中国科学院高能物理研究所

Theoretical studies on high-energy plasma wakefield acceleration

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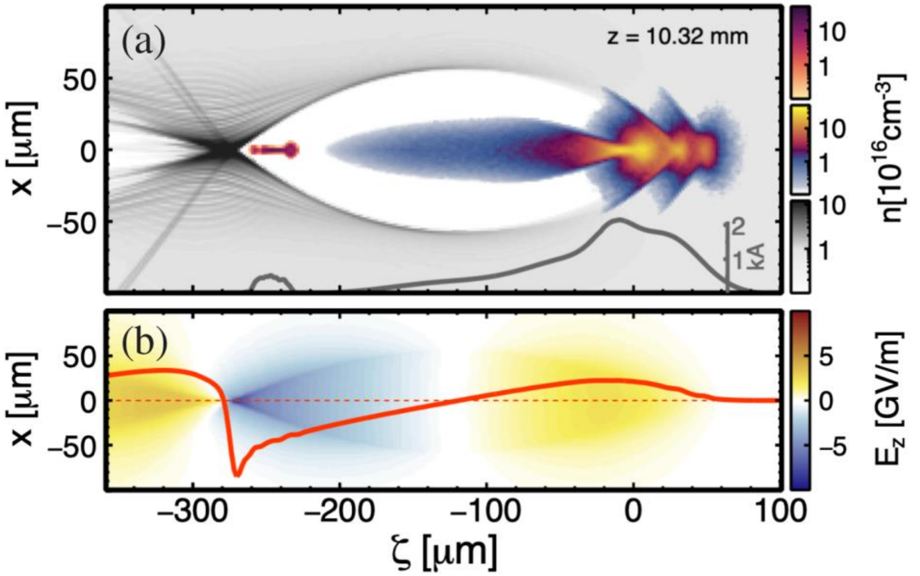
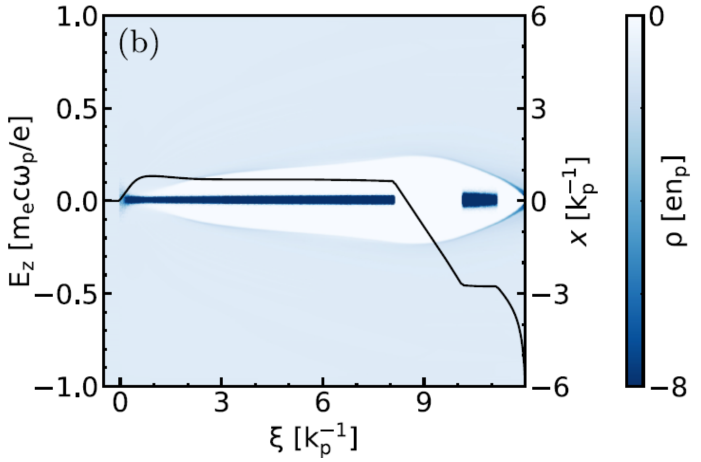
Background



A. KNETSCH et al. PHYS. REV. ACCEL. BEAMS 24, 101302 (2021)



Xiaoning Wang et al., International Journal of Modern Physics A 37, 2246003 (2022)

- Beam-driven plasma wakefield accelerator (PWFA) has ~10 GeV/m gradient
- Driver loses energy and trailer gains energy
- $\text{Trasformer ratio} = \frac{\text{trailer gain}}{\text{diver lose}} \sim 1$ for regular PWFAs
- Need $\text{trasformer ratio} > 1$ for high energy PWFAs
- Long driver + current profile design to flatten deceleration field is important for high transformer ratio

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2. Modeling of long-term 3D betatron oscillation and radiation reaction in PWFAs

Background

- Prediction of blowout radius is important in PWFA
- The pseudo-potential on axis $\psi_a \sim r_c^2/4$, longitudinal field $E_z = \frac{d\psi_a}{d\xi}$
- Prediction of E_z is critical for high transformer ratio PWFA
- Beam current is proportional to

$$\Lambda(\xi) = \int_0^\infty n_b(\xi, r')\, r' dr'$$

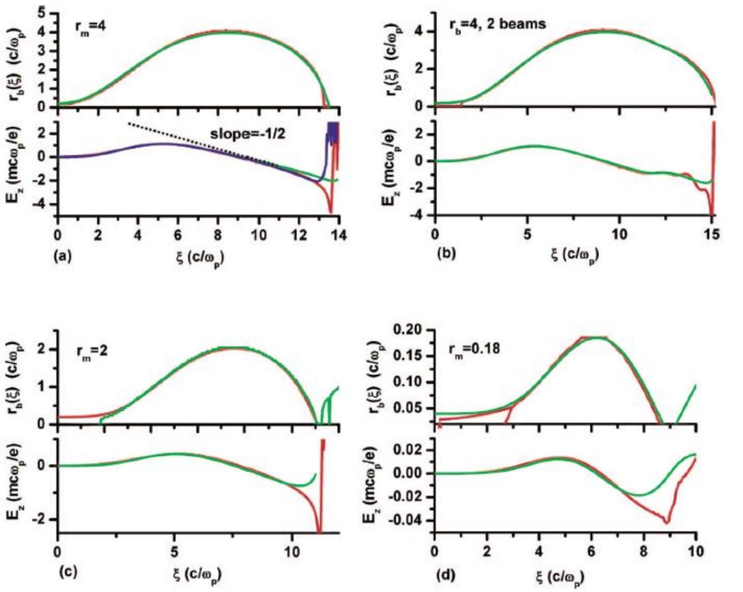
- For a long time, balance blowout radius has been believed to be

$$r_n = \sqrt{2\Lambda}$$

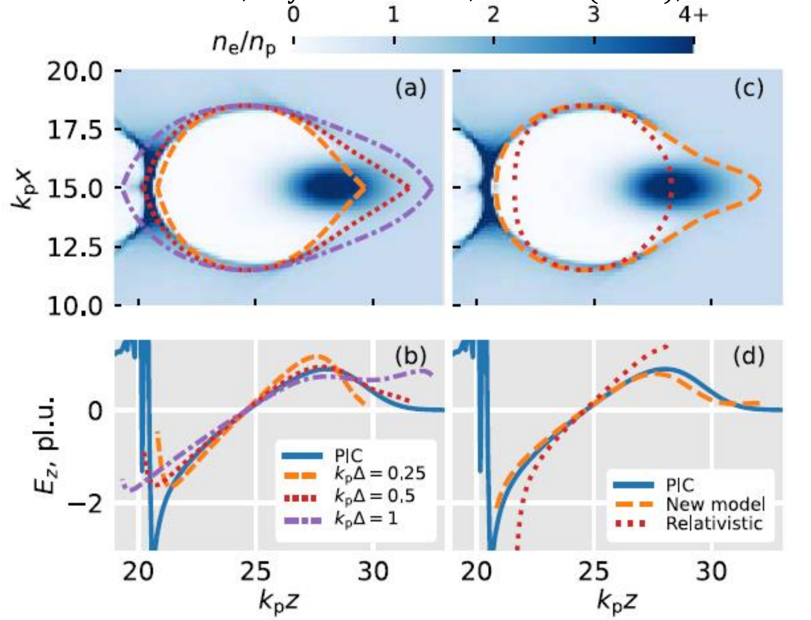
- In 2023, A. Golovanov et al. established a new model based on energy conserving and δ-sheath. The balancing blowout radius according to this model is

$$r_{\delta 0} = 2\sqrt{\Lambda}$$

- Simulations show, the actual balancing radius is between them



W. Lu et al., Phys. Plasmas 13, 056709 (2006);



A. Golovanov et al., PhysRevLett.130.105001 (2023)

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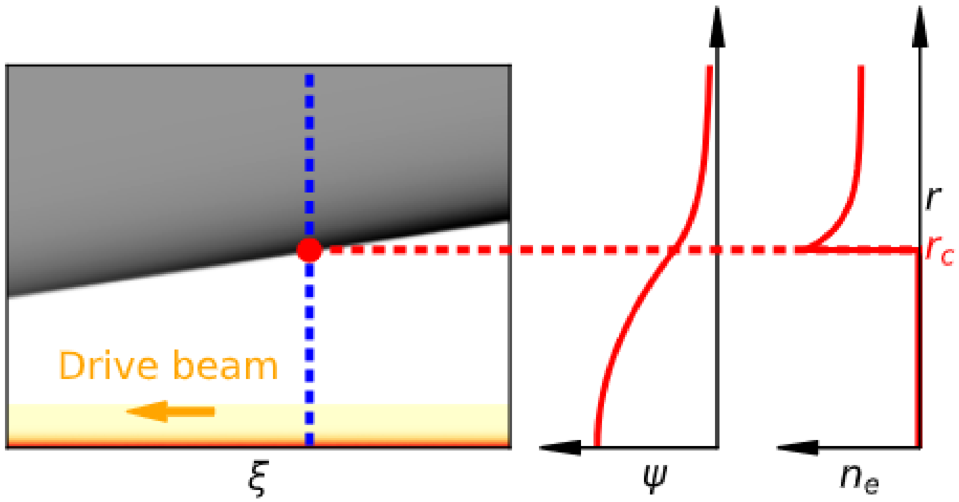
Adiabatic sheath model



- Consider a plasma channel with slow-varying current of the drive beam
- Blowout radius r_c varies slowly, force balancing is achieved everywhere
- Sheath electrons has v_z, but transverse velocity is negligible
- Contant of motion $\gamma - v_z\gamma = 1 + \psi$
- The equations

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) = S(\xi, r) = \begin{cases} 1, & r < r_c(\xi) \\ 1 - n_e(1 - v_z), & r \geq r_c(\xi) \end{cases}$$

$$F_r|_{r>r_c} = \frac{1}{r}\left[-\frac{r^2}{2} + (1 - v_z)\Lambda + \int_0^r n_e(r')r'dr' - v_z\int_0^r n_e(r')v_z(r')r'dr'\right] = 0$$

$$v_z = \frac{2}{1 + (1 + \psi)^2} - 1$$

- Boundary conditions

$$\frac{\partial}{\partial r}\psi|_{r=r_c(\xi)} = -\frac{r_c}{2},$$

$$\lim_{r\to\infty}\psi = 0$$

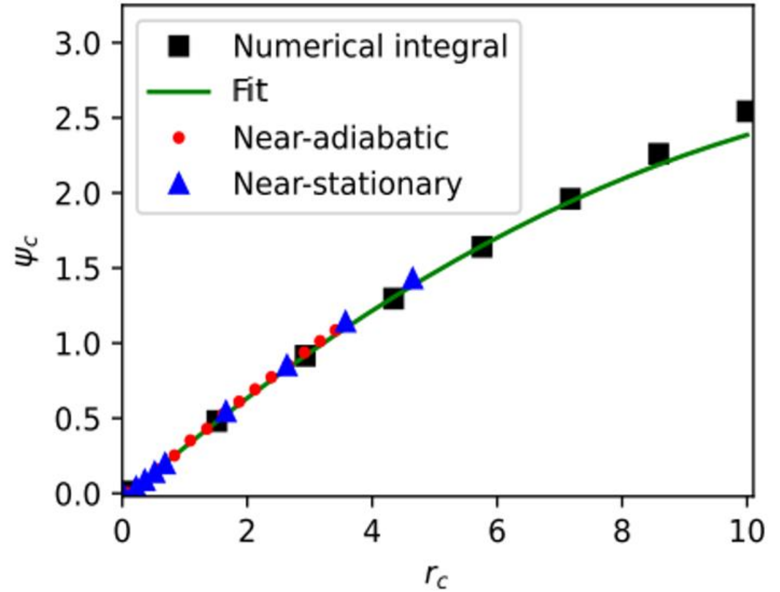
Solve the equations using shooting method

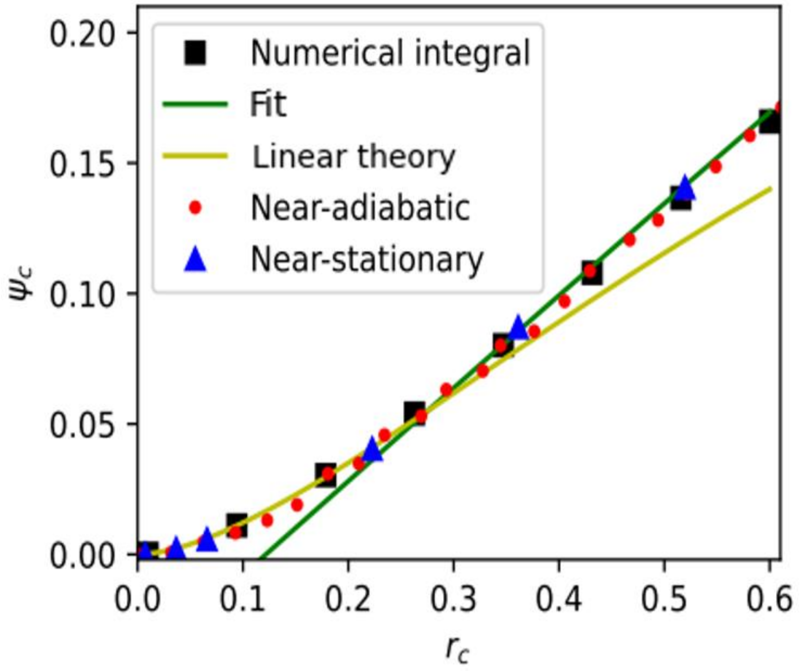
- The equations can be solved numerically using shooting method
- Meanwhile, under small blowout limit, analytical solution can be obtained

$$\psi_c = \begin{cases} \dfrac{r_c^2}{2} K_0(r_c), & r_c \lesssim 0.3 \\ -0.012 r_c^2 + 0.363 r_c - 0.044, & 0.3 \lesssim r_c \lesssim 8 \end{cases}$$

- The blowout radius can be determined as

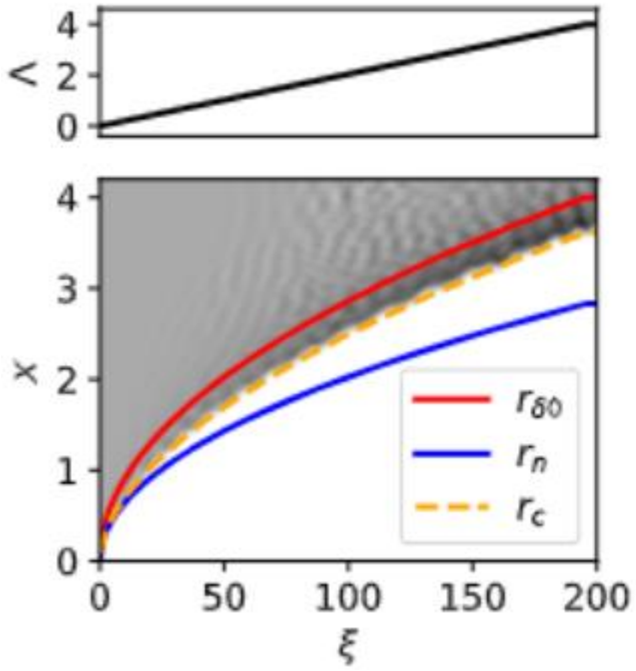
$$r_c = \sqrt{2[1 - v_z(r = r_c)]\Lambda} = 2\sqrt{\frac{\Lambda}{1 + \frac{1}{(1+\psi_c)^2}}}$$



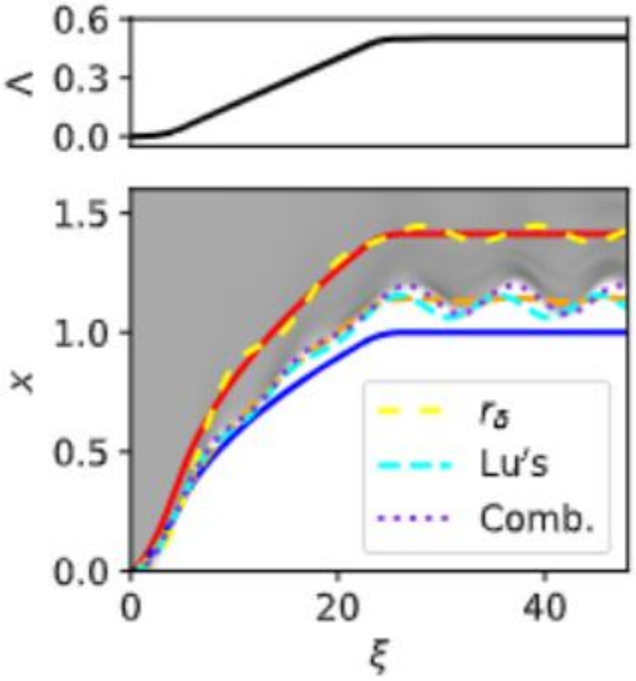


Comparison with simulations

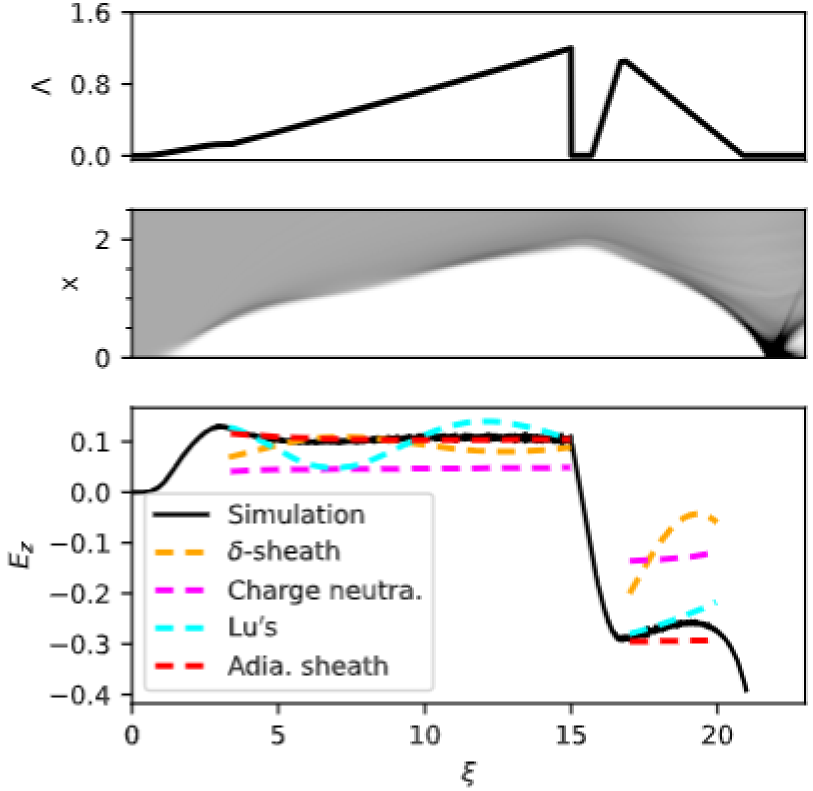
- Our model can better predict the simulations
- This model can be used for high transformer ratio PWFA design



Near-adiabatic blowout



Near-stationary blowout



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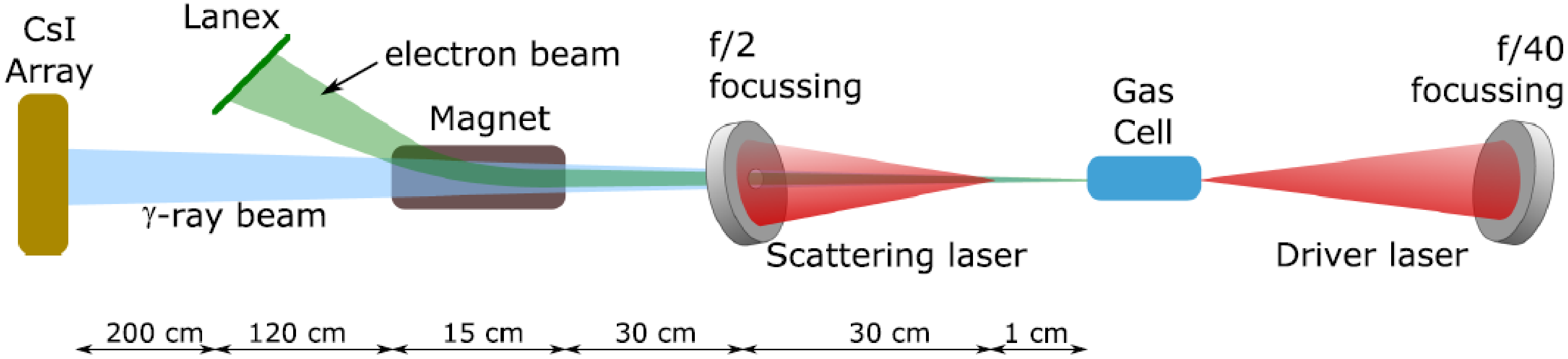
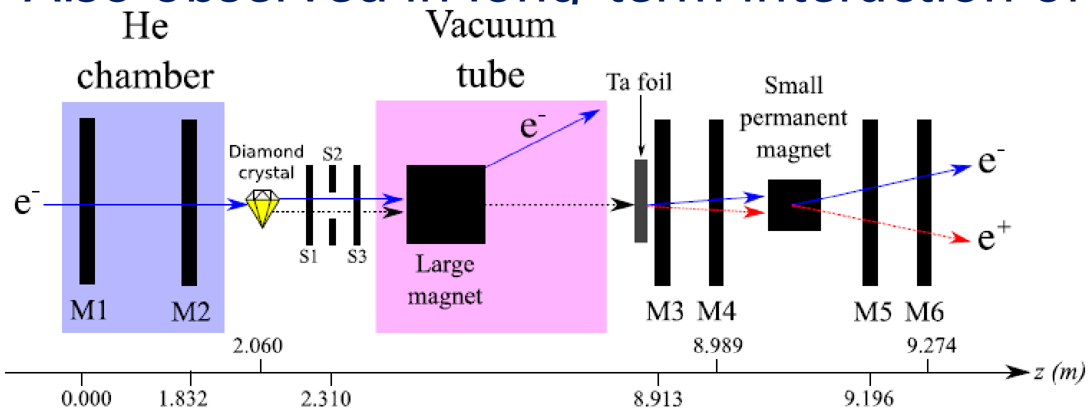
Background

- Radiation reaction when laser electron beam collide



J. M. Cole et al., Phys. Rev. X 8, 011020 (2018)
K. Poder et al ., Phys. Rev. X 8, 031004 (2018)

- Also observed in long-term interaction of e beam with crystal



C. F. Nielsen et al., New J. Phys. 23, 085001 (2021)

- RR Should be considered in future high energy and long distance wakefield accelerators!

3D betatron oscillation model

- One longitudinal and two transverse dimensions
- E and B fields

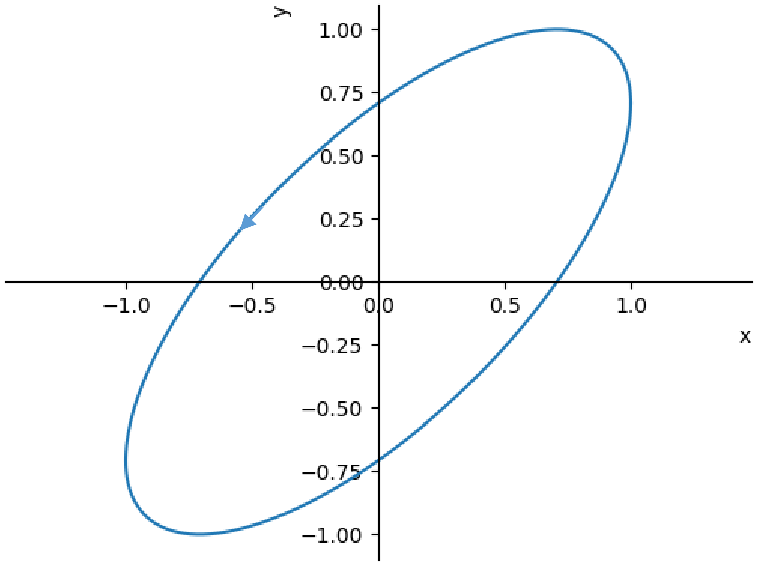
$$\begin{aligned} E_z &= E_{z0} + \lambda\zeta_1, \\ \vec{E}_\perp &= \kappa^2 (1-\lambda)\,\vec{r}, \\ B_\theta &= -\kappa^2\lambda r, \end{aligned}$$

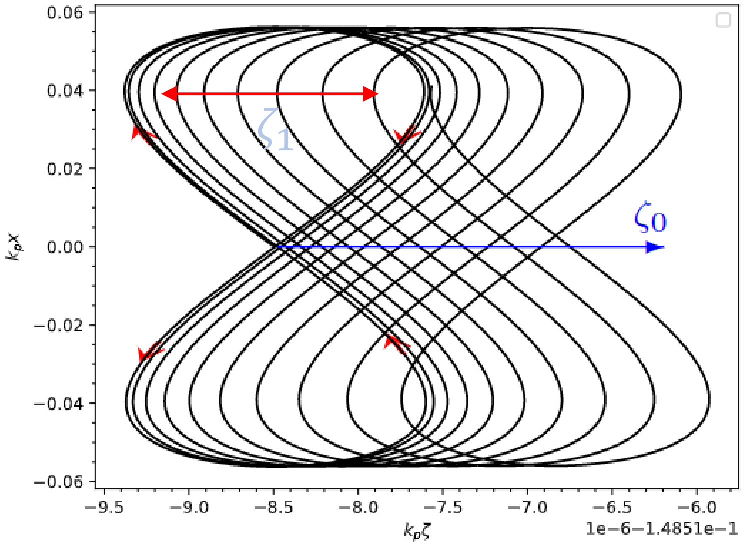
- Forces

$$\begin{aligned} f_z &= -E_{z0} - \lambda\zeta_1 + \kappa^2\lambda(x\beta_x + y\beta_y) + f_z^{\rm rad}, \\ f_x &= -\kappa^2 (1-\lambda+\lambda\beta_z)\,x + f_x^{\rm rad}, \\ f_y &= -\kappa^2 (1-\lambda+\lambda\beta_z)\,y + f_y^{\rm rad}, \end{aligned}$$

- Equations of motion

$$\begin{aligned} \dot{\gamma} &= -E_{z0}\beta_{z0} + \left(\frac{\lambda\beta_{z0}}{4} + \kappa^2\lambda - \kappa^2\right)(x\beta_x + y\beta_y) - \frac{2}{3}k_p r_e \gamma^2\kappa^4\left(x^2+y^2\right), \\ \dot{p_z} &= -E_{z0} + \lambda\left(\frac{1}{4} + \kappa^2\right)(x\beta_x + y\beta_y) - \frac{2}{3}k_p r_e \gamma^2\kappa^4\left(x^2+y^2\right), \\ \dot{p_x} &= -\kappa^2 x + \frac{\kappa^2\lambda}{2}\left(\langle\gamma\rangle^{-2} + \beta_x^2 + \beta_y^2\right)x - \frac{2}{3}k_p r_e \gamma^2\kappa^4\left(x^2+y^2\right)\beta_x, \\ \dot{p_y} &= -\kappa^2 y + \frac{\kappa^2\lambda}{2}\left(\langle\gamma\rangle^{-2} + \beta_x^2 + \beta_y^2\right)y - \frac{2}{3}k_p r_e \gamma^2\kappa^4\left(x^2+y^2\right)\beta_y, \end{aligned}$$





Long-term equations

- Equations are written using the averaging method

$$\langle\dot{\gamma}\rangle = -E_{z0}\beta_{z0} - \frac{1}{3}k_p r_e \kappa^3 \langle\gamma\rangle^{\frac{3}{2}} (S_x + S_y),$$

Energy

$$\langle\dot{\zeta}\rangle = \frac{1}{2}\gamma_w^{-2} - \frac{1}{2}\langle\gamma\rangle^{-2} - \frac{1}{4}\kappa \langle\gamma\rangle^{-\frac{3}{2}} (S_x + S_y),$$

Longitudinal phase

$$\dot{S}_x = -\frac{1}{4}k_p r_e \kappa^3 \langle\gamma\rangle^{\frac{1}{2}} \left(S_x^2 + \frac{4-\cos 2\Delta\Phi}{3} S_x S_y\right) - \frac{1}{8}\left[\frac{1}{4}\lambda\beta_{z0} - \kappa^2(1-2\lambda)\right]\langle\gamma\rangle^{-2} S_x S_y \sin 2\Delta\Phi,$$

Phase space area

$$\begin{aligned}\dot{\Phi}_x =& \frac{1}{24}k_p r_e \kappa^3 \langle\gamma\rangle^{\frac{1}{2}} S_y \sin 2\Delta\Phi + \frac{1}{64}\lambda\beta_{z0}\langle\gamma\rangle^{-2}(S_x + S_y\cos 2\Delta\Phi) \\ &- \frac{1}{16}\kappa^2\langle\gamma\rangle^{-2}[S_x + 2\lambda S_y + (1-2\lambda)S_y\cos 2\Delta\Phi] - \frac{1}{4}\kappa\lambda\langle\gamma\rangle^{-\frac{5}{2}},\end{aligned}$$

Betatron phase

$$\frac{d\Delta\Phi}{dt} = -\frac{1}{24}k_p r_e \kappa^3 \langle\gamma\rangle^{\frac{1}{2}} (S_y + S_x)\sin 2\Delta\Phi + \frac{1}{8}\left[\frac{1}{4}\lambda\beta_{z0} - \kappa^2(1-2\lambda)\right]\langle\gamma\rangle^{-2}(S_y - S_x)\sin^2\Delta\Phi.$$

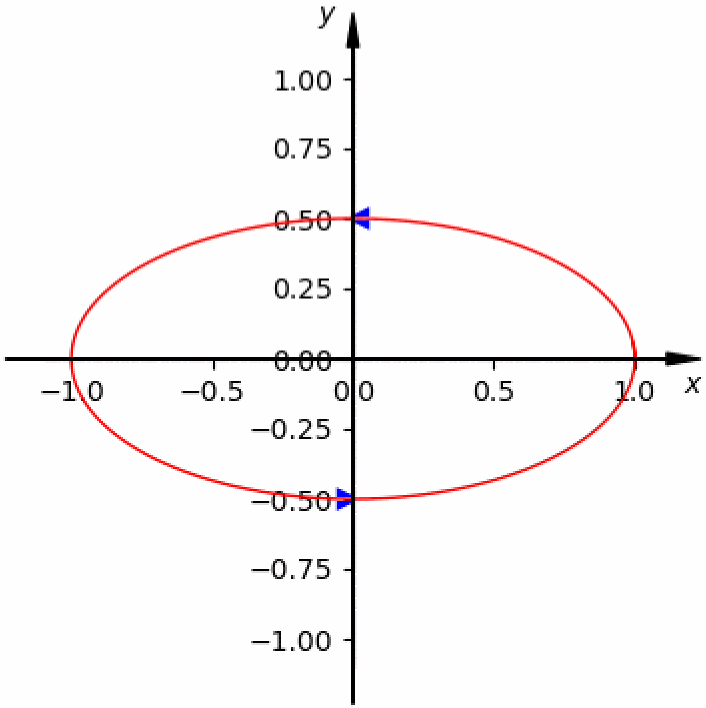
Relative betatron phase

Yulong Liu and Ming Zeng, Phys. Rev. Accel. Beams 26, 031301 (2023)

Predict new phemonenon

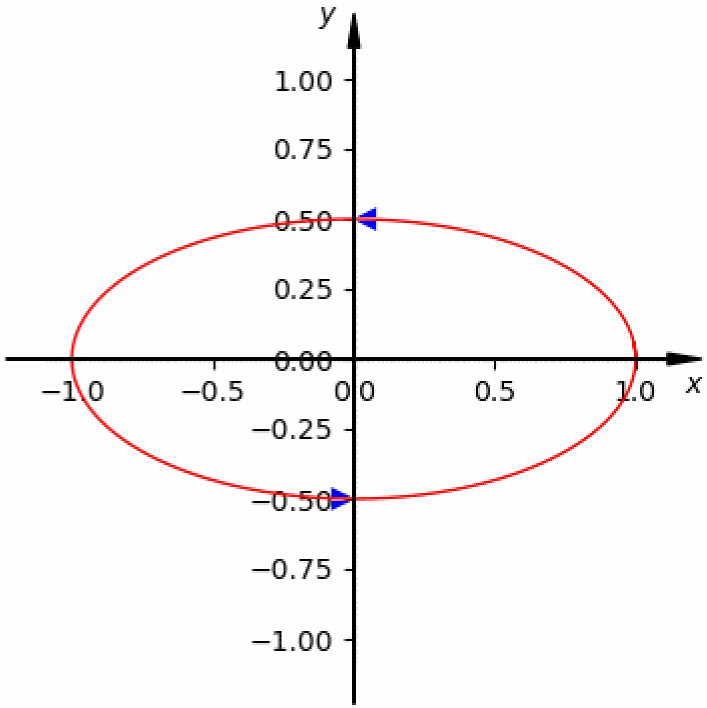
- Betatron obit precession (low energy regime)

$$\Omega = -\frac{1}{8}\left[\frac{\lambda\beta_{z0}}{4} - \kappa^2(1-2\lambda)\right]\langle\gamma\rangle^{-2}\langle L_z\rangle$$



- Betatron obit tapering (high energy regime)

$$\dot{R} = -\frac{1}{6}k_p r_e \kappa^3 \langle\gamma\rangle^{\frac{1}{2}} R\left(S_x - S_y\right) < 0$$



Summary

- PWFA to high energies is our goal
- Adiabatic sheath model for better prediction of blowout radius, which can be used for high transformer ratio PWFAs
- Long-term betatron oscillation and radiation reaction have been modeled