Measuring QED Radiative Bhabha to 10^{-4} with the LumiCal

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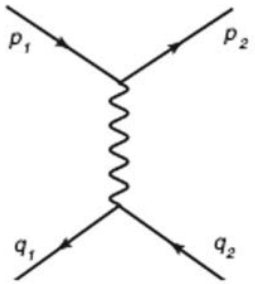
广州 2025.11.08

Radiative Bhabha elastic scattering

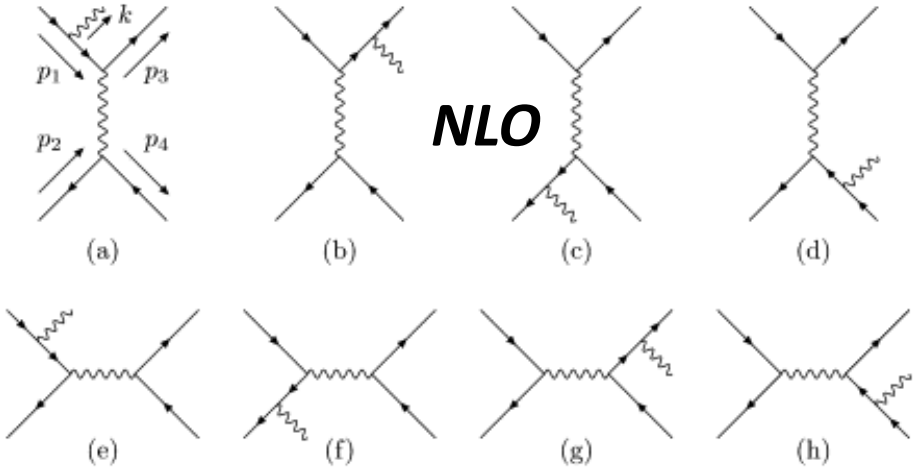


$t-channel$

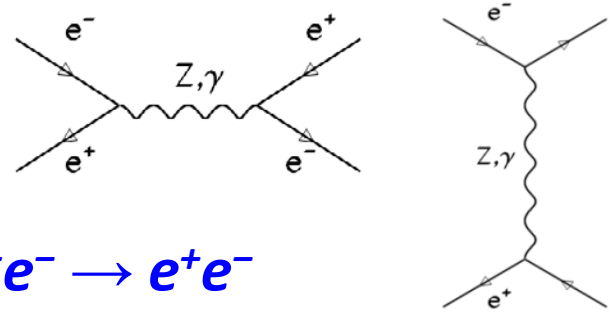
LO

$s-channel$

NLO



Luminosity precision to SM



SM $e^+e^- \rightarrow Z \rightarrow q\bar{q}$, *R(s) ratio*

QED **Luminosity** ***by counting Bhabha*** $e^+e^- \rightarrow e^+e^-$

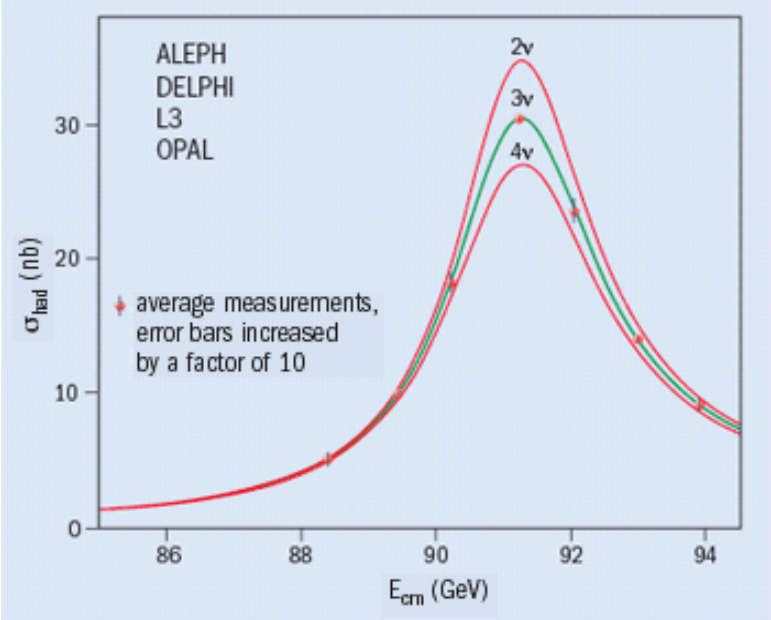
LEP: 17 Million Z (4 IP)

L = 4.3 $10^{31}/cm^2s$ (E=46GeV)

= $1x10^{32}/cm^2s$ (E=100 GeV)

N_ν = 2.9840 ± 0.0082

M_Z = 91187.5 ± 2.1 MeV	2.3×10^{-5}
G_Z = 2495.2 ± 2.3 MeV	1‰
N_ν = 2.9840 ± 0.0082	
Precision luminosity	**$3.4x10^{-4}$**



CERNCOURIER 2 November 2005

QED precision on Bhabha $e^+e^- \to e^+e^-(n\gamma)$

Methods used for multiple photon corrections

1. SF: analytical collinear QED Structure Functions
2. **YFS exponentiation** **Small angle** **0.054%** **BHLUMI (LEP)**
3. **PS: Parton Shower** **Large angle** **0.1%** **BabaYaga@NLO (Flavor F.)**

e^+e^- collision luminosity *by coybtubg Bhabha events*

$$\int \mathcal{L}\, dt = N_{\rm obs}/\sigma_{\rm th}$$

$$\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}}$$

Luminosity errors:
Experiment
Theory

collinear log : $L \equiv \log \frac{s}{m_e^2}$

G. Montagna
Ustron, 2015

$L = \log(s/m_e^2) \simeq 15$ *Large angle @ Flavor*
$L = \log(|t|/m_e^2) \simeq 17$ *Small angle @ LEP*
$L = \log(|t|/m_e^2) \simeq 20$ *Small angle @ $t\bar{t}$ thresh.*

Flavor Factories

collinear log : $L \equiv \log \frac{s}{m_e^2}$

C.M. Carloni Calame
ECFA Higgs CERN 2021

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

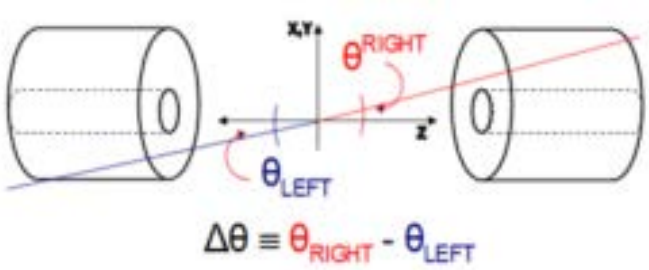
Red: matched PS, SF + NLO

LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

Typically at flavour factories (on integrated Bhabha σ)

Bhabha event counting to 10^{-4}

SM an order improvement to LEP



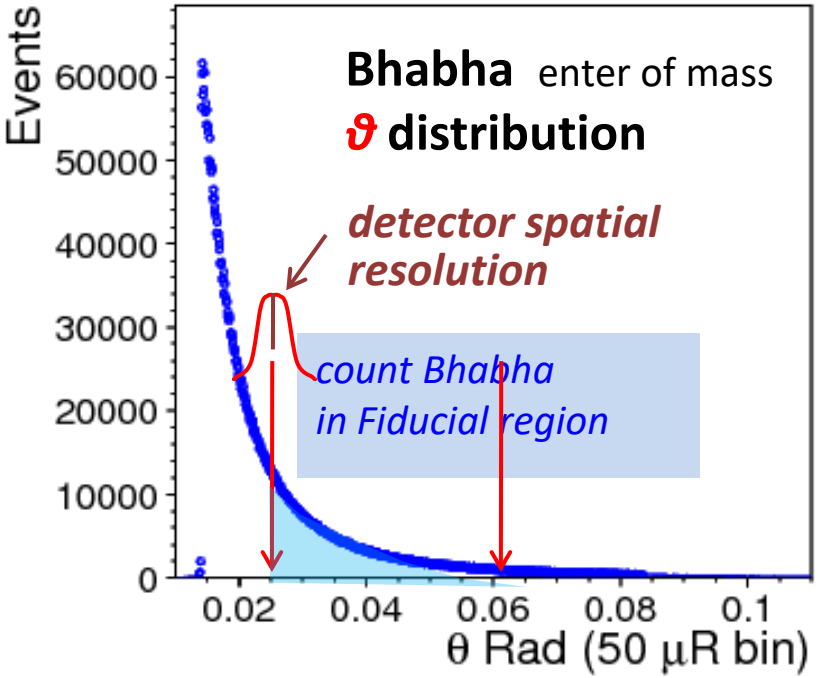
Luminosity $\mathcal{L}$ is derived by

$e^+e^- \rightarrow e^+e^-(n\gamma)$

$$\mathcal{L} = \frac{1}{\varepsilon}\frac{N_{\rm acc}}{\sigma^{\rm vis}} \qquad \sigma = \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta^2_{min}} - \frac{1}{\theta^2_{max}}\right)$$

Bhabha detected for

- ***a pair of back-back electrons,***
- ***precision ϑ of e,e(γ) in fiducial region***



$\delta L/L \sim 2\,\delta\vartheta/\vartheta_{min}$

$\delta L/L = 10^{-4}$

at ***$z = \pm 1000$ mm, $\vartheta_{min} = 20$ mRad***

→ ***$\delta\vartheta = 1$ μRad,*** or ***$dr = 1$ μm***

error due to offset on Z

→ ***50 μm*** *on Z eq.* ***$dr = \delta z \times \vartheta = 1$ μm***

Luminosity systematics ***due to event counting in/out fiducial edge***

→ ***offset on the mean of θ_{min}***

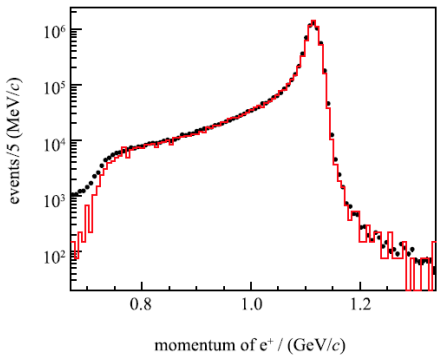
Bhabha experimental results $e^+e^- \to e^+e^-(\gamma)$

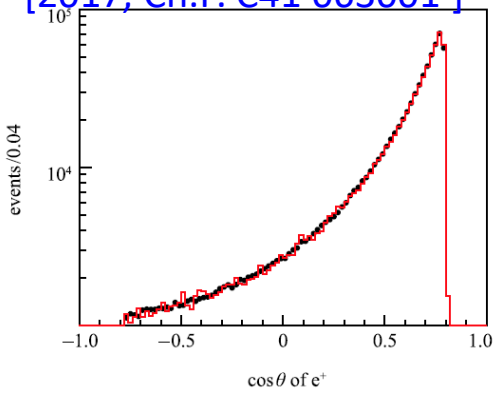
BESIII Luminosity $(\gamma)e^+e^-$, $(\gamma)\gamma\gamma$

Systematic error **~0.7%**

√s = 2.23 - 4.59 GeV

[2017, Ch.P. C41 063001]





TASSO Bhabha

Systematic error **~3%**

√s = 12 - 47 GeV

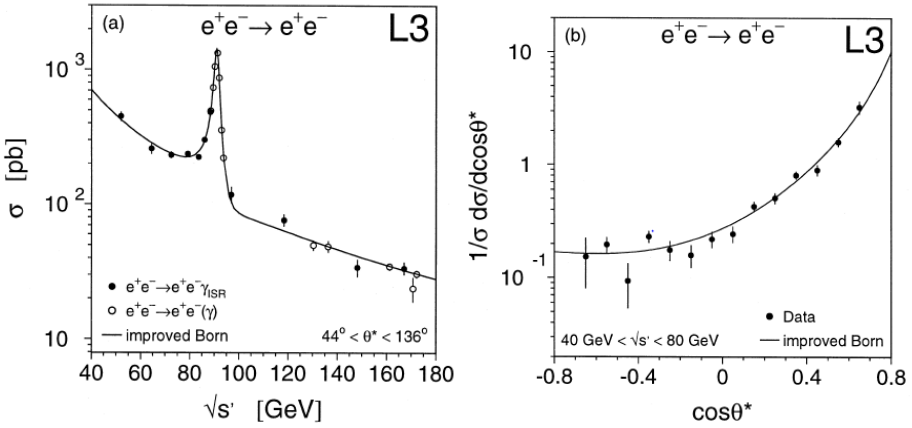
Table 1. Data samples used for the analysis $e^+e^- \to e^+e^-$

$\langle\sqrt{s}\rangle$ (GeV)	$\int \mathcal{L}\,dt$ (pb^{-1})	N_{Bhabha}
14.0	1.7	10730
22.0	2.7	7106
34.8	174.5	166348
38.3	8.9	6035
43.6	37.1	22951

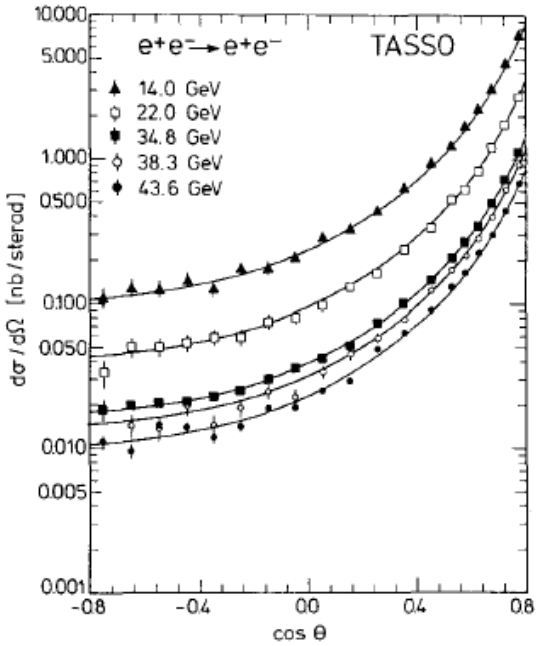
L3 radiative Bhabha with ISR

Systematic error at **~1% level**

√s= 50 ~ 170 GeV, 232 pb^{-1}, 2856 event



[1998, PLB 439, 183]



[1988, ZPC 37, 171]

Challenge: QED α^2L^2 shall be measured

Compare √s = 92.3 GeV

BHLUMI: YFS exponentiation $e^+e^- \rightarrow e^+e^-(n\gamma)$

ReneSANCe: NLO calculation $e^+e^- \rightarrow e^+e^-(\gamma)$

BHLUMI 4.04

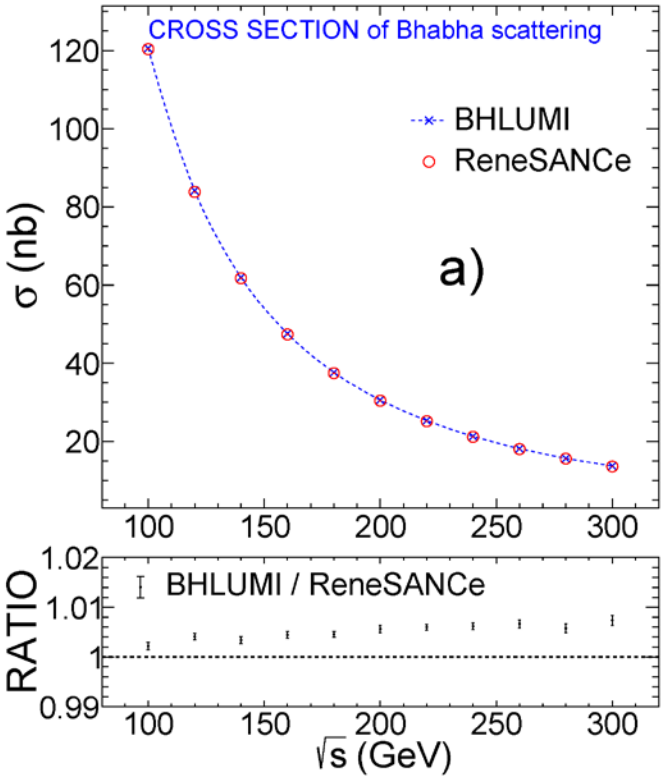
2020 systematic ***0.037%***

[PLB 803 (2020) 135319]

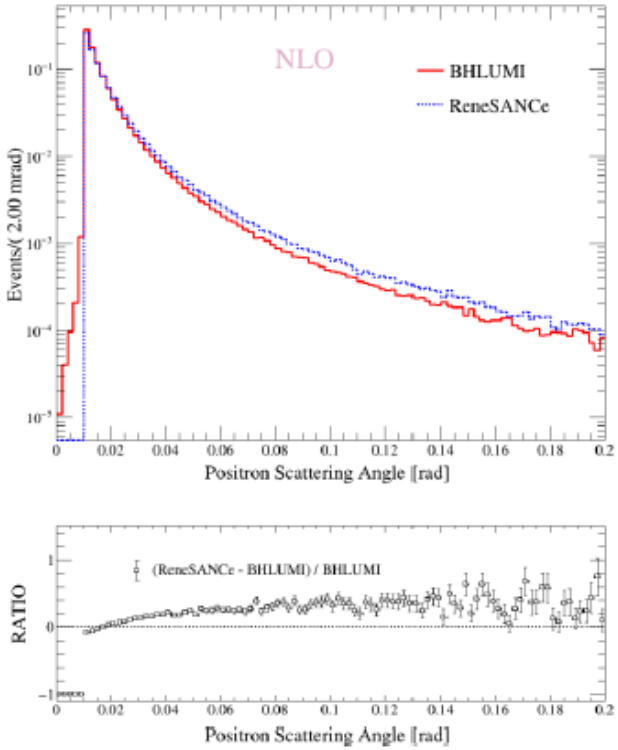
ReneSANCe

[CPC 256 (2020) 107455]

Bhabha Cross section



Bhabha √s= 92.3 GeV
e^+ theta angle, all events



Discrepancy due to 0γ events

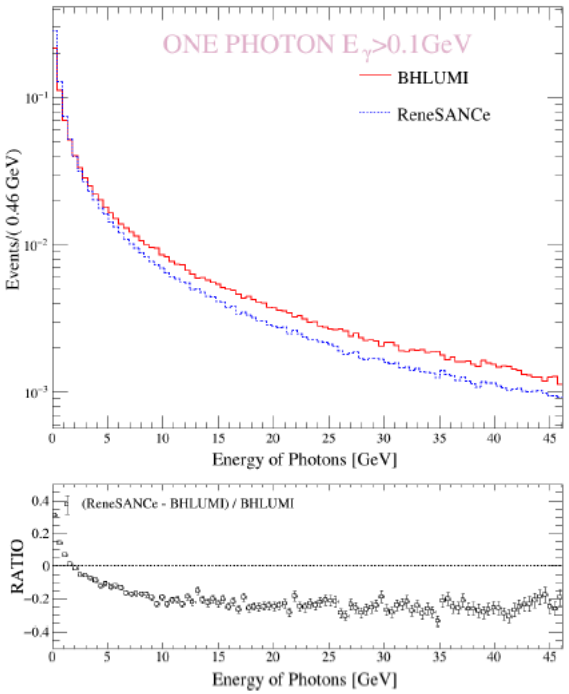
Poster
Jilin U.
J. Gong

Challenge: QED α^2L^2 shall be measured

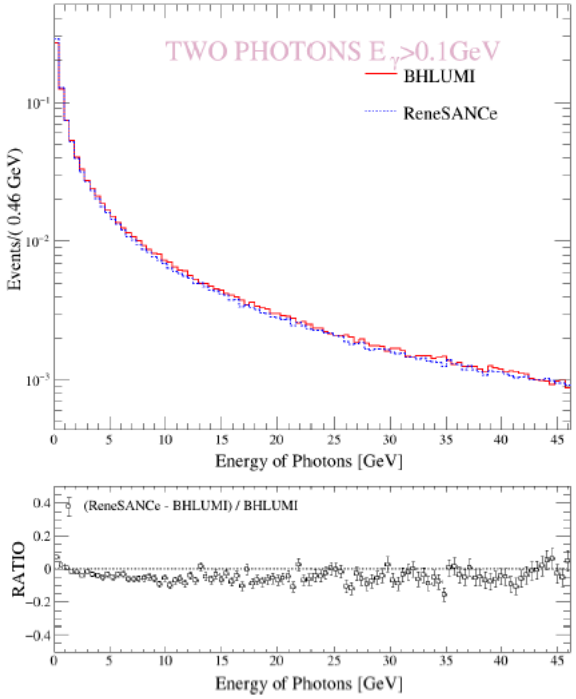
BHLUMI: $e^+e^- \rightarrow e^+e^-(n\gamma)$
ReneSANCe: $e^+e^- \rightarrow e^+e^-(\gamma)$

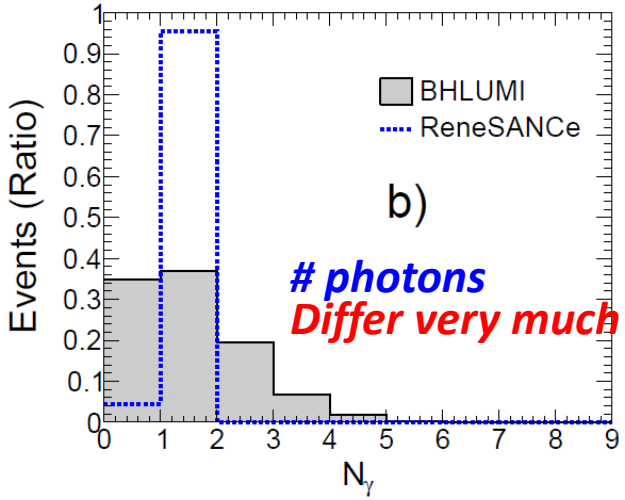
Poster
Jilin U.
J. Gong

Photon events
1γ events



all photons filled





 # photons Differ very much

BHLUMI E(γ)>5MeV

Event final states	BHLUMI generated
e^+e^-	36.4%
$e^+(e^-\gamma)$ or $(e^+\gamma)e^-$	47.8%
$(e^+\gamma)(e^-\gamma)$,	15.8%

Bhabha $e^+e^- \rightarrow e^+e^-(n\gamma)$ at CEPC

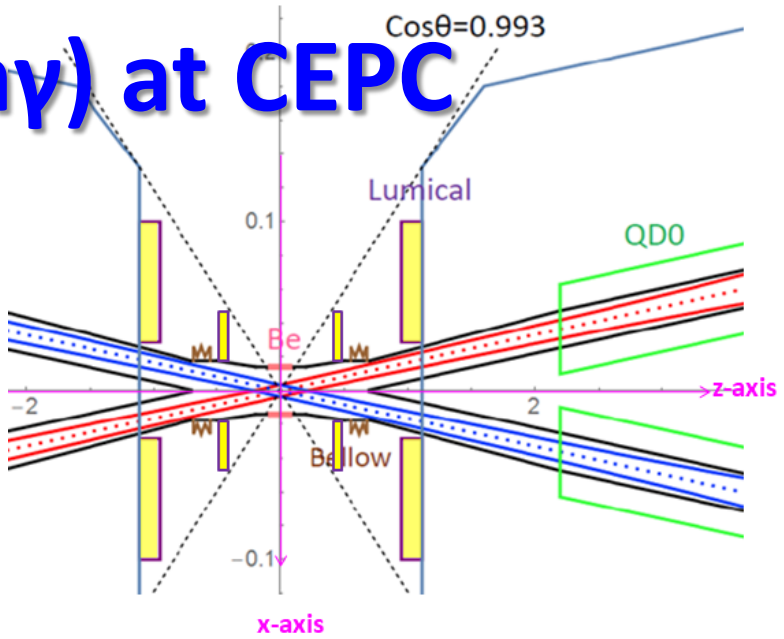
LEP Luminosity template

BHLUMI demo.f cuts

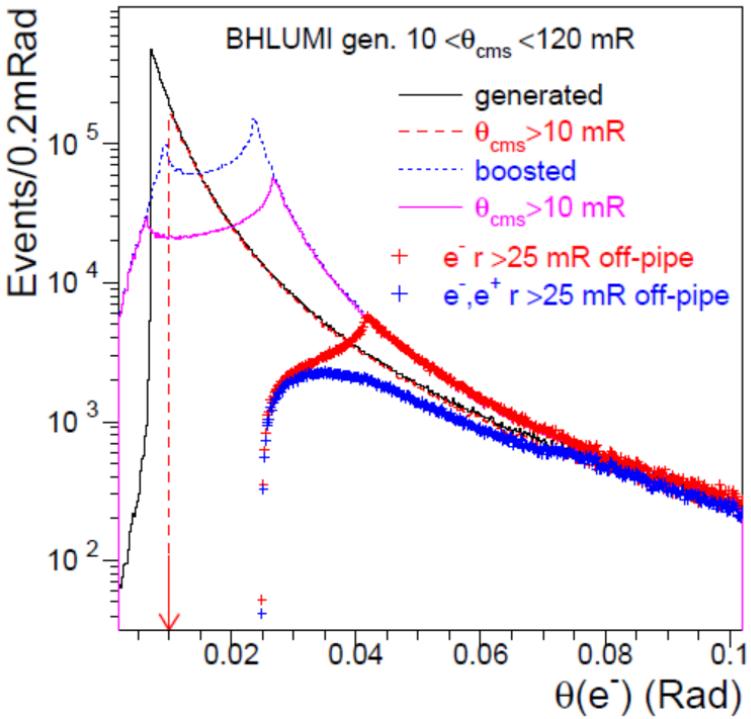
- **ACC 0** CMS 10 mRad < $\theta(e^{\pm})$ < 80 mRad
- **ACC 1** .and. s'(P2,Q2)/s(P1,Q1) >0.5

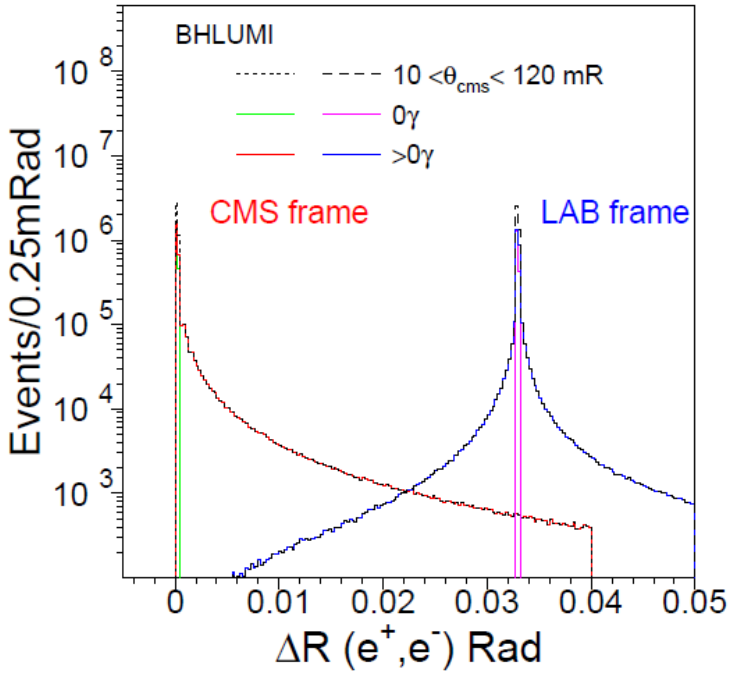
Beam crossing, 33 mRad

➔ Boost in x direct
e^+, e^- offset by 33 mRad



events with 0 photos
Show δ back-back distribution



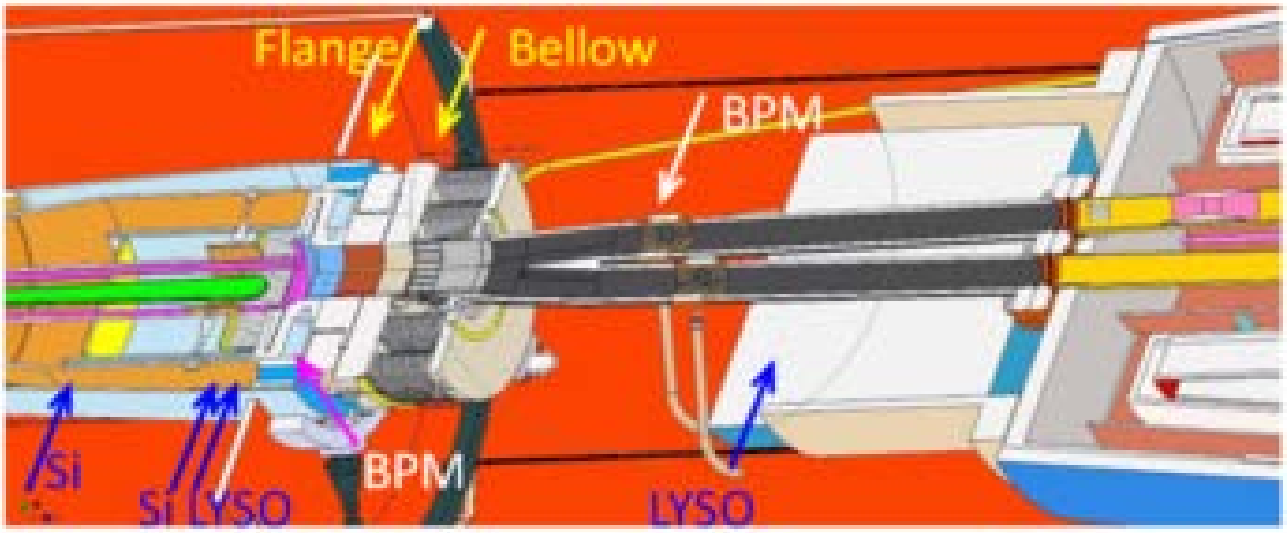


CEPC LumiCal design

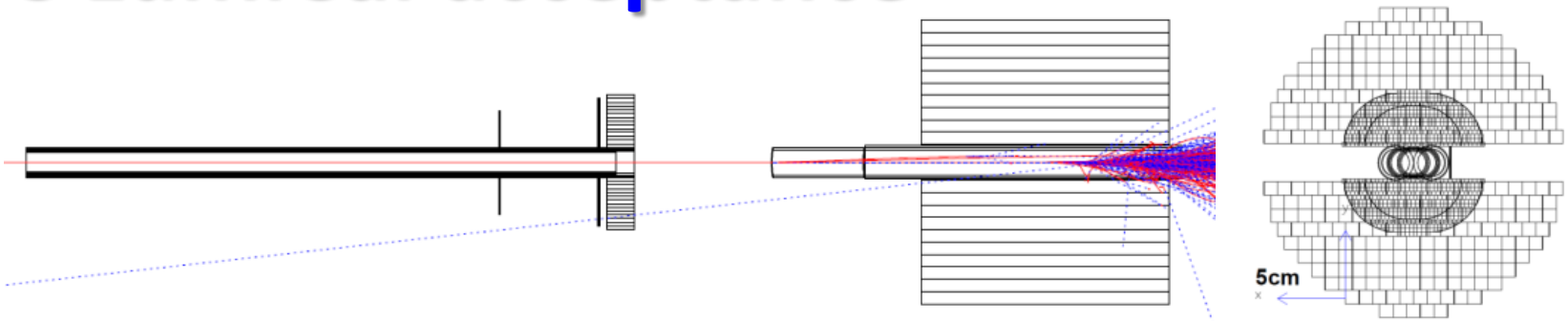
- ***L=2x10^{36}***/cm^{2}s^1 **@Z-pole,**
  - ***ø 20 mm*** racetrack, beam-crossing ***33 mRad***
  - IP bunch : ***σ_x σ_y σ_z = 6 µm, 35 nm, 9 mm***
  - Bunch crossing: ***23 ns***

- ***before Flange*** z = 560~700 mm
  - **Low-mass beampipe window**: **Be 1mm thick** traversing @22 mRad L= 45 mm, **= 0.13 X_0 (Be), 0.50 X_0 (Al)**
  - **Two Si-wafers** for $e^{\pm}$ impact θ
  - **2X_0 LYSO** = 23 mm

- ***behind Bellow*** z= 900~1100 mm
  - **Flange+Bellow : ~60 mm, 4.3 X_0**
  - **13X_0 LYSO 150 mm**



CEPC LumiCal acceptance



BHLUMI event distribution detecting back-to-back e^+, e^- pair

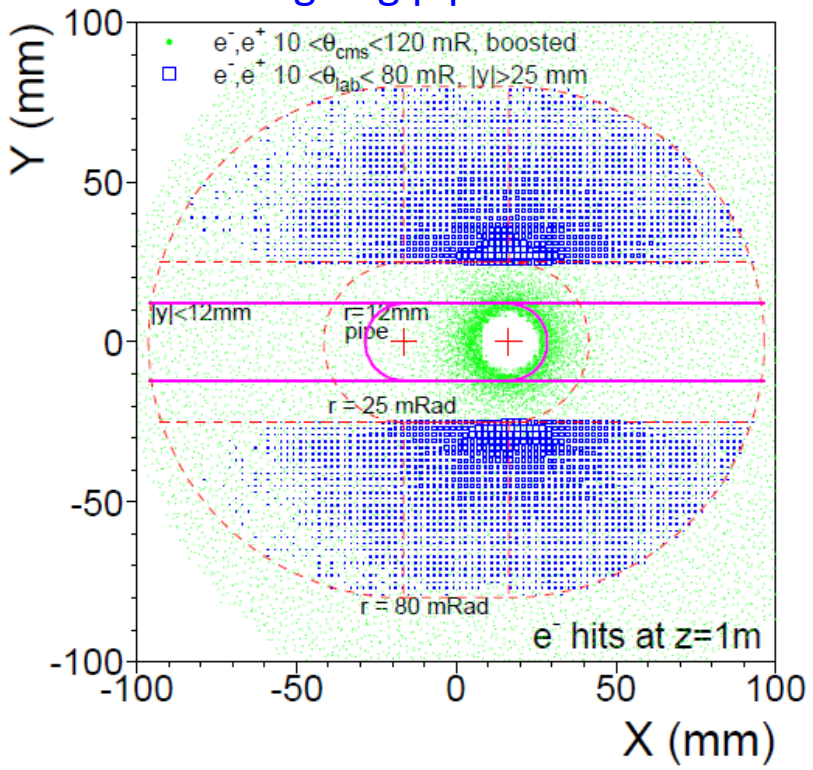
@|z|=1000mm

1) Θ > 25mRad outside pipe centers
2) |y|>25 mm
3) Events in shaded area counted for Xsec

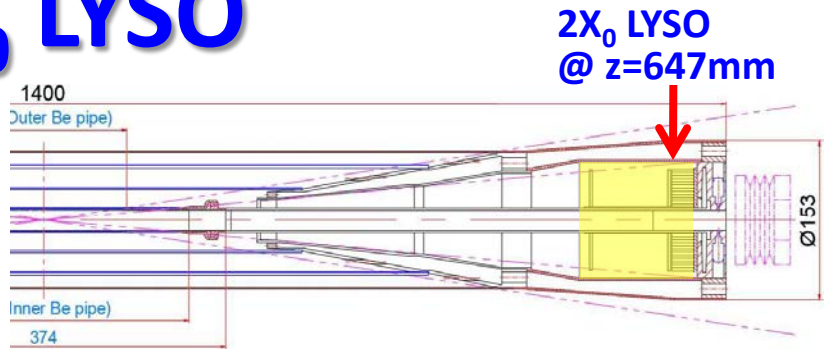
LumiCal acceptance at |z|=1000mm

e^+, e^- back-to-back detected	
θ>25 mRad	θ>25mR & |y|>25mm
85.4 nb	**78.0 nb**

e^+, e^- back-to-back symmetric to out-going pipe center



CEPC LumiCal Front 2X$_0$ LYSO

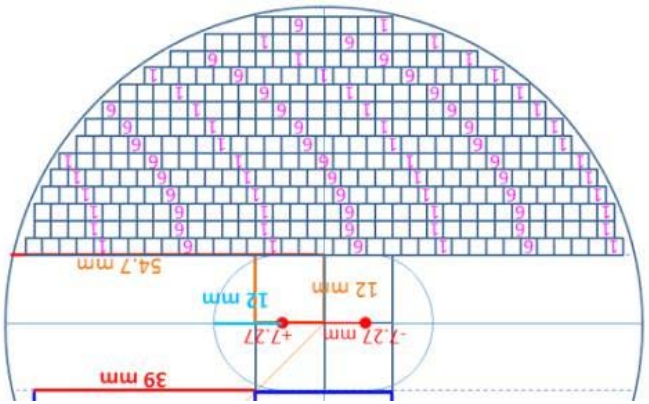


Bhabha hits on LYSO, |y|>12mm

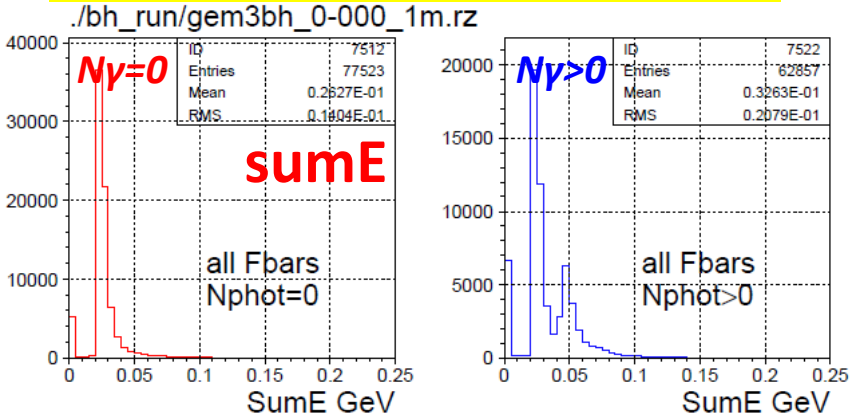
Incident particles are e$^{\pm}$,(γ)

- **GEANT sum dE/dx in each LYSO bars**
  3x3mm^2, 23 mm long, 2X$_0$
- **Deviation to e$^{\pm}$ truth** (impact hit >E$_b$/2)
  mostly < 0.2mm
- **Hit distributions in a Bar**
  distributed due to Bhabha θ, w./w.o. photon

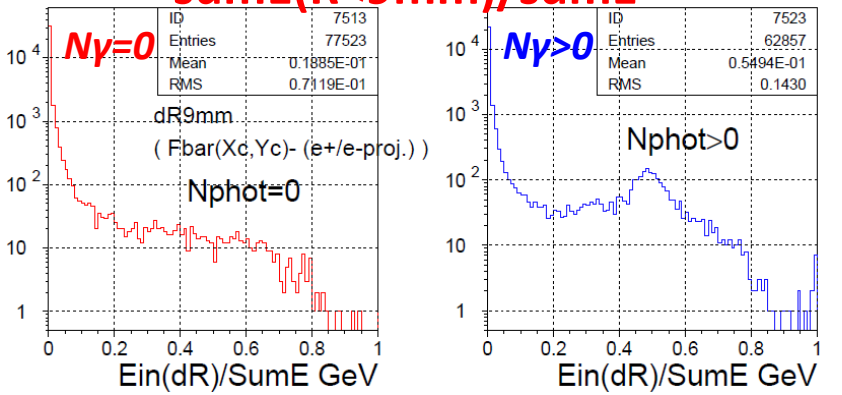
sum dE/dx all LYSO bars (a plane)
- **e$^{\pm}$ one track : sumE min. 20 MeV**
- **(e$^{\pm}$ + FSRγ) : two MIPs, sumE x2**



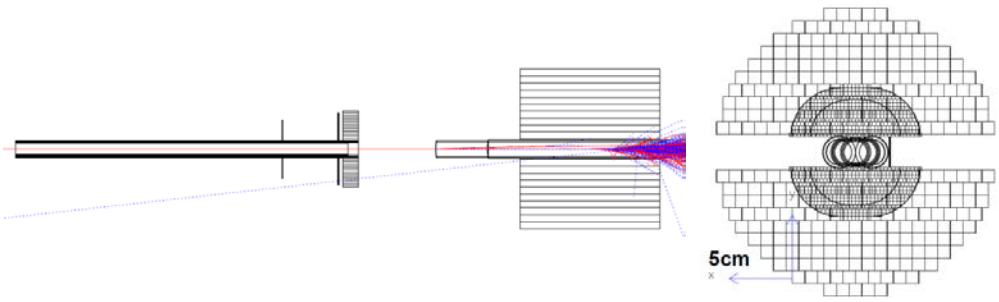
BHLUMI events, GEANT simulation

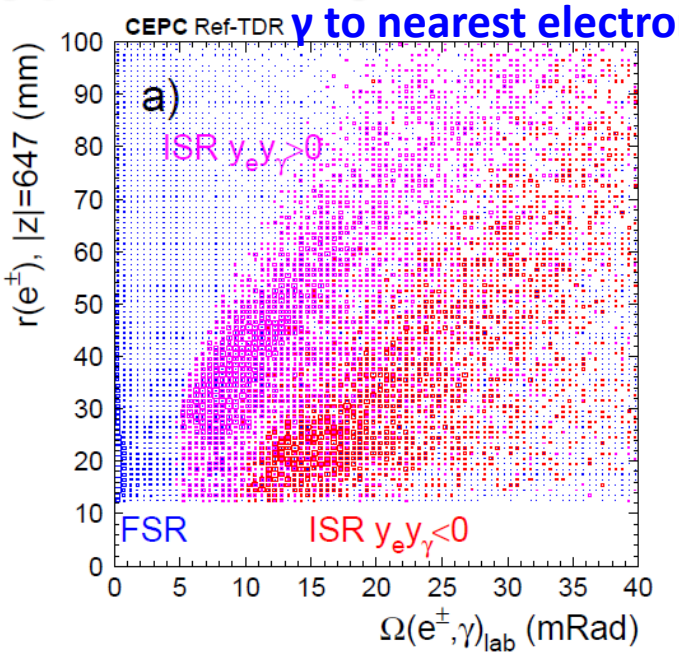


sumE(R<9mm)/sumE



Photons in $e^+e^- \rightarrow e^+e^-(n\gamma)$ @Z-pole





Bhabha events in LumiCal acceptance

e^+,e^-,γ : $|y|>12$ mm at LYSO front face $\pm z=647$mm

±z Hemispheres	BHLUMI generated	& P2,Q2 |y|>12mm
$e^\pm$	60.3 %	3.87 %
$e^\pm\gamma$	39.7 %*	3.16 %

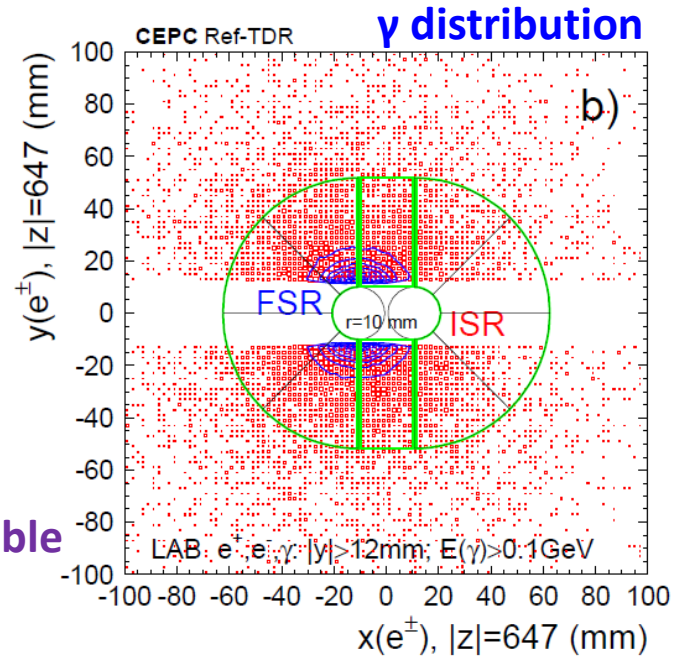
*ISR 20.3%, FSR 19.4%

Detectable Bhabha, e^+,e^-,γ : $|y|>12$ mm

±z Hemispheres	P2,Q2 |y|>12mm	& E(γ)>0.1GeV |y(γ)|>12mm
$e^\pm$	55.1 %	14.7 %
$e^\pm\gamma$	44.9 %	ISR 0.89 % FSR 13.8 % **FSR 2.96%***

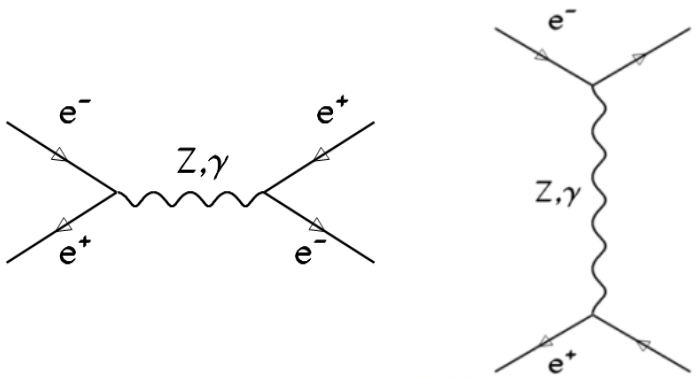
measurable

***FSR $\Omega(e^\pm,\gamma)$ > 5 mRad**



Luminosity precision @ flavor machine

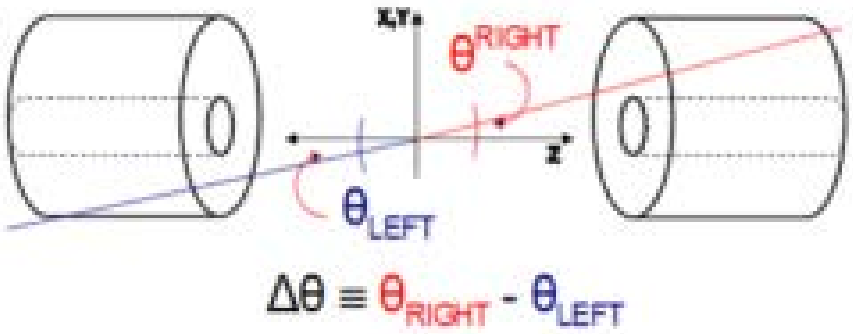
Bhabha $e^+e^- \to e^+e^-(n\gamma)$

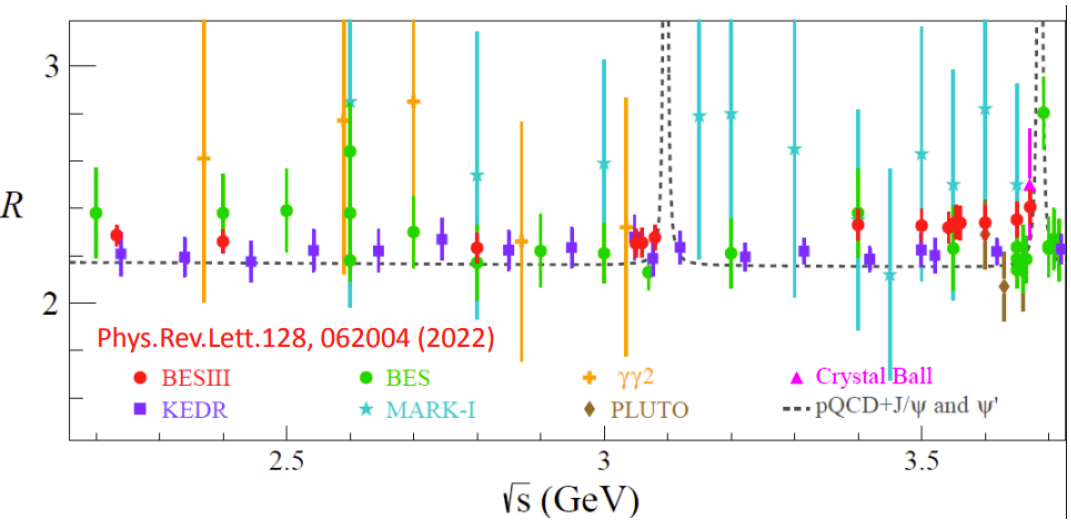


$R(s)$ ratio for SM predictions
$a_\mu = (g_\mu\text{-}2)/2$ and $\Delta\alpha_{\rm had}(M_Z)$

$$a_\mu = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} \mathrm{d}s\, K(s)\frac{R(s)}{s}$$

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{Re}\int_{m_\pi^2}^{\infty}\frac{R(s)\mathrm{d}s}{s(s-M_Z^2-i\epsilon)}$$





BESII in 2–5 GeV, precision 6%
BESIII 2022 3%

CM frame BHWIDE cross-sections

BHWIDE demo.f parameters

```
For the bosons we have (everything in GeV):
 mass of the   Z   =   91.1880    total width of the Z =  2.4972819
 mass of the   W   =   80.3352    <==> sin**2(theta-w) =  0.2238667
 mass of the Higgs =  125.2000

Some coupling strengths:
                  1/alfa =    137.036
the QED correction factor =      1.0017421
             alfa-strong =      0.119
the QCD correction factor =      1.0398948
```

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||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||
Xsection_tot =     34034450.347498 [pb]
error        =        90962.430061 [pb]
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**************************************************************
*     MC Event Generator for Wide-Angle Bhabha Scattering    *
*                    BHWIDE version 1.06                     *
**************************************************************
********************* December 2024 **************************
**************************************************************
*              Last modification: 04.12.2024                 *
**************************************************************
*                         AUTHORS:                           *
*          S. Jadach              (Deceased)                 *
*          W. Placzek             (wieslaw.placzek@uj.edu.pl) *
*          B.F.L. Ward            (bfl_ward@baylor.edu)      *
**************************************************************
*                         PAPERS:                            *
* [1] S. Jadach, W. Placzek, B.F.L. Ward,                    *
*     Phys. Lett. B390 (1997) 298; hep-ph/9608412.           *
**************************************************************
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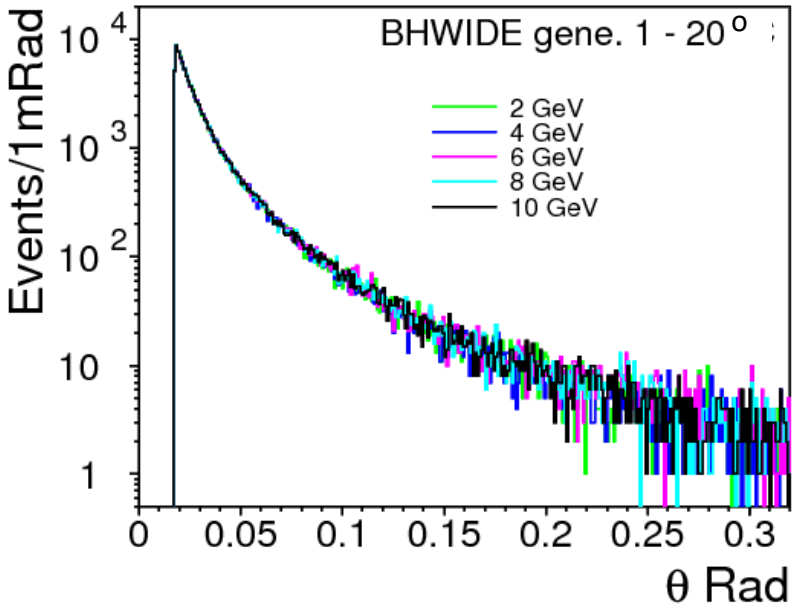
CM frame cross section σ; scattered e± > 0.1 GeV; back-to-back 0 - π					
√s GeV	2	4	6	8	10
1 – 20 deg	851000 nb	214200 nb	95300 nb	53700 nb	34400 nb
20 – 160 deg	1800 nb	455 nb	204 nb	115 nb	73.9 nb

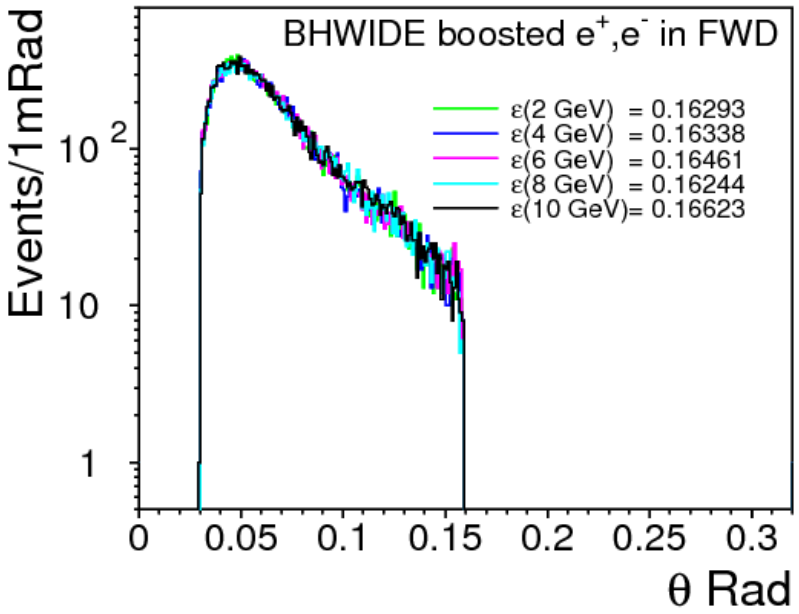
Bhabha √s dependency

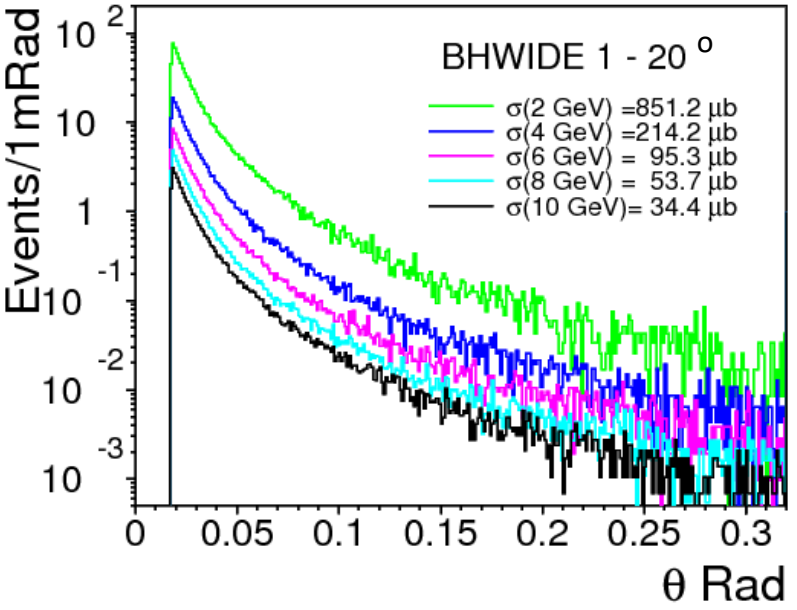
BHWIDE center-of-mass

√s = 2 - 10 GeV, θ range 1 – 20°

- **θ distribution: same, well overlap**
- **Cross section higher at low**
- **60 mRad beam-crossing boost**
- **Events both e^+ e^- in FWD** @ |z| = 500mm
  off beampipe Ø = 30 mm; r < 80mm
  acceptance: ε ~ 16.3 %



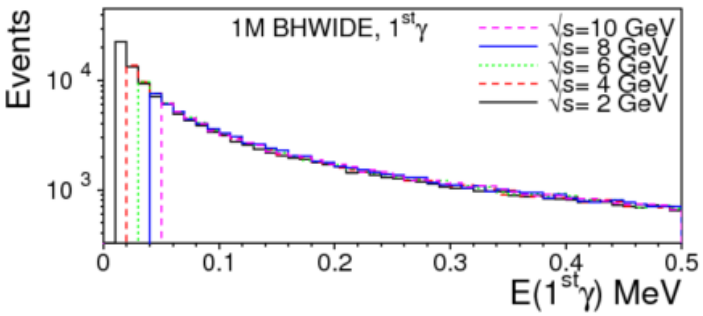


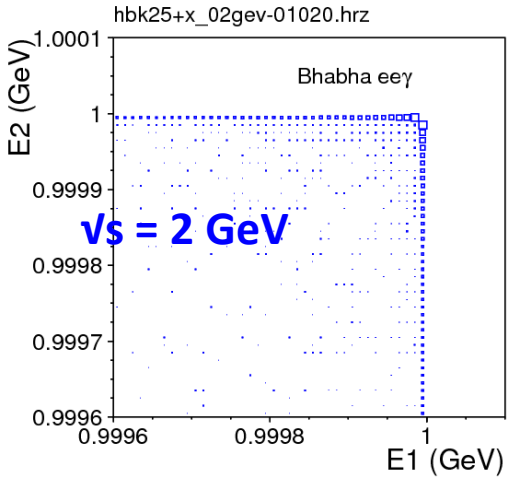


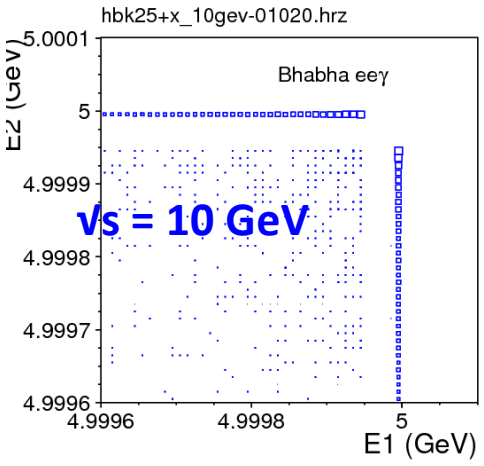
BHWIDE radiative Bhabha $e^+e^-(n\gamma)$

Compare E(γ) in 1M events, √s = 2 – 10 GeV

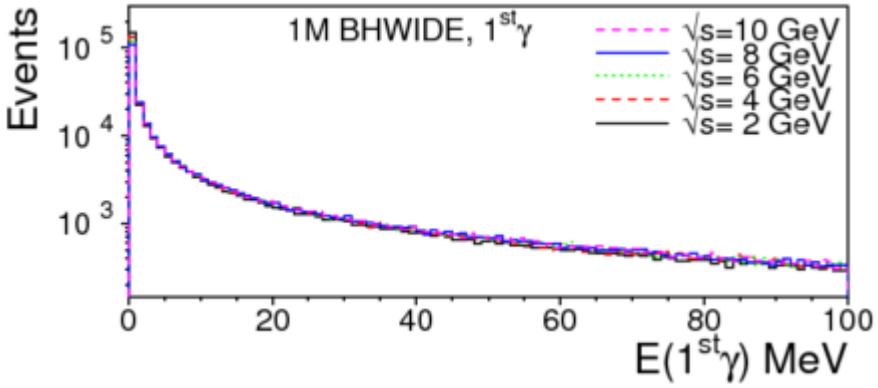
E (γ) cut = 1 x 10^{-5} x E_{beam}

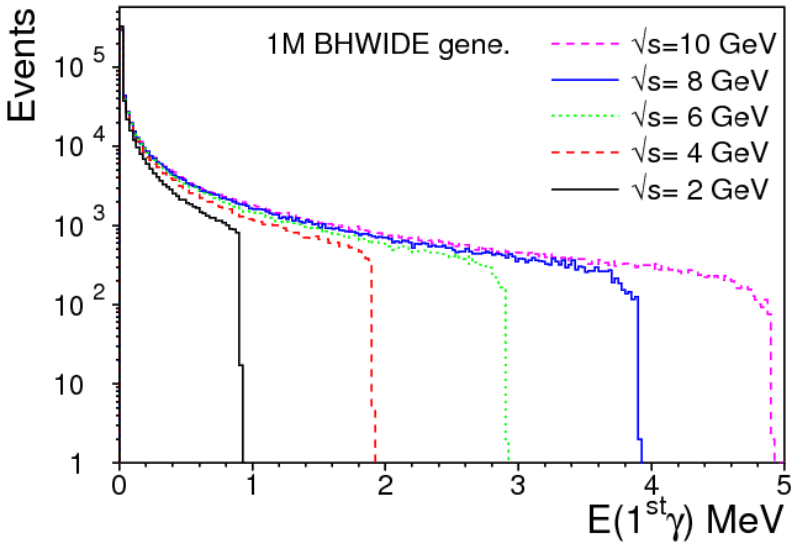






1st γ distribution, (1M @ 2 – 10 GeV) low E, indep. of √s, extend to E_{beam}



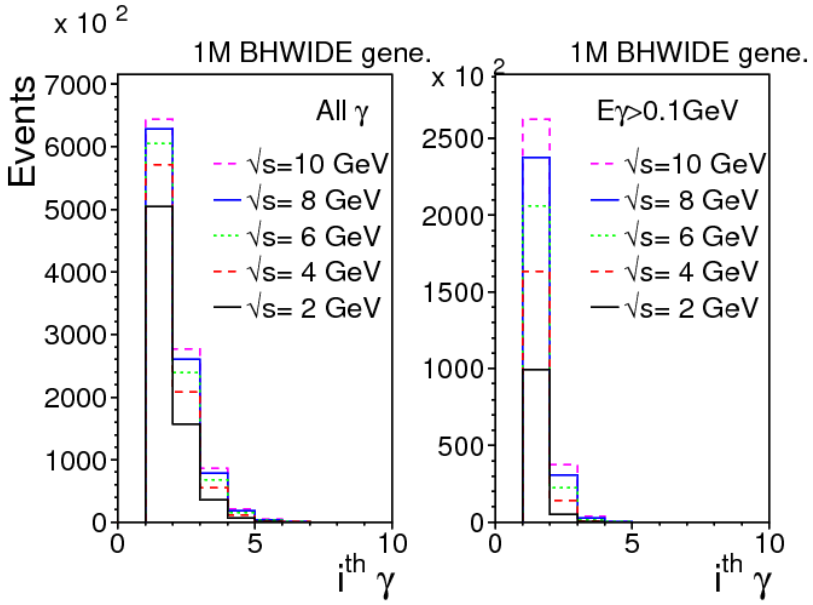


BHWIDE radiative Bhabha $e^+e^-(n\gamma)$

Yennie-Frautschi-Suura (YFS) exponentiation method

➔ **nγ in Poisson distribution**

➔ **nγ generated at vertex, no correlation with electrons**



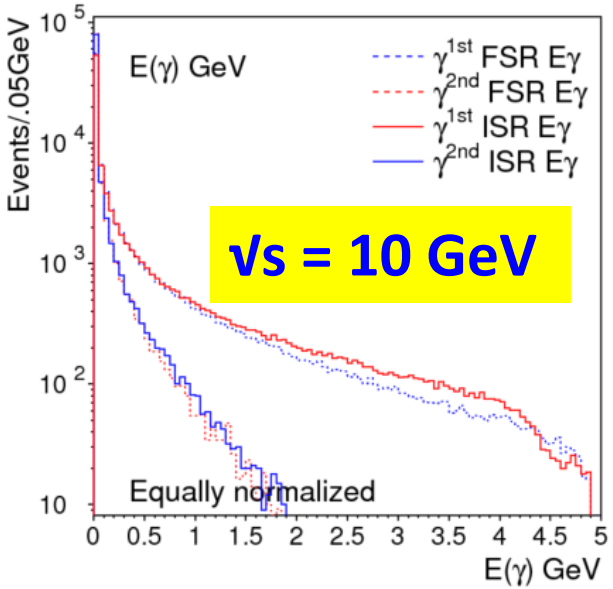
Photons

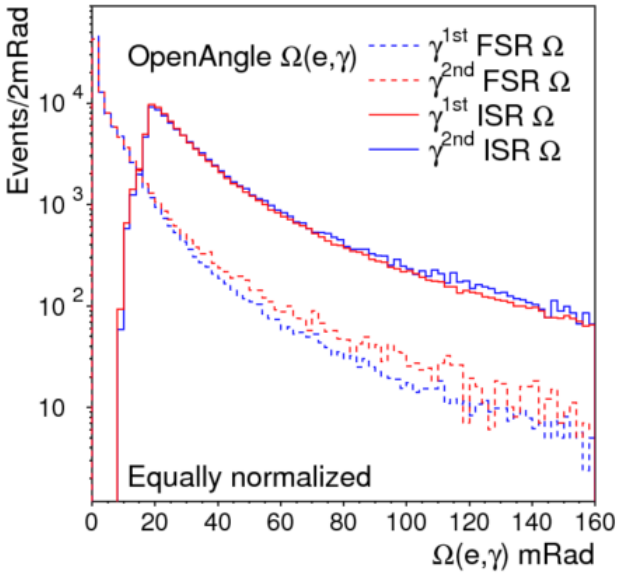
ordered by Eγ

ISR/FSR by

OpenAngle Ω(e,γ)

closer to inci/scat $e^{\pm}$



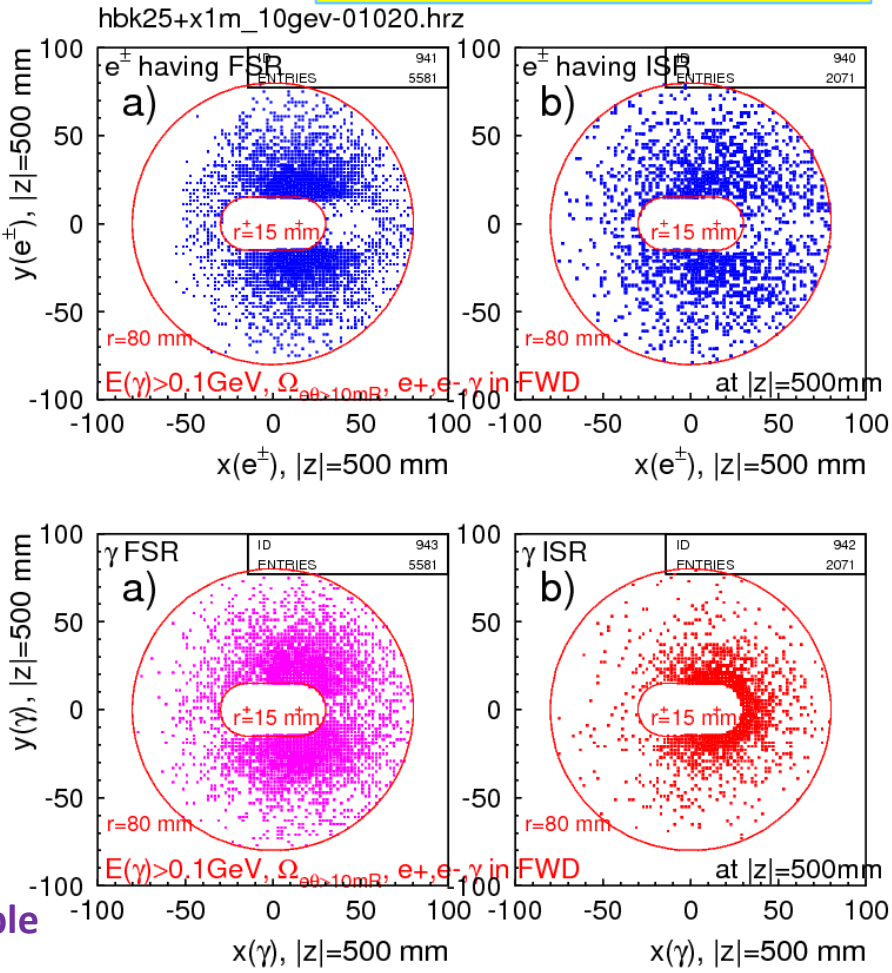


Rad-Bhabha distributions $e^+e^- \rightarrow e^+e^-(n\gamma)$

BHWIDE $\sqrt{s}$ = 10 GeV $e^+e^- \rightarrow e^+e^-(n\gamma)$

- **Detect rad-Bhabha, scattered e^+, e^-, γ^{1st}**
- **FWD acceptance:**
  off-pipe ϕ30mm, $x_c = \pm 15$mm
  external r<80mm
- **γ^{1st} selection:**
  $E\gamma > 0.1$ GeV,
  opening angle $\Omega(e^{\pm}, \gamma^{1st}) > 10$ mRad
- **event rate in Bhabha**
  to both e^+, e^- detected in FWD
  each z-side, **$e^{\pm}$ with γ^{1st} measured**

- $e^{\pm}$ with near γ^{1st} (FSR) near beam-pipe
- $e^{\pm}$ with far γ^{1st} (ISR) Loose scattered



BHWIDE scattered $e^{\pm} > 0.1$ GeV; $\Omega(e^+e^-)$: 0-π $\sigma(1\text{-}20^o)$= **34.4k nb**, e^+e^- in FWD rate= 16.5%		
Detected in each z-side both e^+e^- in FWD		each z-side, $e^{\pm}, \gamma^{1s}$ in FWD $E_\gamma > 0.1$GeV $\Omega(e^{\pm}, \gamma^{1st}) > 10$mR
$e^{\pm}, 0\gamma$	56.1 %	
$e^{\pm}, n\gamma$	43.9 %	**ISR 0.63 %** **FSR 1.7 %**

measurable

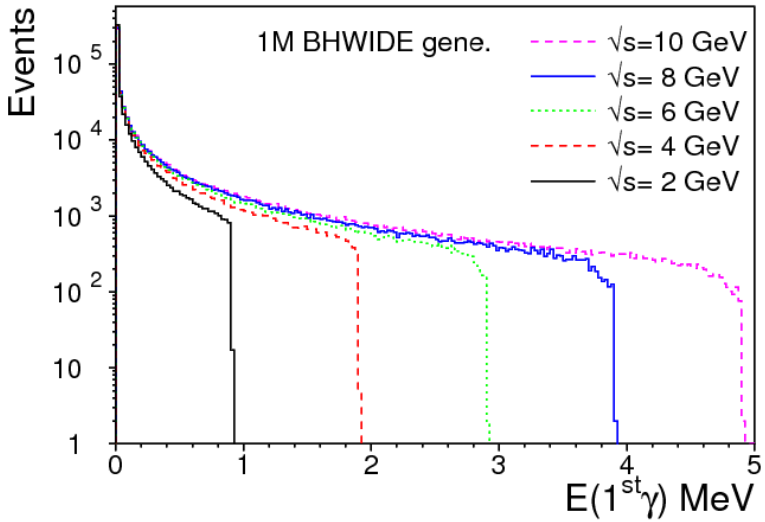
Perspective for measuring Bhabha @ STCF

BHWIDE $\sqrt{s}$ **= 2 – 10 GeV**

scattered $e^{\pm}$ > 0.1 GeV; $\Omega(e^+e^-)$: 0-π

CMS generted for σ(1-20°)

Acceptance e^+e^- in FWD **= 16.5%**



Both e^+e^- fall in FWD

event ratio with

- **0/nγ** generated
- **γ^{1st}** Eγ>0.1 GeV, $\Omega(e^{\pm},\gamma^{1st})$>10 mR

Detected in each z-side	√s =	10 GeV	8 GeV	6 GeV	4 GeV	2 GeV
both e^+e^- in FWD	$e^{\pm}$,0γ	56.1 %	57.1 %	59.1 %	61.6 %	66.0 %
	$e^{\pm}$,nγ	43.9 %	42.9 %	40.9 %	38.4 %	34.0 %
$e^{\pm},\gamma^{1s}$ in FWD E_γ> 0.1GeV, $\Omega(e^{\pm},\gamma^{1st})$>10mR	$e^{\pm},\gamma^{1s}$	ISR 0.63 % FSR 1.69 %	ISR 0.64 % FSR 1.62 %	ISR 0.58 % FSR 1.47 %	ISR 0.50 % FSR 1.26 %	ISR 0.40 % FSR 0.94 %

Discussion

- **Measuring Radiative Bhabha**,
  evaluated with BHLUMI for CEPC @ √s 92 GeV
  evaluated with BHWIDE for flavor @ √s ~ 5 GeV
- Forward **LumiCal** can measure **Bhabha with FSR**
  test **QED on NLO** final state to **~ 0.1 %** aims for **0.01 %**
- **e/γ** discrimination realized by **Calo Crystal** technology
  LYSO crystal strip 2x2 mm^2 cells with SiPM