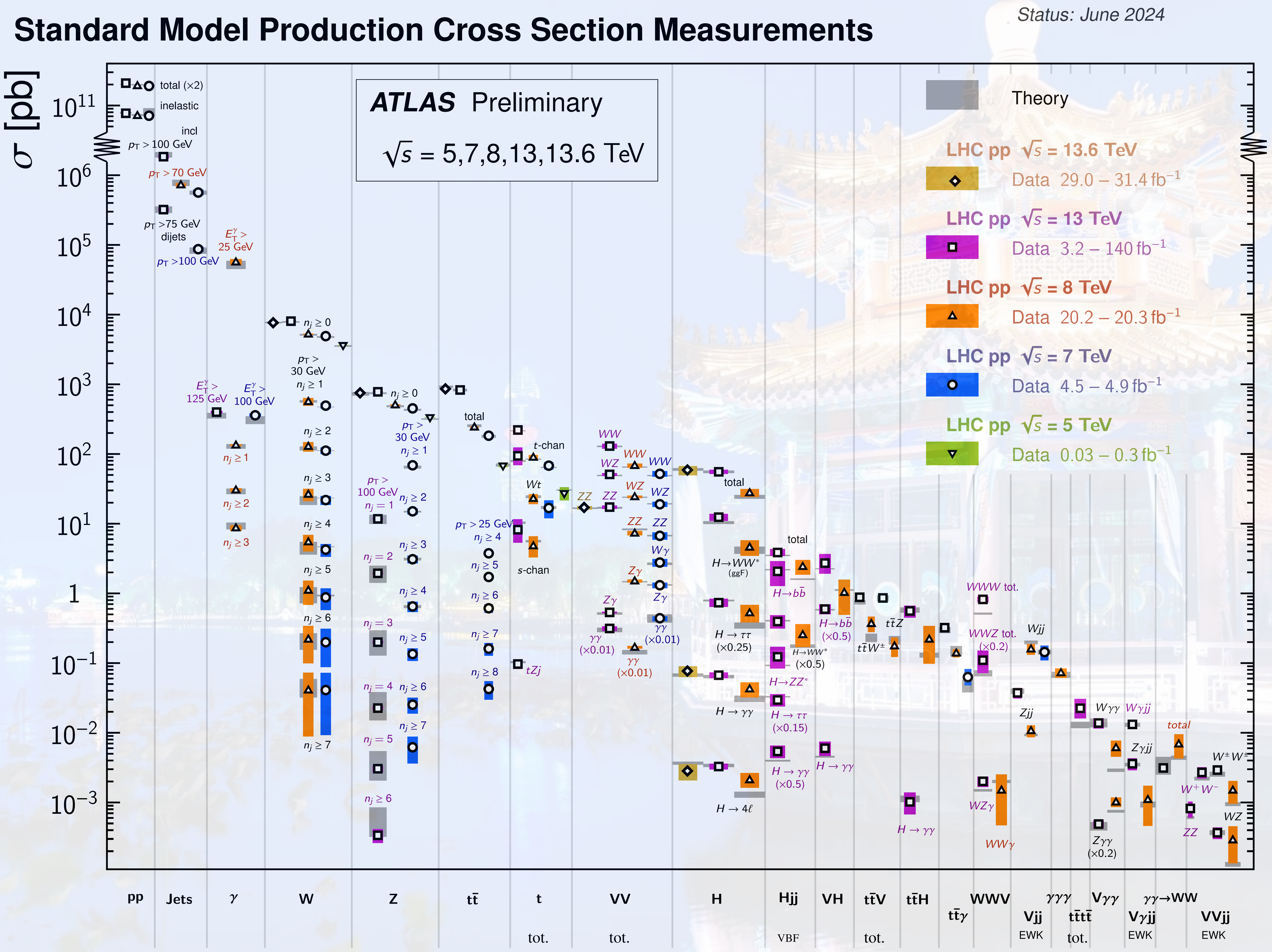
JET PRODUCTION AT N3LO FROM e^+e^- COLLIDERS

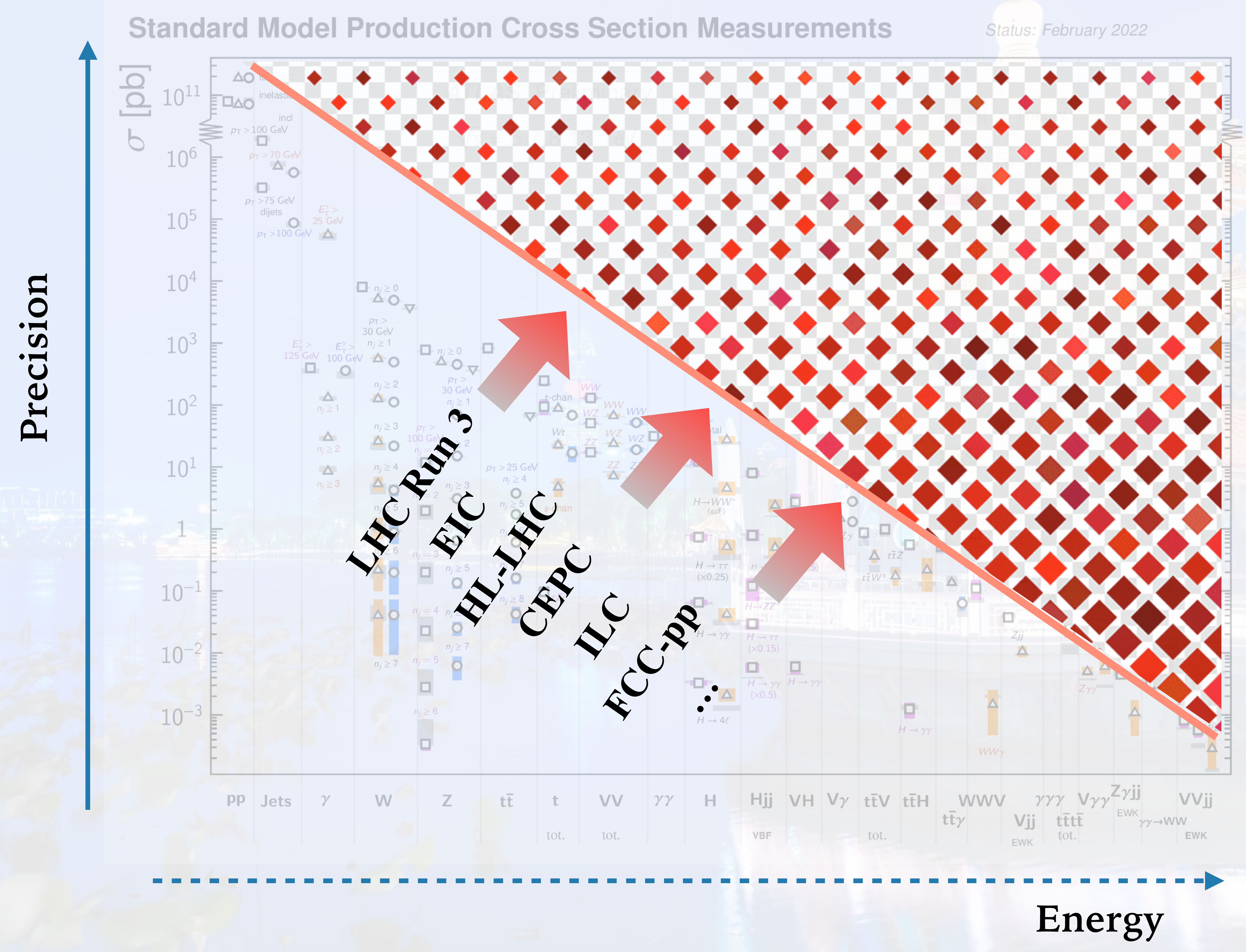
CPEC Workshop 2025

Xuan Chen
Shandong University
Guangzhou, 10 November, 2025

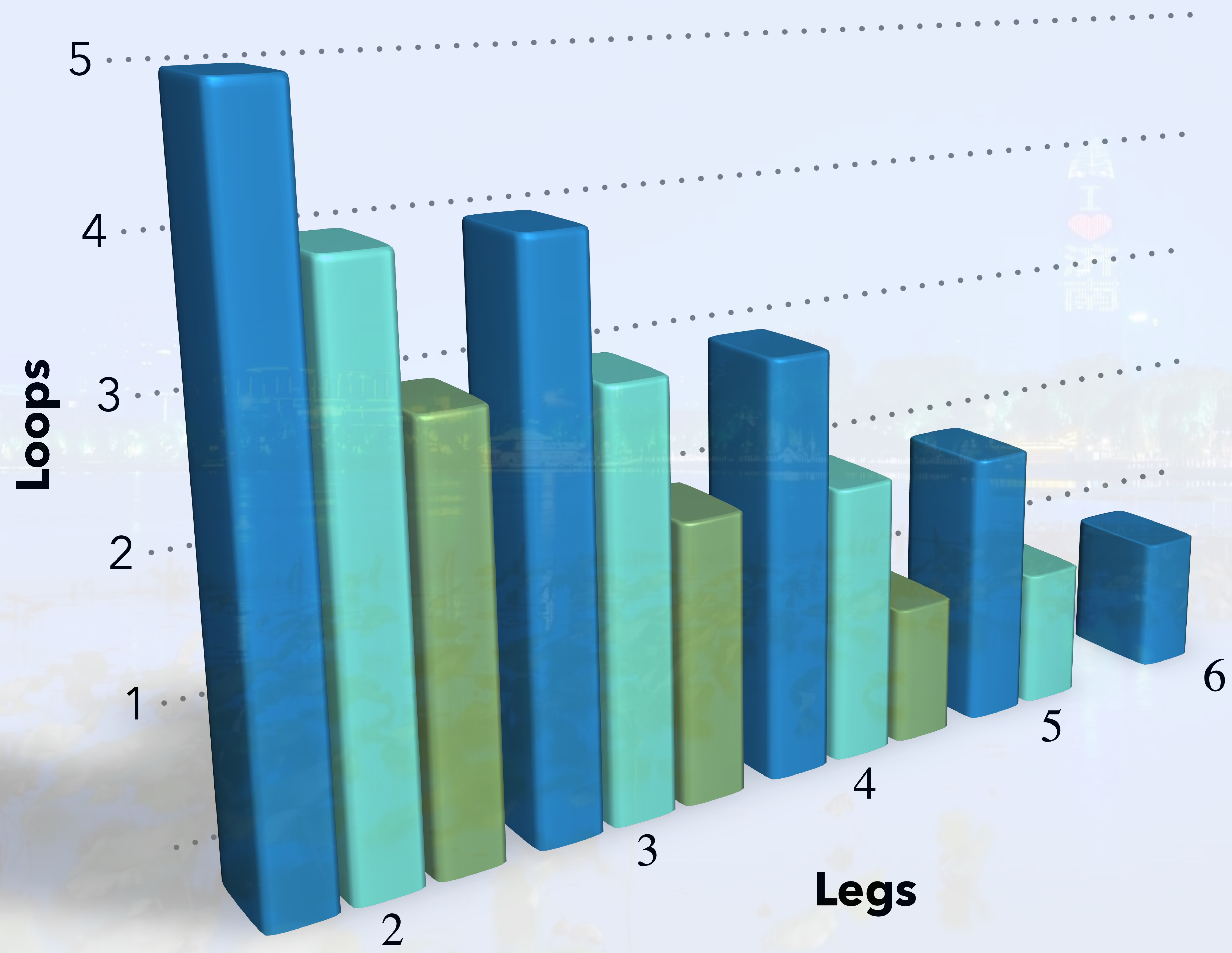


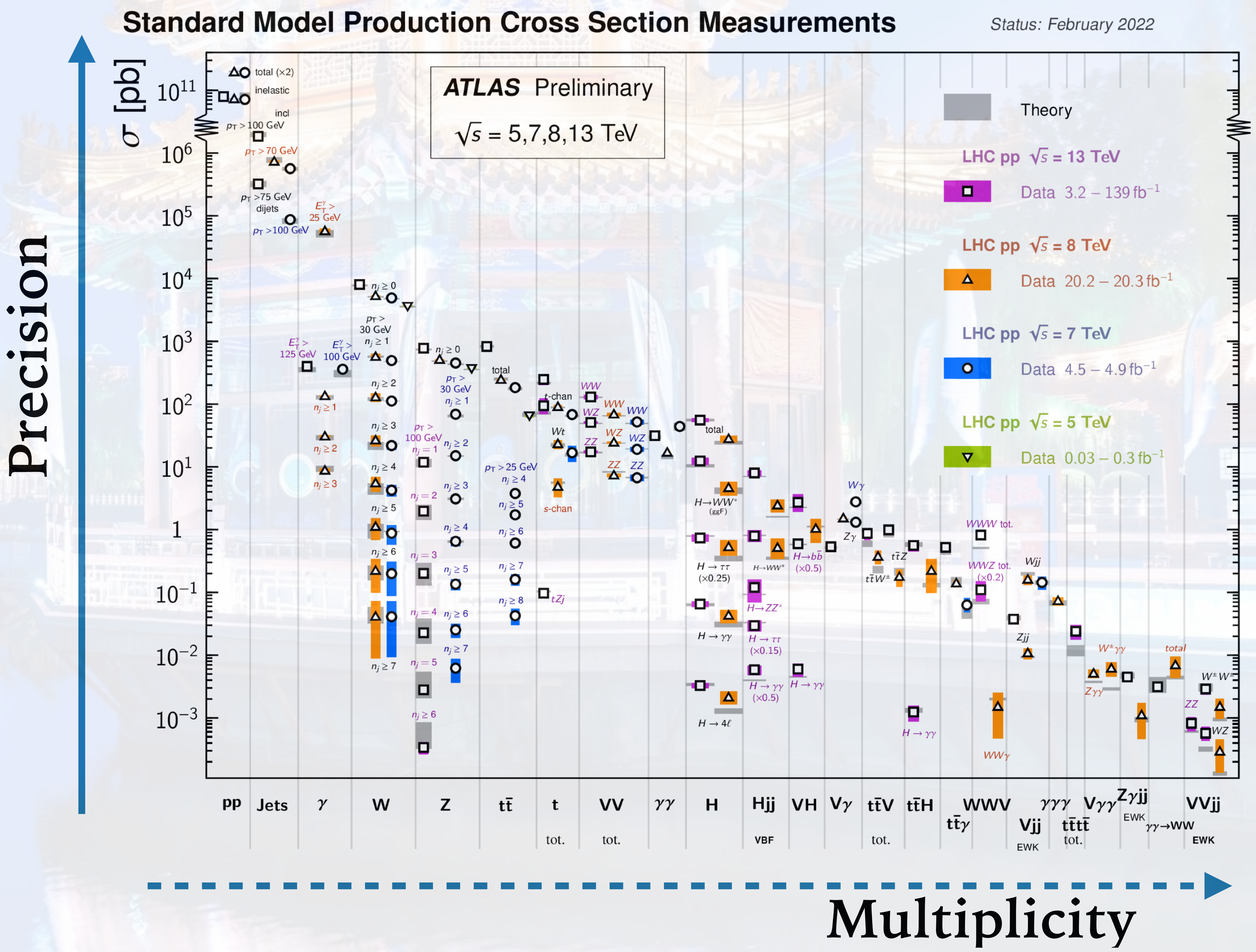
Precision (Rate)

Energy (Multiplicity)

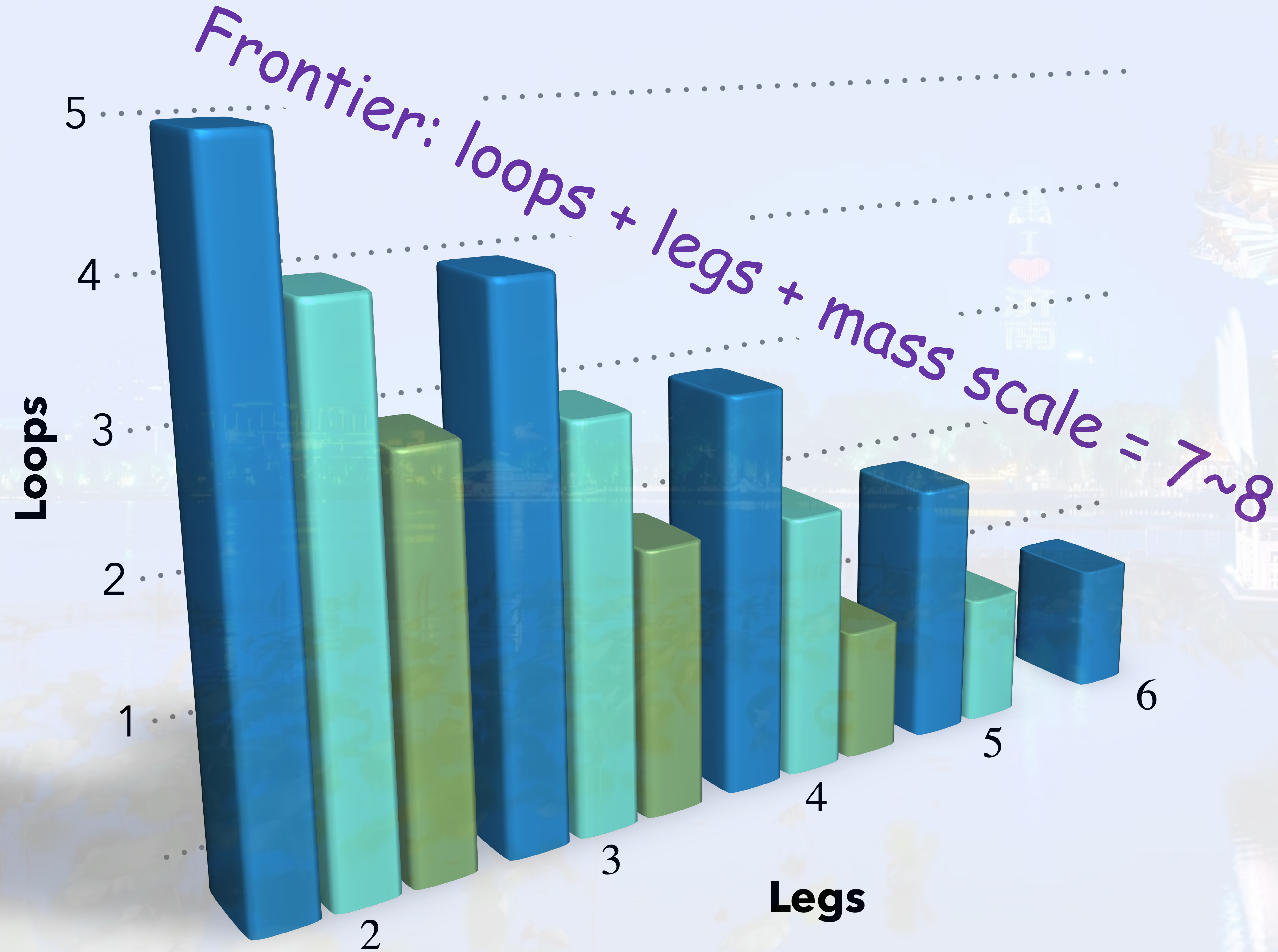

Standard Model Production Cross Section Measurements
Status: February 2022
σ [pb]
Precision
Energy
LHC Run 3
EIC
HL-LHC
CEPC
ILC
FCC-pp
...
pp
Jets
γ
W
Z
tt̄
t
VV
γγ
H
Hjj
VH
Vγ
tt̄V
tt̄H
WWV
tt̄γ
γγγ
Vjj
tt̄tt̄
Vγγ
Zγjj
γγ→WW
VVjj

Perturbative QFT for Precision Predictions



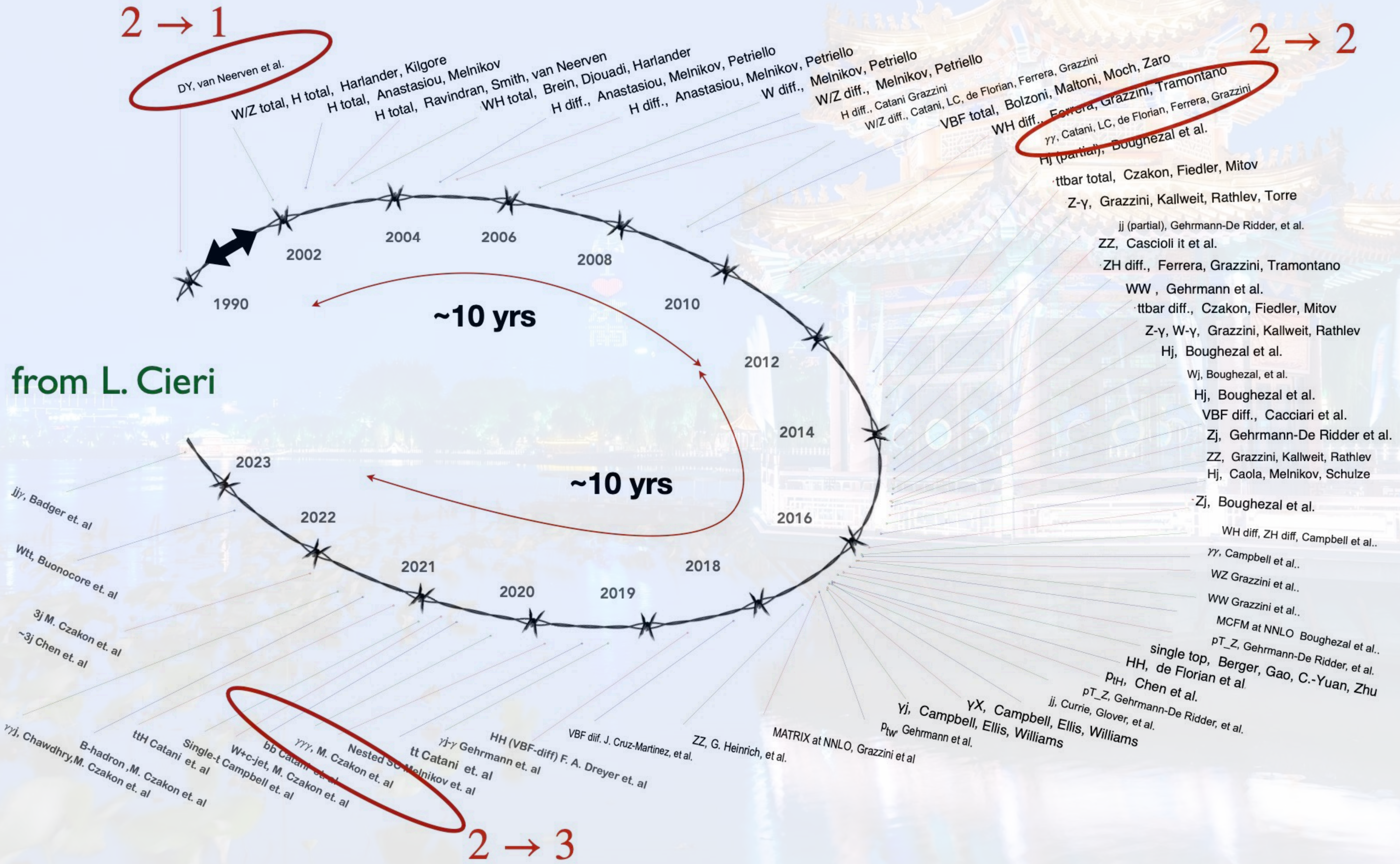


Perturbative QFT for Precision Predictions



Generalised polylogarithms
Riemann zeta values
Elliptic functions
…
Unitarity
Generalised Unitarity
Recursion
Twistors
Differential equations
Integrand/Integral
Sector decomposition
Numerical unitarity
Finite field
Auxiliary mass flow
Neural network amplitude
…

Perturbative QCD @ NNLO



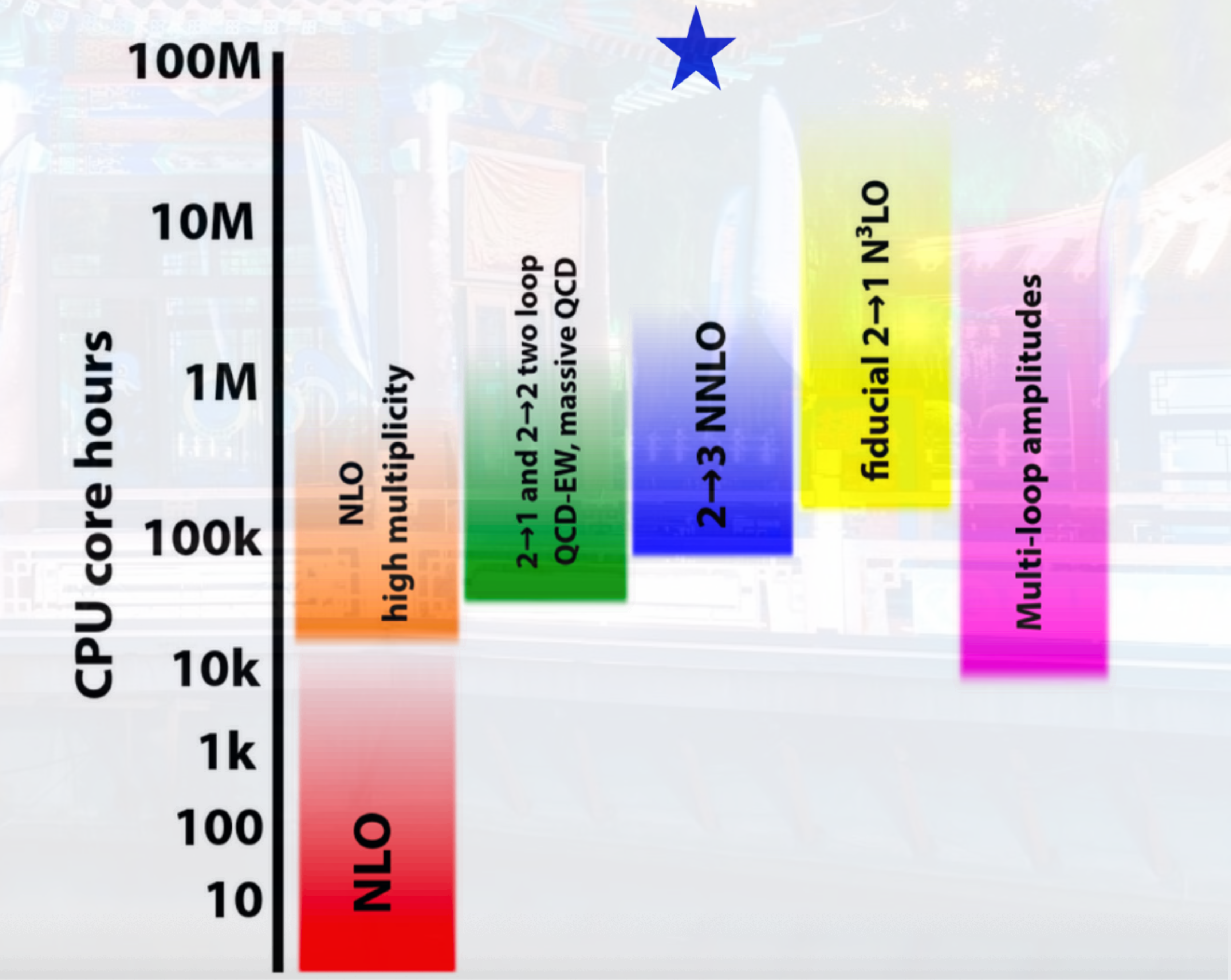
State-of-the-Art QCD Calculations @ NNLO

- NNLO QCD predictions for 2 → 2 processes (NNLO revolution since 2015)
  - Accomplished during past 10 years on case-by-case basis
  - As parton-level event generators (fully differential final state information)
  - Current frontier at NNLO 2 → 3

- Typical size of corrections and uncertainty
  - NLO corrections: 10~100%, uncertainty: 10~30%
  - NNLO corrections: 2~15%, uncertainty: 3~8%
  - expect N3LO to yield uncertainty at level of 1%

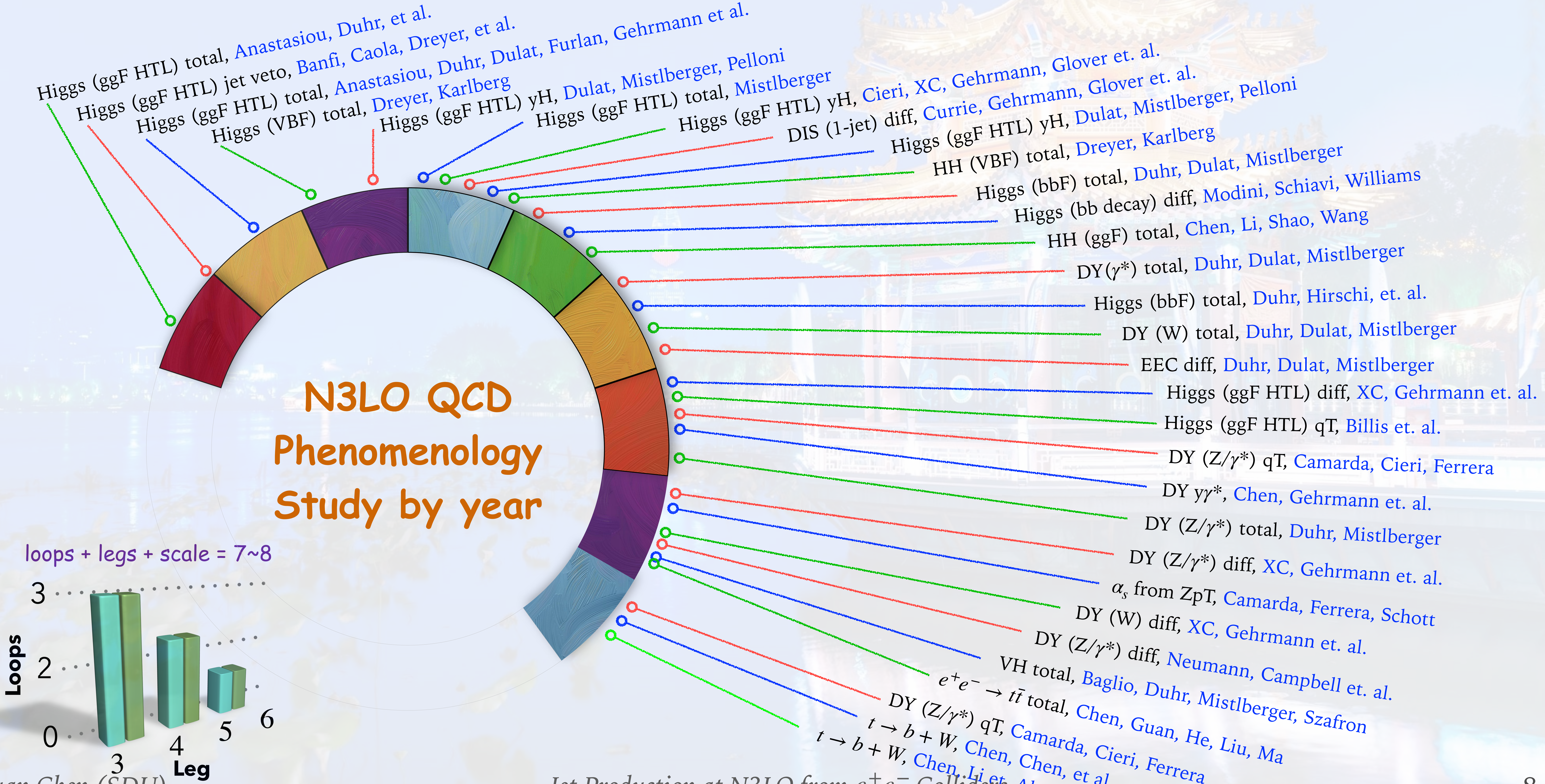
- So, is NNLO **solved**?
  - In principle yes: STRIPPER, given the relevant amplitudes and enough computational resources, the NNLO calculation is streamlined.
  - But:
    - Prohibitive computational cost (loop AMP, IR subtraction)
    - Missing cross-validation (many years between 1st and 2nd)
    - Still a long way to automated NNLO event generation

pp → jjj event shapes with STRIPPER

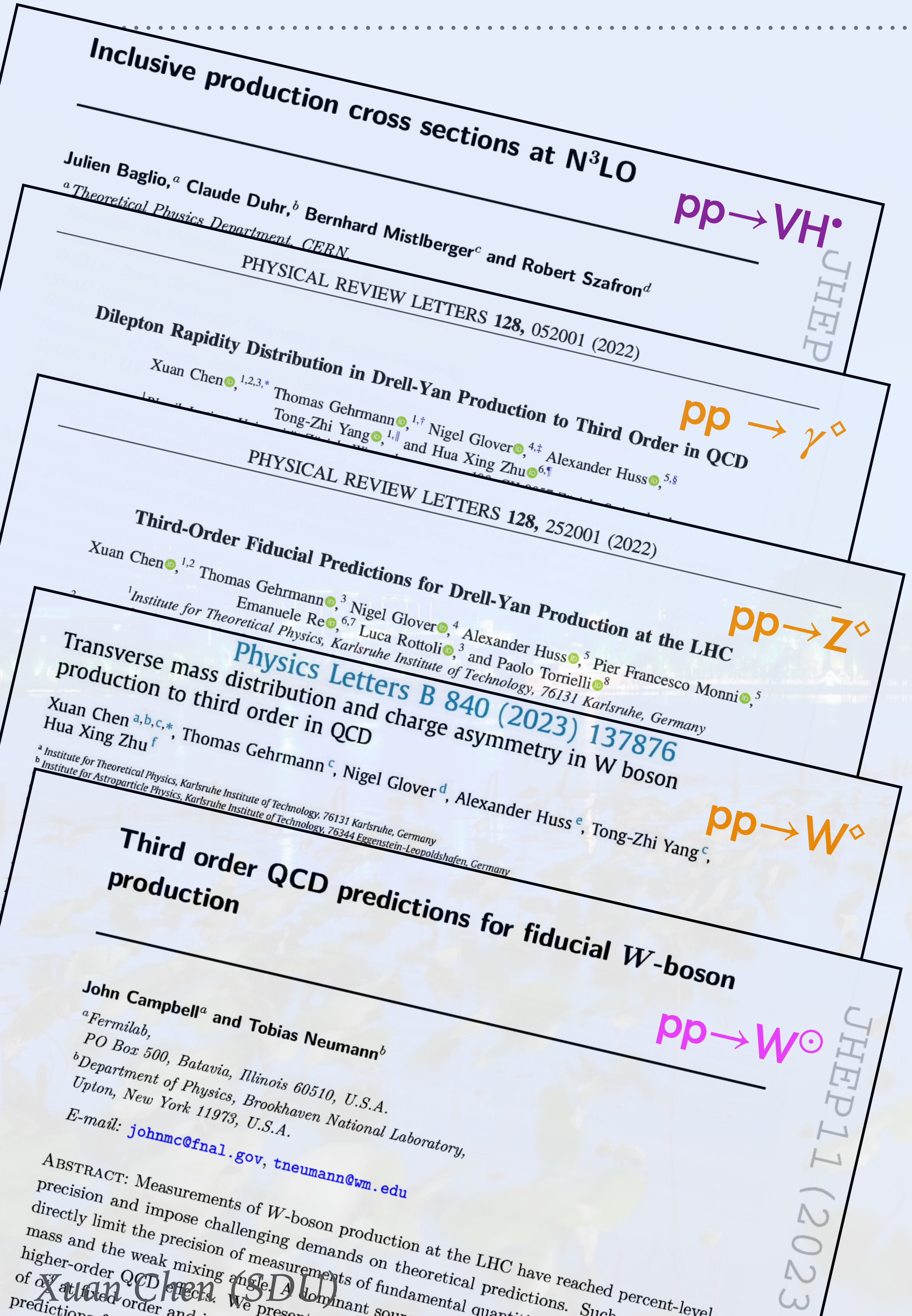


Snowmass White Paper, Comput. Softw. Big Sci. 6 (2022)

Perturbative QCD @ N3LO

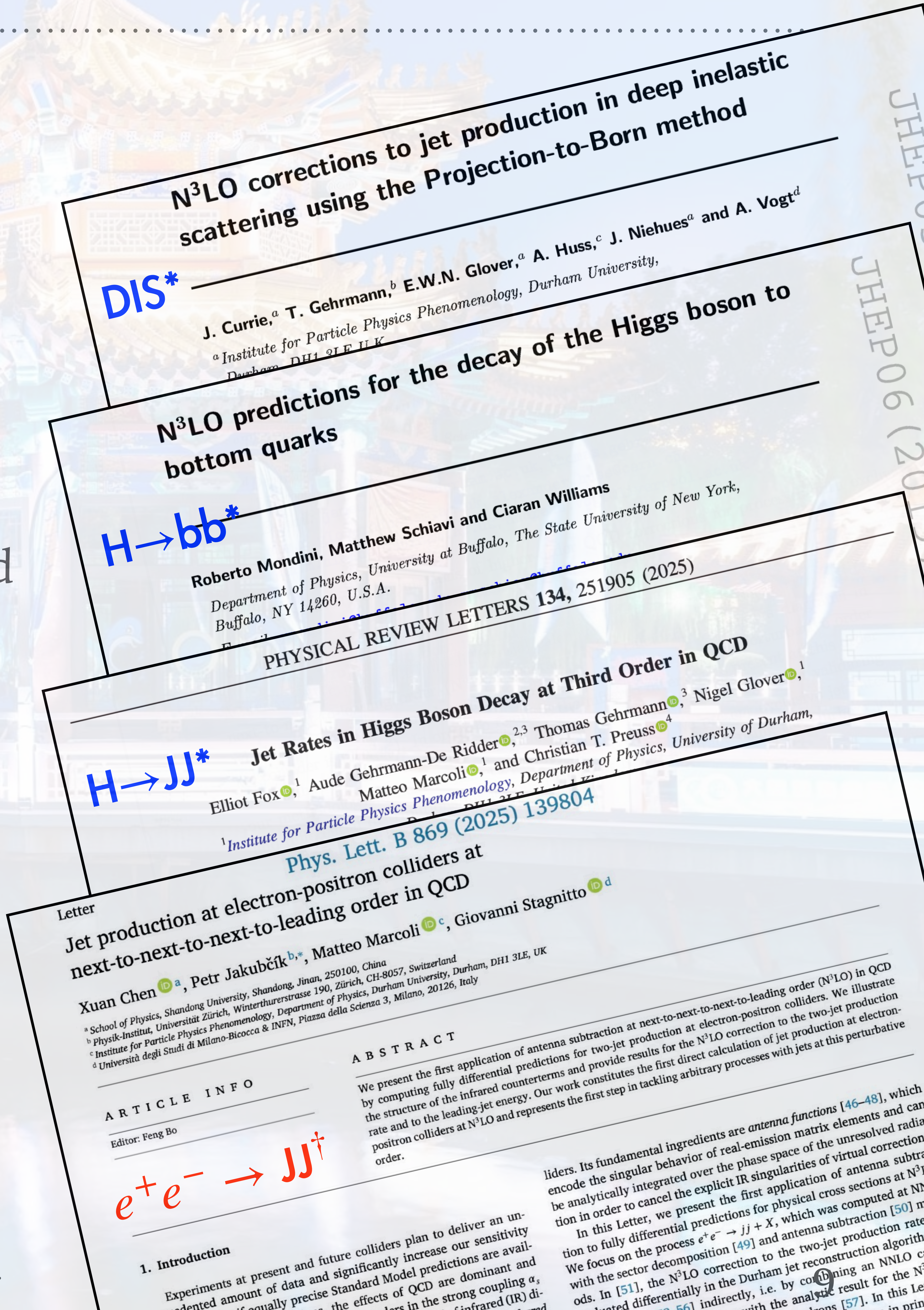


State-of-the-Art QCD Calculations @ N3LO



- Several phenomenologically relevant results despite the extreme complexity.
- Available techniques are applicable to limited cases with high quality EXP data.
- New approaches must be developed for more complicated scattering.

(10 M → X00 k CPU hours)



- • Inclusive
- ◇ qT slicing
- ⊙ τ slicing
- * Projection-2-Born
- † Antenna subtraction

NNLOJET: Parton Level Event Generator

NNLO
JET

A parton-level event generator for jet cross sections at NNLO QCD accuracy

About NNLOJET is a parton-level event generator for jet cross sections using the antenna subtraction method. It can be used to compute a large number of jet cross sections and related observables in e^+e^-, ep and pp collisions at next-to-next-to-leading order in QCD. NNLOJET contains routines for Monte Carlo phase-space integration, event handling and analysis.

Citation If you are using NNLOJET for a scientific paper, please cite:

A. Huss et al. (NNLOJET Collaboration)
NNLOJET: a parton-level event generator for jet cross sections at NNLO QCD accuracy
arXiv:2503.22804 [INSPIRE]

Please also cite the revelant references for each processes (as included in the .bib file which is automatically written when running NNLOJET through the automatic workflow)

License GNU General Public License (GPL) v3.0

Contact Please send comments, questions and suggestions to nnlojet-support@cern.ch

https://nnlojet.hepforge.org/index.html

A.Huss, L.Bonino, O.Braun-White, S.Caletti, XC, J.Cruz-Martinez, J.Currie, W.Feng, G.Fontana, E.Fox, R.Gauld, A.Gehrmann-De Ridder, T. Gehrmann, E.W.N.Glover, M.Höfer, P.Jakubcik, M.Jaquier, M.Löchner, F.Lorkowski, I.Majer, M.Marcoli, P.Meinzinger, J.Mo, T. Morgan, J.Niehues, J.Pires, C.Preuss, A.Rodriguez Gracia, K.Schönwald, R.Schürmann, V.Sotnikov, G.Stagnitto, D.Walker, J.Whitehead, T.Z.Yang, H.Zhang,

- NNLO parton level event generator
  - Based on antenna subtraction
- Provides infrastructure
  - Process management
  - Phase space, histogram routines
  - Validation and testing
  - Parallel computing (MPI) support
  - Typical runtimes: 60 k ~ 250 k core-hours

NNLOJET: Parton Level Event Generator

➤ Processes implemented:

***e^+e^-* scattering**	Jet production	• $e^+e^- \to$ 2jets • $e^+e^- \to$ 3jets
***ep* scattering**	Jet production	• ep → lepton + 1jet • ep → lepton + 2jets
***pp* scattering**	Jet production	• pp → 1jet + X • pp → 2jets
	Vector boson (+ jet) production	• pp → $(\gamma^* Z)$ + 0jet • pp → $(\gamma^* Z)$ + 1jet • pp → $W^\pm$ + 0jet • pp → $W^\pm$ + 1jet
	Photon (+ jet) production	• pp → γ + X • pp → γ + 1jet • pp → $\gamma\gamma$
	Higgs (+ jet) production	• pp → H + X • pp → H + 1jet

- ➤ Open-source code release: NNLOJET v1.0.2
  - ➤ Analytic matrix elements and subtraction
  - ➤ Download from nnlojet.hepforge.org
- ➤ Runcard options:
  - ➤ Process/sub-process selection
  - ➤ Generic histogramming
  - ➤ Multi-run feature: e.g. jet radius
  - ➤ Example runcards for published studies
- ➤ Cluster workflow management: Dokan
  - ➤ Automated resource allocation
  - ➤ Works with slurm and htcondor (lxplus)
  - ➤ Combination of results, quality control

https://github.com/aykhuss/dokan

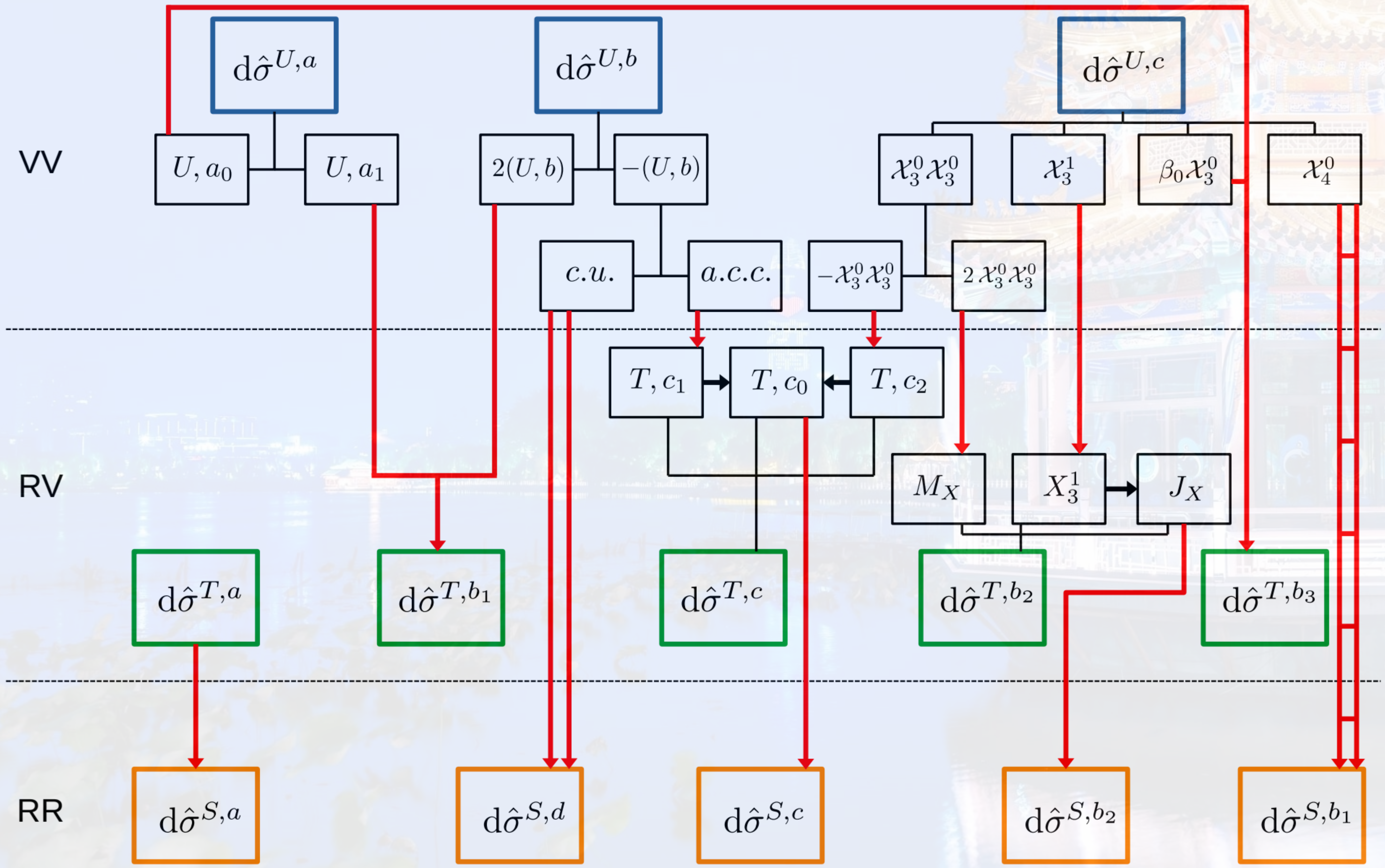
dokan (土管)

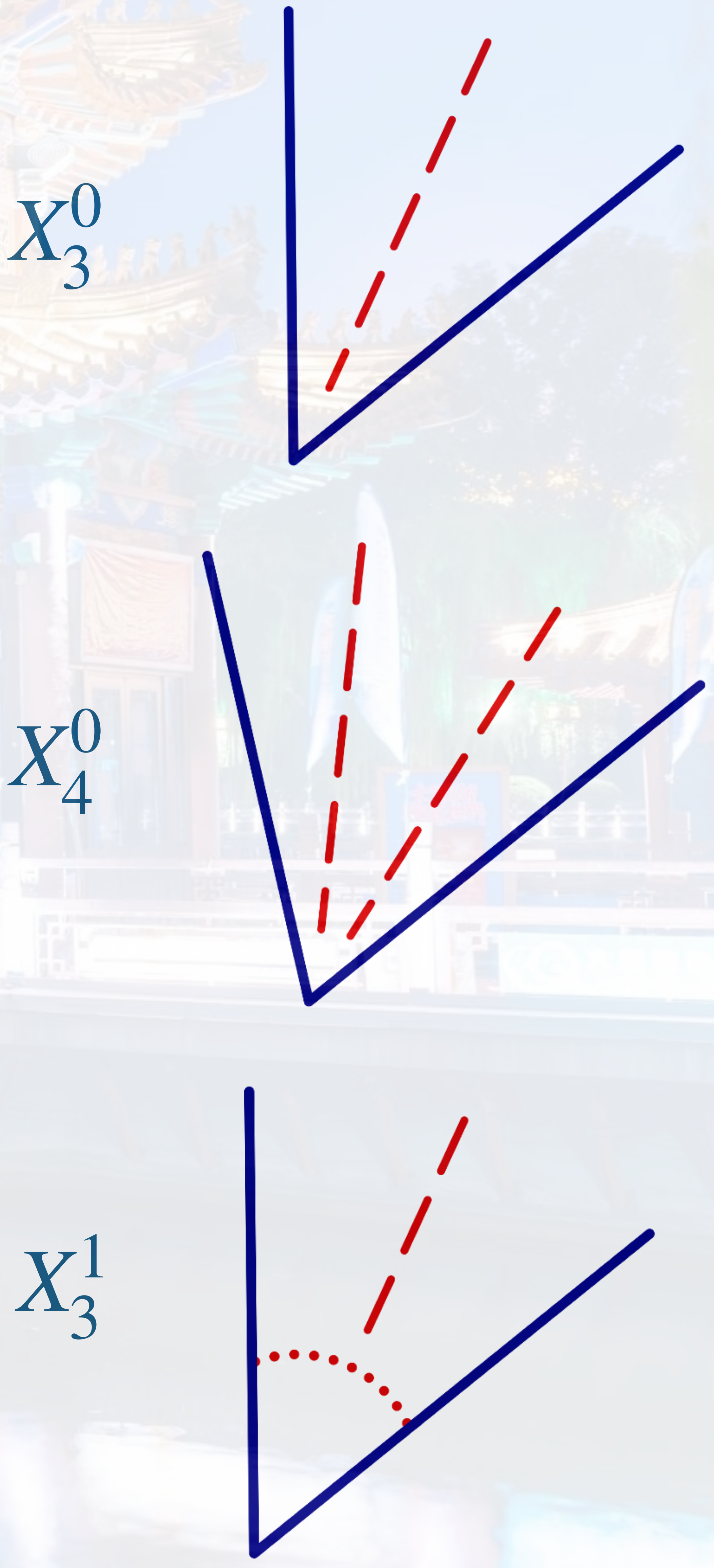
pypi v1.0.0 python missing

A pipeline for automating the NNLOJET workflow

#	LO	R	V	RR	RV	VV
1	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	WRM A[0] D[4]	PRD A[1] D[0]	PRD A[0] D[1]
2	PRD A[0] D[1]	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]	WRM A[1] D[3]	PRD A[0] D[1]
3	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
4	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]
5	-	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
6	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]	PRD A[0] D[1]
7	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
8	-	-	-	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
9	-	-	-	PRD A[0] D[0]	PRD A[1] D[0]	PRD A[0] D[1]
10	-	-	-	PRD A[1] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
11	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
12	-	-	-	PRD A[0] D[1]	-	-
13	-	-	-	PRD A[0] D[0]	PRD A[0] D[1]	PRD A[0] D[1]
14	-	-	-	PRD A[0] D[0]	-	-
15	-	-	-	PRD A[0] D[1]	WRM A[1] D[3]	PRD A[0] D[1]
16	-	-	-	PRD A[1] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
17	-	-	-	PRD A[0] D[0]	WRM A[1] D[3]	PRD A[0] D[1]
18	-	-	-	PRD A[1] D[0]	-	-
19	-	-	-	PRD A[0] D[1]	-	-
20	-	-	-	PRD A[0] D[1]	-	-
21	-	-	-	WRM A[1] D[3]	-	PRD A[0] D[1]
22	-	-	-	PRD A[0] D[1]	-	PRD A[0] D[1]
23	-	-	-	PRD A[1] D[0]	-	PRD A[0] D[1]
24	-	-	-	WRM A[1] D[3]	-	PRD A[0] D[1]
25	-	-	-	WRM A[1] D[3]	-	-
26	-	-	-	PRD A[1] D[0]	-	-
27	-	-	-	PRD A[0] D[1]	-	-

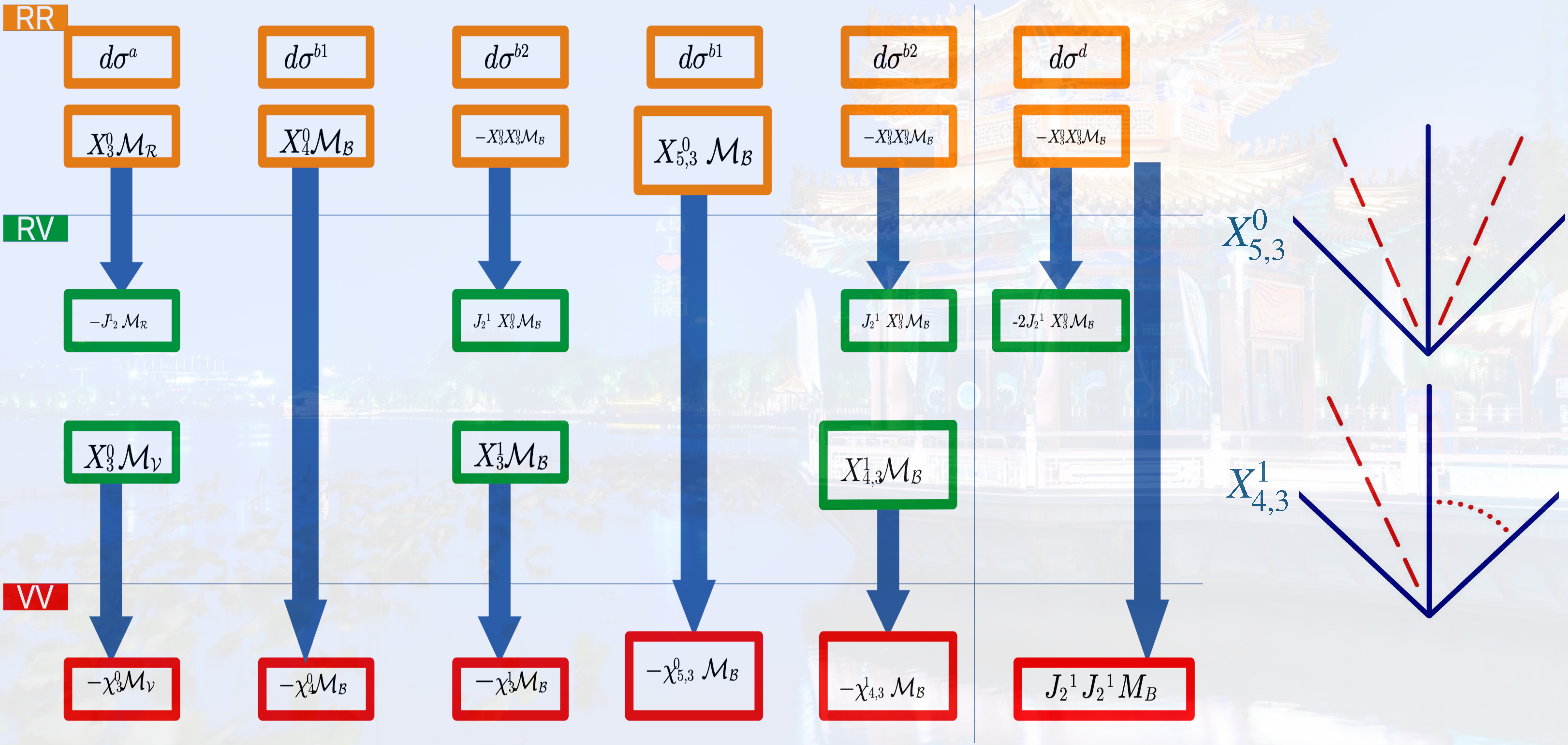
Antenna Subtraction @ NNLO





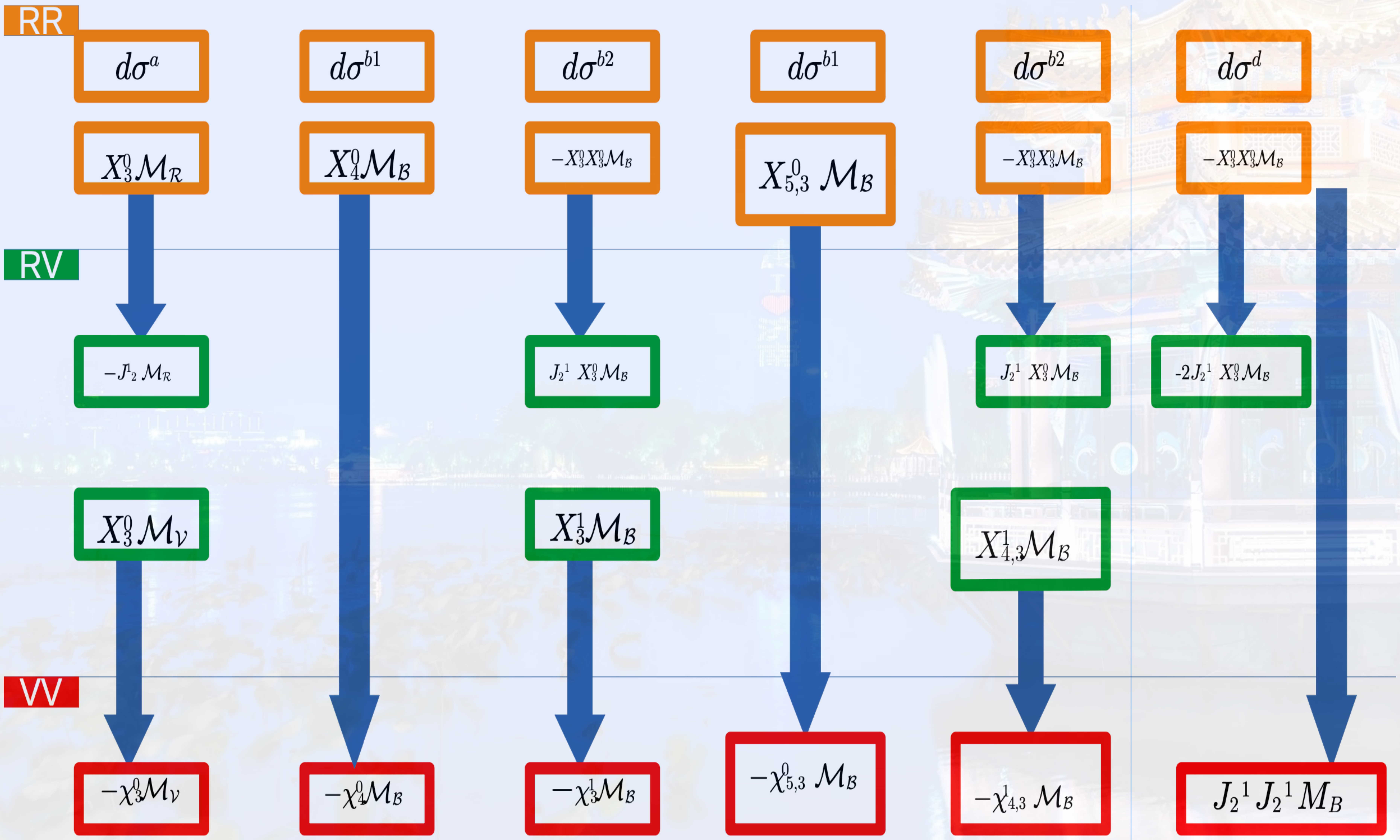
XC, Gehrmann, Glover, Huss, Marcoli JHEP 10 (2022) 099

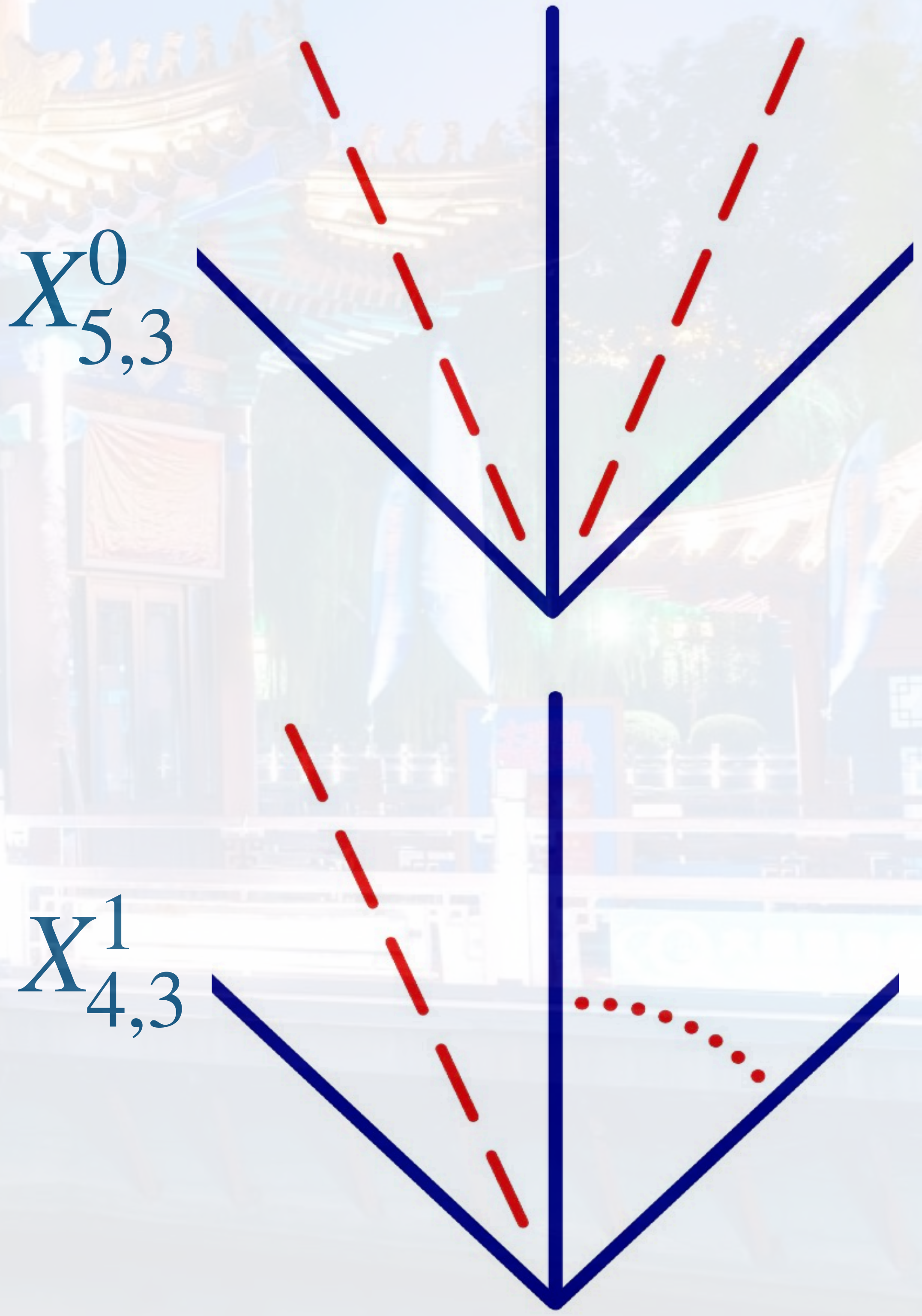
Generalized Antenna @ NNLO



Fox, Glover, Marcoli JHEP 12 (2024) 225

Generalized Antenna @ NNLO





$e^+e^- \to 3$ jets @ NNLO

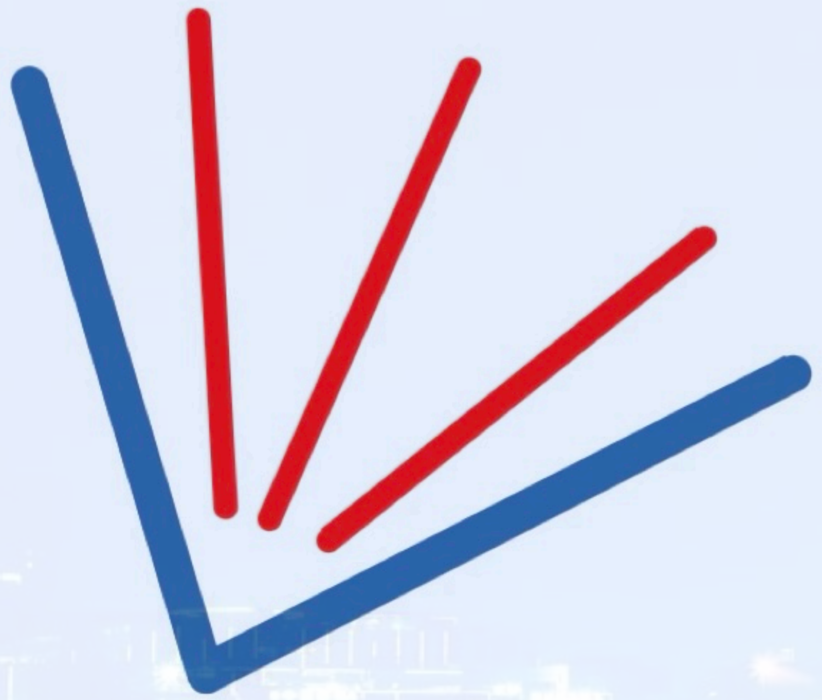
MC efficiency boosted by a factor of 10

Fox, Glover, Marcoli JHEP 12 (2024) 225

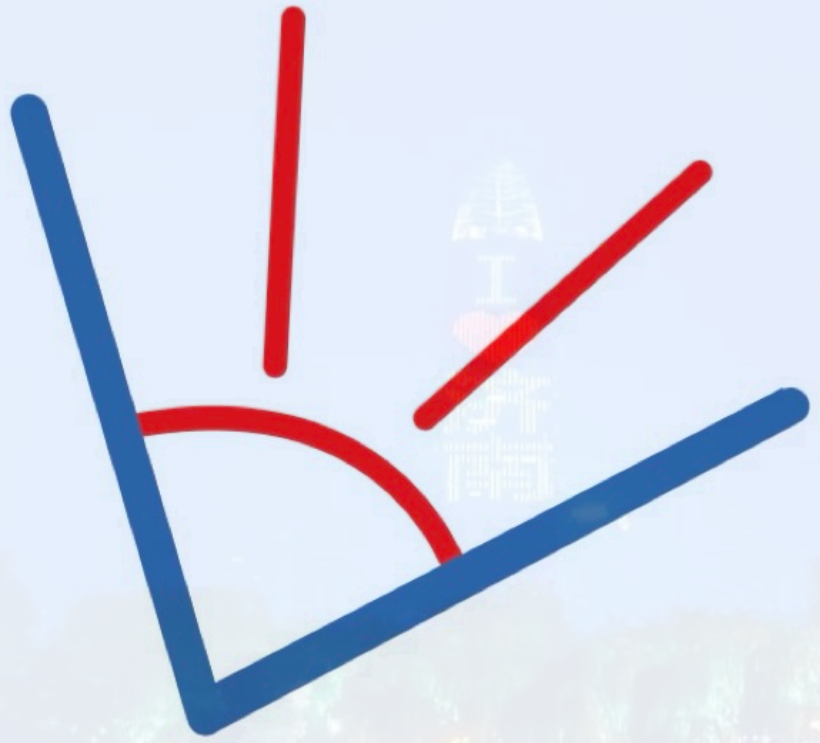
Antenna Subtraction @ N3LO

Topology of X_5^0, X_4^1, X_3^2 antenna functions:

X_5^0



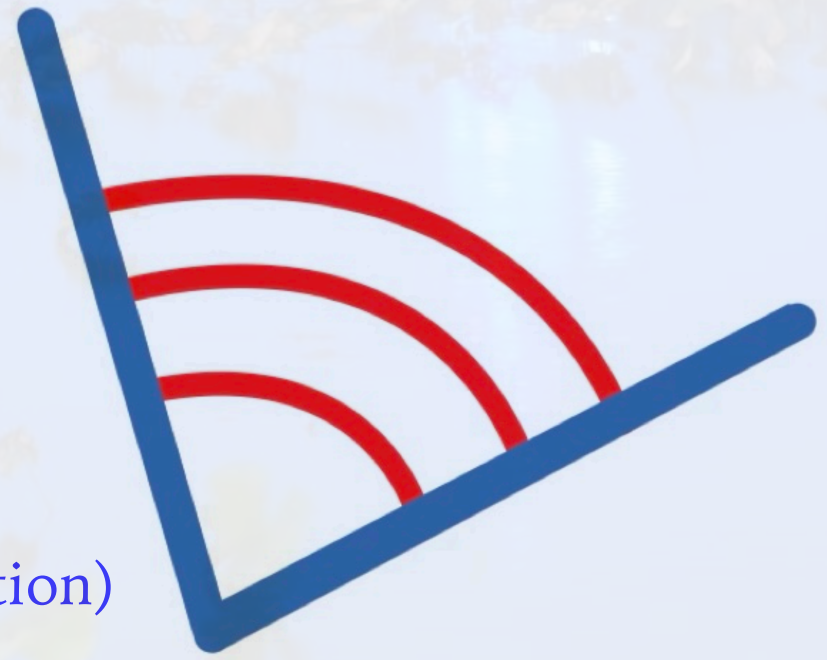
X_4^1



X_3^2

X_2^3

(Only for validation)



Integration of X_5^0, X_4^1, X_3^2 finished for all final states:

$\gamma^* \to q\bar{q}$

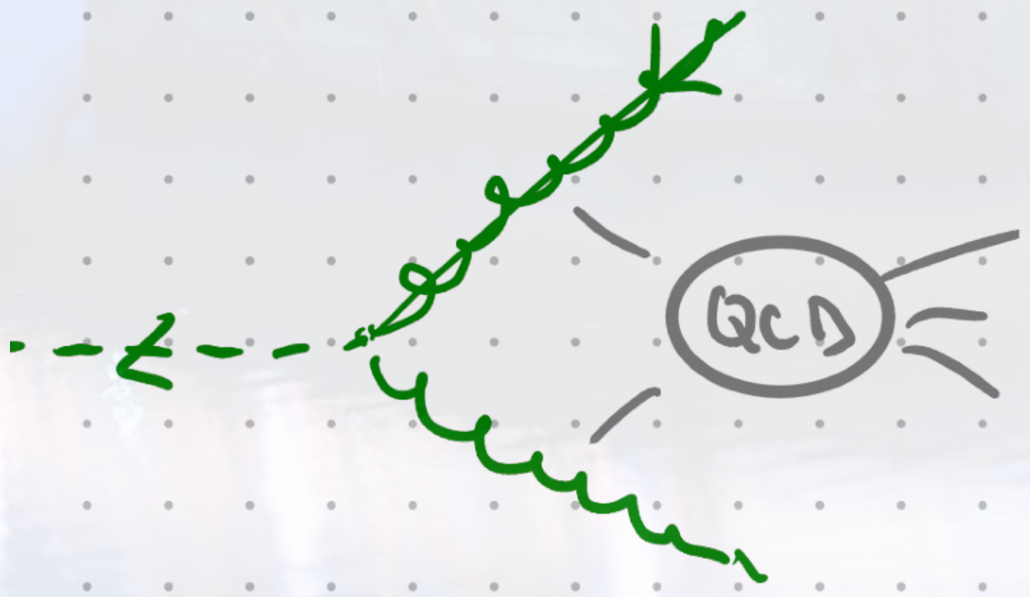
Jakubcik, Marcoli, Stagnitto

JHEP 01 (2023) 168

$H \to gg$

XC, Jakubcik, Marcoli, Stagnitto

JHEP 06 (2023) 192



$\chi \to \tilde{g}g$

XC, Jakubcik, Marcoli, Stagnitto

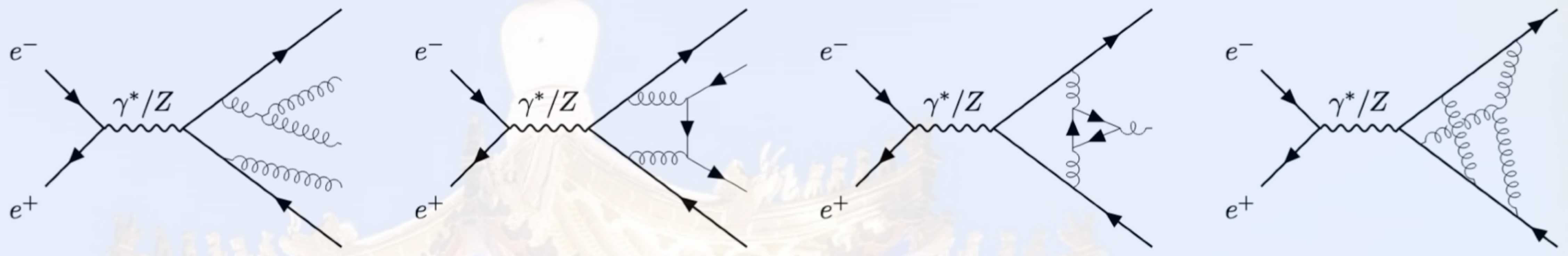
JHEP 12 (2023) 198

Application of NNLOJET in e^+e^- Colliders
(Close Relation to Antenna Subtraction)

$e^+e^- \to$ di-jet @ N3LO	(Antenna Subtraction) 2505.10618
H decay $\to$ di-jet @ N3LO	(Generalized Antenna) 2502.17333, 2508.14282, 2510.11665
$e^+e^- \to$ ZH @ NNLO	(Generalized Antenna) 2510.20485

$e^+e^- \to$ JJ @ N3LO

- Simple process:
  - Only $\gamma^* \to q\bar{q}$ N3LO antenna functions
  - Only dipole-like correlations at N3LO
  - Recycle ingredients from $e^+e^- \to JJJ$ @ NNLO
- Goals:
  - Establish N3LO antenna subtraction framework
    - Extension of NNLO framework
    - Introduce sector antenna mapping to remove the requirement of sub-antenna functions
  - Exploration of numerical challenges:
    - One-loop double-unresolved regions
    - Two-loop single-unresolved regions
    - Rescue-system to trigger:
      1, Quadruple precision
      2, Taylor expansion of special functions
  - Preparation of computational framework:
    - Spike tests of multiple unresolved IR limits
    - Phase space generators
    - Code generation for N3LO MC.



(a) triple-real (b) double-real-virtual (c) real-double-virtual (d) triple-virtual

$$\mathrm{d}\sigma_{N^3LO} = \int_n [\mathrm{d}\sigma^{VVV} - \mathrm{d}\sigma^{W}] + \int_n [\mathrm{d}\sigma^{RVV} - \mathrm{d}\sigma^{U}]$$

triple-virtual subtraction term

double-virtual real subtraction term

$$+ \int_{n+1} [\mathrm{d}\sigma^{RRV} - \mathrm{d}\sigma^{T}] + \int_{n+2} [\mathrm{d}\sigma^{RRR} - \mathrm{d}\sigma^{S}]$$

double-real-virtual subtraction term

triple-real subtraction term

$$\mathrm{d}\sigma^{S} = \mathrm{d}\sigma^{S_1} + \mathrm{d}\sigma^{S_2} + \mathrm{d}\sigma^{S_3}$$

$$\mathrm{d}\sigma^{T} = \mathrm{d}\sigma^{V_1S_1} + \mathrm{d}\sigma^{V_1S_2} - \int_1 \mathrm{d}\sigma^{S_1}$$

$$\mathrm{d}\sigma^{U} = \mathrm{d}\sigma^{V_2S_1} - \int_1 \mathrm{d}\sigma^{V_1S_1} - \int_2 \mathrm{d}\sigma^{S_2}$$

$$\mathrm{d}\sigma^{W} = -\int_1 \mathrm{d}\sigma^{V_2S_1} - \int_2 \mathrm{d}\sigma^{V_1S_2} - \int_3 \mathrm{d}\sigma^{S_3}$$

XC, Jakubcik, Marcoli, Stagnitto, Phys. Lett. B. *869 (2025) 139804*

XC, Marcoli, 2507.12537

$e^+e^- \to$ JJ @ N3LO

- Fully working subtraction terms for all partonic channels:
  - Spike test in IR limits of RRR, RRV and RVV

$$t_{RRR} = \log_{10}\left(\left|1 - \frac{M_{RRR}}{S_{RRR}}\right|\right)$$

  - Green → Blue → Red: deeper in IR divergence, better cancellation between ME and Subtraction terms.
  - Use sector antenna mapping in RRR and RRV:

$\tilde{X}_4$ sector: $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}$ @RRV

(a) $s_{12}s_{34} \le s_{13}s_{24}$: $\{4 \to 2\}$ mapping with ordering $\{p_1^h, p_2, p_3, p_4^h\}$.

(b) $s_{12}s_{34} > s_{13}s_{24}$: $\{4 \to 2\}$ mapping with ordering $\{p_1^h, p_3, p_2, p_4^h\}$.

$\tilde{\tilde{X}}_5$ sector: $e^+e^- \to q\bar{q}\tilde{g}\tilde{g}\tilde{g}$ @RRR

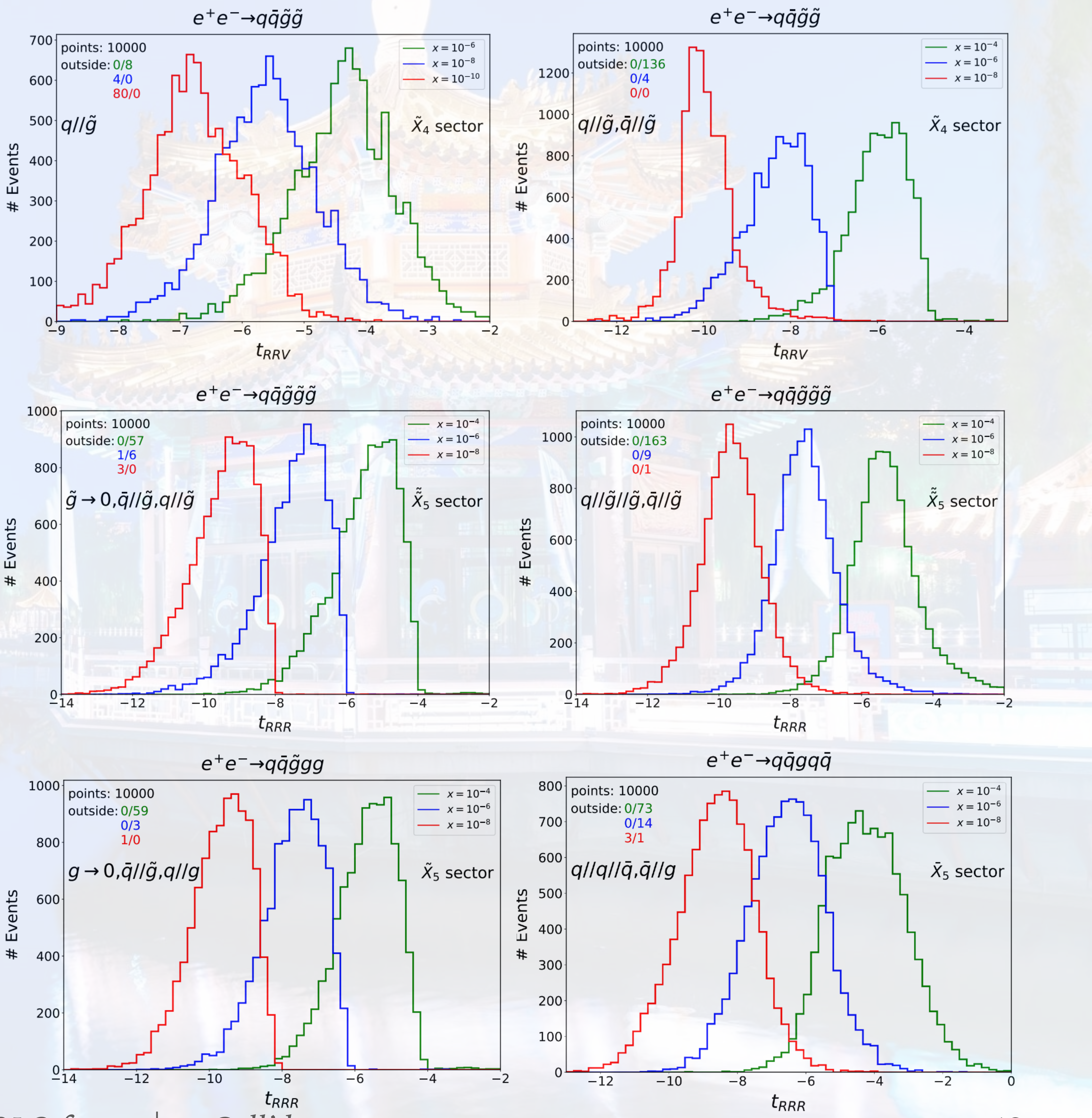
$\tilde{X}_5$ sector: $e^+e^- \to q\bar{q}\tilde{g}gg$ @RRR

$\bar{X}_5$ sector: $e^+e^- \to q\bar{q}\tilde{g}q\bar{q}$ @RRR

- Explicit pole cancellation checked analytically for RRV, RVV and VVV

XC, Jakubcik, Marcoli, Stagnitto, Phys. Lett. B. *869 (2025) 139804*

XC, Marcoli, 2507.12537



$e^+e^- \to$ JJ @ N3LO

➤ Basic checks: inclusive cross section

N3LO coefficient:

$$\sigma^{(3)} = \sigma^{(0)}\left(\frac{\alpha_s}{2\pi}\right)^3(-105 \pm 11)$$

Monte Carlo error:

Not so small for inclusive quantities due to large cancellations.

Not the most clever way to compute inclusive cross sections.

XC, Jakubcik, Marcoli, Stagnitto, Phys. Lett. B. *869 (2025) 139804*

$$\sigma^{(3)} = \sigma^{(0)}\left(\frac{\alpha_s}{2\pi}\right)^3(-102.14\cdots)$$

Chetyrkin, Künn, Kwiatkowski, Phys. Rept. *277 (1996) 189*

N3LO 2-jet rate:

Exclusive n-jet rate @ N3LO: $$R_n^{(3)}(y_{cut}) = \frac{\Gamma^{(3)}_{\gamma^*\to n\ jets}(y_{cut})}{\Gamma^{(3)}_{\gamma^*\to hadrons}}$$

For back-to-back QCD emissions, we have at least two jets $\to n \geq 2$

$$R_2^{(3)}(y_{cut})\Gamma^{(3)}_{\gamma^*\to hadrons} = \int_0^{y_{cut}} \frac{d\sigma}{dy_{23}}dy_{23}$$

with $y_{ij} = \frac{2\min(E_i^2, E_j^2)}{Q^2}(1-\cos\theta_{ij})$

$$R_3^{(3)}(y_{cut})\Gamma^{(3)}_{\gamma^*\to hadrons} = \int_{y_{cut}}^1 \frac{d\sigma}{dy_{23}}dy_{23} - \int_{y_{cut}}^1 \frac{d\sigma}{dy_{34}}dy_{34}$$

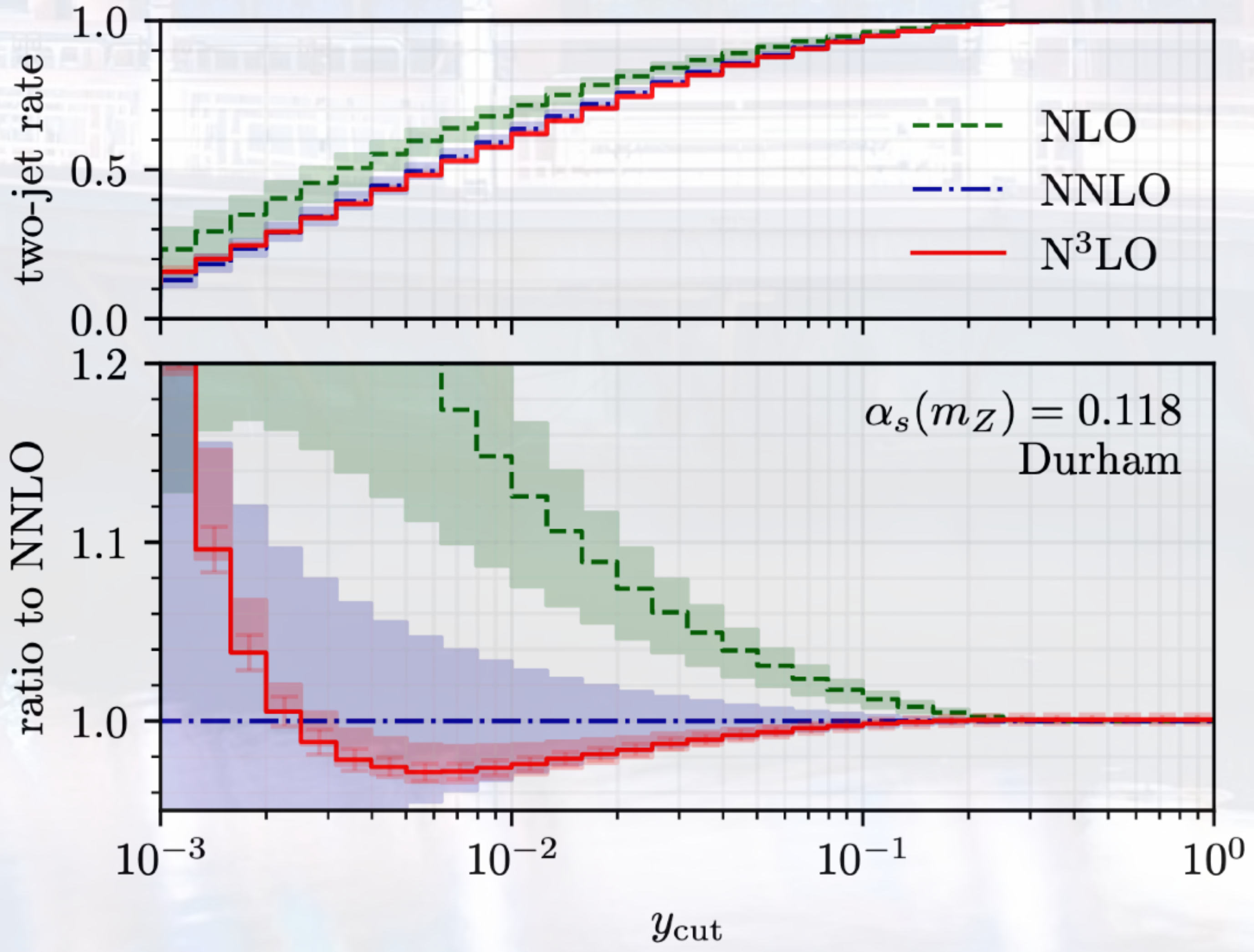
$$R_4^{(3)}(y_{cut})\Gamma^{(3)}_{\gamma^*\to hadrons} = \int_{y_{cut}}^1 \frac{d\sigma}{dy_{34}}dy_{34} - \int_{y_{cut}}^1 \frac{d\sigma}{dy_{45}}dy_{45}$$

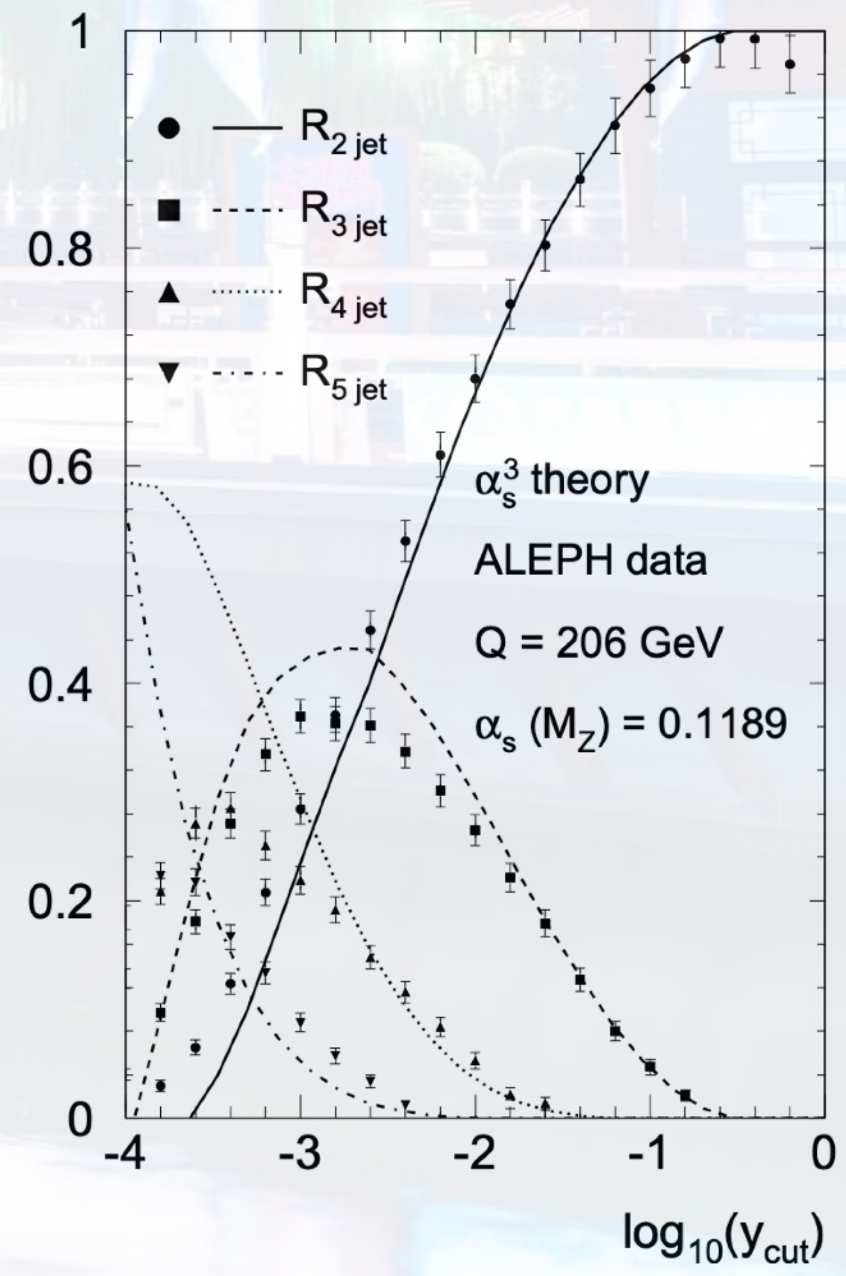
$$R_5^{(3)}(y_{cut})\Gamma^{(3)}_{\gamma^*\to hadrons} = \int_{y_{cut}}^1 \frac{d\sigma}{dy_{45}}dy_{45}$$

$$\Rightarrow \sum_{n=2}^{m+2} R_n^{(m)}(y_{cut}) = 1$$

Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Phys. Rev. Lett. *100 (2008) 172001*

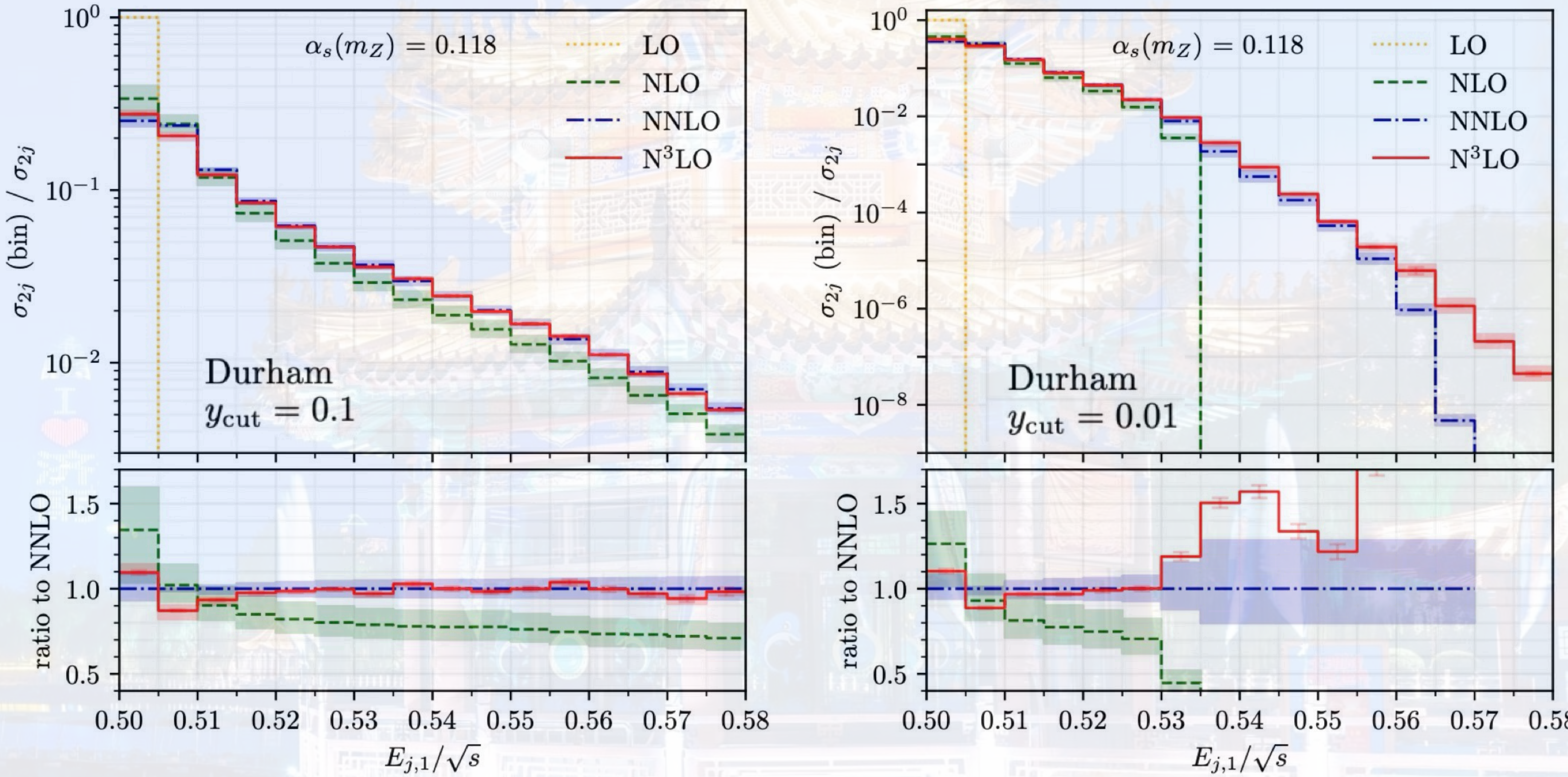
Full agreement between direct and indirect calculation of $R_2^{(3)}(y_{cut})$



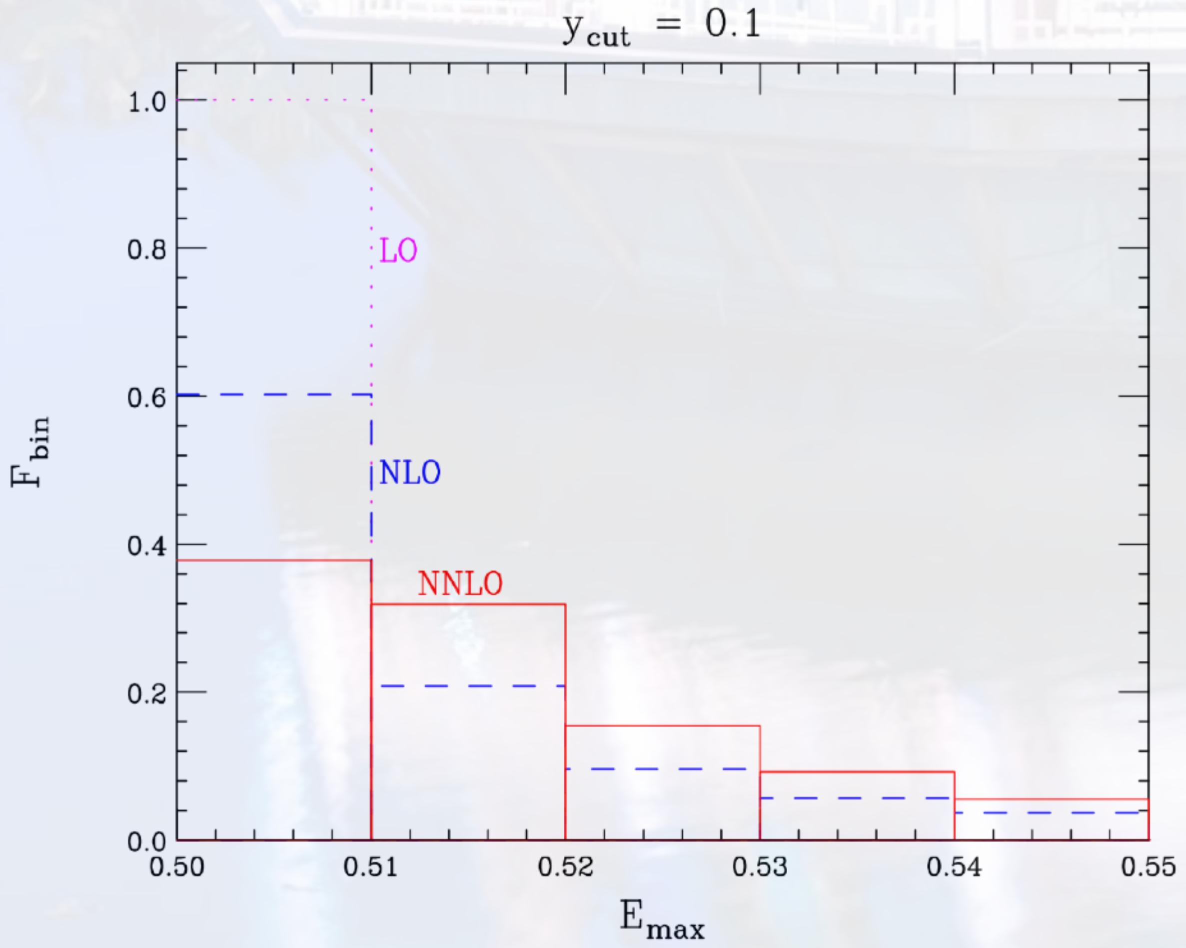


$e^+e^- \to$ JJ @ N3LO

- Differential observable:
  - Leading-jet energy E_{j1}
    - Defined on 2-jet events, bin-integrated and normalized to exclusive 2-jet XS
    - Lower orders vanish faster at high E_{j1} for smaller y_{cut} due to energetic leading jet recoil against multiple emissions
    - Very good convergence for large y_{cut}
    - The distribution can be obtained by combining $e^+e^- \to$ JJ @ NNLO and N3LO inclusive XS
- Future plan:
  - Include in public release
  - Jet forward-backward asymmetry
  - $e^+e^- \to$ JJJ @ N3LO



XC, Jakubcik, Marcoli, Stagnitto, Phys. Lett. B. *869 (2025) 139804*



Full agreement up to NNLO with

Anastasiou, Melnikov, Petriello, Phys. Rev. Lett. *93 (2004) 032002*

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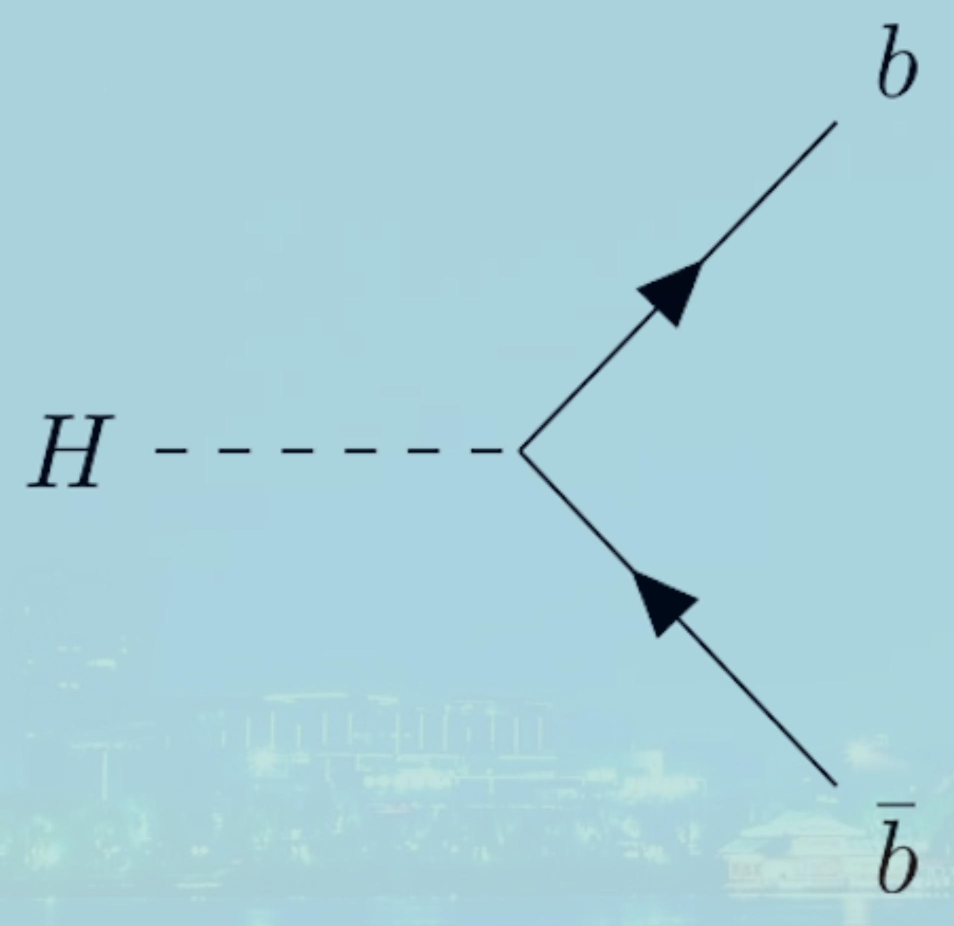
Higgs → JJ @ N3LO

Marcoli's slide @ Loop Summit 2

We consider a Higgs boson at rest (neglect production mode) decaying hadronically.

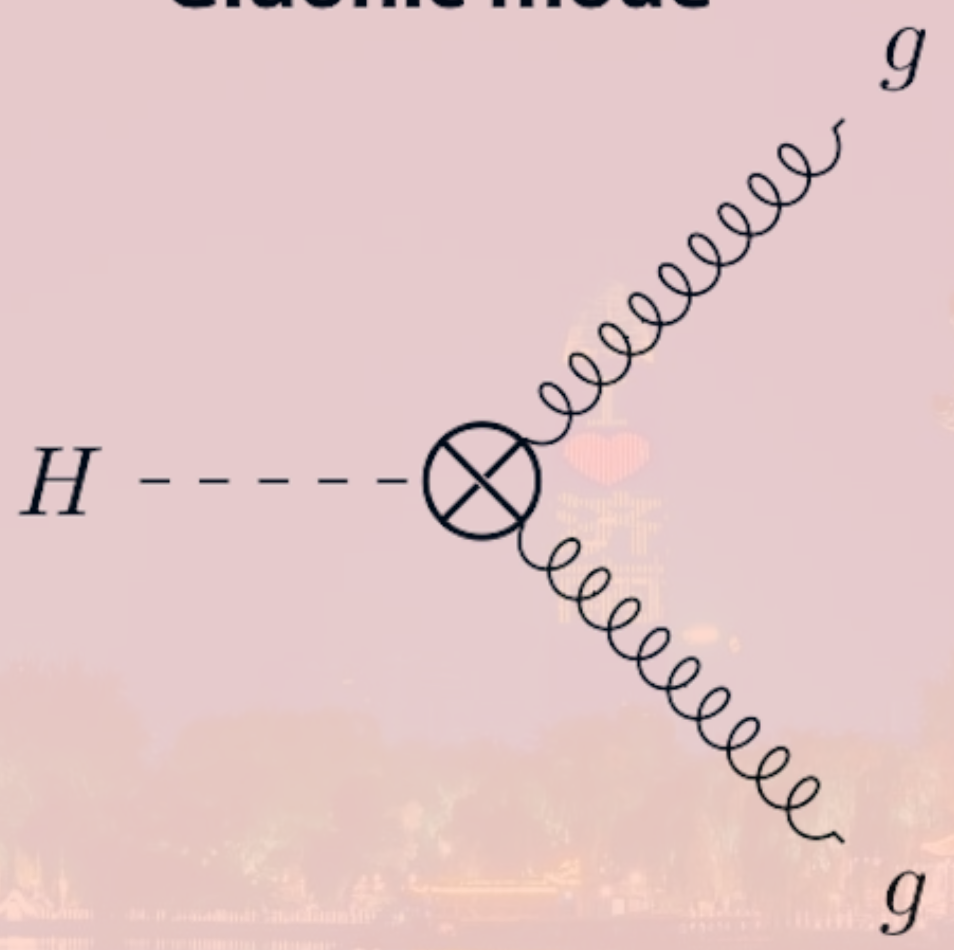
Yuakawa mode



$$\Gamma^{(0)}_{H\to b\bar{b}} = \frac{m_b^2(\mu_R) m_H N_c}{8\pi v^2}$$

- massless b (apart from Yuakwa interaction) ***
- analogous contribution from charm

Gluonic mode



$$\Gamma^{(0)}_{H\to gg} = \frac{\alpha_s^2(\mu_R) m_H^3 (N_c^2 - 1)}{576\pi^3 v^2}$$

- effective vertex: infinite top mass limit
- finite t, b and c mass and EW vertex corrections included by rescaling

Inclusive decay widths at order k in QCD:

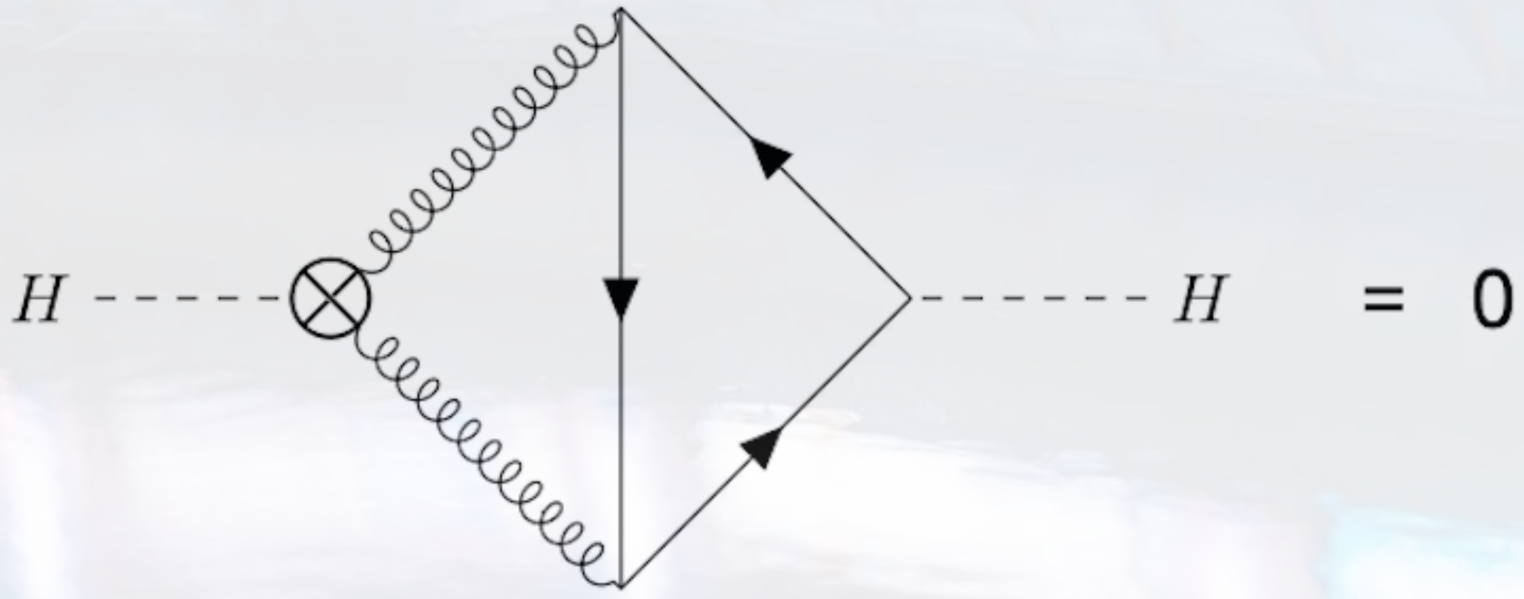
$$\Gamma^{(k)}_{H\to b\bar{b}} = \Gamma^{(0)}_{H\to b\bar{b}} \left(1 + \sum_{n=1}^{k} \alpha_s^n(\mu_R) C^{(n)}_{b\bar{b}}\right)$$

$$\Gamma^{(k)}_{H\to gg} = \Gamma^{(0)}_{H\to gg} \left(1 + \sum_{n=1}^{k} \alpha_s^n(\mu_R) C^{(n)}_{gg}\right)$$

Expansion coefficients know up to k=4

Herzog, Ruijl, Ueda, Vermaseren, Vogt JHEP 08 (2017) 113

*** The interference between the two modes vanishes. We verified that it is anyway negligible for the observables we consider.



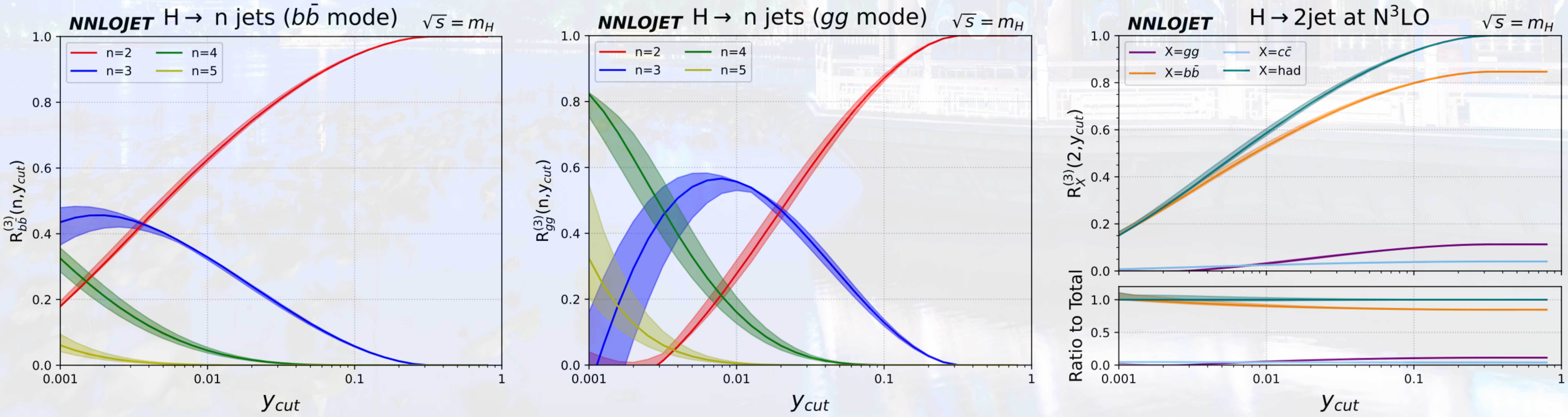
Higgs → JJ @ N3LO

- We compute the decay of a Higgs boson into three jets up to NNLO in QCD
- From this we can extract the 3-jet rate at NNLO and 2-jet rate at N3LO
- Previous calculation in the Yukawa mode. Novel results in the gluonic mode.

Mondini, Williams, JHEP 06 (2019) 120
Mondini, Schiavi, Williams, JHEP 06 (2019) 079

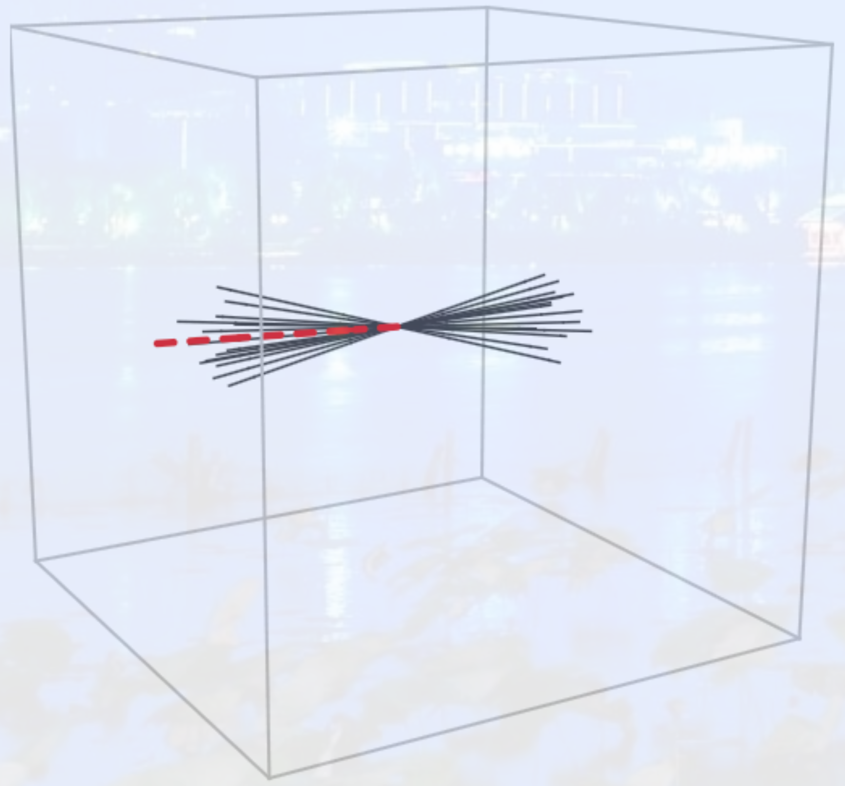
Fox, Gehrmann-De Ridder, Gehrmann, Glover, Marcoli, Preuss
Phys.Rev.Lett. 134 (2025)251905

Physical parameters: $y_b(m_H) = m_b(m_H)/vev = 0.011309$ $y_c(m_H) = m_c(m_H)/vev = 0.0024629$
$m_H = 125.09$ GeV $m_Z = 91.2$ GeV $vev = 246.22$ GeV $\alpha_s(m_z) = 0.118$ $m_t(m_H) = 166.48$ GeV



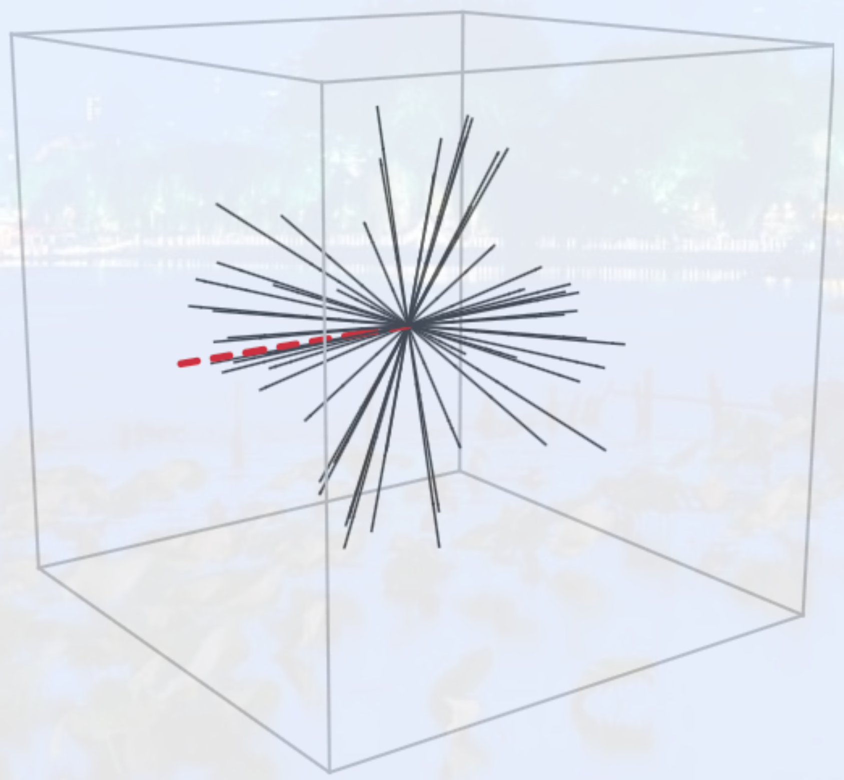
Higgs → JJ @ N3LO

- Thrust distribution from Higgs decay
  - Fraction of gg mode enhanced in the high multiplicity hard region
  - Perturbative predictions for the gluonic mode breaks down earlier than for the Yukawa one in pencil-like regions
  - All-order resummation effects are important in the back-to-back region ($\tau \to 0$)



$T = 0.998\,, \quad \tau = 0.002$

pencil-like
back to back

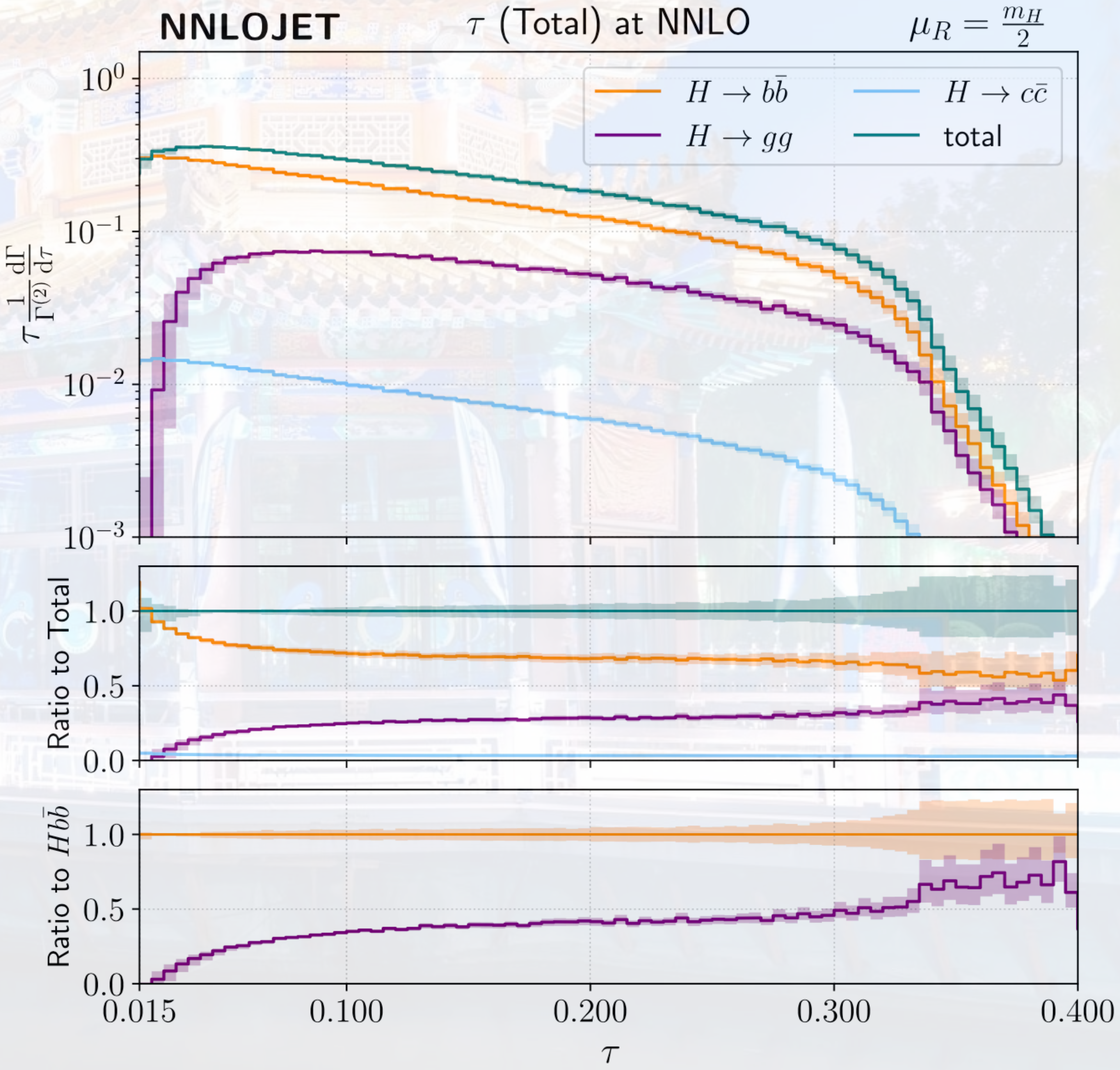


$T = 0.65\,, \quad \tau = 0.35$

spherical
isotropic

$$T = \max_{\vec{n}} \left(\frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i} \right)$$

$$\tau \equiv 1 - T = \min_{\vec{n}} \left(1 - \frac{\sum_i \vec{p}_i \cdot \vec{n}}{\sum_i \vec{p}_i} \right)$$



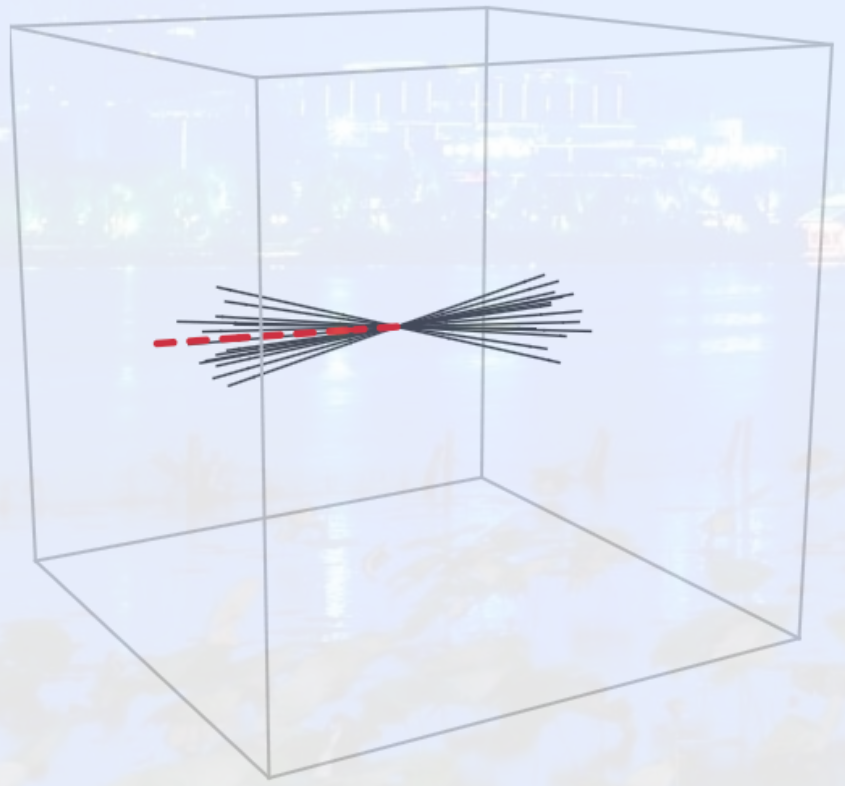
pencil-like
back to back

spherical
isotropic

Fox, Gehrmann-De Ridder, Gehrmann, Glover, Marcoli, Preuss [2508.14282]

Higgs → JJ @ N3LO

- ➤ Thrust distribution from Higgs decay
  - ➤ Fraction of gg mode enhanced in the high multiplicity hard region
  - ➤ Perturbative predictions for the gluonic mode breaks down earlier than for the Yukawa one in pencil-like regions
  - ➤ All-order resummation effects are important in the back-to-back region ($\tau \to 0$)



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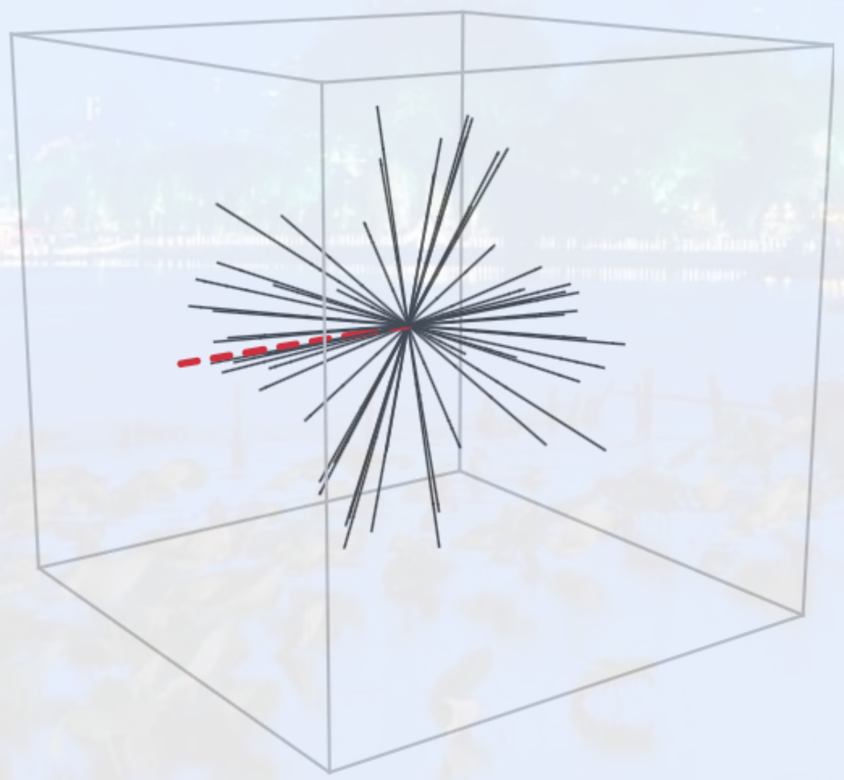

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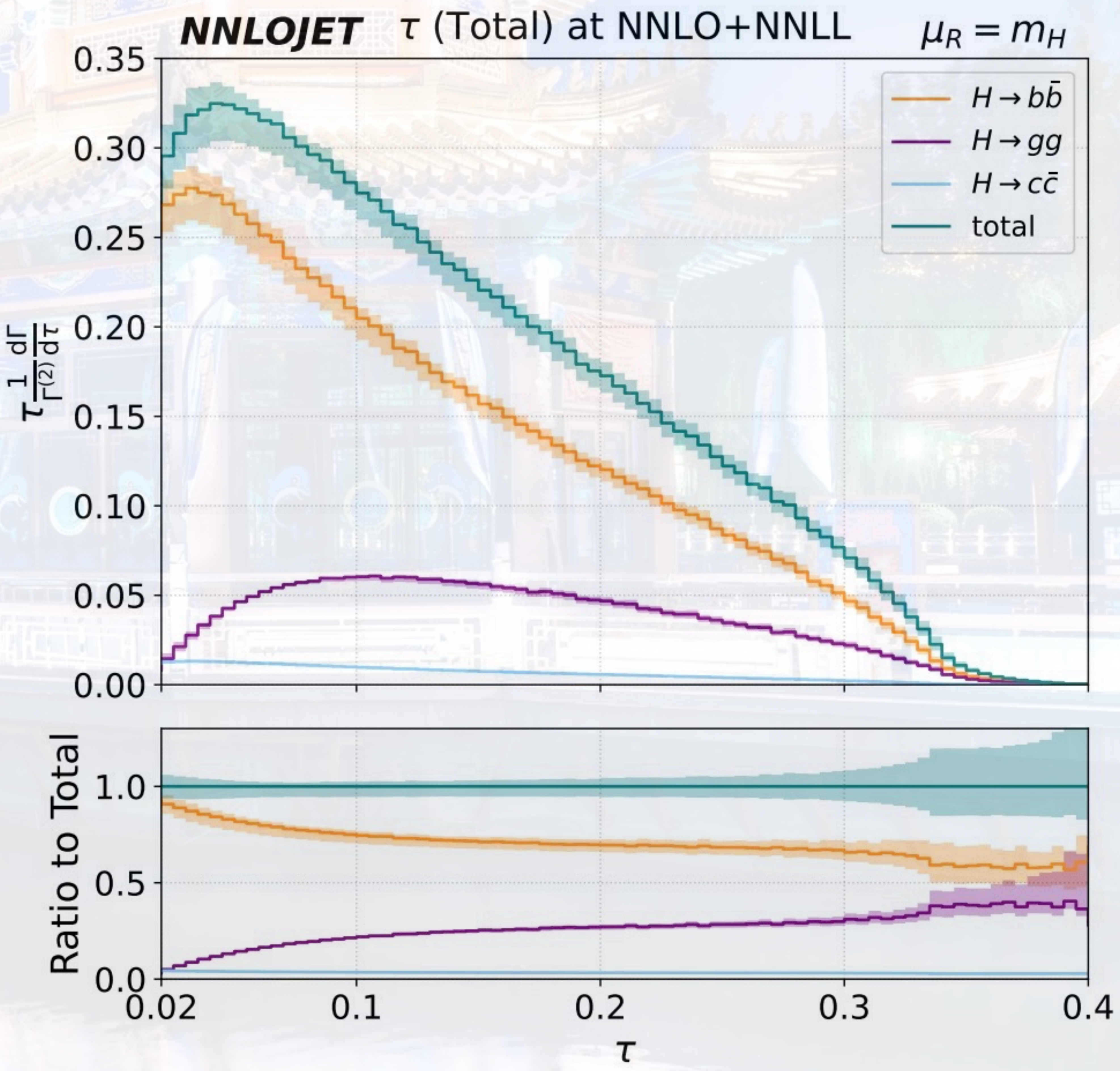


Fox, Gehrmann-De Ridder, Gehrmann, Glover, Marcoli, Preuss [2510.11665]

Application of NNLOJET in e^+e^- Colliders

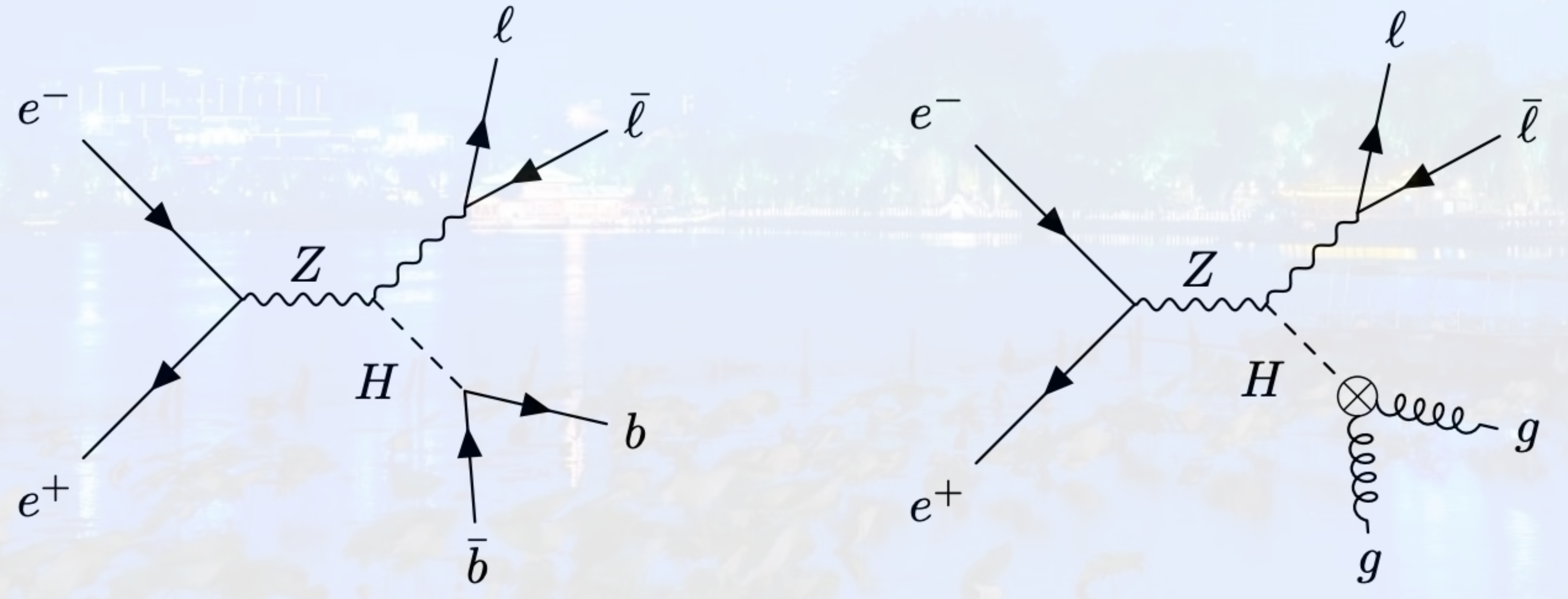
(Close Relation to Antenna Subtraction)

$e^+e^- \to$ di-jet @ N3LO	(Antenna Subtraction) 2505.10618
H decay $\to$ di-jet @ N3LO	(Generalized Antenna) 2502.17333, 2508.14282, 2510.11665
$e^+e^- \to$ ZH @ NNLO	(Generalized Antenna) 2510.20485

$e^+e^- \to$ ZH @ NNLO

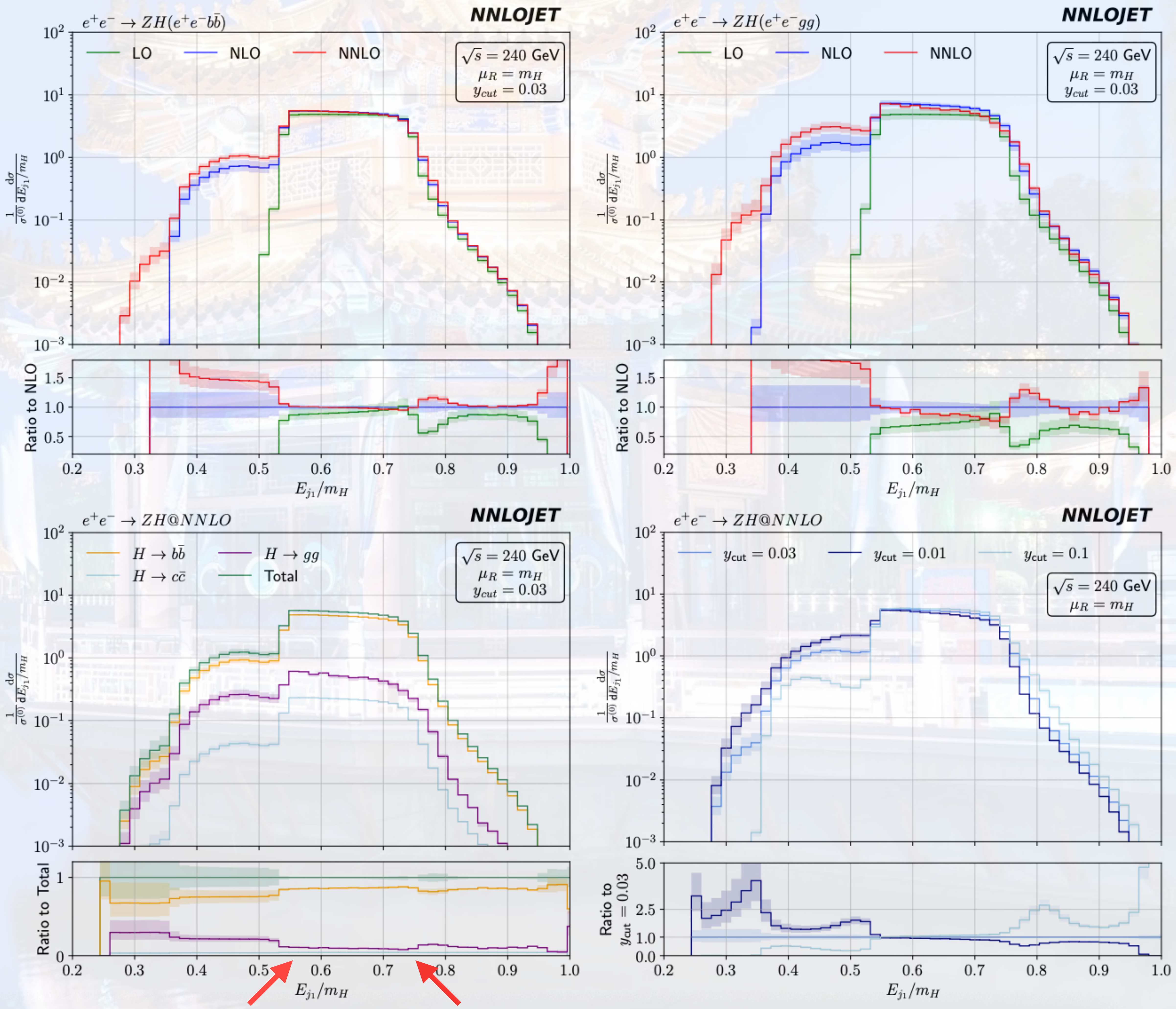
- $e^+e^- \to ZH \to l^+l^- +$ Jets
  - The main Higgs production channel
  - Consider production@LO + decay@NNLO
  - Boosted decay frame leads to rich phenomenology
  - Comparison between Yukawa and gluonic mode

Leading Order Kinematics



$$\frac{E_\pm^{lab.}}{m_H} = \frac{1}{2}\left(\gamma \pm \sqrt{\gamma^2-1}\,\left|cos\theta\right|\right) \qquad cos\theta_{j_1 j_2} = -\frac{1-2\beta^2+\beta^2 cos^2\theta}{1-\beta^2 cos^2\theta}$$

$$\text{with} \qquad \gamma = \frac{s+m_H^2-m_Z^2}{2m_H\sqrt{s}}, \qquad \beta = \sqrt{1-\frac{1}{\gamma^2}}$$



≈ 0.54 ≈ 0.75

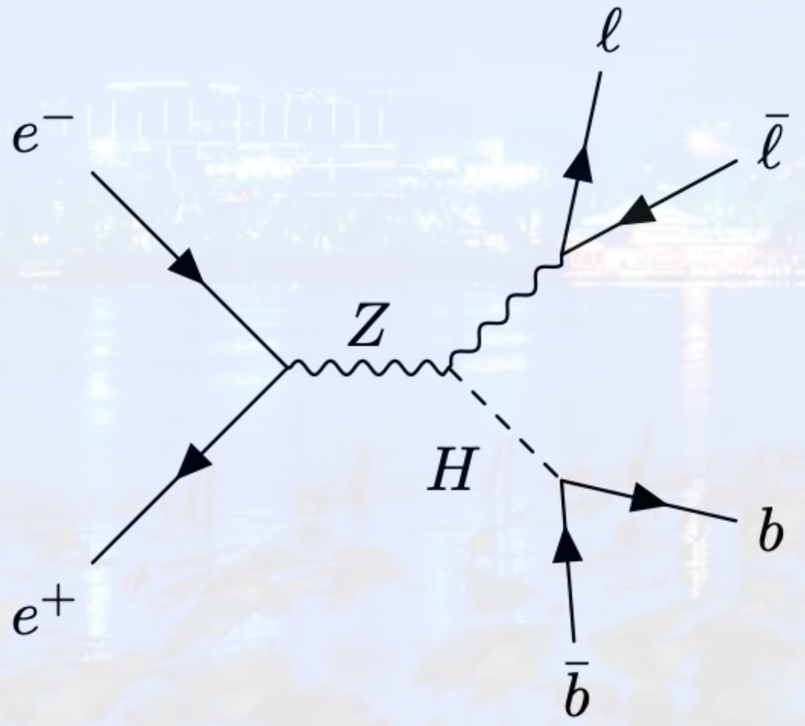
$E_+^{lab.}$ distributions

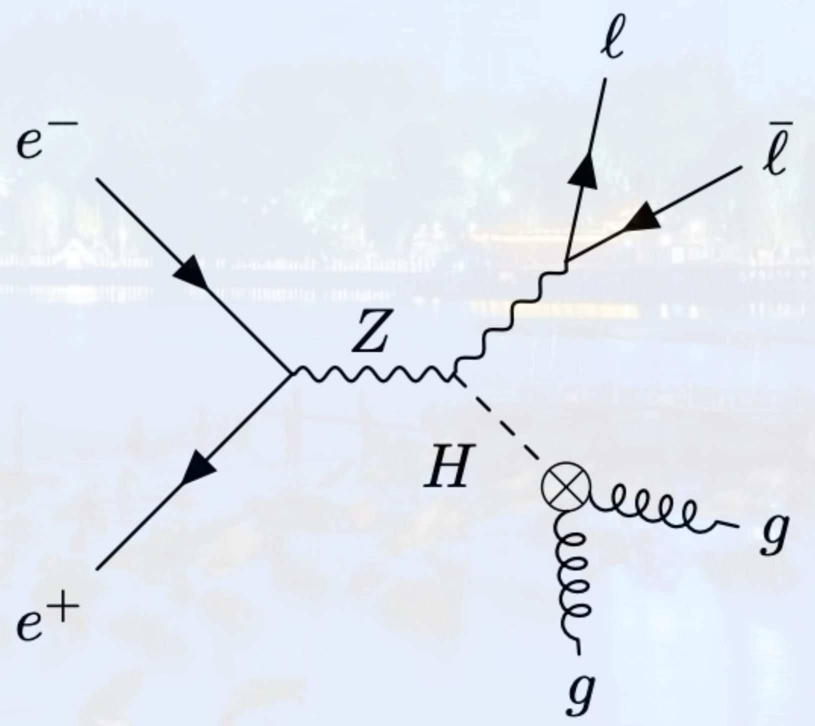
Caletti, Gehrmann-De Ridder, Marcoli [2510.20485]

$e^+e^- \to$ ZH @ NNLO

- $e^+e^- \to ZH \to l^+l^- +$ Jets
  - The main Higgs production channel
  - Consider production@LO + decay@NNLO
  - Boosted decay frame leads to rich phenomenology
  - Comparison between Yukawa and gluonic mode

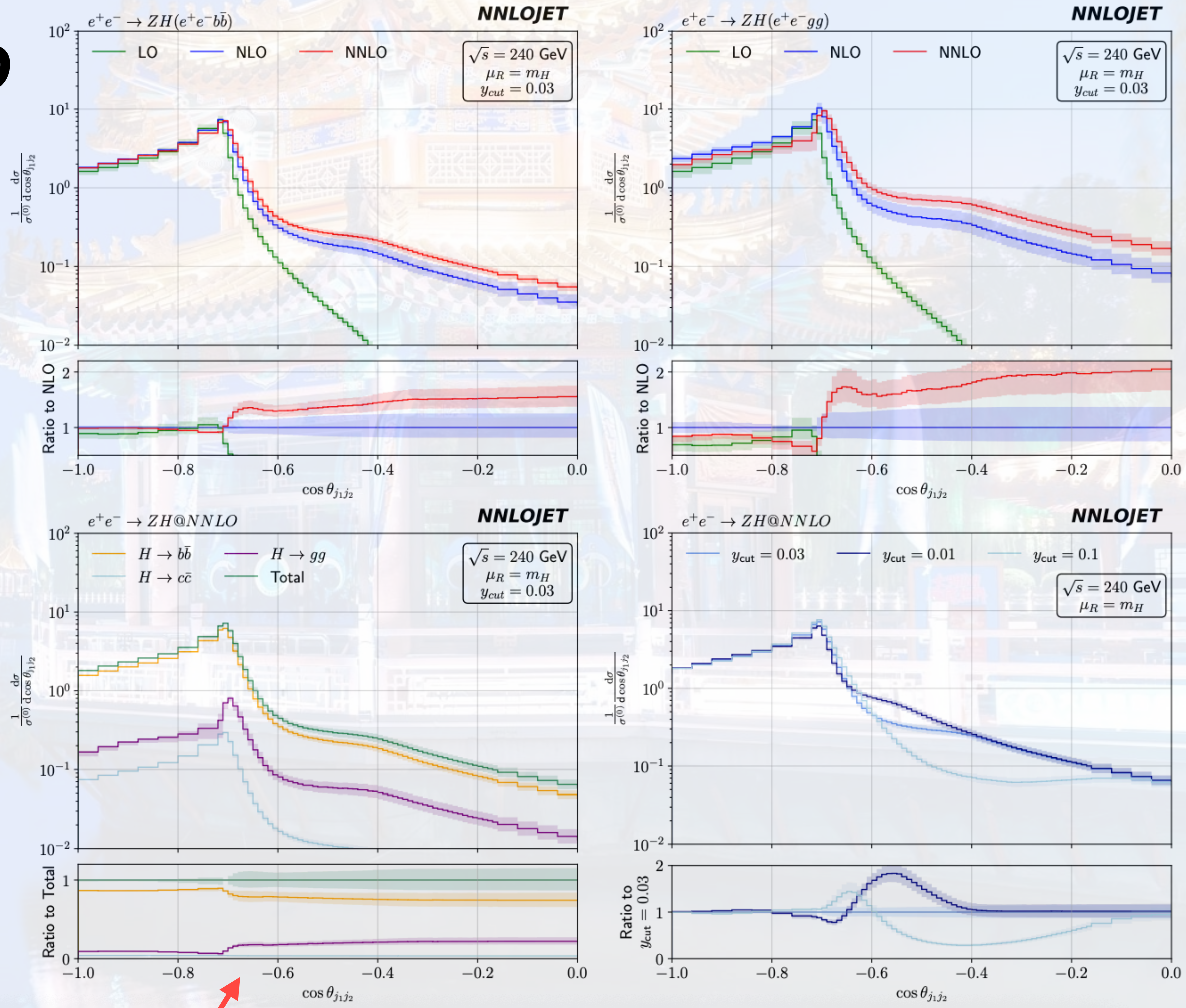
Leading Order Kinematics





$$\frac{E_{\pm}^{lab.}}{m_H} = \frac{1}{2}\left(\gamma \pm \sqrt{\gamma^2 - 1}\,\left|cos\theta\right|\right) \qquad cos\theta_{j_1 j_2} = -\frac{1 - 2\beta^2 + \beta^2 cos^2\theta}{1 - \beta^2 cos^2\theta}$$

with $\gamma = \dfrac{s + m_H^2 - m_Z^2}{2 m_H \sqrt{s}}$, $\quad \beta = \sqrt{1 - \dfrac{1}{\gamma^2}}$



≈ -0.71

$cos\theta_{j_1 j_2}$ distributions

Caletti, Gehrmann-De Ridder, Marcoli [2510.20485]

FUTURE PROSPECTS

- Precision is not the ultimate goal → identify anomaly then understand
- The most famous **failed experiment**: Michelson–Morley in 1887, foundation of special relativity. → 1907 Nobel Prize to Albert A. Michelson . 
- "… it seems probable that **most of the grand underlying principles have been firmly established** and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice. … An eminent physicist remarked that **the future truths of physical science are to be looked for in the sixth place of decimals.**" —— Albert A. Michelson, 1894 University of Chicago

- NNLO QCD precision maybe **solved**, but still not easily accessible. Several ongoing efforts towards aotumation and generalization. N3LO is very challenging, but first steps have been made in this direction.
- **Generalized antenna functions** yield a simpler and more efficient formulation of final-state IR subtraction.
- First application of antenna subtraction to a **fully-differential N3LO calculation**. Gradual extension to more complicated processes is desired in the future.


Thank You for Your Attention

BACK UP SLIDES

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^{3(4)}LL}$

FO	α_s^n	$H(m_V,\mu)$	$I^{(n)}_{i/j}(x,b)$	$\ln W(x_a, x_b, m_V, \vec{b}, \mu = b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu}/\bar{\mu}\left(A(\alpha_s(\bar{\mu}))\ln\frac{m_V^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu}))\right)$						
$\frac{d\hat{\sigma}^V_{NLO}}{dq_T}$	NLO	✓	✓	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1				
$\frac{d\hat{\sigma}^V_{NNLO}}{dq_T}$	N2LO	✓	✓	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1			
$\frac{d\hat{\sigma}^V_{N^3LO}}{dq_T}$	N3LO	✓	✓	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1		
$\frac{d\hat{\sigma}^V_{N^4LO}}{dq_T}$	N4LO	✓	✗	$\ln^5(b^2m_V^2)$	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1	
…	…			…	…	…	…		…	…
$\frac{d\hat{\sigma}^V_{N^kLO}}{dq_T}$	NKLO			$\ln^{k+1}(b^2m_V^2)$	$\ln^k(b^2m_V^2)$	$\ln^{k-1}(b^2m_V^2)$	$\ln^{k-2}(b^2m_V^2)$	$\ln^{k-3}(b^2m_V^2)$	…	…
…	…			…	…	…	…		…	…
Resum				LL	NLL	NNLL	N3LL	N4LL	…	$N^{k+1}LL$
A				A1 ✓	A2 ✓	A3 ✓	A4 ✓	A5 ✗	…	A_{k+2}
B					B1 ✓	B2 ✓	B3 ✓	B4 ✓	…	B_{k+1}

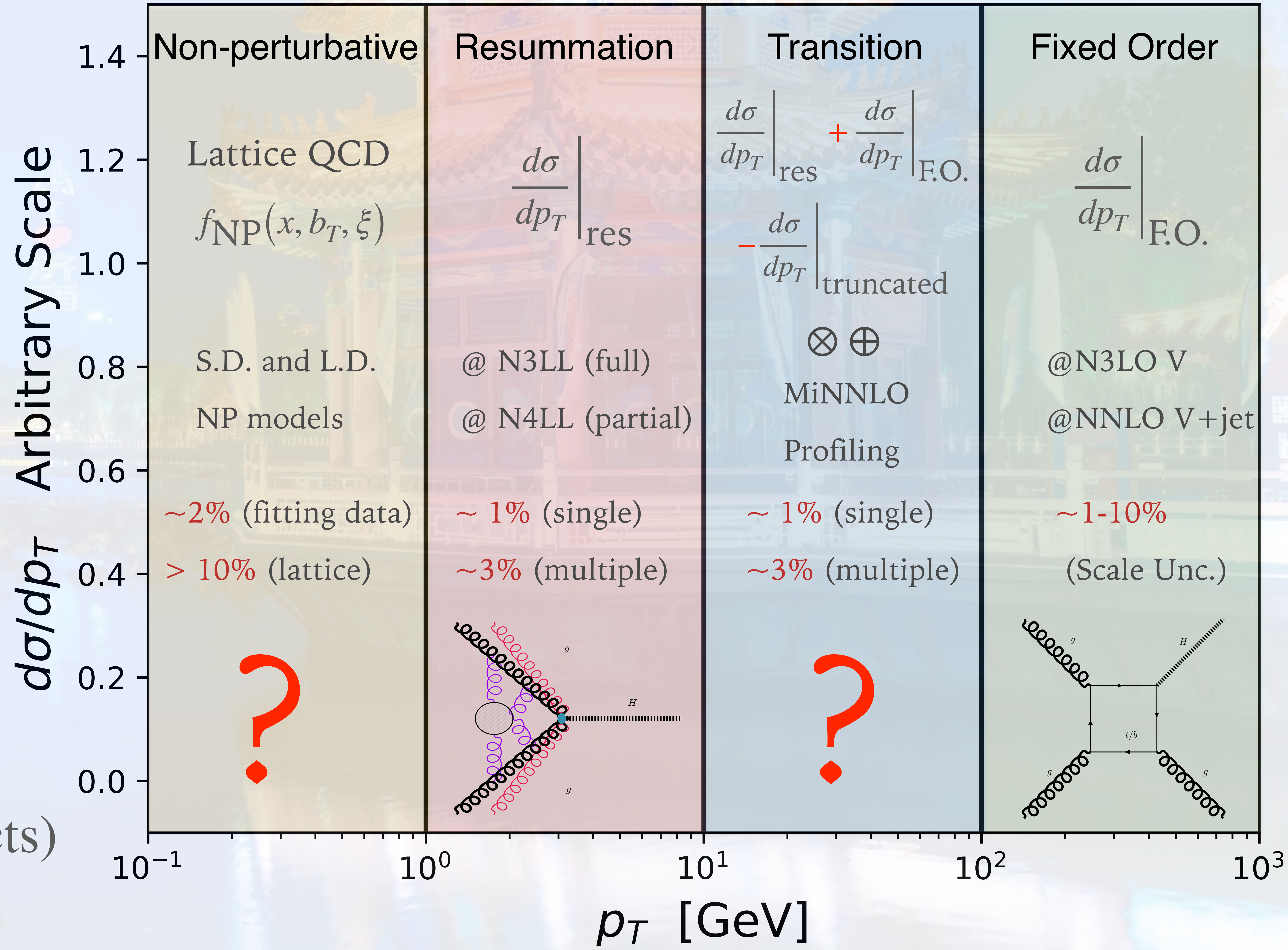
PREDICTIONS OF COLOURLESS PT AT HADRON COLLIDER

- Beyond QCD improved parton model
  - pQCD describes the tail of spectrum
  - Large logarithmic divergence

$$\ln\frac{p_T}{Q} \text{ as } p_T \to 1 \text{ GeV}$$

  - Various LP resummation schemes
  - Multiple solutions in transition region
  - Non-perturbative effects ~ 1 GeV (Short distance and long distance effects)

p_T Spectrum = multi-scale problem



PERTURBATIVE QFT FOR PRECISION PREDICTIONS

➤ State-of-the-art differential N3LO predictions ($2 \to 1$)

- Fully differential N3LO Drell-Yan production (via γ^*) (XC, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang, H. X. Zhu 2021)
- Apply qt-slicing at N3LO with SCET factorisation and expand to N3LO:

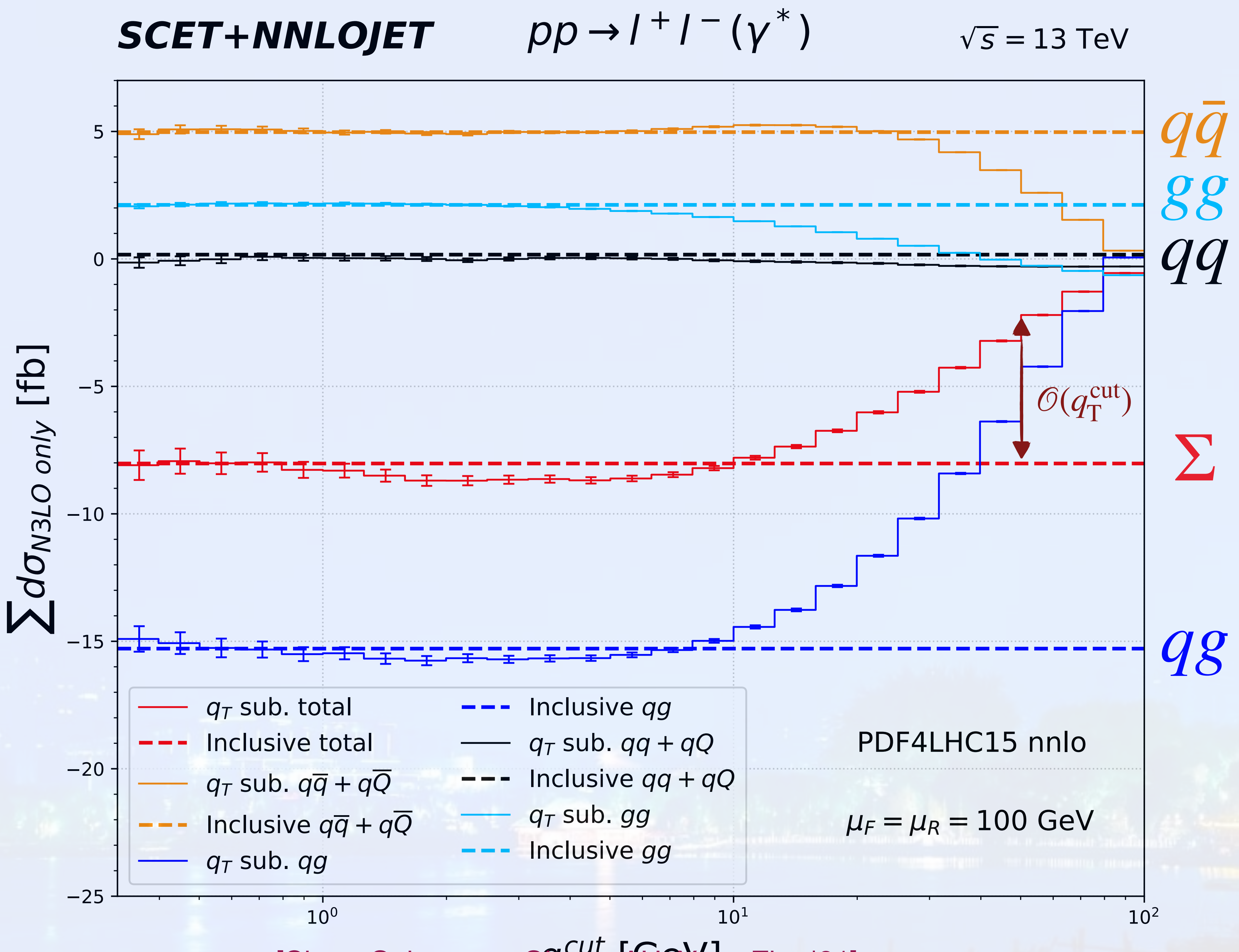
$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\rm LO}^{\gamma^*} H_{q\bar{q}} \Big[\sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}\left(z_1, b_T^2, \mu\right) f_{k/h_1}(x_1/z_1, \mu) \times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}\left(z_2, b_T^2, \mu\right) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}\left(b_\perp, \mu\right) + (q \leftrightarrow \bar{q}) \Big] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- All factorised functions are recently known up to N3LO:
  1) 3-loop hard function $H_{q\bar{q}}^{(3)}$ (T. Gehrmann, E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus 2010)
  2) Transverse-momentum-dependent (TMD) soft function $S(b_\perp, \mu)$ at α_s^3 (Y. Li, H.X. Zhu 2016)
  3) Matching kernel of TMD beam function I_{qk} at α_s^3 (M.-X. Luo, T.-Z. Yang, H. X. Zhu, Y. J. Zhu 2019, M. A. Ebert, B. Mistlberger, G. Vita 2020)
- Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma^{\gamma^*}_{N^3LO} = [\mathscr{H}^{\gamma^*} \otimes d\sigma^{\gamma^*}]_{N^3LO}\Big|_{\delta(p_{T,\gamma^*})} + \left[d\sigma^{\gamma^*+jet}_{NNLO} - d\sigma^{\gamma^*\ CT}_{N^3LO}\right]_{p_{T,\gamma^*} > qt_{cut}} + \mathcal{O}(qt_{cut}^2/Q^2)$$

$pp \to \gamma^*/Z$ @ N3LO

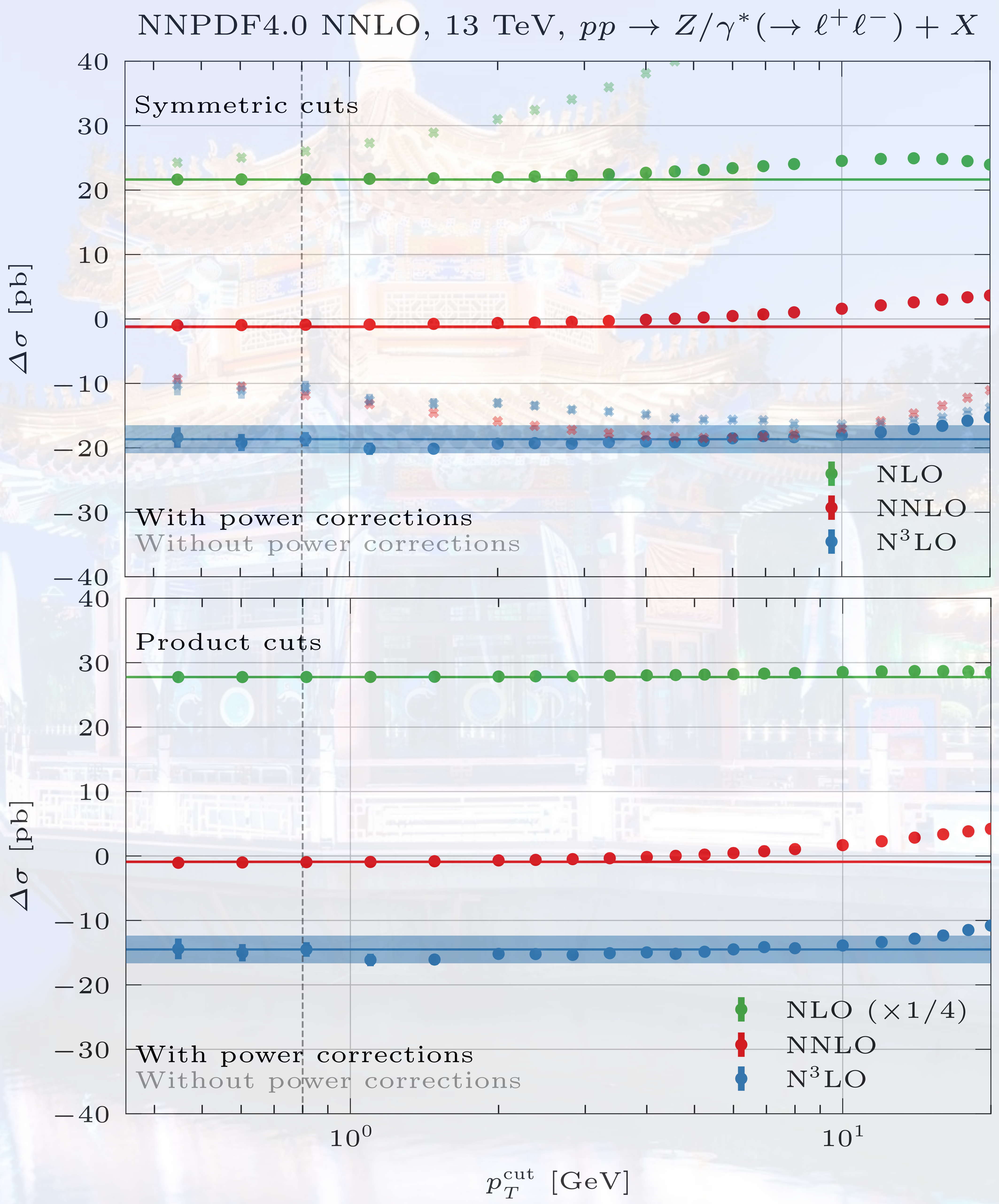
XC, T. Gehrmann, et. al. PRL. 128, 052001 (2022)



[Chen, Gehrmann, Glover, Huß, Yang Zhu '21]

Fixed order	$\sigma_{pp\to\gamma^*}$ (fb)		
LO	$339.62^{+34.06}_{-37.48}$		
NLO	$391.25^{+10.84}_{-16.62}$		
NNLO	$390.09^{+3.06}_{-4.11}$		
N^{3}LO	$382.08^{+2.64}_{-3.09}$ [14]		
N^{3}LO only	$q_T^{\rm cut} = 0.63$ GeV	$q_T^{\rm cut} \to 0$ fit	[14]
qg	$-15.32(32)$	$-15.34(54)$	-15.29
$q\bar{q}+q\bar{Q}$	$+5.06(12)$	$+5.05(12)$	$+4.97$
gg	$+2.17(6)$	$+2.19(6)$	$+2.12$
$qq+qQ$	$+0.09(13)$	$+0.09(17)$	$+0.17$
Total	$-7.98(36)$	$-8.01(58)$	-8.03

C. Duhr, F. Dulat, B. Mistlberger. PRL. **125**, 172001 (2020)



XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

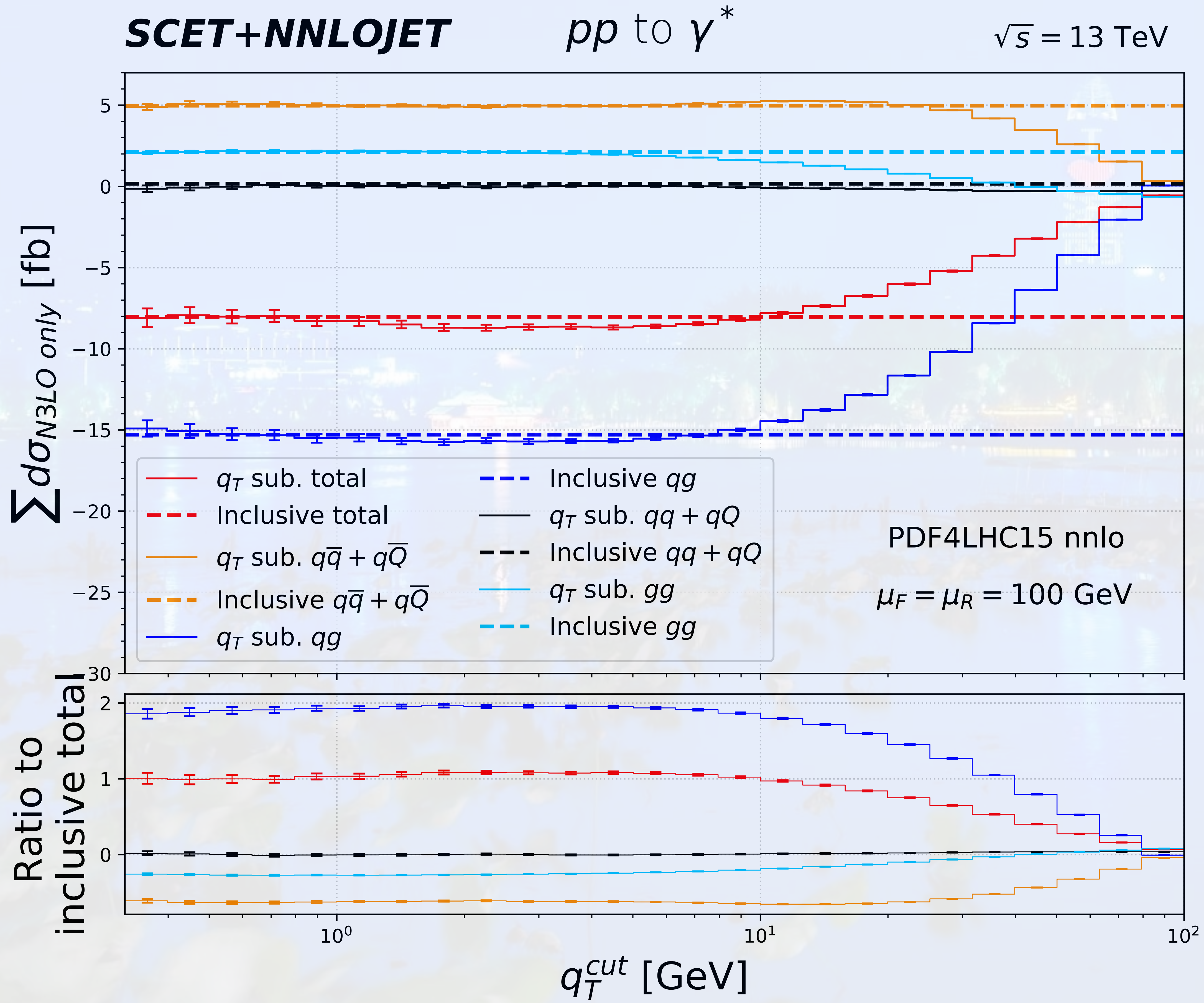
➤qT slicing at N3LO for neutral and charged current production (NNLOJET)

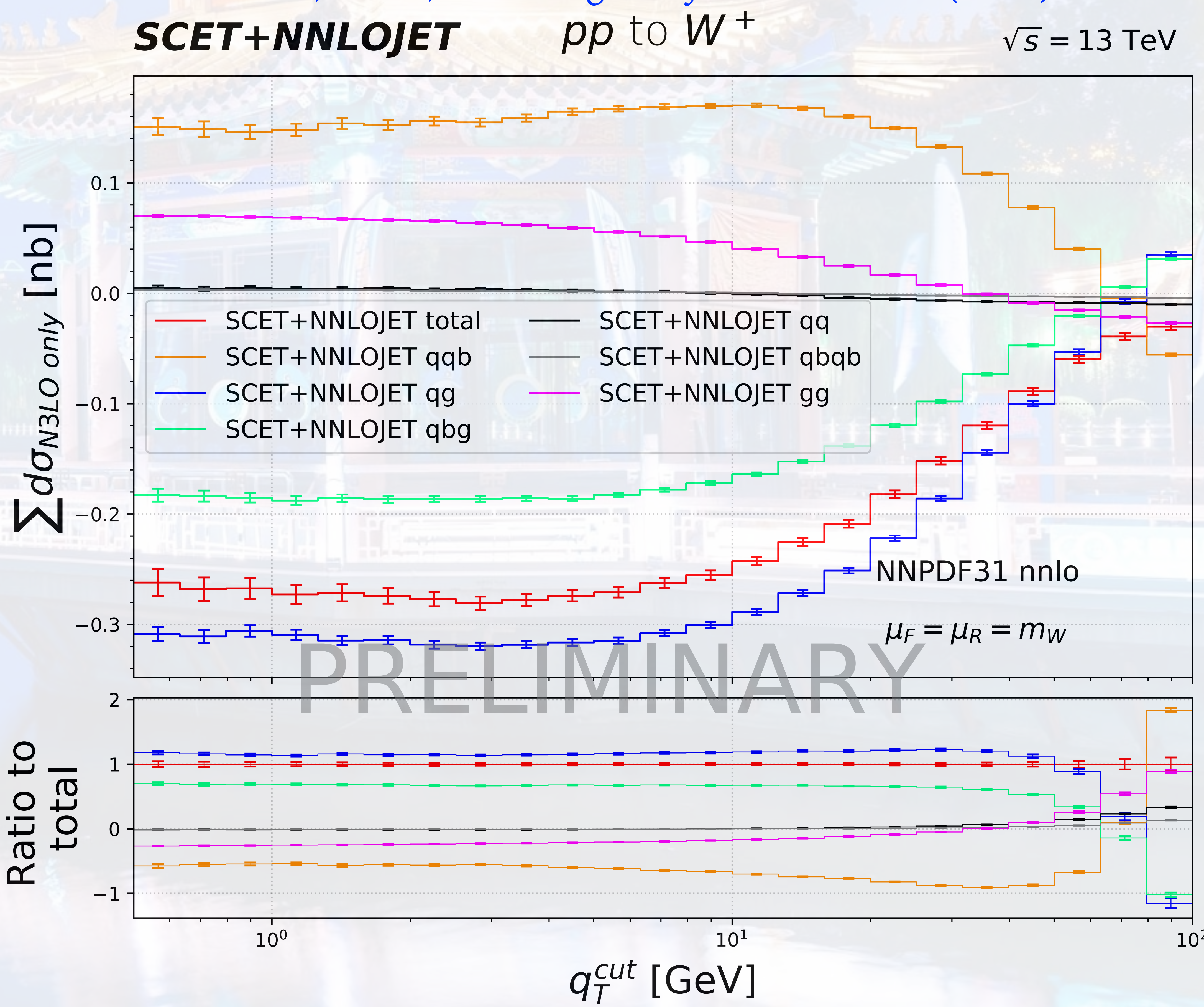
$$\sum d\sigma^{V}_{N3LO} \equiv \sum_{dp_{T,V}} d\sigma^{V+jet}_{NNLO}/dp_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{dp_{T,V}} d\sigma^{V\,SCET}_{N^3LO}/dp_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

NC and CC Validated against inclusive XS within ± 5% uncertainty

$$\Delta\sigma^{\gamma^*}_{N^3LO} = -7.98 \pm 0.36\, fb \quad vs. \quad -8.03\, fb$$

Duhr, Dulat, Mistlberger *Phys.Rev.Lett*. 125 (2020)





XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Rev.Lett*. 128 (2022) 5

XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Lett.B 840 (2023)*

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

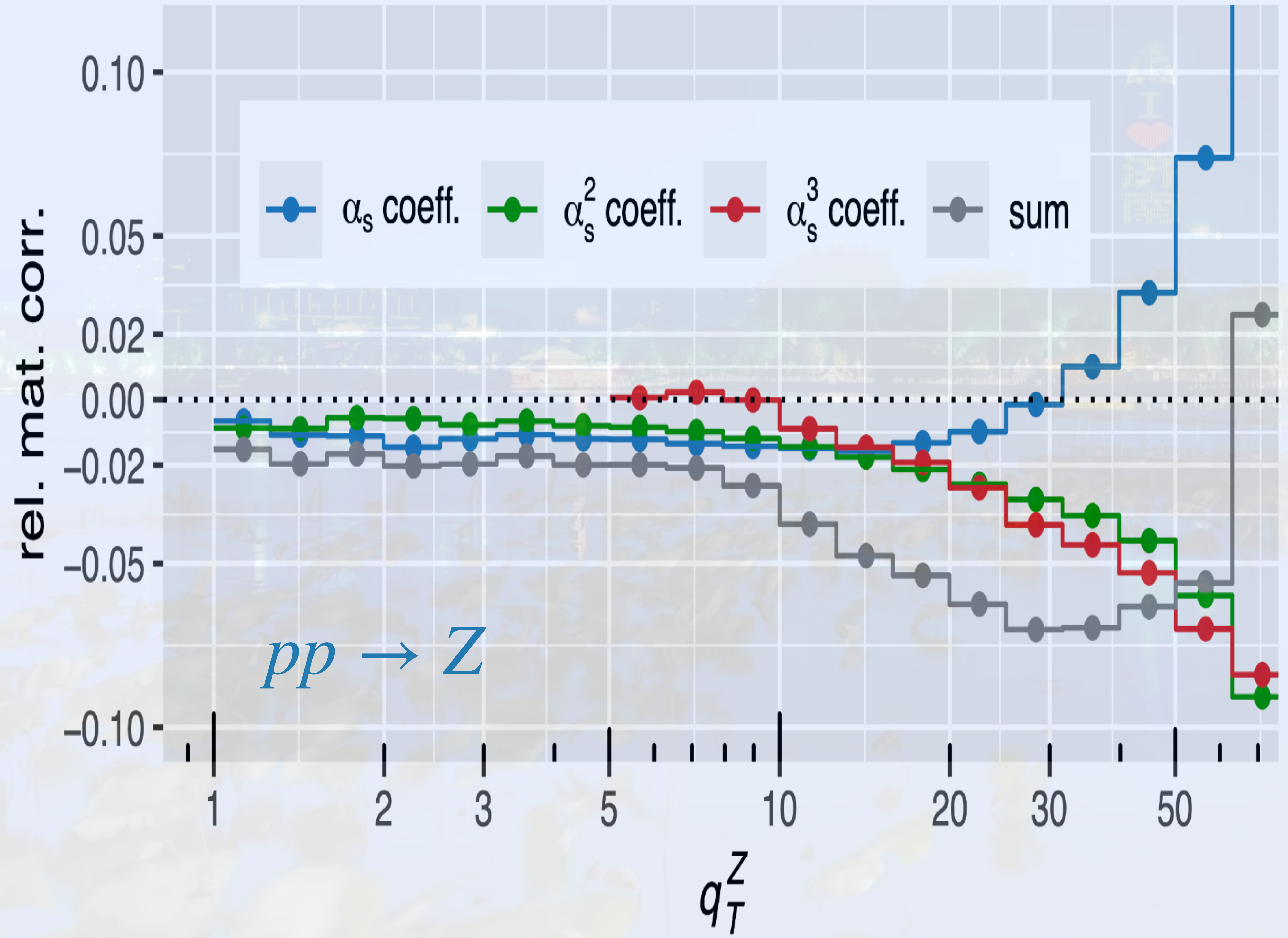
➤qT slicing at N3LO for neutral and charged current production (MCFM)

$$\sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V}_{N3LO} \equiv \sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V+jet}_{NNLO}/\mathrm{d}p_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V\,SCET}_{N^3LO}/\mathrm{d}p_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

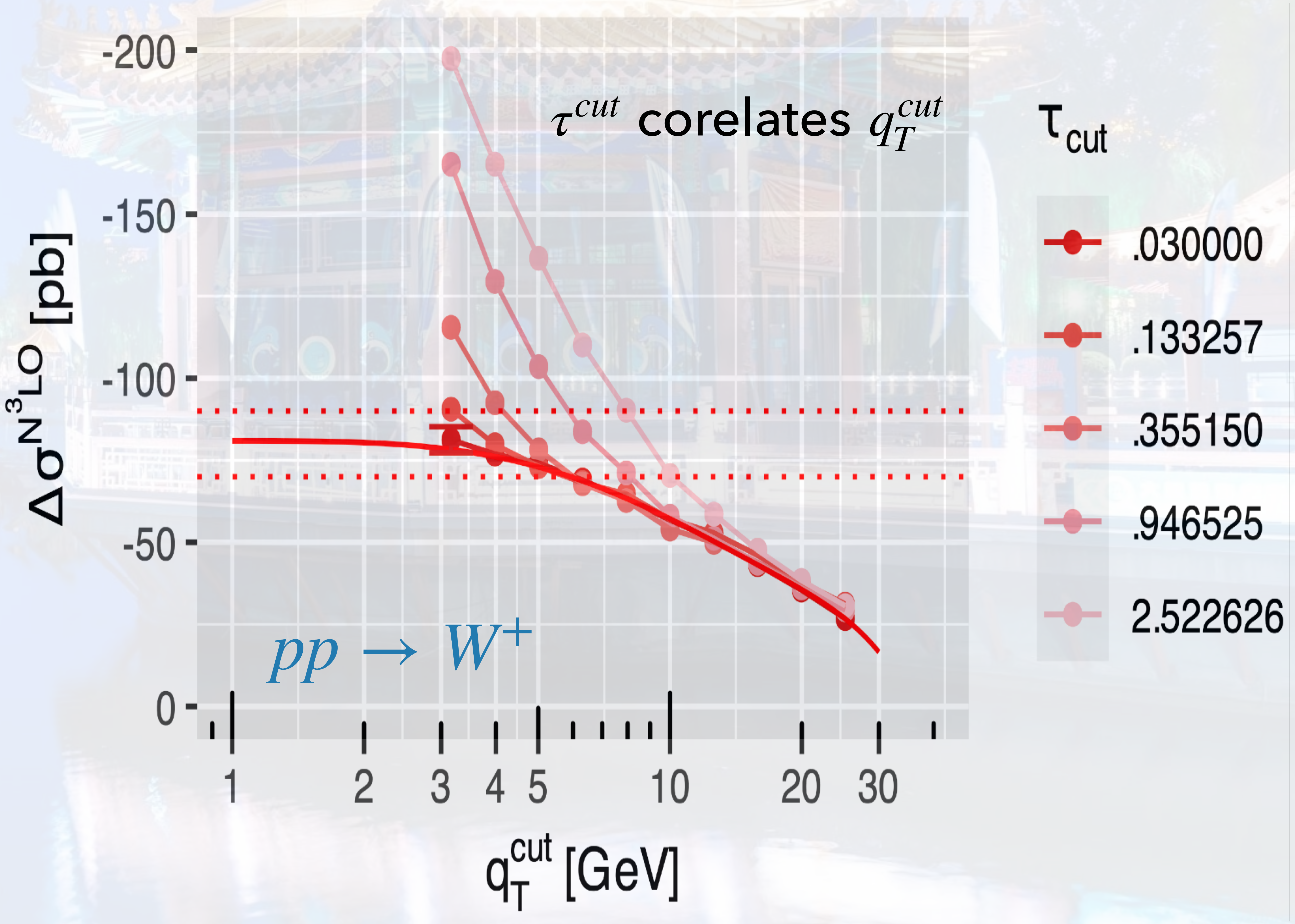
NC MCFM: $-22.6\,\mathrm{pb} \pm 1.4\,\mathrm{pb}\,(\mathrm{num.}) \pm 1\,\mathrm{pb}\,(\mathrm{slicing})$

NC NNLOJET: $-18.7\,\mathrm{pb} \pm 1.1\,\mathrm{pb}\,(\mathrm{num.}) \pm 0.9\,\mathrm{pb}\,(\mathrm{slicing})$

CC agree to inclusive XS within ± 60% uncertainty of $\Delta(\alpha_s^3)$



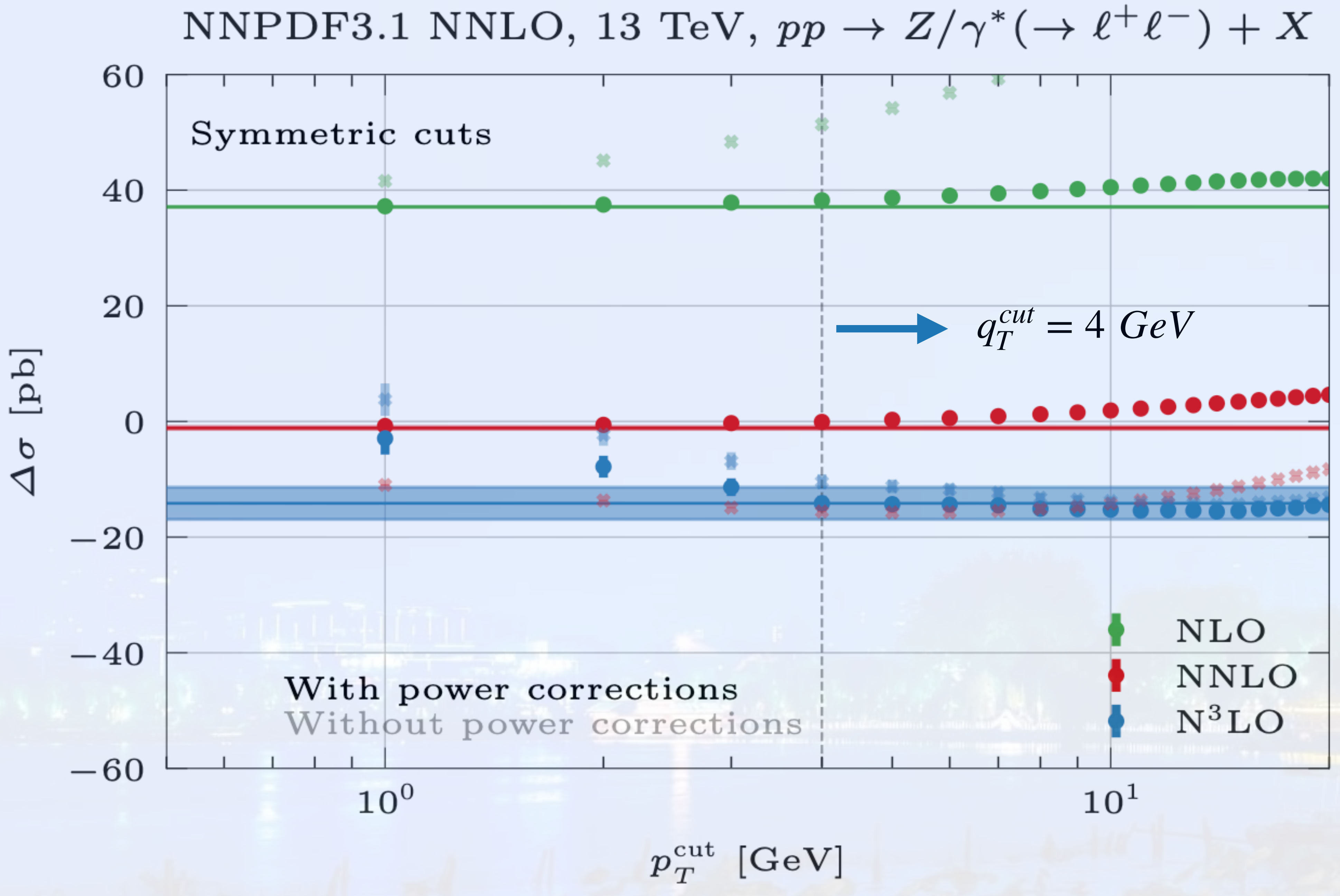
Neumann and Campbell *Phys.Rev.D* 107 (2023) 1



Neumann and Campbell *JHEP 11 (2023) 127*

Precision Predictions at Hadron Collider

$2 \to 1$ @ N3LO (+ N3LL) QCD



XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

- Solid horizontal lines: NLO, NNLO at 1 GeV, N3LO at 4 GeV with MC error.
  - N3LO shows no plateau in 1905.05171
- Pale dots are values used by DYTurbo in 2103.04974 and 2303.12781 (taken from 1905.05171).
  - Fiducial power corrections are not included.
  - Leads to 30% difference of N3LO coefficients at $q_T^{cut} = 4\ GeV$.
- Solid dots are corrected values with fiducial power correction.
  - Central value shifts 2 pb starting from NLO (the dominant error).
  - ± 2.1 pb uncertainty from MC and q_T^{cut} (estimated from [3,5] GeV region).
  - Not consistent with DYTurbo update result of ± 0.7 pb uncertainty.

DYTurbo result with fiducial power correction

Order	N^{3}LO
q_T subtr. ($q_T^{cut} = 4\,\text{GeV}$)	$747.1 \pm 0.7\,\text{pb}$
recoil q_T subtr.	$745.7 \pm 0.7\,\text{pb}$

S. Camarda, L. Cieri, G. Ferrera Eur.Phys.J.C 82 (2022) 6

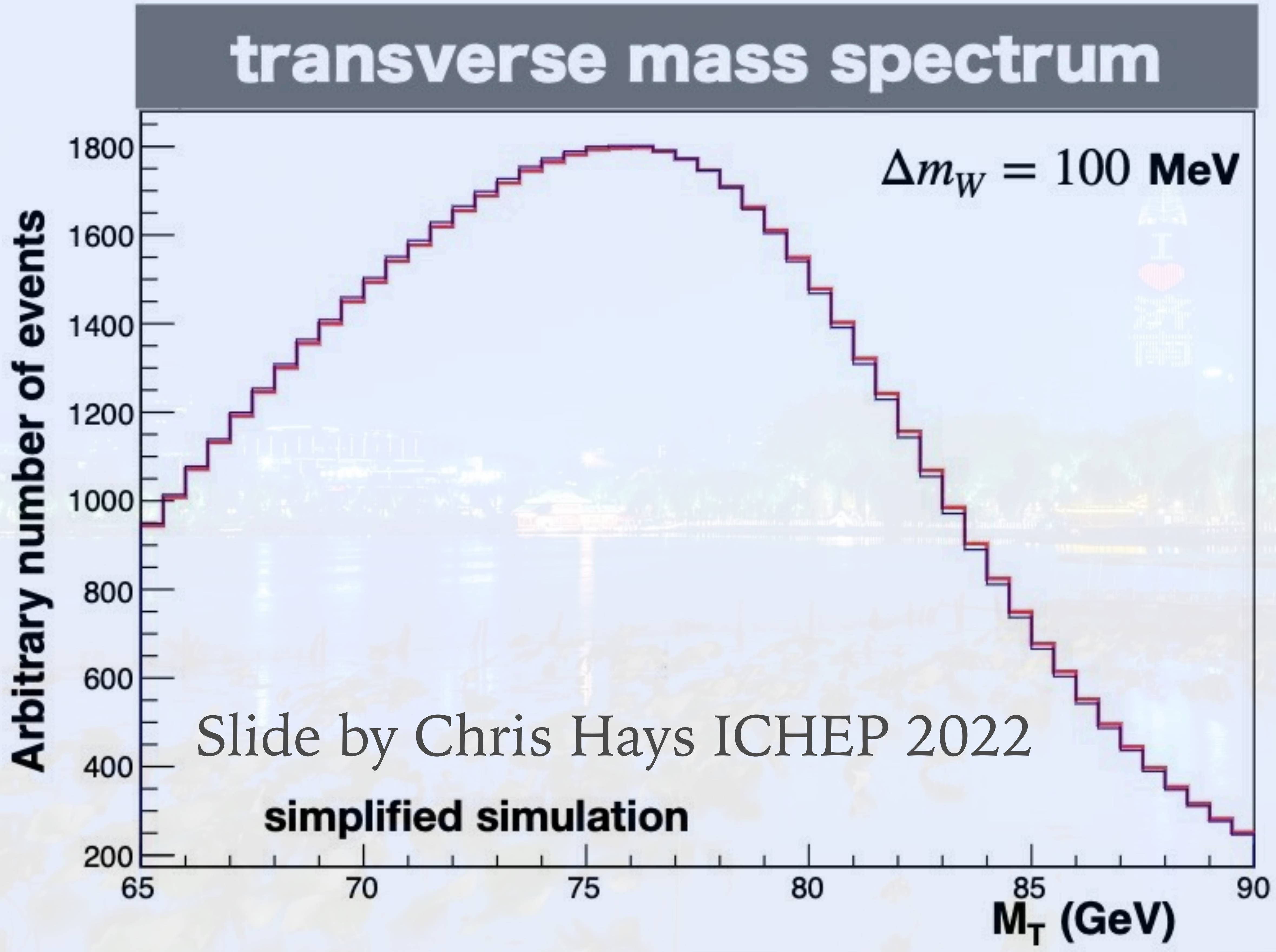
DYTurbo result without fiducial power correction cited in ATLAS α_s fitting

Order	NLO	NNLO	N^{3}LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	766.3 ± 1	757.4 ± 2	746.1 ± 2.5
Order	NLL+NLO	NNLL+NNLO	N^{3}LL+N^{3}LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	773.7 ± 1	759.8 ± 2	749.6 $\pm$2.5

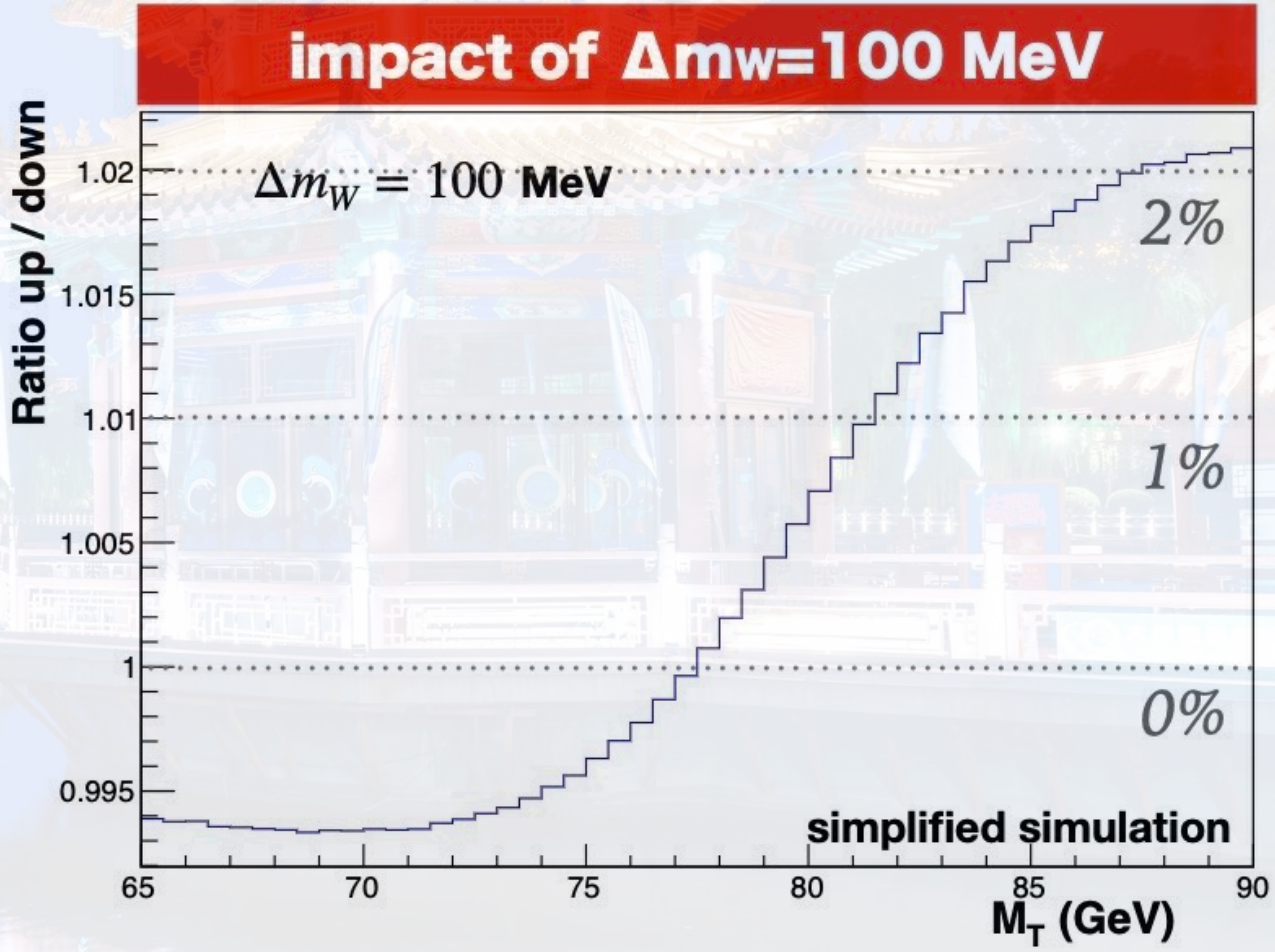
*S. Camarda, L. Cieri, G. Ferrera Phys. Rev. D **104**, L111503 (2021)*

W MASS IN CDFII MEASUREMENT

➤ $d\sigma/dm_T^W$ two templates with $\Delta m_W = 100$ MeV



Slide by Chris Hays ICHEP 2022



$\Delta m_W =$ 100 MeV ~ 0.5-2% change in $d\sigma/dm_T^W$ → $\Delta m_W =$ 10 MeV ~ 0.1% precision in $d\sigma/dm_T^W$

PRECISION PREDICTIONS IN CDF II

➤CDF II use ResBos to generate theory templates

➤NLO+NNLL accuracy for W/Z production

Balazs, Brock, Landry, Nadolsky and Yuan`97 to`03

➤CSS factorisation and resummation of p_T in b space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}^2\vec{p}_T\,\mathrm{d}y\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \sigma_0 \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} e^{-S(b)}$$

$$\times\, C \otimes f(x_1,\mu)\, C \otimes f(x_2,\mu) + Y(Q,\vec{p}_T,x_1,x_2,\mu_R,\mu_F)$$

Collins, Soper and Sterman`85

➤Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

$$S(b) = S_{\mathrm{NP}} S_{\mathrm{Pert}}\,,$$

Collins and Soper `77

$$S_{\mathrm{Pert}}(b) = \int_{C_1^2/(b^*)^2}^{C_2^2Q^2} \frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right) A(\bar{\mu},C_1) + B(\bar{\mu},C_1,C_2)\right]$$

$$S_{\mathrm{NP}} = \left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$$

S_{NP} assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan `02

➤Use data driven method:

Fix	g1	g2	g3	α_s
p_T^Z	Global fit `03	CDFII fit	Global fit `03	CDFII fit
p_T^Z/p_T^W			Global fit `03	

Global fit by Brock, Landry, Nadolsky and Yuan `03

$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

➤Scale uncertainty of p_T^Z/p_T^W by DYQT

Bozzi, Catani, Ferrera, de Florian, Grazzini `09 `11

$m_T^W \sim 3.5$ MeV, $p_T^l \sim 10.1$ MeV, $p_T^\nu \sim 3.9$ MeV

Not included in final result CDF sm`22