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Determination of the Strong Coupling Constant from Inclusive Semi-leptonic B Meson Decays

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arXiv:2412.02480,

with Long Chen, Jinfei Wu, Xinchou Lou, Xiang Chen, Xin Guan, Yan-Qing Ma, Manqi Ruan.

The 2025 International Workshop on the High Energy Circular Electron Positron Collider

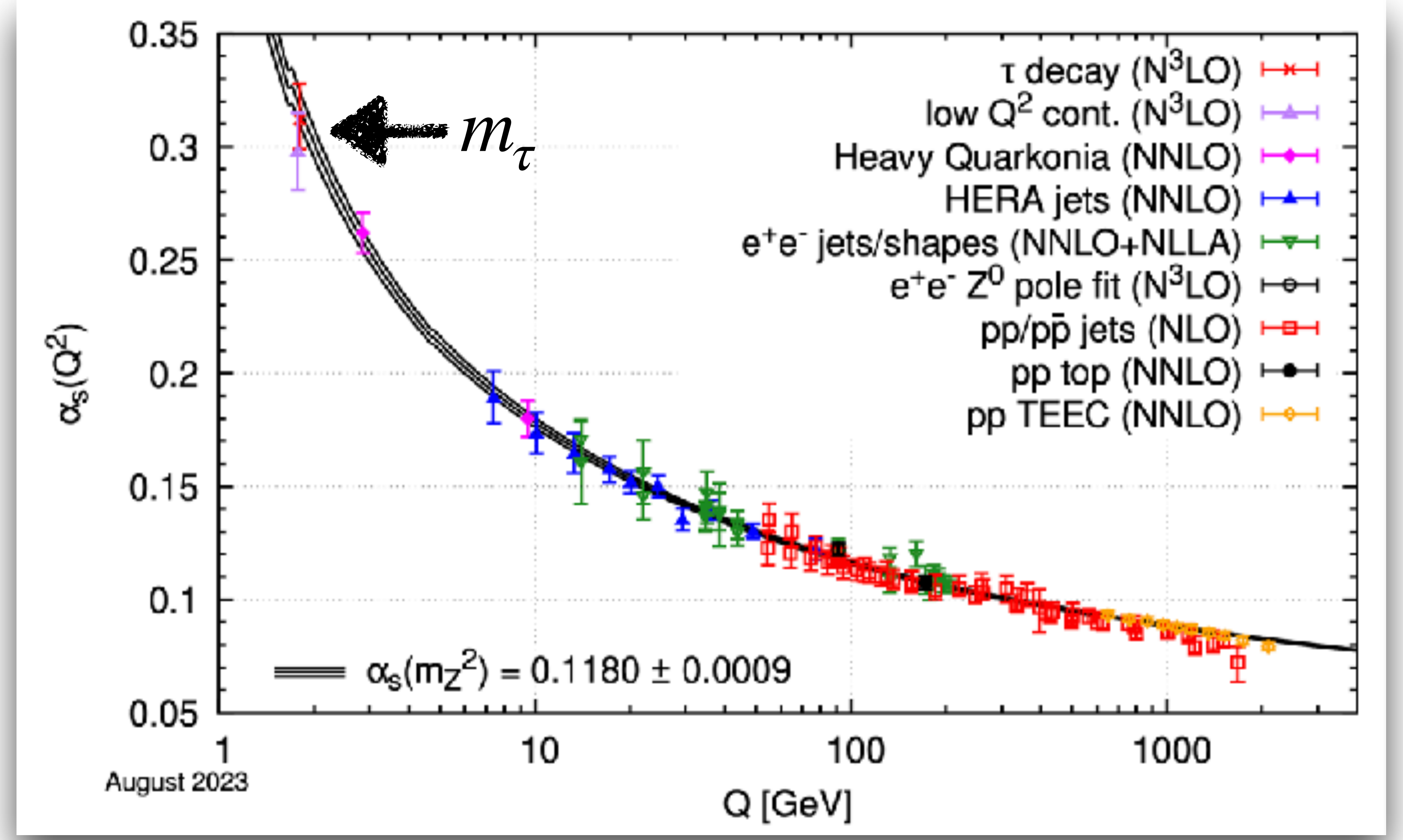
The strong coupling constant, α_s

- Strong coupling constant α_s
  - encodes the strength of strong interaction,
  - is a fundamental parameter in the Standard Model (SM) and the Quantum Chromodynamics (QCD).
- However, the α_s is less understood compared with the other couplings.

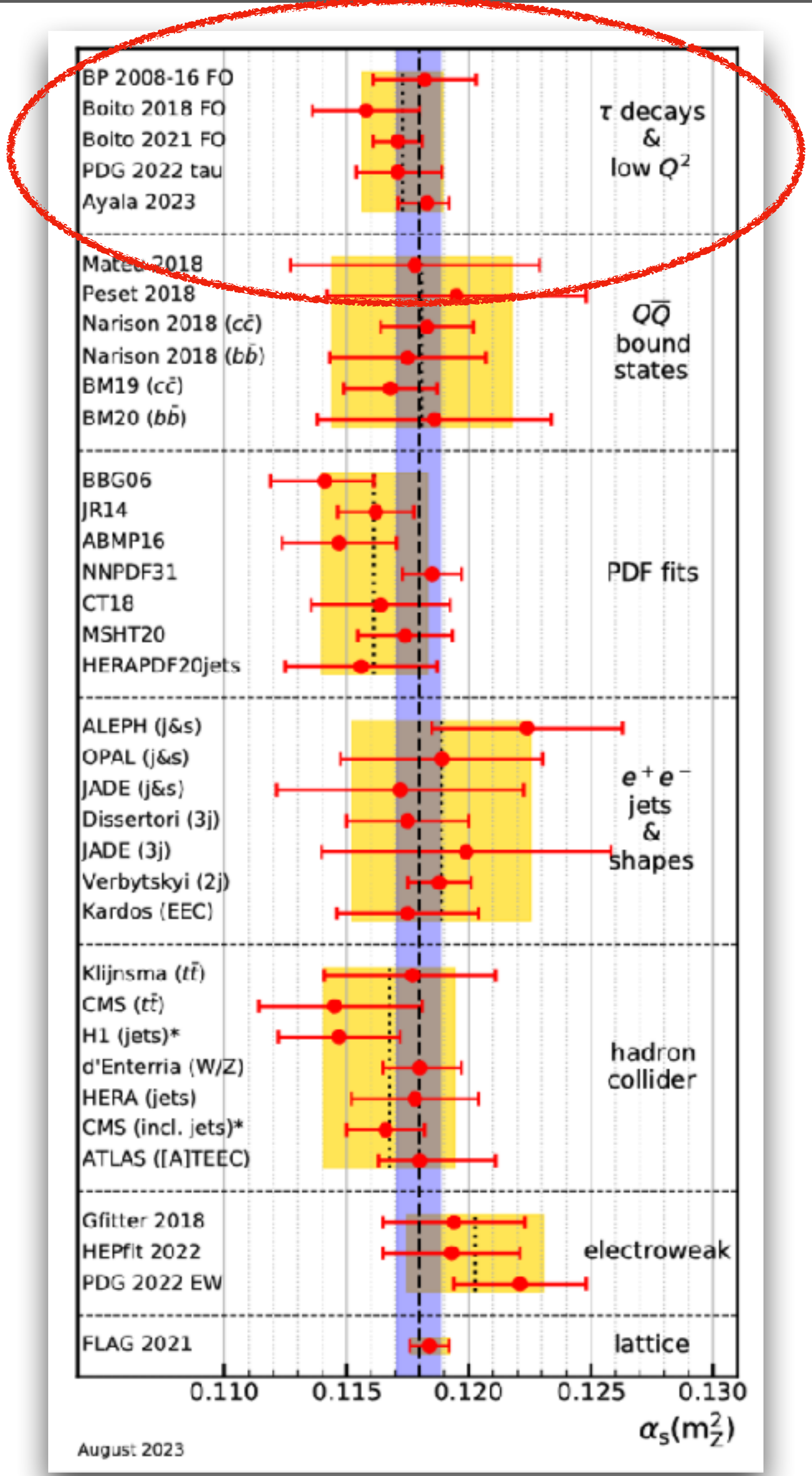
α	$7.2973525693(11) \times 10^{-3}$	1.5×10^{-10}
$G_F/(\hbar c)^3$	$1.1663788(6) \times 10^{-5}$ GeV^{-2}	5.1×10^{-7}
m_Z	91.1880(20) GeV/c^2	2.2×10^{-5}
m_W	80.3692(133) GeV/c^2	1.7×10^{-4}
$\alpha_s(m_Z^2)$	0.1180(9)	7.6×10^{-3}
m_H	125.20(11) GeV/c^2	8.7×10^{-4}
m_e	0.51099895000(15) MeV/c^2	2.9×10^{-10}
m_μ	0.1056583755(23) GeV/c^2	2.1×10^{-8}
m_τ	1.77693(9) GeV/c^2	5.1×10^{-5}
$\overline{m}_u$(2 GeV)	2.16(7) MeV/c^2	3.2×10^{-2}
$\overline{m}_d$(2 GeV)	4.70(7) MeV/c^2	1.5×10^{-2}
$\overline{m}_s$(2 GeV)	93.5(8) MeV/c^2	8.6×10^{-3}
$\overline{m}_c(\overline{m}_c)$	1.2730(46) GeV/c^2	3.6×10^{-3}
$\overline{m}_b(\overline{m}_b)$	4.183(7) GeV/c^2	1.7×10^{-3}
m_t	172.57(29) GeV/c^2	1.7×10^{-3}

The strong coupling constant, α_s

- The value of α_s decreases with the increasing energy scale μ, described by the Renormalization Group Equation (RGE).
- This running behavior corresponds to fundamental properties of asymptotic freedom and color confinement.
  - ***Measuring $\alpha_s(\mu)$ over a wide range of energy scale is crucial for understanding and testing QCD.***
- An uncertainty δ on a measurement of $\alpha_s(\mu)$, at a scale μ, translates to an uncertainty $\delta' = (\alpha_s^2(m_Z)/\alpha_s^2(\mu)) \cdot \delta$.
  - ***Determining $\alpha_s(\mu)$ in the low energy scale provides precise $\alpha_s(m_Z)$.***



Phys.Rev.D 110 (2024) 3, 030001



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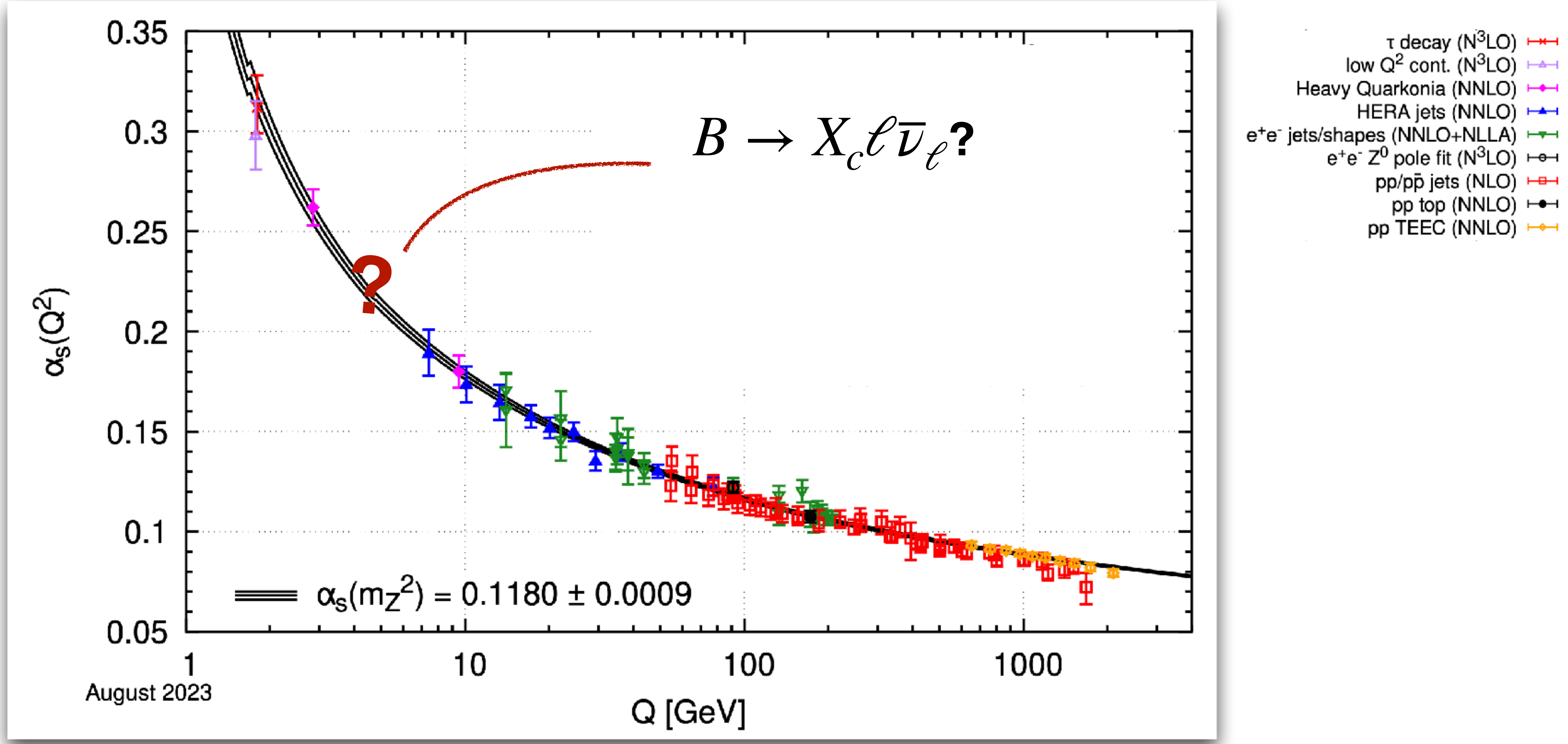
The strong coupling constant, α_s

Is there alternative way for α_s determination?



Heavy quark expansion for $B \to X_c \ell \bar{\nu}_\ell$

In the **Heavy Quark Expansion (HQE)** framework, the inclusive semileptonic B decay widths can be expressed as double expansions in α_s and quark masses (m_b):

$$\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \frac{G_F^2 |V_{cb}|^2 m_b^5 A_{\mathrm{ew}}}{192\pi^3} \Big[$$

$$\mathbf{c}_0 + \mathbf{c}_1 \frac{\alpha_s}{\pi} + \mathbf{c}_2 (\frac{\alpha_s}{\pi})^2 + \mathbf{c}_3 (\frac{\alpha_s}{\pi})^3 + \mathcal{O}(\alpha_s^4)$$ ⋯⋯ leading-order power correction

$$+ C_{\mu_\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2}$$ ⋯⋯ 2nd-order power correction

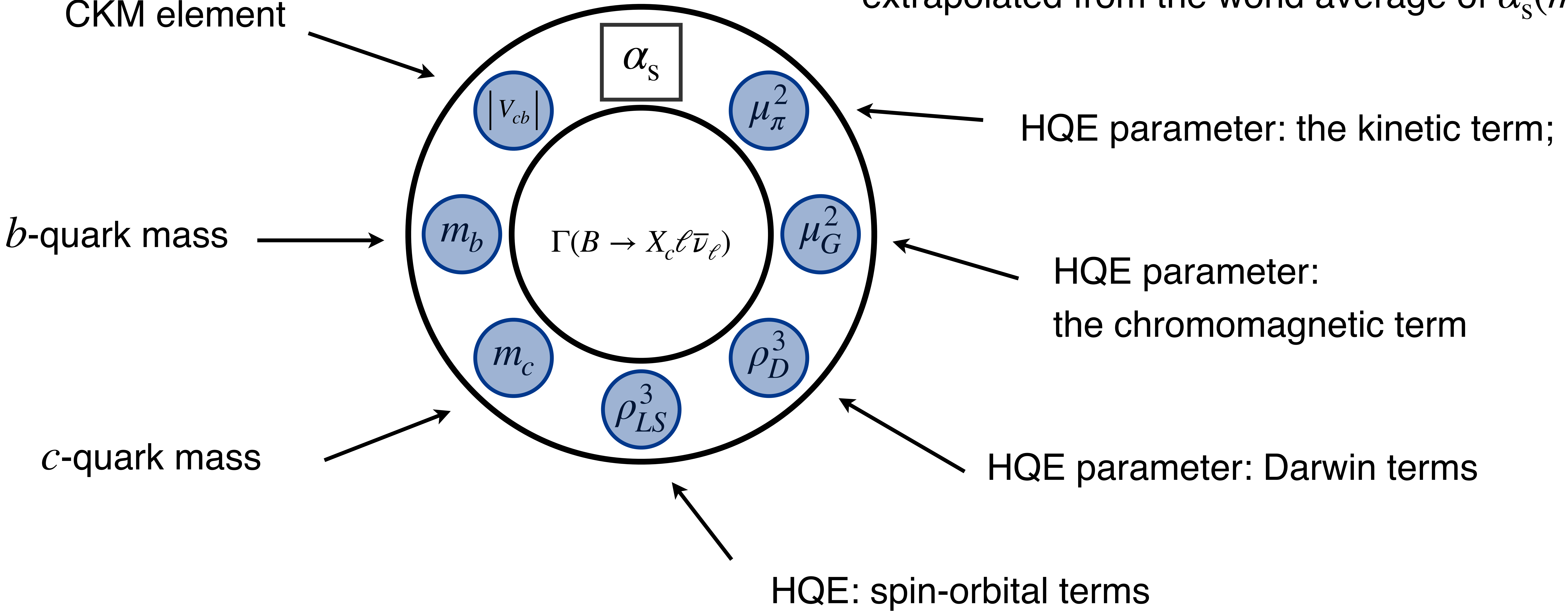
$$+ C_{\rho_D} \frac{\rho_D^3}{2m_b^3} + C_{\rho_{LS}} \frac{\rho_{LS}^3}{2m_b^3}$$ ⋯⋯ 3rd-order power correction

$$+ \cdots \Big]$$ ⋯⋯ higher-order power corrections

Heavy quark expansion for $B \to X_c \ell \bar{\nu}_\ell$

$$\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \frac{G_F^2 |V_{cb}|^2 m_b^5 A_{\rm ew}}{192\pi^3}\left[\sum_{i=0,1,2,\cdots} c_i \left(\frac{\alpha_s}{\pi}\right)^i + C_{\mu_\pi}\frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G}\frac{\mu_G^2}{2m_b^2} + C_{\rho_D}\frac{\rho_D^3}{2m_b^3} + C_{\rho_{LS}}\frac{\rho_{LS}^3}{2m_b^3} + \cdots\right]$$

The $B \to X_c \ell \bar{\nu}_\ell$ process was used to determine $|V_{cb}|$, m_b and m_c, with α_s **fixed** at the value extrapolated from the world average of $\alpha_s(m_Z)$

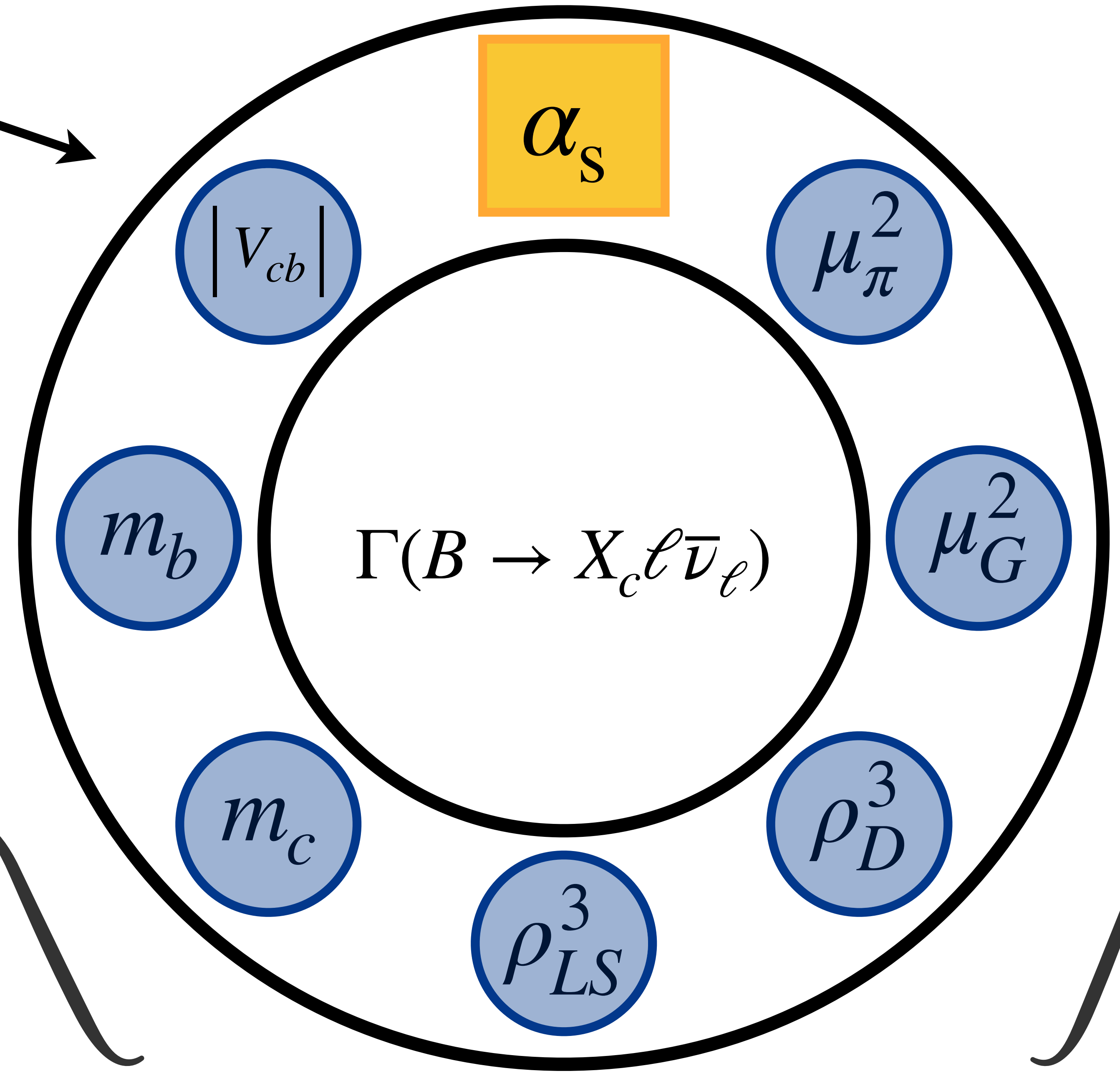


Heavy quark expansion for $B \to X_c \ell \bar{\nu}_\ell$ and $D \to X_s \ell \bar{\nu}_\ell$

$$\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \frac{G_F^2 |V_{cb}|^2 m_b^5 A_{\mathrm{ew}}}{192\pi^3}\left[\sum_{i=0,1,2,\cdots} c_i \left(\frac{\alpha_s}{\pi}\right)^i + C_{\mu_\pi}\frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G}\frac{\mu_G^2}{2m_b^2} + C_{\rho_D}\frac{\rho_D^3}{2m_b^3} + C_{\rho_{LS}}\frac{\rho_{LS}^3}{2m_b^3} + \cdots\right]$$

Extract α_s from $B \to X_c \ell \bar{\nu}_\ell$, by fixing the other parameters at external determinations?

Exclusive B decays
W decays, etc



B/D meson masses,
Lattice QCD,
etc.

HQE parameters:
Differential decay width of $B \to X_c \ell \bar{\nu}$
Lattice QCD

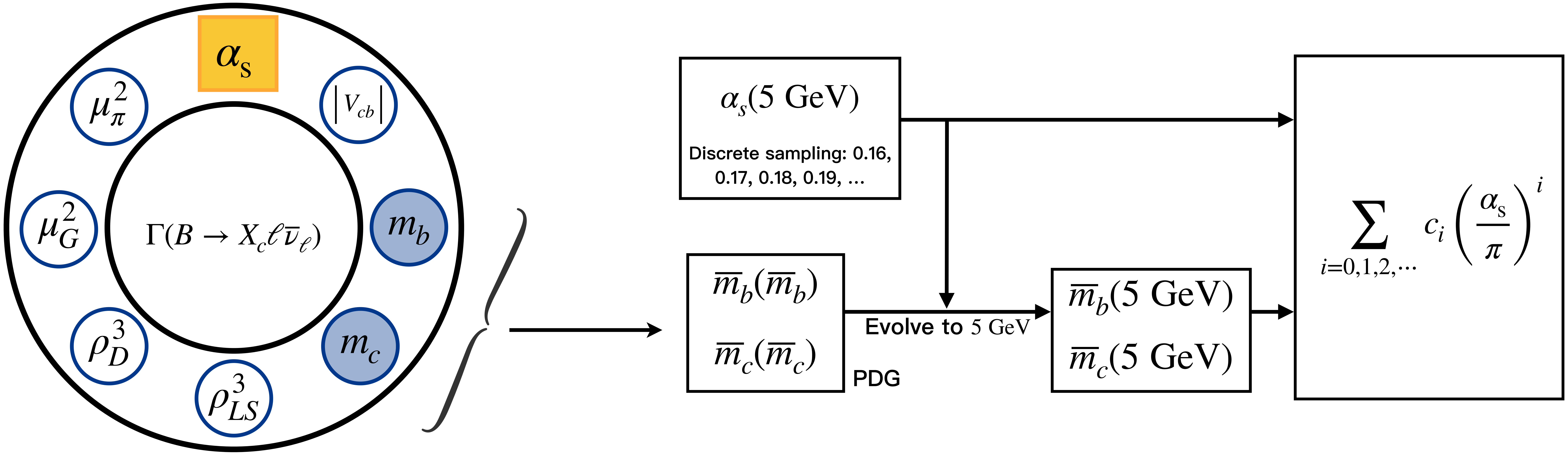
A numerical relation between $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$ and α_s

$$\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \frac{G_F^2 |V_{cb}|^2 m_b^5 A_{ew}}{192\pi^3}\left[\sum_{i=0,1,2,\cdots} c_i \left(\frac{\alpha_s}{\pi}\right)^i + C_{\mu_\pi}\frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G}\frac{\mu_G^2}{2m_b^2} + C_{\rho_D}\frac{\rho_D^3}{2m_b^3} + C_{\rho_{LS}}\frac{\rho_{LS}^3}{2m_b^3} + \cdots\right]$$

Leading-order power correction:

$\mathbf{c}_i$: calculated to 4th order, depending on m_b, m_c.[1,2]

reformulated consistently in terms of the **$\overline{\text{MS}}$-renormalized** quark masses $\overline{m}_b(\mu)$, $\overline{m}_c(\mu)$ and $\alpha_s(\mu)$, with $\mu = 5\,\text{GeV}$.

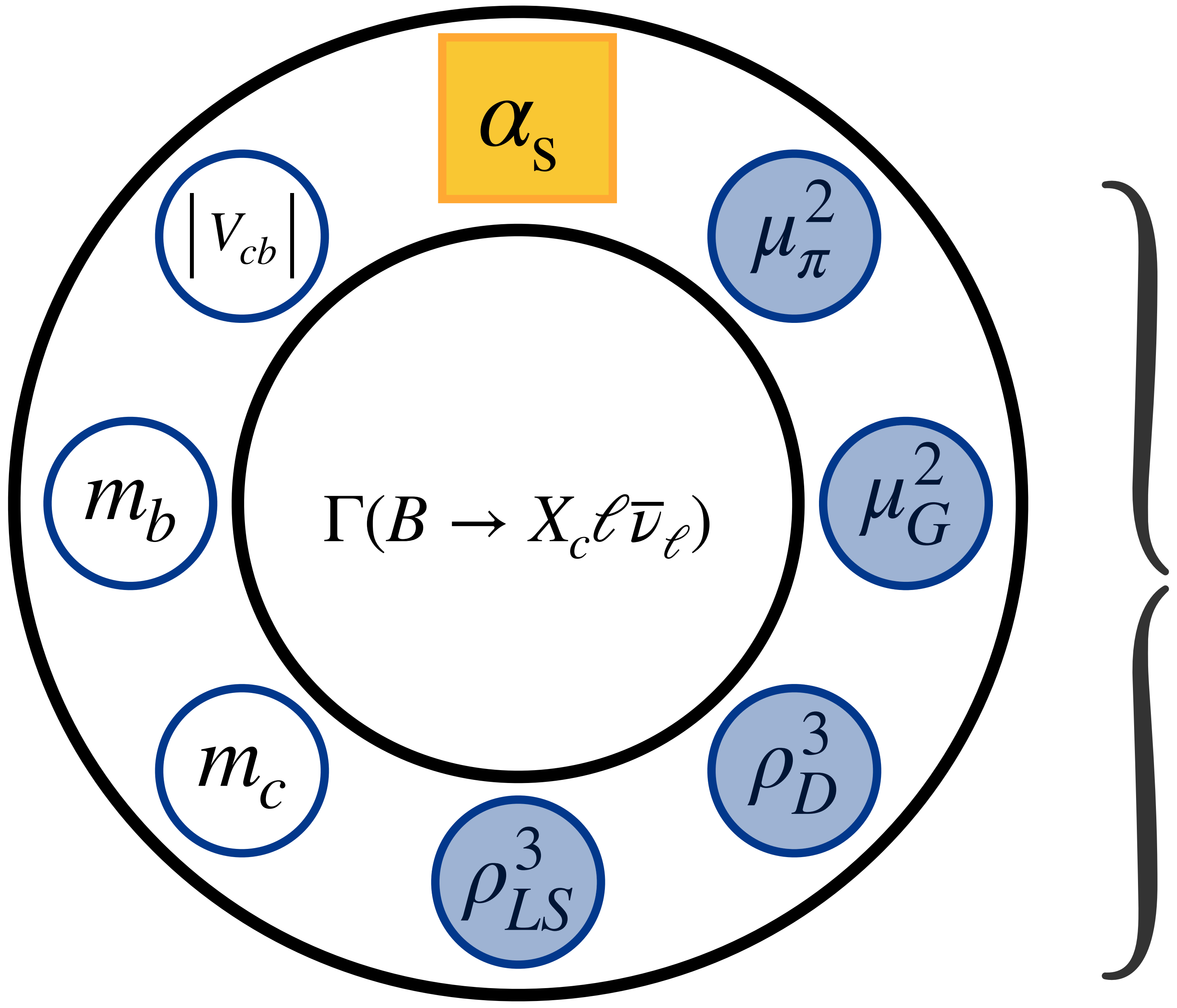


[1] https://doi.org/10.1103/PhysRevD.78.114015
[2] https://doi.org/10.1103/PhysRevD.104.016003

A numerical relation between $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$ and α_s

$$\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \frac{G_F^2 |V_{cb}|^2 m_b^5 A_{\rm ew}}{192\pi^3}\left[\sum_{i=0,1,2,\cdots} c_i \left(\frac{\alpha_s}{\pi}\right)^i + C_{\mu_\pi}\frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G}\frac{\mu_G^2}{2m_b^2} + C_{\rho_D}\frac{\rho_D^3}{2m_b^3} + C_{\rho_{LS}}\frac{\rho_{LS}^3}{2m_b^3} + \cdots\right]$$

- **2nd and 3rd order power corrections:**
  - decrease the decay width by $\sim 7\,\%$ [arXiv:1009.4622].



- The the coefficients are also series of α_s, in the **kinetic scheme**, [1]
  - $C_{\mu_\pi} = 2\mathbf{c}_0\left(\frac{1}{2} - 0.99\frac{\alpha_s}{\pi}\right)$, $C_{\mu_G} = -2\mathbf{c}_0\left(1.94 + 3.46\frac{\alpha_s}{\pi}\right)$, etc.
- The non-perturbative parameters: [2]
  - $\mu_\pi^2 = 0.477 \pm 0.056\,\mathrm{GeV}^2$, $\mu_G^2 = 0.306 \pm 0.050\,\mathrm{GeV}^2$, etc
- The higher order power corrections are estimated around -7%, with the coefficients estimated using their first terms.
- **Truncation error:** $\mathcal{O}(\alpha_s/m_b^{2,3})$ terms and $\mathcal{O}(1/m_b^{4,5})$ terms, $\sim 2.3\%$.

[1] https://doi.org/10.1007/JHEP01(2014)147
[2] https://doi.org/10.1016/j.physletb.2021.136679

Determining α_s from $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$

$$\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right) = \frac{G_F^2 |V_{cb}|^2 m_b^5 A_{\rm ew}}{192\pi^3}\left[\sum_{i=0,1,2,\cdots} c_i \left(\frac{\alpha_s}{\pi}\right)^i + C_{\mu_\pi}\frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G}\frac{\mu_G^2}{2m_b^2} + C_{\rho_D}\frac{\rho_D^3}{2m_b^3} + C_{\rho_{LS}}\frac{\rho_{LS}^3}{2m_b^3} + \cdots\right]$$

Experimental inputs: the lifetime and the branching ratios,

$$\tau_{B^\pm} = 1.638 \pm 0.004\,\text{ps}, \mathscr{B}(B^\pm \to X_c \ell \nu) = 10.8 \pm 0.4\,\%\,, \tau_{B^0} = 1.517 \pm 0.004\,\text{ps}, \mathscr{B}(B^0 \to X_c \ell \nu) = 10.1 \pm 0.4\,\%\,.$$

TABLE I. The parameters used during the construction of the theoretical model.

Parameter	Notation	Value & error	Note
Fermi coupling constant	G_F	$1.16637886 \times 10^{-5}$ GeV^{-2}	[50]
Electroweak correction factor	$A_{\rm ew}$	1.014	[51]
CKM matrix element	$\lvert V_{cb}\rvert$	0.0398 ± 0.0006	[50]
b-quark mass in $\overline{\rm MS}$	$\overline{m}_b(\overline{m}_b)$	$4.18^{+0.03}_{-0.02}$ GeV	[50]
c-quark mass in $\overline{\rm MS}$	$\overline{m}_c(\overline{m}_c)$	1.27 ± 0.02 GeV	[50]
HQE parameters	μ_π^2	0.477 ± 0.056 GeV^2	[28]
	μ_G^2	0.306 ± 0.050 GeV^2	[28]
	ρ_D^3	0.185 ± 0.031 GeV^2	[28]
	ρ_{LS}^3	-0.130 ± 0.092 GeV^2	[28]
b-quark mass in kinetic scheme	$m_b^{\rm kin}$	4.573 ± 0.012 GeV	[28]

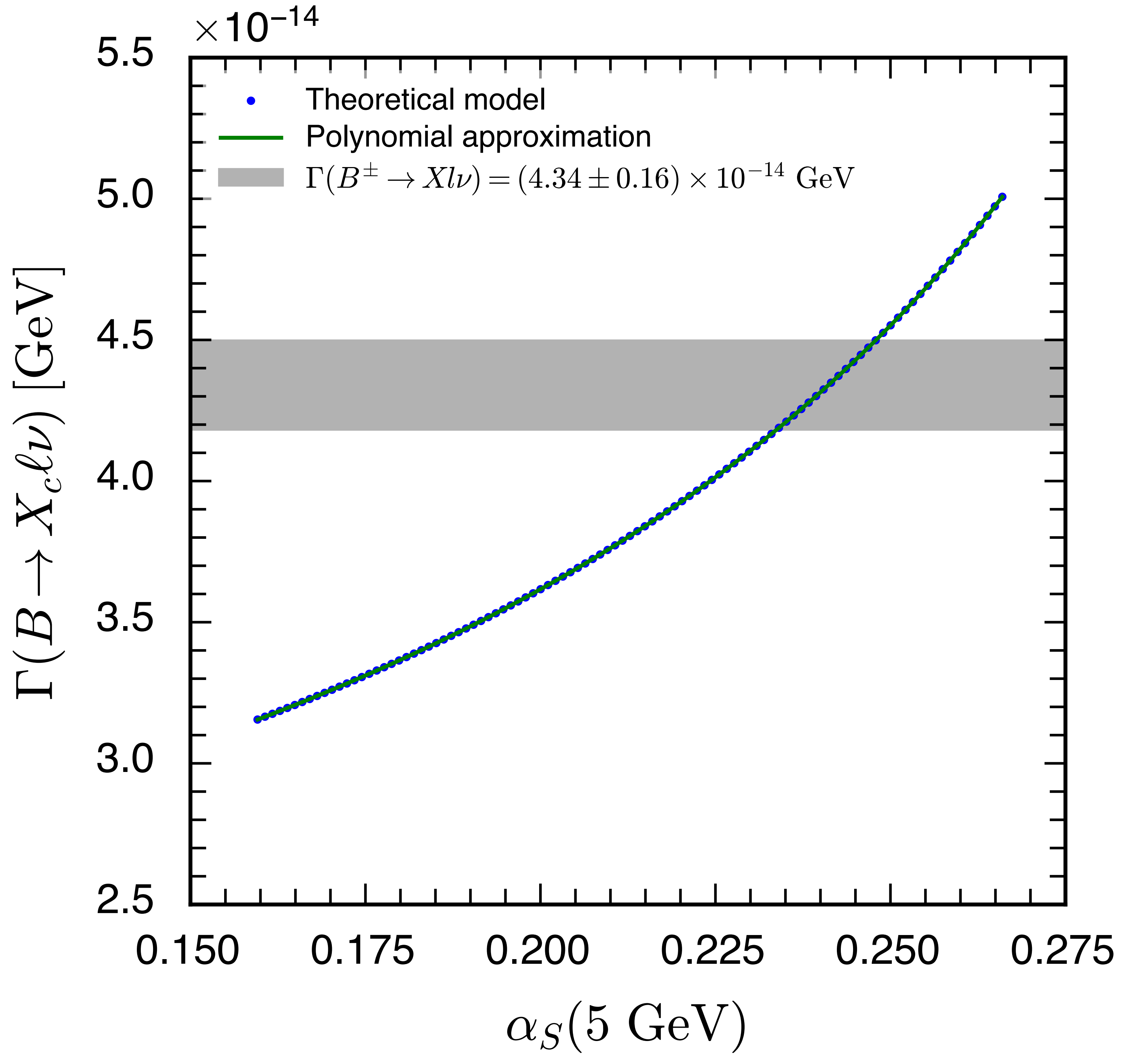


FIG I. The numerical dependence of $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$ versus $\alpha_s(5\,\text{GeV})$, compared with experimental measurement of $B^\pm$.

Determining α_s from $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$

The χ^2 fit, defined as $\chi^2(\alpha_s) = \dfrac{[\Gamma_B - \hat{\Gamma}_B(\alpha_s)]^2}{\sigma^2_{\Gamma_{B,\mathrm{exp}}} + \sigma^2_{\Gamma_{B,\mathrm{theo}}}}$, yields

$$\alpha_s(5\,\mathrm{GeV}) = 0.246 \pm 0.013$$

Combining the two fits gives $\alpha_s(5\,\mathrm{GeV}) = 0.245 \pm 0.009$, corresponding to $\alpha_s(m_Z) = 0.1266 \pm 0.0023$.

This resulting precision is comparable to the PDG averages of $\alpha_s(m_Z)$ obtained from other experimental methods.

TABLE. The relative uncertainty contributions to the theoretical prediction of $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$ and the $\alpha_s(5\,\mathrm{GeV})$ fitting result using $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$. Values in the parenthesis are the perspective values considering future improvements.

	Γ_{sl} prediction [%]	$\alpha_s(5\,\mathrm{GeV})$ [%]
$\lvert V_{cb}\rvert = 0.0398 \pm 0.0006$	3.0 (1.4)	2.1 (1.0)
$\overline{m}_b(\overline{m}_b) = 4.18^{+0.03}_{-0.02}$ GeV	3.0 (1.1)	2.1 (0.8)
$\overline{m}_c(\overline{m}_c) = 1.27 \pm 0.02$ GeV	2.1 (1.4)	1.4 (1.0)
R-scale $\mu = 5^{+5}_{-2.5}$ GeV	4.4 (2.2)	3.1 (1.6)
High-order power corrections	2.3 (2.3)	1.6 (1.6)
$\tau_{B^\pm} = 1.638 \pm 0.004$ ps	-	0.2
$\mathcal{B}(B^\pm \to X_c \ell \nu) = 10.8 \pm 0.4$ %	-	3.0 (2.2)
Sum	6.9(3.2)	5.7 (3.5)

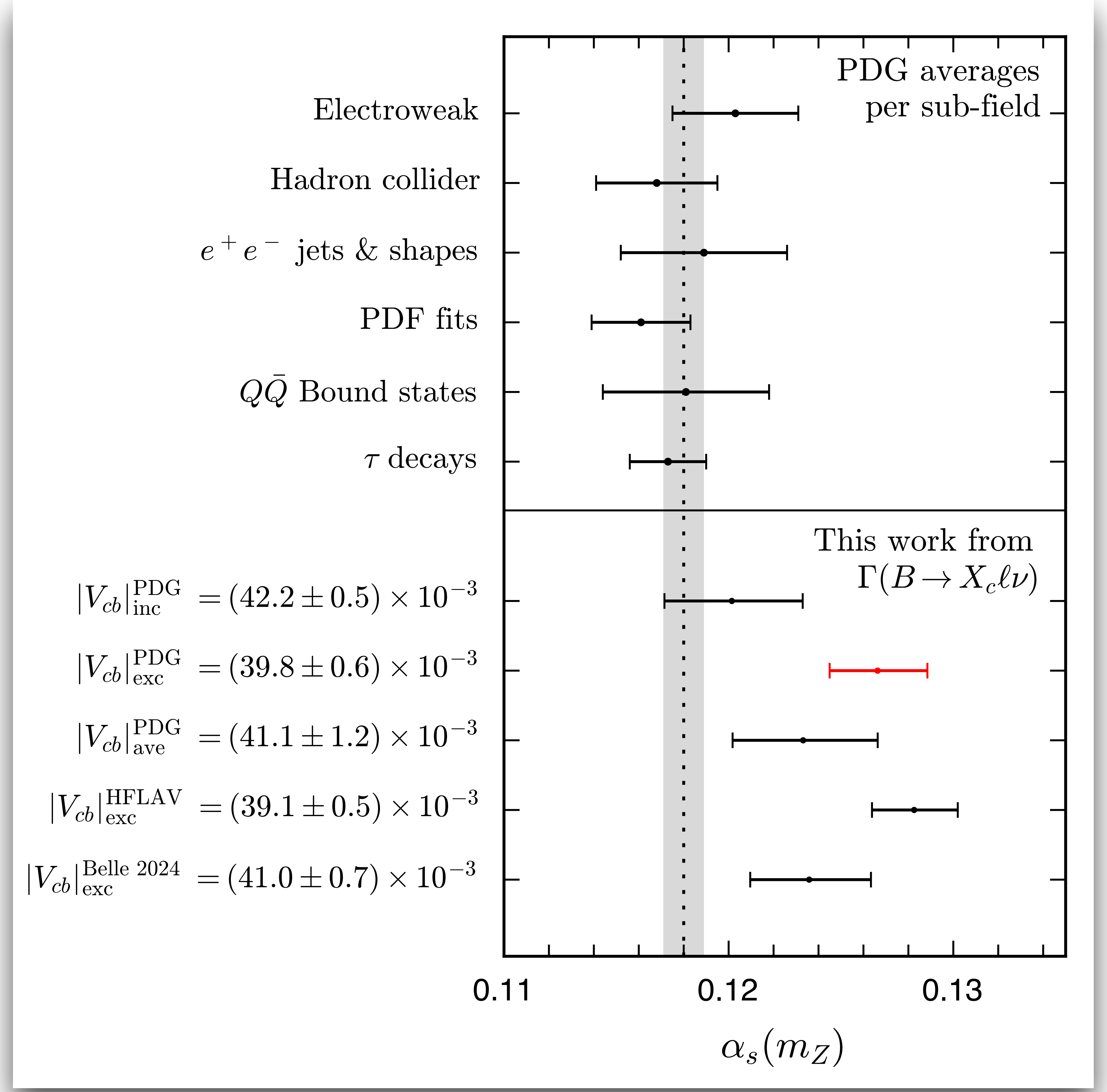


FIG. The comparison of the $\alpha_s(m_Z)$ pre-averages from six experimental subfields in PDG and the extrapolated values obtained in this work. The running of α_s along the energy scale is conducted using the RunDec package [arXiv:hep-ph/0004189]

Determining α_s from $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$

- **Primary uncertainties:**
  - the perturbative expansion;
  - Branching ratio
  - the value of $|V_{cb}|$
- ***The resulting $\alpha_s(5\ \mathrm{GeV})$ is larger than the world average value by ~2σ, which is correlated with the $|V_{cb}|$ puzzle.***

TABLE. The relative uncertainty contributions to the theoretical prediction of $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$ and the $\alpha_s(5\,\mathrm{GeV})$ fitting result using $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$. Values in the parenthesis are the perspective values considering future improvements.

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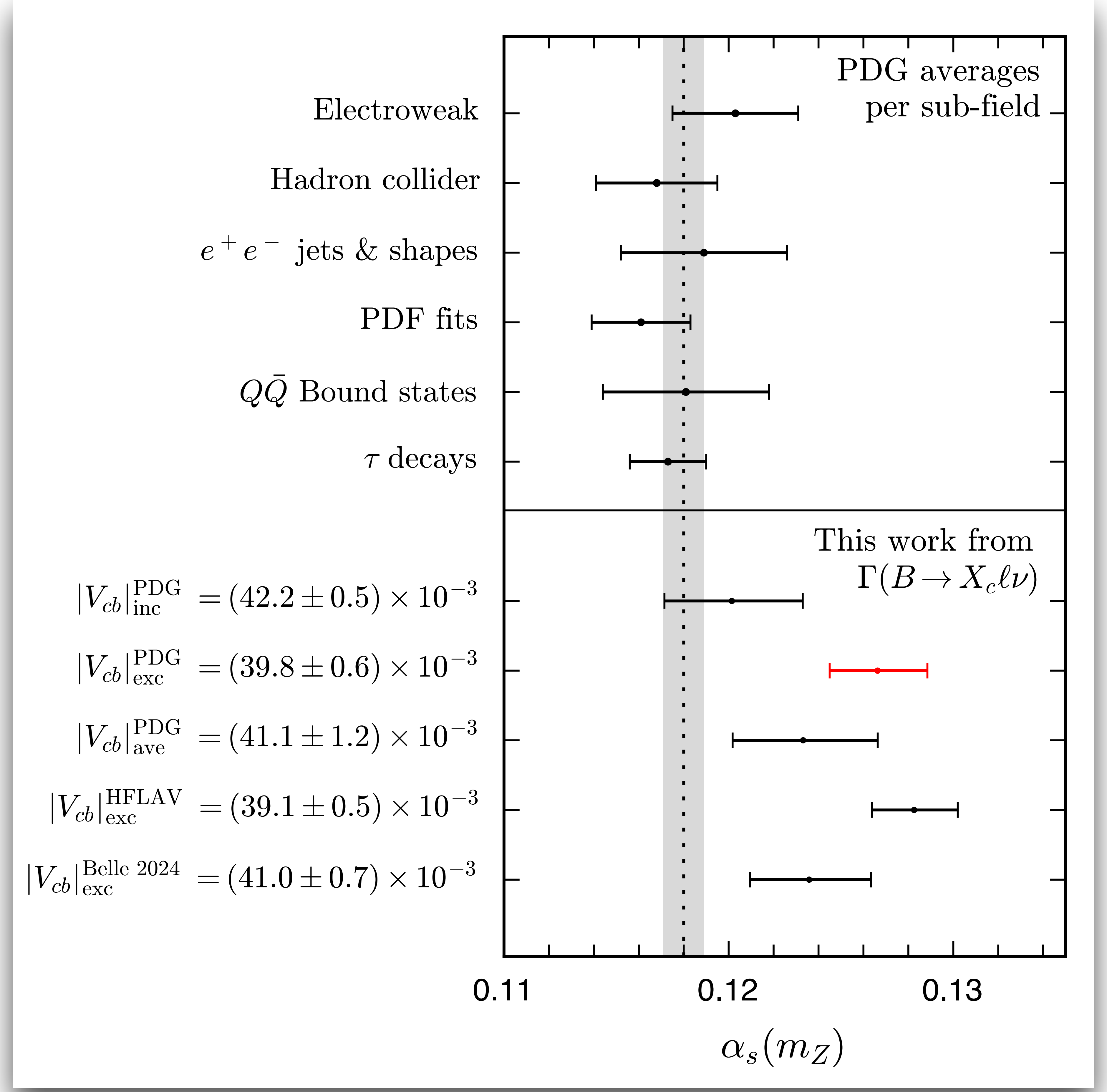

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Determining α_s from $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$

- **Prospects**:
  - $|V_{cb}|$ and B decay should be deeper understood at Belle II and **future Z factories**.
- other improvement from:
  - higher-order perturbation calculation,
  - lattice determinations of b- and c-quark masses.

TABLE. The relative uncertainty contributions to the theoretical prediction of $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$ and the $\alpha_s(5\,\text{GeV})$ fitting result using $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$. Values in the parenthesis are the perspective values considering future improvements.

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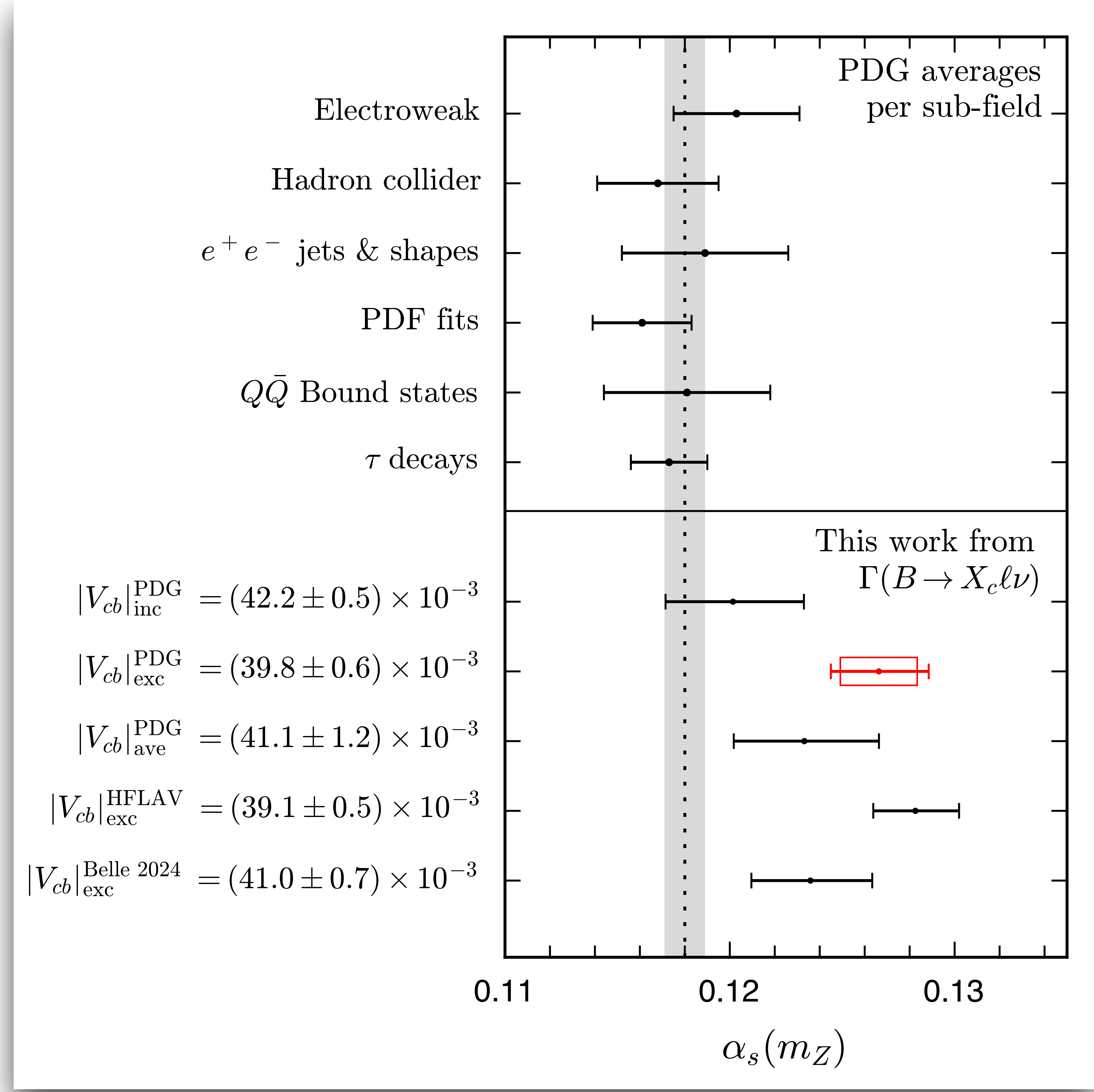

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Determining α_s from $\Gamma\left(D \to X_s \ell \bar{\nu}_\ell\right)$

Another work on $D \to X_c \ell \bar{\nu}$ [arXiv:2406.16119], using

$$\Gamma\left(D \to X_s \ell \bar{\nu}_\ell\right) = \frac{G_F^2 |V_{cs}|^2 m_c^5 A_{\text{ew}}}{192\pi^3} \Big[f_0 + f_1 \frac{\alpha_s}{\pi} + f_2 (\frac{\alpha_s}{\pi})^2 + \mathcal{O}(\alpha_s^3) + f_{\mu_\pi} \frac{\mu_\pi^2}{2m_c^2} + f_{\mu_G} \frac{\mu_G^2}{2m_c^2} + f_{\rho_D} \frac{\rho_D^3}{2m_c^3} + f_{\rho_{LS}} \frac{\rho_{LS}^3}{2m_c^3} + \frac{32\pi^2}{m_c^3} B_{\text{WA}} + \cdots \Big]$$

yields,

$$\alpha_s(m_c) = 0.445 \pm 0.009_{\text{exp}} \pm 0.081_{m_c} \pm 0.056_{\text{trun}} \pm 0.057_{\text{others}}$$

The HQE expressions of the semi-leptonic D & B decay widths share the same parameters.

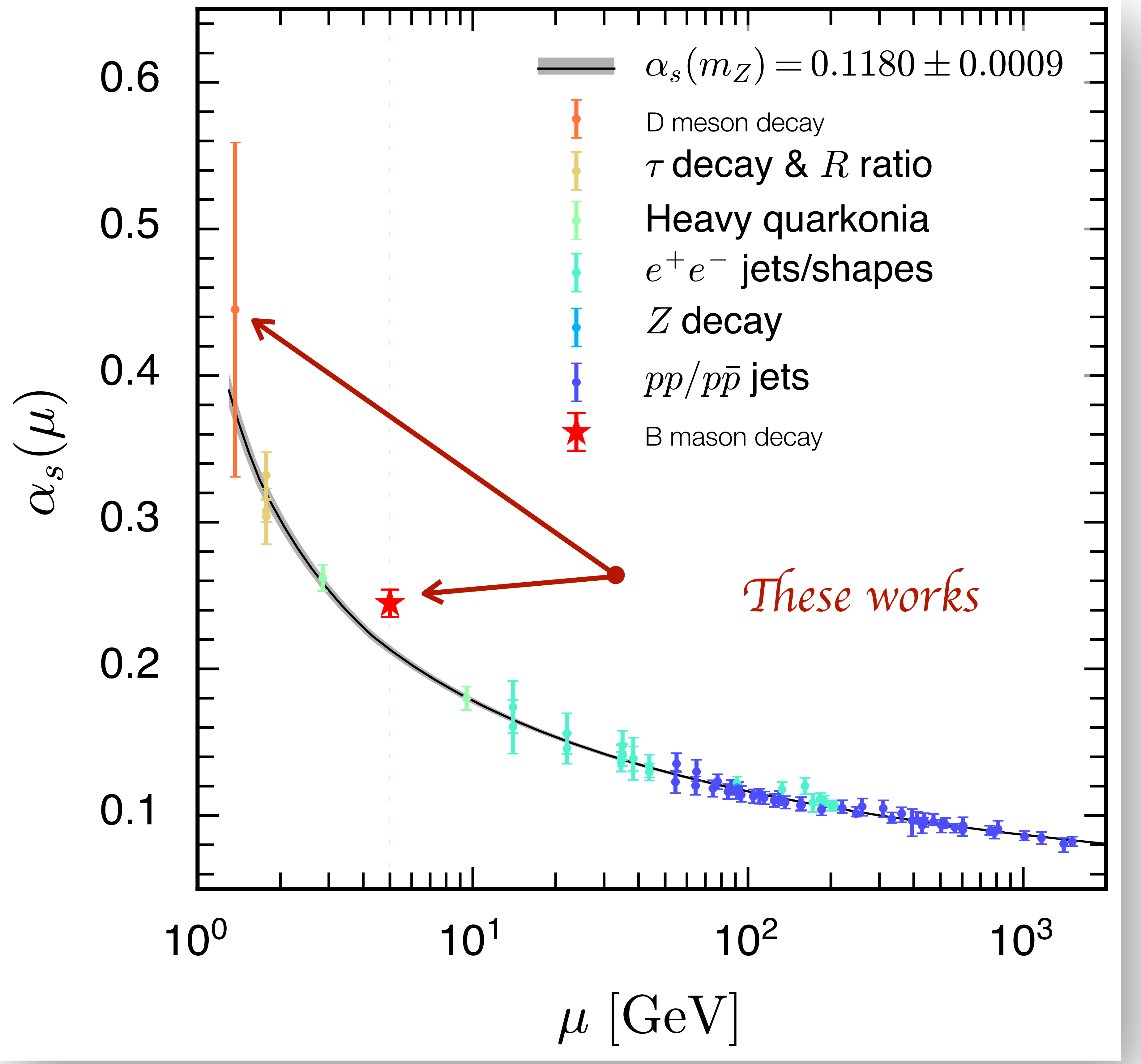
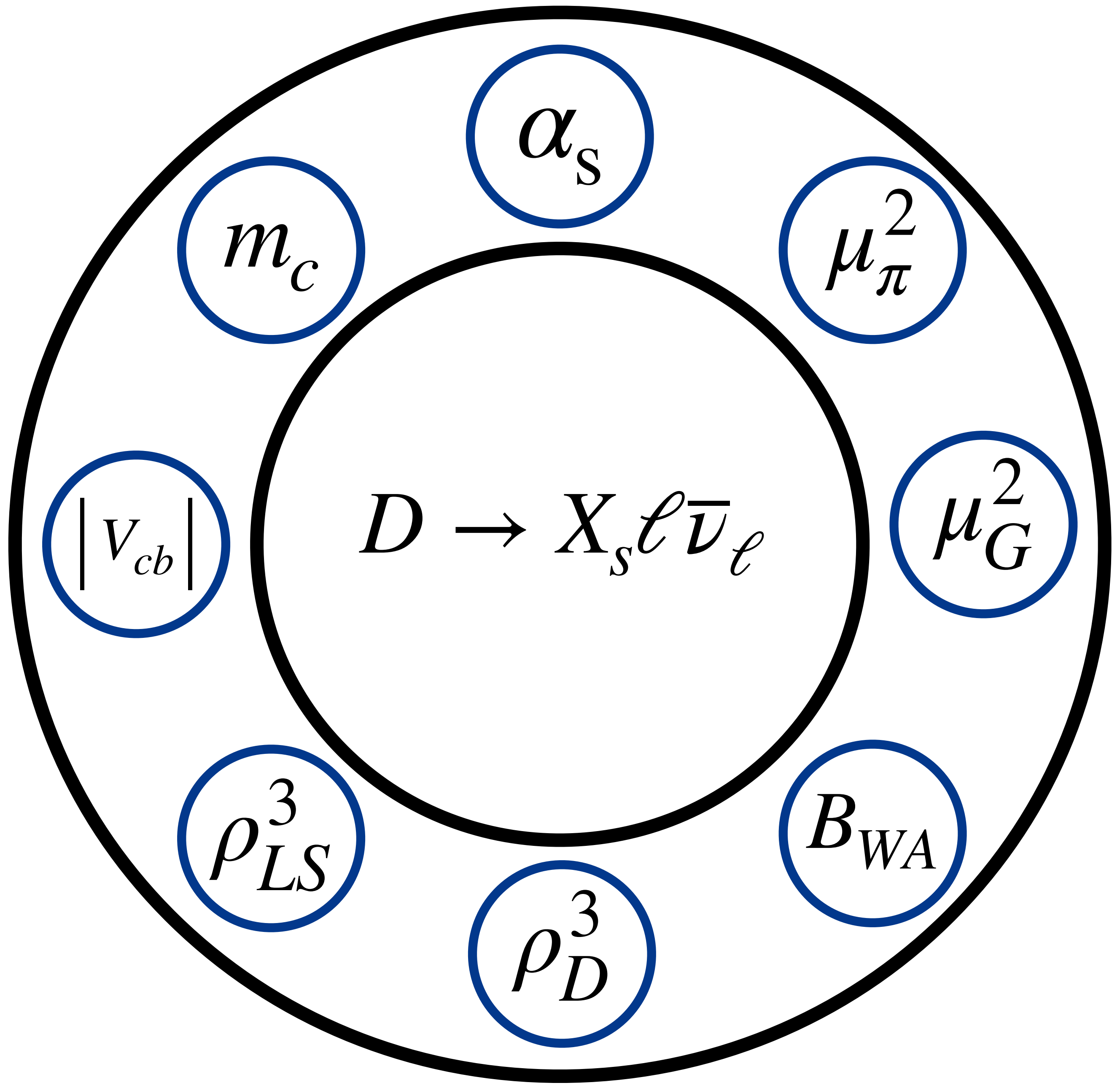


FIG. Comparison of the $\alpha_s(5\,\text{GeV})$ and $\alpha_s(m_c)$ results with the α_s determinations at different energy scales.

Discussion

Recent studies determine the HQE parameters from $D \to X_c \ell \overline{\nu}$, providing good precision.

[*JHEP* 05 (2025) 061], [*Eur.Phys.J.C* 85 (2025) 9, 1011]



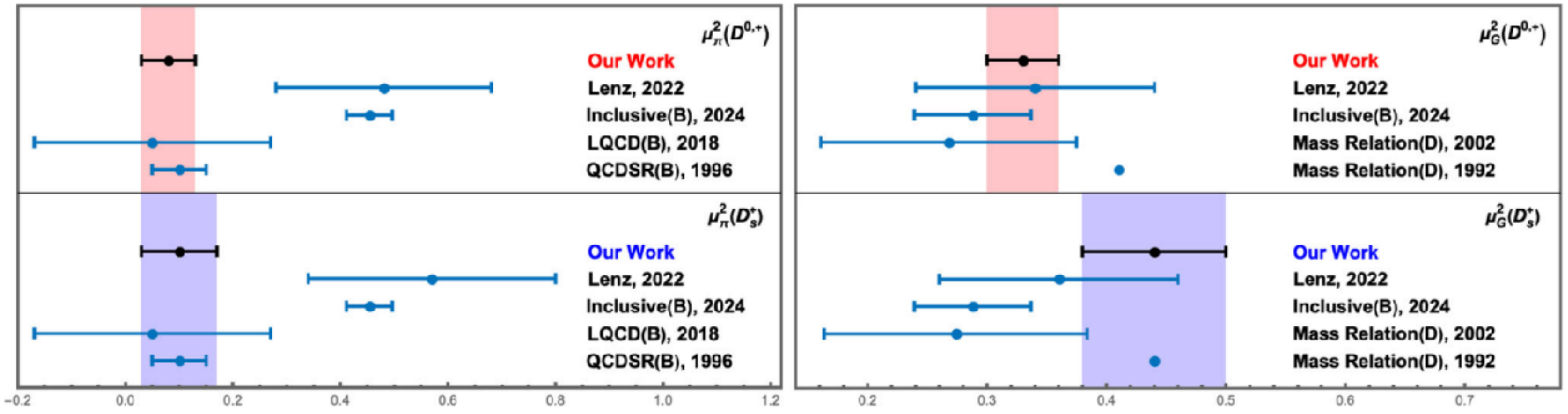


Fig. 1 The comparison between the results for μ_π^2 (left) and μ_G^2 (right) in "Our work" and those obtained in the literature. The "Inclusive(B), 2024" [38] results are obtained in a global fit to semi-leptonic inclusive B decay observables for the corresponding B meson parameters in the kinetic mass scheme, which are related to the D meson ones by the heavy quark symmetry. The "LQCD(B), 2018" [50] and "QCDSR(B), 1996" [51] results for μ_π^2 are also for the B meson, calculated by using lattice QCD and QCD sum rules, respectively. The "Mass Relation(D), 2002" [52] and "Mass Relation(D), 1992" [53] results for μ_G^2 are obtained by using the two versions of mass relations between the $D_{(s)}$ and $D_{(s)}^*$ mesons

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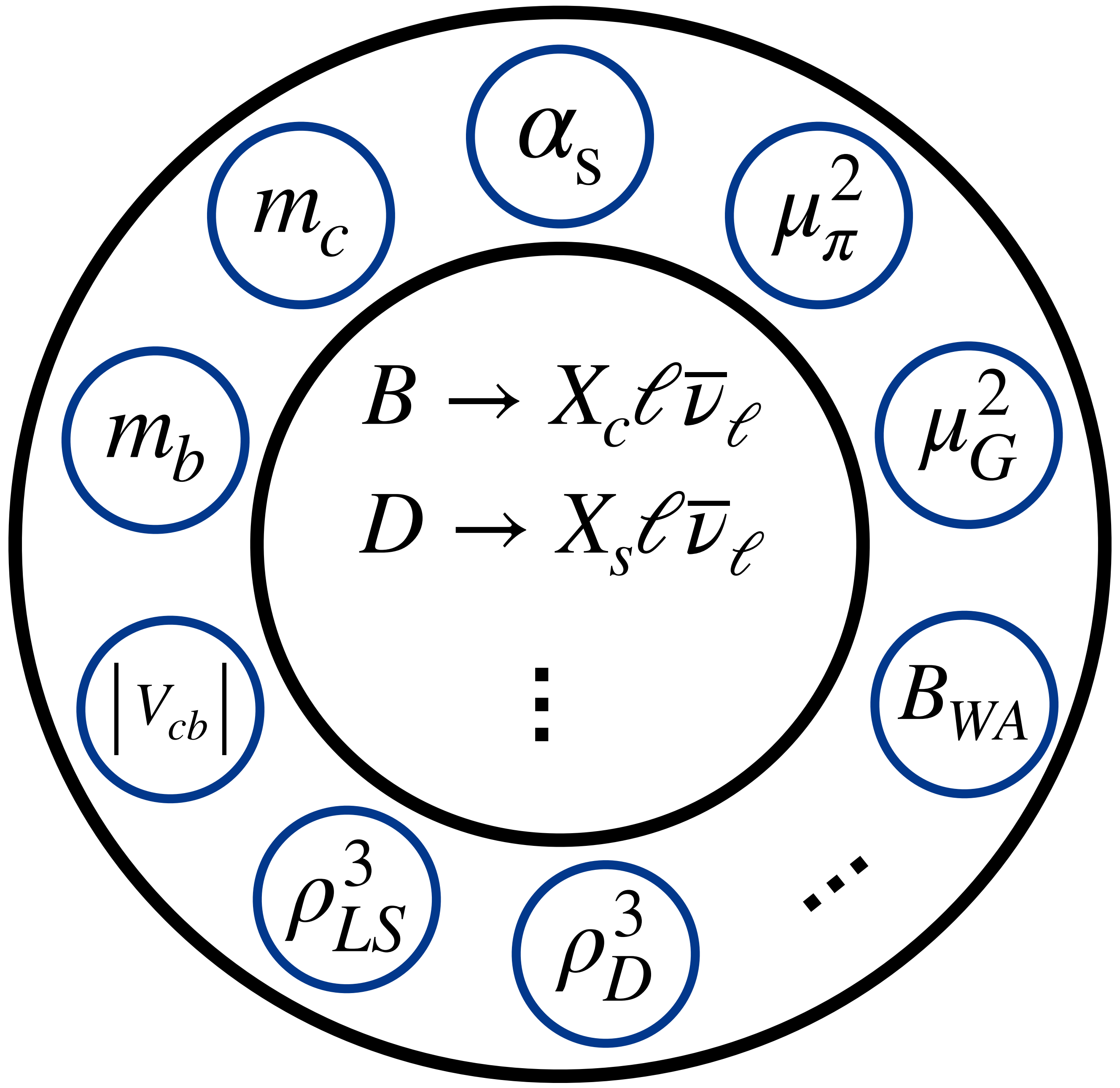
Discussion

Eventually, a global fit combing multi-processes, multiple observables is expected,

- Simultaneously extract α_s and other parameters.
- Validate the consistency of the HQE theory.

CEPC Tera-Z provides valuable experimental input !



Particle	BESIII	STCF (1 ab^{-1})	Belle II (50 ab^{-1} on $\Upsilon(4S)$)	LHCb (300 fb^{-1})	CEPC (TDR)
B^0, $\bar{B}^0$	-	-	5.4×10^{10}	3×10^{13}	4.8×10^{11}
$B^\pm$	-	-	5.7×10^{10}	3×10^{13}	4.8×10^{11}
B_s^0, $\bar{B}_s^0$	-	-	6.0×10^8 (5 ab^{-1} on $\Upsilon(5S)$)	1×10^{13}	1.2×10^{11}
$B_c^\pm$	-	-	-	1×10^{11}	7.2×10^8
Λ_b^0, $\bar{\Lambda}_b^0$	-	-	-	2×10^{13}	1×10^{11}
D^0, $\bar{D}^0$	1.2×10^8	7.2×10^9	4.8×10^{10}	7×10^{14}	8.3×10^{11}
$D^\pm$	1.2×10^8	5.6×10^9	4.8×10^{10}	3×10^{14}	4.9×10^{11}
$D_s^\pm$	1×10^7	1.8×10^9	1.6×10^{10}	1×10^{14}	1.8×10^{11}
$\Lambda_c^\pm$	0.3×10^7	1.1×10^9	1.6×10^{10}	1×10^{14}	6.2×10^{10}
$\tau^+\tau^-$	3.6×10^8	3.6×10^9	4.5×10^{10}		1.2×10^{11}

[2025 Chinese Phys. C 49 103003] Expected yields of b-hadrons, c-hadrons, and τ leptons at BESIII, STCF, Belle II, LHCb Upgrade II, and CEPC.

Summary

As a fundamental parameter of QCD, α_s widely entangled in the observables of semileptonic decays of heavy mesons.

- The HQE expression of $\Gamma\left(B \to X_c \ell \bar{\nu}_\ell\right)$ shows enough sensitivity to support a determination of α_s with a precision comparable to the existing approaches.
  - A 2σ larger $\alpha_s(m_Z)$ is obtained, which is correlated to the $|V_{cb}|$ puzzle.
- Eventually, a global fit combing multi-processes, multiple observables is expected.
- CEPC provides valuable experimental input:
  - Precise $|V_{cb}|$ determination.
  - Provide experimental data of semileptonic B/D decays.

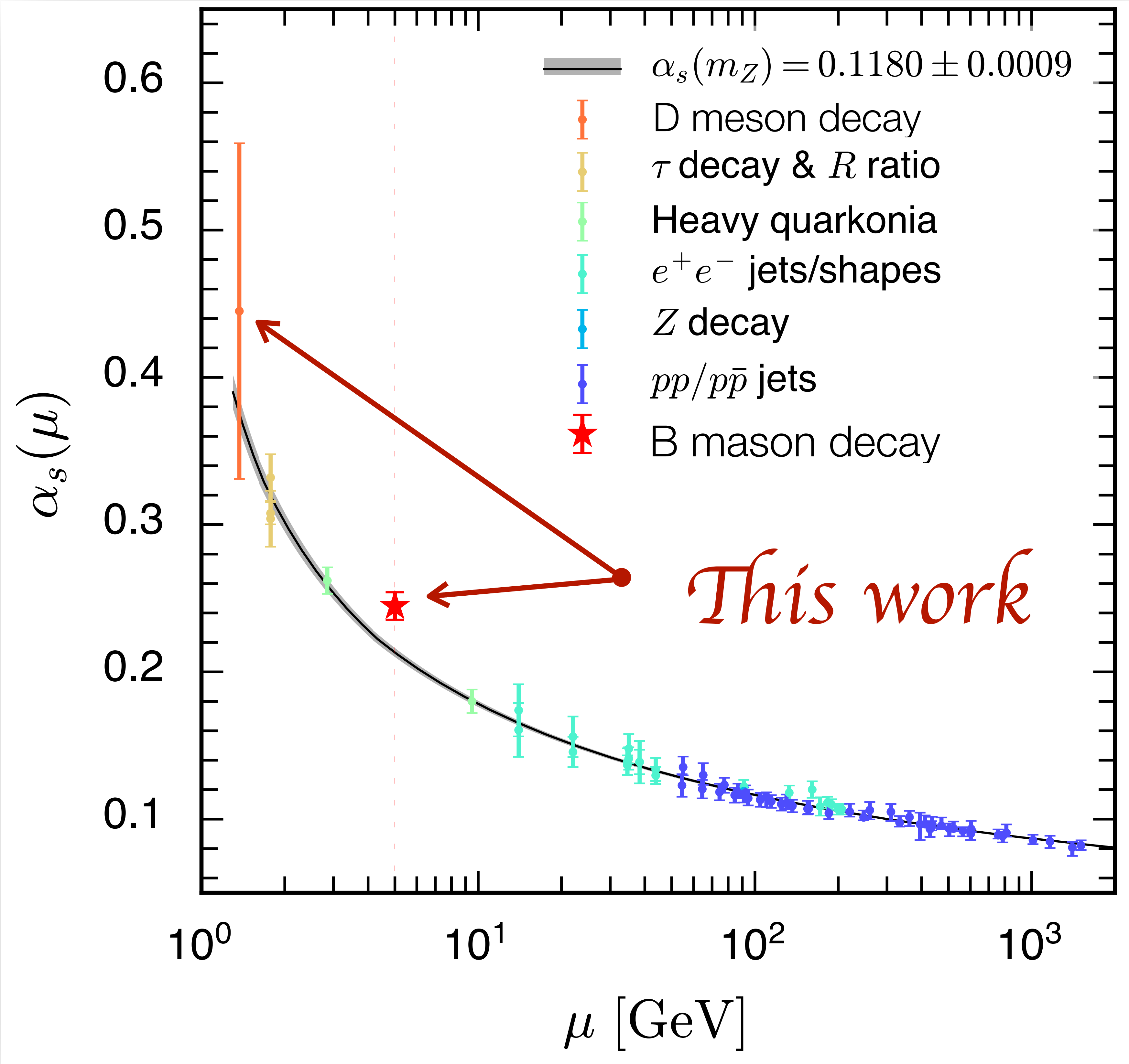


FIG. The $\alpha_s(5\,\mathrm{GeV})$ and $\alpha_s(m_c)$ result compared with the α_s measurements at other energy scales.

Thanks for your attention

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arXiv:2412.02480,
with Long Chen, Jinfei Wu, Xinchou Lou, Xiang Chen, Xin Guan, Yan-Qing Ma, Manqi Ruan.

The 2025 International Workshop on the High Energy Circular Electron Positron Collider