CKM determination from W decays with Jet tagging at CEPC

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Outline:

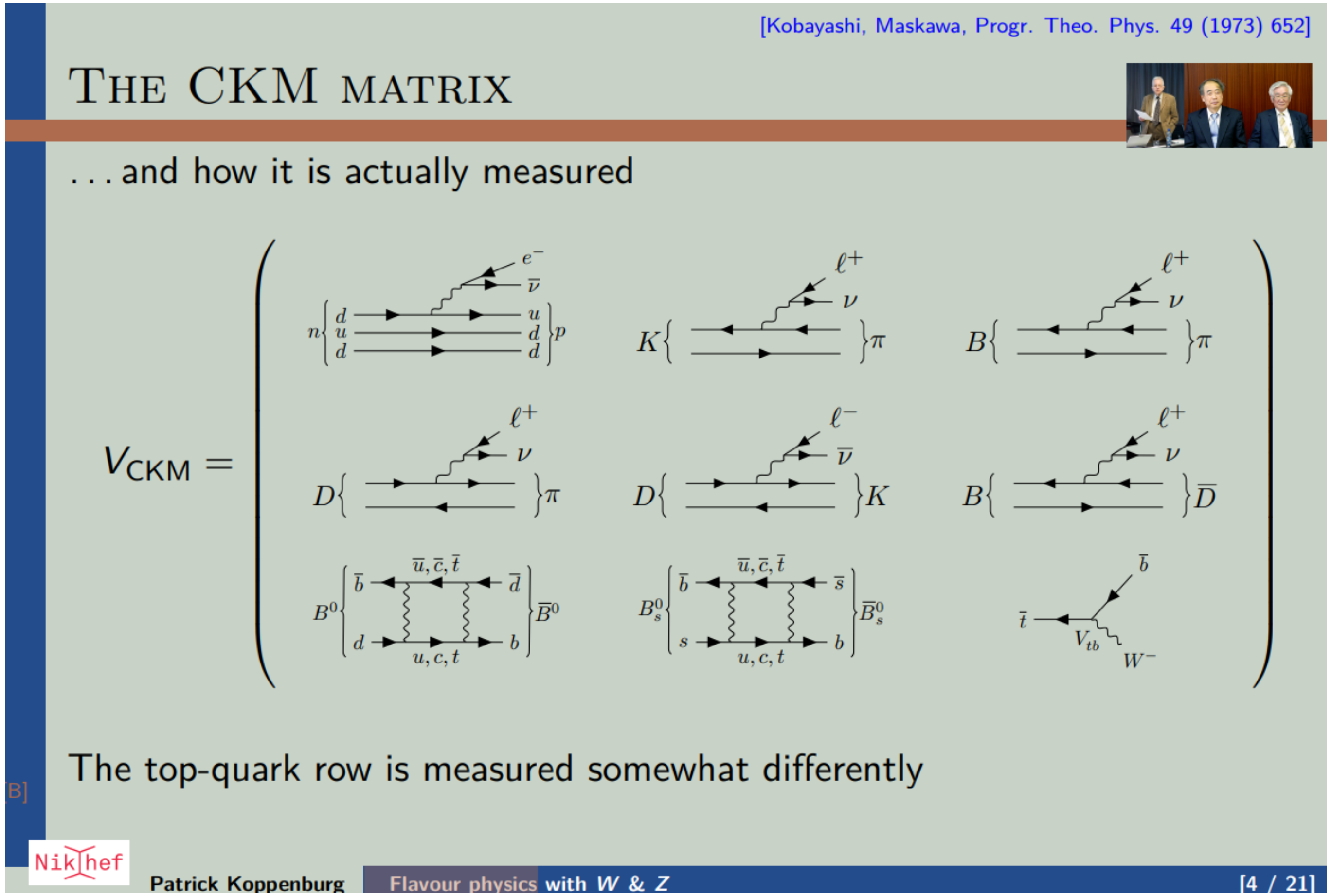
1. Motivation
2. Experiment setup
3. Event selection
4. Jet flavor tagging
5. CKM measurement
6. Summary

- **CKM Matrix and Flavor Physics**
  - The CKM matrix controls all quark flavor-changing transitions
  - It determines the magnitude of CP violation within the Standard Model

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- **Importance of Precision**
  - Tests CKM unitarity ($\Sigma|V_{ij}|^2 = 1$) provides a probe beyond the Standard Model
  - Precision measurements are essential for isolating new physics in rare decays

- How the CKM elements are currently measured
  - Rely on hadronic decay measurements that introduce significant theoretical uncertainties from non-perturbative QCD effects



From Patrick Koppenburg at ESPP June 25

- The CKM elements current measured values

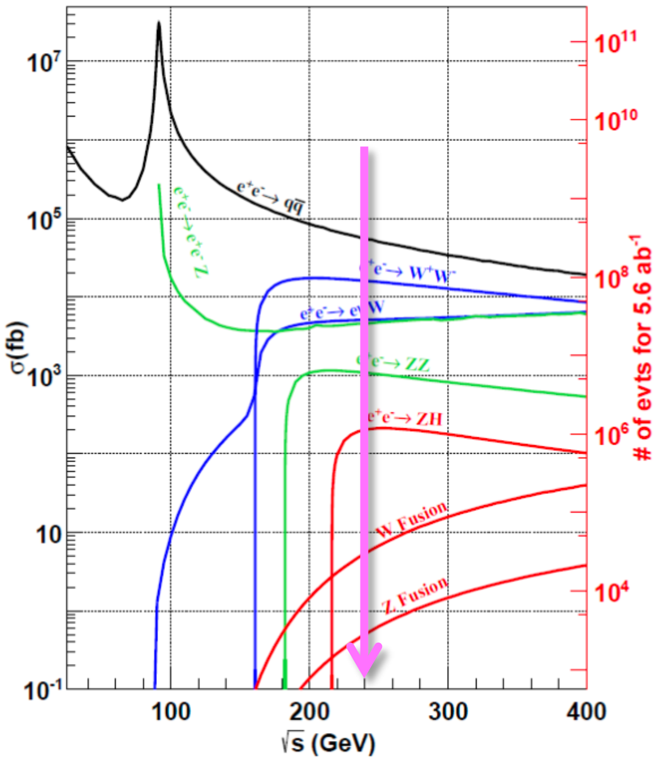
CKM element	Current Value	Relative Precision	Methods
$\|V_{ud}\|$	0.97373 ± 0.00031	0.032%	Superallowed β decay
$\|V_{us}\|$	0.2250 ± 0.004	0.18%	$K \to \pi l \upsilon, K \to \mu\nu$
$\|V_{ub}\|$	$(3.67 \pm 0.15) \times 10^{-3}$	4.1%	$B \to \pi\ell\nu$
$\|V_{cd}\|$	0.2181 ± 0.0050	2.30%	$D \to \mu\nu, D \to \tau\nu$
$\|V_{cs}\|$	0.972 ± 0.007	0.72%	$D \to k\ell\nu$
$\|V_{cb}\|$	$(39.8 \pm 0.6) \times 10^{-3}$	1.5%	Semileptonic B decays

Cited from PDG

- **W Decays as a clean probe**
  - The decay rate $\Gamma(W \to q_i q_j) \propto |V_{ij}|^2$ offers a direct measurement
  - This is a direct method, avoiding hadronic uncertainties
- LEP2(1998) measured $|V_{cs}| = 0.91\ ^{+0.15}_{-0.14}(\text{stat}) \pm (0.05)(\text{syst})$, using W decays at $\sqrt{s} = 161$ and 172 GeV with approximately 100 W pairs events

$$\mathcal{B}\left(W \to q_i q_j\right) = \frac{\left|V_{ij}\right|^2}{\sum_{lm}|V_{lm}|^2}\mathcal{B}(W \to qq) \approx \frac{1}{2}\left|V_{ij}\right|^2 \mathcal{B}(W \to qq)$$

- CEPC(future) provides high statistics of W pair events at $\sqrt{s} = 240$ GeV, about **170 million**
- The **CEPC Reference Detector** integrates state-of-the-art technologies to deliver unprecedented precision CEPC TDR



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2. Experiment setup

- MC simulation method: WHIZARD, DELPHES fast simulation
- Signal processes (**take the simplest channel**):

Process $e^+e^- \to WW$, $\boldsymbol{W \to \mu\nu}$	**Expected events**	**Simulated events**
$W \to ud$	5,468,964	100 k
$W \to us$	292,366	100 k
$W \to ub$	77	100 k
$W \to cd$	292,366	100 k
$W \to cs$	5,451,766	100 k
$W \to cb$	10,146	100 k

- Background processes:

Process	Expected Events	Simulated events
$e^+e^- \rightarrow qq$ $e^+e^- \rightarrow ll$ $e^+e^- \rightarrow \nu\nu$	1,082,130,000 697,130,000 1,081,992,000	100 k 100 k 100 k
$e^+e^- \rightarrow ZZ\_sl$	11,129,800	100 k
$e^+e^- \rightarrow WW \rightarrow e\nu\ qq$	11,522,660	100 k
$e^+e^- \rightarrow WW \rightarrow \tau\nu\ qq$	11,522,660	100 k
$e^+e^- \rightarrow single\ Z$	9,233,200	100 k
$e^+e^- \rightarrow single\ W \rightarrow e\nu\ \mu/\tau\ \nu$ $e^+e^- \rightarrow single\ W \rightarrow e\nu qq$	17,452,600 52,252,400	100 k 100 k
$e^+e^- \rightarrow llH$ $e^+e^- \rightarrow \nu\nu H$ $e^+e^- \rightarrow qqH$	411,200 925,800 2,736,200	100 k 100 k 100 k

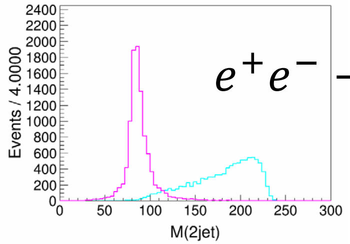
- Selection criteria:

Basic event selection criteria	Definition
N(jet)=2	Number of jet
N(tracks)	Number of charged tracks
N(ℓ)	Number of lepton
E(all tracks)	Energy of all tracks
E_max(γ)	Energy of highest-energy photon

Muon Identification	Definition
N(μ)	Number of muon
P_max(μ)	Momentum of highest-momentum muon (leading muon)
P(all μ)	Momentum of all muon
Angle(P_max(μ) - 2jet)	Angle between leading muon and 2jet
E_ratio(L_μ)	The energy of the leading muon divided by the combined energy within a 30◦ cone

2jet System Characterization	Definition
M(2jet)	Invariant mass of 2jet
Y23	Quantifies the transition between 2-jet and 3-jet

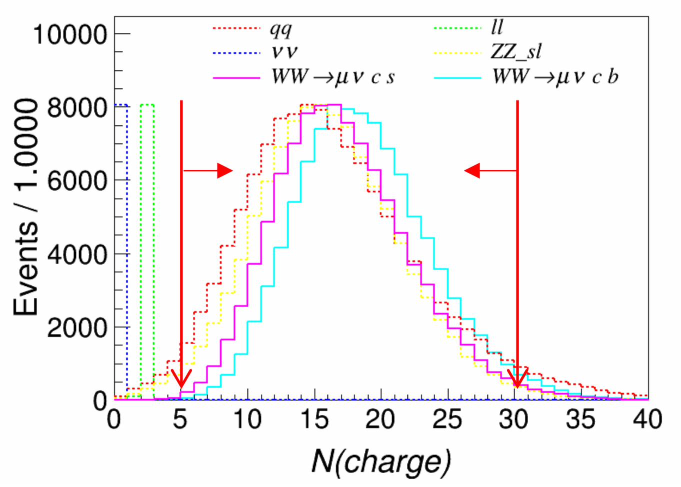
Use the ee-kt algorithm to reconstruct jet, exclude a leading muon

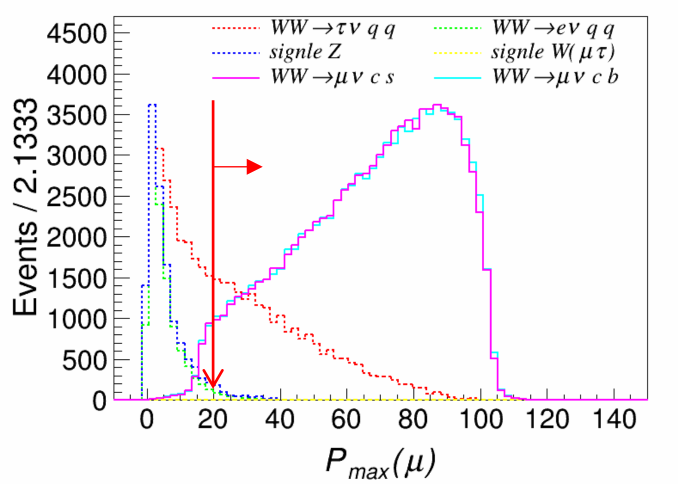


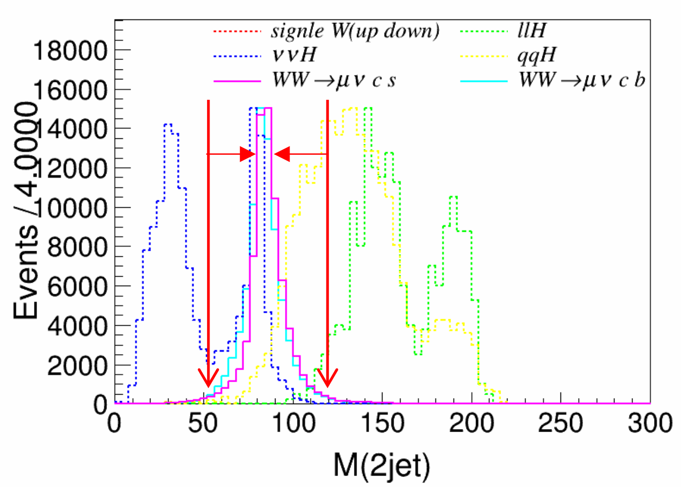
$e^+e^- \rightarrow WW \rightarrow \mu\nu\ cs$

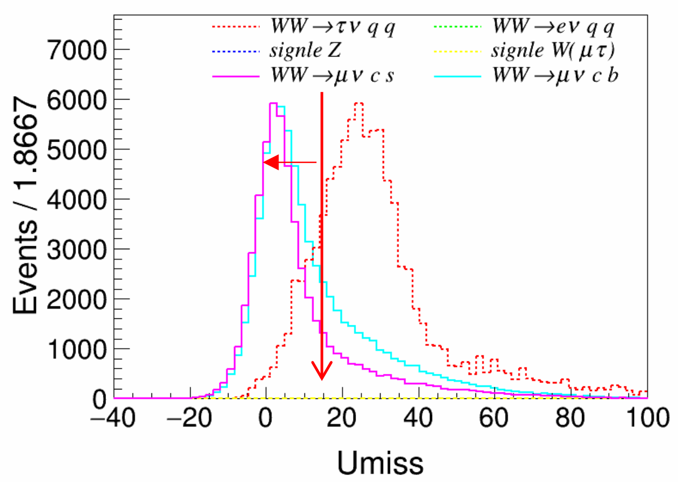
Missing Energy and Vertexing requirement	Definition
cosθ(miss)	Cosine of the angle between missing momentum and beam axis
Umiss	Umiss = Emiss - Pmiss
D0Dz	$(D_0/\sigma_{D_0})^2+(D_z/\sigma_{D_z})^2$ Impact parameter significance

- Check of event selection:









3.Event selection

Cut flow	$WW \to \mu\nu cs$	$WW \to \tau\nu qq$	$e^+e^- \to ZZ\_sl$	$e^+e^- \to qqH$
Begin	100,000	100,000	100,000	100,000
N(jet)=2	100,000	100,000	99,996	100,000
5 ≤ N(tracks) ≤ 30	98,952	99,550	97,645	37,150
N(ℓ) ≥1	98,928	45,203	54,493	26,779
50 ≤ E(all tracks) ≤ 200 GeV	97,766	41,957	44,667	26,571
E_max(γ) ≤ 60 GeV	97,421	41,756	43,195	26,472
N(μ) ≥ 1	97,419	19,391	36,724	15,547
P_max(μ) ≥ 20 GeV	95,132	9,778	27,659	5,482
P(all μ) ≤ 100 GeV	90,523	9,776	10,013	5,421
80° ≤ Angle(P_max(μ) - 2jet) ≤ 170°	82,920	8,504	3,105	3,600
E_ratio(L_μ) ≥ 0.9	79,711	8,183	1,835	2,863
50 ≤ M(2jet) ≤ 120 GeV	77,813	7,926	1,016	947
Y23 ≤ 0.1	76,599	6,454	895	654
|cosθ(miss)| ≤ 0.9	70,998	4,964	99	587
Umiss ≤ 15 GeV	56,194	878	35	94
D0Dz ≤ 10	55,808	229	35	90

- Selected signals:

Process $e^+e^- \to WW,\ W \to \ \mu\nu$	**Expected events**	**After Cut (efficiency)**
$W \to ud$	5,468,964	3,303,254 (60.4%)
$W \to us$	292,366	171,326 (58.6%)
$W \to ub$	77	38 (49.8%)
$W \to cd$	292,366	167,233 (57.2%)
$W \to cs$	5,451,766	3,042,085 (55.8%)
$W \to cb$	10,146	4,718 (46.5%)

- Sum of all signal events: 6,688,654

- Survived backgrounds:

Process	Expected Events	After Cut (efficiency)
$e^+e^- \rightarrow qq$ $e^+e^- \rightarrow ll$ $e^+e^- \rightarrow \nu\nu$	1,082,130,000 697,130,000 1,081,992,000	0 0 0
$e^+e^- \rightarrow ZZ\_sl$	11,129,800	3,896 (0.035%)
$e^+e^- \rightarrow WW \rightarrow e\nu\ qq$	11,522,660	0
$e^+e^- \rightarrow WW \rightarrow \tau\nu\ qq$	11,522,660	26,387 (0.229%)
$e^+e^- \rightarrow single\ Z$	9,233,200	0
$e^+e^- \rightarrow single\ W \rightarrow e\nu\ \mu/\tau\ \nu$ $e^+e^- \rightarrow single\ W \rightarrow e\nu qq$	17,452,600 52,252,400	0 0
$e^+e^- \rightarrow llH$ $e^+e^- \rightarrow \nu\nu H$ $e^+e^- \rightarrow qqH$	411,200 925,800 2,736,200	37 (0.009%) 0 2,463 (0.09%)

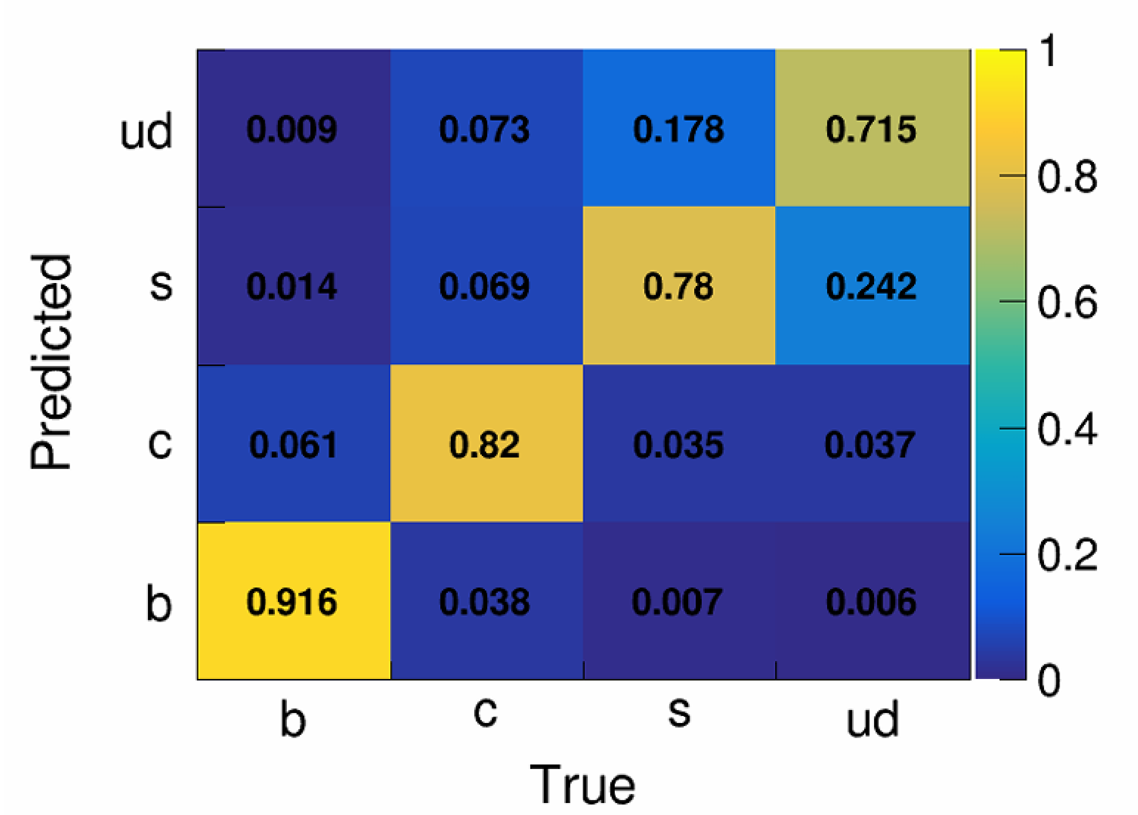
- Sum of all the backgrounds: 32,783 (background ratio 0.5%)

- Mathine Learning: ParticleNet, an advanced neural network architecture
- 4-class jet classification: **b c s ud**
- Combined **ud** category: u and d are hard to distinguish, appear simultaneously, and tagging u and d is not important for the measurement
- 4-class jet samples are from signal process $e^+e^- \to WW \to \mu\nu\, qq$
- Each jet has 1 million entries, 80% for training, 20% for validation

- Each jet has about 15-20 particles, each particle has the following variables
- One jet characterized by ~150-200 input variables

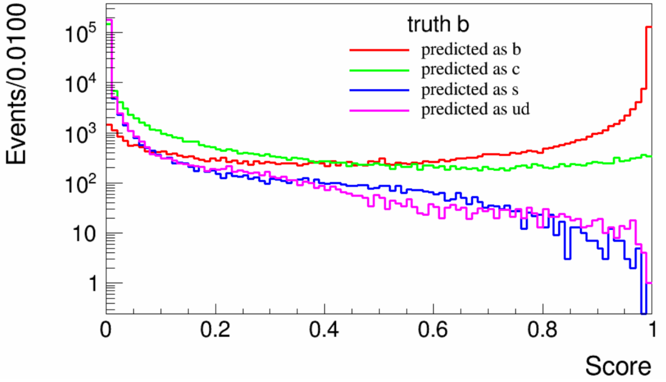
Input variables:	Definition
D0/D0_err	Impact parameters
Z0/Z0err	
Prob	$\chi = (D_0/\sigma_{D_0})^2 + (D_z/\sigma_{D_z})^2$ $prob = \int_{\chi}^{\infty} p(\chi, 2) d\chi$
logPt	Log transverse momentum and energy of particle
logE	
log(Pt/Pt_jet)	Log of Pt fraction in a jet
Log(E/E_jet)	Log of energy fraction in a jet
Px, Py, Pz, E	4-momentum

- Tagging result: accuracy 80.8%
- Confusion matrix: good separation sufficient for the following analysis



4. Jet Flavor Tagging

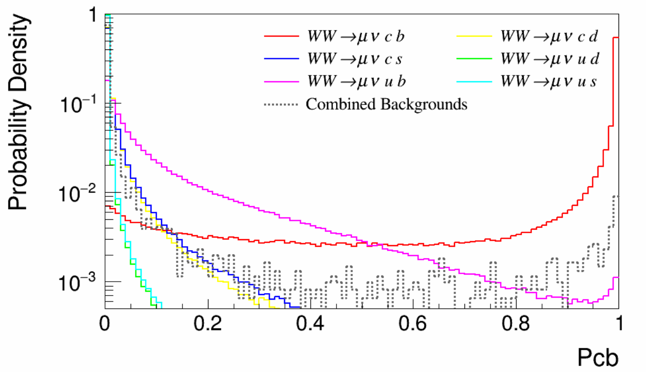
- After apply the ML model, each jet has score when predicted as c, b, s, ud



- Then define the event type by: $P_1(\text{i})$ and $P_2(j)$ represent score of predict jet1 and jet2 as quark i j

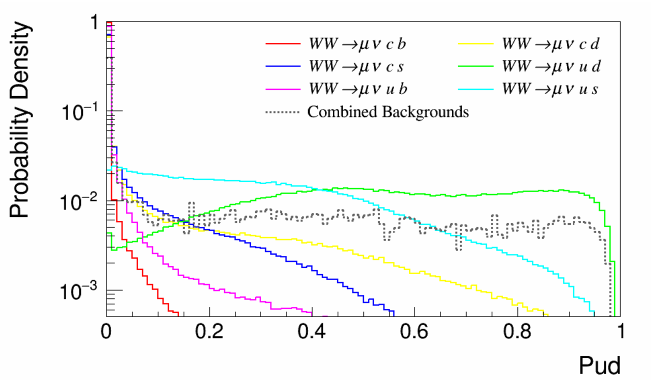
$$P_{cb} = \frac{P_1(c)P_2(b) + P_2(c)P_1(b)}{[P_1(c)P_2(s) + P_2(c)P_1(s)] + [P_1(c)P_2(b) + P_2(c)P_1(b)] + \cdots}$$

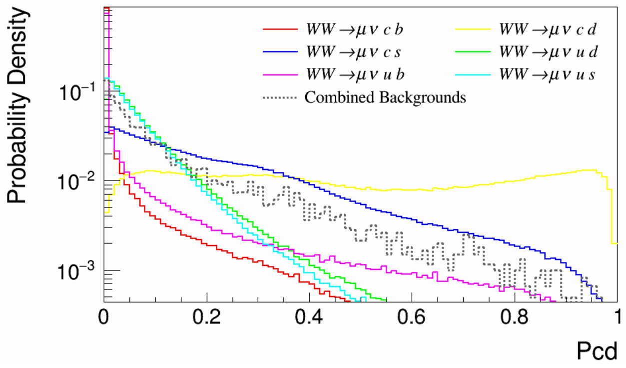
- For $e^+e^- \to WW \to \mu\nu\, \boldsymbol{cb}$ event, the P_{cb} value is close to 1, other type to 0
- Similar define P_{cd}, P_{cs}, P_{ud}, P_{us}, P_{ub}

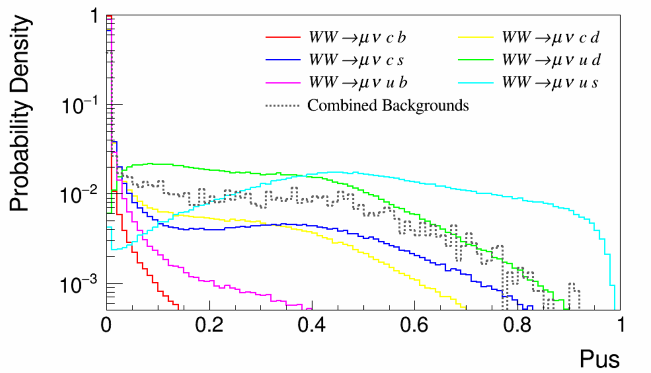


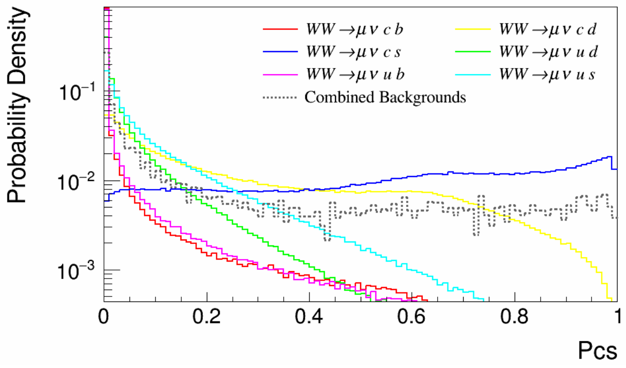
4. Jet Flavor Tagging

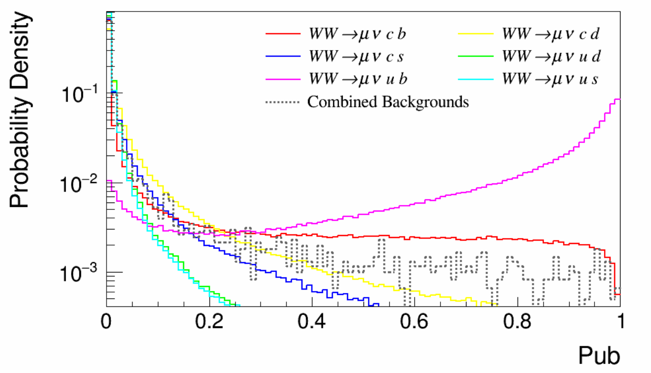
- Shapes of P_{ij} for signal and background processes

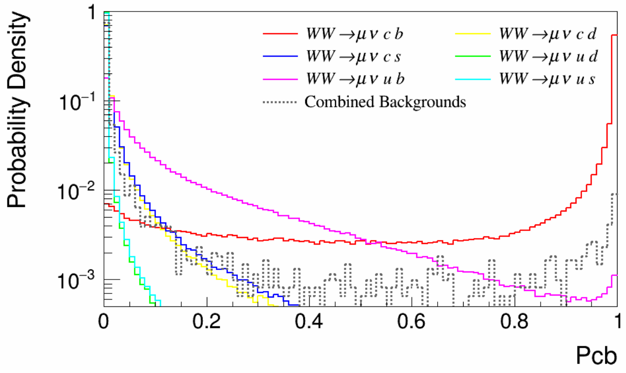






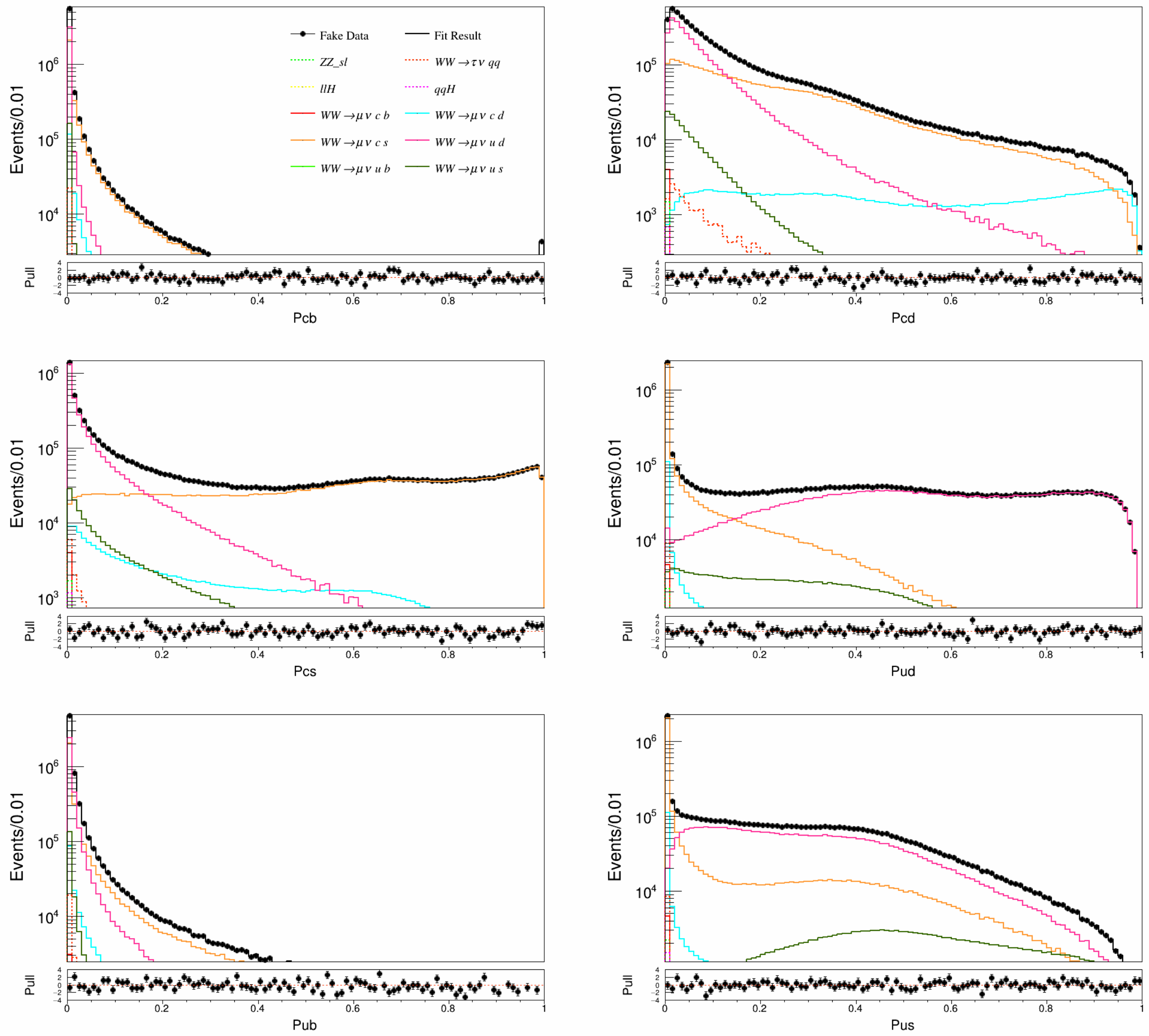




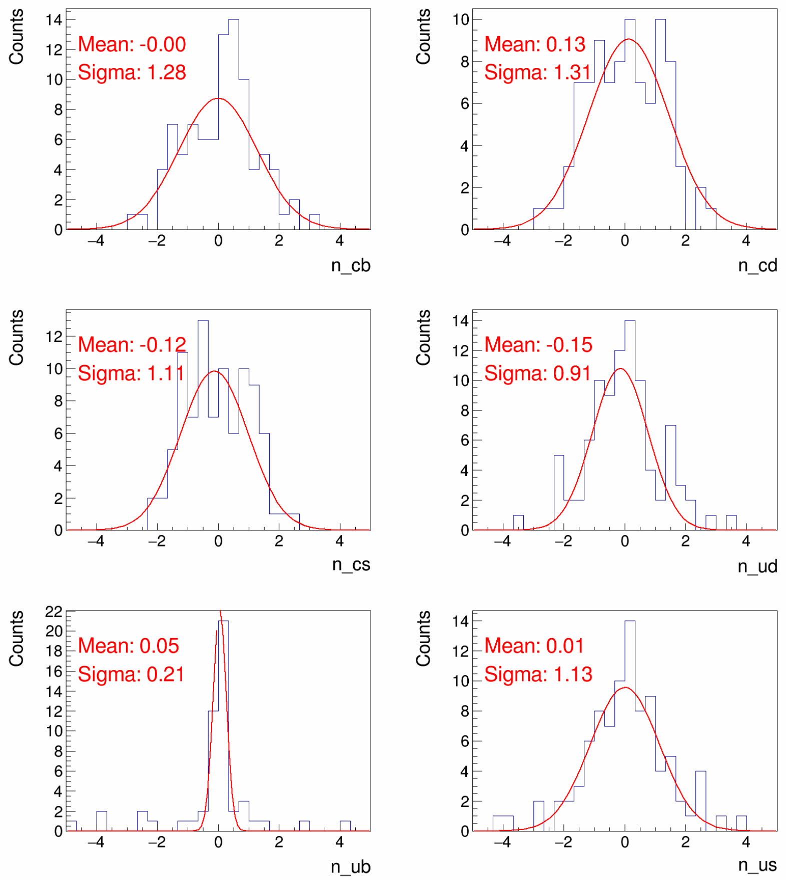


- Fake data, includes expected signal and background events
- **Simultaneous fit** to 6 distributions ($P_{cd}, P_{cs}, P_{cb}, P_{ud}, P_{us}, P_{ub}$) , each composed of 6 signal and 4 background components ($e^+e^- \to ZZ\_sl$, $e^+e^- \to WW \to \tau\nu\, qq$, $e^+e^- \to llH$, $e^+e^- \to qqH$)
- The shape of signal, the shape and strength of background are fixed; the signal strength are fitted parameter

- Simultaneous fit result:



Pull distribution:



- Simultaneous fit result: $N(W \to q_i q_j)$

$$\mathcal{B}\left(W \to q_i q_j\right) = \frac{N(W \to q_i q_j)}{N(e^+e^- \to WW) \cdot \mathcal{B}(W \to \mu\nu) \cdot \epsilon}$$

$$\mathcal{B}\left(W \to q_i q_j\right) \approx \frac{1}{2}\left|V_{\mathrm{ij}}\right|^2 \mathcal{B}(W \to qq)$$

$$\left|V_{\mathrm{ij}}\right| = \sqrt{\frac{2\,N(W \to q_i q_j)}{N(e^+e^- \to WW) \cdot \mathcal{B}(W \to \mu\nu) \cdot \mathcal{B}(W \to qq) \cdot \epsilon}}$$

$$\frac{\Delta\left|V_{\mathrm{ij}}\right|}{\left|V_{\mathrm{ij}}\right|} = \frac{1}{2}\frac{\Delta N(W \to q_i q_j)}{N(W \to q_i q_j)}$$

5.CKM Measurement

$$|V_{ij}| = \sqrt{\frac{2\,N(W \to q_i q_j)}{N(e^+e^- \to WW)\cdot \mathcal{B}(W \to \mu\nu)\cdot \mathcal{B}(W \to qq)\cdot \epsilon}}$$

	$W \to ud$	$W \to us$	$W \to ub$	$W \to cd$	$W \to cs$	$W \to cb$
$N(W \to q_i q_j) \pm \Delta N(W \to q_i q_j)$	$3.3025\times10^6 \pm 1.48\times10^3$	$1.7157\times10^5 \pm 1.33\times10^3$	57 ± 0.3	$1.6598\times10^5 \pm 1.18\times10^3$	$3.0437\times10^6 \pm 1.45\times10^3$	4769 ± 119
$N(e^+e^- \to WW)$	171,980,000					
$\mathcal{B}(W \to \mu\nu)$	10%					
$\mathcal{B}(W \to qq)$	67.41%					
ϵ	60.4%	58.6%	49.8%	57.2%	55.8%	46.5%
$\|V_{ij}\| \pm \Delta\|V_{ij}\|$	0.971218 ± 0.000218	0.224743 ± 0.000871	0.0044436 ± 0.0000117	0.223740 ± 0.000796	0.970058 ± 0.000231	0.042063 ± 0.000525
$\Delta\|V_{ij}\|/\|V_{ij}\|$	0.023%	0.388%	0.264%	0.36%	0.024%	1.3%

	$\|V_{ud}\|$	$\|V_{us}\|$	$\|V_{ub}\|$	$\|V_{cd}\|$	$\|V_{cs}\|$	$\|V_{cb}\|$
$\Delta\|V_{ij}\|/\|V_{ij}\|$ at CEPC	0.023%	0.388%	0.264%	0.36%	0.024%	1.3%
Current precision	0.032%	0.18%	4.1%	2.3%	0.72%	1.5%

- The precision has improved significantly for $|V_{ub}|, |V_{cd}|, |V_{cs}|$

- At CEPC, leveraging high-statistics data, and state-of-the-art machine learning for jet flavor tagging, a remarkable improvement in the precision of key CKM elements is projected

- This direct and clean approach will be a powerful tool in the search for physics beyond the Standard Model

- To do: simultaneous fit optimization, systematic study

Thanks!

Backup

CKM global fit : Summer 2025 inputs

	CKM	Process	Observables			Non-perturbative theo. inputs		
$\vert V_{xy}\vert$ from (semi) leptonic decays	$\vert V_{ud}\vert$	$0^+ \to 0^+ \beta$	$\vert V_{ud}\vert_{\text{nucl}}$	=	$0.97373 \pm 0.00009 \pm 0.00053$	Nuclear matrix elements		
	$\vert V_{us}\vert$	$K \to \pi \ell \nu_\ell$	$\vert V_{us}\vert_{\text{SL}} f_+^{K\to\pi}(0)$	=	0.21635 ± 0.00038	$f_+^{K\to\pi}(0)$	=	$0.9675 \pm 0.0011 \pm 0.0023$
		$K \to e\nu_e$	$\mathcal{B}(K \to e\nu_e)$	=	$(1.582 \pm 0.007) \cdot 10^{-5}$			
		$K \to \mu\nu_\mu$	$\mathcal{B}(K \to \mu\nu_\mu)$	=	0.6356 ± 0.0011	f_K	=	$155.47 \pm 0.18 \pm 0.57$ MeV
		$\tau \to K\nu_\tau$	$\mathcal{B}(\tau \to K\nu_\tau)$	=	$(0.6986 \pm 0.0085) \cdot 10^{-2}$			
	$\frac{\vert V_{us}\vert}{\vert V_{ud}\vert}$	$K \to \mu\nu_\mu / \pi \to \mu\nu_\mu$	$\frac{\mathcal{B}(K\to\mu\nu_\mu)}{\mathcal{B}(\pi\to\mu\nu_\mu)}$	=	1.3367 ± 0.0028	f_K/f_π	=	$1.1987 \pm 0.0009 \pm 0.0007$ ↑
		$\tau \to K\nu_\tau / \tau \to \pi\nu_\tau$	$\frac{\mathcal{B}(\tau\to K\nu_\tau)}{\mathcal{B}(\tau\to\pi\nu_\tau)}$	=	$(6.437 \pm 0.092) \cdot 10^{-2}$			
	$\vert V_{cd}\vert$	νN	$\vert V_{cd}\vert_{\text{not lattice}}$	=	0.230 ± 0.011			
		$D \to \tau\nu_\tau$	$\mathcal{B}(D \to \tau\nu_\tau)$	=	$(1.20 \pm 0.27) \cdot 10^{-3}$			
		$D \to \mu\nu_\mu$	$\mathcal{B}(D \to \mu\nu_\mu)$	=	$(3.77 \pm 0.17) \cdot 10^{-4}$	f_{D_s}/f_D	=	$1.1784 \pm 0.0006 \pm 0.0033$
		$D \to \pi\ell\nu_\ell$	$\vert V_{cd}\vert_{\text{SL}} f_+^{D\to\pi}(0)$	=	0.1426 ± 0.0018	$f_+^{D\to\pi}(0)$	=	$0.624 \pm 0.004 \pm 0.006$
	$\vert V_{cs}\vert$	$W \to c\bar{s}$	$\vert V_{cs}\vert_{\text{not lattice}}$	=	$0.94^{+0.32}_{-0.26} \pm 0.13$			
		$D_s \to \tau\nu_\tau$	$\mathcal{B}(D_s \to \tau\nu_\tau)$	=	$(5.39 \pm 0.09) \cdot 10^{-2}$			
		$D_s \to \mu\nu_\mu$	$\mathcal{B}(D_s \to \mu\nu_\mu)$	=	$(5.36 \pm 0.12) \cdot 10^{-3}$	f_{D_s}	=	$249.09 \pm 0.25 \pm 0.65$ MeV
		$D \to K\ell\nu_\ell$	$\vert V_{cs}\vert_{\text{SL}} f_+^{D\to K}(0)$	=	0.7180 ± 0.0033	$f_+^{D\to K}(0)$	=	$0.742 \pm 0.002 \pm 0.004$
	$\vert V_{ub}\vert$	semileptonic B	$\vert V_{ub}\vert_{\text{SL}}$	=	$(3.86 \pm 0.07 \pm 0.12) \cdot 10^{-3}$	form factors, shape functions		
		$B \to \tau\nu_\tau$	$\mathcal{B}(B \to \tau\nu_\tau)$	=	$(1.12 \pm 0.19) \cdot 10^{-4}$	f_{B_s}/f_B	=	$1.2118 \pm 0.0020 \pm 0.0058$
	$\vert V_{cb}\vert$	semileptonic B	$\vert V_{cb}\vert_{\text{SL}}$	=	$(41.22 \pm 0.24 \pm 0.37) \cdot 10^{-3}$	form factors, OPE matrix elements		
	$\vert V_{ub}/V_{cb}\vert$	semileptonic Λ_b	$\frac{\gamma(\Lambda_b\to p\mu^-\bar{\nu}_\mu)_{q^2>15}}{\gamma(\Lambda_b\to\Lambda_c\mu^-\bar{\nu}_\mu)_{q^2>7}}$	=	$(0.918 \pm 0.083) \cdot 10^{-2}$	$\frac{\zeta(\Lambda_b\to p\mu^-\bar{\nu}_\mu)_{q^2>15}}{\zeta(\Lambda_b\to\Lambda_c\mu^-\bar{\nu}_\mu)_{q^2>7}}$	=	$1.471 \pm 0.096 \pm 0.290$
		semileptonic B_s	$\frac{\gamma(B_s\to K^+\mu^-\bar{\nu}_\mu)_{q^2>15}}{\gamma(B_s\to D_s^+\mu^-\bar{\nu}_\mu)_{q^2>7}}$	=	$(3.25 \pm 0.28) \cdot 10^{-3}$	$\frac{\zeta(B_s\to K^+\mu^-\bar{\nu}_\mu)_{q^2>7}}{\zeta(B_s\to D_s^+\mu^-\bar{\nu}_\mu)_{q^2>7}}$	=	$0.363 \pm 0.001 \pm 0.065$
		inclusive	$\vert V_{ub}/V_{cb}\vert_{\text{incl}}$	=	$0.100 \pm 0.006 \pm 0.003$			
UT angles	α	$B \to \pi\pi,\ \rho\pi,\ \rho\rho$	branching ratios, CP asymmetries			isospin symmetry		
	β	$B \to (c\bar{c})K$	$\sin(2\beta)_{[c\bar{c}]}$	=	0.710 ± 0.011	subleading penguins neglected		
		$B^0 \to D^{(*)}h^0$	$\cos(2\beta)$	=	0.91 ± 0.25			
	γ	$B \to D^{(*)}K^{(*)}$	γ	=	$(66.4^{+2.7}_{-2.8})^\circ$	GGSZ, GLW, ADS methods		
	ϕ_s	$B_s \to J/\psi(KK, \pi\pi)$	$(\phi_s)_{b\to c\bar{c}s}$	=	-0.052 ± 0.013			
Oscillations	$V_{tq}^* V_{tb}$	Δm_d	Δm_d	=	$0.5069 \pm 0.0019\ \text{ps}^{-1}$	$\hat{B}_{B_s}/\hat{B}_{B_d}$	=	$1.007 \pm 0.010 \pm 0.014$
		Δm_s	Δm_s	=	$17.766 \pm 0.006\ \text{ps}^{-1}$	$\hat{B}_{B_s}$	=	$1.313 \pm 0.012 \pm 0.030$
		$B_s \to \mu\mu$	$\mathcal{B}(B_s \to \mu\mu)$	=	$(3.34 \pm 0.27) \cdot 10^{-9} [\times(1-0.063)]$	f_{B_s}	=	$228.75 \pm 0.69 \pm 1.87$ MeV
Kaons	$V_{td}^* V_{ts}$ and $V_{cd}^* V_{cs}$	ε_K	$\vert\varepsilon_K\vert$	=	$(2.228 \pm 0.011) \cdot 10^{-3}$	$\hat{B}_K$	=	$0.7692 \pm 0.0019 \pm 0.0120$ ↑
						κ_ε	=	$0.940 \pm 0.013 \pm 0.023$

From [CKM 2025 workshop]

Backup

Table 1.4: CEPC upgraded operation plan at different center-of-mass energies ($\sqrt{s}$), and corresponding anticipated peak instantaneous luminosity ($\mathcal{L}$), total integrated luminosity ($\int \mathcal{L}$) and event yields. (*) The maximum instantaneous luminosity achievable at the Z pole operation is dependent on the detector solenoid magnetic field. The value reported here assumes a 2 Tesla solenoid. (†) The cross section of W^+W^- at threshold is about 8 pb. An additional 3.2×10^8 W^+W^- events will be produced during the Higgs boson factory operation.

Operation mode	$\sqrt{s}$ (GeV)	SR power (MW)	$\mathcal{L}$ (10^{34} cm^{-2}s^{-1})	$\int \mathcal{L}$/year (ab^{-1}, 2 IPs)	Years	Total $\int \mathcal{L}$ (ab^{-1}, 2 IPs)	Event yields
H	240	50	8.3	2.2	10	21.6	4.3×10^6
Z	91	50	192(*)	50	2	100	4.1×10^{12}
W^+W^-	155-170	50	26.7	6.9	1	6.9	5.5×10^7
$t\bar{t}$	360	50	0.8	0.2	5	1.0	0.6×10^6

From CEPC TDR

$W^- \to$	$\bar{u}d$	$\bar{u}s$	$\bar{u}b$	$\bar{c}d$	$\bar{c}s$	$\bar{c}b$
BR	31.8%	1.7%	4.5×10^{-6}	1.7%	31.7%	5.9×10^{-4}

From Arxiv:2401.07564v3

How to quantify meaning of ‘large χ^2’

- Probability distr. for χ^2 is given by

$$\chi^2 = \sum_i \left(\frac{y_i - \mu_i}{\sigma_i} \right)^2 \quad \Longrightarrow \quad p(\chi^2, N) = \frac{2^{-N/2}}{\Gamma(N/2)} \chi^{N-2} e^{-\chi^2/2}$$

- To make judgement on goodness-of-fit, relevant quantity is integral of above:

$$P(\chi^2; N) = \int_{\chi^2}^{\infty} p(\chi^{2\prime}; N) \, d\chi^{2\prime}$$

- What does χ^2 probability $P(\chi^2,N)$ mean?
  - It is the probability that a function which does genuinely describe the data on N points would give a χ^2 probability as large or larger than the one you already have.
    - Since it is a probability, it is a number in the range [0-1]