Effective field theory for type II seesaw model —symmetric phase v.s. broken phase—

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arXiv:2504.02580 [hep-ph]

Collaborated with

Yi Liao (SCNU)
Xiao-Dong Ma (SCNU)

SMEFT v.s. HEFT

Standard Model Effective Field Theory (SMEFT)

$$\mathscr{L}_{\rm SMEFT} = (D_\mu H)^\dagger D^\mu H + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$

Symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$

Field contents SM fields *before* the EW symmetry breaking: $H,\ W_\mu^a,\ B_\mu,\ \cdots$

Power counting $1/\Lambda$ expansion

Higgs Effective Field Theory (HEFT)

$U = \exp\left(\frac{i}{v}\varphi^a\tau^a\right)$

φ^a : would-be NG boson

$$\mathscr{L}_{\rm HEFT} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{v^2}{4}{\rm Tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right) + \cdots$$

Symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ * $SU(2)_L \times U(1)_Y$ is non-linearly realized

Field contents SM fields *after* the EW symmetry breaking: $h,\ W_\mu^\pm,\ Z_\mu,\ A_\mu, \cdots$

Power counting Loop expansion

What is each of SMEFT and HEFT particulary good at?

SMEFT v.s. HEFT

SM + real singlet scalar

$$\mathscr{L} = (D_\mu H)^\dagger D^\mu H + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S)$$

$S \in (\mathbf{1},\mathbf{1},0)$, real

Buchalla et al. [1608.03564](https://arxiv.org/abs/1608.03564)

$$V(H,S) = -\frac{\mu_1^2}{2}H^\dagger H - \frac{\mu_2^2}{4}S^2 + \frac{\lambda_1}{4}(H^\dagger H)^2 + \frac{\lambda_2}{16}S^4 + \frac{\lambda_3}{4}H^\dagger H S^2$$

SMEFT

E

$M_{\tilde{s}}$

$S = v_s + \tilde{s}$

Integrate out $\tilde{s}$

$$\mathscr{L}_{\rm NLO} = -\frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}(H^\dagger H)\Box(H^\dagger H)$$

$$H = \left(0, \frac{v+h}{\sqrt{2}}\right)^T$$

$$(D_\mu H)^\dagger D^\mu H - \frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}(H^\dagger H)\Box(H^\dagger H)$$

$$\mathscr{L}_{\rm SMEFT} = (D_\mu H)^\dagger D^\mu H + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$

HEFT

$M_{\tilde{s}}$

$S = v_s + \tilde{s}$

$$H = \left(0, \frac{v+h}{\sqrt{2}}\right)^T$$

$$\begin{pmatrix} h' \\ \tilde{s}' \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}\begin{pmatrix} h \\ \tilde{s} \end{pmatrix}$$

Integrate out $\tilde{s}'$

$$\frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(2c_\chi\frac{h}{v} + \left(c_\chi^4 - s_\chi^3 c_\chi \frac{v}{v_s}\right)\left(\frac{h}{v}\right)^2 + \mathscr{O}(h^3)\right)$$

$$\mathscr{L}_{\rm HEFT} = \frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + 2\kappa_W\frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right) + \cdots$$

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$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$(D_\mu H)^\dagger D^\mu H - \frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}(H^\dagger H)\Box(H^\dagger H)$$

$$\supset m_W^2 W_\mu^- W^{+\mu}\left((2-\chi^2)\frac{h}{v} + (1-2\chi^2)\left(\frac{h}{v}\right)^2 + \mathscr{O}(h^3)\right)$$

$$\chi^2 \doteq \frac{v^2}{2}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}$$

HEFT

$M_{\tilde{s}}$

$S = v_s + \tilde{s}$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$\begin{pmatrix} h' \\ \tilde{s}' \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}\begin{pmatrix} h \\ \tilde{s} \end{pmatrix}$$

Integrate out $\tilde{s}'$

$m_W^2 W_\mu^- W^{+\mu}$

$$\frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(2c_\chi\frac{h}{v} + \left(c_\chi^4 - s_\chi^3 c_\chi \frac{v}{v_s}\right)\left(\frac{h}{v}\right)^2 + \mathscr{O}(h^3)\right)$$

$$c_\chi \simeq 1 - \frac{\chi^2}{2}, \quad s_\chi \simeq 0$$

SMEFT v.s. HEFT

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E

SMEFT

$M_{\tilde{s}}$

$S = v_s + \tilde{s}$

SMEFT

Applicable only in

$v_s \gg v_{\rm EW}$

Integrate out $\tilde{s}$

$$\mathscr{L}_{\rm NLO} = -\frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}(H^\dagger H)\Box(H^\dagger H)$$

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HEFT

$M_{\tilde{s}}$

HEFT

Applicable even in

$v_s \gtrsim v_{\rm EW}$

$S = v_s + \tilde{s}$

$$H = \left(0, \frac{v+h}{\sqrt{2}}\right)^T$$

$$\begin{pmatrix} h' \\ \tilde{s}' \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}\begin{pmatrix} h \\ \tilde{s} \end{pmatrix}$$

Integrate out $\tilde{s}'$

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$$c_\chi \simeq 1 - \frac{\chi^2}{2}, \quad s_\chi \simeq 0$$

SMEFT v.s. HEFT

	Good	Bad
SMEFT	Easy to handle	Applicable only in $\Lambda_{\text{EFT}} \gg v_{\text{EW}}$
HEFT	Applicable even in $\Lambda_{\text{EFT}} \gtrsim v_{\text{EW}}$	Complex to handle $\text{Tr}[(D_\mu U)^\dagger D^\mu U]$ $U = \exp\left(\frac{i}{v}\varphi^a \tau^a\right)$

We suggest altanative EFT overcoming the shortcoming of both EFTs

"Broken phase EFT (bEFT)" Phys. Rev. D 112 (2025) no.3, 035005, Y. Liao, X. D. Ma, and **YU**

i) Integrate out heavy fields in the broken phase, $\tilde{s}'$, just like HEFT

$$\tilde{s}' = h \sin\chi + \tilde{s}\cos\chi \qquad S = v_s + \tilde{s} \qquad H = \left(0, \frac{v+h}{\sqrt{2}}\right)^T$$

ii) Take unitary gauge: $U = 1$

iii) In E.o.M., only expand $\tilde{s}'$ w.r.t. $1/M^2$, not c_i

E.o.M. $\left(-\partial^2 - M^2 + c_1\right)\tilde{s}' + c_0 + c_2\tilde{s}'^2 + c_3\tilde{s}'^3 = 0$

$$c_i = c_i(M^2, \tilde{h}', \text{Tr}[D_\mu U^\dagger D^\mu U])$$

$$\left(\begin{array}{l} c_i = \cancel{c_{i0} + c_{i2}}M^2 \\ \tilde{s}' = \tilde{s}'_0 + \dfrac{1}{M^2}\tilde{s}'_2 + \dfrac{1}{M^4}\tilde{s}'_4 + \cdots \end{array}\right.$$

SMEFT v.s. HEFT

- Specific models

	SMEFT	HEFT
SM + singlet scalar	G. Buchalla, et. al., 1608.03564	
SM + triplet scalar	T. Cohen, et. al., 2008.08597	H. Song, et. al., 2412.00355
type-II seesaw model	X. Li, et. al., 2201.05082	
Two Higgs doublet model	S. Dawson, et. al., 2305.07689 Ian Banta, et. al., 2304.09884	G. Buchalla, et. al., 2312.13885

- General discussion

Models that cannot be matched to SMEFT but only to HEFT

R. Alonso et. al., 1605.03602 — Target space of the scalar kinetic term

A. Falkowski et. al., 1902.05936 — Singularity of scalar potential

T. Cohen, et. al., 2008.08597 — Both scalar kinetic term and potential

- Bottom-up approach

R. Gómez-Ambrosio et. al., 2204.01763

R. Gómez-Ambrosio et. al., 2207.09848

A. Salas-Bernardez, et. al., 2211.09605

R. L. Delgado, et. al., 2311.04280

S. Mahmud, et. al., 2406.03522

Why type-II seesaw model?

New scalar’s VEV must be much smaller than the EW scale

$$\rho = \frac{v_H^2 + 2v_\Delta^2}{v_H^2 + 4v_\Delta^2} = 1.00038 \pm 0.00020 \quad \Rightarrow \quad v_\Delta \lesssim 10^{-2}\, v_{\rm EW} \qquad v_{\rm EW} = 246\,{\rm GeV}$$

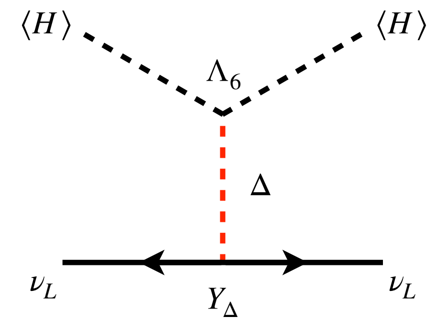
Matching type-II seesaw onto bEFT & SMEFT

Type-II seesaw model

- In order to realize tiny neutrino mass, triplet scalar $\Delta \in (\mathbf{3}, 1)$ is introduced.

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

- Neutrino mass is generated by the $U(1)_L$ violating interactions



$$\mathscr{L}_{\not{L}} = \Lambda_6 H^T \Delta^\dagger H - Y_\Delta^{ij} \bar{L}_i^C \Delta L_j + \mathrm{H.c.}$$

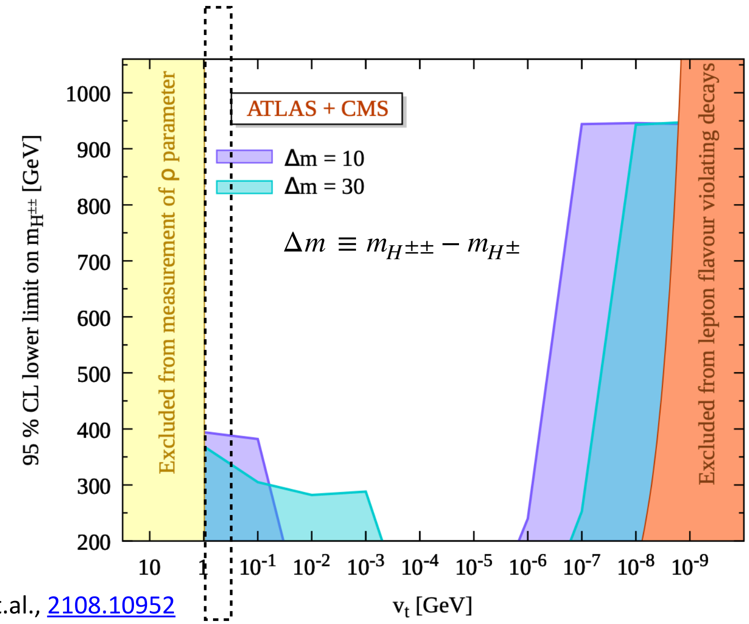
- Triplet VEV induces large deviations in ρ parameter from unity

$$\rho = \frac{v_H^2 + 2v_\Delta^2}{v_H^2 + 4v_\Delta^2} = 1.00038 \pm 0.00020$$

$$\Rightarrow \quad v_\Delta \lesssim 10^{-2}\, v_{\rm EW} \qquad v_{\rm EW} = 246\,\mathrm{GeV}$$

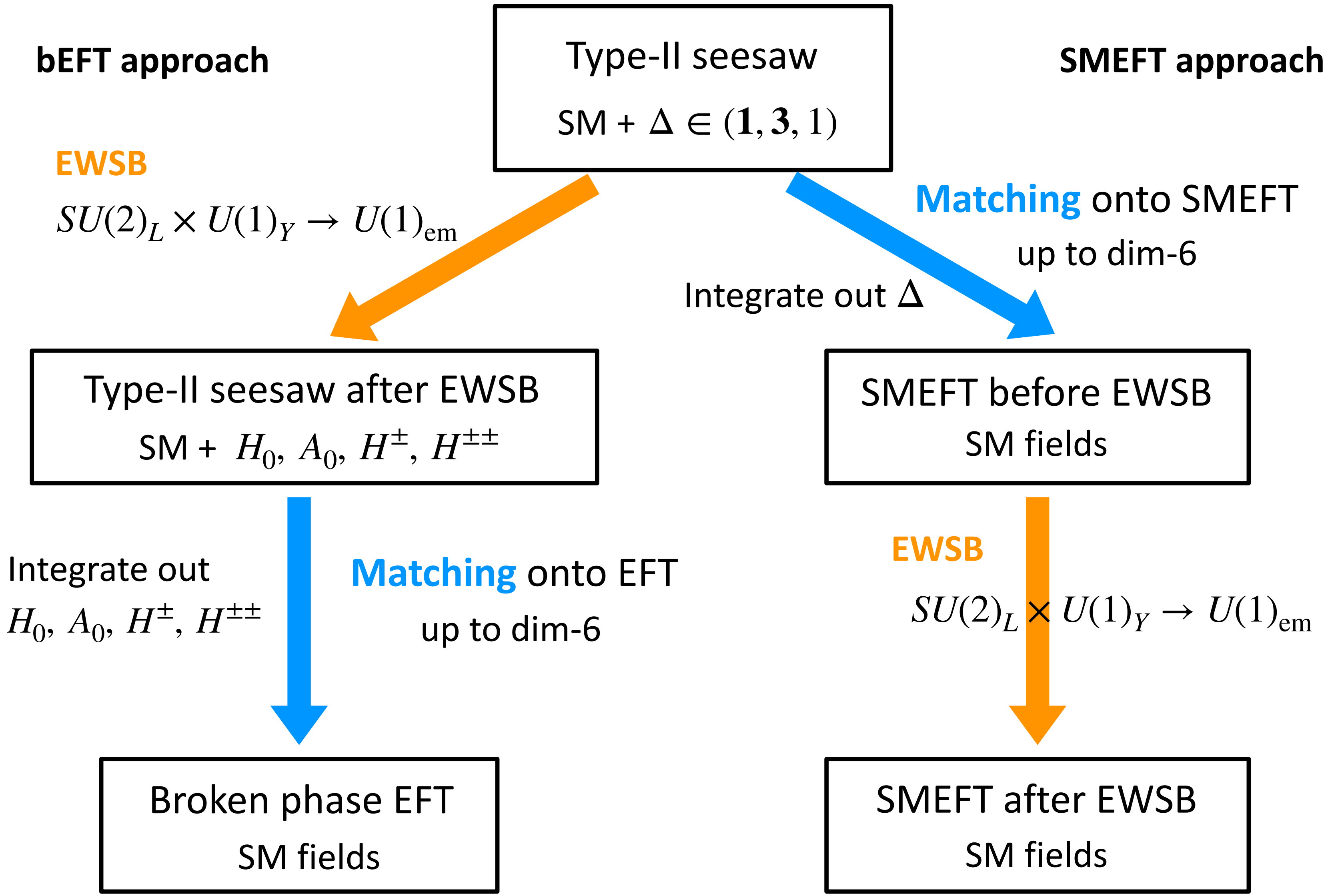
We perform double expansion

w.r.t. $\epsilon \equiv v_\Delta / v_{\rm EW}$ and $1/M_\Delta$



S. Ashanujjaman, et.al., 2108.10952

Strategy



bEFT approach
Type-II seesaw
SM + $\Delta \in (\mathbf{1}, \mathbf{3}, 1)$
SMEFT approach
EWSB
$SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{em}}$
Matching onto SMEFT
up to dim-6
Integrate out Δ
Type-II seesaw after EWSB
SM + H_0, A_0, $H^{\pm}$, $H^{\pm\pm}$
SMEFT before EWSB
SM fields
Integrate out
H_0, A_0, $H^{\pm}$, $H^{\pm\pm}$
Matching onto EFT
up to dim-6
EWSB
$SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{em}}$
Broken phase EFT
SM fields
SMEFT after EWSB
SM fields

Matching onto bEFT

Difficulty

In the broken phase, there are so many fields to be integrated out: $\{H_0,\ A_0,\ H^\pm,\ H^{\pm\pm}\}$

Solution

There is "Z_2 symmetry" in the $\mathscr{L}_{\mathrm{UV}}$

$U(1)_L$ violating terms

$$V(H,\Delta) = V_{\mathrm{SM}}(H) + \lambda HH\Delta\Delta + (\Delta\Delta)^2 + \Lambda_6 H\Delta H + Y_\Delta L\Delta L$$

$V(H,\Delta)$ is invariant under

$$\Lambda_6 \to -\Lambda_6 \qquad Y_\Delta \to -Y_\Delta \qquad \Delta \to -\Delta$$

After EWSB, this "Z_2 symmetry" still holds

$$\epsilon \to -\epsilon \qquad Y_\Delta \to -Y_\Delta \qquad \Phi_{\mathrm{heavy}} \to -\Phi_{\mathrm{heavy}}$$

$\Phi_{\mathrm{heavy}} \in \{H_0, A_0, H^\pm, H^{\pm\pm}\}$

$$\epsilon := \frac{v_\Delta}{\sqrt{v_H^2 + 2v_\Delta^2}}$$

This suggests the solution of E.o.M. for Φ_{heavy} must take the form of

$$\Phi_{\mathrm{heavy}}[\phi_{\mathrm{SM}}] \ \propto\ \epsilon^m \left(Y_\Delta \bar{L}L\right)^n \qquad m+n = \mathrm{odd}$$

We keep only ϵ^2 order

$$H_0^4 \ \propto\ \epsilon^2 \left(\frac{Y_\Delta}{M_\Delta^2}\right)^2,\ \epsilon \left(\frac{Y_\Delta}{M_\Delta^2}\right)^3,\ \left(\frac{Y_\Delta}{M_\Delta^2}\right)^4$$ can be neglected

Matching onto SMEFT

This is already done by IHEP group

X. Li, D. Zhang, S. Zhou, JHEP04(2022)038, 2201.05082

Symbol	Operator	dimensionless WC
	dim 5	
$O_{pr}^{(5)}$	$\bar{L}^p \tilde{H} \tilde{H}^T L^{rC}$	$-\frac{\Lambda_6}{2M_\Delta} Y_\Delta^{*pr}$
	dim 6	
O_H	$(H^\dagger H)^3$	$\frac{1}{2}(8\lambda - \lambda_4 + \lambda_5)\frac{\Lambda_6^2}{M_\Delta^2} - \frac{\Lambda_6^4}{M_\Delta^4}$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\frac{\Lambda_6^2}{2M_\Delta^2}$
O_{HD}	$(H^\dagger D_\mu H)^\dagger (H^\dagger D^\mu H)$	$\frac{\Lambda_6^2}{M_\Delta^2}$
O_{eH}^{pr}	$(H^\dagger H)(\bar{L}^p H e_R^r)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y_l^{pr}$
O_{uH}^{pr}	$(H^\dagger H)(\bar{Q}^p \tilde{H} u_R^r)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y_{\rm u}^{pr}$
O_{dH}^{pr}	$(H^\dagger H)(\bar{Q}^p H d_R^r)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y_{\rm d}^{pr}$
$O_{\ell\ell}^{prst}$	$(\bar{L}^p \gamma_\mu L^r)(\bar{L}^s \gamma^\mu L^t)$	$\frac{1}{4} Y_\Delta^{*ps} Y_\Delta^{rt}$

We rewrite the SMEFT in the broken phase.

VEV shift:

$$\mathscr{L}_{\rm dim6}^{\rm smeft} \supset m_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 - C_H (H^\dagger H)^3$$

new VEV: $\langle H^\dagger H \rangle = \frac{v^2}{2}\left(1 + \frac{3C_H}{2\lambda}\right) \equiv \frac{v_{\rm ew}^2}{2}$

Normalization:

$$\mathscr{L}_{\rm dim6}^{\rm smeft} \supset (D_\mu H)^\dagger D^\mu H + \frac{C_{H\Box}}{M_\Delta^2}(H^\dagger H)\Box(H^\dagger H) + \frac{C_{HD}}{M_\Delta^2}(H^\dagger D_\mu H)^\dagger (H^\dagger D^\mu H)$$

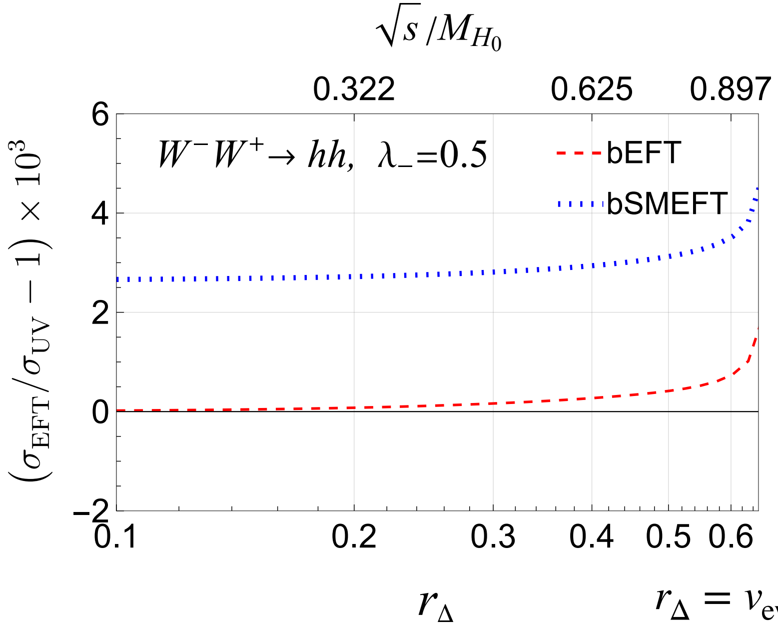
$$\xrightarrow[H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}]{} \frac{1}{2}\left[1 - 2c_{H,\rm kin}\left(1 + \frac{h}{\tilde{v}_T}\right)^2\right]\partial_\mu h \partial^\mu h$$

To make h canonically normalized,

$$\tilde{h} = \frac{1}{\sqrt{1 - 2c_{H,\rm kin}}} h$$

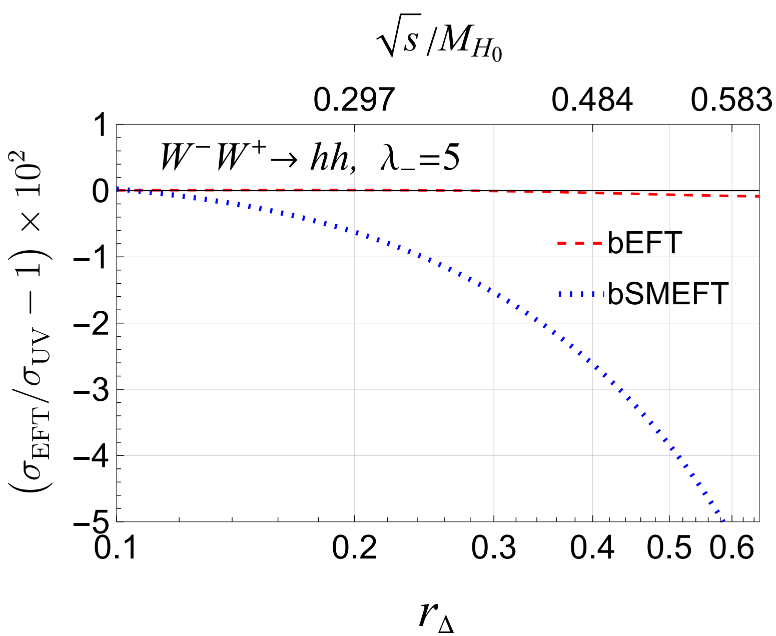
Numerical evaluations

$W^-W^+ \to hh$



$r_\Delta = v_{\rm ew}/M_\Delta$

H_0 : 640 GeV
A_0 : 640 GeV
$H^\pm$: 663 GeV
$H^{\pm\pm}$: 685 GeV



H_0 : 826 GeV
A_0 : 826 GeV
$H^\pm$: 844 GeV
$H^{\pm\pm}$: 862 GeV

$$
\begin{aligned}
&V(H,\Delta)\\
&= -m_H^2(H^\dagger H) + \frac{\lambda}{2}(H^\dagger H)^2\\
&\quad + M_\Delta^2 \mathrm{Tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2\\
&\quad + \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 - \mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right)\\
&\quad + \lambda_4(H^\dagger H)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5 H^\dagger[\Delta^\dagger,\Delta]H\\
&\quad + \left(\frac{\Lambda_6}{\sqrt{2}} H^T i\sigma_2 \Delta^\dagger H + \mathrm{h.c.}\right)
\end{aligned}
$$

$\sqrt{s} = 400\ \mathrm{GeV}$,

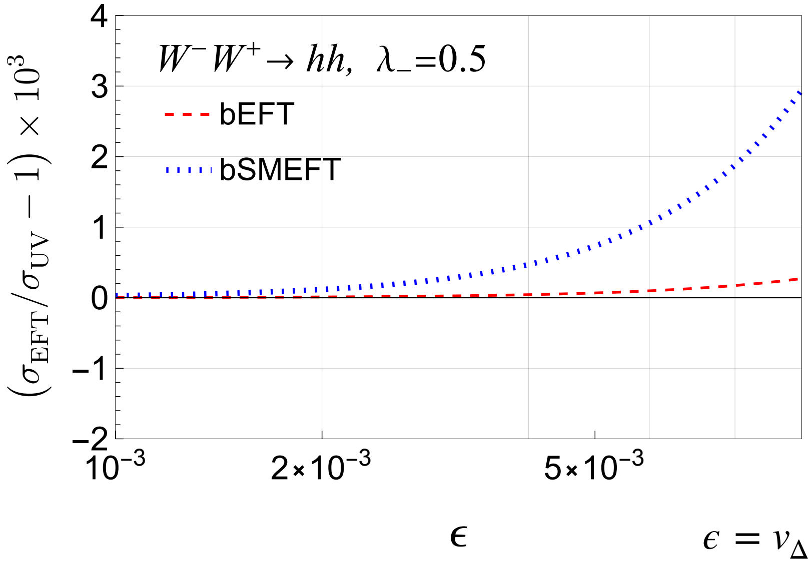
$\epsilon = v_\Delta / v_{\rm ew} = 10^{-2}$,

$\lambda_- := (\lambda_4 - \lambda_5)/2$

$\lambda_1 = \lambda_2 = \lambda_5 = 1$

bEFT reproduces UV theory more accurately than SMEFT when $v_{\rm ew}/M_\Delta$ is large.

$W^-W^+ \to hh$



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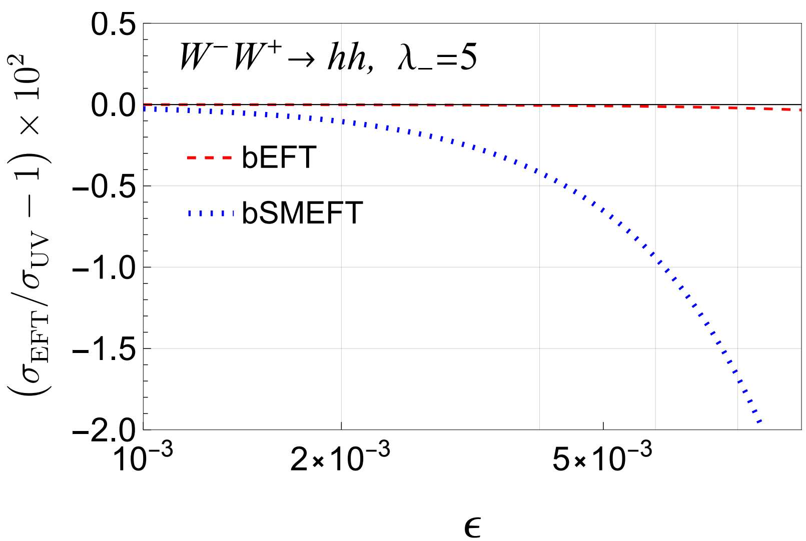

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$$\sqrt{s} = 400\,\mathrm{GeV},$$

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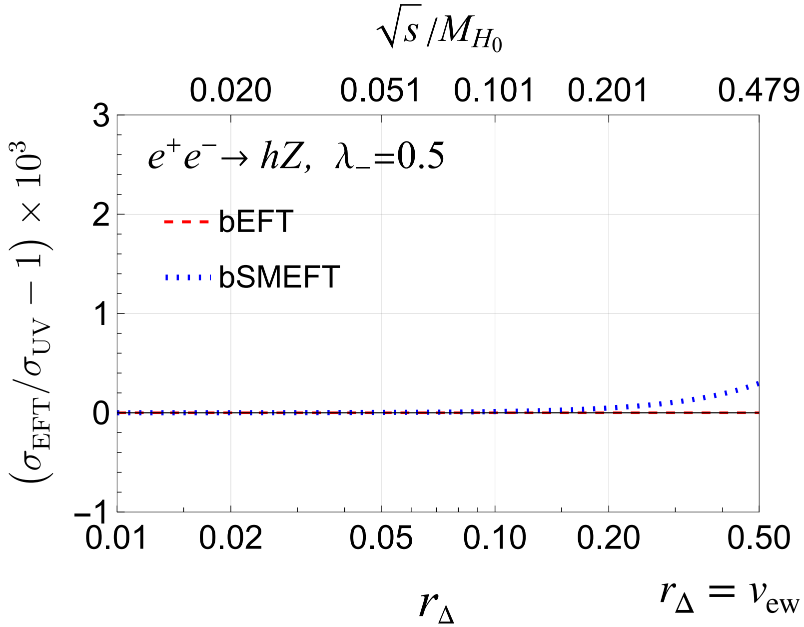
$e^+e^- \to Zh$



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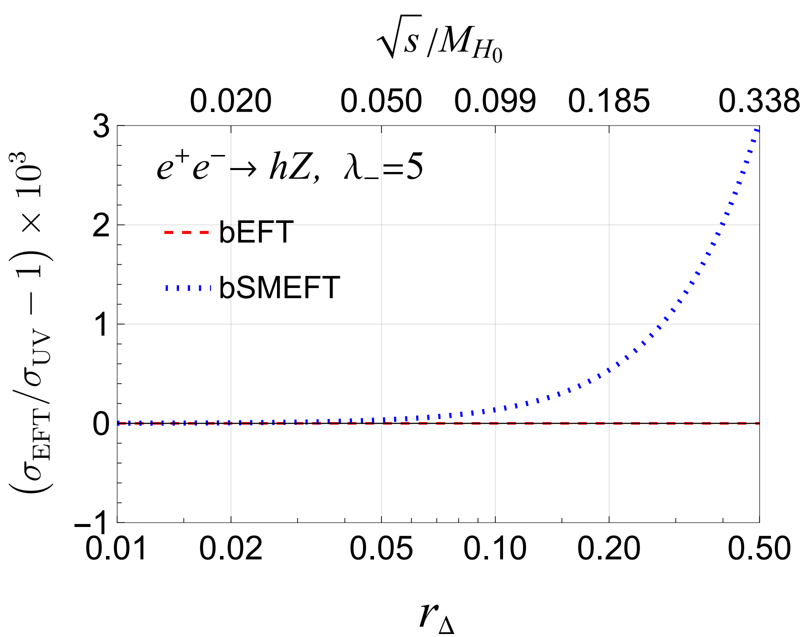

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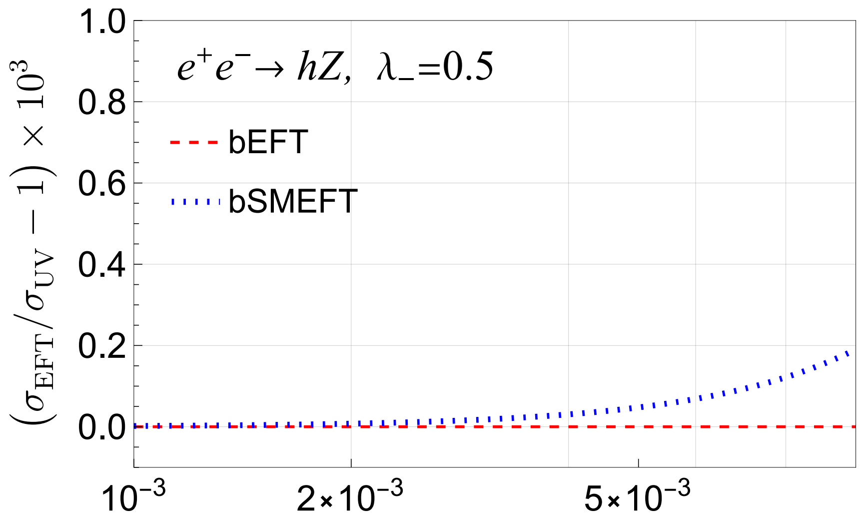

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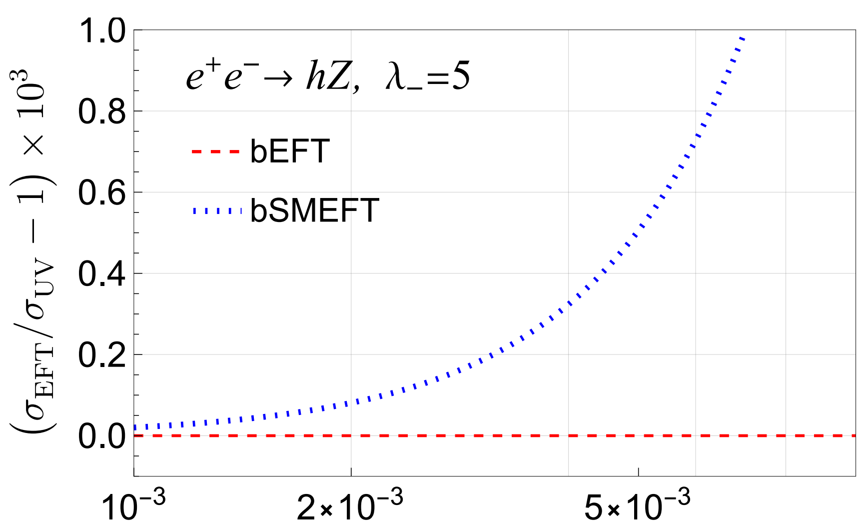

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&\quad + \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 - \mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right) \\
&\quad + \lambda_4 (H^\dagger H)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5 H^\dagger[\Delta^\dagger,\Delta]H \\
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\end{aligned}
$$

$$\sqrt{s} = 400\,\mathrm{GeV},$$

$$r_\Delta = v_\Delta / M_\Delta = 0.4$$

$$\lambda_- := (\lambda_4 - \lambda_5)/2$$

$$\lambda_1 = \lambda_2 = \lambda_5 = 1$$

bEFT reproduces UV theory more accurately than SMEFT when $v_\Delta / v_{\rm ew}$ is large.

Why bEFT is better than SMEFT ?

✔ WCs of dim-3 operators are same in the UV and the bEFT

$$\mathscr{L}_{\rm dim3} \supset C_{hW^2}\, hW^-_\mu W^{+\mu} + C_{hZ^2}\, hZ_\mu Z^\mu + C_{\bar\nu\nu}\, \overline{\nu^c_L}\nu_L$$

$$C^{\rm UV}_{hW^2} = C^{\rm bEFT}_{hW^2} \neq C^{\rm SMEFT}_{hW^2}$$
$$C^{\rm UV}_{hZ^2} = C^{\rm bEFT}_{hZ^2} \neq C^{\rm SMEFT}_{hZ^2}$$
$$C^{\rm UV}_{\bar\nu\nu} = C^{\rm bEFT}_{\bar\nu\nu} \neq C^{\rm SMEFT}_{\bar\nu\nu}$$

✔ Operators absent in SMEFT but present in bEFT

dim 4 $Z_\mu Z^\mu Z_\nu Z^\nu$

dim 5

$\Delta L = 0$ $\quad hW^-_\mu W^{+\mu} W^-_\nu W^{+\nu} \quad hW^-_\mu W^+_\nu W^{-\mu} W^{+\nu} \quad hZ_\mu Z^\mu Z_\nu Z^\nu \quad hW^-_\mu W^{+\mu} Z_\nu Z^\nu$

$hW^-_\mu W^+_\nu Z^\mu Z^\nu \quad ihW^-_\mu W^+_\nu Z^{\mu\nu} \quad i(h\overleftrightarrow{D}_\mu W^-_\nu)W^{+\mu}Z^\nu \quad W^-_\mu W^{+\mu}(\bar\psi_L\psi_R) \quad Z_\mu Z^\mu(\bar\psi_L\psi_R)$

$\Delta L = 2$ $\quad W^-_\mu W^{+\mu}(\nu_L^T C\nu_L) \quad W^+_\mu W^{+\mu}(e_L^T C e_L) \quad W^+_\mu Z^\mu(\nu_L^T C e_L) \quad Z_\mu Z^\mu(\nu_L^T C\nu_L)$

dim 6

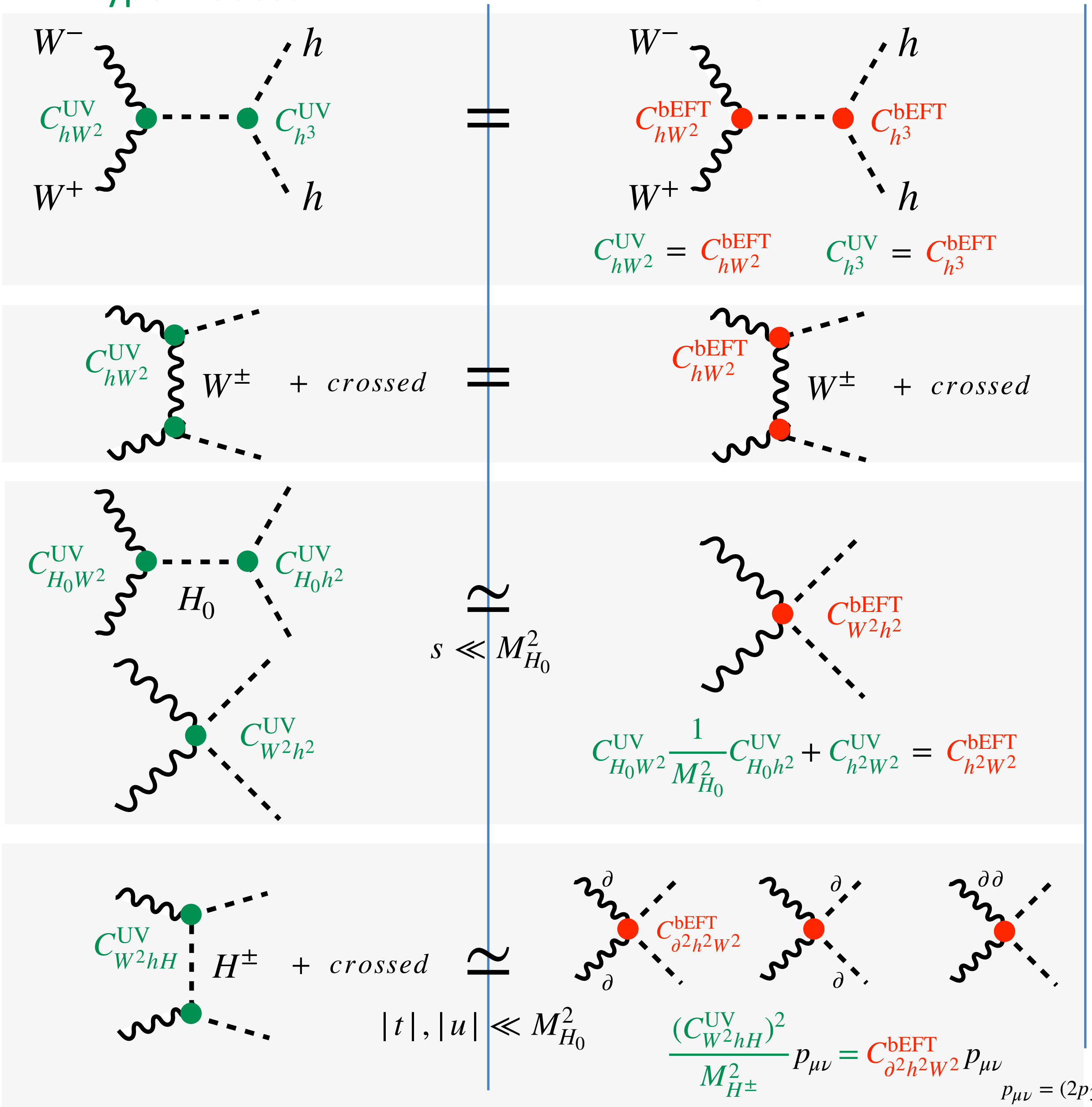
$\Delta L = 0$ $\quad (\partial_\mu h)(\partial_\nu h)W^{-\mu}W^{+\nu} \quad hh(D_\mu W^{-\mu})(D_\nu W^{+\nu}) \quad hh(D_\mu D_\nu W^{-\mu})W^{+\nu} \ \ldots$

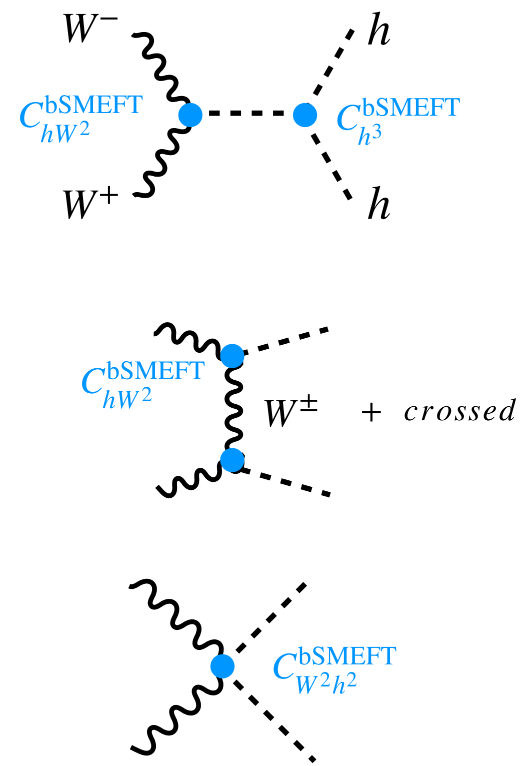
In the SMEFT, non-derivative quartic gauge interactions arise from dim-8

$$\mathscr{L}^{\rm smeft}_{\rm dim8} = \left[(D_\mu H)^\dagger D^\mu H\right]^2 \supset W^-_\mu W^{+\mu} W^-_\nu W^{+\nu}$$

Why bEFT is better than SMEFT ?

type-II seesaw | bEFT | Broken phase SMEFT





- ✔ UV and bEFT have one-to-one correspondence
- ✔ SMEFT treat heavy scalars as one multiplet, so cannot take into account scalar mixing at the lower order of perturbation

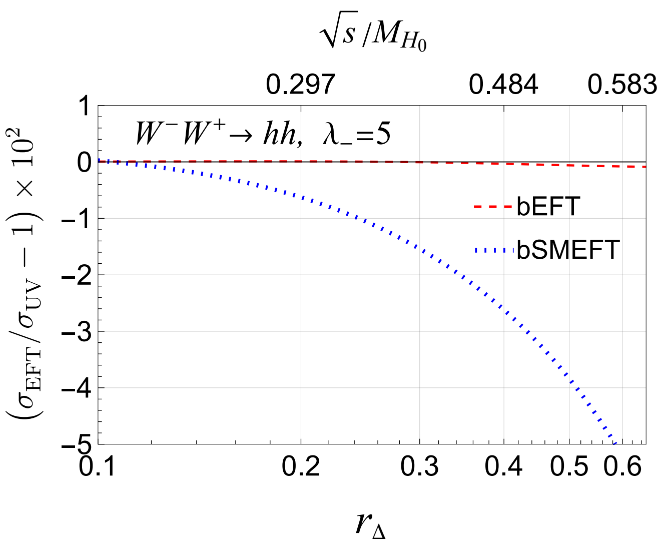
$$p_{\mu\nu} = (2p_3 - p_1)_\mu(2p_4 - p_2)_\nu + (2p_4 - p_1)_\mu(2p_3 - p_2)_\nu$$

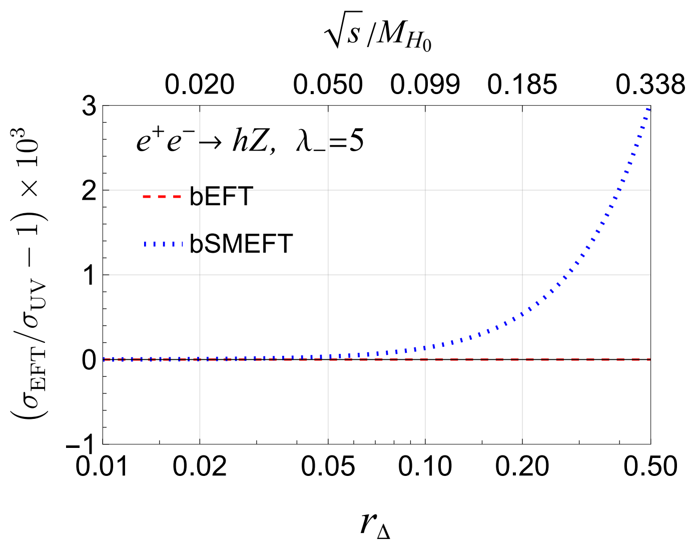
Summary

- We suggest altanative EFT overcoming the shortcoming of both SMEFT and HEFT

- We match type-II seesaw model onto SMEFT and bEFT and examine which EFT more accurately reproduces UV theory.

- We performed numerical calculations and found that bEFT reproduces UV theory more accurately than SMEFT in the regime that v_T/M_Δ is close to unity.





Back Up

SMEFT v.s. HEFT

- Specific models

	SMEFT	HEFT
SM + singlet scalar	G. Buchalla, et. al., 1608.03564	
SM + triplet scalar	T. Cohen, et. al., 2008.08597	H. Song, et. al., 2412.00355
type-II seesaw model	X. Li, et. al., 2201.05082	
Two Higgs doublet model	S. Dawson, et. al., 2305.07689 Ian Banta, et. al., 2304.09884	G. Buchalla, et. al., 2312.13885

- General discussion

Models that cannot be matched to SMEFT but only to HEFT

R. Alonso et. al., 1605.03602 Target space of the scalar kinetic term

A. Falkowski et. al., 1902.05936 Singularity of scalar potential

T. Cohen, et. al., 2008.08597 Both scalar kinetic term and potential

- Bottom-up approach

R. Gómez-Ambrosio et. al., 2204.01763

R. Gómez-Ambrosio et. al., 2207.09848

A. Salas-Bernardez, et. al., 2211.09605

R. L. Delgado, et. al., 2311.04280

S. Mahmud, et. al., 2406.03522

SMEFT v.s. HEFT

	Good	Bad
SMEFT	Easy to handle	Applicable only in $\Lambda_{\mathrm{EFT}} \gg v_{\mathrm{EW}}$
HEFT	Applicable even in $\Lambda_{\mathrm{EFT}} \gtrsim v_{\mathrm{EW}}$	Complex to handle $\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]$ $\quad U = \exp\left(\frac{i}{v}\varphi^a \tau^a\right)$

We suggest altanative EFT overcoming the shortcoming of both EFTs

Broken phase EFT (bEFT)

Integrate out heavy fields in the broken phase, $\tilde{s}'$, just like HEFT

$$\tilde{s}' = h \sin\chi + \tilde{s}\cos\chi \qquad S = v_s + \tilde{s} \qquad H = \left(0, \frac{v+h}{\sqrt{2}}\right)^T$$

In E.o.M., only expand $\tilde{s}'$ w.r.t. $1/M^2$, not c_i

$$c_i = \cancel{c_{i0} + c_{i2}M^2}$$

$$\left(-\partial^2 - M^2 + c_1\right)\tilde{s}' + c_0 + c_2\tilde{s}'^2 + c_3\tilde{s}'^3 = 0$$

$$\tilde{s}' = \tilde{s}'_0 + \frac{1}{M^2}\tilde{s}'_2 + \frac{1}{M^4}\tilde{s}'_4 + \cdots$$

SMEFT v.s. HEFT

SM + real singlet scalar

$$\mathscr{L} = (D_\mu H)^\dagger D^\mu H + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S)$$

$S \in (\mathbf{1},\mathbf{1},0)$, real

Buchalla et al. 1608.03564

$$V(H,S) = -\frac{\mu_1^2}{2}H^\dagger H - \frac{\mu_2^2}{4}S^2 + \frac{\lambda_1}{4}(H^\dagger H)^2 + \frac{\lambda_2}{16}S^4 + \frac{\lambda_3}{4}H^\dagger H S^2$$

How to integrate $\tilde{s}'$ out:

➡ Solving E.o.M.

$$\left(-\partial^2 - M^2 + c_1\right)\tilde{s}' + c_0 + c_2\tilde{s}'^2 + c_3\tilde{s}'^3 = 0$$

$c_i\ (i = 0,\cdots,3)$ is the function of

$$M^2,\ \tilde{h}',\ \mathrm{Tr}[D_\mu U^\dagger D^\mu U]$$

① Expand c_i and $\tilde{s}'$ w.r.t. $1/M^2$

$$c_i = c_{i0} + c_{i2}M^2$$

$$\tilde{s}' = \tilde{s}'_0 + \frac{1}{M^2}\tilde{s}'_2 + \frac{1}{M^4}\tilde{s}'_4 + \cdots$$

② Substitute ① into E.o.M. to get $\tilde{s}'_i$ order by order

HEFT

$M_{\tilde{s}}$

$S = v_s + \tilde{s}$

$$H = \left(0,\ \frac{v+h}{\sqrt{2}}\right)^T$$

$$\begin{pmatrix} h' \\ \tilde{s}' \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}\begin{pmatrix} h \\ \tilde{s} \end{pmatrix}$$

Integrate out $\tilde{s}'$

$m_W^2 W_\mu^- W^{+\mu}$

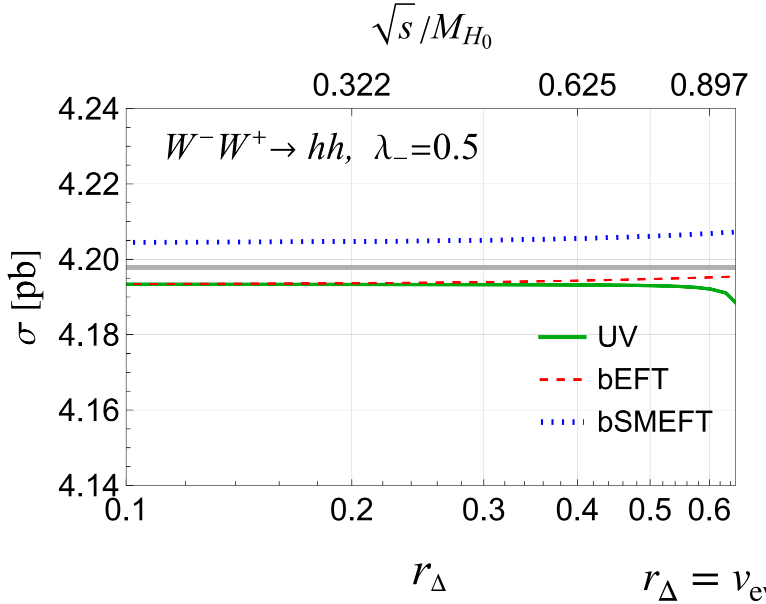
$$\frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(2c_\chi\frac{h}{v} + \left(c_\chi^4 - s_\chi^3 c_\chi \frac{v}{v_s}\right)\left(\frac{h}{v}\right)^2 + \mathcal{O}(h^3)\right)$$

Equations

$$\left(-\partial^2 - M^2 + \cdots\right)\tilde{s}' + \frac{\delta V}{\delta \tilde{s}'} + \cdots = 0$$

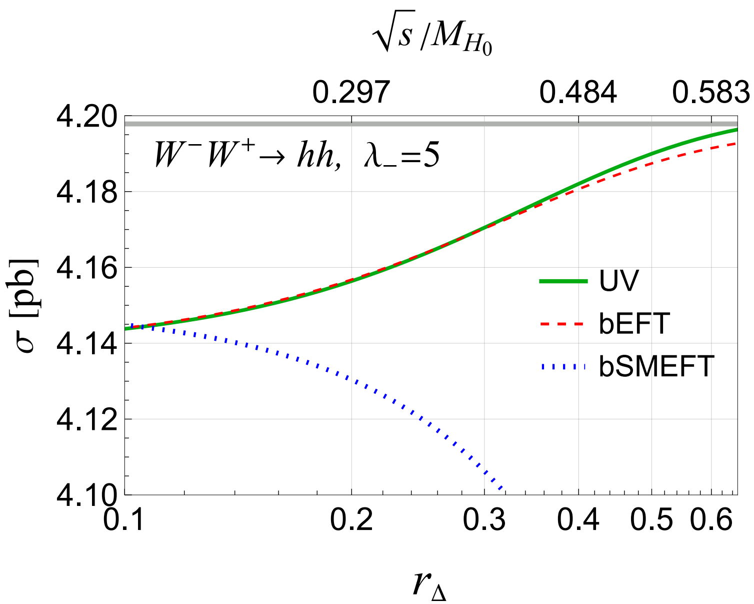
$$\frac{1}{2}\tilde{s}'(-\partial^2 - M^2)\tilde{s}' - \frac{\delta V}{\delta \tilde{s}'}$$

$W^-W^+ \to hh$



H_0 : 640 GeV
A_0 : 640 GeV
$H^{\pm}$: 663 GeV
$H^{\pm\pm}$: 685 GeV

$r_\Delta = v_{\rm ew}/M_\Delta$



H_0 : 826 GeV
A_0 : 826 GeV
$H^{\pm}$: 844 GeV
$H^{\pm\pm}$: 862 GeV

$$
\begin{aligned}
&V(H,\Delta)\\
&= -m_H^2(H^\dagger H) + \frac{\lambda}{2}(H^\dagger H)^2\\
&\quad + M_\Delta^2 \mathrm{Tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2\\
&\quad + \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 - \mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right)\\
&\quad + \lambda_4 (H^\dagger H)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5 H^\dagger[\Delta^\dagger,\Delta]H\\
&\quad + \left(\frac{\Lambda_6}{\sqrt{2}} H^T i\sigma_2 \Delta^\dagger H + \mathrm{h.c.}\right)
\end{aligned}
$$

$\sqrt{s} = 400\,\mathrm{GeV}$,

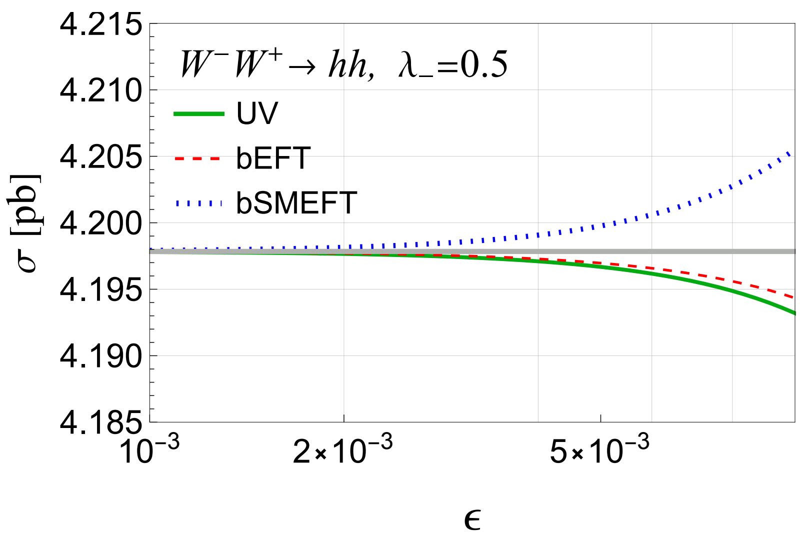
$\epsilon = v_\Delta/v_{\rm ew} = 10^{-2}$,

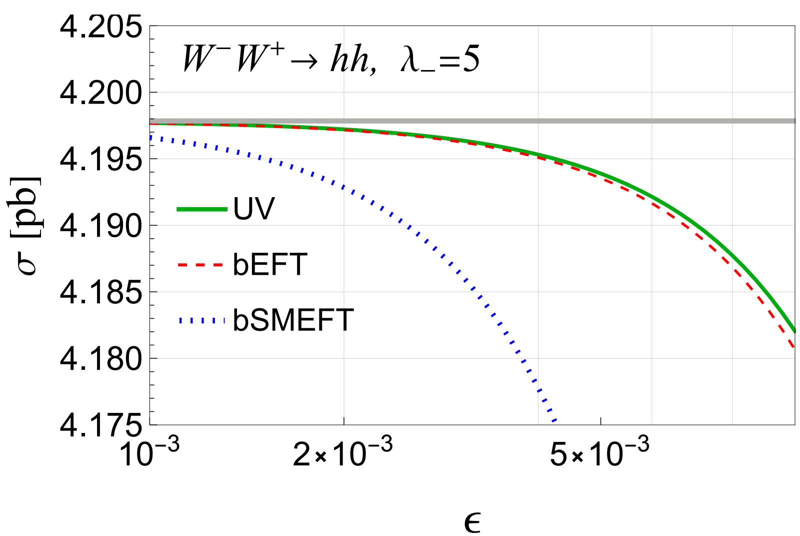
$\lambda_- := (\lambda_4 - \lambda_5)/2$

$\lambda_1 = \lambda_2 = \lambda_5 = 1$

bEFT reproduces UV theory more accurately than SMEFT when $v_{\rm ew}/M_\Delta$ is large.

$W^-W^+ \to hh$





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$$\sqrt{s} = 400\,\mathrm{GeV},$$

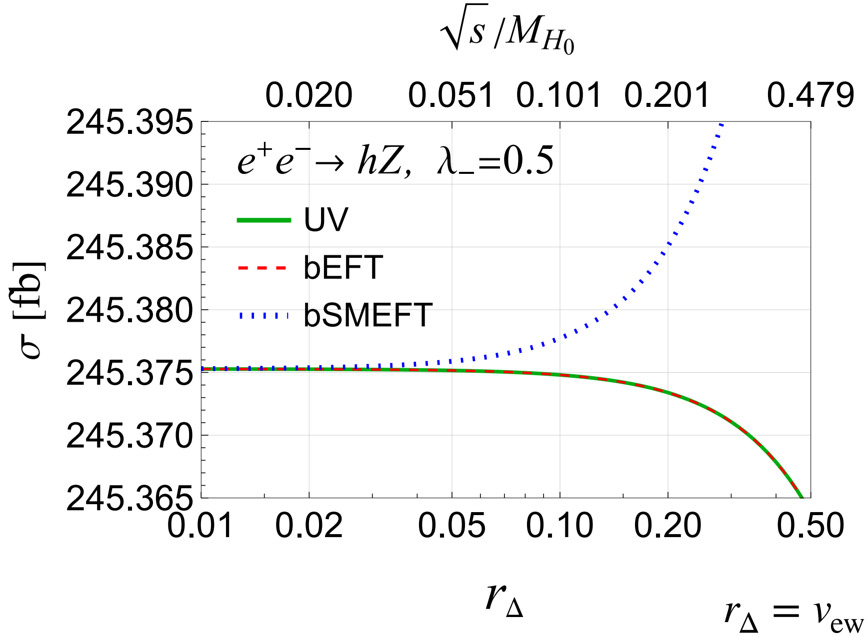
$$r_\Delta = v_\Delta / M_\Delta = 0.4$$

$$\lambda_- := (\lambda_4 - \lambda_5)/2$$

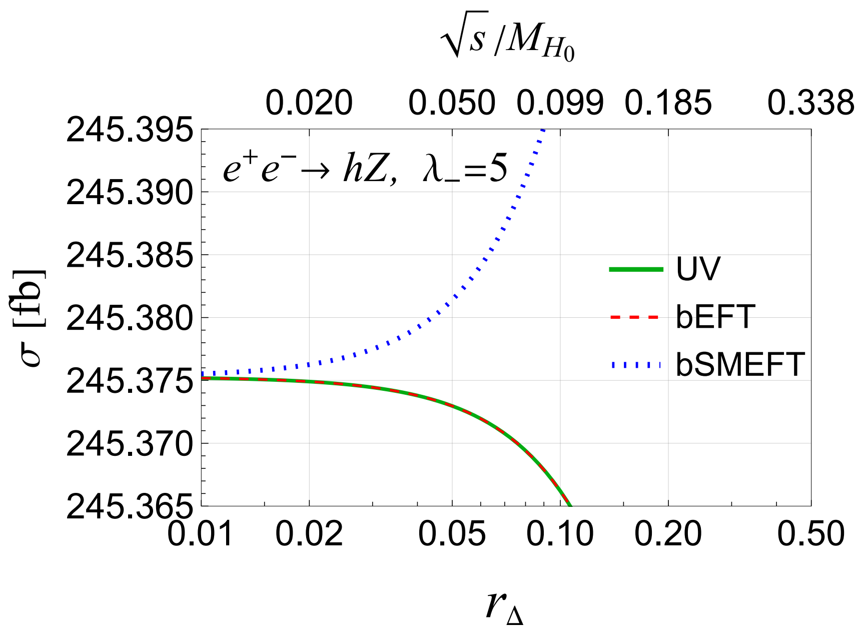
$$\lambda_1 = \lambda_2 = \lambda_5 = 1$$

HEFT reproduces UV theory more accurately than SMEFT when v_{ew}/M_Δ is large.

$e^+e^- \to Zh$



$r_\Delta = v_{\mathrm{ew}}/M_\Delta$



$$
\begin{aligned}
&V(H,\Delta)\\
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&\quad + \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 - \mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right)\\
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&\quad + \left(\frac{\Lambda_6}{\sqrt{2}} H^T i\sigma_2 \Delta^\dagger H + \mathrm{h.c.}\right)
\end{aligned}
$$

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$\epsilon = v_\Delta / v_{\mathrm{ew}} = 10^{-2}$,

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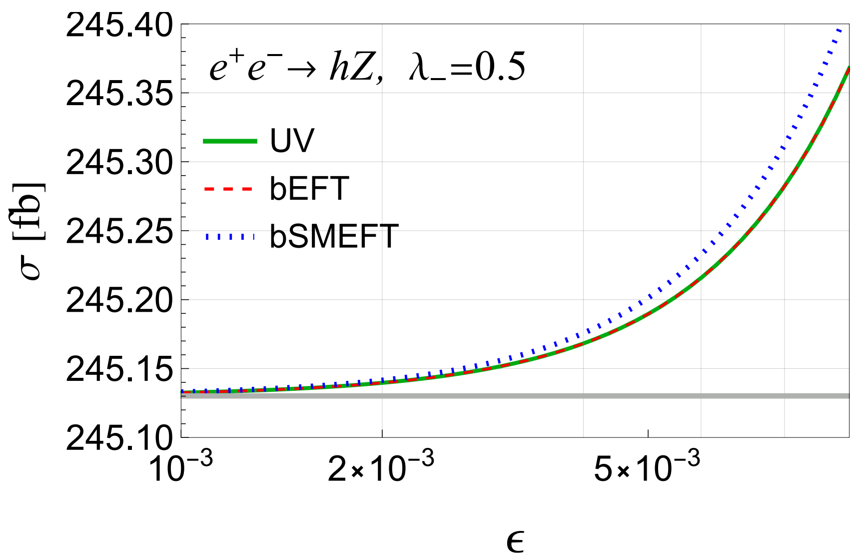
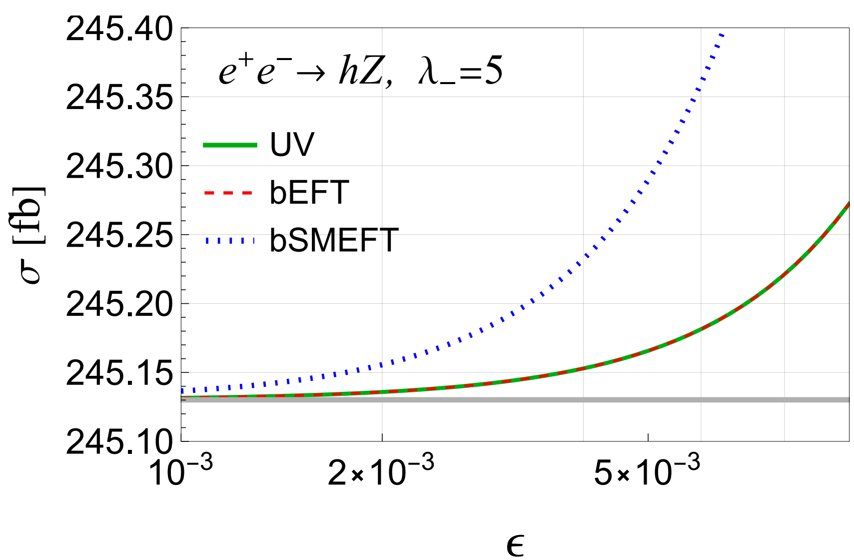
HEFT reproduces UV theory more accurately than SMEFT when v_{ew}/M_Δ is large.

$e^+e^- \to Zh$





$$
\begin{aligned}
&V(H,\Delta)\\
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&\quad + M_\Delta^2 \mathrm{Tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2\\
&\quad + \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 - \mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right)\\
&\quad + \lambda_4(H^\dagger H)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5 H^\dagger[\Delta^\dagger,\Delta]H\\
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\end{aligned}
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$\sqrt{s} = 400\,\mathrm{GeV}$,

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$\lambda\_ := (\lambda_4 - \lambda_5)/2$

$\lambda_1 = \lambda_2 = \lambda_5 = 1$

HEFT reproduces UV theory more accurately than SMEFT when v_{ew}/M_Δ is large.

Matching onto HEFT

Difficulty #1

In the broken phase, $SU(2)_L \times U(1)_Y$ is not really broken, but is non-linearly realized

Polar decomposition, $\widetilde{\Delta} = U\left(\Delta^a \frac{\sigma^a}{2}\right)U^\dagger$ $\quad$ $\widetilde{\Sigma} = \frac{1}{\sqrt{2}}\big((v+h)\mathbf{1} + i\rho^a\sigma^a\big)U$

Real triplet case

H. Song, et. al., 2412.00355

Integrating out heavy fields from $(\star)$, we get the HEFT for type-II seesaw model.

Solution #1

We just take a unitary guage : $U = \mathbf{1}_{2\times 2}$

Difficulties in HEFT

Difficulty #1

In the broken phase, $SU(2)_L \times U(1)_Y$ is not really broken, but is non-linearly realized

$\boxed{\text{SM + singlet scalar}}$ $\quad \mathscr{L} = (D_\mu H)^\dagger D^\mu H + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S)$

Substituting polar decomposition, $H = \frac{v+h_1}{\sqrt{2}} U \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\quad S = \frac{v_s + h_2}{\sqrt{2}}$ $\quad \begin{pmatrix} h \\ \tilde{s} \end{pmatrix} = \begin{pmatrix} c_\chi & -s_\chi \\ s_\chi & c_\chi \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$

we get

$$\mathscr{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \tilde{s} \partial^\mu \tilde{s} - V(h,\tilde{s})$$
$$+ \frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + \frac{2c_\chi}{v}h + \frac{2s_\chi}{v}\tilde{s} + \frac{c_\chi^2}{v^2}h^2 + \frac{s_\chi^2}{v^2}\tilde{s}^2 + \frac{2s_\chi c_\chi}{v^2}h\tilde{s}\right) \qquad \ldots (\star)$$

$(\star)$ is invariant under non-linerly realized $SU(2)_L \times U(1)_Y$

$$U \to \mathfrak{g}_L U \mathfrak{g}_Y^\dagger \qquad \mathfrak{g}_L = \exp\left(i\theta_L^a \frac{\sigma^a}{2}\right) \qquad \mathfrak{g}_Y = \exp\left(i\theta_Y \frac{\sigma^3}{2}\right)$$

Integrating out $\tilde{s}$ from $(\star)$, we get

$$\mathscr{L}_{\mathrm{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right) + \cdots$$

Difficulty #2

In the broken phase, there are so many fields to be integrated out

Symmetric phase: $\Delta \in (\mathbf{1}, \mathbf{3}, 1)$ $\qquad$ Broken phase: $\{H_0,\ A_0,\ H^\pm,\ H^{\pm\pm}\}$

HEFT

Higgs sector & Yukawa sector in HEFT

Higgs sector in SM

$$\mathcal{L}_{\mathrm{Higgs}}^{\mathrm{SM}} = (D_\mu H)^\dagger D^\mu H + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2$$

$$= \frac{1}{2}\mathrm{Tr}\left[(D_\mu \hat{\Phi})^\dagger D^\mu \hat{\Phi}\right] + \frac{\mu^2}{2}\mathrm{Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right] - \frac{\lambda}{4}\mathrm{Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right]^2$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　但し $\tilde{H} := i\sigma_2 H^*$

$$D_\mu \hat{\Phi} = \partial_\mu \hat{\Phi} - i g W_\mu^a \frac{\tau^a}{2}\hat{\Phi} + i g' \hat{\Phi} B_\mu \frac{\tau^3}{2}$$

Higgs sector in HEFT

h : singlet under non-linearly realized SU(2)xU(1)
(125GeV Higgs)

$w^\pm, z$: transform nontrivially under non-linearly realized SU(2)xU(1)
(would-be Goldstone)

➡ We must separate h from $w^\pm, z$ because they transform differently

Polardecomposition : $\hat{\Phi} = (v+h)U$　　$U = \exp\left(\frac{w^\pm + z^0}{v}\right)$

$$\mathcal{L}_{\mathrm{Higgs}}^{\mathrm{SM}} = \frac{v^2}{4}\left(1+\frac{h}{v}\right)^2 \mathrm{Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V_{\mathrm{SM}}(h)$$

Higgs sector & Yukawa sector in HEFT

Higgs sector in SM

$$\mathcal{L}_{\text{Higgs}}^{\text{SM}} = (D_\mu H)^\dagger D^\mu H + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2$$

$$= \frac{1}{2}\mathrm{Tr}\left[(D_\mu \hat{\Phi})^\dagger D^\mu \hat{\Phi}\right] + \frac{\mu^2}{2}\mathrm{Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right] - \frac{\lambda}{4}\mathrm{Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right]^2$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　　但し $\tilde{H} := i\sigma_2 H^*$

$$D_\mu \hat{\Phi} = \partial_\mu \hat{\Phi} - igW_\mu^a \frac{\tau^a}{2}\hat{\Phi} + ig'\hat{\Phi} B_\mu \frac{\tau^3}{2}$$

Higgs sector in HEFT

h : singlet under non-linearly realized SU(2)xU(1)
(125GeV Higgs)

$w^\pm, \; z$: transform nontrivially under non-linearly realized SU(2)xU(1)
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➡ We must separate h from $w^\pm, z$ because they transform differently

Polar decomposition : $\hat{\Phi} = (v + h) U$　　　$U = \exp\left(\frac{w^\pm + z^0}{v}\right)$

$$\mathcal{L}_{\text{Higgs}}^{\text{SM}} = \frac{v^2}{4}\left(1 + \frac{h}{v}\right)^2 \mathrm{Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V_{\text{SM}}(h)$$

Higgs sector & Yukawa sector in HEFT

Higgs sector in SM

$$\mathcal{L}_{\rm Higgs}^{\rm SM} = (D_\mu H)^\dagger D^\mu H + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2$$

$$= \frac{1}{2}{\rm Tr}\left[(D_\mu \hat{\Phi})^\dagger D^\mu \hat{\Phi}\right] + \frac{\mu^2}{2}{\rm Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right] - \frac{\lambda}{4}{\rm Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right]^2$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　但し $\tilde{H} := i\sigma_2 H^*$

$$D_\mu \hat{\Phi} = \partial_\mu \hat{\Phi} - i g W_\mu^a \frac{\tau^a}{2}\hat{\Phi} + i g' \hat{\Phi} B_\mu \frac{\tau^3}{2}$$

Higgs sector in HEFT

h : singlet under non-linearly realized SU(2)xU(1)

(125GeV Higgs)

$w^\pm, z$: transform nontrivially under non-linearly realized SU(2)xU(1)

(would-be Goldstone)

We must separate h from $w^\pm, z$ because they transform differently

Polar decomposition : $\hat{\Phi} = (v+h)U$　　$U = \exp\left(\frac{w^\pm + z^0}{v}\right)$

$$\mathcal{L}_{\rm Higgs}^{\rm HEFT} = \frac{v^2}{4} F(h)\, {\rm Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$

Higgs sector & Yukawa sector in HEFT

SM Yukawa

$$\mathcal{L}^{\rm SM}_{\rm Yukawa} = -y_u\,\bar{q}_L\tilde{H}u_R - y_d\,\bar{q}_L H d_R - y_e\,\bar{l}_L\tilde{H}e_R + \text{h.c.}$$

$$= -y_u\,\bar{q}_L\hat{\Phi}P_+q_R - y_d\,\bar{q}_L\hat{\Phi}P_-q_R - y_e\,\bar{l}_L\hat{\Phi}P_-l_R + \text{h.c.}$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　但し $\tilde{H} := i\sigma_2 H^*$

$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$, $l_R = \begin{pmatrix} 0 \\ e_R \end{pmatrix}$: right-handed fermions　　$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$: projection operator

HEFT Yukawa

Substituting $\hat{\Phi} = (v+h)U$ into $\mathcal{L}^{\rm SM}_{\rm Yukawa}$, we get

$$\mathcal{L}^{\rm SM}_{\rm Yukawa} = -v\,y_u\left(1+\frac{h}{v}\right)\bar{q}_L UP_+q_R - v\,y_d\left(1+\frac{h}{v}\right)\bar{q}_L UP_-q_R - v\,y_e\left(1+\frac{h}{v}\right)UP_-l_R + \text{h.c.}$$

Replacement

$$\mathcal{L}^{\rm HEFT}_{\rm Yukawa} = -v\,Y_u(h)\,\bar{q}_L UP_+q_R - v\,Y_d(h)\,\bar{q}_L UP_-q_R - v\,Y_e(h)\,UP_-l_R + \text{h.c.}$$

Buchalla et al., 1307.5017

HEFT (LO)

$$\mathcal{L}_{\mathrm{HEFT,\,LO}} = \mathcal{L}_0 + \mathcal{L}_{Uh}$$

$$\mathcal{L}_{Uh} = \frac{v^2}{4} F(h) \mathrm{Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$ Higgs sector

$$- v\left[Y_u(h)\, \bar{q}_L\, U P_+ q_R + Y_d(h)\, \bar{q}_L\, U P_- q_R + Y_e(h)\, \bar{l}_L\, U P_- l_R + \mathrm{h.c.}\right]$$ Yukawa項

$$U = \exp\left(\frac{w^\pm + z^0}{v}\right)$$

$$SU(2)_L \times U(1)_Y \quad : \quad \hat{\Phi} \to \mathfrak{g}_L \hat{\Phi} \mathfrak{g}_Y^\dagger \quad \longleftrightarrow \quad h \to h\,, \quad U \to \mathfrak{g}_L\, U\, \mathfrak{g}_Y^\dagger$$

$$\mathcal{L}_0 = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$
$$+ \bar{q}_L i\not{D} q_L + \bar{l}_L i\not{D} l_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + \bar{e}_R i\not{D} e_R$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in \mathbf{2}_{1/6} \qquad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \in \mathbf{2}_{-1/2}$$

Identical to those in SM

$$u_R \in \mathbf{1}_{2/3} \qquad d_R \in \mathbf{1}_{-1/3} \qquad e_R \in \mathbf{1}_{-1}$$

CCWZ

Concept of Higgs Effective Field Theory

Symmetry : $O(4)$

Matter : $\vec{\phi} = (\phi^1, \phi^2, \phi^3, \phi^4)^T \in \mathbf{4}$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{1}{2}\mu^2(\vec{\phi})^2 - \frac{\lambda}{4}(\vec{\phi})^4$$

$\langle \phi \rangle \neq 0$

$O(4) \to O(3)$

Symmetry : $O(3)$

Matter : $\underbrace{(\pi^1, \pi^2, \pi^3)}_{\text{Goldstone}} \in \mathbf{3}\,,\ \sigma \in \mathbf{1}$

$$\vec{\phi} = (v + \sigma) e^{i\frac{\pi^a}{v} X^a} \vec{F}$$

$$\mathcal{L} = \frac{1}{2}\left(1 + \frac{\sigma}{v}\right)^2 \left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{(\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 - |\vec{\pi}|^2 (\partial_\mu \vec{\pi})^2}{3v^2} + \cdots\right] + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - V(\sigma)$$

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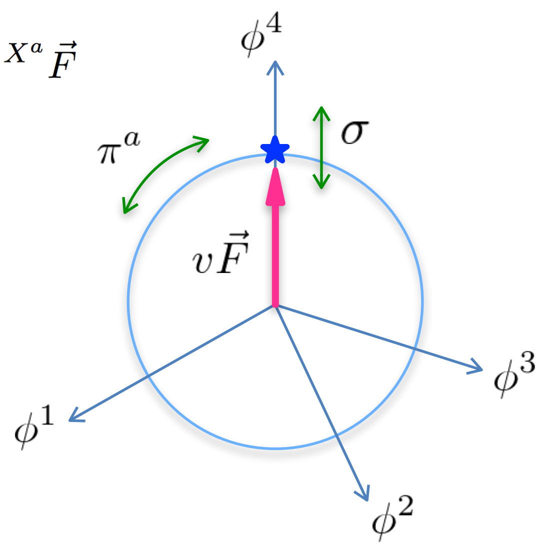

X^a : broken generator

$\vec{F} = (0, 0, 0, 1)^T$

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?

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$O(4) \to O(3)$

All we know

Symmetry : $O(3)$

Matter : $(\pi^1, \pi^2, \pi^3) \in \mathbf{3}\,, \quad \sigma \in \mathbf{1}$

Goldstone

$$\vec{\phi} = (v + \sigma) e^{i\frac{\pi^a}{v} X^a} \vec{F}$$

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Let's assume we don't know UV theory

EFT approach

Write down all the $O(3)$ invariant operators

Q1. Should we write $(\vec{\pi} \cdot \vec{\pi})\,, \quad (\vec{\pi} \cdot \vec{\pi})^2\,, \quad (\vec{\pi} \cdot \vec{\pi})^3$? No

Q2. Should we write $|\vec{\pi}|^2 (\partial_\mu \vec{\pi})^2$? Yes

What are the criteria for inclusion or exclusion of Goldstone operator?

Concept of Higgs Effective Field Theory

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?

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What did we miss ?

If the symmetry $\mathcal{H}$ is due to a SSB of the larger symmetry $\mathcal{G}$,
we should write invariance under the non-linearly realized $\mathcal{G}$, rather than $\mathcal{H}$.

We should write non-linear $O(4)$ invariance,

rather than $O(3)$ invariance

Concept of Higgs Effective Field Theory

Symmetry : $O(4)$

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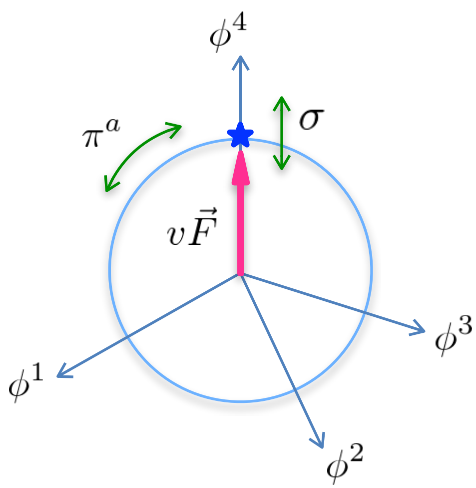
What is non-linearly realized symmetry ?



$$O(4): \quad \vec{\phi} \ \to\ \mathfrak{g} \cdot \vec{\phi} \qquad \mathfrak{g} \in O(4)$$

Substituting $\vec{\phi} = (v + \sigma) e^{i\frac{\pi^a}{v}X^a} \vec{F}$

$$(v + \sigma) e^{i\frac{\pi^a}{v}X^a} \vec{F} \ \to\ (v + \sigma)\, \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h} \cdot \vec{F} \qquad \mathfrak{h} \in O(3)$$

$$O(4): \quad \sigma \to \sigma \qquad e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h} = e^{i\frac{\pi'^a}{v}X^a} \cancel{e^{i\varphi^a S^a}} \cancel{\cdot\, \mathfrak{h}}$$

Concept of Higgs Effective Field Theory

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What is non-linearly realized symmetry ?

$O(4)$: $\quad \vec{\phi} \to \mathfrak{g} \cdot \vec{\phi}$ $\qquad \mathfrak{g} \in O(4)$

Substituting $\vec{\phi} = (v + \sigma) e^{i\frac{\pi^a}{v}X^a} \vec{F}$

$$(v + \sigma) e^{i\frac{\pi^a}{v}X^a} \vec{F} \to (v + \sigma)\, \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h} \cdot \vec{F} \qquad \mathfrak{h} \in O(3)$$

$$\pi'^a = \overbrace{v + \mathcal{M}^a{}_b \pi^b + \mathcal{N}^a{}_{bc} \pi^b \pi^c + \mathcal{O}(\pi^3)}^{\text{non-linear function of } \pi}$$

Non-linearly realized $O(4)$: $\quad \sigma \to \sigma \qquad e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h} = e^{i\frac{\pi'^a}{v}X^a} \cancel{e^{i\varphi^a S^a}} \cancel{\cdot\, \mathfrak{h}}$

Concept of Higgs Effective Field Theory

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?

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Let's assume we don't know UV theory

Concept of Higgs Effective Field Theory

Symmetry : $O(4)$ $\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{1}{2}\mu^2(\vec{\phi})^2 - \frac{\lambda}{4}(\vec{\phi})^4$

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Symmetry : Non-linear $O(4)$

Matter : $\sigma \to \sigma$

$e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h}$

Let’s assume we don’t know UV theory

correct EFT is ...

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} F(h) \left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{(\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 - |\vec{\pi}|^2 (\partial_\mu \vec{\pi})^2}{3v^2} + \cdots\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$

$F(h)$, $V(h)$: arbitrary function of h