Lam-Tung relation breaking effects and weak dipole moments at colliders

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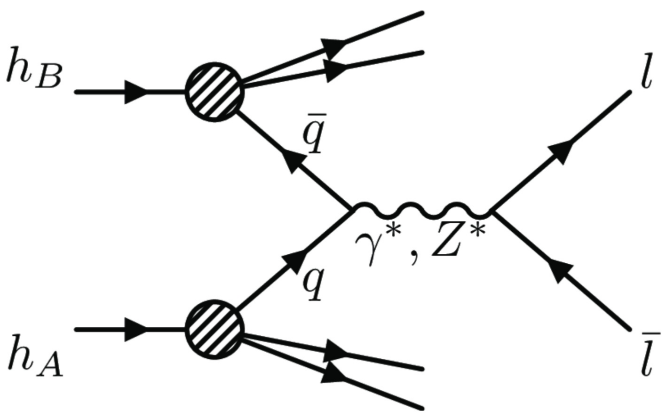
Based on: Xu Li, Bin Yan, C.-P. Yuan, PRD 111 (2025) 073007
Guanghui Li, Xu Li, Bin Yan, PLB 870 (2025) 139931

The Drell-Yan process

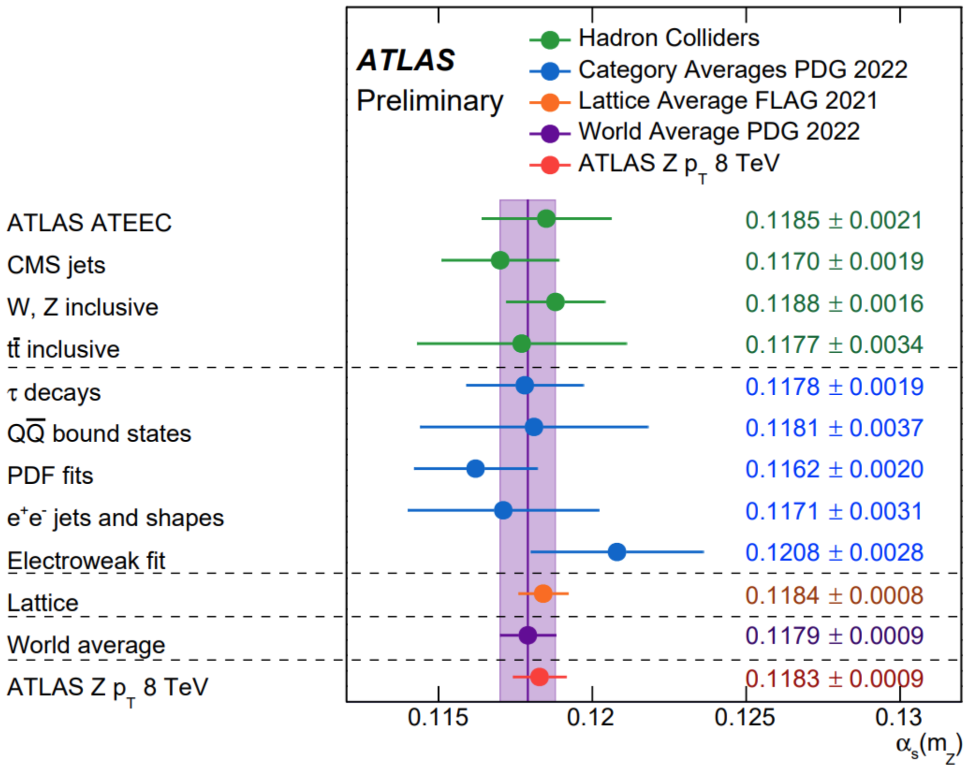
A "Standard candle" at the LHC

- Large cross section
- Clean leptonic signature



Precise Z boson measurements:

- test pQCD, constrain PDFs, weak mixing angle, strong coupling



The most precision measurements

Y. Fu, R. Brock, D. Hayden, C.-P. Yuan, PRD 2024,
S. Yang, Y. Fu, M. Liu, L. Han, T. Hou, C.-P. Yuan, PRD 2022,
S. Yang et al, EPJC 2022
ATLAS-CONF-2023-015

The Drell-Yan process

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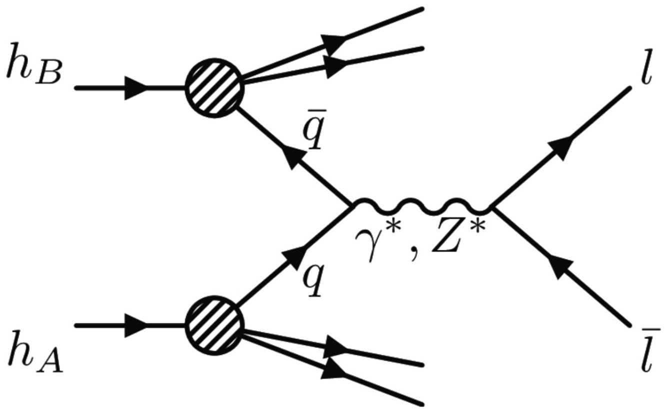


Precise Z boson measurements:

- Detector calibration, luminosity monitor,…

W-mass @ CDF, Science 376, 170–176 (2022)

- Searches for BSM physics

S. Alioli, W. Dekens, M. Girard, E. Mereghetti, JHEP 2018
S. Alioli, R. Boughzal, E. Mereghetti, F. Petriello, PLB 2020
R. Boughezal, Y. Huang, F. Petriello, PRD 2022, 2023
X. Li, K. Mimasu, K. Yamashita, C. Yang, C. Zhang, S.-Y. Zhou, JHEP 2022
S. Grossi and R. Torre 2404.10569

Retain full differential information on the leptons (spin information):

- Can be encoded in eight angular coefficients

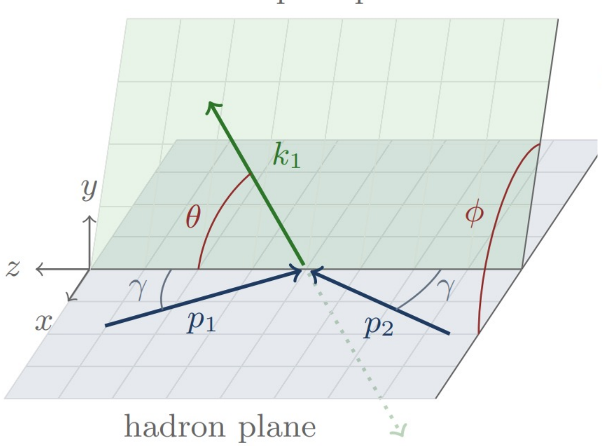
Novel angular dependence from NP:
S. Alioli, R. Boughzal, E. Mereghetti, F. Petriello, PLB 2020

- Directly probe production dynamics

R. Gauld et al, JHEP 2017, N3LO

Angular coefficients

Decomposition in terms of spherical harmonics in Collins-soper frame:



$$\frac{d\sigma}{d^4q\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{\text{unpol.}}}{d^4q}\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0\,(1-3\cos^2\theta) + A_1\,\sin(2\theta)\cos\phi + \frac{1}{2}A_2\,\sin^2\theta\,\cos(2\phi) + A_3\,\sin\theta\,\cos\phi + A_4\,\cos\theta + A_5\,\sin^2\theta\,\sin(2\phi) + A_6\,\sin(2\theta)\,\sin\phi + A_7\,\sin\theta\,\sin\phi\Big\},$$

$Y_{l,m}(\theta,\phi), l = 0,1,2$

Production dynamics: A_i

Lepton kinematics: $Y_{l,m}(\theta,\phi)$

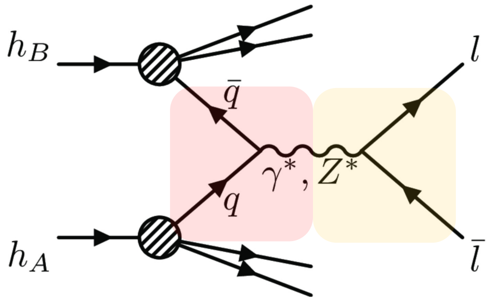
$l = 0$: $m = 0$

$l = 1$: $m = \pm 1, 0$

$l = 2$: $m = \pm 2, \pm 1, 0$

What's the physical meaning of these angular coefficients?

Angular coefficients



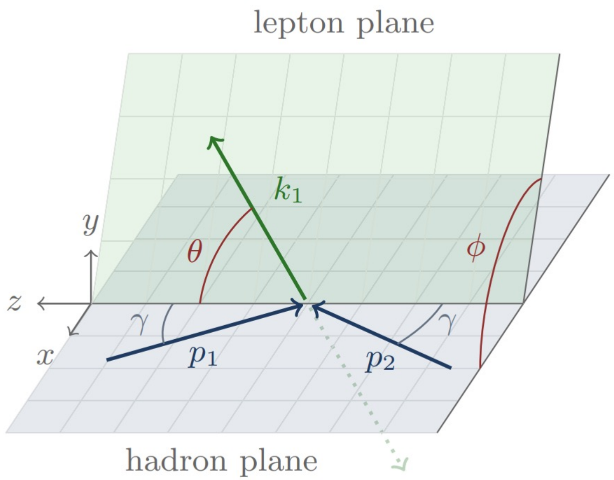
$$\rho_{\lambda_Z \lambda'_Z} = \begin{pmatrix} \frac{1-\delta_L}{3} + \frac{J_3}{2} & \frac{J_1+2Q_{xz}-i(J_2+2Q_{yz})}{2\sqrt{2}} & \lambda_T - iQ_{xy} \\ \frac{J_1+2Q_{xz}+i(J_2+2Q_{yz})}{2\sqrt{2}} & \frac{1+2\delta_L}{3} & \frac{J_1-2Q_{xz}-i(J_2-2Q_{yz})}{2\sqrt{2}} \\ \lambda_T + iQ_{xy} & \frac{J_1-2Q_{xz}+i(J_2-2Q_{yz})}{2\sqrt{2}} & \frac{1-\delta_L}{3} - \frac{J_3}{2} \end{pmatrix}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\ \mathrm{d}\cos\theta\ \mathrm{d}\phi} = \frac{3}{16\pi}\frac{\mathrm{d}\sigma^{\text{unpol.}}}{\mathrm{d}^4q}\Big\{(1+\cos^2\theta) + \frac{1}{2}\ A_0\ (1-3\cos^2\theta)$$
$$+ A_1\ \sin(2\theta)\cos\phi + \frac{1}{2}\ A_2\ \sin^2\theta\ \cos(2\phi)$$
$$+ A_3\ \sin\theta\ \cos\phi + A_4\ \cos\theta + A_5\ \sin^2\theta\ \sin(2\phi)$$
$$+ A_6\ \sin(2\theta)\ \sin\phi + A_7\ \sin\theta\ \sin\phi\Big\},$$

$$\frac{\Gamma}{\Omega_f^*} \propto \frac{|B_+|^2+|B_-|^2}{2}\Big[\frac{2}{3} + \frac{\delta_L}{3}(1-3\cos^2\theta_f^*) + \lambda_T\sin^2\theta_f^*\cos 2\phi_f^*$$
$$+ Q_{yz}\sin 2\theta_f^*\sin\phi_f^* + Q_{xz}\sin 2\theta_f^*\cos\phi_f^* + Q_{xy}\sin^2\theta_f^*\sin 2\phi_f^*\Big]$$
$$+ \frac{|B_+|^2-|B_-|^2}{2}(J_1\sin\theta_f^*\cos\phi_f^* + J_2\sin\theta_f^*\sin\phi_f^* + J_3\cos\theta_f^*).$$

$A_0 \rightarrow \delta_L$
$A_1 \rightarrow Q_{xz}$
$A_2 \rightarrow \lambda_T$
$A_3 \rightarrow J_1$
$A_4 \rightarrow J_3$
$A_5 \rightarrow Q_{xy}$
$A_6 \rightarrow Q_{yz}$
$A_7 \rightarrow J_2$

Angular coefficients and polarization



$$\frac{d\sigma}{d^4q\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{\text{unpol.}}}{d^4q}\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0\,(1-3\cos^2\theta) + A_1\,\sin(2\theta)\cos\phi + \frac{1}{2}A_2\,\sin^2\theta\,\cos(2\phi) + A_3\,\sin\theta\,\cos\phi + A_4\,\cos\theta + A_5\,\sin^2\theta\,\sin(2\phi) + A_6\,\sin(2\theta)\,\sin\phi + A_7\,\sin\theta\,\sin\phi\Big\},$$

Z-boson density matrix:

$$\rho_{\lambda_Z\lambda_Z'} = \begin{pmatrix} \frac{1-\delta_L}{3}+\frac{J_3}{2} & \frac{J_1+2Q_{xz}-i(J_2+2Q_{yz})}{2\sqrt{2}} & \lambda_T - iQ_{xy} \\ \frac{J_1+2Q_{xz}+i(J_2+2Q_{yz})}{2\sqrt{2}} & \frac{1+2\delta_L}{3} & \frac{J_1-2Q_{xz}-i(J_2-2Q_{yz})}{2\sqrt{2}} \\ \lambda_T + iQ_{xy} & \frac{J_1-2Q_{xz}+i(J_2-2Q_{yz})}{2\sqrt{2}} & \frac{1-\delta_L}{3}-\frac{J_3}{2} \end{pmatrix}$$

$A_0 \rightarrow \delta_L$ — Difference between longitudinal and transverse polarization of Z boson

$A_1 \rightarrow Q_{xz}$
$A_3 \rightarrow J_1$
$\boldsymbol{A_6 \rightarrow Q_{yz}}$
$\boldsymbol{A_7 \rightarrow J_2}$

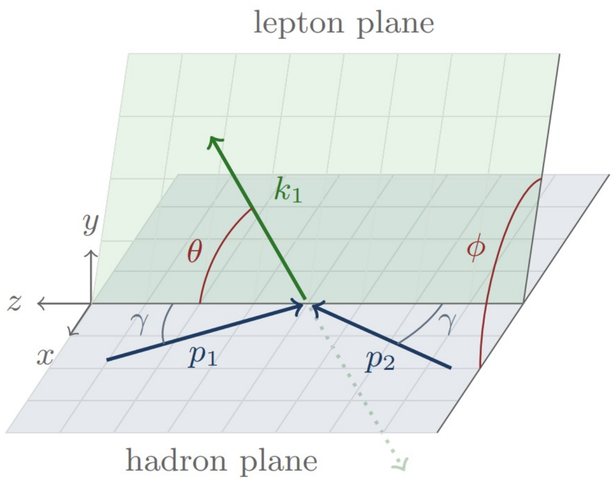
Interference between transverse and longitudinal Z boson

$A_2 \rightarrow \lambda_T$
$\boldsymbol{A_5 \rightarrow Q_{xy}}$

Linear polarization of Z boson

$A_4 \rightarrow J_3$ — Parity violation effects

Lam-Tung relation and polarization



$$\frac{d\sigma}{d^4q\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{\text{unpol.}}}{d^4q}\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0\,(1-3\cos^2\theta) + A_1\,\sin(2\theta)\cos\phi + \frac{1}{2}A_2\,\sin^2\theta\,\cos(2\phi) + A_3\,\sin\theta\,\cos\phi + A_4\,\cos\theta + A_5\,\sin^2\theta\,\sin(2\phi) + A_6\,\sin(2\theta)\,\sin\phi + A_7\,\sin\theta\,\sin\phi\Big\},$$

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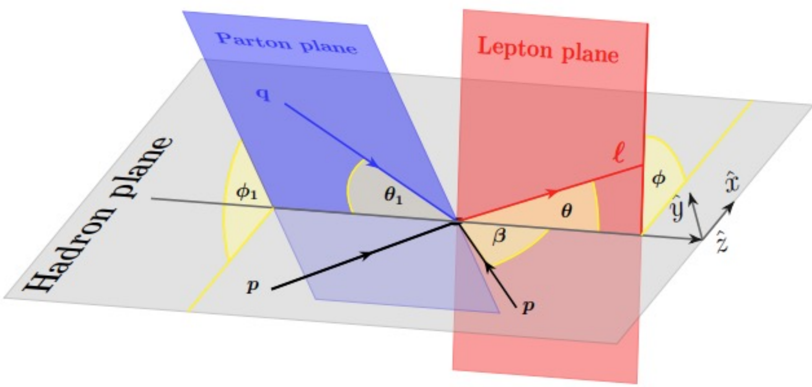
$$\frac{\Gamma}{\Omega_f^*} \propto \frac{|B_+|^2+|B_-|^2}{2}\Big[\frac{2}{3}+\frac{\delta_L}{3}(1-3\cos^2\theta_f^*)+\lambda_T\sin^2\theta_f^*\cos2\phi_f^* + Q_{yz}\sin2\theta_f^*\sin\phi_f^*+Q_{xz}\sin2\theta_f^*\cos\phi_f^*+Q_{xy}\sin^2\theta_f^*\sin2\phi_f^*\Big] + \frac{|B_+|^2-|B_-|^2}{2}(J_1\sin\theta_f^*\cos\phi_f^*+J_2\sin\theta_f^*\sin\phi_f^*+J_3\cos\theta_f^*).$$

Lam-Tung relation: $A_0 = A_2$

Linear and Longitudinal polarization of Z boson

- Spin ½ nature of quarks @ LO
- Vector coupling of spin-1 gluon @ NLO
- Violation @ NNLO and beyond

Lam-Tung relation and polarization



Collins-Soper frame

$$\frac{d\sigma}{d^4q\, d\cos\theta\, d\phi} = \frac{3}{16\pi}\frac{d\sigma^{\text{unpol.}}}{d^4q}\Big\{(1+\cos^2\theta) + \frac{1}{2}A_0\,(1-3\cos^2\theta) + A_1\,\sin(2\theta)\cos\phi + \frac{1}{2}A_2\,\sin^2\theta\,\cos(2\phi) + A_3\,\sin\theta\,\cos\phi + A_4\,\cos\theta + A_5\,\sin^2\theta\,\sin(2\phi) + A_6\,\sin(2\theta)\,\sin\phi + A_7\,\sin\theta\,\sin\phi\Big\},$$

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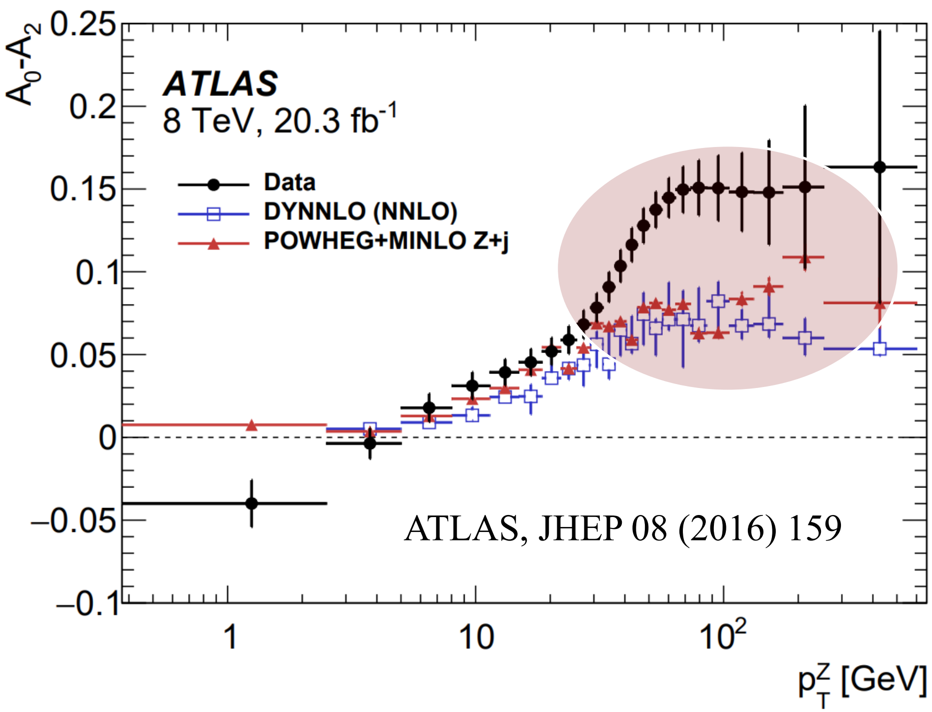
Lam-Tung relation: $A_0 = A_2$

Linear and Longitudinal polarization of Z boson

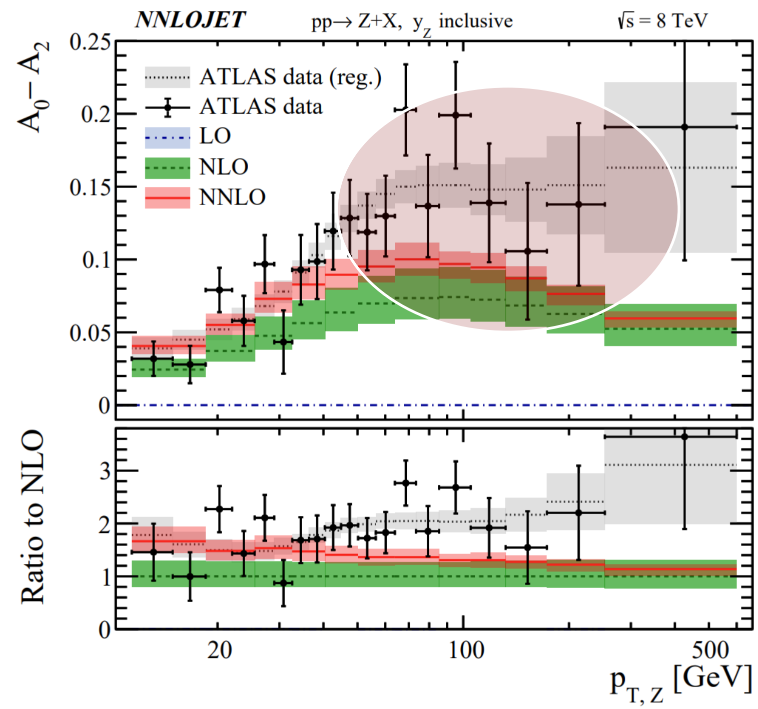
$A_0 \neq A_2$ @ NNLO in QCD
non-coplanarity between the hadron and parton planes

J.C. Peng et al, PLB 758,384 (2016)

Lam-Tung relation and polarization



ATLAS, JHEP 08 (2016) 159



R. Gauld et al, JHEP 2017, N3LO

These results are confirmed by CMS (PLB750, 154 (2015)) and LHCb (PRL 129 (2022) 091801) collaborations

The discrepancy with the SM prediction
NP effects or non-perturbative effects ?

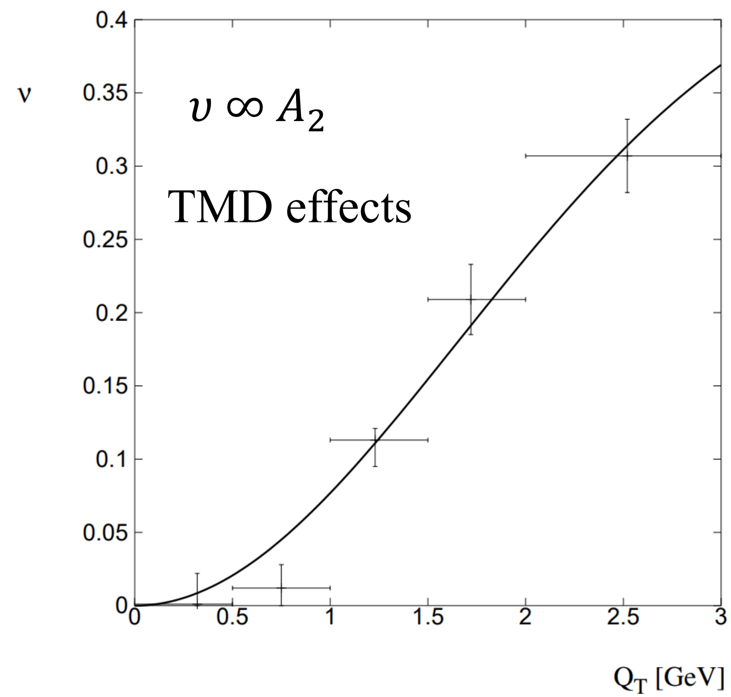
Boer-Mulders function

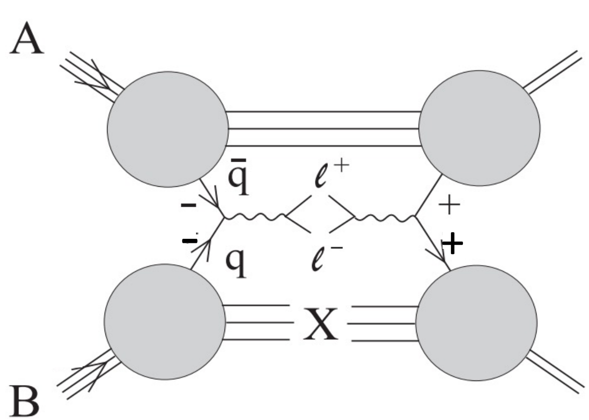
The $cos2\phi$ dependence can be induced by the Boer-Mulders function

Leading Quark TMDPDFs — Nucleon Spin, Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^{\perp}$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^{\perp}$ = Worm-gear
T	$f_{1T}^{\perp}$ = Sivers	$g_{1T}^{\perp}$ = Worm-gear	h_1 = Transversity; $h_{1T}^{\perp}$ = Pretzelosity

Boer, PRD 60 (1999) 014012





Transversely polarized quark

Lam-Tung relation and NP

Center-of-mass frame:

$$\frac{d\sigma}{d\Omega} = a\cos\hat{\theta} + b\cos^2\hat{\theta} + c\cos^3\hat{\theta} + d$$

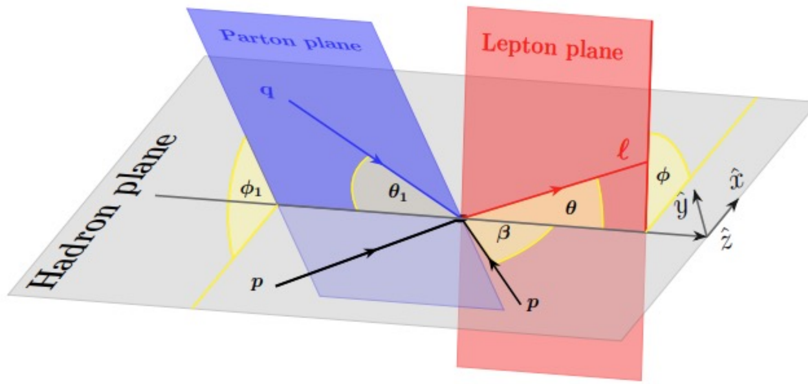
$$\cos\hat{\theta} = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi-\phi_1)$$

$$A_0 = \left\langle \frac{2(d-b)+4b\sin^2\theta_1}{b+3d} \right\rangle,$$

$$A_2 = \left\langle \frac{4b\sin^2\theta_1\cos 2\phi_1}{b+3d} \right\rangle.$$

$$\langle P_l(\cos\theta,\phi)\rangle = \frac{\int P_l(\cos\theta,\phi)\, d\sigma d\cos\theta d\phi}{\int d\sigma d\cos\theta d\phi}$$

J.C. Peng et al, PLB 758,384 (2016)

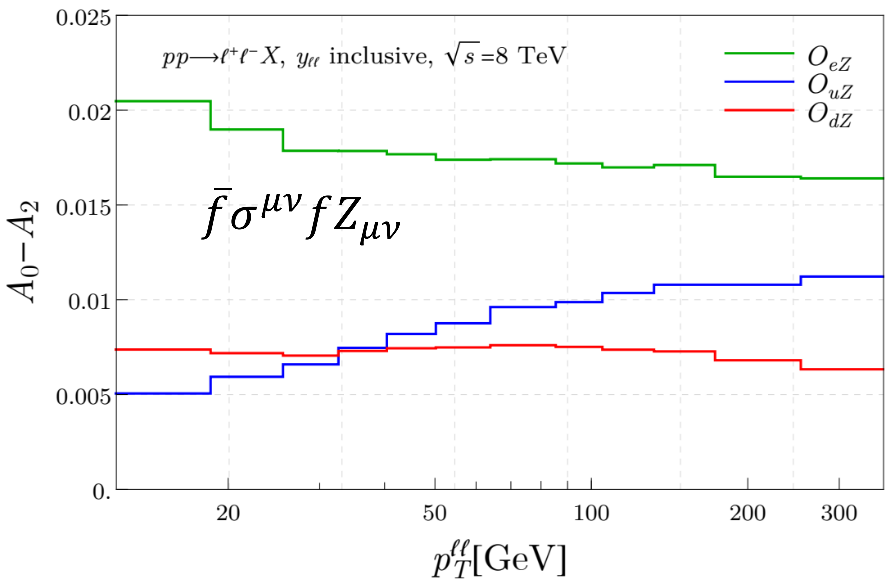


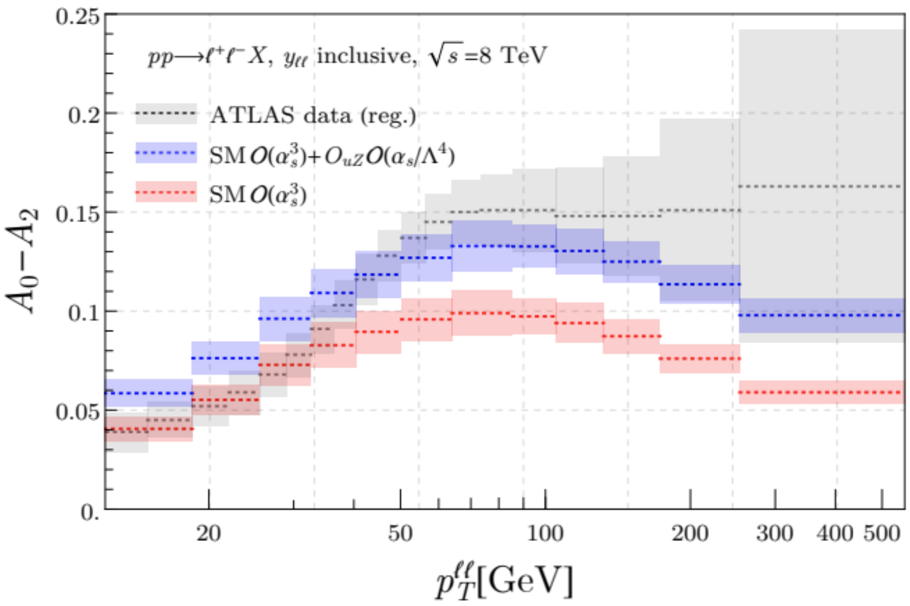
$A_0 \neq A_2$

- Coplanarity case: $b \neq d$
- Non-coplanarity case: $\phi_1 \neq 0$, NNLO and beyond or by the nonperturbative effects

Xu Li, Bin Yan, C.-P. Yuan, PRD 111 (2025) 073007

Lam-Tung relation and polarization

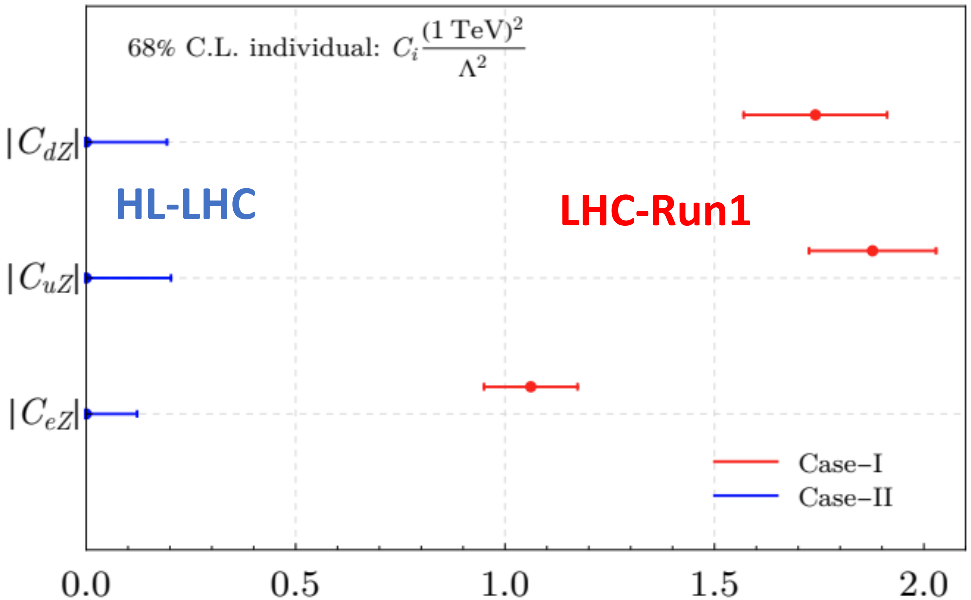


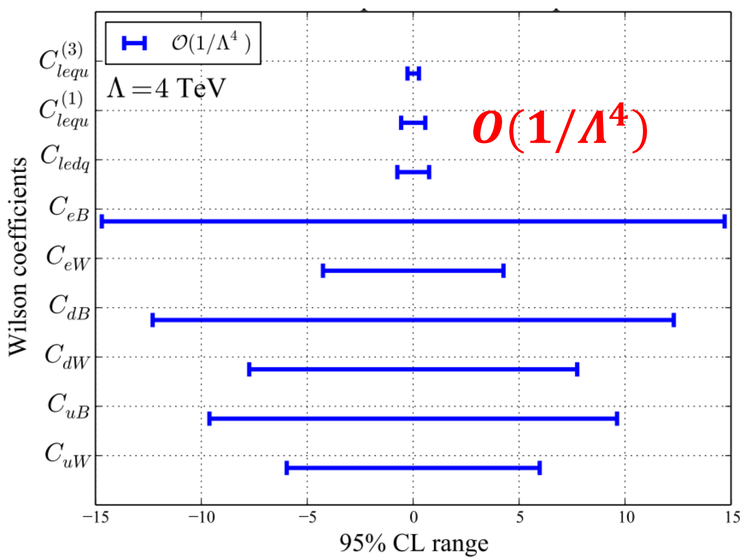


- The discrepancy in Lam-Tung relation could be explained by electroweak dipole interactions (transversely polarized quark or lepton)
- It could be more significant in high-invariant mass region

Xu Li, Bin Yan, C.-P. Yuan, PRD 111 (2025) 073007

Lam-Tung relation and polarization





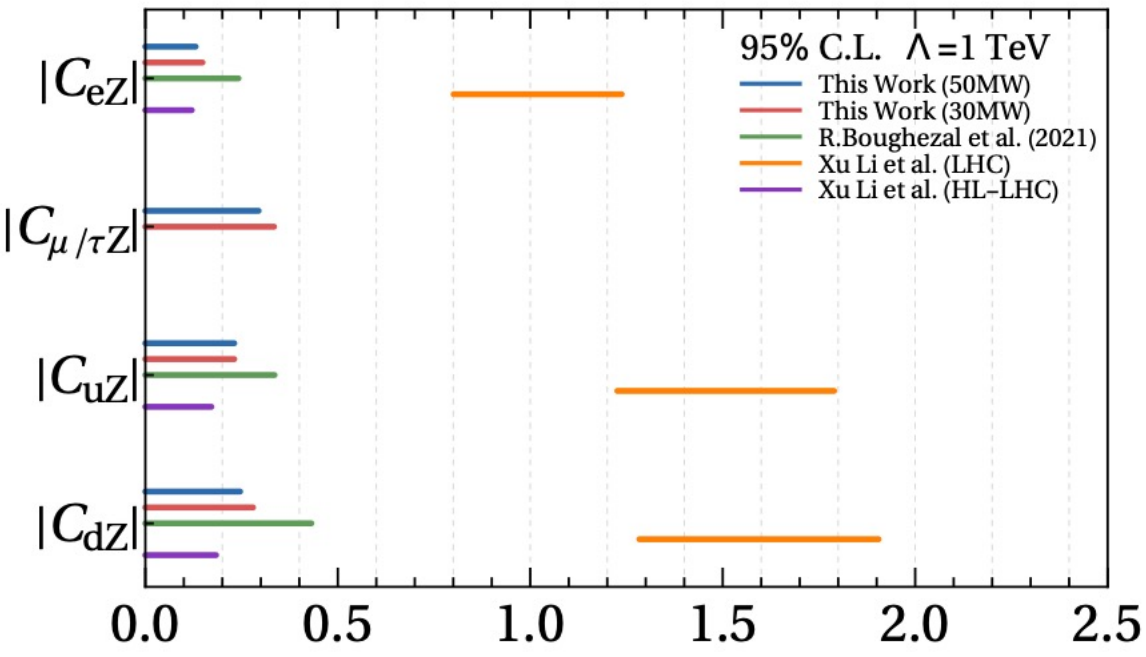
R. Boughezal et al. *Phys.Rev.D* 104 (2021) 9, 095022

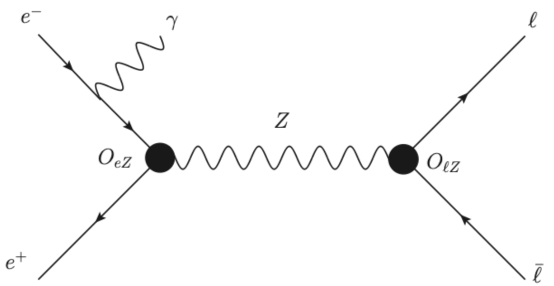
- The accuracy from A0-A2 would be comparable to the results from cross section, but the violation effects will dominantly depend on the dipole interactions.

Xu Li, Bin Yan, C.-P. Yuan, PRD 111 (2025) 073007

How to cross-check this?

- Possible additional non-perturbative effects?
- Both quark and lepton dipole operators contribute to A0-A2: how to separate their effects?
- Probing dipole couplings of heavy-flavors?

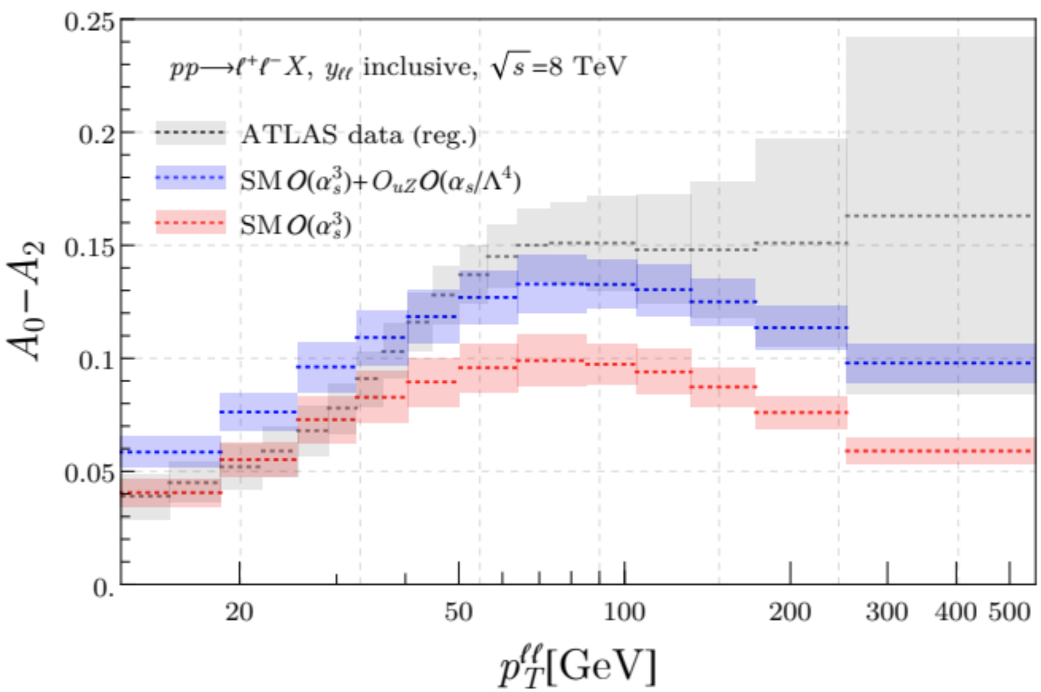




Guanghui Li, Xu Li, Bin Yan,
PLB 870 (2025) 139931

Summary

- The violation of Lam-Tung relation in the Drell-Yan process can be emerged due to the non-coplanarity between the hadron plane and parton plane (NNLO and beyond)
- The discrepancy in ATLAS data could be explained by electroweak dipole interactions in coplanarity case and can be cross-checked at the CEPC



Thank you!