Global Analyses of Collinear Fragmentation Functions from the NPC Collaboration

XiaoMin Shen(沈晓民)

Institute of Modern Physics, CAS

On behalf of

the Non-Perturbative Physics Collaboration (NPC)

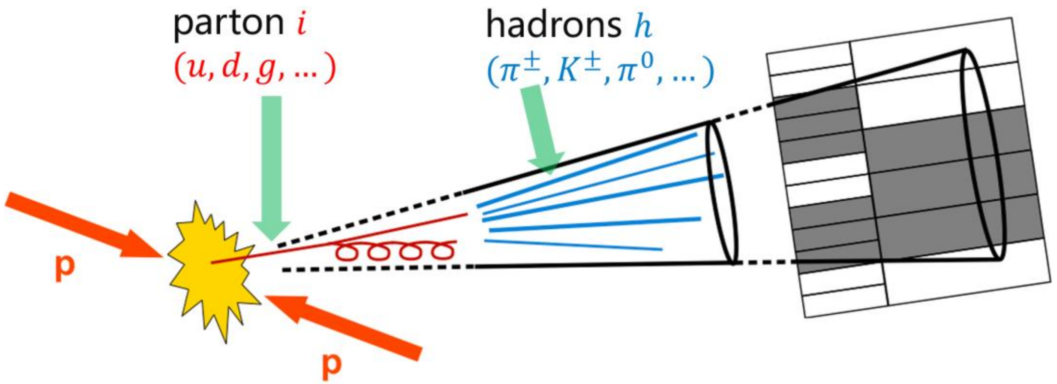
INTERNATIONAL WORKSHOP

ON THE HIGH ENERGY CIRCULAR ELECTRON POSITRON COLLIDER

CEPC 2025

IMP

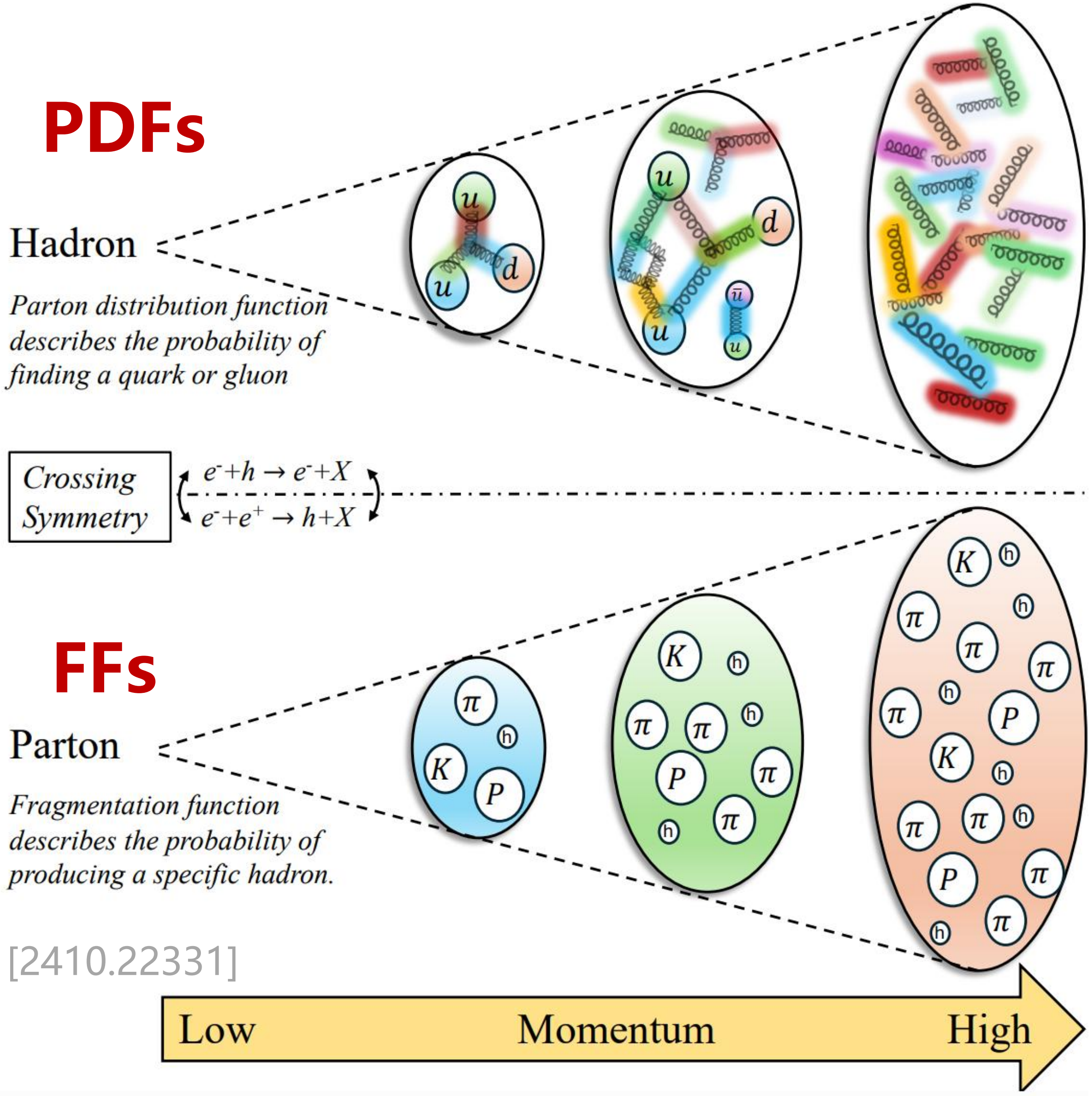
Fragmentation Functions (FFs)



To describe hadronization

parton i ⟶ hadron h

$$D_{h/i}(z \equiv p_h^+ / p_i^+)$$

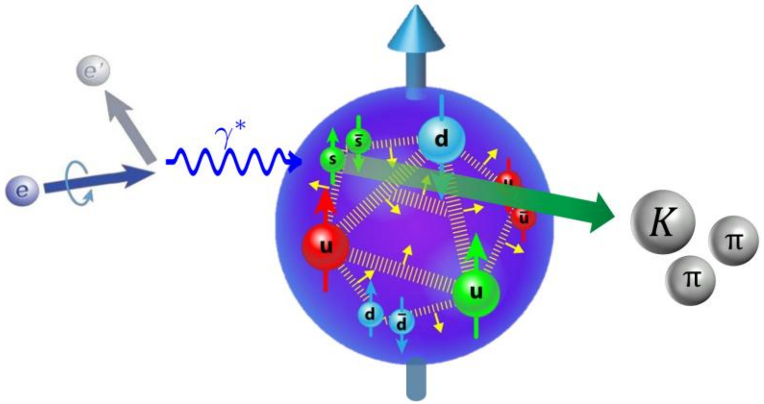


[2410.22331]

Why FFs: key ingredients of QCD factorization framework

1969	1971	1977	1982	1989
Feynman, '69 Parton model	Bjorken et al., '71 **FFs proposed**	Field, Feynman, '77 First determination	Collins, Soper, '82 Definition in QCD	Collins et al., '89 **Factorization thm.**

- Semi-Inclusive DIS (SIDIS) : e + N -> e + **h** + X



$$\frac{d^3\sigma_h}{dx_B dy dz_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x, y, z) \otimes D_{h/j}(z)$$

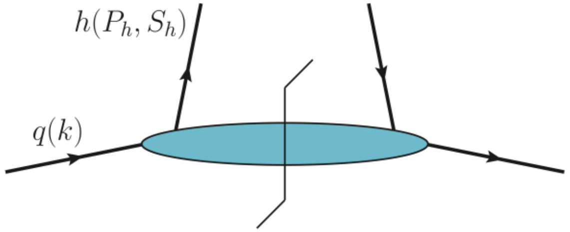
PDF　　pQCD　　FF

- $e^+e^- \to h + X$ (SIA), **pQCD** $\otimes$ **FF** [see Tongzhi's talk in this session]
- $pp \to h + X$, **PDF** $\otimes$ **PDF** $\otimes$ **pQCD** $\otimes$ **FF**

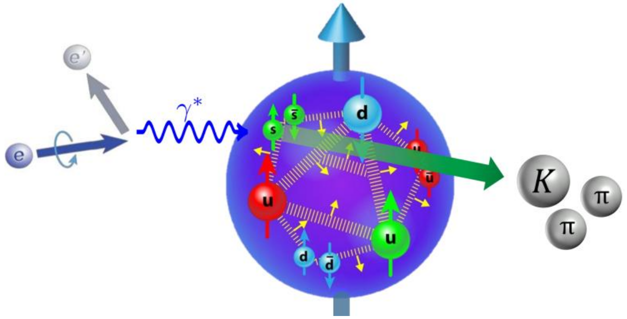
Determination of FFs from global data fit

❖ Field theory definiton of the collinear (integrated) quark FFs [Collins, Soper '82]

$$D_{h/q}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^-\xi^+/z}\,\mathrm{Tr}\Big[\langle 0|\mathcal{W}(\infty^+,\xi^+)\ \psi_q(\xi^+,0^-,\vec{0}_T)\,|\,P_h,S_h;X\rangle \times \langle P_h,S_h;X\,|\,\bar{\psi}_q(0^+,0^-,\vec{0}_T)\ \mathcal{W}(0^+,\infty^+)\,|\,0\rangle\ \gamma^-\Big]$$



❖ **Global data fits** based on factorization formula **ep(SIDIS) + ee(SIA) + pp collisions**



$$\frac{d^3\sigma_h}{dx_B\,dy\,dz_h} = f_{i/p}(x)\otimes\hat{\sigma}_{j\leftarrow i}(x,y,z)\otimes D_{h/j}(z)$$

measurement input/output pQCD to be determined. Universal !

Outline

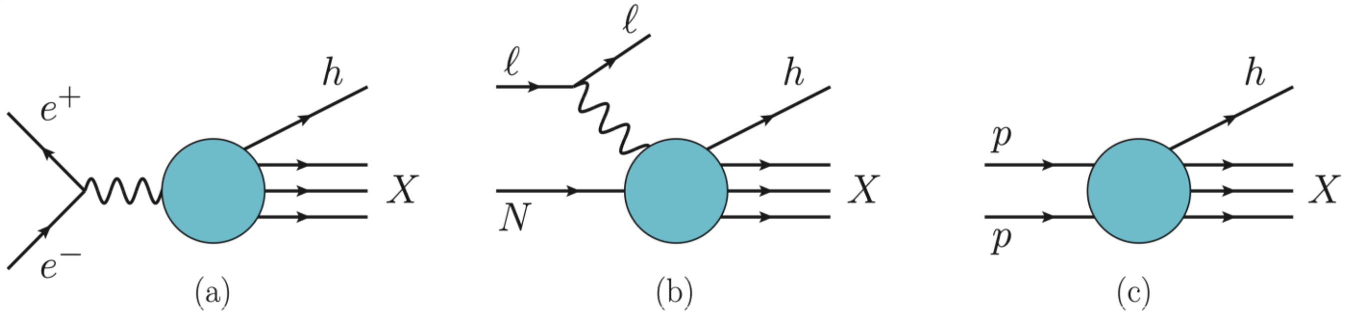
- Introduction
- Global fits of FFs at NLO from NPC
- Global fits of FFs at NNLO
- Impact of data from future lepton colliders on FFs

NPC=Non-Perturbative Physics Collaboration

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
hadron in jet	✗	✗	✗	✗	✗	✓
FFs (charged h)	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p	$\pi^{\pm}$	$\pi^{\pm}$,$K^{\pm}$	$\pi^{\pm}$,$K^{\pm}$,p
FFs (neutral h)			η			K^0,η,Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

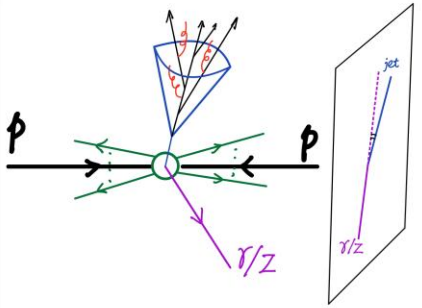
Only some of the recent global analyses are shown here.

NPC FFs analyses incorporate various types of data

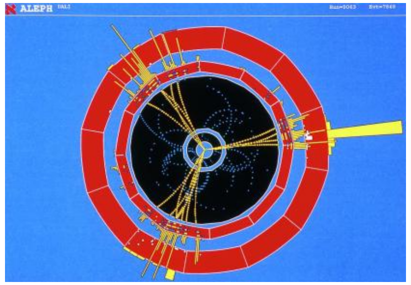


ee(SIA) ep(SIDIS) pp

single-inclusive hadron production



pp



ee

hadron-in-jet measurements

- Hadron-in-jet data provides direct probe of z dependence $$\frac{p_{T,h}}{p_{T,j}} \xrightarrow{\text{LO}} z$$

- All theoretical predictions calculated with **FMNLO.**

[Liu, **XS**, Zhou, Gao, 2305.14620 (JHEP)]

figure credit: A. Metz, A. Vossen,1607.02521; ALEPH Collaboration

Experiments	N_{pt}	χ^2	χ^2/N_{pt}
ATLAS jets †	446	350.8	0.79
ATLAS $Z/\gamma+$jet †	15	31.8	2.12
CMS $Z/\gamma+$jet †	15	17.3	1.15
LHCb $Z+$jet	20	30.6	1.53
ALICE inc. hadron	147	150.6	1.02
STAR inc. hadron	60	42.2	0.70
pp sum	703	623.3	0.89
TASSO	8	7.0	0.88
TPC	12	11.6	0.97
OPAL	20	16.3	0.81
OPAL (202 GeV) †	17	24.2	1.42
ALEPH	42	31.4	0.75
DELPHI	78	36.4	0.47
DELPHI (189 GeV)	9	15.3	1.70
SLD	198	211.6	1.07
SIA sum	384	353.8	0.92
H1 †	16	12.5	0.78
H1 (asy.) †	14	12.2	0.87
ZEUS †	32	65.5	2.05
COMPASS (06l)	124	107.3	0.87
COMPASS (16p)	97	56.8	0.59
SIDIS sum	283	254.4	0.90
Global total	1370	1231.5	0.90

NPC23 $\pi^{\pm}, K^{\pm}, p$ FFs fit

collaboration	year	$\sqrt{s}$[GeV]	χ^2	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$
TASSO	1985	14	5.65	9	0.63
TASSO	1985	22	5.87	6	0.98
TASSO	1985	34	16.03	13	1.23
TASSO	1990	14.8	12.56	9	1.40
TASSO	1990	21.5	3.78	6	0.63
TASSO	1990	34.5	17.51	13	1.35
TASSO	1990	35	14.76	13	1.14
TASSO	1990	42.6	33.60	13	2.58
TPC	1984	29	2.75	8	0.34
MARK II	1985	29	12.65	17	0.74
HRS	1987	29	33.16	12	2.76
CELLO	1990	35	2.71	9	0.30
TOPAZ	1995	58	0.29	4	0.07
OPAL	1991	91.2	7.75	7	1.11
OPAL	1995	91.2	13.63	16	0.85
OPAL	2000	91.2	8.62	16	0.54
ALEPH	1998	91.2	6.39	16	0.40
ALEPH	2000	91.2	12.72	14	0.91
ALEPH jet 1	2000	91.2	14.91	12	1.24
ALEPH jet 2	2000	91.2	8.21	13	0.63
ALEPH jet 3	2000	91.2	8.55	11	0.78
DELPHI	1995	91.2	7.55	13	0.58
SLD	1999	91.2	7.39	9	0.82
SLD c-tagged	1999	91.2	17.44	9	1.94
SLD b-tagged	1999	91.2	11.12	9	1.24
SIA sum			285.60	277	1.03
ZEUS $Q^2 \in 160, 640\,\text{GeV}^2$	2012	318	4.41	5	0.88
ZEUS $Q^2 \in 640, 2560\,\text{GeV}^2$	2012	318	3.26	5	0.65
ZEUS $Q^2 \in 2560, 10240\,\text{GeV}^2$	2012	318	2.74	2	1.37
SIDIS sum			10.41	12	0.87
ALICE $N^{13\,\text{TeV}}_{K^0_S}/N^{7\,\text{TeV}}_{K^0_S}$	2021	13000 & 7000	2.88	10	0.29
ALICE $N_{K^0_S}/N_{\pi^{\pm}}$	2021	13000	5.79	15	0.39
pp sum			8.67	25	0.35
total sum			**304.68**	**314**	**0.97**

NPC23 K^0 FFs fit

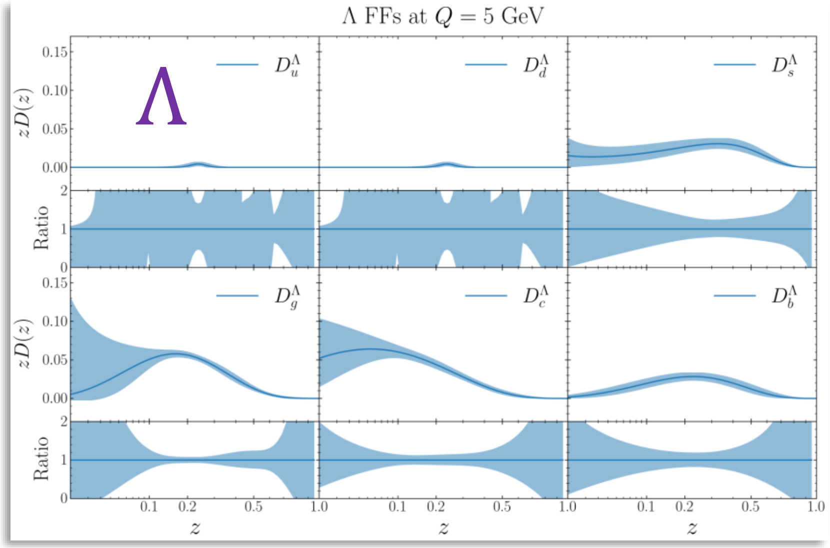
The NPC FF sets at NLO

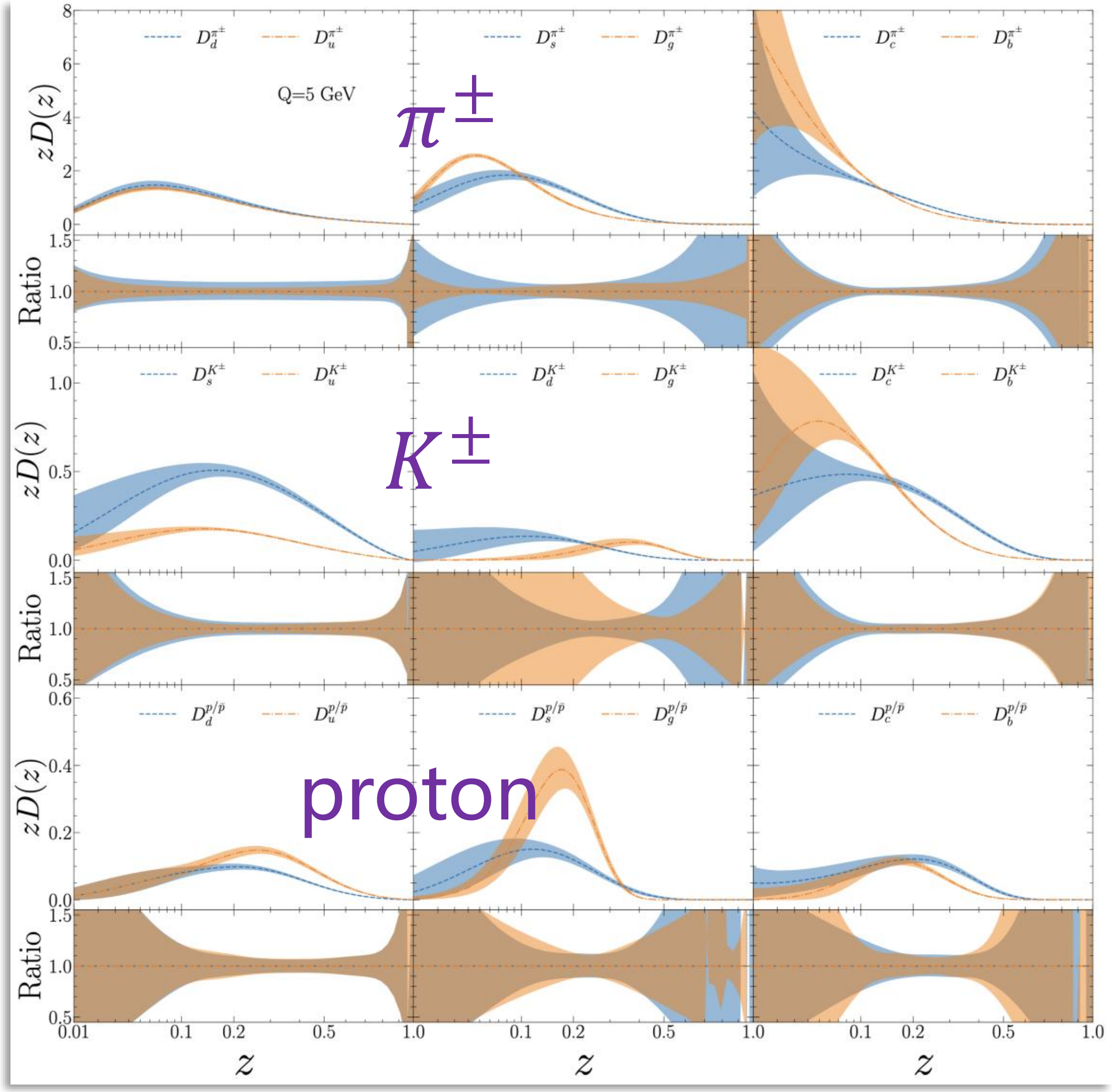
Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903,* '24
Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019,* '24 (Editors' suggestion)
Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045, '25*

… are publicly available

LHAPDF 6.5.5

Main page	PDF sets	Class hierarchy	Examples	More...
2070000	NPC23_PIp_nlo	(tarball)	(info file)	127
2070200	NPC23_KAp_nlo	(tarball)	(info file)	127
2070400	NPC23_PRp_nlo	(tarball)	(info file)	127
2070600	NPC23_PIm_nlo	(tarball)	(info file)	127
2070800	NPC23_KAm_nlo	(tarball)	(info file)	127
2071000	NPC23_PRm_nlo	(tarball)	(info file)	127
2071200	NPC23_PIsum_nlo	(tarball)	(info file)	127
2071400	NPC23_KAsum_nlo	(tarball)	(info file)	127
2071600	NPC23_PRsum_nlo	(tarball)	(info file)	127
2071800	NPC23_CHHAp_nlo	(tarball)	(info file)	127
2072000	NPC23_CHHAm_nlo	(tarball)	(info file)	127
2072200	NPC23_CHHAsum_nlo	(tarball)	(info file)	127





Combination of all types of hadron production data leads to **good constraints** on FFs.

Both charged and neutral hadron FFs determined

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓
hadron in jet	✗	✗	✗	✗	✗	✓
FFs (charged h)	$\pi^\pm, K^\pm, p$	$\pi^\pm, K^\pm$	$\pi^\pm, K^\pm, p$	$\pi^\pm$	$\pi^\pm, K^\pm$	$\pi^\pm, K^\pm, p$
FFs (neutral h)			η			K^0, η, Λ
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO

- FFs determination at NLO from Nonperturbative Physics Collaboration (NPC)
  - NPC23 FFs to light **charged** hadrons:

    Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903,* '24
    Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019,* '24 (Editors' suggestion)

  - NPC23 FFs to light **neutral** hadrons:

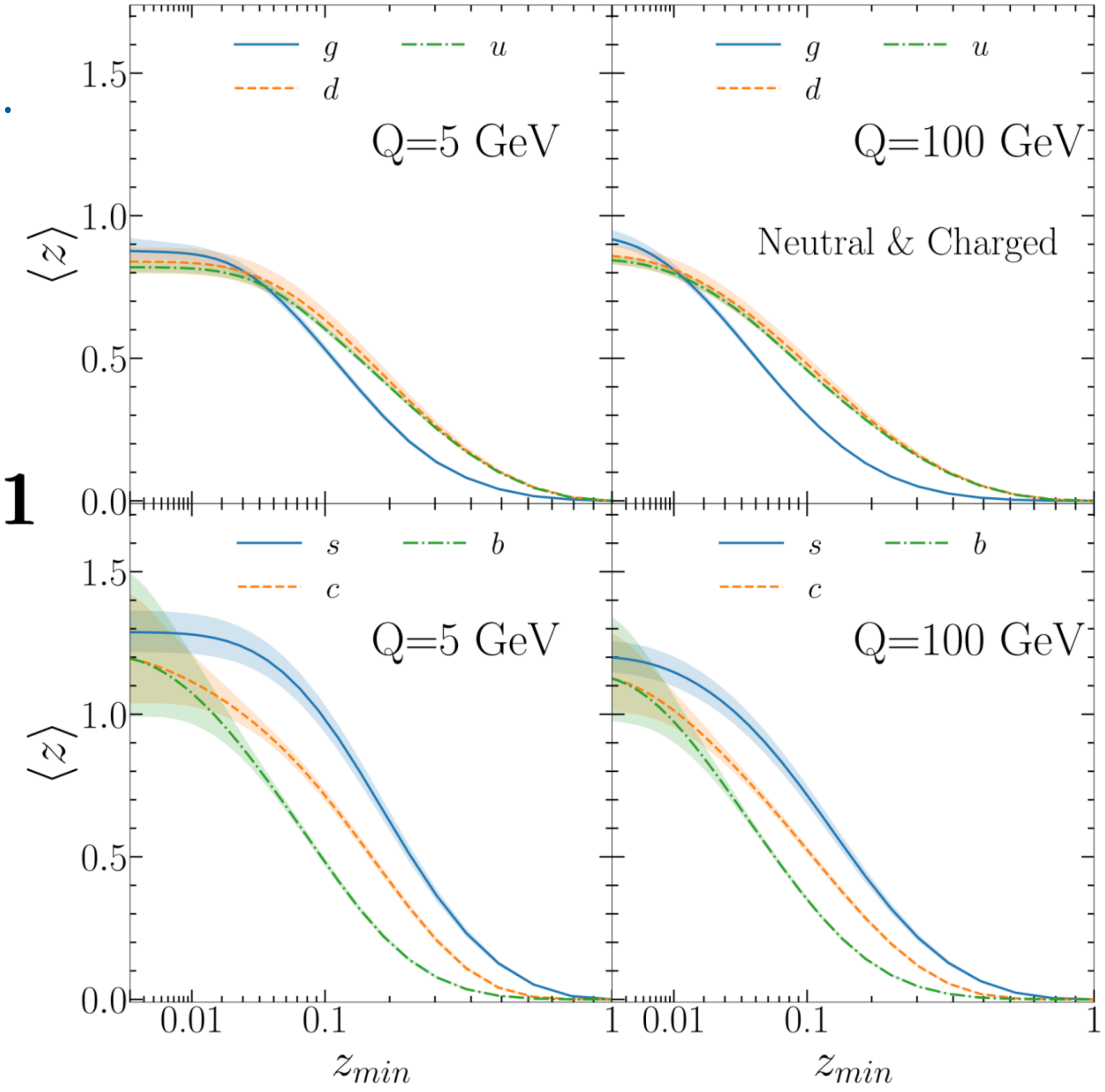
    Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045, '25*

Test sum rule using neutral + charged hadron FFs

parton i ➡ hadrons $h = \pi^{\pm}, \pi^0, K^{\pm}, K^0, \cdots$

- The **momentum sum rule**:

$$\lim_{z_{\min}\to 0} \sum_h \int_{z_{min}}^{1} \left[z D_{h/i}(z) \right] \mathrm{d}z = 1$$



Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903, '24*
Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045, '25*

Outline

- Introduction
- NPC analyses of FFs at NLO
- NPC analyses of FFs at NNLO + constraints on PDFs
- Impact of data from future colliders

collaboration	NNFF	JAM	DSS+	BDSSV	MAP	NPC	NPC
SIA (ee)	✓	✓	✓	✓	✓	✓	✓
SIDIS (ep)	✗	✓	✓	✓	✓	✓	✓
pp incl. hadron	✗	✗	✓	✗	✗	✓	✗
hadron in jet	✗	✗	✗	✗	✗	✓	✗
FFs	$\pi^\pm$,$K^\pm$,p	$\pi^\pm$,$K^\pm$	$\pi^\pm$,$K^\pm$,p,$h^\pm$ η	$\pi^\pm$	$\pi^\pm$,$K^\pm$	$\pi^\pm$,$K^\pm$,p,$h^\pm$ K^0,η,Λ	$\pi^\pm$,$K^\pm$
pQCD order	NNLO	NLO	NLO	appr. NNLO	appr. NNLO	NLO	NNLO

[Gao, **XS**, Xing, Zhao, Zhou, ***PRL*** *135, 041902*, 2025]

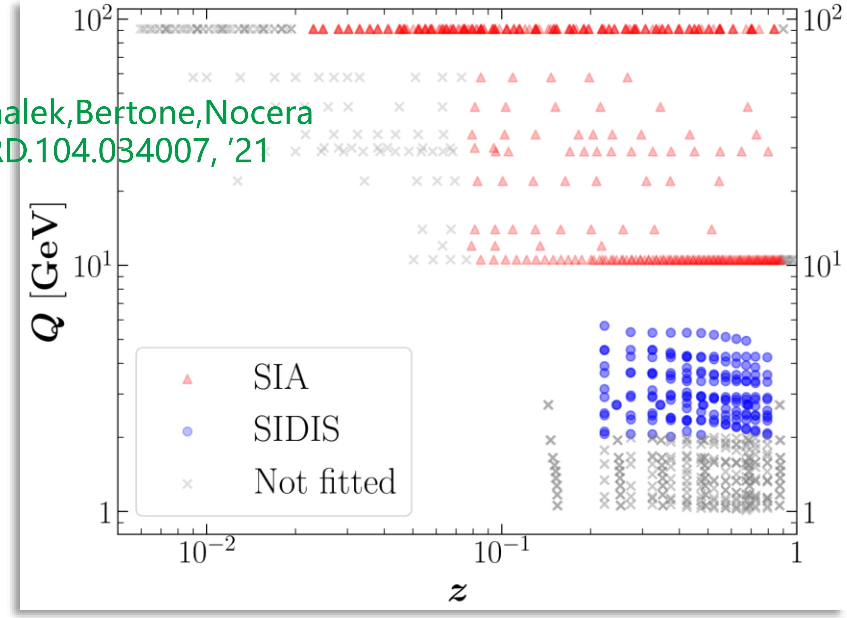
Global analysis of FFs at full NNLO: the datasets

SIA(e^+e^-) data used in the fit:

exp.	$\sqrt{s}$/GeV	lum.(n_Z)	year	final states	hadrons
DELPHI	189	157.7 pb^{-1}	2002	inc. had.	$\pi^\pm, K^\pm$
OPAL	m_Z	780 000	1994	$Z \to q\bar{q}$	$\pi^\pm, K^\pm$
ALEPH	m_Z	520 000	1995	$Z \to q\bar{q}$	$\pi^\pm, K^\pm$
DELPHI	m_Z	1 400 000	1998	$Z \to q\bar{q}$	$\pi^\pm, K^\pm$
				$Z \to b\bar{b}$	$\pi^\pm, K^\pm$
SLD	m_Z	400 000	2004	$Z \to q\bar{q}$	$\pi^\pm, K^\pm$
				$Z \to b\bar{b}$	$\pi^\pm, K^\pm$
				$Z \to c\bar{c}$	$\pi^\pm, K^\pm$
TASSO	44	34 pb^{-1}	1989	inc. had.	$\pi^\pm, \pi^0$
TASSO	34	77 pb^{-1}	1989	inc. had.	$\pi^\pm, K^\pm$
TPC/2γ	29	70 pb^{-1}	1988	inc. had.	$\pi^\pm, K^\pm$
Belle	10.52	68 fb^{-1}	2013	inc. had.	$\pi^\pm, K^\pm$
BaBar	10.54	0.91 fb^{-1}	2013	inc. had.	$\pi^\pm, K^\pm$
BESIII	2.0-3.671	253 pb^{-1}	2025	inc. had.	$\pi^\pm, K^\pm$

[BESIII, **PRL**135, 151901, 2025]

Khalek,Bertone,Nocera
PRD.104.034007, '21



SIA(e^+e^-)

SIDIS / BESIII

- Kinematic cuts in our analyses:
  - $Q > 3$ GeV (SIA)
  - $Q > \ 2$ GeV(SIDIS)
  - $z > 0.01, E_h > E_{h,min}$(0.8 GeV by default)

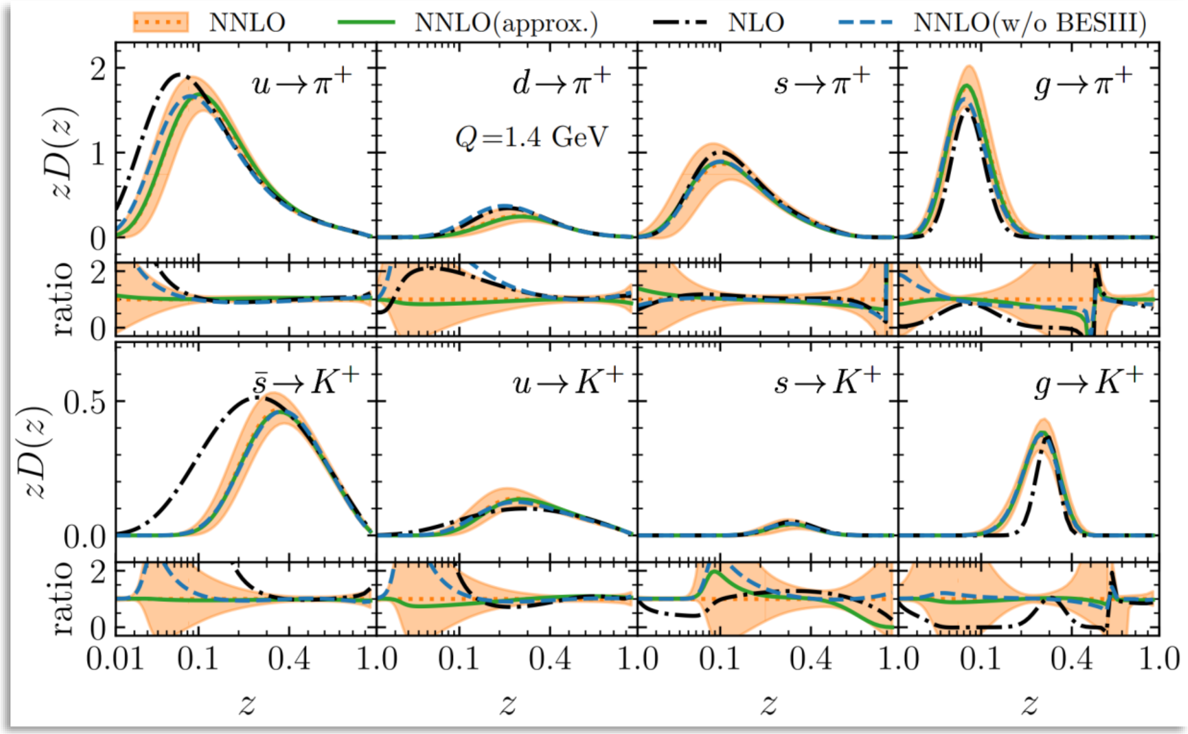
the **first** test on **universality of FFs** at Q~3 GeV using both **ee** and **SIDIS** data

Global analysis of FFs at full NNLO: results

➢ Fit quality of the NNLO analyses

	BESIII		COMPASS		B-factories		HE-SIA		global		
$E_{h,\min}$[GeV]	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$	$N_{\rm pt}$	χ^2	$\chi^2/N_{\rm pt}$
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

energy cut of the identified hadron



LHAgrids of our FFs have been submitted to the LHAPDF repository.

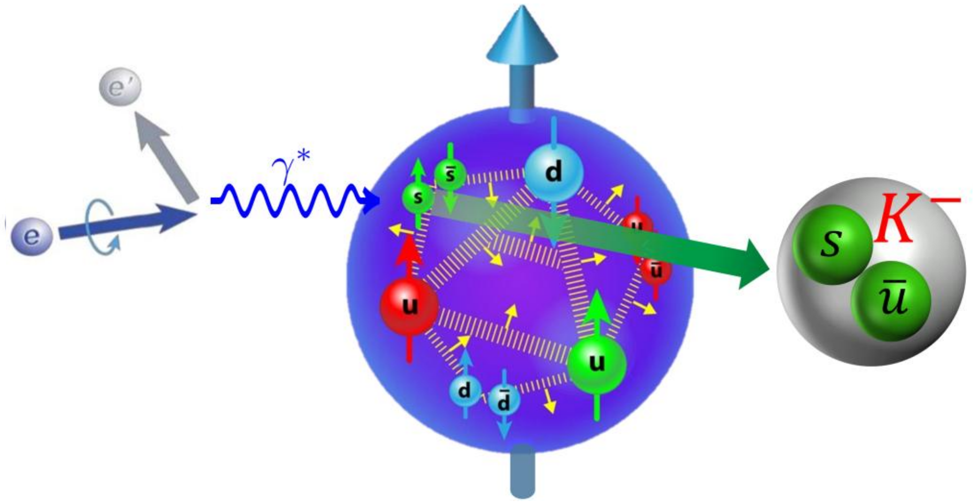
https://www.lhapdf.org/pdfsets.html

Application: constraining proton PDFs at NNLO

SIDIS may also constrain PDFs:

$$\frac{d^3\sigma_h}{dx_B dy dz_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x, y, z) \otimes D_{h/j}(z)$$

unpolarized PDF FF



- LO xsec of SIDIS off an isoscalar target (COMPASS)

$$\frac{d\sigma^{K^+}}{dxdydz} - \frac{d\sigma^{K^-}}{dxdydz}$$
$$\sim 2\Big(u_v(x) + d_v(x)\Big)\Big(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z)\Big)$$
$$+\Big(s(x) - \bar{s}(x)\Big)\Big(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z)\Big) + \cdots$$

PDF FF

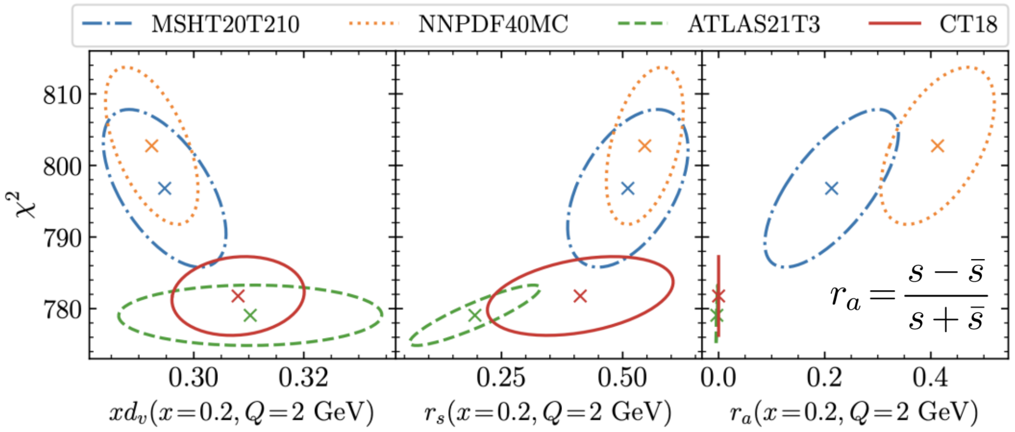
is sensitive to **strangeness asymmetry**

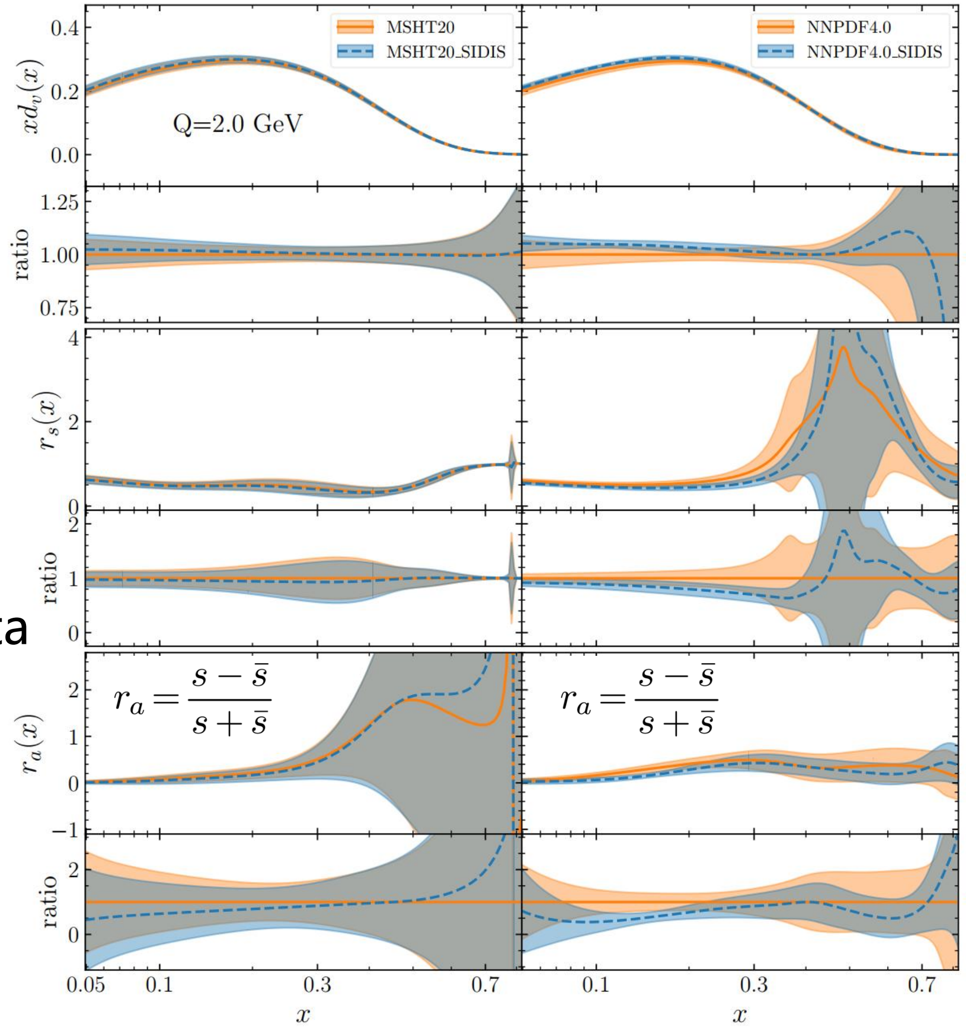
$$r_a = \frac{s - \bar{s}}{s + \bar{s}}$$

Application: constraining proton PDFs at NNLO

- Correlation between χ^2 and PDFs

[Gao, **XS**, Xing, Zhao, Zhou, ***PRL 135, 041902***, 2025]



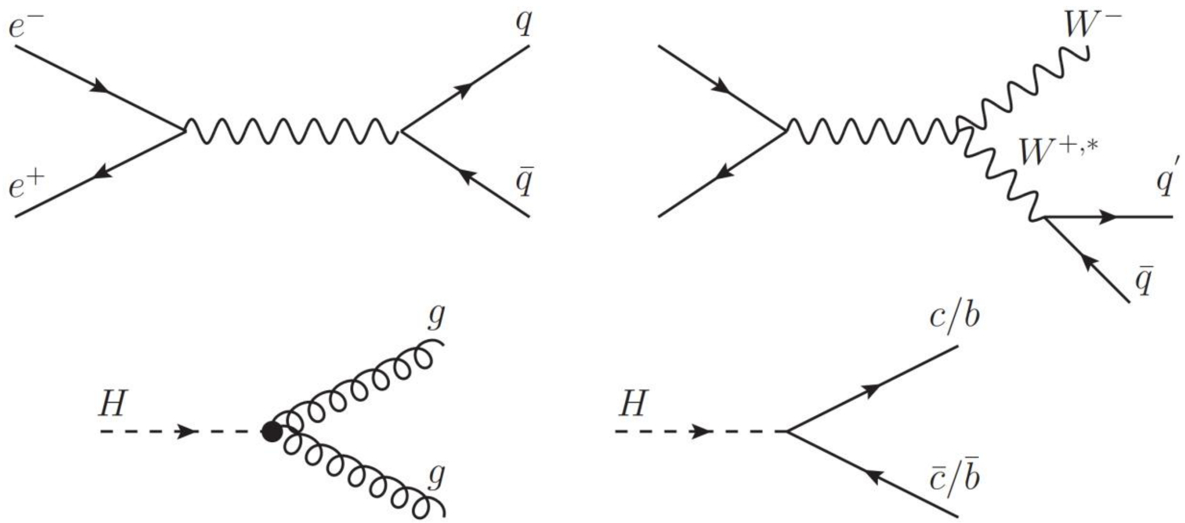


- **Modified PDFs** which reflect the impact of SIDIS data
  - Reweighting of the NNPDF4.0 PDF set
  - Profiling of the MSHT20 PDF set

PDF sets before and after reweighting/profiling

Outline

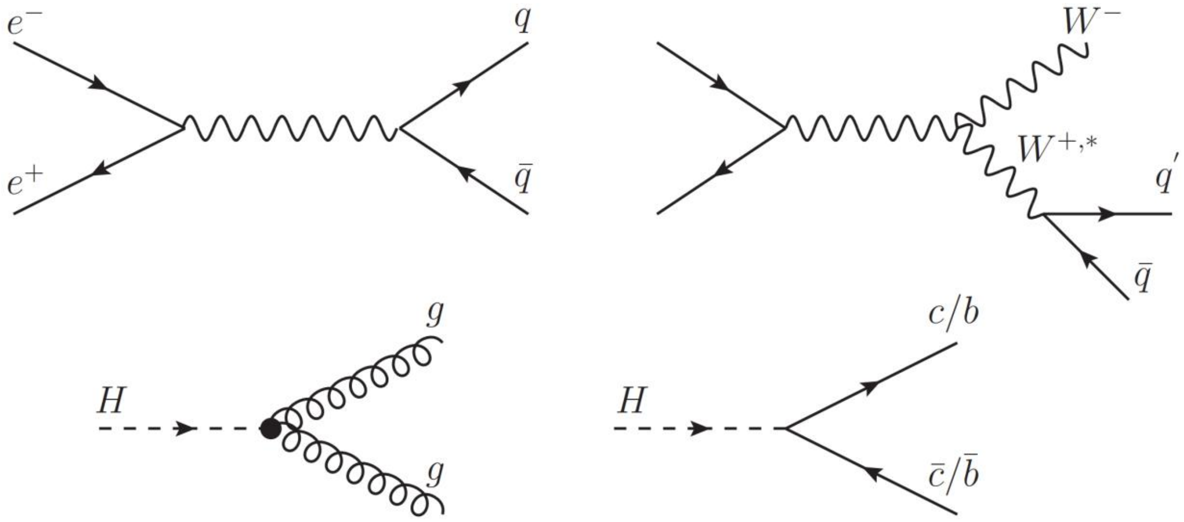
- Introduction
- NPC analyses of FFs at NLO
- NPC analyses of FFs at NNLO + constraints on PDFs
- Impact of data from future lepton colliders (CEPC, FCC-ee, ILC) on FFs



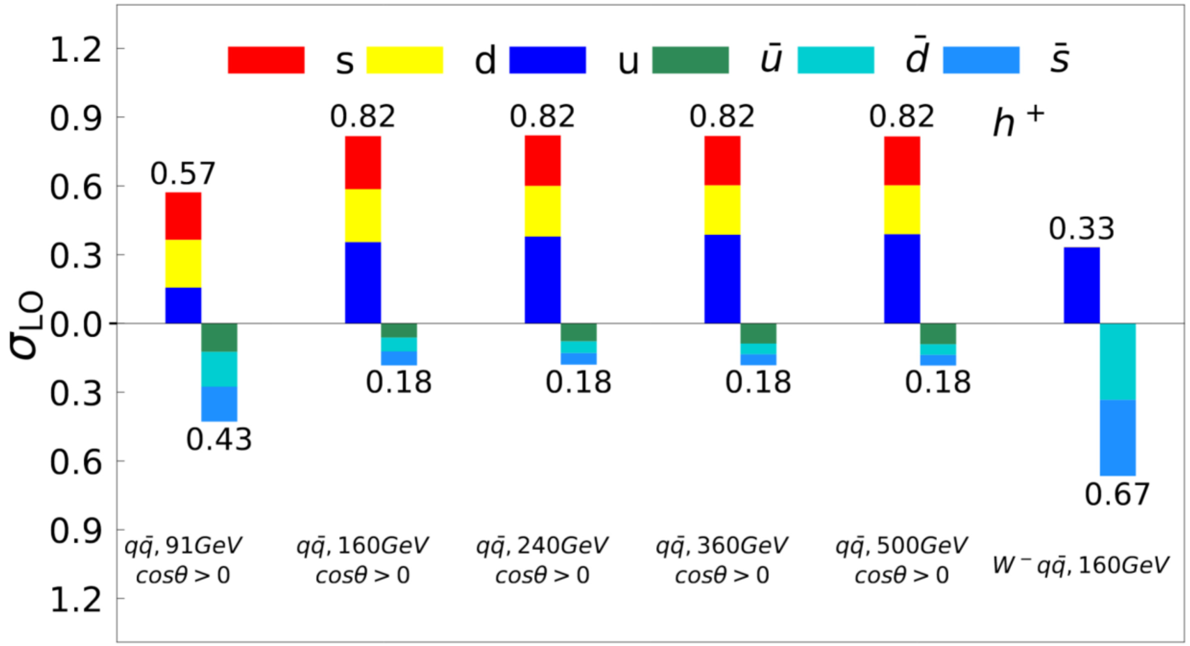
[Jun Gao, Bin Zhou, JHEP 02 (2025) 003]

Opportunities with future lepton colliders

… for flavor separation and determination of gluon FFs



Processes sensitive to FFs



relative size of light (anti)quark productions rate

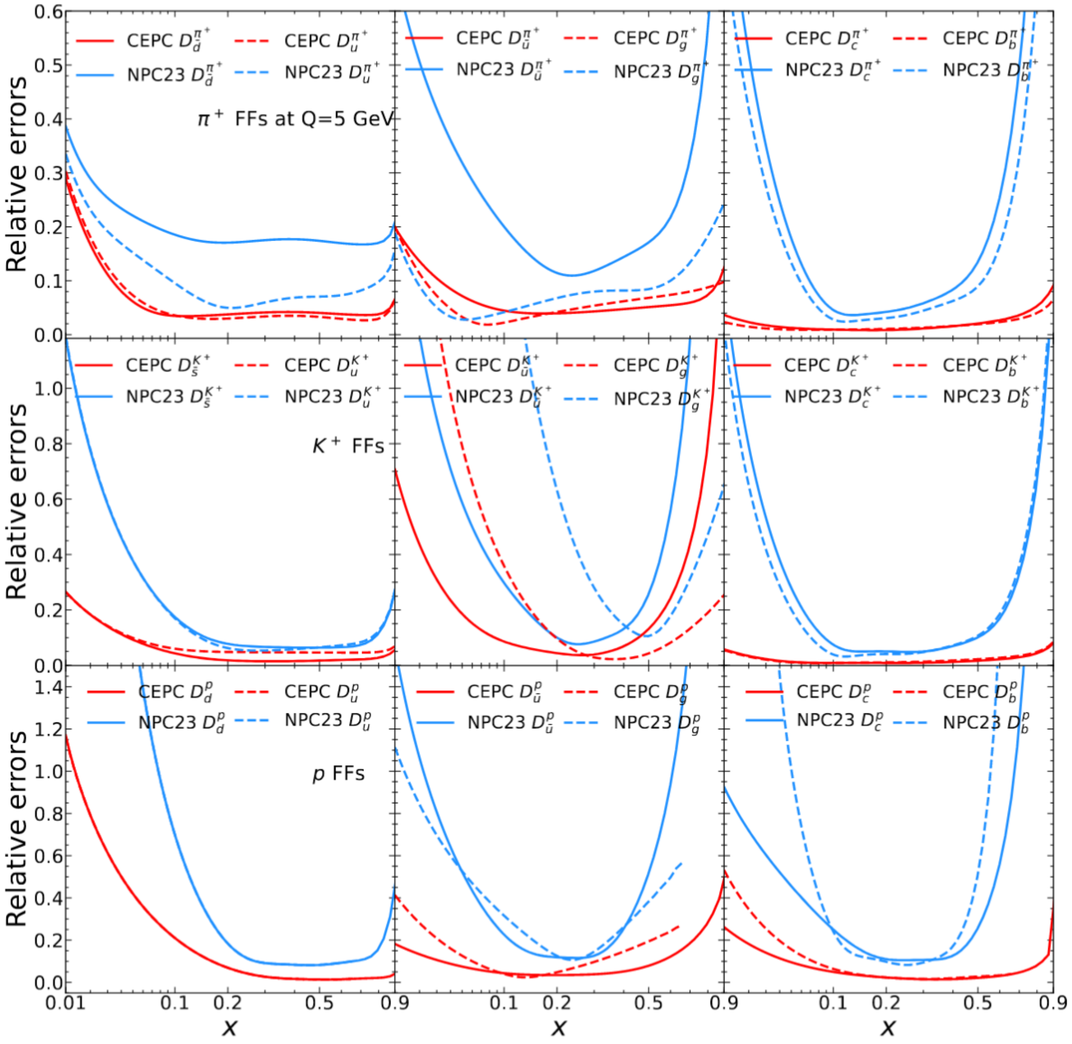
FFs determination using only pseudo-data at future ee colliders

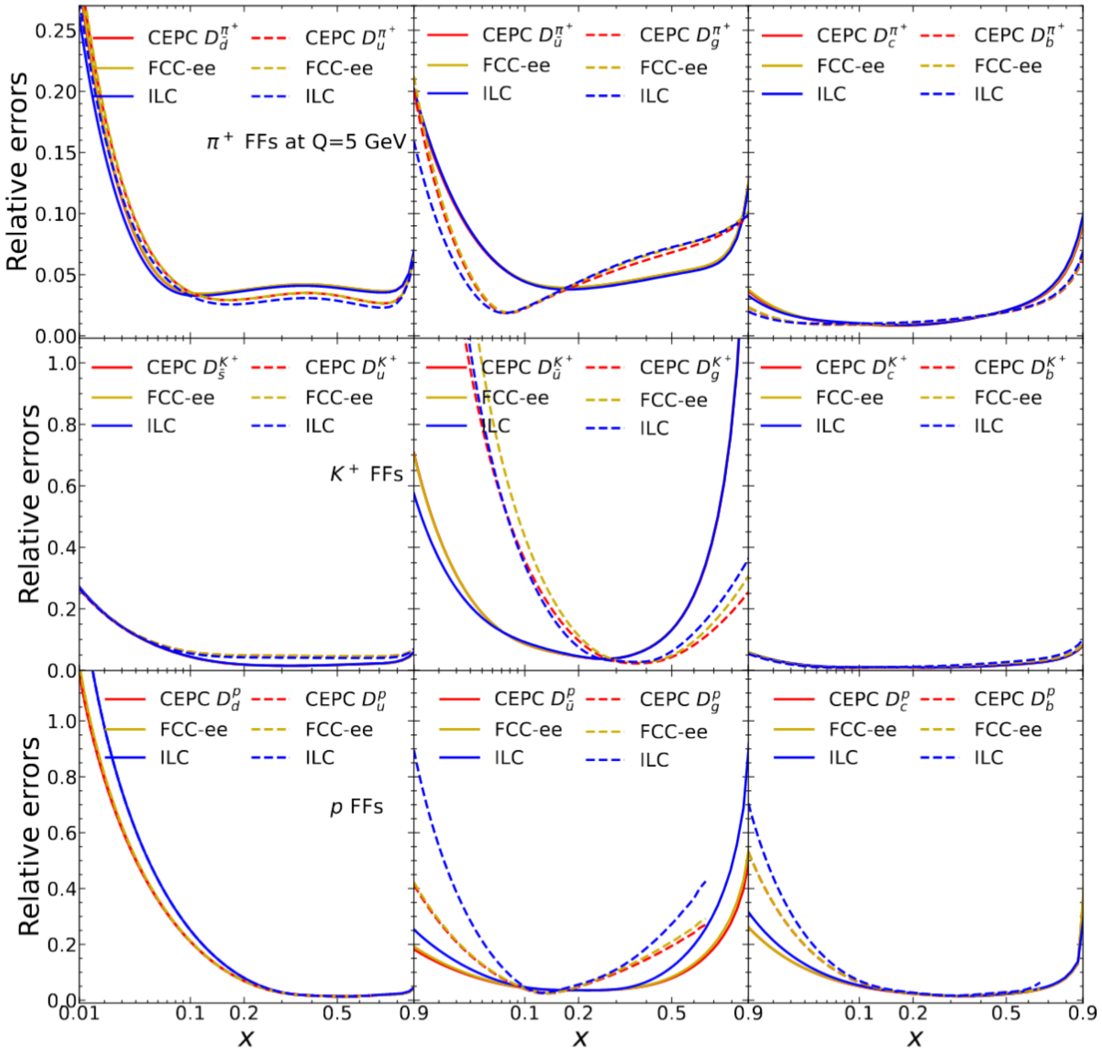
e^+e^- annihilation							
$\sqrt{s}$ (GeV)	luminosity (ab^{-1})			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC-ee	ILC				
91.2	60	150	-	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	132
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	65
160	4.2	-	-	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	168
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	83
161	-	10	-	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	168
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	83
240	13	5	-	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	186
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	92
250	-	-	2	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	186
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	92
350	-	0.2	0.2	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	198
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	98
360	0.65	-	-	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	198
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	98
365	-	1.5	-	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	198
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	98
500	-	-	4	$q\bar{q}$	$\cos(\theta) > 0$	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	198
				$c\bar{c}/b\bar{b}$	-	$\pi^\pm, K^\pm, p/\bar{p}$	98
W boson decay channels							
$\sqrt{s}$ (GeV)	# events (million)			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC-ee	ILC				
80.419	116	68	62	$W^- W^{+*} \to W^- q\bar{q}$	-	$\pi^+, \pi^-, K^+, K^-, p, \bar{p}$	120
	58	34	31	$W^- W^{+*} \to W^- c\bar{s}$			
Higgs boson decay channels							
$\sqrt{s}$ (GeV)	# events (million)			final state	kinematic cuts	hadrons	N_{pt}
	CEPC	FCC-ee	ILC				
125	0.23	0.09	0.07	gg	-	$\pi^\pm, K^\pm, p/\bar{p}$	77
	0.08	0.03	0.02	$c\bar{c}$			
	1.53	0.59	0.47	$b\bar{b}$			

- binning (in momentum fraction): same as SLD measurement at Z pole [PRD 59,052001]
- central values calculated using the NPC23 FFs
- statistical uncertainties evaluated from the expected number of events
- systematic uncertainties set to baseline measurement of SLD at Z pole.

Significant reduction of FFs uncertainties observed

[Jun Gao, Bin Zhou, JHEP 02 (2025) 003]





Summary

- NPC collaboration has delivered precise and comprehensive FF sets at NLO.
- We present the **first** global (SIA+SIDIS) FFs determination at full NNLO.
- CEPC can significantly reduce the uncertainties of FFs in a wide kinematic range.

collaboration	NPC	NPC
SIA (ee)	✓	✓
SIDIS (ep)	✓	✓
pp incl. hadron	✓	✗
hadron in jet	✓	✗
FFs	$\pi^\pm$,$K^\pm$,p K^0,η,Λ	$\pi^\pm$,$K^\pm$
pQCD order	NLO	NNLO

FF sets from NPC available from https://www.lhapdf.org/pdfsets.html

NLO charged hadron:

Gao, Liu, **XS**, Xing, Zhao, ***PRL*** *132, 261903*, 2024

Gao, Liu, **XS**, Xing, Zhao, *PRD 110, 114019*, 2024

NLO neutral hadron:

Gao, Liu, Li, **XS**, Xing, Zhao, Zhou, *PRD 112, 054045*, 2025

NNLO:

Gao, **XS**, Xing, Zhao, Zhou, ***PRL*** *135, 041902*, 2025

Thank you for your attention!

Backup slides

NPC collaboration gathering on July 19th 2025

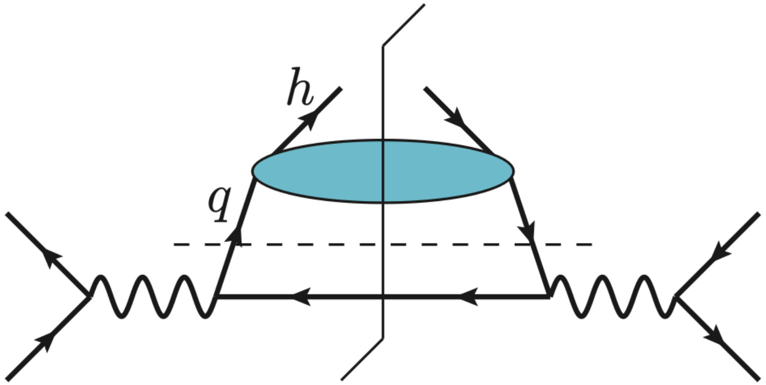
in neighborhood of Huizhou city (host of EICc)

Jun Gao, ChongYang Liu, Meng Yang Li, **XiaoMin Shen**, **HongXi Xing**, **YuXiang Zhao**, Bin Zhou, YiYu Zhou
Shanghai JiaoTong Univ., South China Normal Univ., Institute of Modern Physics, CAS

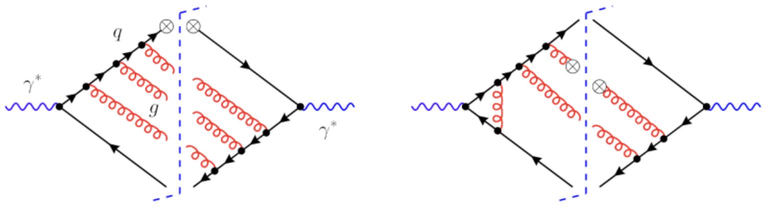
NPC=Non-Perturbative Physics Collaboration

Slide from Jun Gao

Recent progresses from pQCD

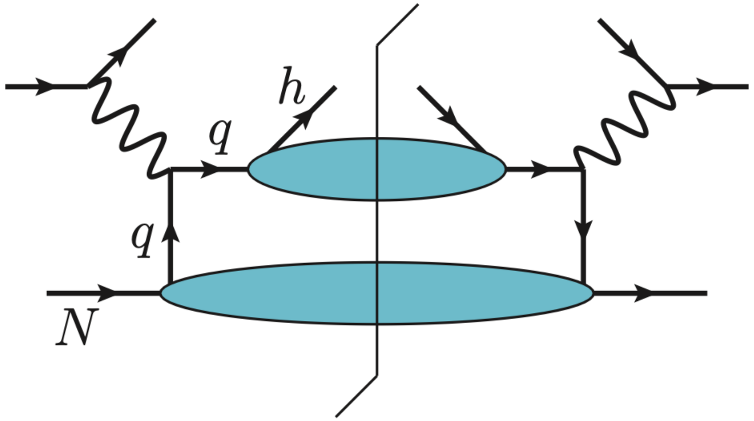


❖SIA(e^+e^-) at N3LO



[He, Xing, Yang, Zhu, PRL.135.101901(2025)]

See Tongzhi Yang's talk today

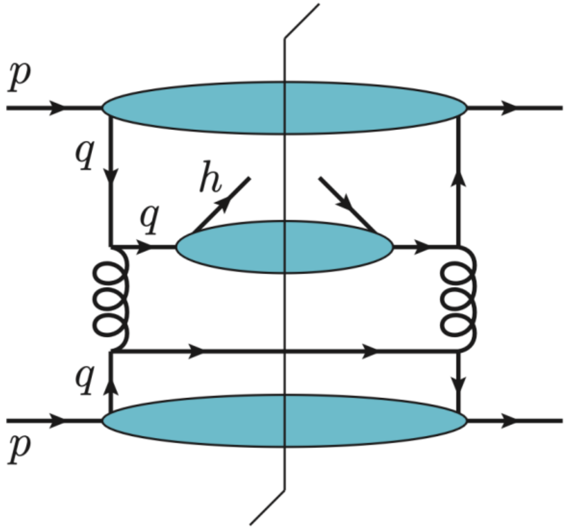


❖ SIDIS(ep) at NNLO

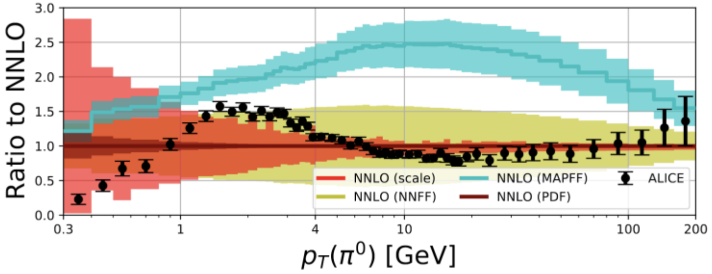
[Bonino, Gehrmann, et al.

& Goyal, Moch, et al.

PRL.132.251901, '24, PRL.132.251902, '24, PRL.133.211904, '24, PRL.133.211905, '24, 2504.05376]



❖ pp at NNLO



[Czakon, Generet, Mitov, Poncelet, 2503.11489]

Figure credit: A. Metz, A. Vossen,1607.02521

Global analysis of FFs at full NNLO: theoretical prediction

FFs at starting scale $Q_0 = 1.4$ GeV

- parameterized as

$$zD_i^h(z,Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h}\exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

- charge/isospin symmetries suppress number of free parameters (54 in total)

$$D_{u\to\pi^+}(z,Q) = D_{\bar{u}\to\pi^-}(z,Q)$$

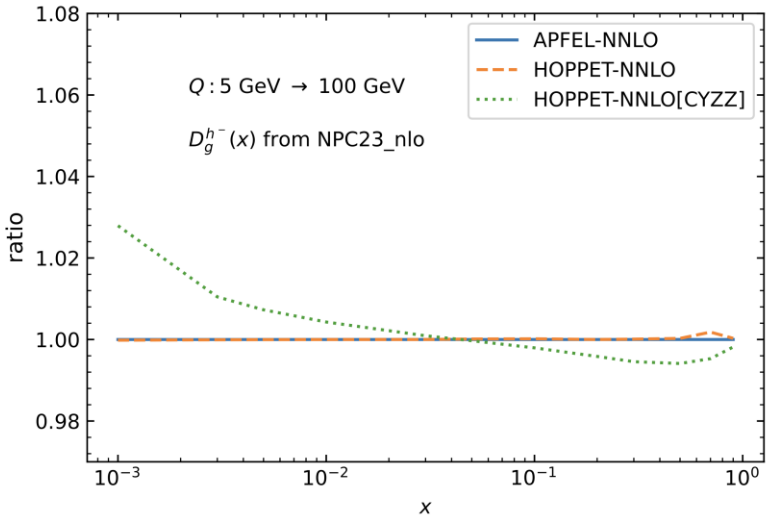
$$D_{u\to\pi^+}(z,Q_0) = D_{\bar{d}\to\pi^+}(z,Q_0)$$

FFs at arbitrary energy scale ($Q_0 \to Q$)

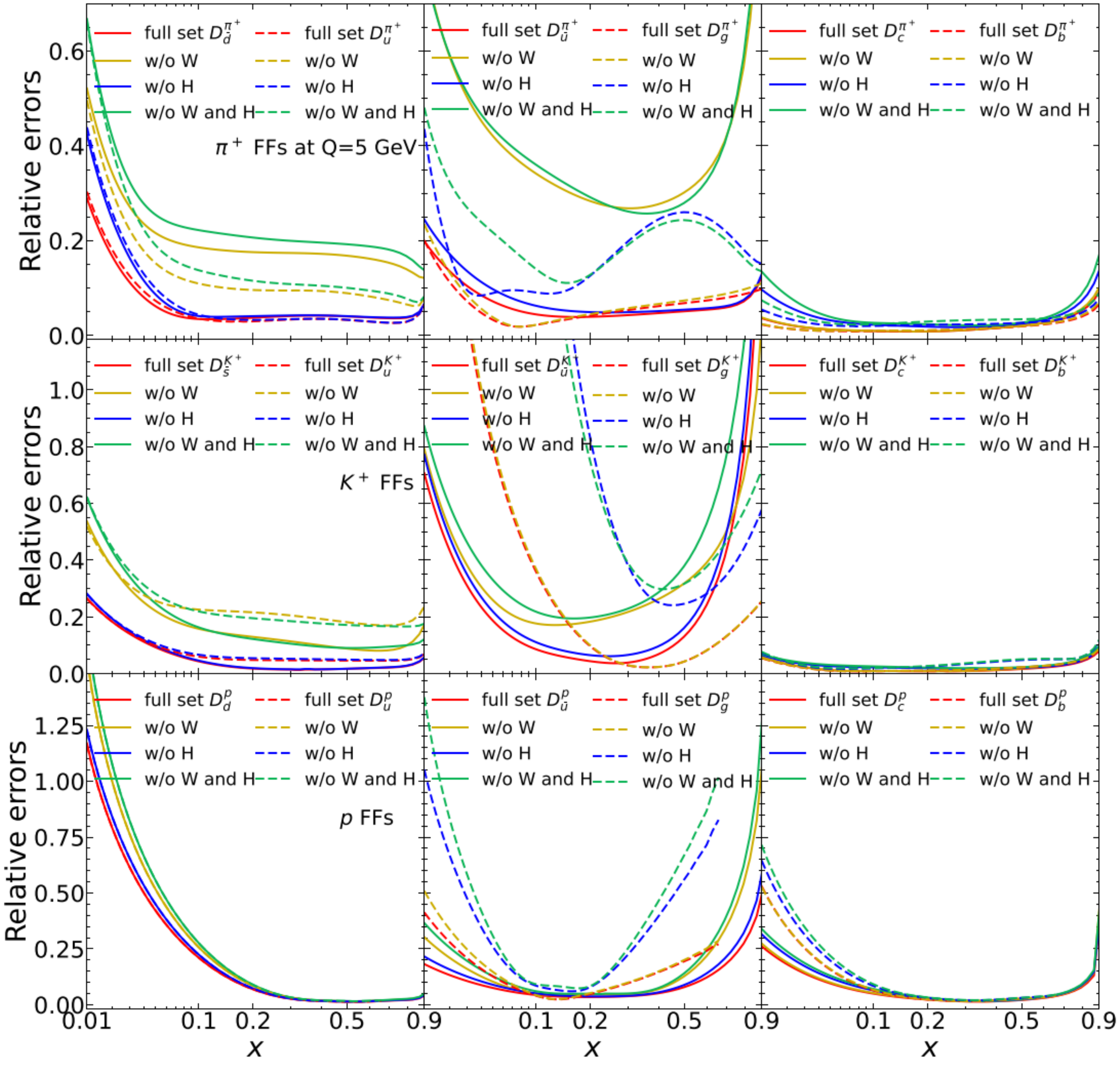
- 3-loop timelike DGLAP evolution

  [Mitov, Moch, Vogt, Almasy]

- $+P_{qg}^{T(2)}$correction[Chen, Yang, Zhu, Zhu, '20]



- Heavy quark FFs are frozen below mass threshold.



full set $D_d^{\pi^+}$
full set $D_u^{\pi^+}$
w/o W
w/o H
w/o W and H
π^+ FFs at Q=5 GeV
full set $D_{\bar{u}}^{\pi^+}$
full set $D_g^{\pi^+}$
full set $D_c^{\pi^+}$
full set $D_b^{\pi^+}$
full set $D_{\bar{s}}^{K^+}$
full set $D_u^{K^+}$
K^+ FFs
full set $D_{\bar{u}}^{K^+}$
full set $D_g^{K^+}$
full set $D_c^{K^+}$
full set $D_b^{K^+}$
full set D_d^{p}
full set D_u^{p}
p FFs
full set $D_{\bar{u}}^{p}$
full set D_g^{p}
full set D_c^{p}
full set D_b^{p}
Relative errors
x