 (0.7)	$\delta\alpha_s$

reduction of the theoretical uncertainties by 1 order of magnitude

necessary to cope with the statistical errors estimated at FCC-ee

→ full program of higher-loop EW calculations:

muon decay amplitude (3L), gauge boson self-energies (4L) and Z-boson decay widths (3L)

The W boson mass: theoretical prediction

<u>on-shell scheme:</u> dominant contributions to Δr

$$\Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\text{rem}}$$

$$\Delta\alpha = \Pi^{\gamma}_{\text{ferm}}(M_Z^2) - \Pi^{\gamma}_{\text{ferm}}(0) \quad \to \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} = 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} \quad \textit{[one-loop]} \quad \sim \frac{m_t^2}{v^2} \sim \alpha_t$$

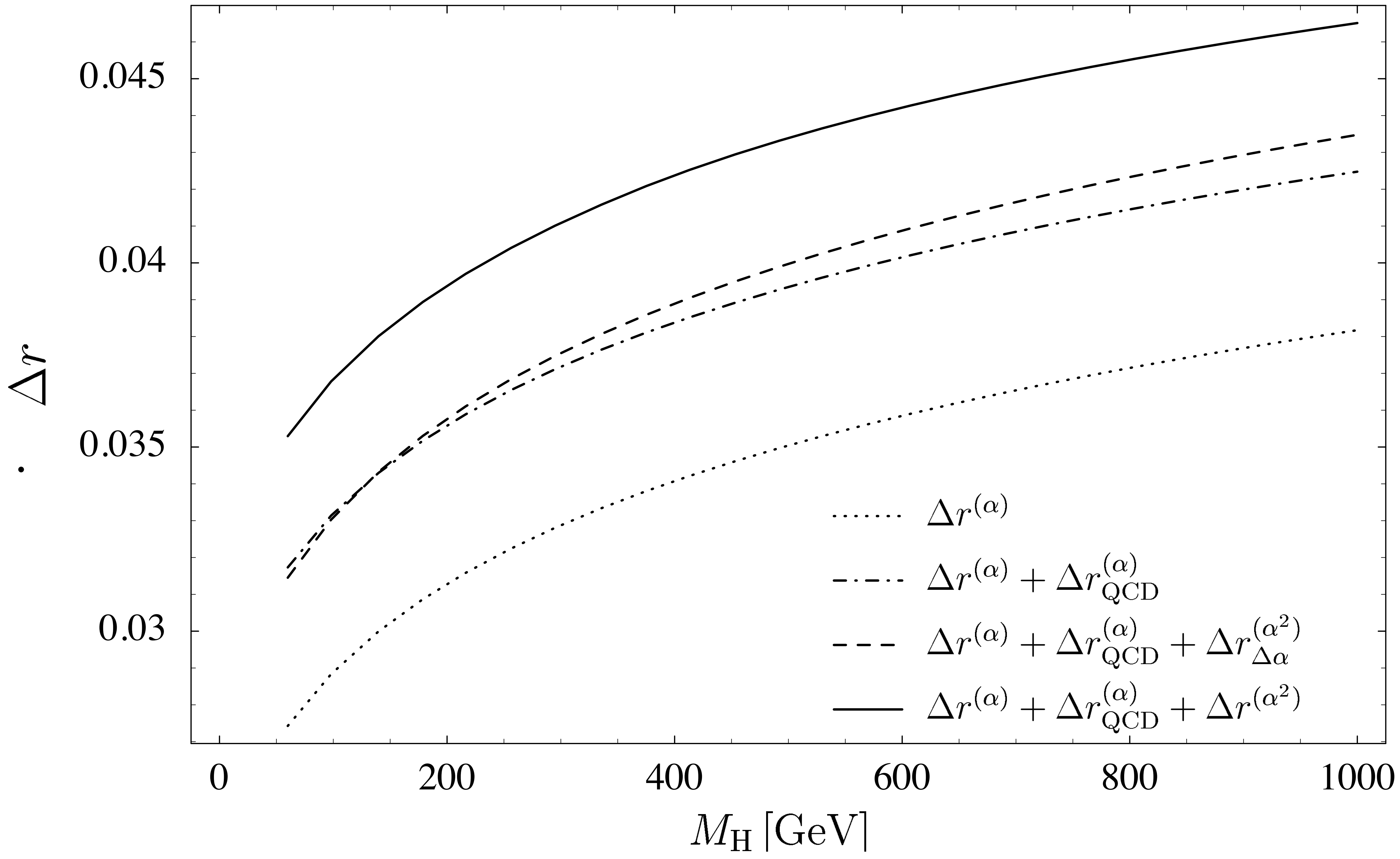
beyond one-loop order: $\sim \alpha^2,\ \alpha\alpha_t,\ \alpha_t^2,\ \alpha^2\alpha_t,\ \alpha\alpha_t^2,\ \alpha_t^3,\ldots$

reducible higher order terms from $\Delta\alpha$ and $\Delta\rho$ via

$$1 + \Delta r \ \to \ \frac{1}{(1-\Delta\alpha)\left(1+\frac{c_w^2}{s_w^2}\Delta\rho\right) + \cdots}$$

$$\rho = 1 + \Delta\rho \ \to \ \frac{1}{1-\Delta\rho}$$

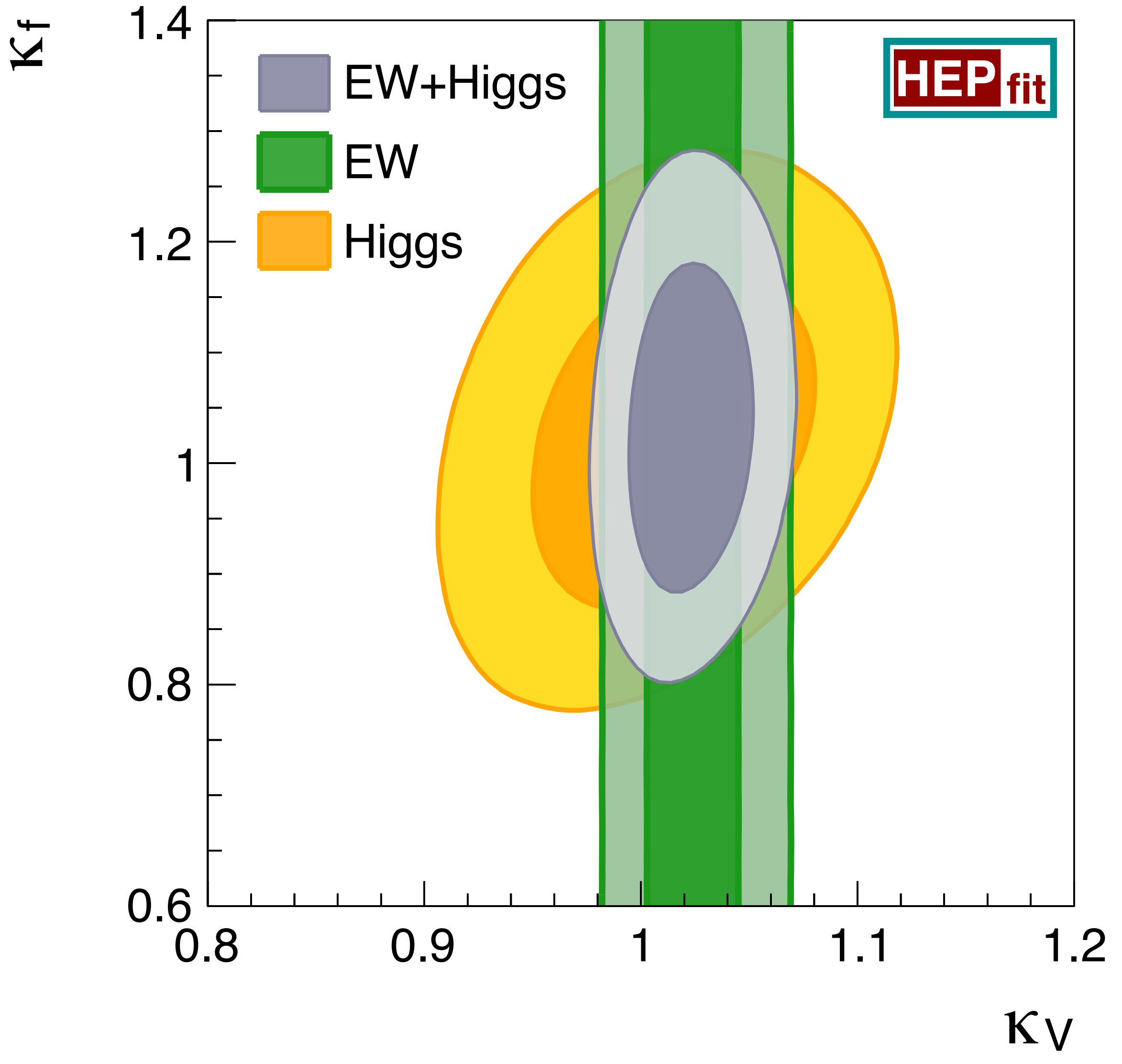
effects of higher-order terms on Δr

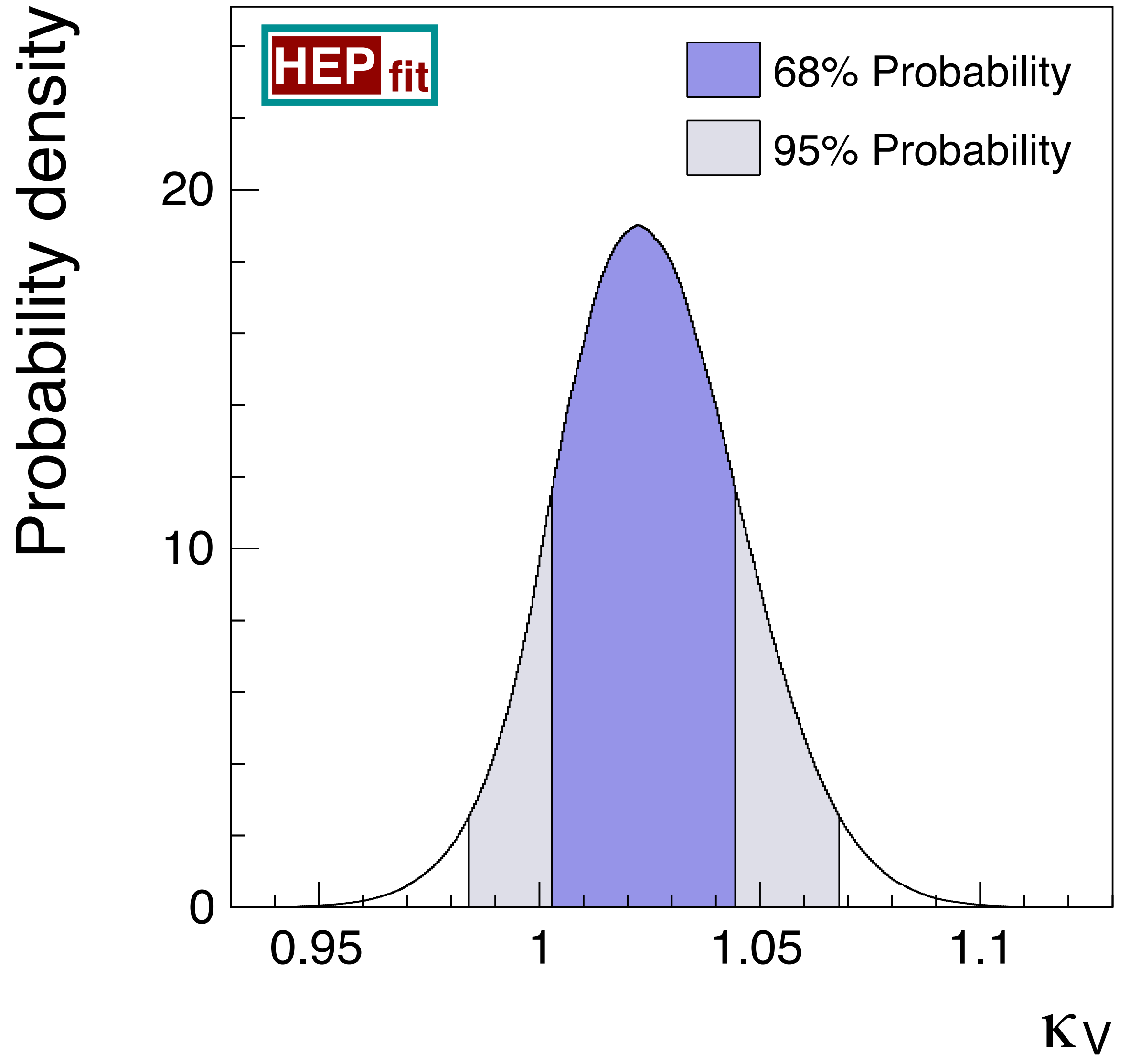


(Consoli, Hollik, Jegerlehner)

Relevance of new high-precision measurement of EW parameters

de Blas et al, arXiv:1608.01509





$$\mathcal{L}_{\rm Eff} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}} \mathcal{L}_d = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \qquad [\mathcal{O}_i] = d \longrightarrow \left(\frac{q}{\Lambda}\right)^{d-4}$$

Effects suppressed by $q = v, E < \Lambda$

Λ: Cut-off of the EFT

$$\mathcal{O}_{\phi WB} = \phi^\dagger \sigma_a \phi B^{\mu\nu} W^a_{\mu\nu}$$

EWSB →

$v^2 B^{\mu\nu} W^3_{\mu\nu}$ gauge boson masses

$vh B^{\mu\nu} W^3_{\mu\nu}$ h→ZZ, γ γ

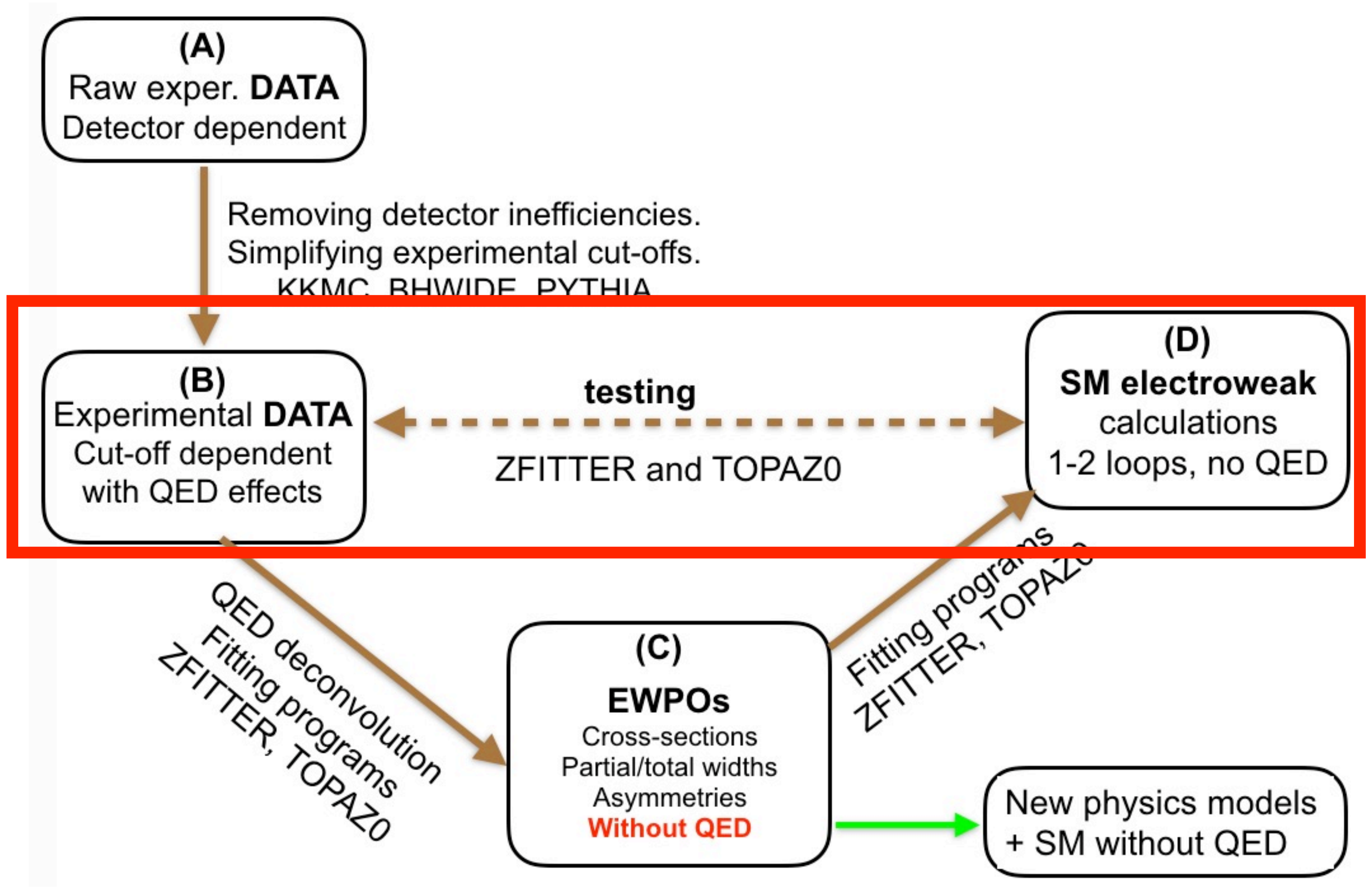
$$M_W^2 = M_Z^2 c^2 \left[1 - \frac{c^2}{c^2 - s^2}\left(\frac{1}{2}C_{\phi D} + 2\frac{s}{c}C_{\phi WB} + \frac{s^2}{c^2}\Delta_{G_\mu}\right)\frac{v^2}{\Lambda^2}\right]$$

A precise measurement of m_W and $\sin^2\theta_{eff}$ constrains several dim-6 operators contributing to Higgs and gauge interaction vertices.

Today still one of the strongest constraints

The LEP/SLD legacy: $\sin^2\theta^{\ell}_{eff}$ determination; two distinct approaches (1)

- SM prediction of cross sections and asymmetries and comparison with data (SM test)

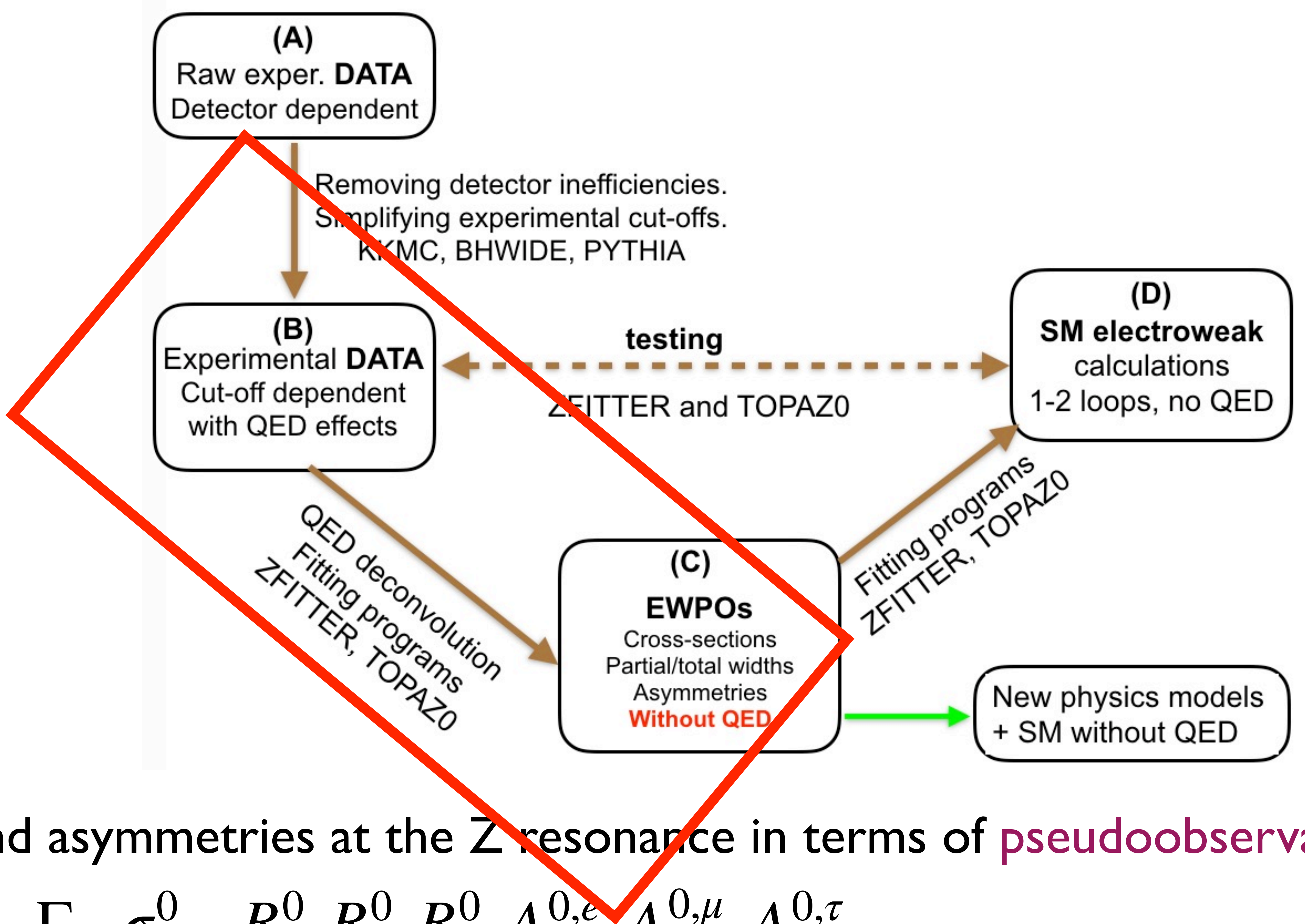


- SM prediction of xsecs and asymmetries computed as a function of $(\alpha, G_\mu, m_Z, m_H, m_t)$
- m_T and m_H fit to the data to maximise the agreement

- $\sin^2\theta^{\ell}_{eff}$ has then been evaluated in the SM using Zfitter/TOPAZ0 with best m_t and m_H values

The LEP/SLD legacy: $\sin^2\theta^\ell_{eff}$ determination; two distinct approaches (2)

· Extraction of $\sin^2\theta^\ell_{eff}$ from pseudo-observables introduced to describe the Z resonance



· parameterisation of xsecs and asymmetries at the Z resonance in terms of pseudoobservables (≠SM)

$$m_Z, \Gamma_Z, \sigma^0_{had}, R^0_e, R^0_\mu, R^0_\tau, A^{0,e}_{FB}, A^{0,\mu}_{FB}, A^{0,\tau}_{FB}$$

· fit of the Z-resonance model to the data → experimental values of the pseudoobservables

· tree-level relation between the experimental Z decay widths (subtracted of QED/QCD effects) and the ratio g_V/g_A

→ algebraic solution for $\sin^2\theta^\ell_{eff}$ → effective angle

Mixed QCD-EW corrections to the Drell-Yan processes

Strong boost of the activities in the theory community in the last 4 years! (references not covering the Monte Carlo developments)

→ mathematical and theoretical developments and computation of universal building blocks

- 2-loop virtual Master Integrals with internal masses

U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193, R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581, M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491, S.Hasan, U.Schubert, arXiv:2004.14908, M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130

- New methods to solve the Master Integrals

M.Hidding, arXiv:2006,05510, D.X.Liu, Y.-Q. Ma, arXiv:2201.11669, T.Armadillo, R.Bonciani, S.Devoto, N.Rana,AV, arXiv: 2205.03345

- Altarelli-Parisi splitting functions including QCD-QED effects

D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612

- renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

→ on-shell Z and W production as a first step towards full Drell-Yan

- pole approximation of the NNLO QCD-EW corrections

S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016, 2401.15682

- analytical total cross section including NNLO QCD-QED and NNLO QED corrections

D. de Florian, M.Der, I.Fabre, arXiv:1805.12214

- ptZ distribution including QCD-QED analytical transverse momentum resummation

L. Cieri, G. Ferrera, G. Sborlini, arXiv:1805.11948

- fully differential on-shell Z production including exact NNLO QCD-QED corrections

M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428

- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections

R. Bonciani, F. Buccioni, R.Mondini, AV, arXiv:1611.00645, R. Bonciani, F. Buccioni, N.Rana, I.Triscari, AV, arXiv:1911.06200, R. Bonciani, F. Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections

F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221, A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671,

Mixed QCD-EW corrections to the Drell-Yan processes

Strong boost of the activities in the theory community in the last 4 years! (references not covering the Monte Carlo developments)

→ complete Drell-Yan

- neutrino-pair production including NNLO QCD-QED corrections
  L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315
- 2-loop NC and CC amplitudes
  M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918 , T.Armadillo, R.Bonciani, S.Devoto, N.Rana,AV, arXiv: 2201.01754, 2405.00612
- NNLO QCD-EW corrections to charged-current DY (2-loop contributions in pole approximation).
  L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539
- NNLO QCD-EW corrections to neutral-current DY
  R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, N.Rana, F.Tramontano, AV, arXiv:2102.12539, F. Buccioni, F. Caola, H.A.Chawdhry, F.Devoto, M.Heller, A.V.Manteuffel, K.Melnikov, R.Roentsch, C.Signorile-Signorile, arXiv:2203.11237

→ mixed QCD-QED resummation

- initial-state corrections
  L. Cieri,G.Ferrera, G.Sborlini,, arXiv:1805.11948, A.Autieri, L. Cieri,G.Ferrera, G.Sborlini,, arXiv:2302.05403
- initial and final state corrections
  L.Buonocore, L'Rottoli, P.Torrielli, arXiv:2404.15112

The double virtual amplitude: UV renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

Complex mass scheme

$$\mu_{W0}^2 = \mu_W^2 + \delta\mu_W^2, \quad \mu_{Z0}^2 = \mu_Z^2 + \delta\mu_Z^2, \quad e_0 = e + \delta e$$

$$\frac{\delta s^2}{s^2} = \frac{c^2}{s^2}\left(\frac{\delta\mu_Z^2}{\mu_Z^2} - \frac{\delta\mu_W^2}{\mu_W^2}\right)$$

the mass counterterms are defined at the complex pole of the propagator

the weak mixing angle is complex valued $c^2 \equiv \mu_W^2/\mu_Z^2$

BFG EW Ward identity → cancellation of the UV divergences combining vertex and fermion WF corrections

The bare couplings of Z and photon to fermions in the (G_μ, μ_W, μ_Z) input scheme are given by

$$\frac{g_0}{c_0} = \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left[1 - \frac{1}{2}\Delta r + \frac{1}{2}\left(2\frac{\delta e}{e} + \frac{s^2-c^2}{c^2}\frac{\delta s^2}{s^2}\right)\right] \equiv \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left(1 + \delta g_Z^{G_\mu}\right)$$

$$g_0 s_0 = \sqrt{4\sqrt{2}G_\mu\mu_W^2 s^2}\left[1 + \tfrac{1}{2}\left(-\Delta r + 2\tfrac{\delta e}{e}\right)\right] \equiv e_{ren}^{G_\mu}\left(1 + \delta g_A^{G_\mu}\right)$$

Gauge boson renormalised propagators

$$\Sigma_{R,T}^{AA}(q^2) = \Sigma_T^{AA}(q^2) + 2\,q^2\,\delta g_A$$

$$\Sigma_{R,T}^{ZZ}(q^2) = \Sigma_T^{ZZ}(q^2) - \delta\mu_Z^2 + 2\,(q^2 - \mu_Z^2)\,\delta g_Z$$

$$\Sigma_{R,T}^{AZ}(q^2) = \Sigma_T^{AZ}(q^2) - q^2\,\frac{\delta s^2}{sc}$$

$$\Sigma_{R,T}^{ZA}(q^2) = \Sigma_T^{ZA}(q^2) - q^2\,\frac{\delta s^2}{sc},$$

After the UV renormalisation, the singular structure is entirely due to IR soft and/or collinear singularities

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$\mathrm{T}r(\gamma_\alpha \ldots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \ldots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \ldots \right)$$

If a_1 is evaluated in a non-consistent way,
then poles might not cancel and the finite part of the xsec might have a spurious contribution

- 't Hooft-Veltman treat γ_5 (anti)commuting in (4) $n-4$ dimensions preserving the cyclicity of the traces (one counterterm is needed)
- Kreimer treats γ_5 anticommuting in n dimensions, abandoning the cyclicity of the traces (→ need of a starting point)
  $\gamma_5 = \frac{i}{4!} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma$

- Heller, von Manteuffel, Schabinger verified that the IR-subtracted squared matrix element are identical in the two approaches

- in the Kreimer scheme, residual ambiguities are removed by choosing a 4-dimensional $\epsilon_{\mu\nu\rho\sigma}$
  → new hybrid scalar products which lead to the evaluation of Feynman integrals in $6 - 2\varepsilon$ and $8 - 2\varepsilon$

Differential equations and IBPs

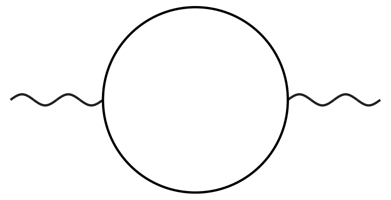
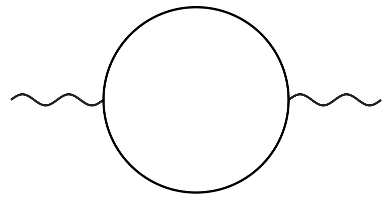
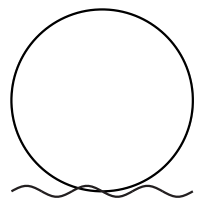
- Not all the Feynman integrals in one amplitude are independent
  → exploit Integration-by-parts (IBP) and Lorentz identities to reduce to a basis of independent Master Integrals

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_1^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0}\,[(k_1+p_1)^2 - m_1^2]^{\alpha_1}\,\ldots\,[(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j}\,\ldots\,[(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

$$\int \frac{d^n k_1}{(2\pi)^n} \int \frac{d^n k_2}{(2\pi)^n} \frac{\partial}{\partial k_2^\mu} \frac{(k_1^\mu,\, k_2^\mu,\, p_r^\mu)}{[k_1^2 - m_0^2]^{\alpha_0}\,[(k_1+p_1)^2 - m_1^2]^{\alpha_1}\,\ldots\,[(k_1+k_2+p_j)^2 - m_j^2]^{\alpha_j}\,\ldots\,[(k_2+p_l)^2 - m_l^2]^{\alpha_l}} = 0$$

- The independent Master Integrals (MIs) satisfy a system of first-order linear differential equations
  with respect to each of the kinematical invariants / internal masses

  When considering the complete set of MIs, the system can be cast in homogeneous form: $d\vec{I}(\vec{s};\varepsilon) = \mathbf{A}(\vec{s};\varepsilon)\cdot\vec{I}(\vec{s};\varepsilon)$

$\frac{d}{dk^2}$  $+ \frac{1}{2}\left[\frac{1}{k^2} - \frac{(D-3)}{(k^2+4m^2)}\right]$  $= -\frac{(D-2)}{4m^2}\left[\frac{1}{k^2} - \frac{1}{(k^2+4m^2)}\right]$ 

- Henn's conjecture (2013): if a change of basis exists which leads to $d\vec{J}(\vec{s};\varepsilon) = \varepsilon\tilde{\mathbf{A}}(\vec{s})\cdot\vec{J}(\vec{s};\varepsilon)$
  then the solution is expressed in terms of iterated integrals (Chen integral representation)
  depending only on the results at previous orders in the ε expansion

A Simple Example

$$\begin{cases} f'(x) + \dfrac{1}{x^2 - 4x + 5} f(x) = \dfrac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f'_{hom}(x) = \sum_{k=0}^{\infty} (k+r)\, c_k\, x^{(k+r-1)}$$

$$\begin{cases} r c_0 = 0 \\ \frac{1}{5} c_0 + c_1 (r+1) = 0 \\ \frac{4}{25} c_0 + \frac{1}{5} c_1 + c_2 (2+r) = 0 \\ \ldots \end{cases}$$

$$f_{hom}(x) = 5 - x - \frac{3}{10} x^2 + \frac{11}{150} x^3 + \ldots$$

Expanded around $x' = 0$

$$f_{part}(x) = f_{hom}(x) \int_0^x dx' \, \frac{1}{(x'+2)} \, f_{hom}^{-1}(x')$$

$$= \frac{1}{2} x - \frac{7}{40} x^2 + \frac{2}{75} x^3 + \ldots$$

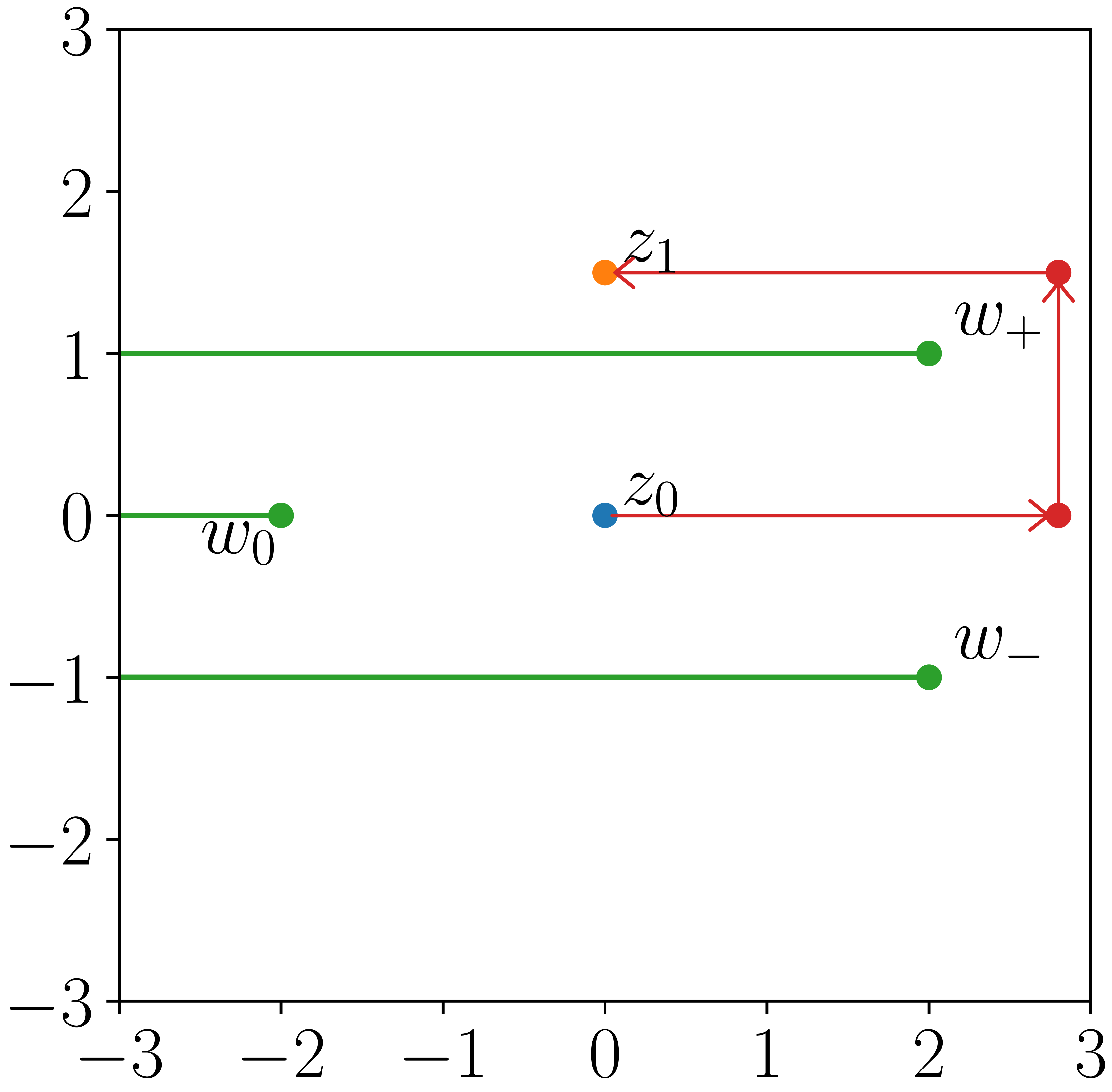
$$f(x) = f_{part}(x) + C f_{hom}(x)$$

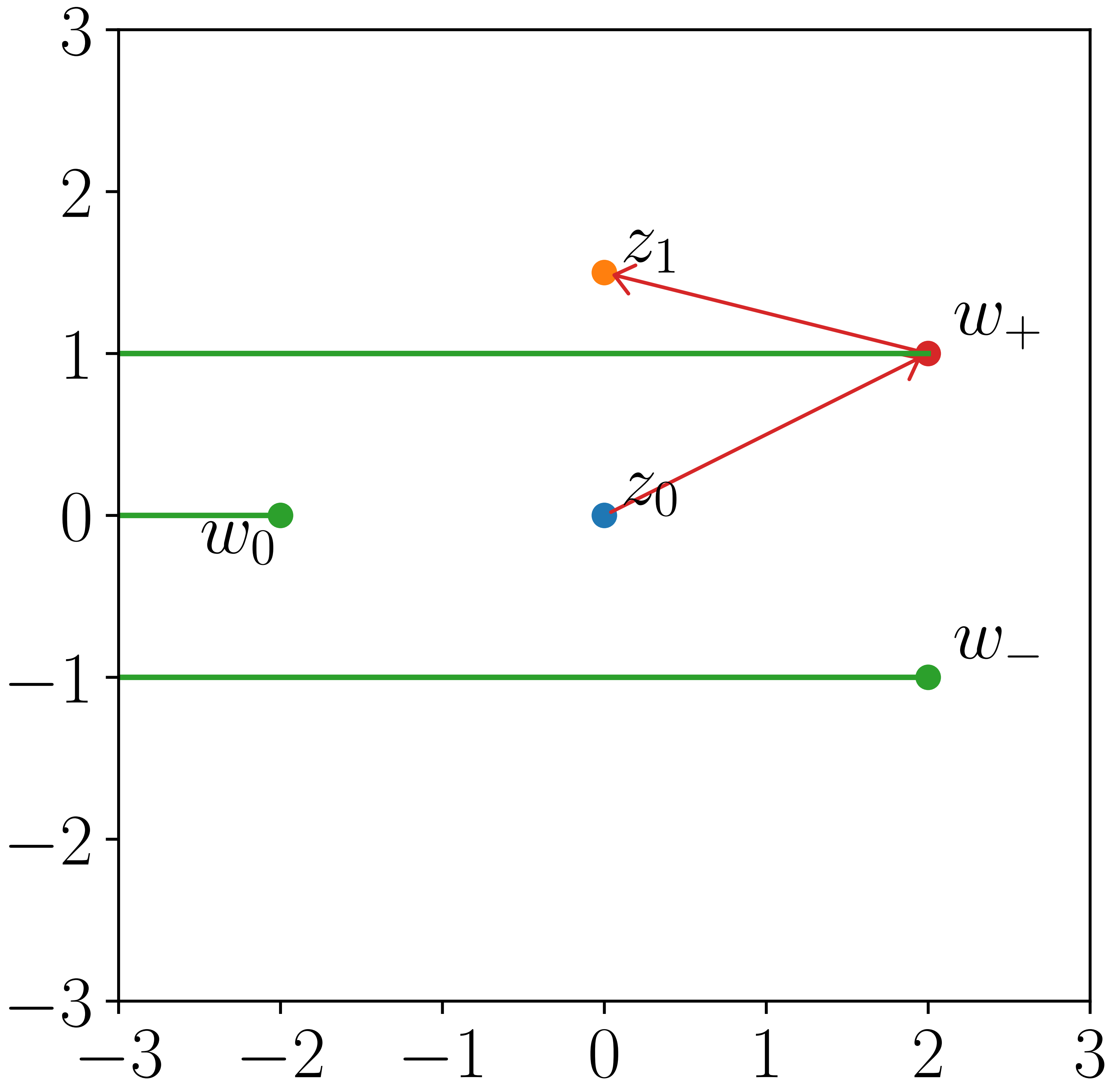
$$f(0) = 1 \rightarrow C = \frac{1}{5}$$

Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

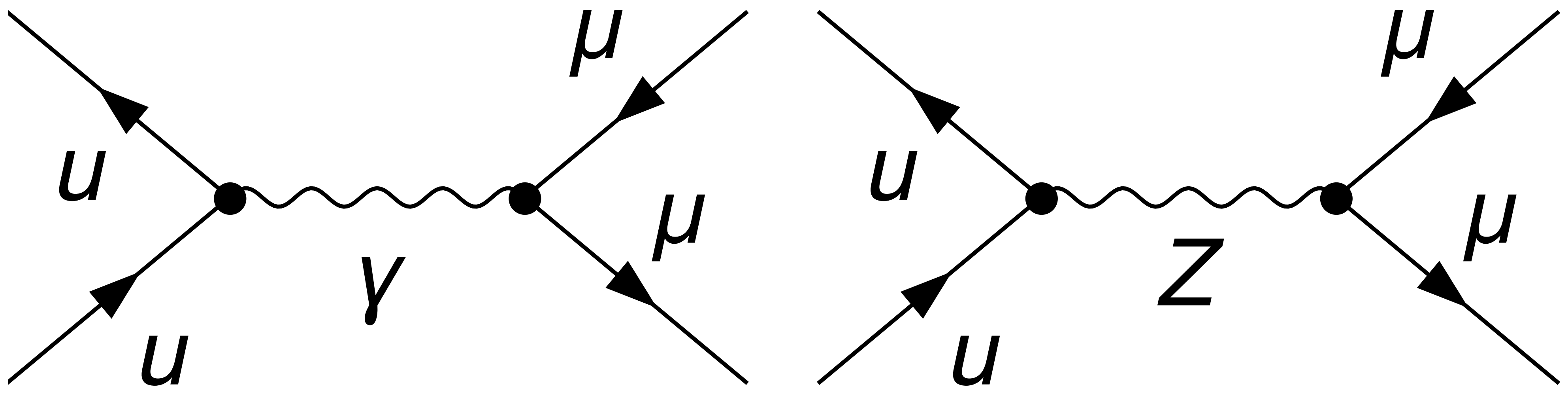
- **Taylor expansion**: **avoids** the singularities;
- **Logarithmic expansion**: uses the singularities as **expansion points**.
- Logarithmic expansion has larger convergence radius but requires longer evaluation time. **We use Taylor expansion as default**.



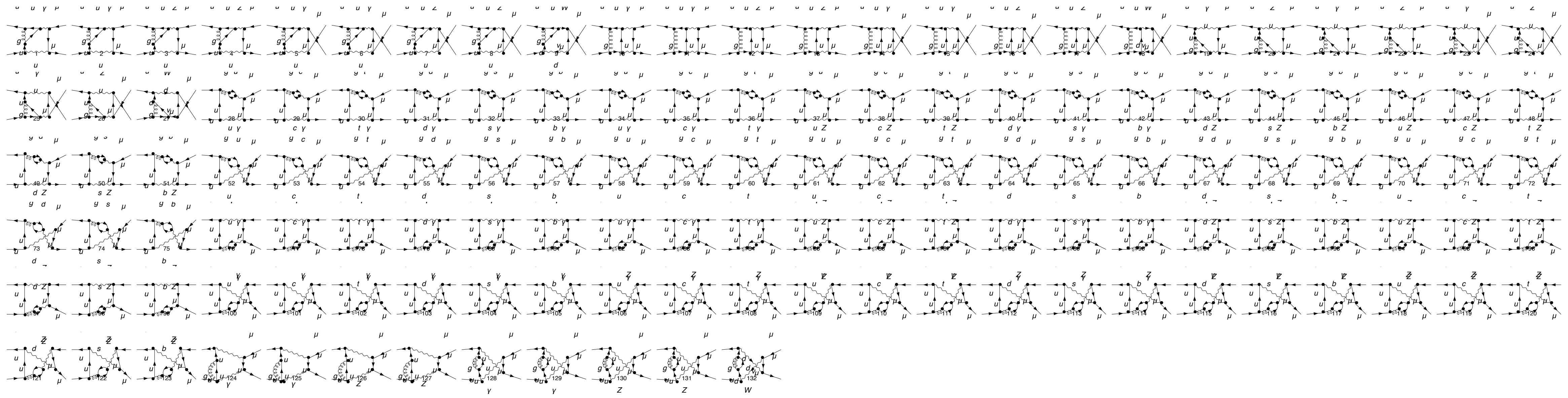


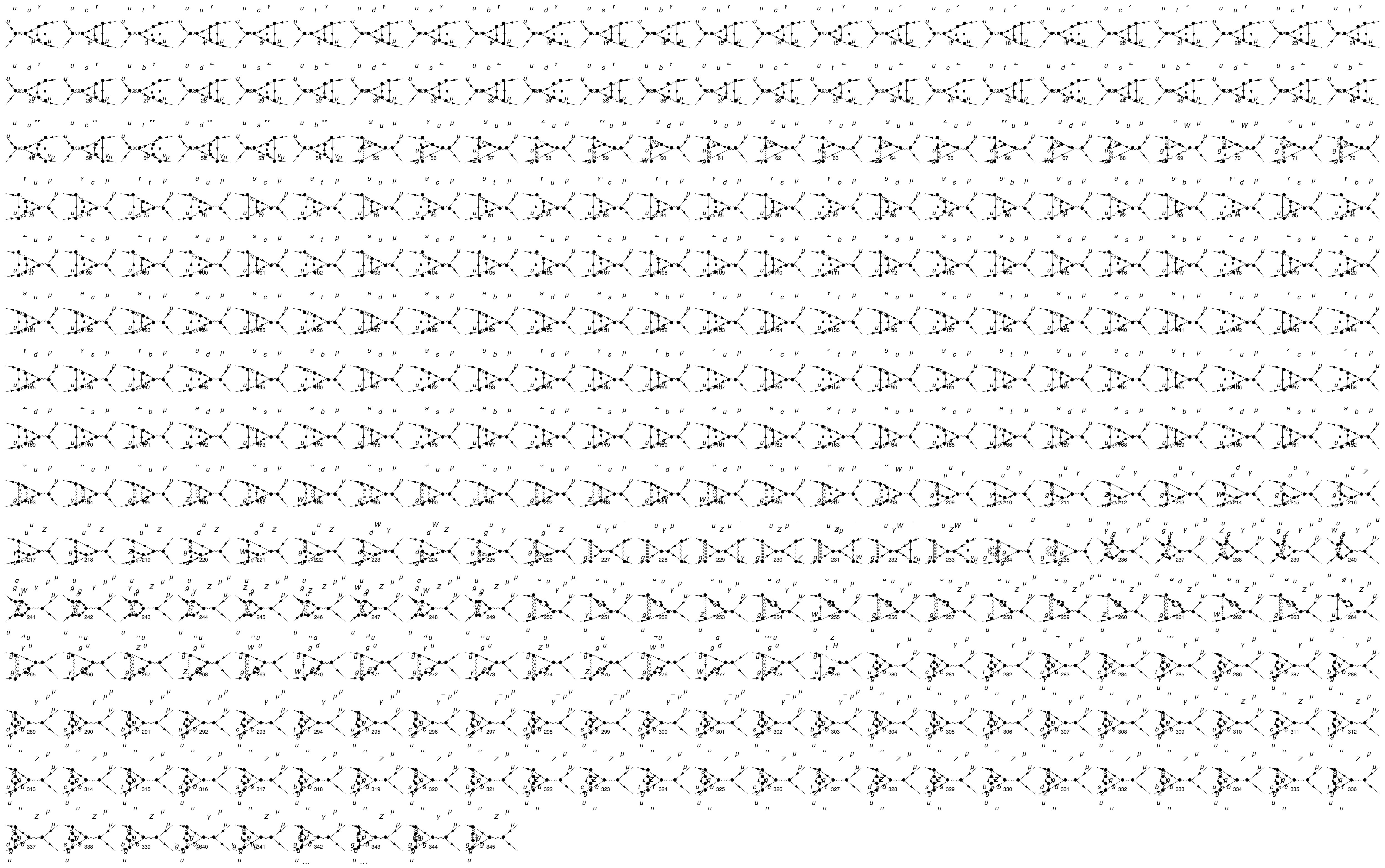
The double virtual amplitude: generation of the amplitude

$\mathscr{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$



$\mathscr{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$ O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)





Structure of the double virtual amplitude

$$2\mathrm{Re}\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^\dagger\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\; \mathscr{I}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε

Their size can rapidly "explode" in the GB range

→ careful work to identify the patterns of recurring subexpressions, keeping the total size in the O(1-10 MB) range

Abiss Mathematica package

Structure of the double virtual amplitude

$$2\mathrm{Re}\left(\mathscr{M}^{(1,1)}(\mathscr{M}^{(0,0)})^\dagger\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathscr{I}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε

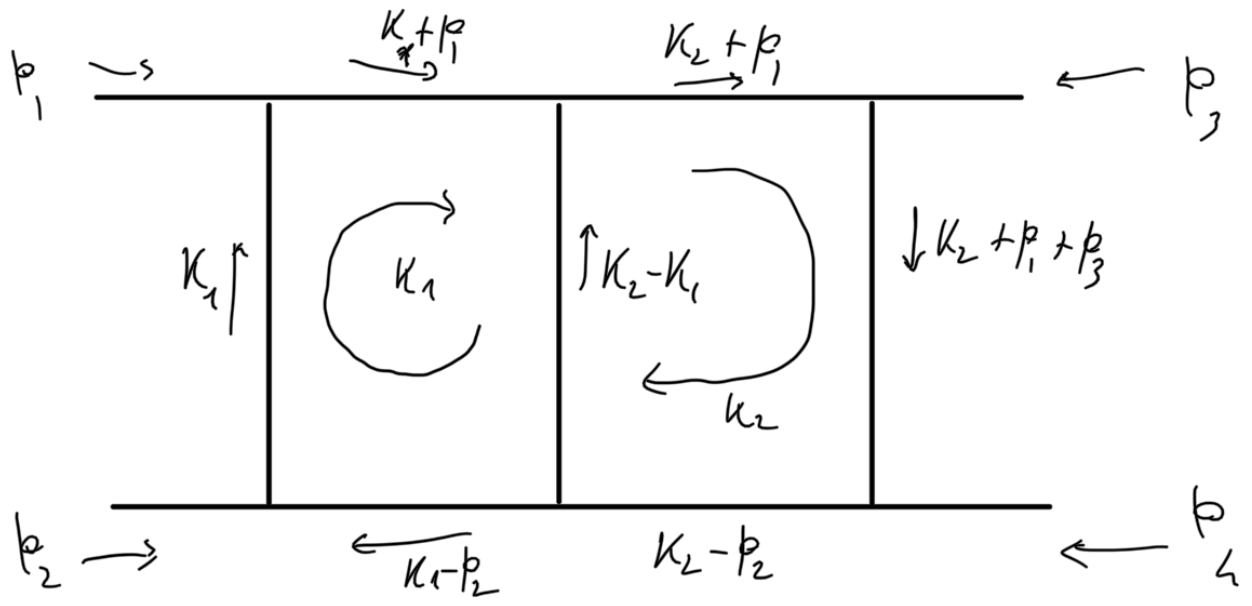
Their size can rapidly "explode" in the GB range

→ careful work to identify the patterns of recurring subexpressions, keeping the total size in the O(1-10 MB) range

Abiss Mathematica package

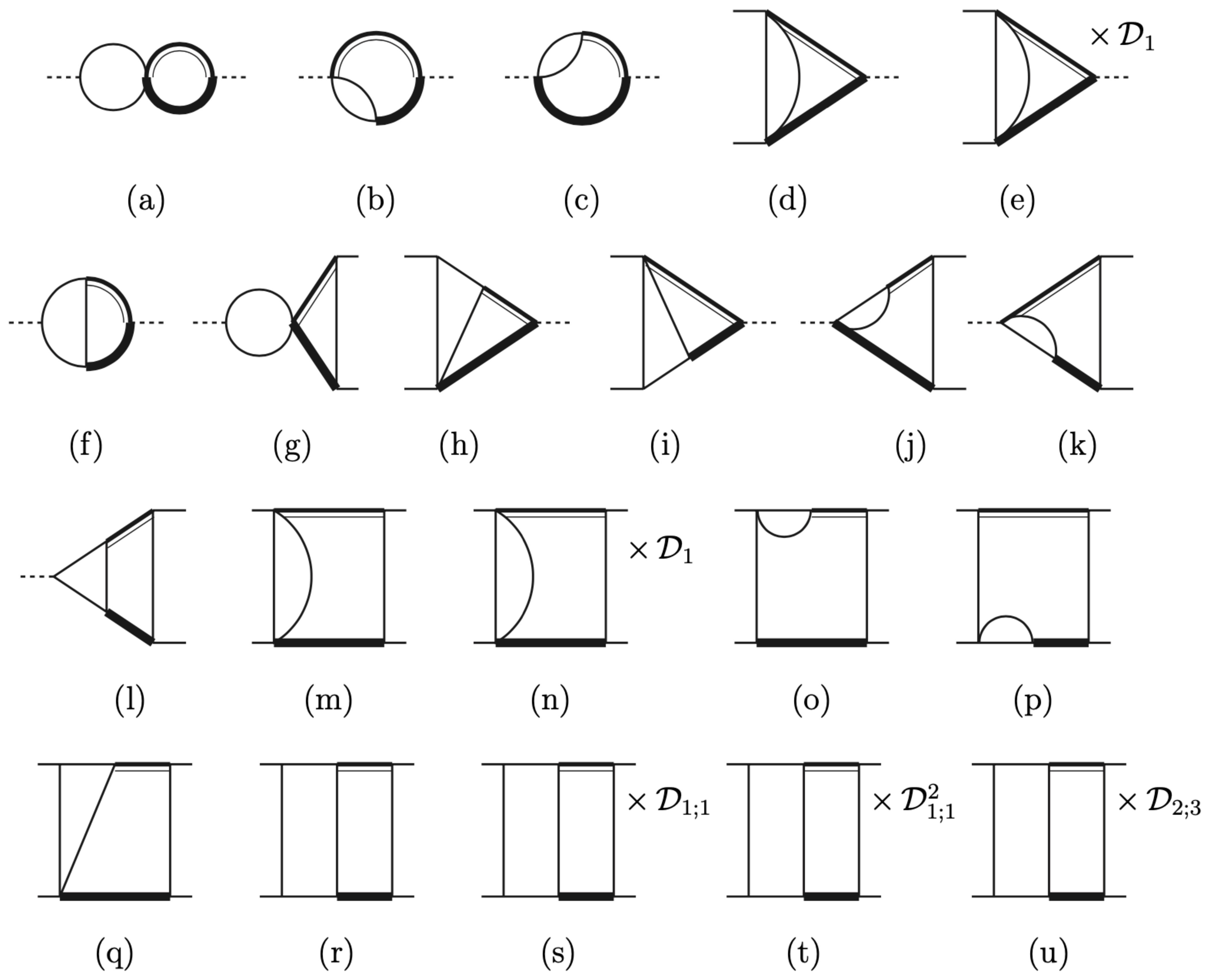
The Feynman Integrals $\mathscr{I}_i$ are one of the major challenges in the evaluation of the virtual corrections

$$\mathscr{I}(p_i\cdot p_j;\vec{m}) = \int\frac{d^nk_1}{(2\pi)^n}\int\frac{d^nk_2}{(2\pi)^n}\frac{1}{[k_1^2-m_0^2]^{\alpha_0}\,[(k_1+p_1)^2-m_1^2]^{\alpha_1}\ \ldots\ [(k_1+k_2+p_j)^2-m_j^2]^{\alpha_j}\ \ldots\ [(k_2+p_l)^2-m_l^2]^{\alpha_l}}$$

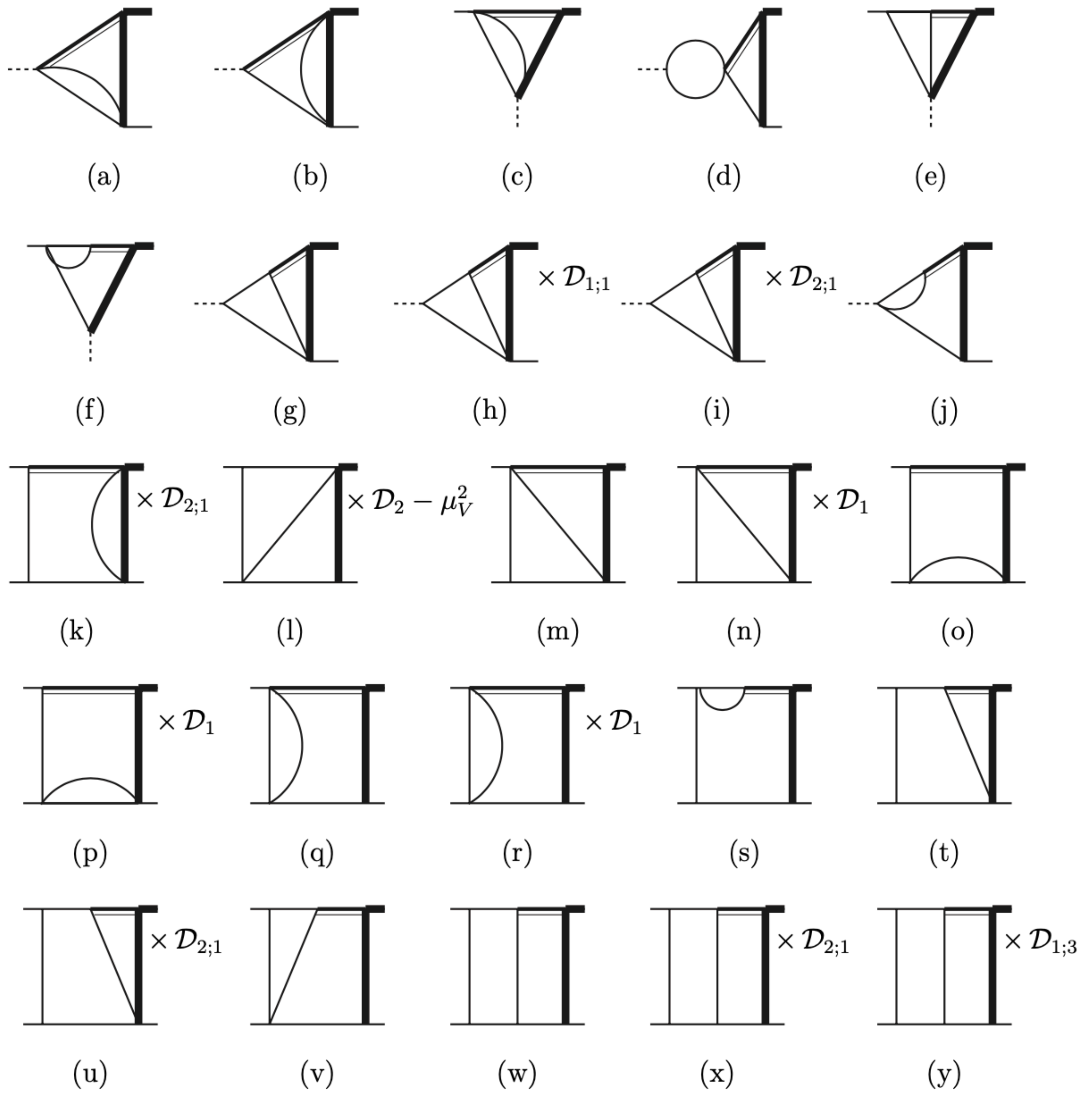


The complexity of the solution grows with the number of energy scales (masses and invariants) upon which it depends

2-loop virtual QCD-EW corrections to CC DY: new Master Integrals



Master Integrals with two different internal masses



Master Integrals with
one W and one internal massive lepton lines

Automated workflow

- All the terms in the amplitude are reduced to Master Integrals with Abiss+KIRA
- The differential equations are written with LiteRed
- The Boundary Conditions are computed with AMFlow
- The Master Integrals are computed with SeaSyde

useful to tackle NNLO-EW corrections
→ relevant at LHC and later at FCC-ee

Evaluation of the Master Integrals by series expansions

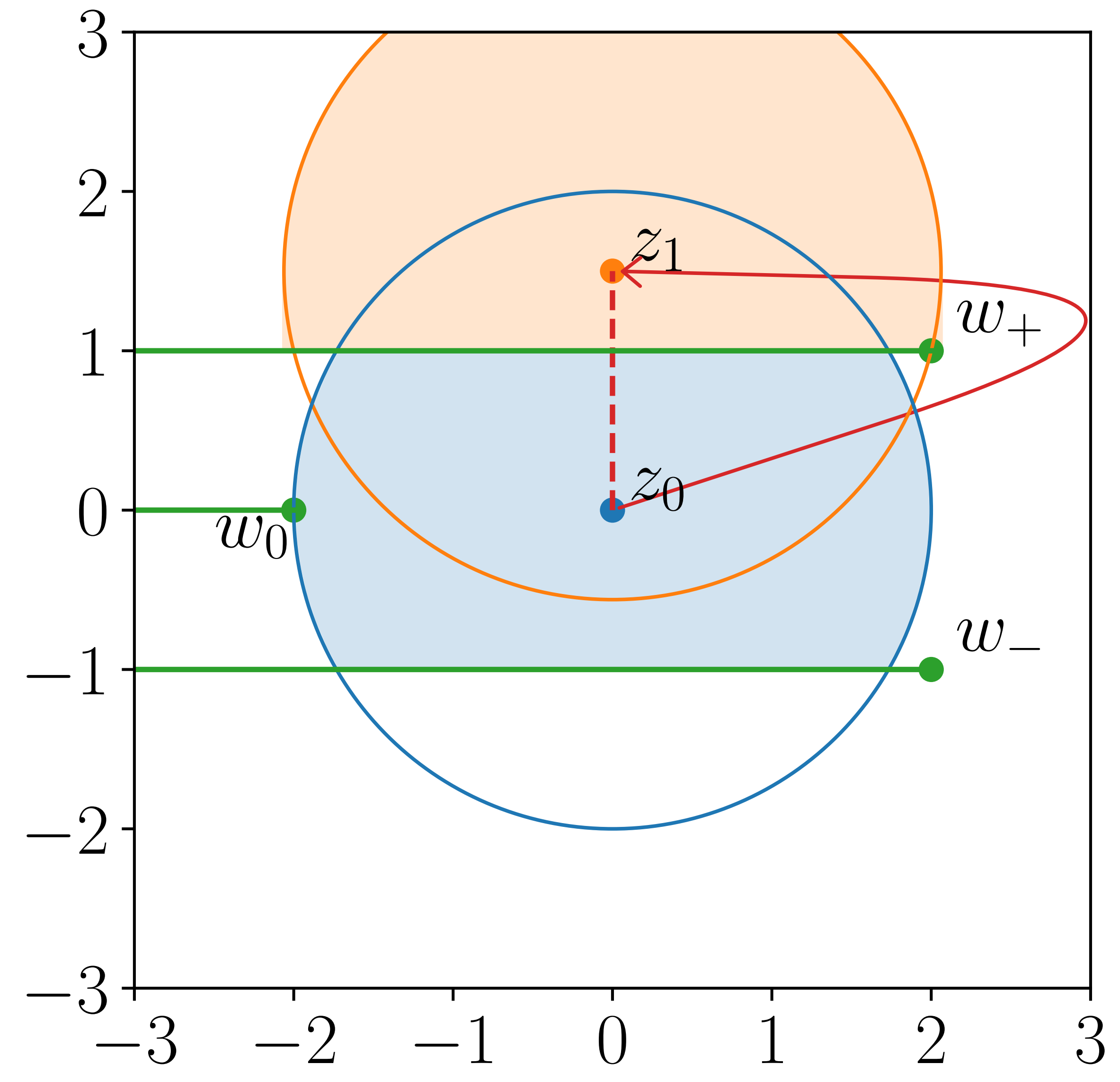
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations.
The MIs are replaced by formal series with unknown coefficients → eqs for the unknown coefficients of the series.
The package `DiffExp` by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.
But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (`SeaSyde`)

We implemented the series expansion approach, for arbitrary complex-valued masses,
working in the complex plane of each kinematical variable, one variable at a time

Complete knowledge about the singular structure of the MI
can be read directly from the differential equation matrix

The solution can be computed with an arbitrary number of significant digits,
but not in closed form → semi-analytical



Evaluation timings and prospects for NNLO-EW studies

The evaluation of Feynman integrals is a very active research field

→ identification and study of new classes of special functions needed to represent the quantum corrections

→ semi-analytical approaches = solution of differential equations by series expansions

→ numerical integration supported by analytical methods to better sample the singular regions

The complexity of the solution/evaluation of these integrals depends

1) on the number of energy scales

→ number of kinematical invariants ($3n-1$ for a $2\to n$ process)

→ number of internal masses (at 2-loop we have up to $k+3$ masses, with k external particles)

2) on the number of Feynman integrals which have to be solved simultaneously (integral family)

3) the size of the expression of the differential equations system

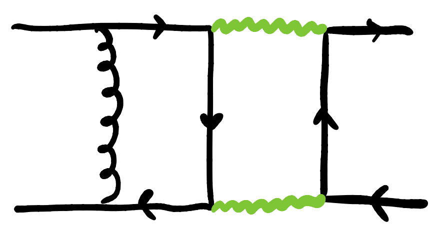
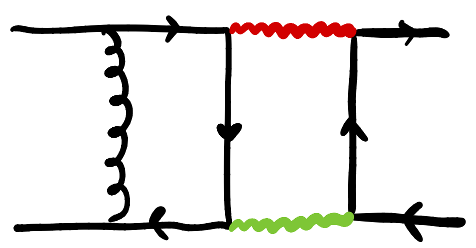
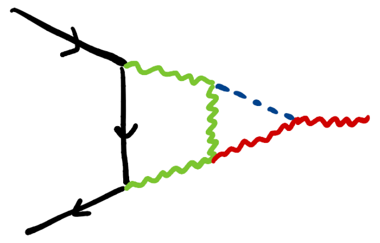
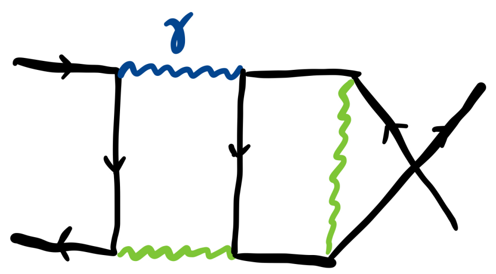
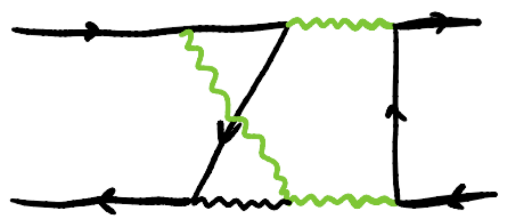
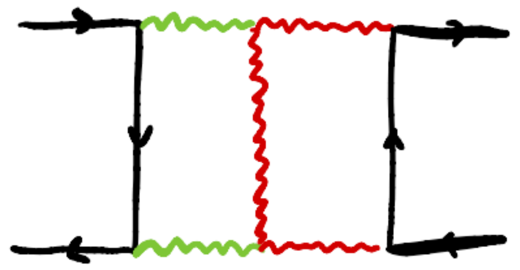
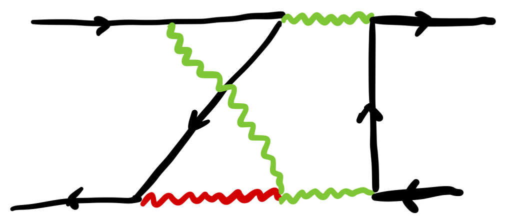
The timing of the evaluation algorithms scales quite violently with these parameters

→ a brute-force approach is not the optimal choice

→ more mathematical and algorithmic developments are needed

Examples

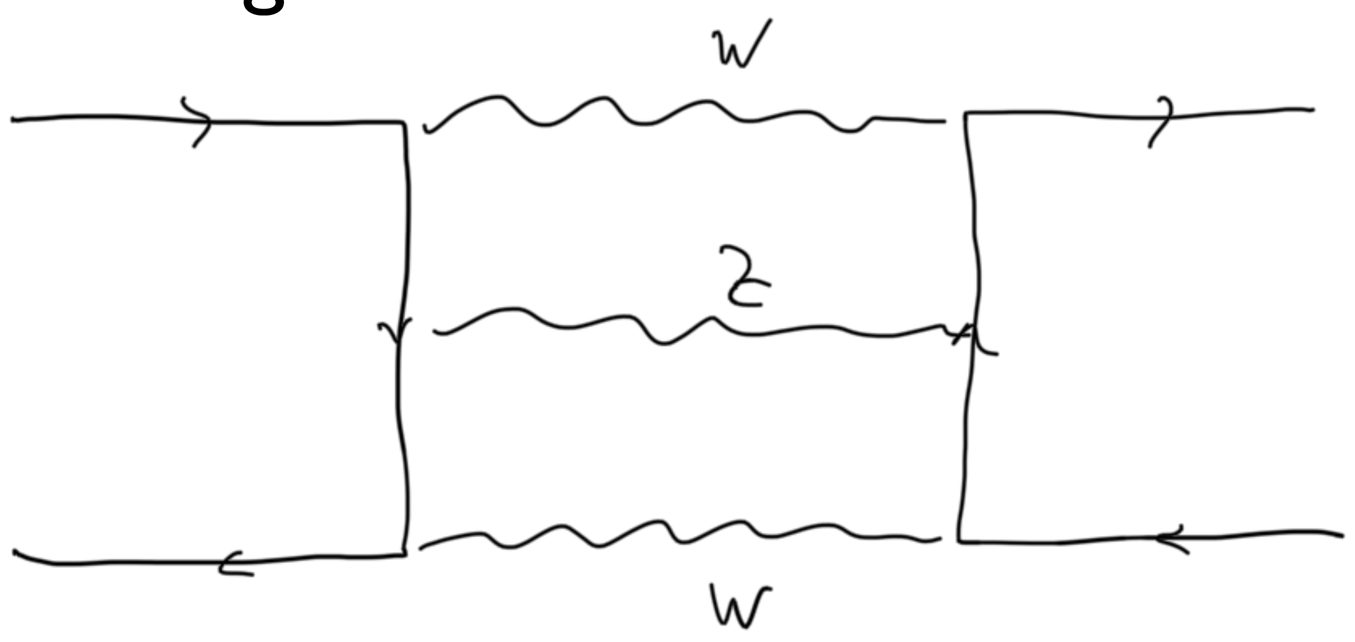
courtesy of T.Armadillo

	NCDY - 2L Mixed	CCDY - 2L Mixed	Z on shell - 2L EW	NCDY - 2L EW	NCDY - 2L EW	NCDY - 2L EW	NCDY - 2L EW
Example Topology							
Number of masters	36	56	51	47	104	126	140
Reduction Kira + Firefly	12 hours (32 core)	16 hours (32 core)	1 day (32 core)	1.5 m (96 core + Ratracer)	30 m (120 core + Ratracer)	8 h (120 core + Ratracer)	54 h (120 core + Ratracer)
AMFlow 1 point	50 min (32 core)	75 min (32 core)	6 h 45 m (32 core)	15 min (96 core)	1 hour (120 core)	4 hours (120 core)	X
Dimension equations	700 Kb	2.1 Mb	/	4 Mb	45 Mb	350 Mb	??
SeaSyde 3250 points	5 days	10 days	/	??	??	??	??

key to overcome these problems: the improvement in the choice of the Master Integrals
systematic usage of polynomial reconstruction

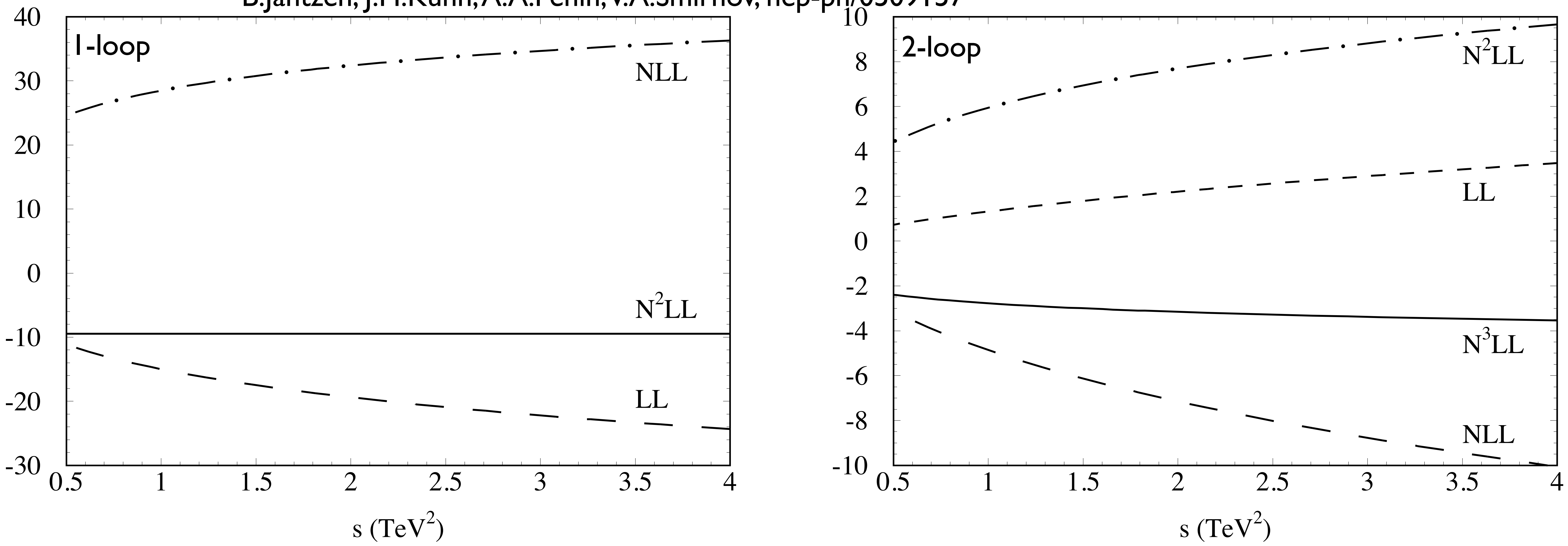
Need for a full NNLO-EW calculation to reduce the uncertainties to sub-percent (per mill) level

The NNLO-EW corrections to scattering processes are still today one of the frontiers in QFT:
challenging Feynman integrals



High-energy limit → enhancement of $\log \frac{s}{m_W^2}$ caveat: alternating signs series

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157



corrections to $e^+e^- \to q\bar{q}$ due to EW Sudakov logs

urgently needed to match sub-percent precision in the TeV region, but also to match CEPC / FCC-ee precision