The Higgs Factory – Gravitational Wave Interface

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- *UMass Amherst (on leave)*
- *Caltech*

About MJRM:

Science

Family

Friends

My pronouns: he/him/his
 # MeToo

CEPC International Workshop
Guangzhou, November 8, 2025

Key Themes for This Talk

- *The possibility of primordial gravitational waves generated from various particle physics dynamics has become an exciting area of exploration*
- *There exist many creative ideas for novel phenomena and dynamics that could have generated GW*
- *Realizing which, if any, of these ideas was realized in nature requires input from additional observables and performing the most rigorous theoretical calculations*
- *The electroweak phase transition provides a unique "laboratory" for testing our theoretical methods and ideas, with LHC and Higgs factory measurements providing key input*

Questions

- *Did the early universe undergo symmetry breaking through condensation ?*

- *What are the experimental signatures of this process ?*

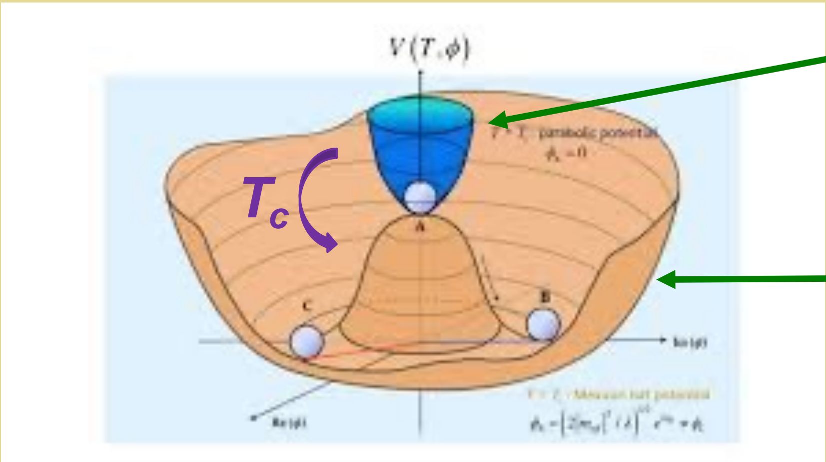
- *What theoretical challenges must be met to address this question ?*

Outline

I. Cosmic Phase Transitions & GW

Spontaneous Symmetry Breaking

Higgs Boson potential energy



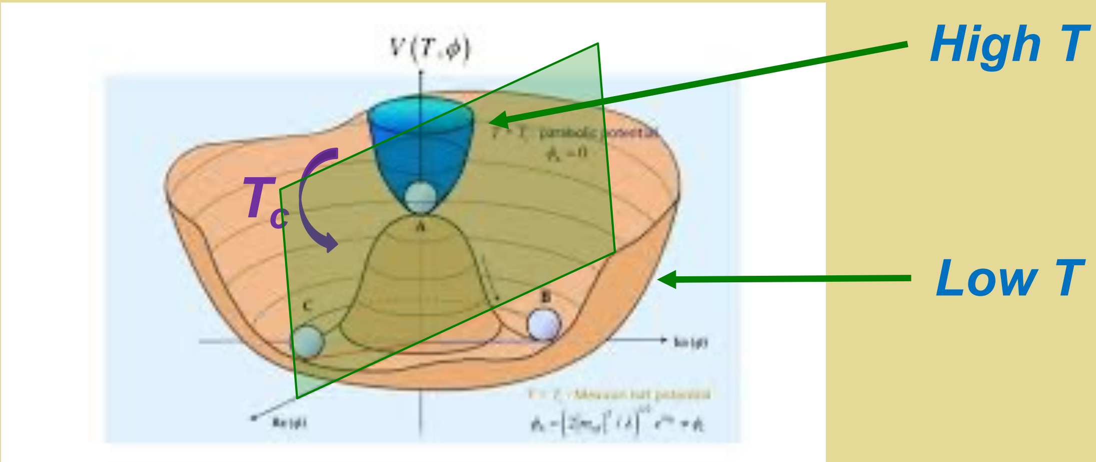
High T

Low T

How did this transition occur ?

Spontaneous Symmetry Breaking

Higgs Boson potential energy



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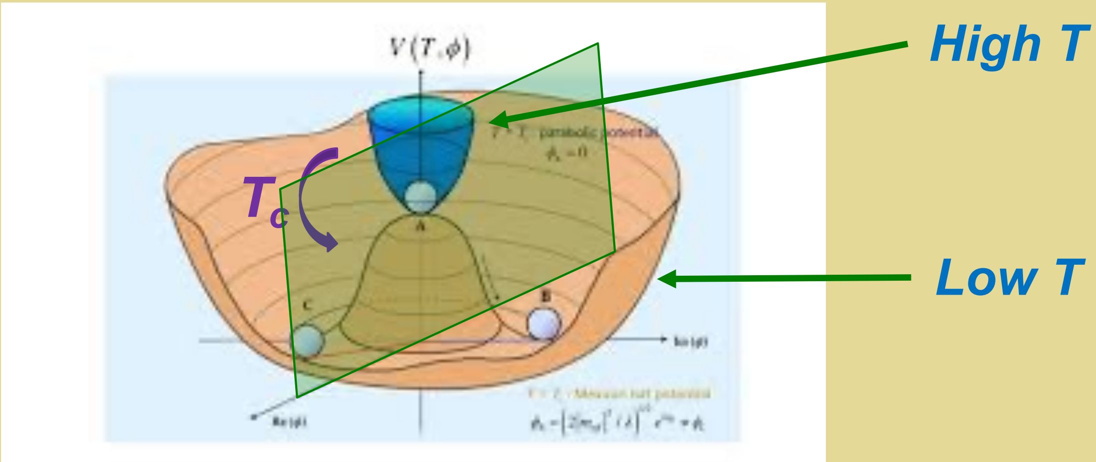

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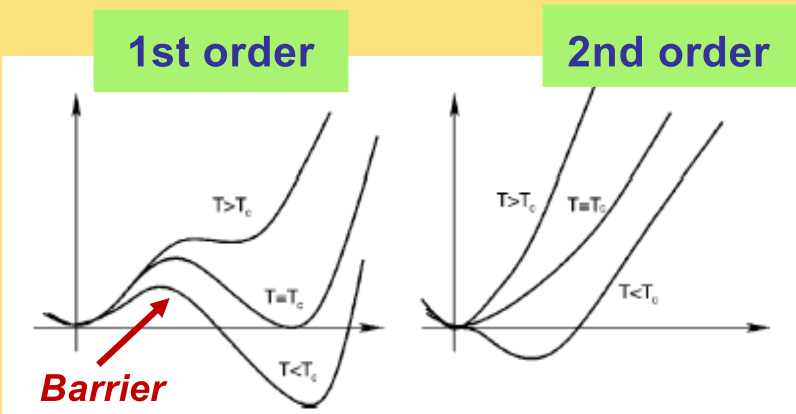
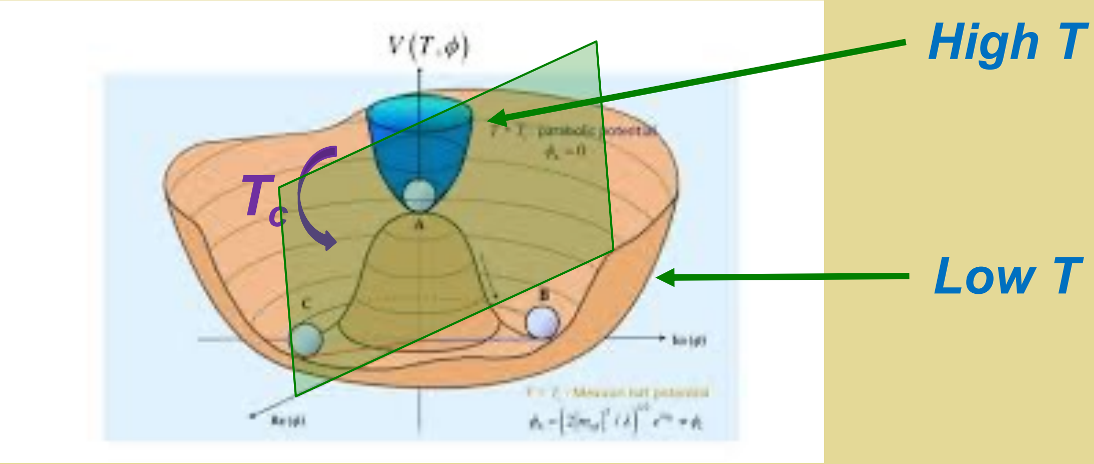


Increasing m_h →

Spontaneous Symmetry Breaking

Higgs Boson potential energy



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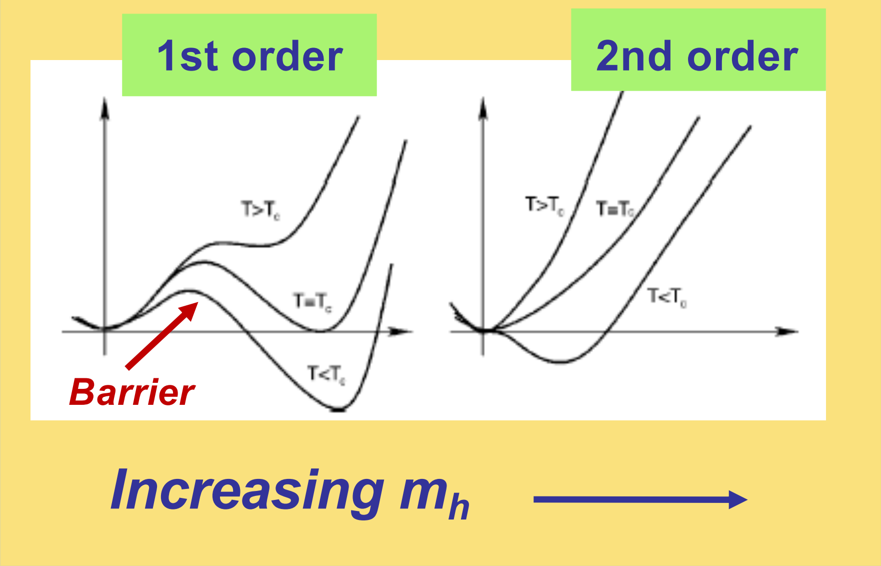


- ***Was there an EW phase transition ?***
- ***Was it 1st order ?***

Phase Transitions

Ehrenfest classification:

1st order PT: discontinuous first derivative of free energy

$$\left(\frac{\partial F}{\partial T}\right)_V = -S \qquad\qquad \left(\frac{\partial G}{\partial T}\right)_P = -S$$

1st order: entropy change

Phase Transitions

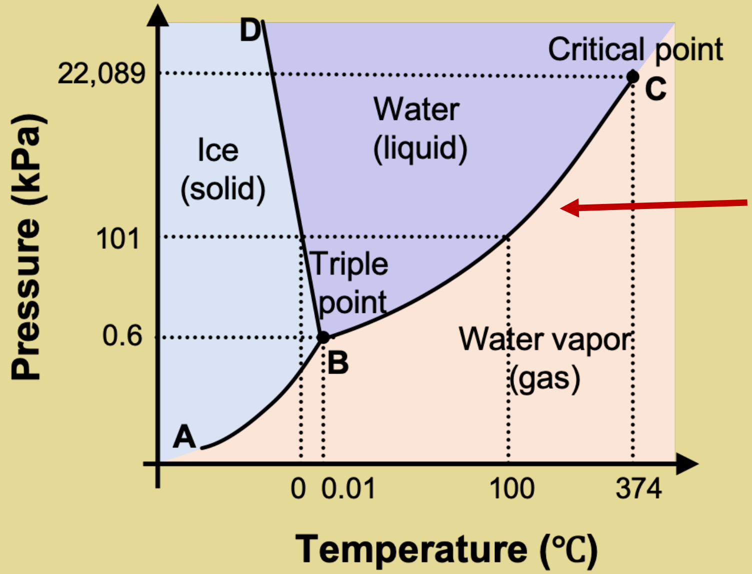
Ehrenfest classification:

1^{st} order PT: discontinuous first derivative of free energy

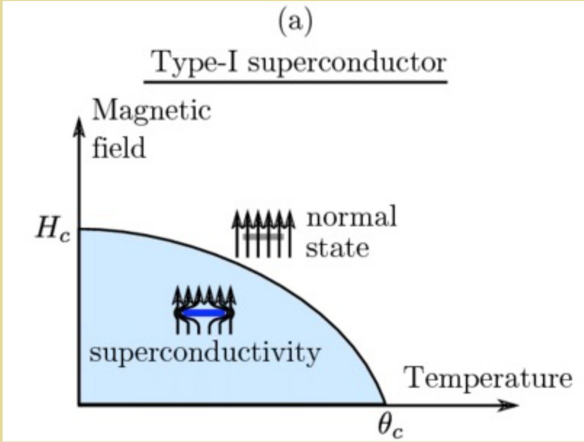
$$\boxed{\left(\frac{\partial F}{\partial T}\right)_V = -S \qquad\qquad \left(\frac{\partial G}{\partial T}\right)_P = -S}$$

- n^{th} order PT: discontinuous n^{th} derivative of free energy
- smooth “crossover” transition: no discontinuities in derivatives of the free energy

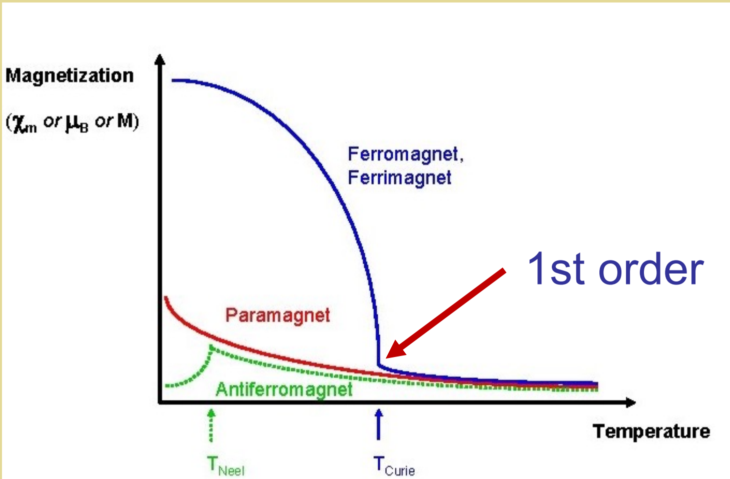
Phase Transitions



1st order

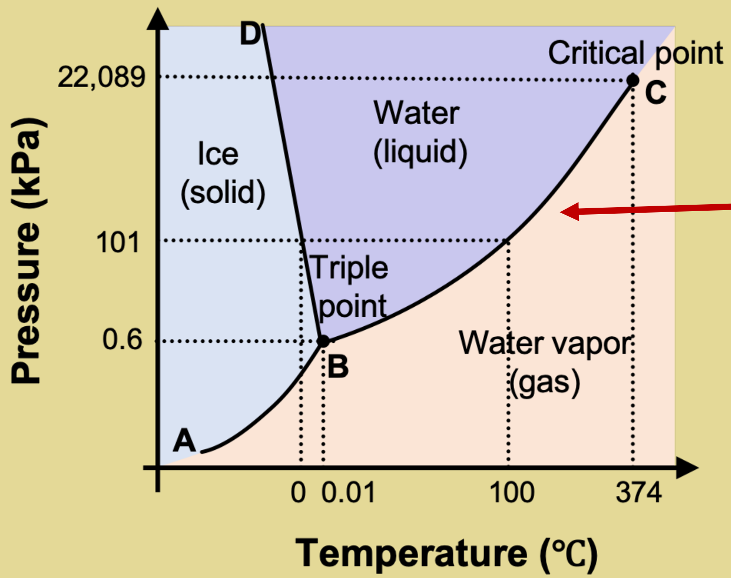


Order parameter



1st order

Phase Transitions



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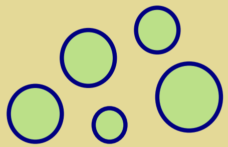

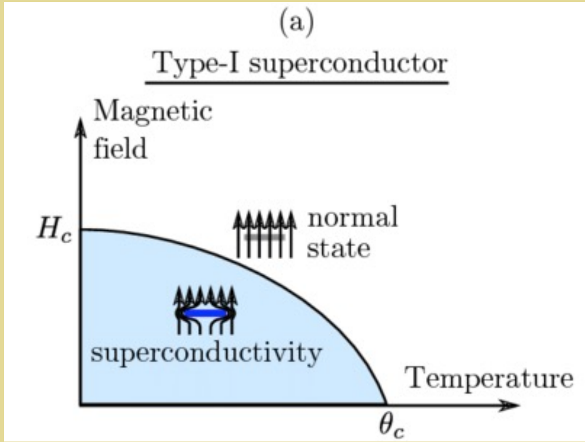

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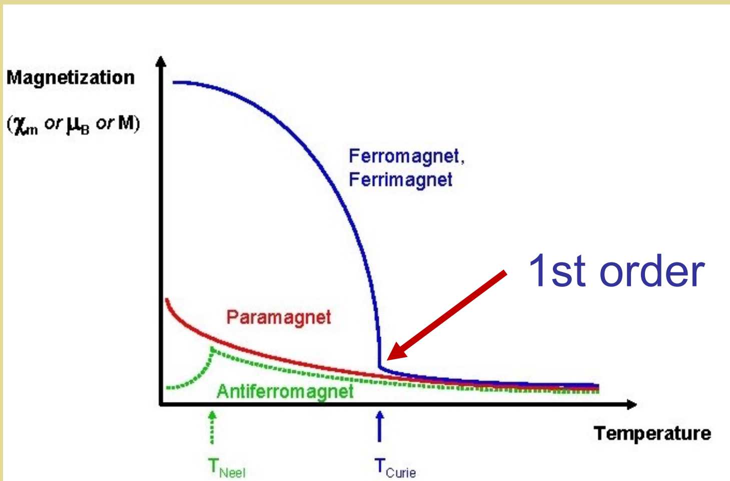


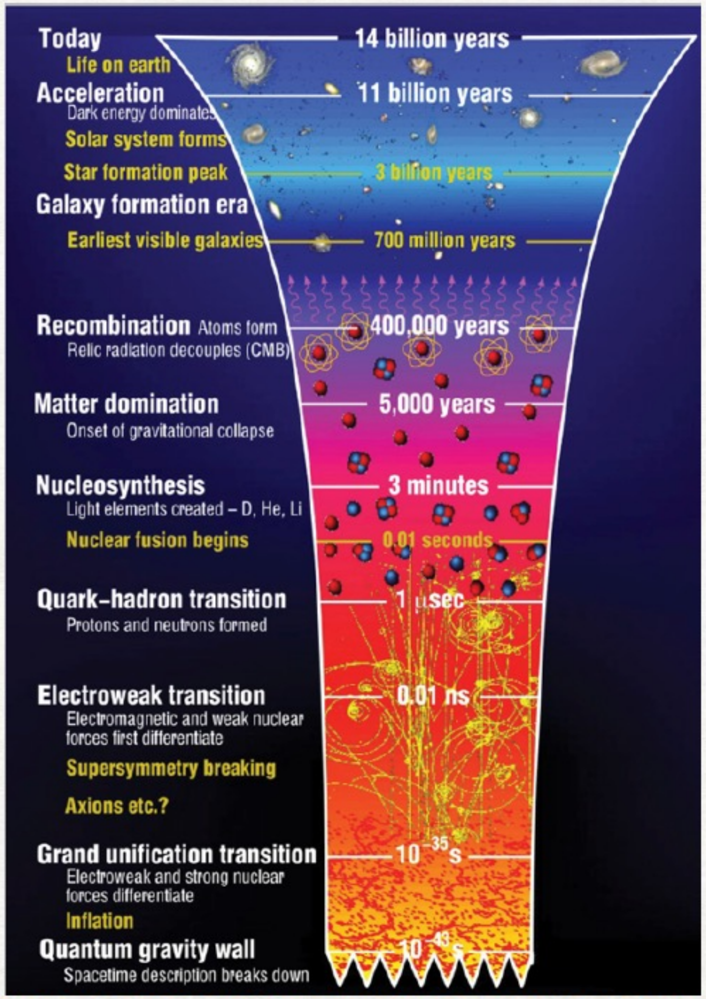
1st order

Thermal History of Symmetry Breaking

QCD →

Higgs →

Inflation →



Credit: PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH 17, 013104 (2021)

Thanks: Yanda Wu SPCS '25

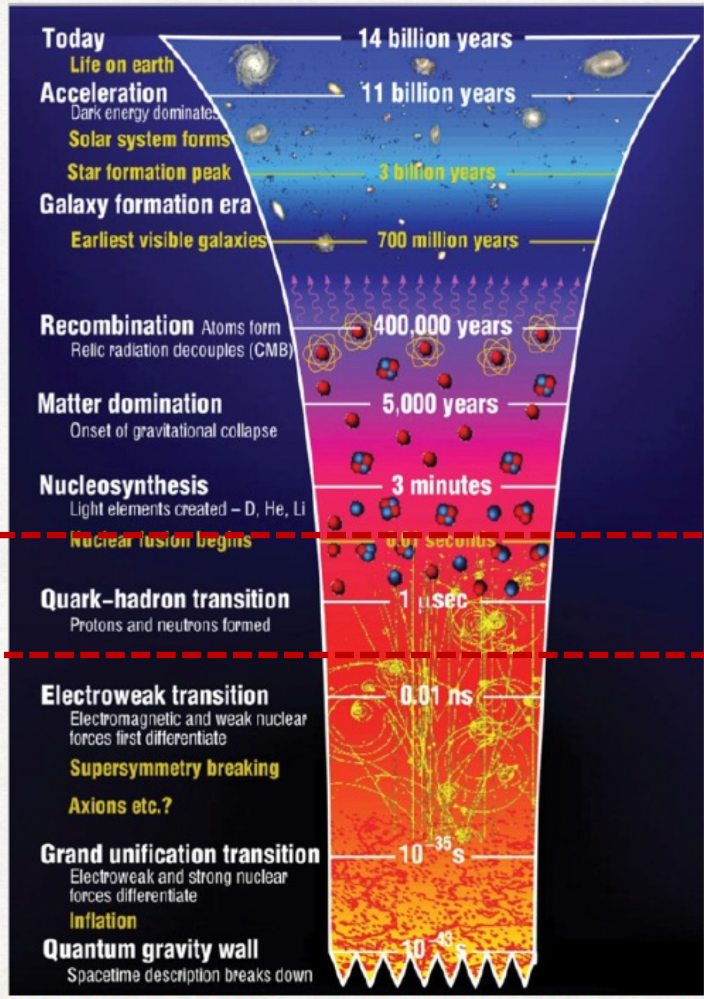
Thermal History of Symmetry Breaking

- ***Free quarks & gluons → hadrons***
- ***Chiral sym breaking → pions…***

QCD →

Higgs →

Inflation →



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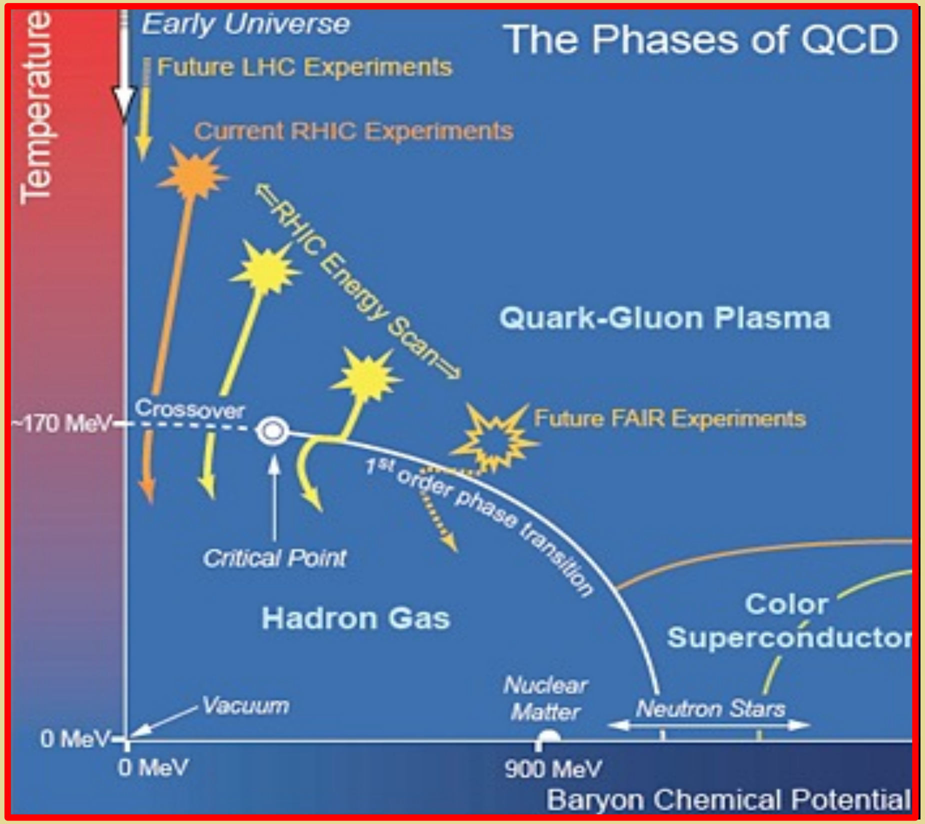


QCD Phase Diagram → EW Theory Analog ?
"Dark Sector" ? Cosmological consequences ?

Thermal History of Symmetry Breaking

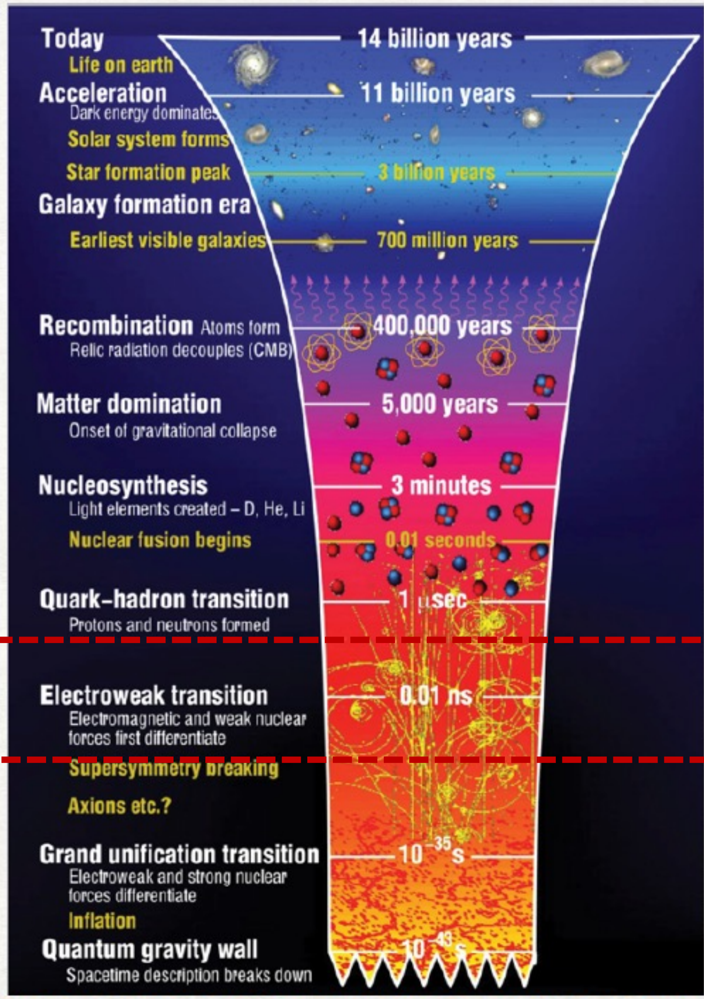
- *Elementary particle masses*

QCD

Higgs

$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

Inflation



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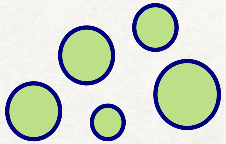
Thermal History of Symmetry Breaking

- ***Elementary particle masses***
- ***Matter-antimatter asym ?***



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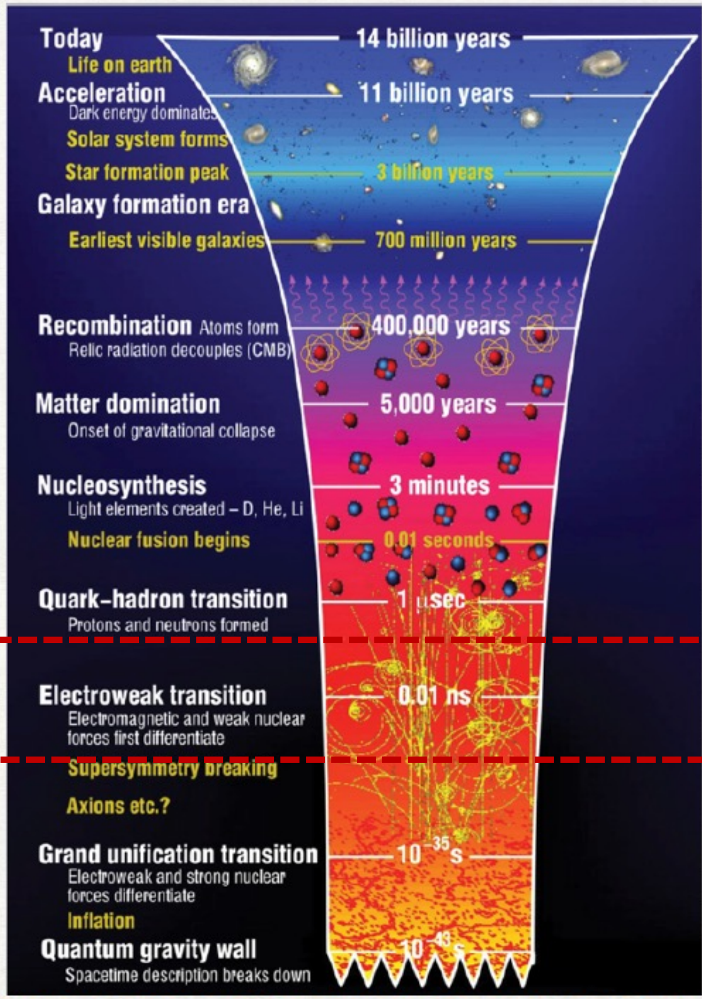

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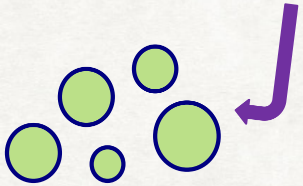

CPV

QCD

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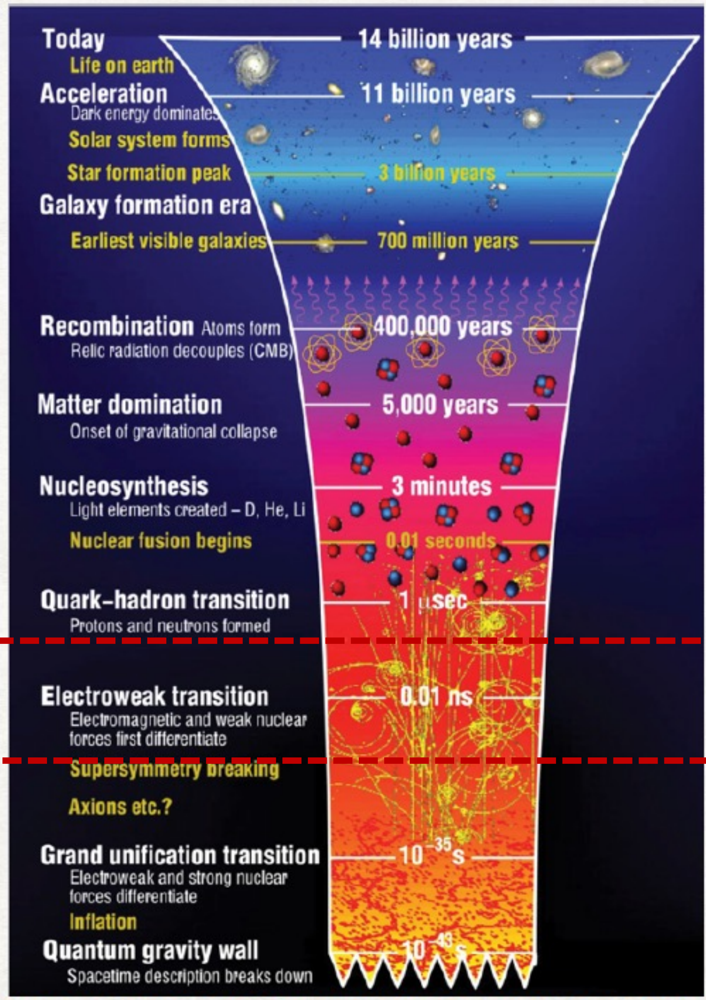

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Thermal History of Symmetry Breaking

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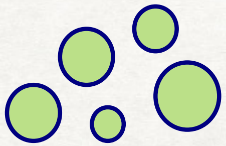


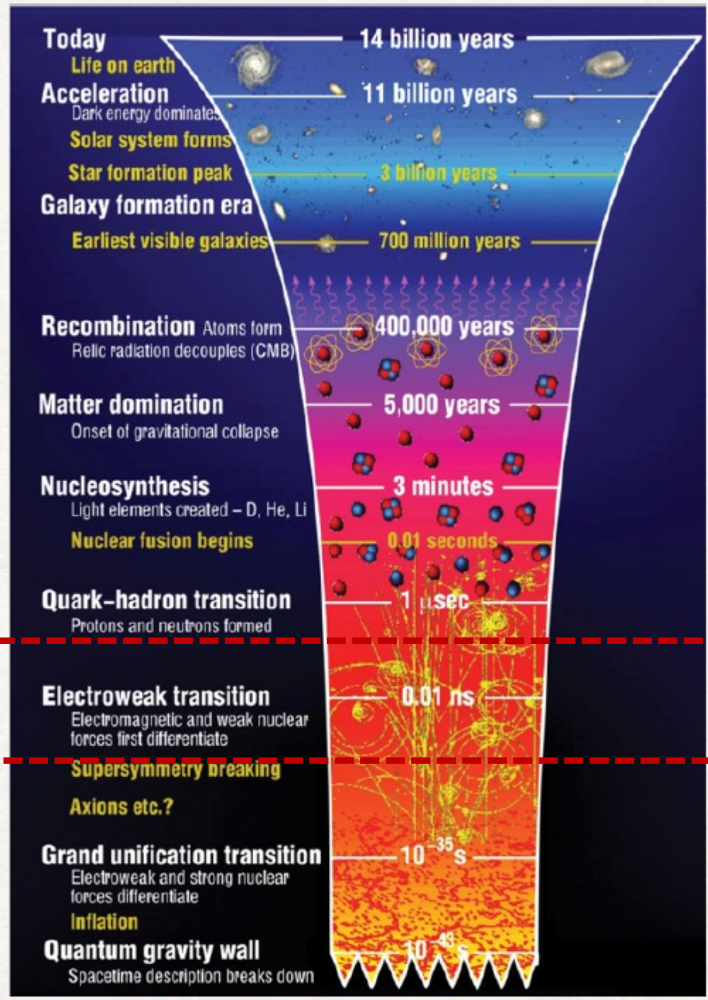
→ *GW*

QCD →

Higgs →

$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

Inflation →



Credit: PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH 17, 013104 (2021)

Thanks: Yanda Wu SPCS '25

Thermal History of Symmetry Breaking

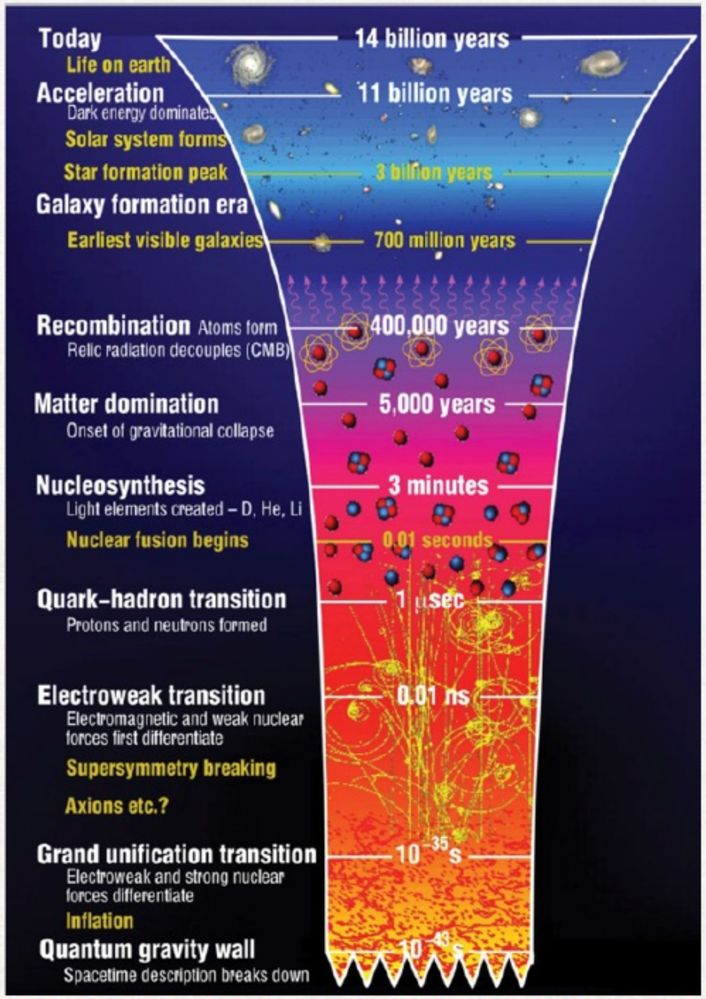
- ***Neutrino masses ?***
- ***Cosmic strings ?***
- ***Dark matter ?***
- ***Primordial black holes ?***
- ***Hubble tension ?***
- ***…***

Other ?

QCD

Higgs

Inflation



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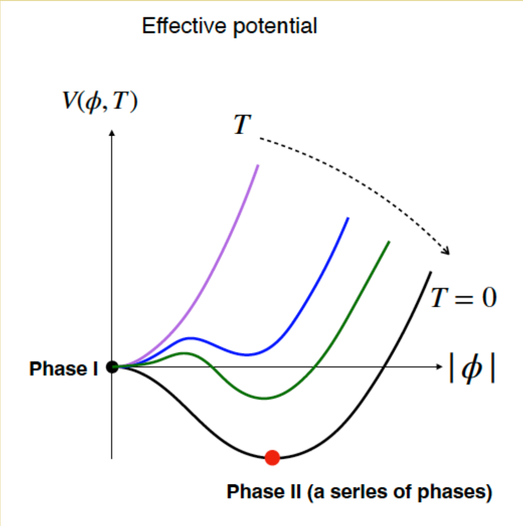
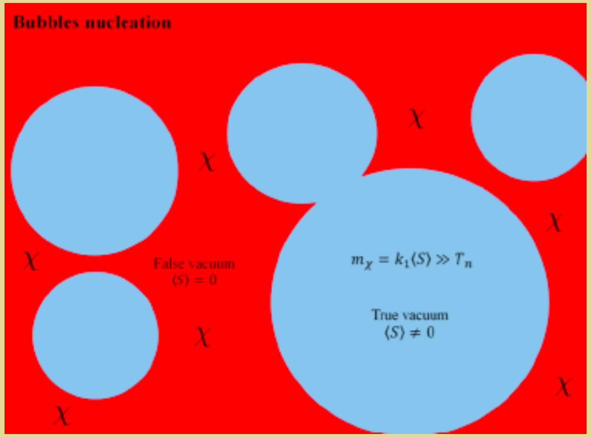
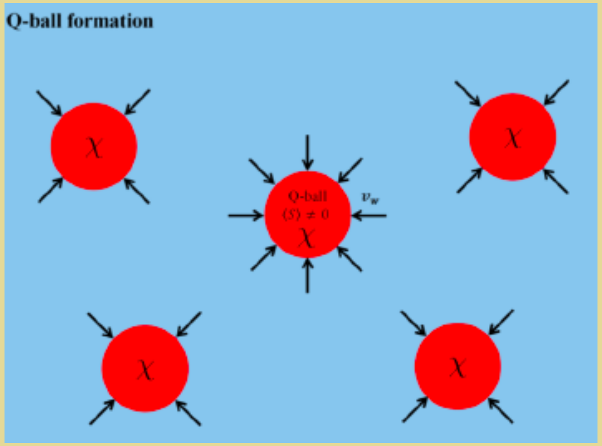
Particle Physics & GW: Spontaneous Symmetry Breaking



Thanks: Ye-Ling Zhou CPCS '24





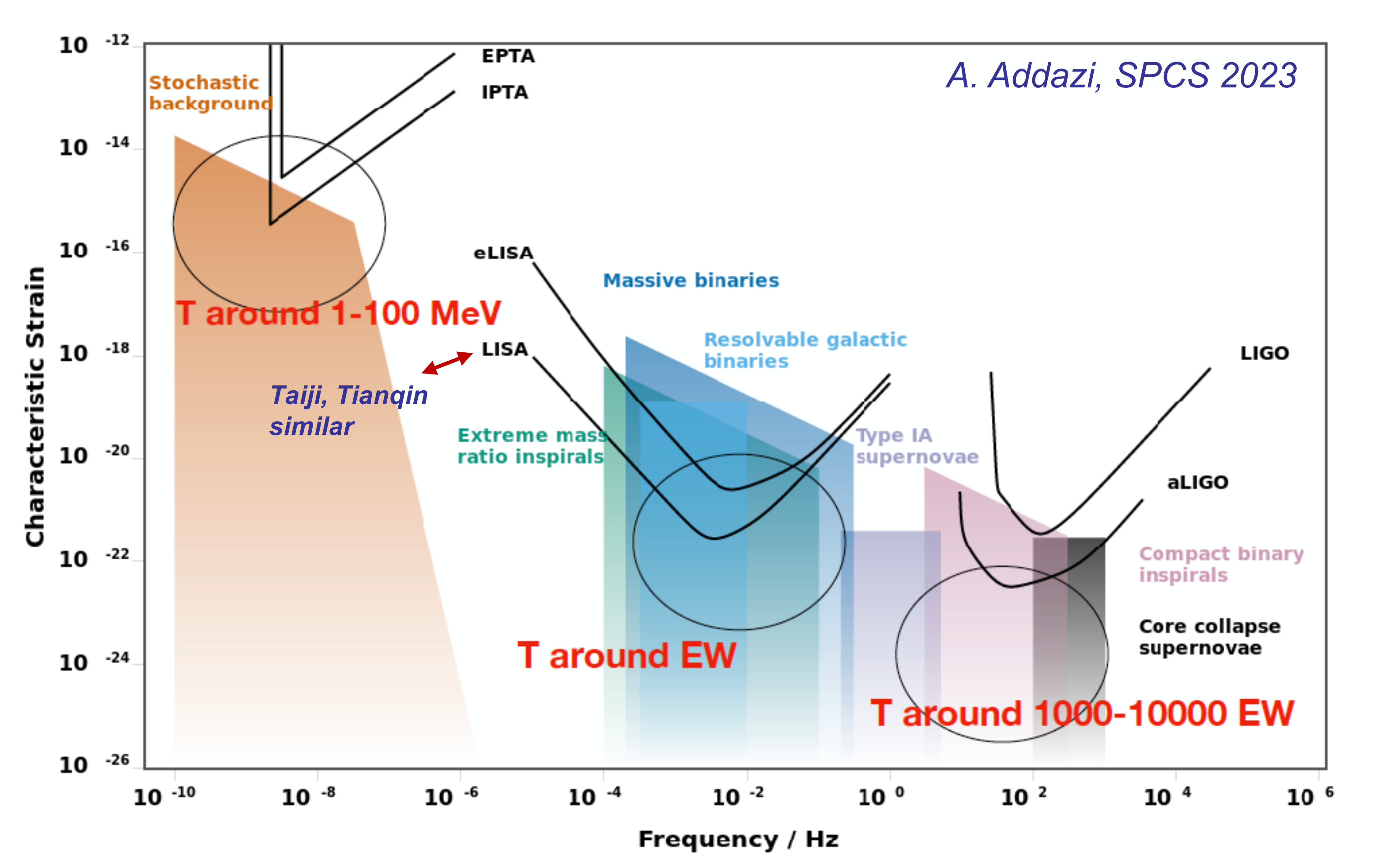
Thanks: Fa Peng Huang CPCS '24

Trapping of heavy particles outside bubbles

→ Q-Ball dark matter

GW from bubble collisions

Gravitational Waves



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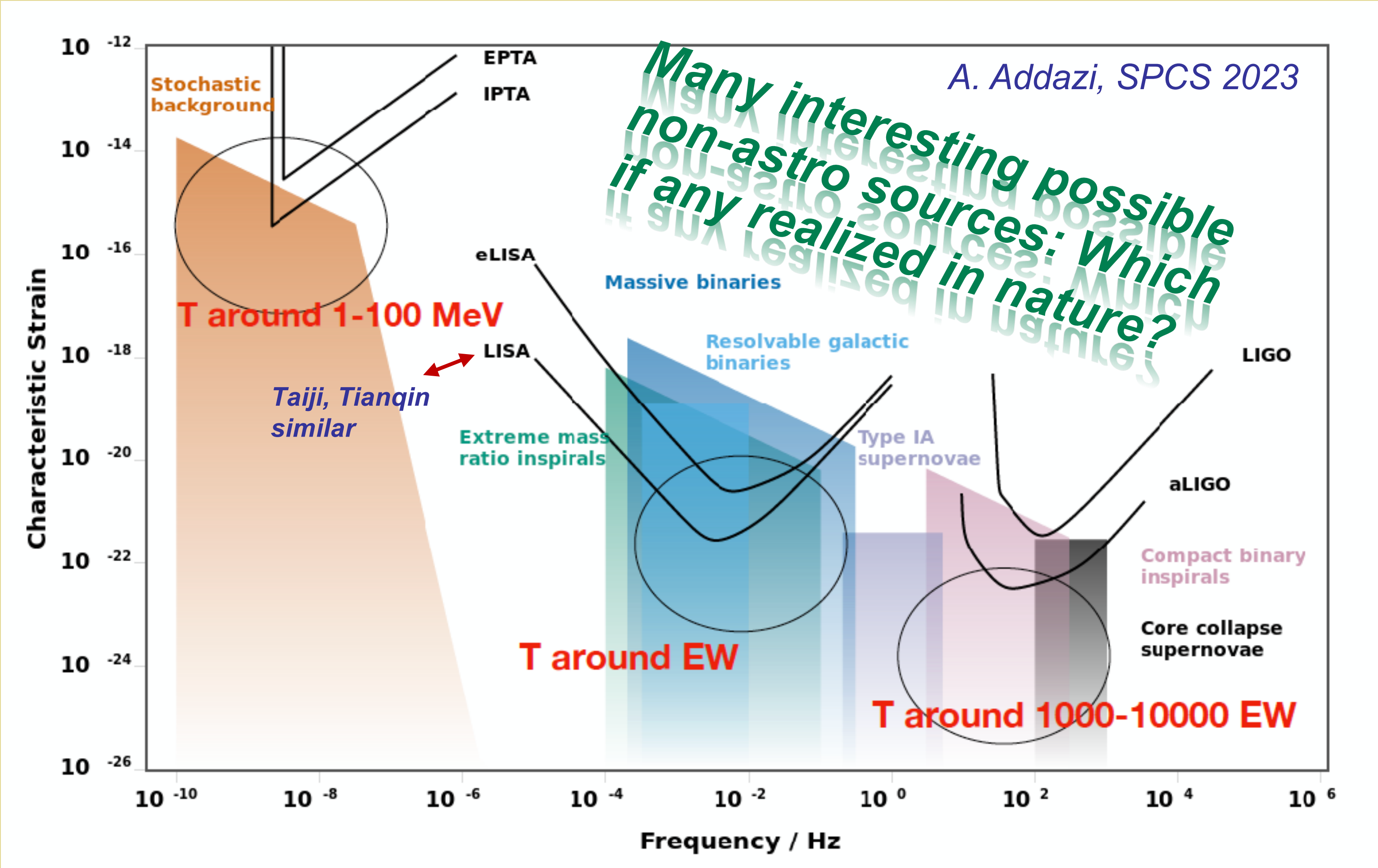

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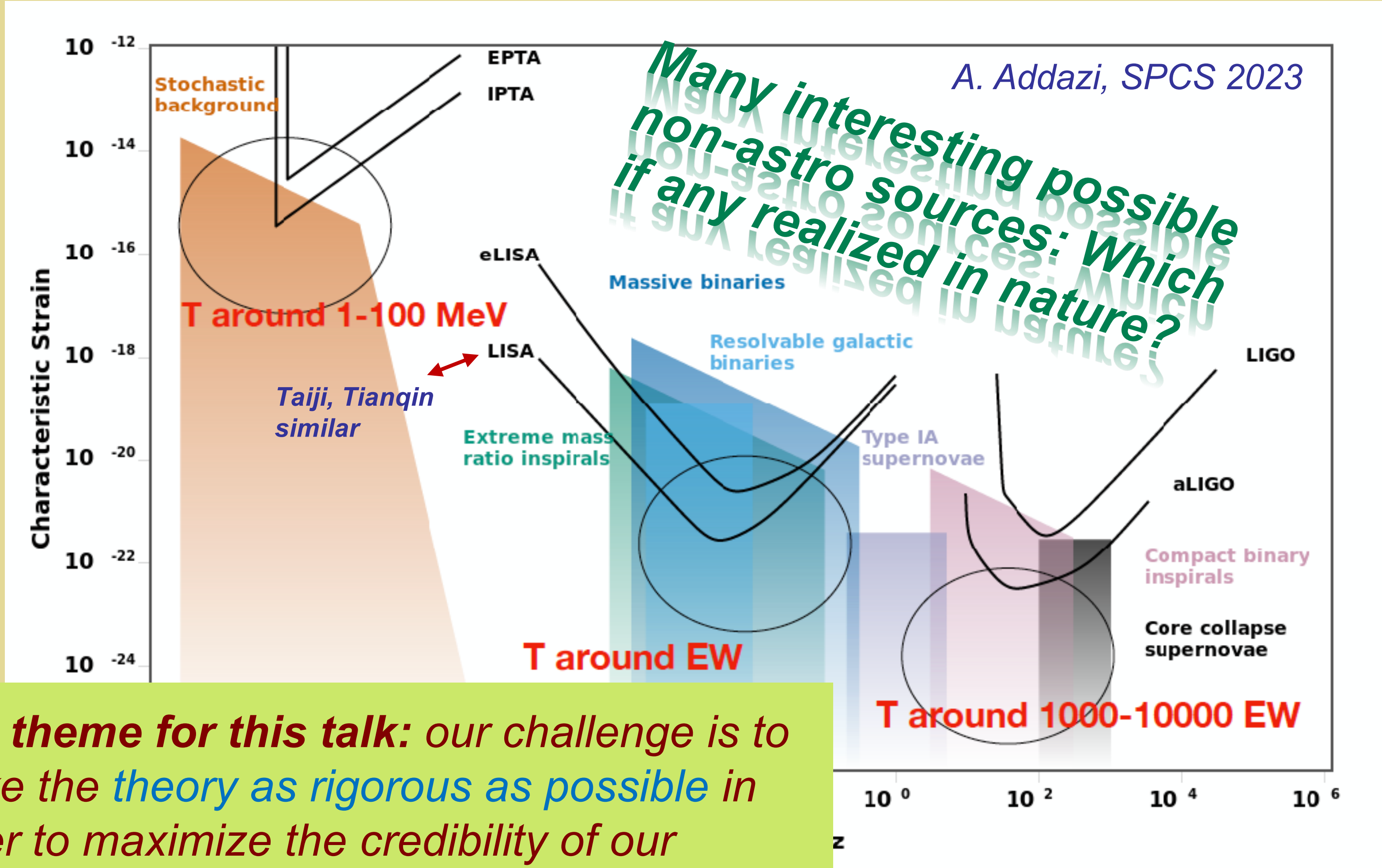
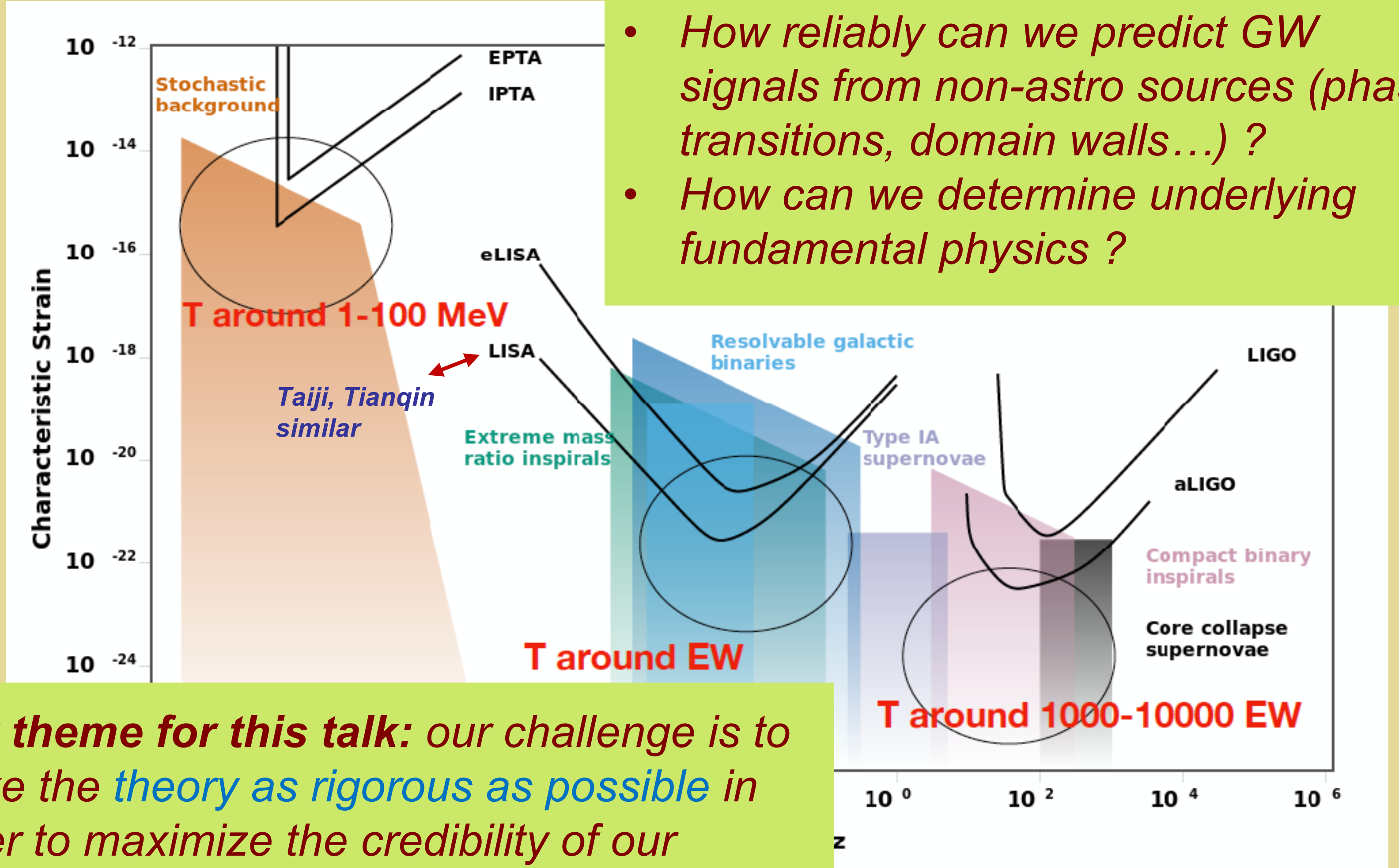


***Key theme for this talk:** our challenge is to make the theory as rigorous as possible in order to maximize the credibility of our creative ideas and enable a robust confrontation with experiment to address this question*

Gravitational Waves

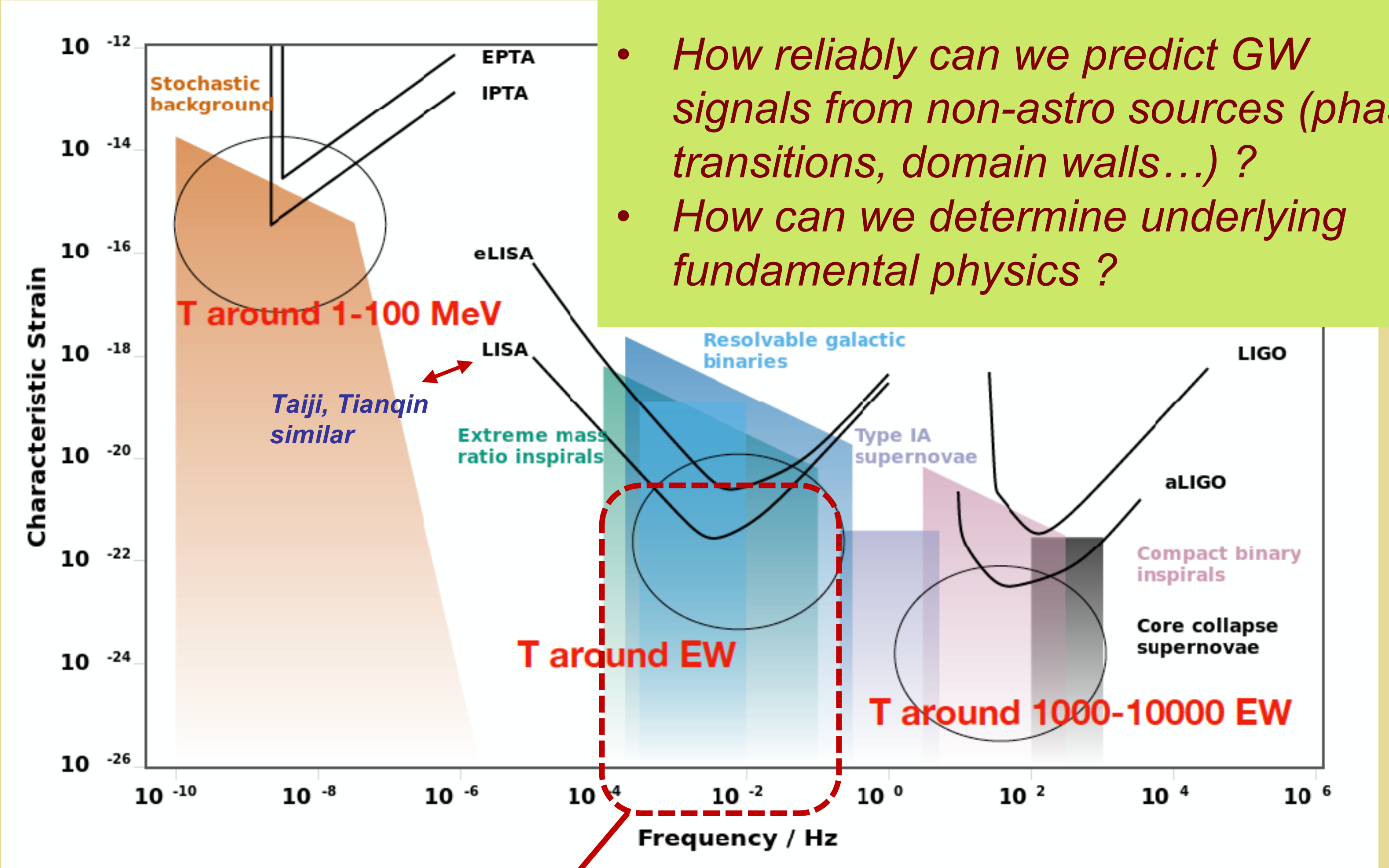


- *How reliably can we predict GW signals from non-astro sources (phase transitions, domain walls…) ?*
- *How can we determine underlying fundamental physics ?*

Key theme for this talk: *our challenge is to make the theory as rigorous as possible in order to maximize the credibility of our creative ideas and enable a robust confrontation with experiment to address this question*

GW: Electroweak Phase Transition

- *How reliably can we predict GW signals from non-astro sources (phase transitions, domain walls…) ?*
- *How can we determine underlying fundamental physics ?*



EWPT laboratory for GW micro-physics: *colliders can probe particle physics responsible for non-astro GW sources → test our framework for GW microphysics at other scales*

II. Theoretical Challenges & Progress

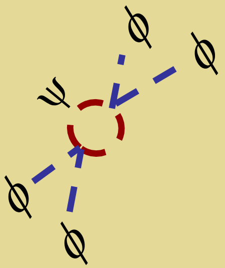
Robust assessment of early universe symmetry-breaking and confrontation between theory and experiment faces significant quantum field theory challenges

QFT Robustness

- ***IR Problem:*** ***limits of perturbation theory***
- ***Nucleation @ finite T:*** ***gauge invariance***
- ***Wall velocity:*** ***microphysics & fluid dynamics***

QFT Robustness

- **IR Problem:** *limits of perturbation theory*

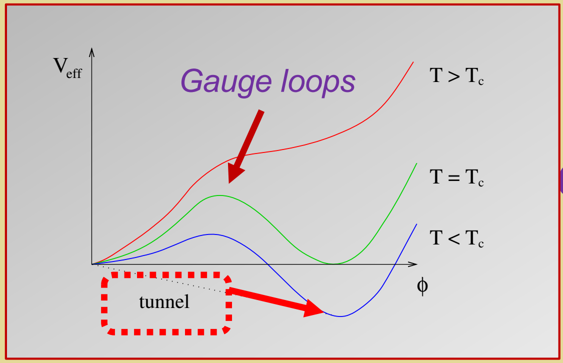


  - *Enhanced low-momentum finite T bosonic loop contributions can break the perturbative expansion → pert theory can't determine first order / crossover boundary*
  - *Combination of lattice computations & thermal EFT essential → 2005.11332, 2203.05889, 2405.01191, 2409.17554*

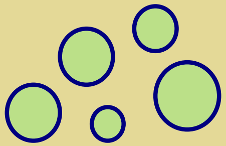
- ***Nucleation @ finite T:*** *gauge invariance*

- ***Wall velocity:*** *microphysics & fluid dynamics*

QFT Robustness



em: limits of perturbation theory

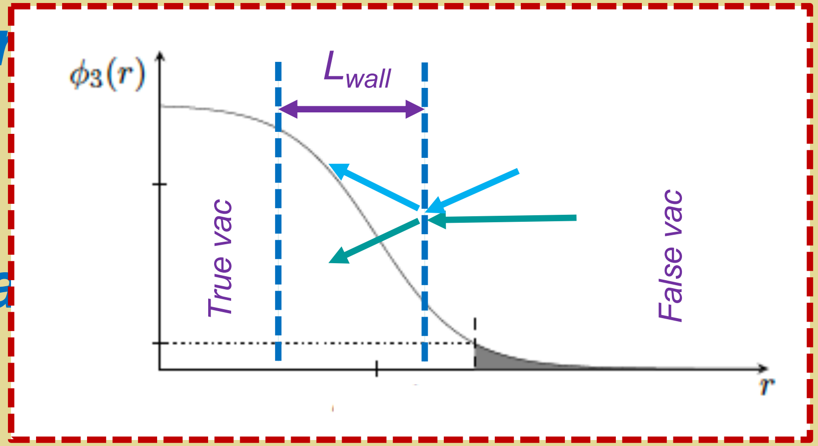


- **Nucleation @ finite T: gauge invariance**
  - ***Radiative (gauge sector loops) potential barrier is gauge-dependent → conventional nucleation rate computation not theoretically robust***
  - ***Reorganization of perturbative expansion yields a gauge-invariant computation → 2112.05472, 2112.08912***
- ***Wall velocity: microphysics & fluid dynamics***

QFT Robustness

- *IR Problem: limits of per*

- *Nucleation @ finite T: ga*



- ***Wall velocity:*** ***microphysics & fluid dynamics***
  - ***Interplay between driving force (vacuum energy difference) and friction pressure (particle-wall interactions) decisive for obtaining v_w → decisive for GW probe sensitivity***
  - ***Rigorous treatment of p_{NORMAL} non-conservation in particle-wall interactions via Kadanoff-Baym equations → 2504.13724***

III. The Electroweak Phase Transition

Was There and Electroweak Phase Transition

- *Higgs discovery $\rightarrow$ What was the thermal history of EWSB ?*

- *Baryogenesis $\rightarrow$ Was the matter-antimatter asymmetry generated in conjunction with EWSB (EW baryogenesis) ?*

- *Gravitational waves $\rightarrow$ If a signal observed in in next generation probes, could a cosmological phase transition be responsible ?*

- *Laboratory for testing phase transition physics $\rightarrow$ How reliable is the theory ?*

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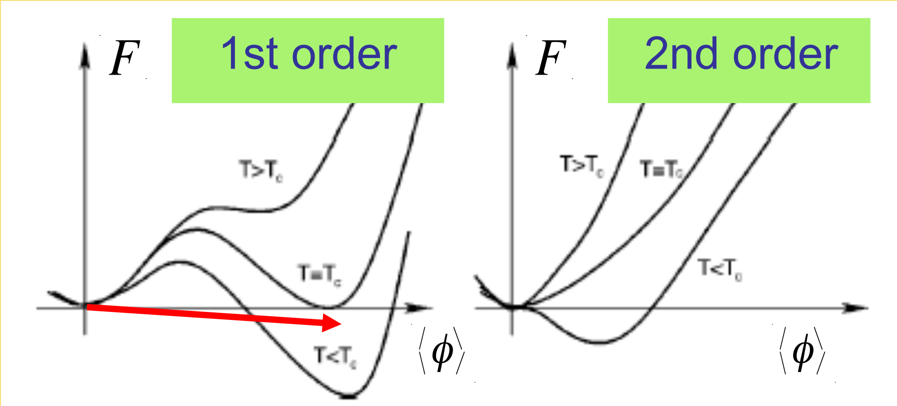
EWSB Transition: St'd Model



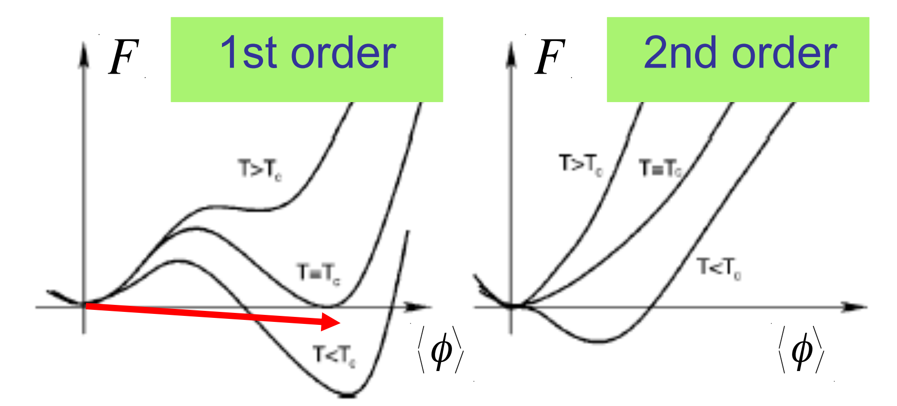
Increasing m_h ⟶

Higgs potential: T=0

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$m_h^2 = 2\lambda\, v^2$$

EWSB Transition: St'd Model



Increasing m_h ⟶

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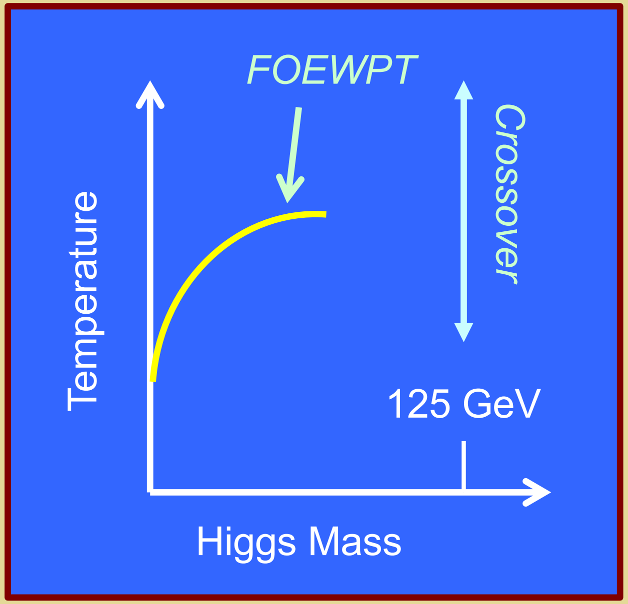
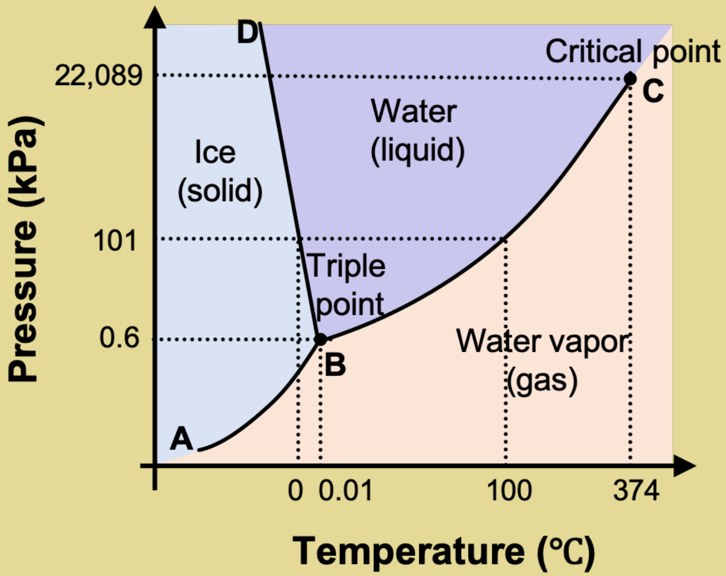
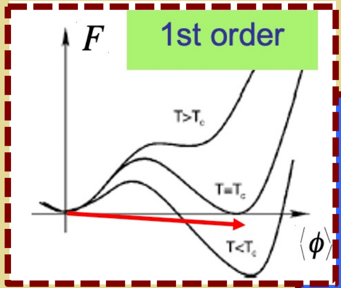


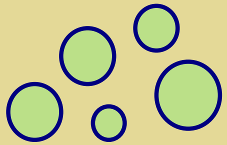
EW Phase Diagram

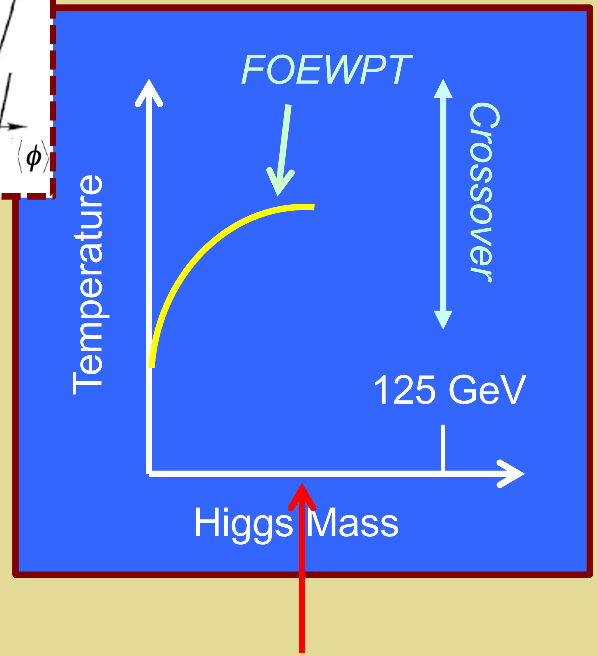
SM EW: Cross over transition

Was There an Electroweak Phase Transition ?









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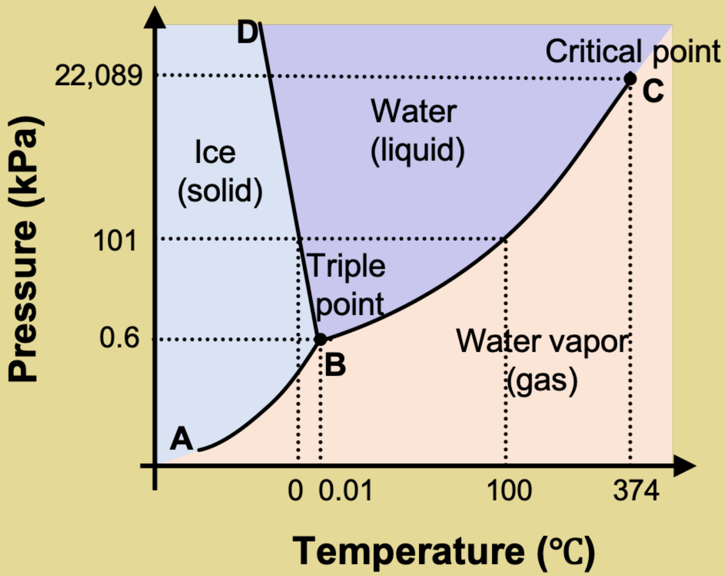

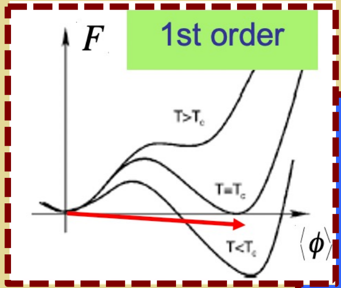

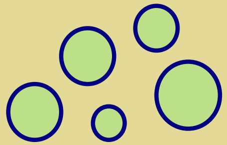

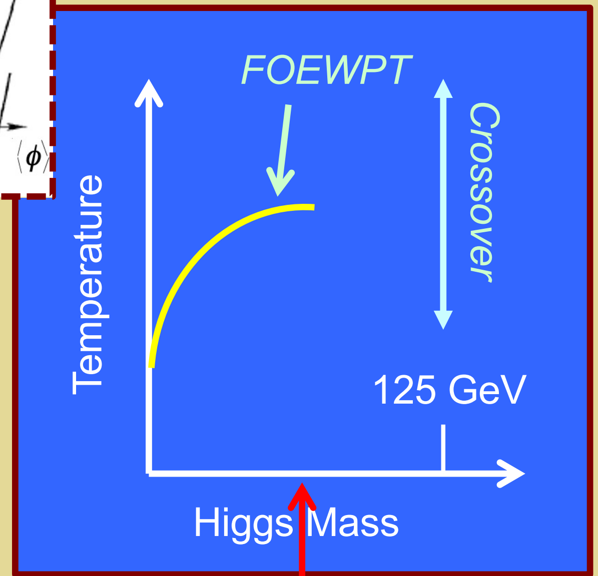

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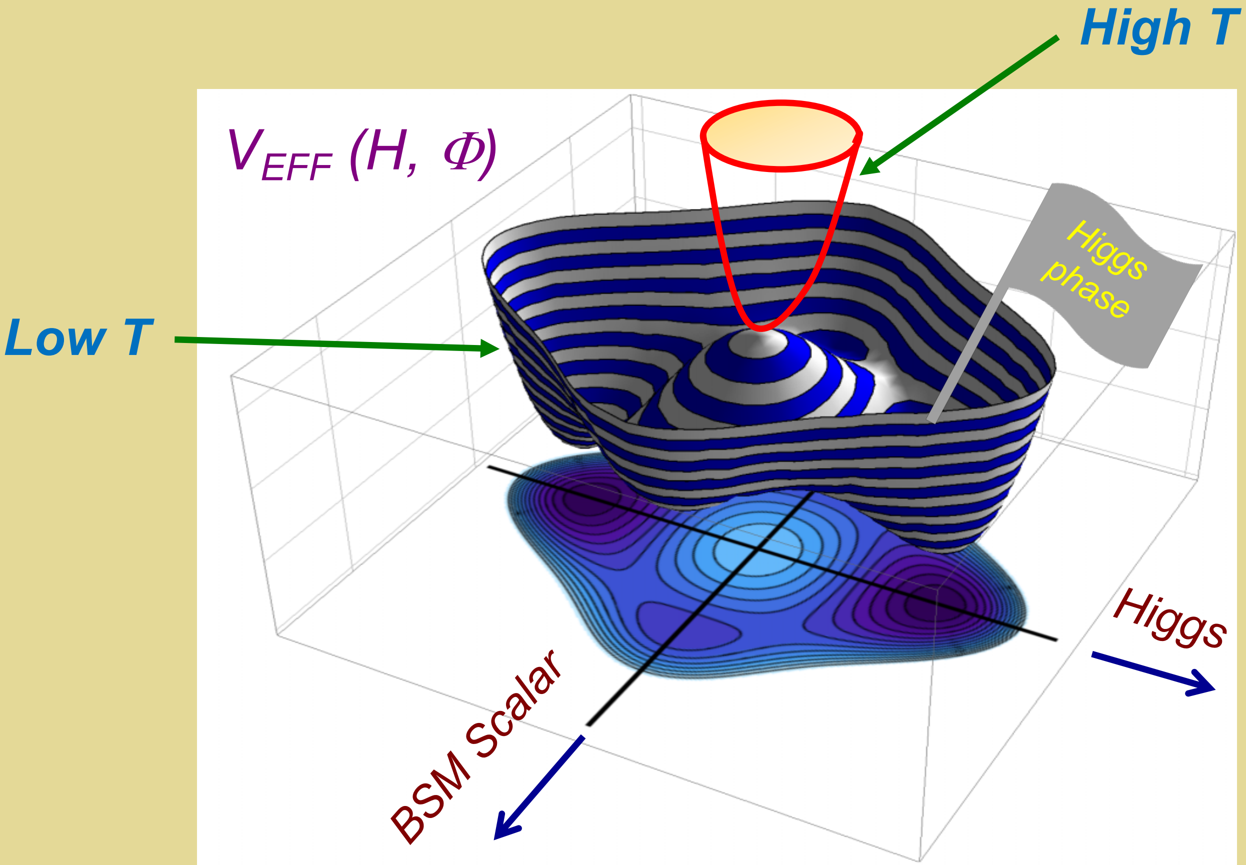
How does this picture change in presence of new TeV scale physics ? What is the phase diagram ? SFOEWPT ?

What Was the EWSB Thermal History ?



Extrema can evolve differently as T evolves → rich possibilities for symmetry breaking

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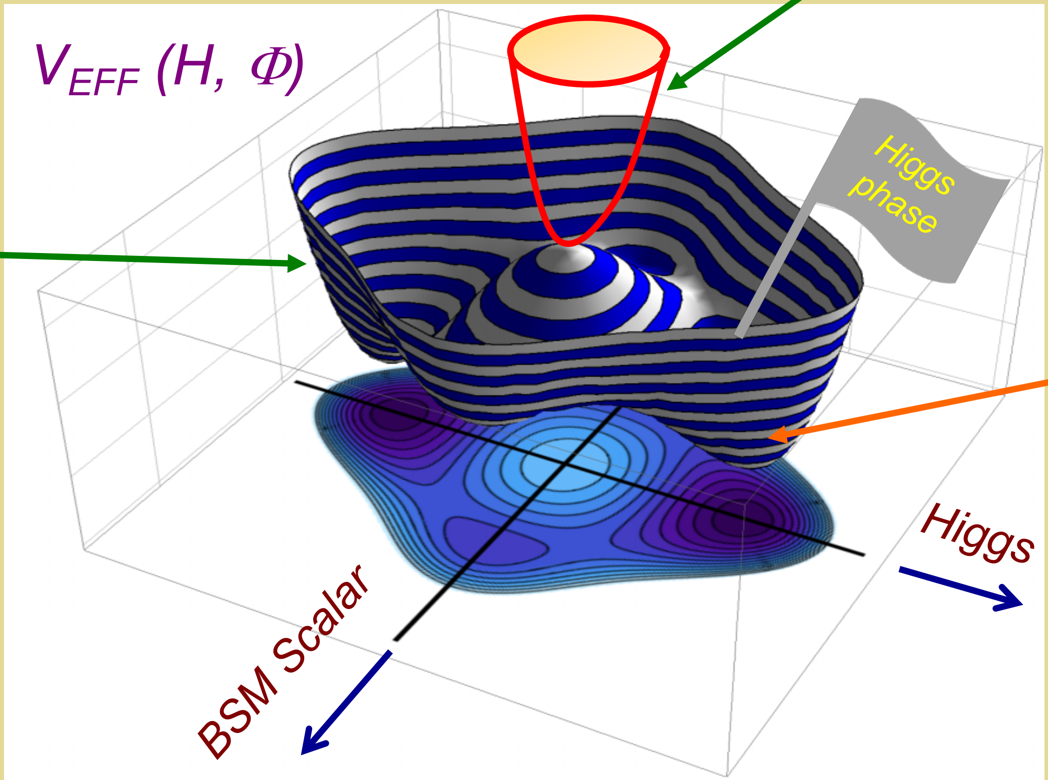

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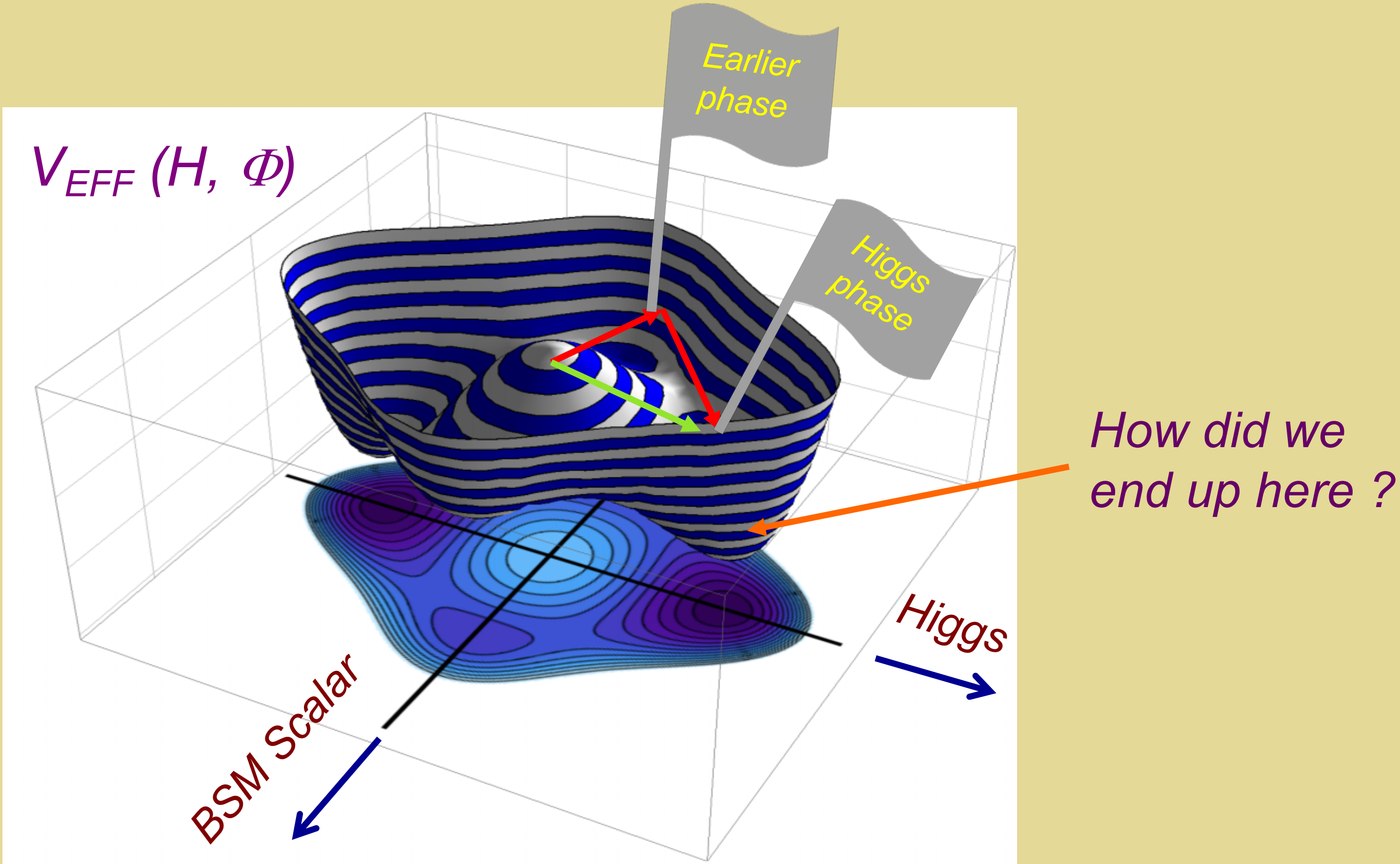

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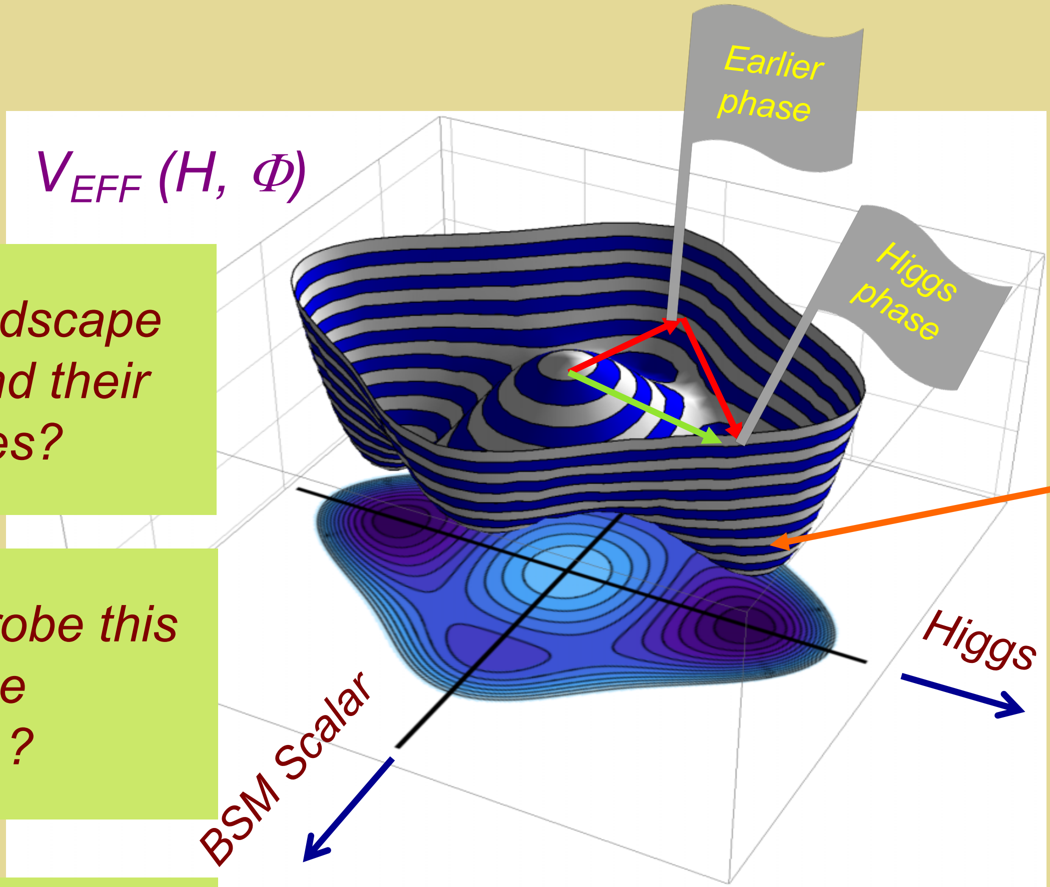


How did we end up here ?

- What is the landscape of potentials and their thermal histories?

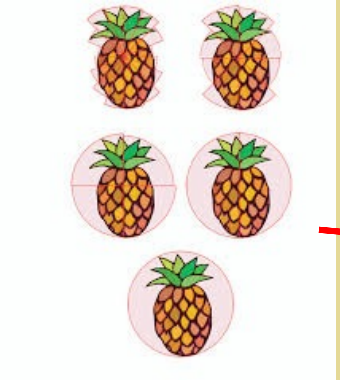
- How can we probe this T > 0 landscape experimentally ?

- How reliably can we compute the thermodynamics ?

n evolve differently as T evolves →
ilities for symmetry breaking

Experimental Probes

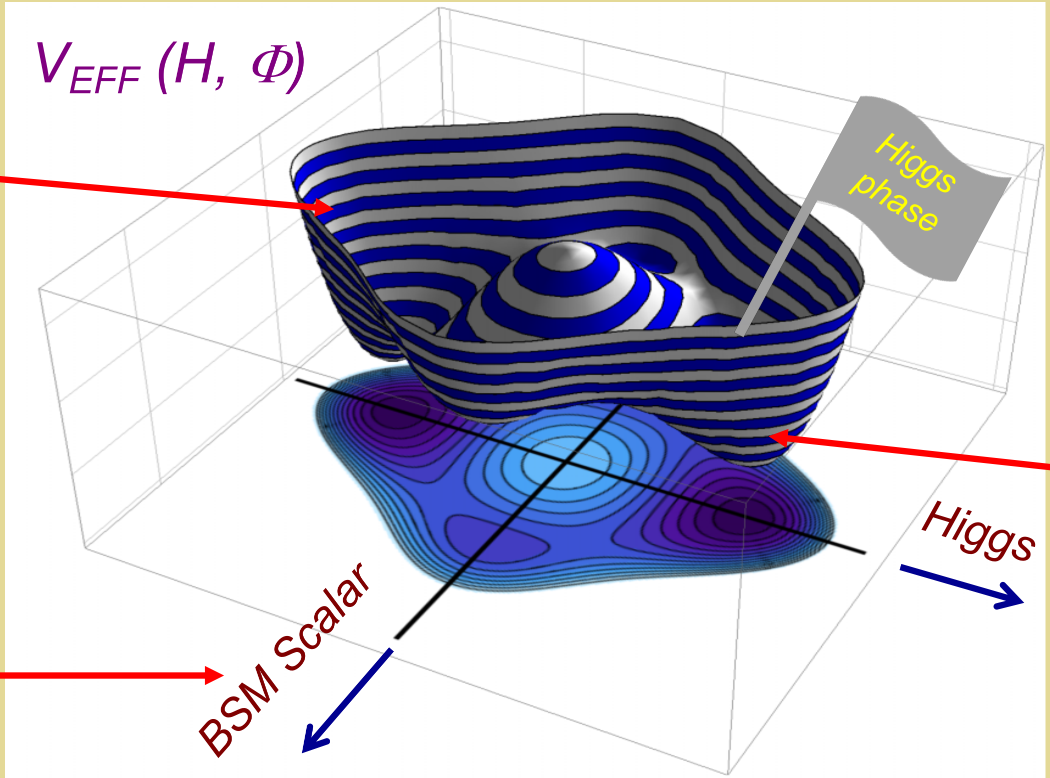
Bubble Collisions



Grav Radiation

Direct Production

BSM Higgs

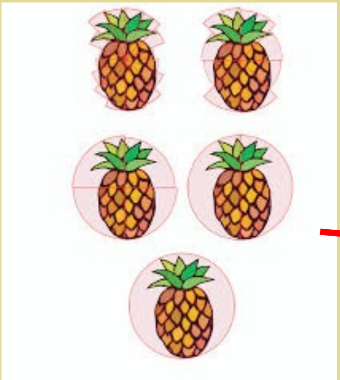


Higgs precision tests

Extrema can evolve differently as T evolves → rich possibilities for symmetry breaking

Experimental Probes

Bubble Collisions



Grav Radiation

Direct Production

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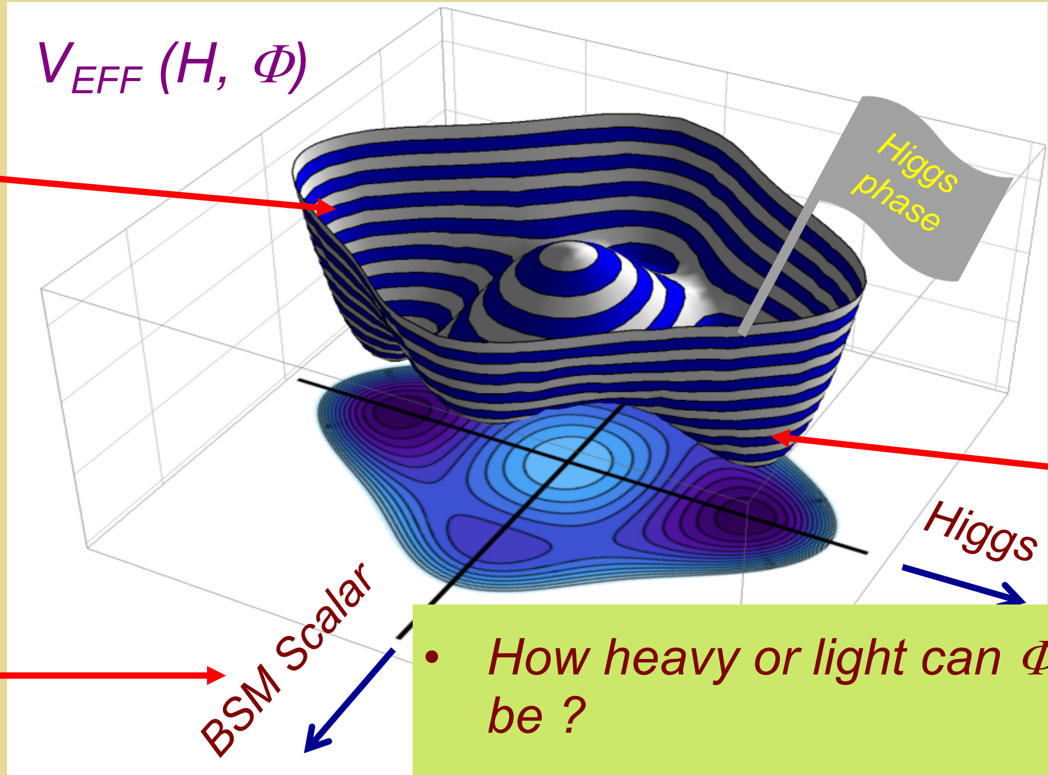

Higgs precision tests

- How heavy or light can Φ be ?
- How coupled to H ?
- Can it be discovered with colliders & GW probes ?

Extrema can evolv

rich possibilities fo

→

MJRM: 1912.07189

T_{EW} → Scale for Colliders & GW probes

High-T SM Effective Potential

$$\boxed{V(h,T)_{\rm SM} = D(T^2 - T_0^2)\, h^2 + \lambda\, h^4 \;+ \ldots}$$

$\boxed{T_0 \sim 140 \text{ GeV}}$ $\boxed{\equiv \quad T_{EW}}$

MJRM: 1912.07189

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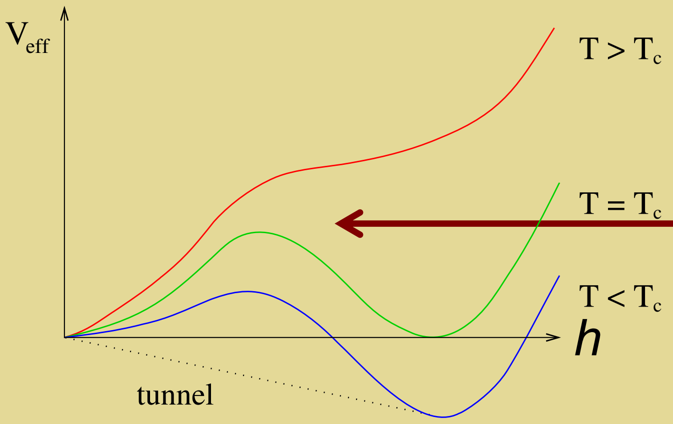
$\boxed{T_0 \sim 140 \text{ GeV}}$ $\boxed{\equiv \quad T_{EW}}$

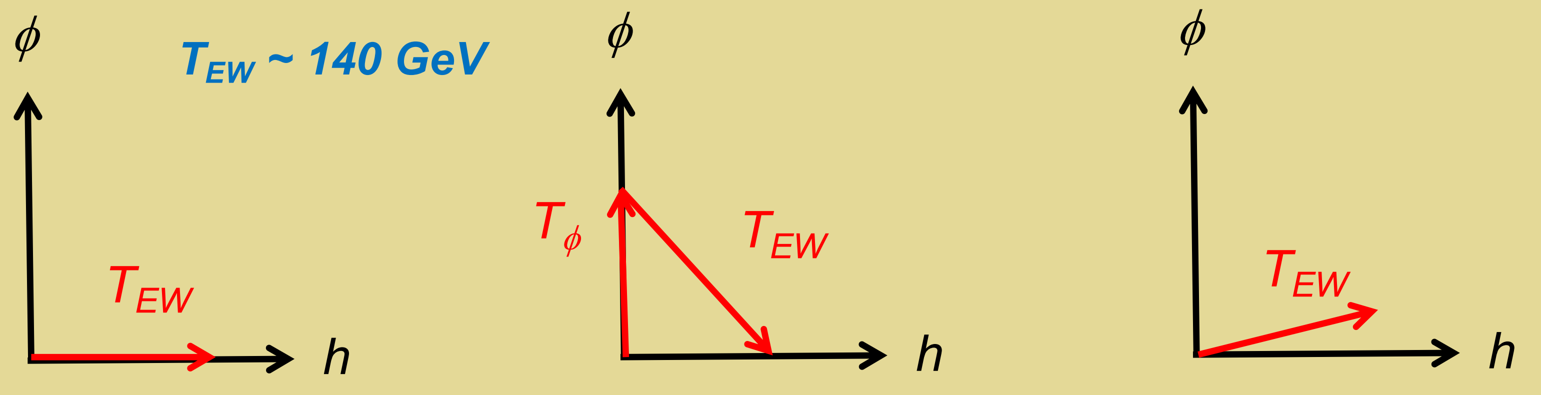
Collider target

MJRM: 1912.07189

First Order EWPT from BSM Physics



Representative thermal histories → barrier for SFOEWPT



$a_2\, H^2\phi^2$: $T > 0$
loop effect

$a_2\, H^2\phi^2$: $T = 0$
tree-level effect

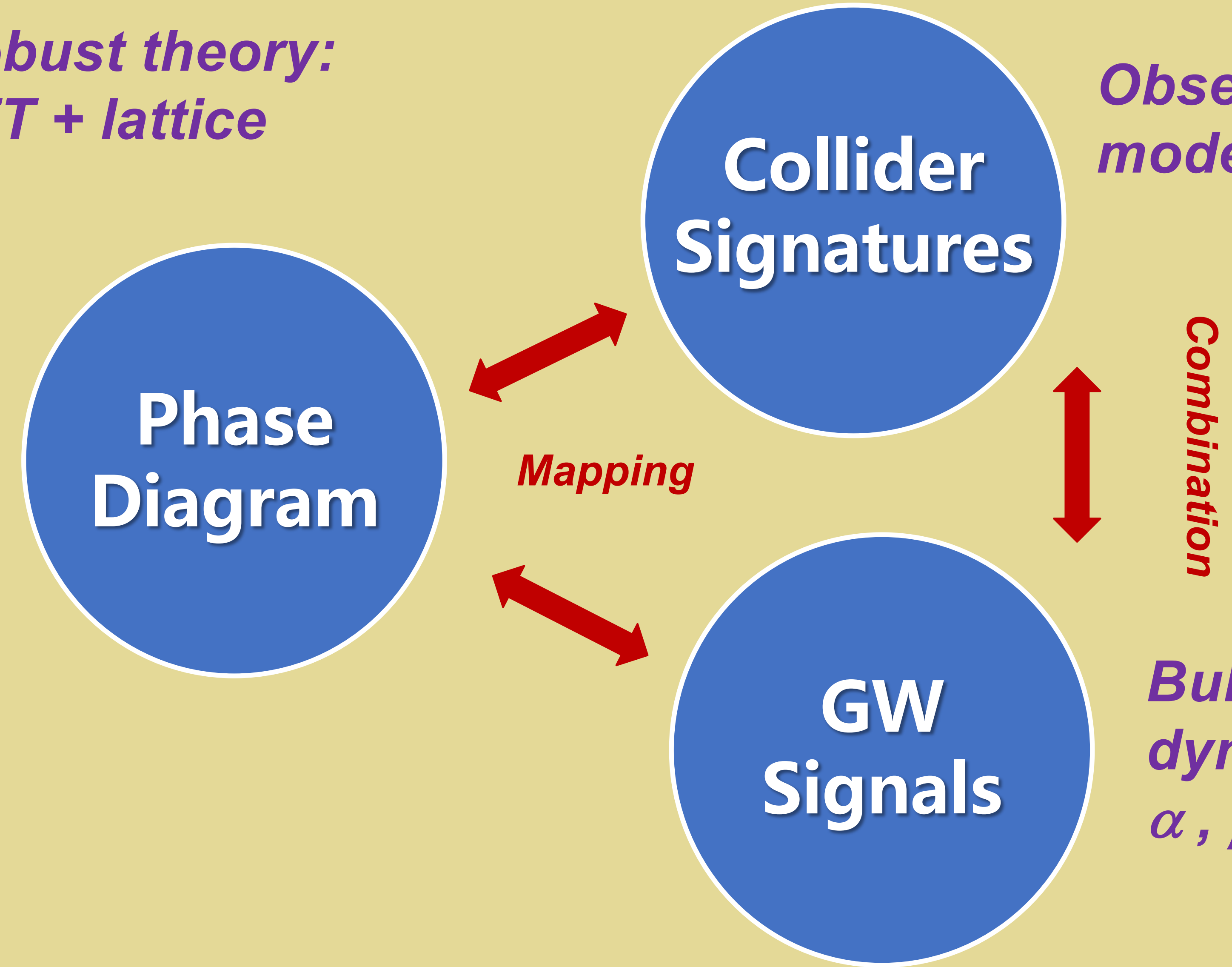
$a_1\, H^2\phi$: $T = 0$
tree-level effect

IV. Higgs Factory-GW Interplay

BSM EWPT: Inter-frontier Connections

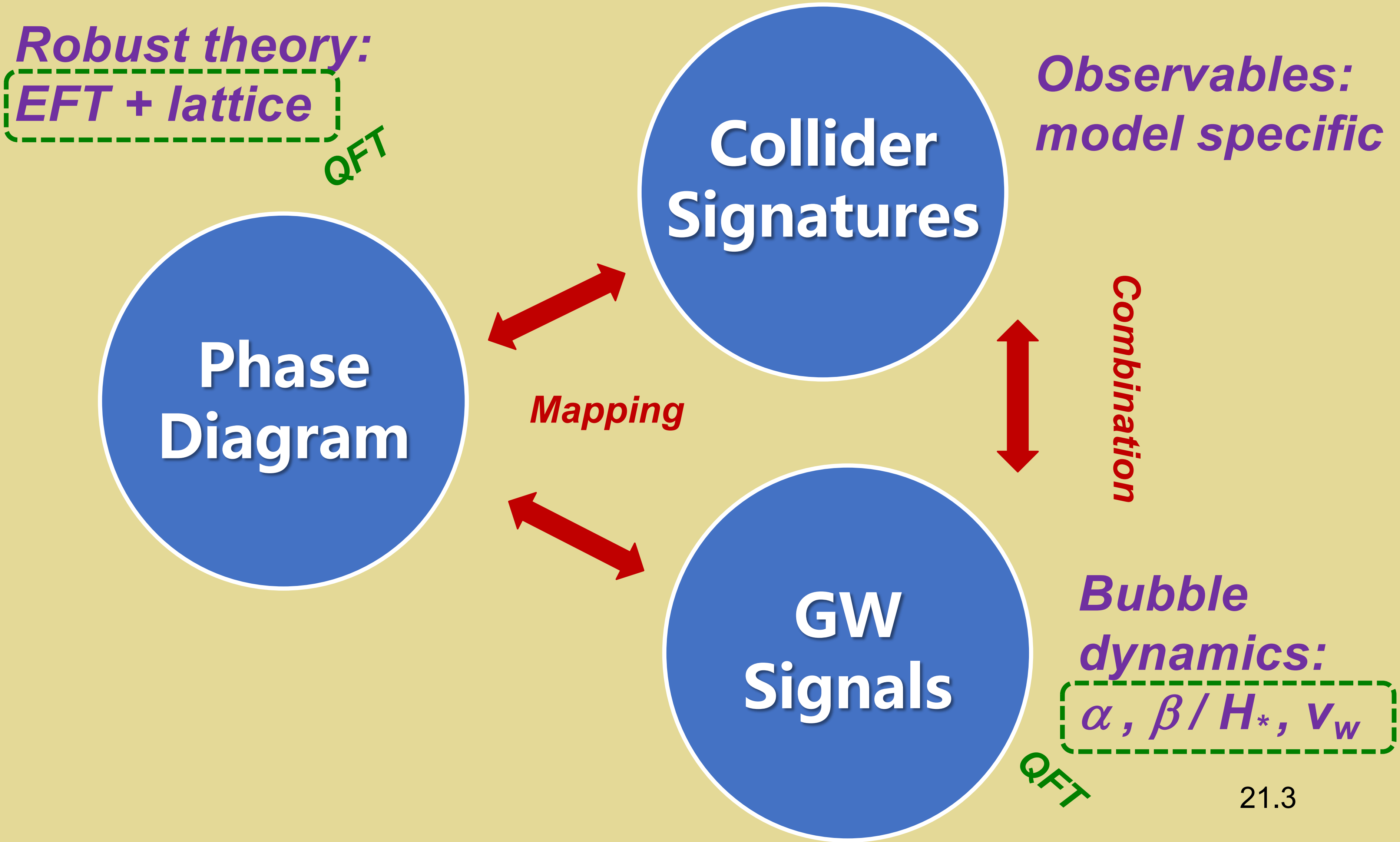
Robust theory:
EFT + lattice

Observables:
model specific



Bubble dynamics:
α , β / H_* , v_w

BSM EWPT: Inter-frontier Connections

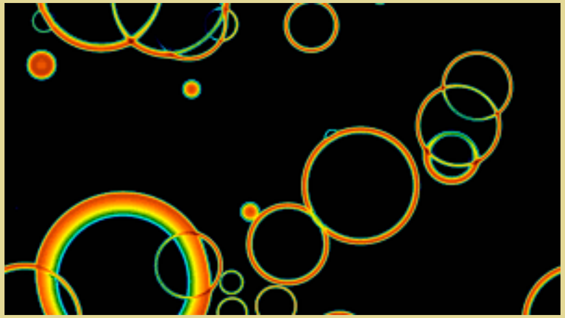


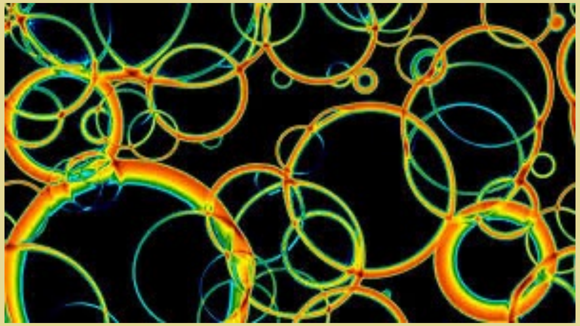
First Order EWPT: Collider Probes

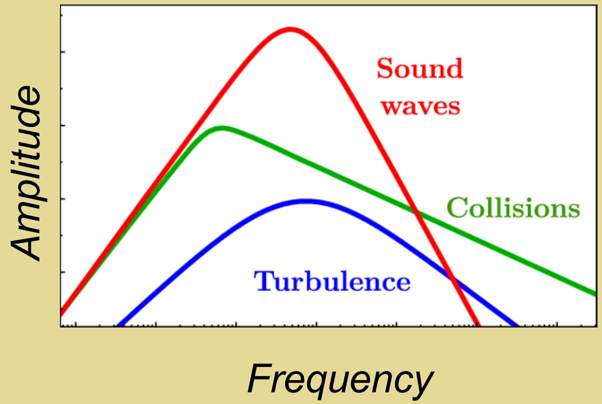
Higgs property	hadron	lepton
• $\Gamma(h \rightarrow \gamma\gamma)$	😊	😊
• Exotic Higgs Decays	😊	😊
• Higgs signal strengths	😊	😊
• Higgs self-coupling	😊	😊
• Resonant di-Higgs production	😊	
• Heavy Higgs $\rightarrow$ VV	😊	
• Associated production (Zh)	😊	😊

Gravitational Radiation

$$\boxed{\Omega_{\rm GW} = \Omega_{\rm coll} + \Omega_{\rm sw} + \Omega_{\rm turb}}$$







*α, β / H_*, v_w ,... → inputs for GW sources: **collisions, sound waves, hydrodynamic turbulence***

Nucleation @ T>0: Gravitational Waves

Amplitude & frequency: latent heat & intrinsic time scale

Normalized latent heat

$$\Delta Q = \Delta F + T\Delta S$$
$$S = -\partial F/\partial T$$
$$F \approx V$$

$$\Delta Q \approx \Delta V - T\partial\Delta V/\partial T$$

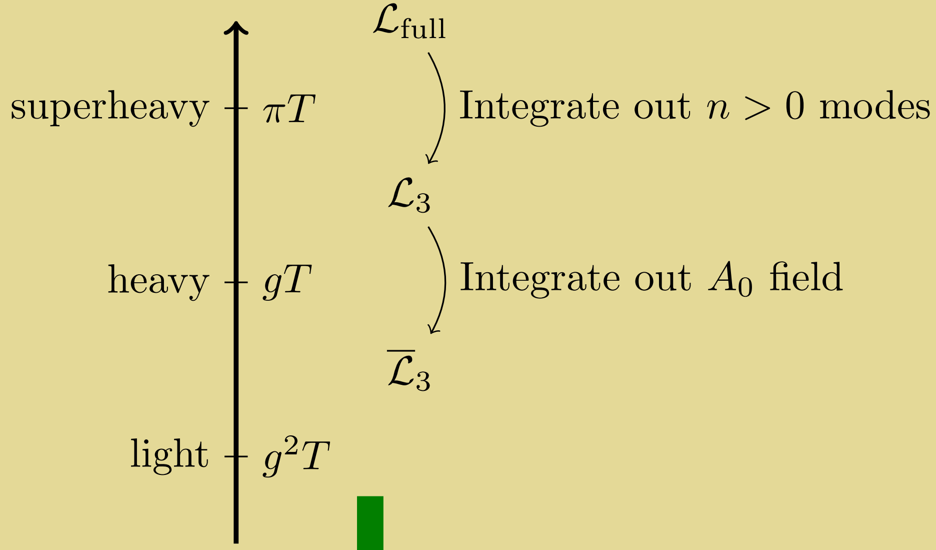
$$\alpha = \frac{30\Delta q}{\pi^2 g_* T^4}$$

Time scale

$$\frac{\beta}{H_*} = T\frac{d}{dT}\frac{S_3}{T}$$

Particle Physics Ingredients @ T>0

Framework: 3-D high-T effective theory



+ Integrate out BSM scalars

$$V(\phi) = \bar{\mu}^2_{\phi,3}\phi^\dagger\phi + \bar{\lambda}_3(\phi^\dagger\phi)^2$$

Non-dynamical BSM scalars

"Standard Model-like"

Particle Physics Ingredients @ T>0

Framework: 3-D high-T effective theory

$\mathcal{L}_{\rm full}$

superheavy — πT — Integrate out $n > 0$ modes

$\mathcal{L}_3$

heavy — gT — Integrate out A_0 field

$\overline{\mathcal{L}}_3$

light — $g^2 T$

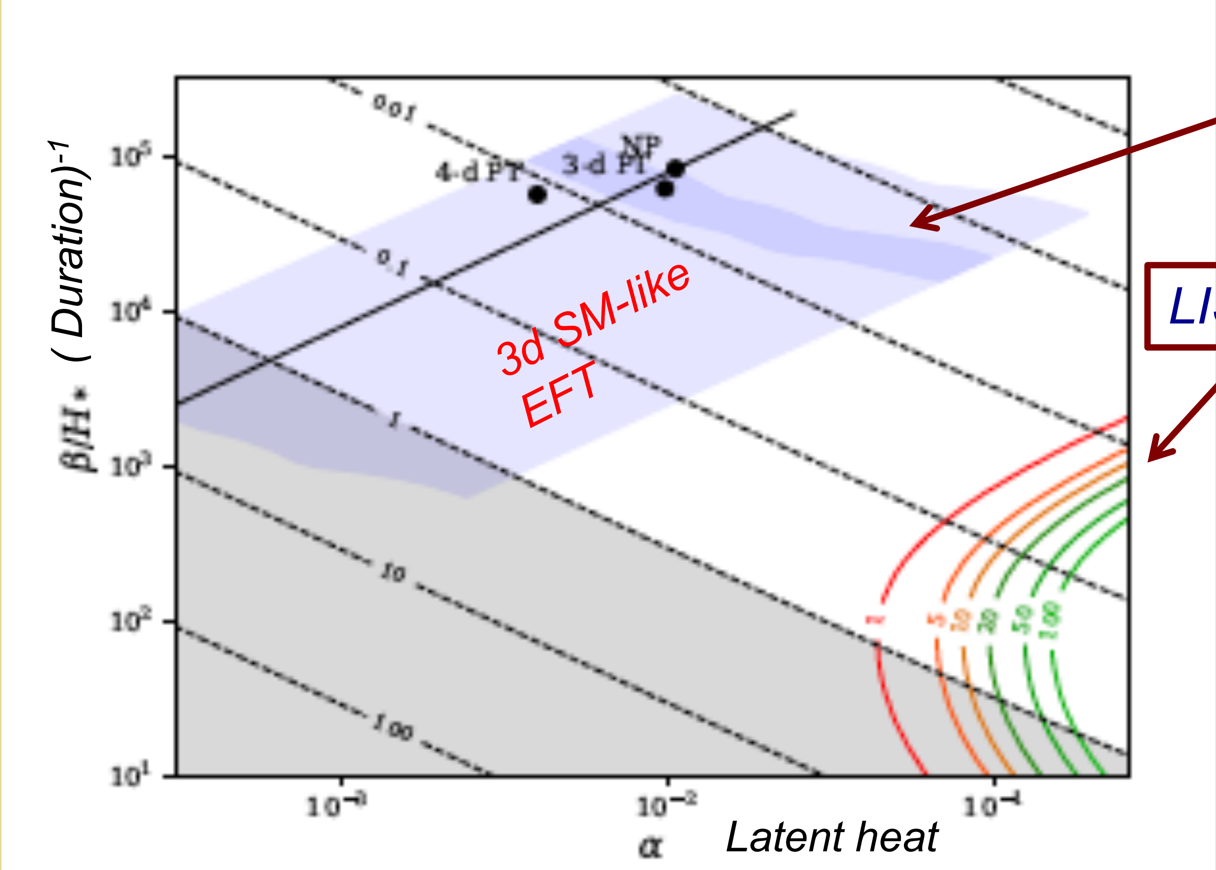
$$V(\phi) = \bar{\mu}^2_{\phi,3}\phi^\dagger\phi + \bar{\lambda}_3(\phi^\dagger\phi)^2$$

+ *V* (*Φ*) + *V* (*ϕ*, *Φ*)$_{portal}$

Dynamical BSM scalars

BSM Scalar: EWPT & GW

High-T dimensional reduction: DR 3d EFT



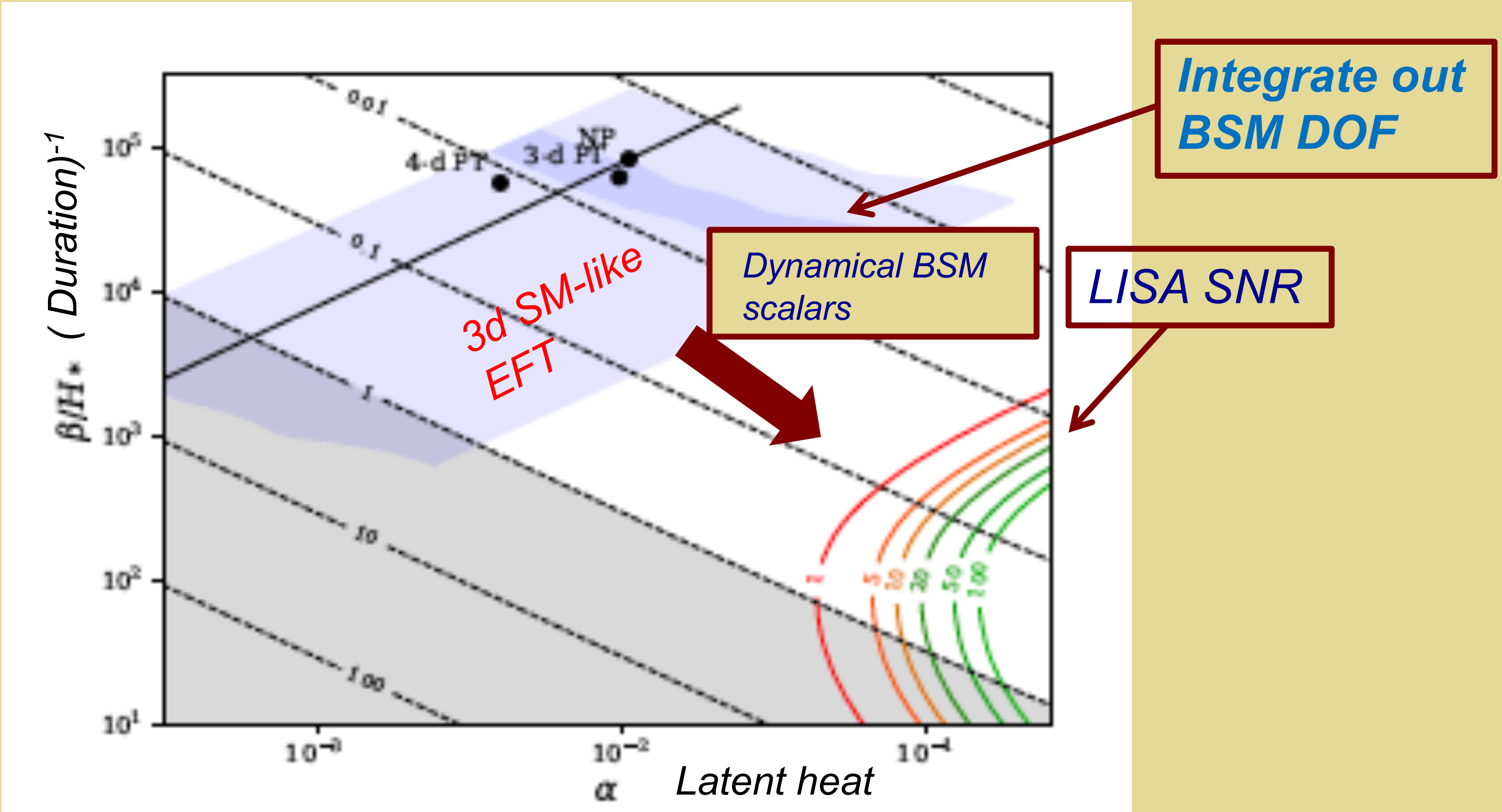
Integrate out BSM DOF

LISA SNR

Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

BSM Scalar: EWPT & GW

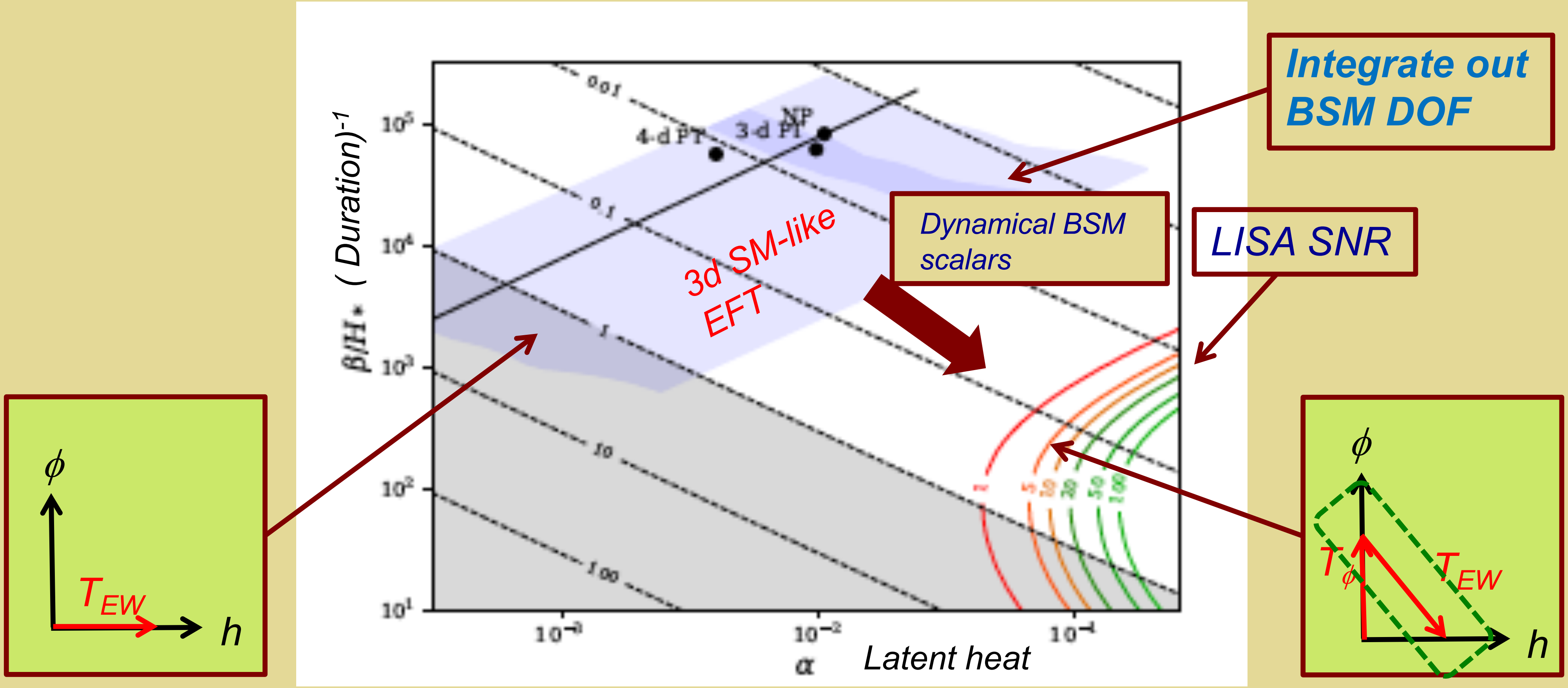
High-T dimensional reduction: DR 3d EFT



Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

BSM Scalar: EWPT & GW

High-T dimensional reduction: DR 3d EFT

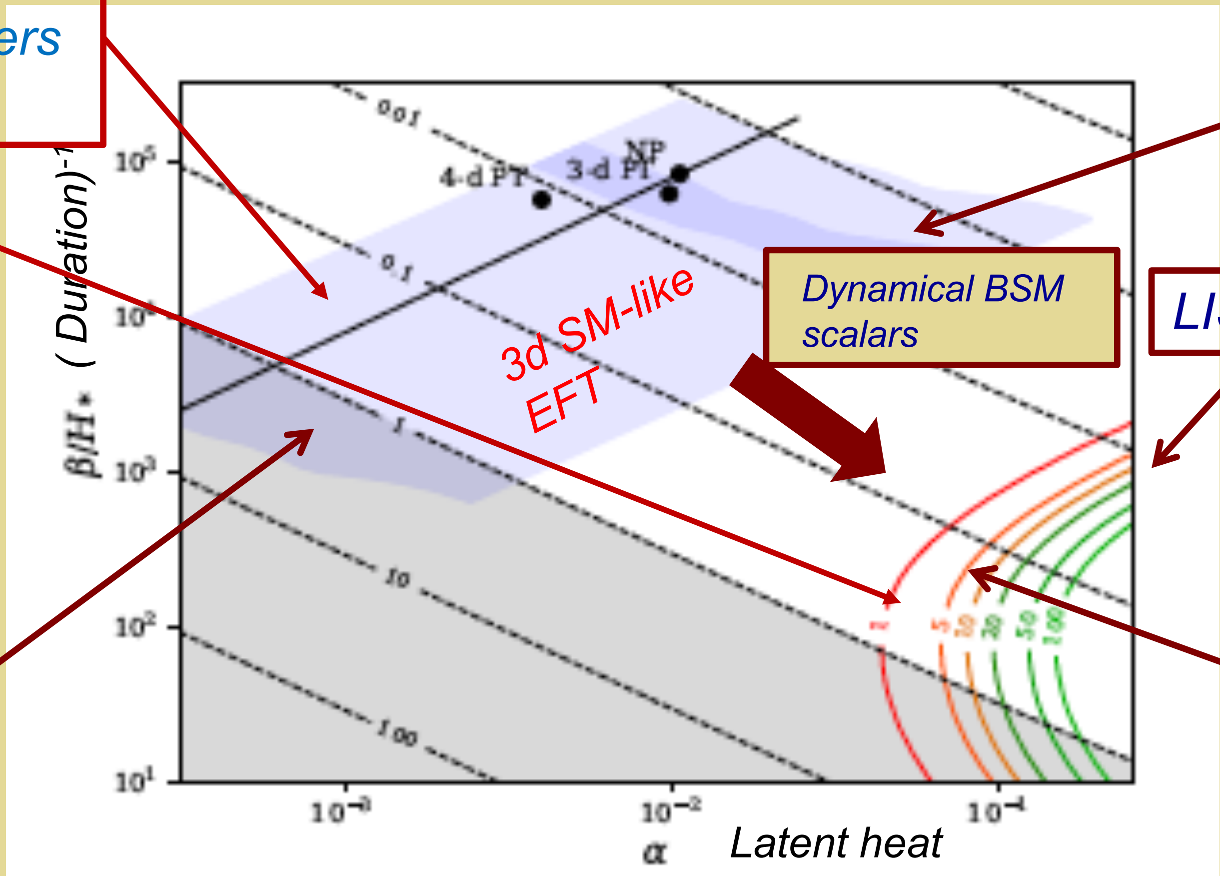


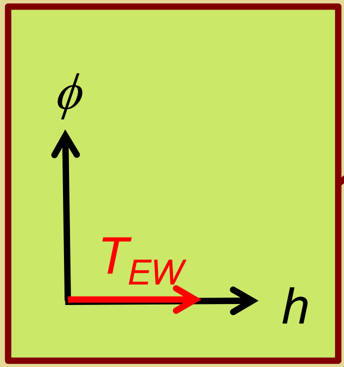
Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

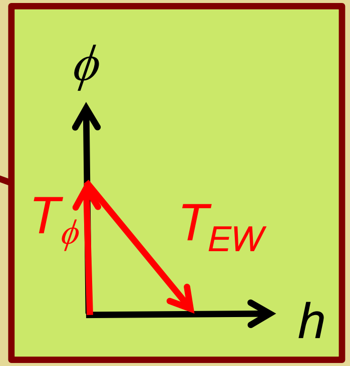
BSM Scalar: EWPT & GW

Collider probes of BSM parameters in $\mathcal{L}_{full}$

Integrate out BSM DOF

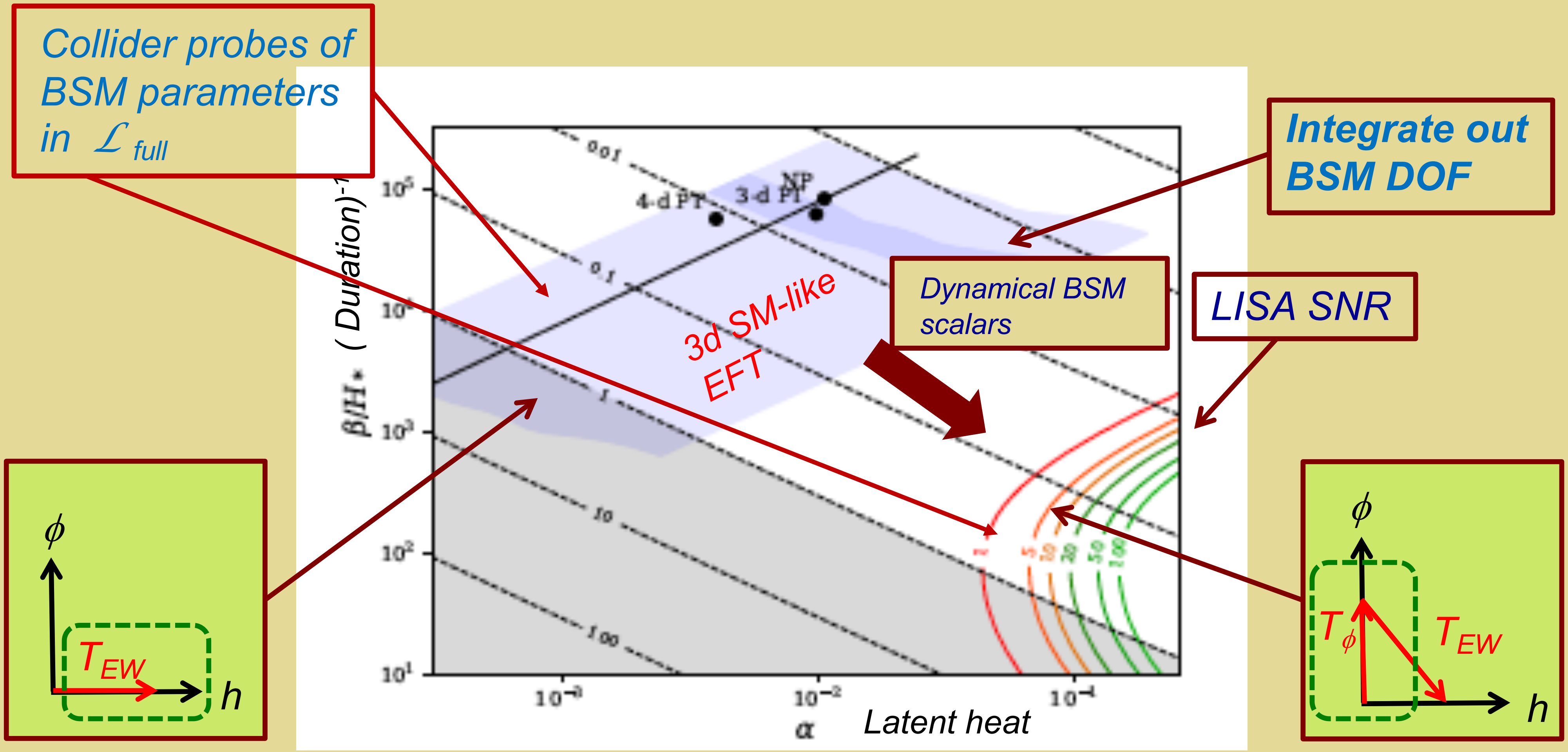






Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

BSM Scalar: EWPT & GW



Radiative barrier from gauge sector

Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

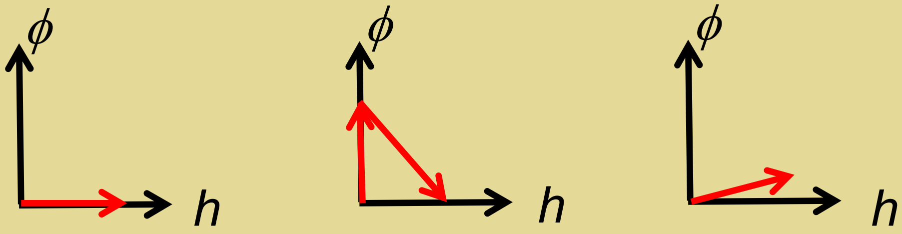
V. Model Illustrations & Results

Theory-Pheno Interface

Simple Higgs portal models:

- ***Real gauge singlet (SM + 1)***
- ***Real EW triplet (SM + 3)***

$$\boxed{V \;\supset\; a_1 H^2\phi + a_2 H^2\phi^2}$$



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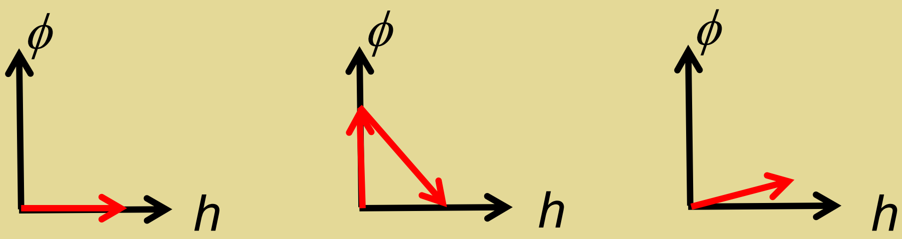
$$\phi = \frac{1}{2}\begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}$$



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Small

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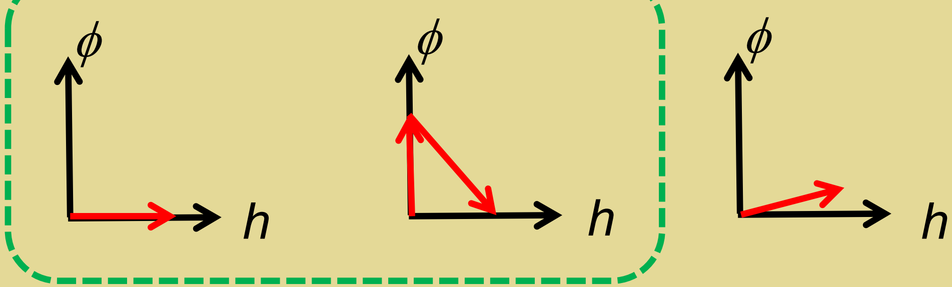

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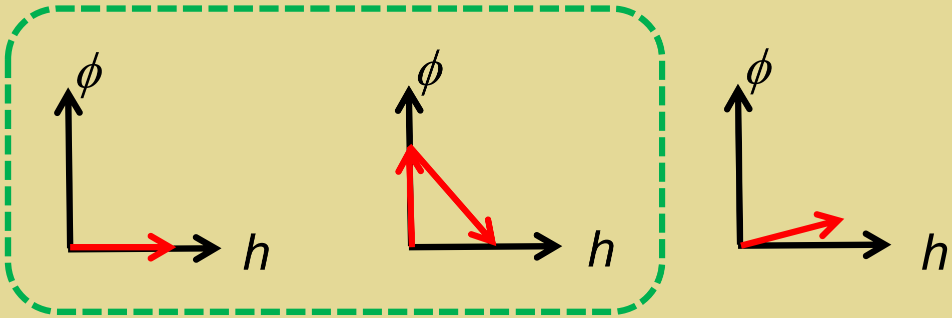
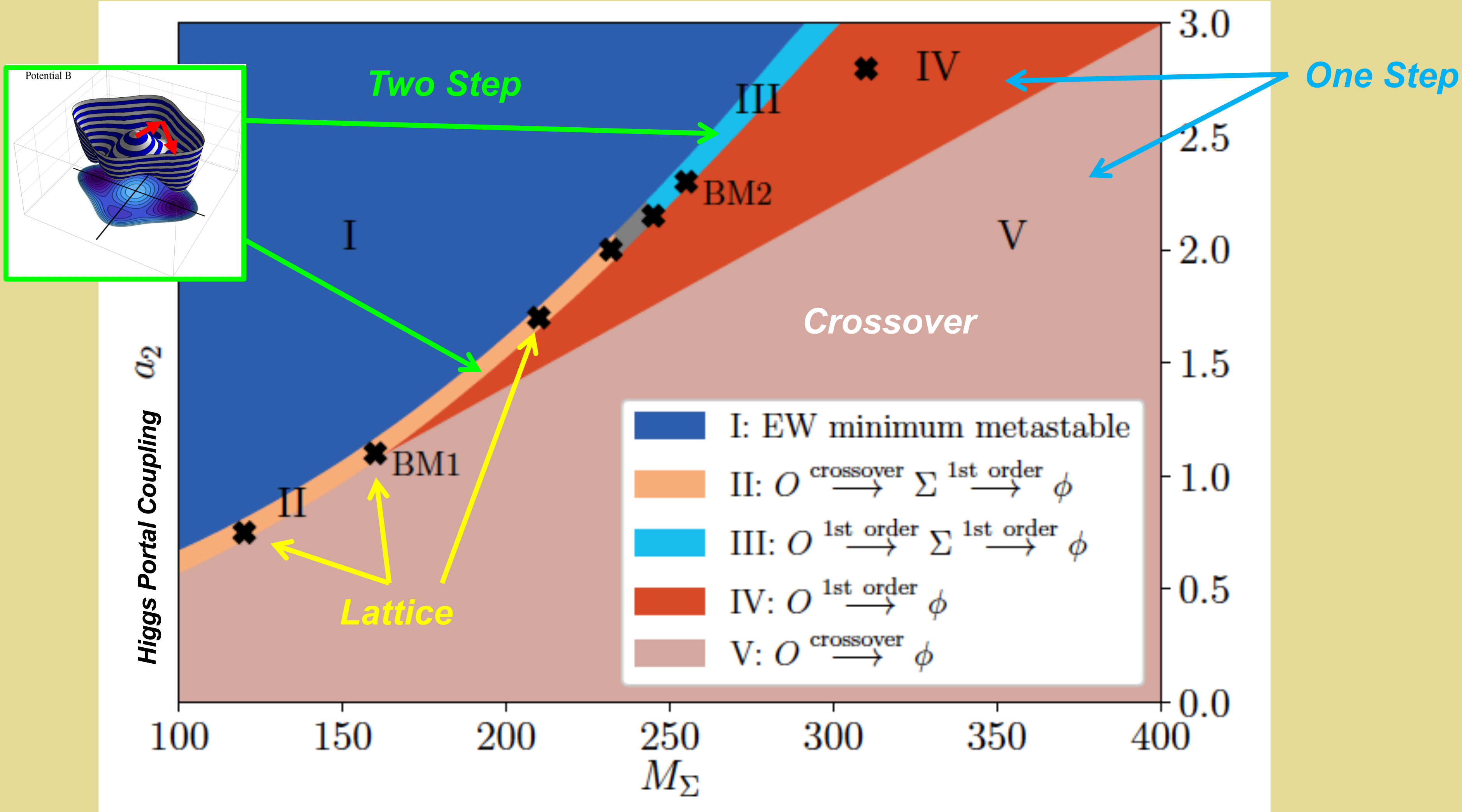


Phenomenology

- *Gravitational waves*
- *Collider: $h \rightarrow \gamma\gamma$, dis charged track, NLO $e^+e^- \rightarrow Zh$…*

Real Triplet & EWPT: Novel EWSB



Niemi, R-M, Tenkanen, Weir 2005.11332

- *1 or 2 step*
- *Non-perturbative*

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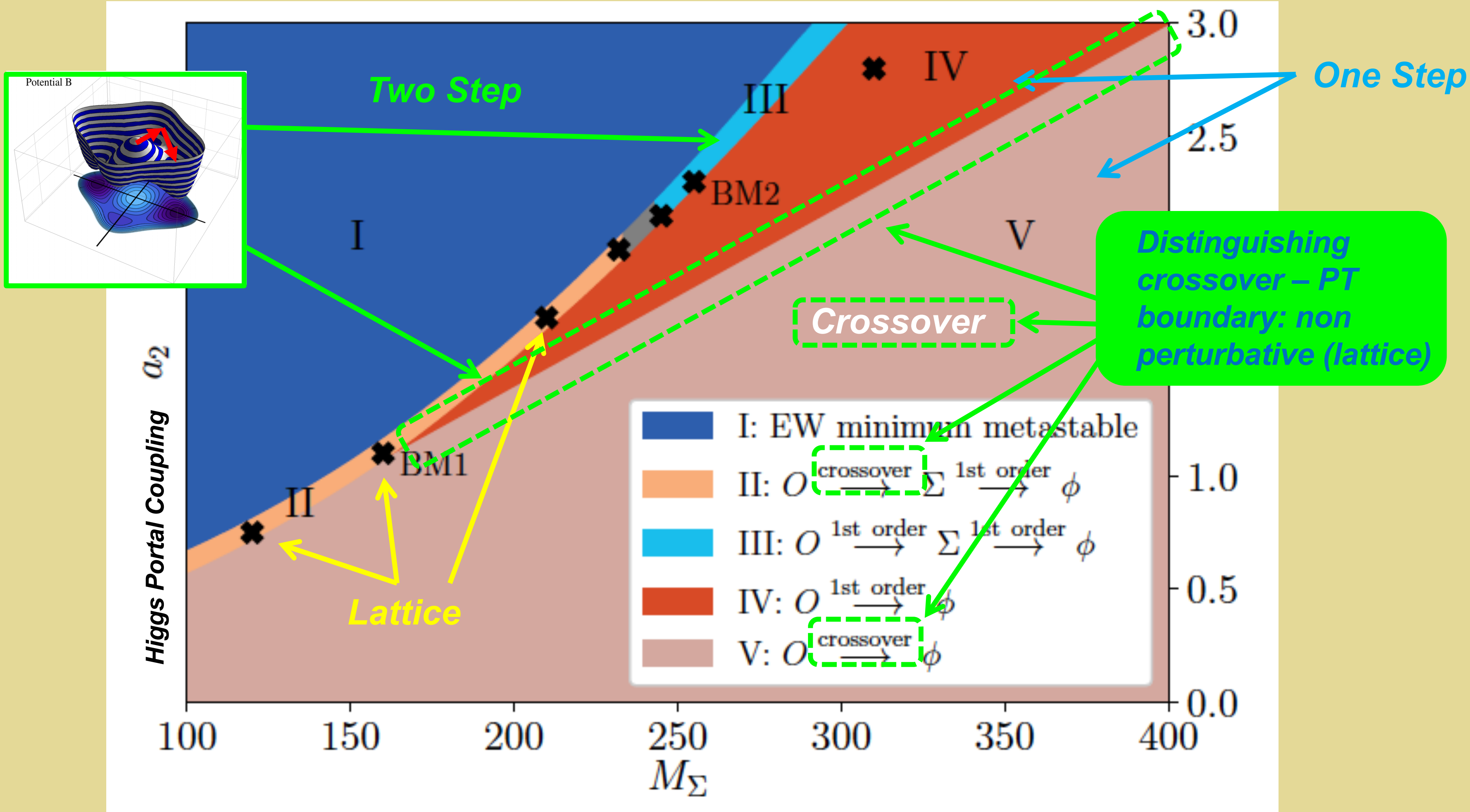

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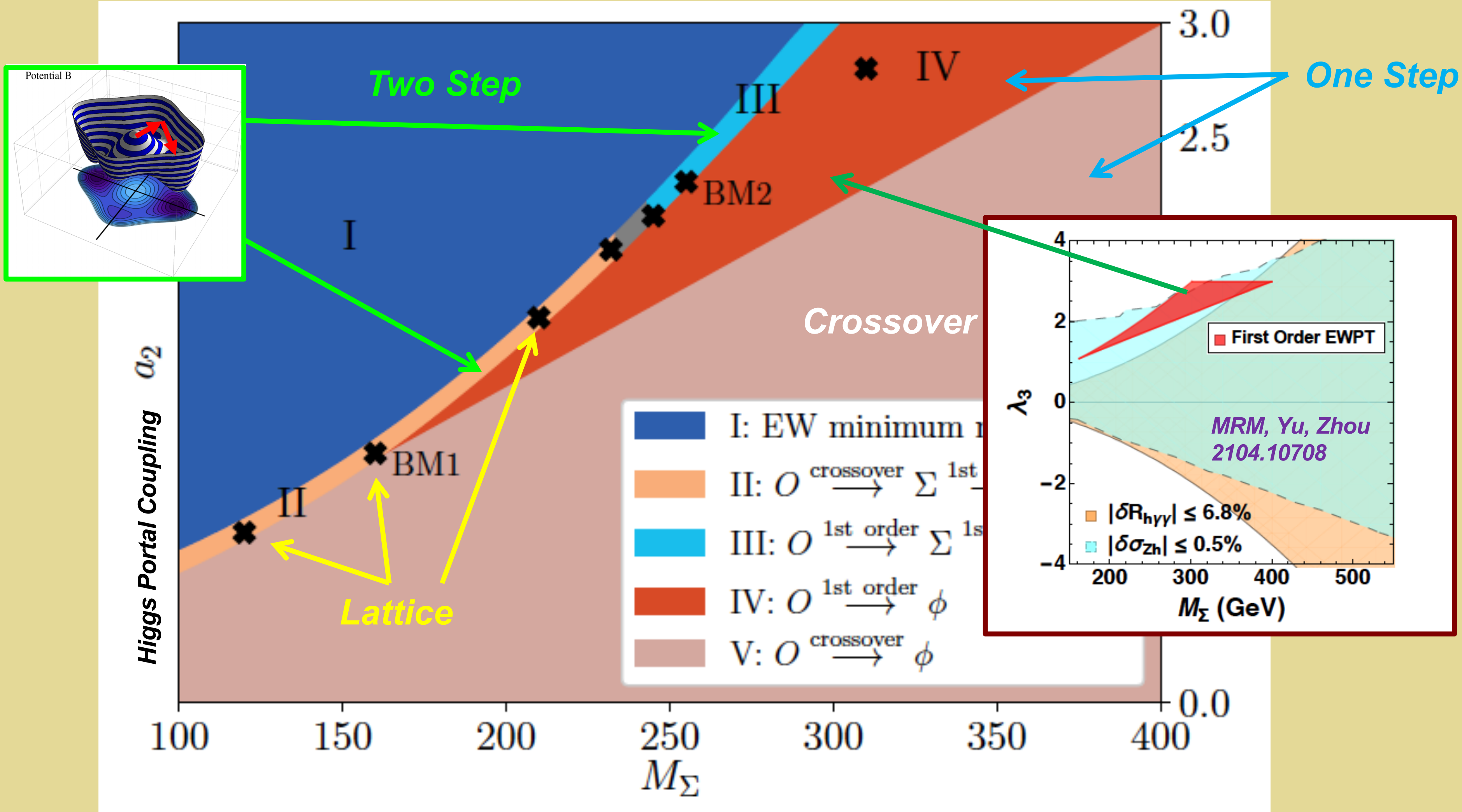
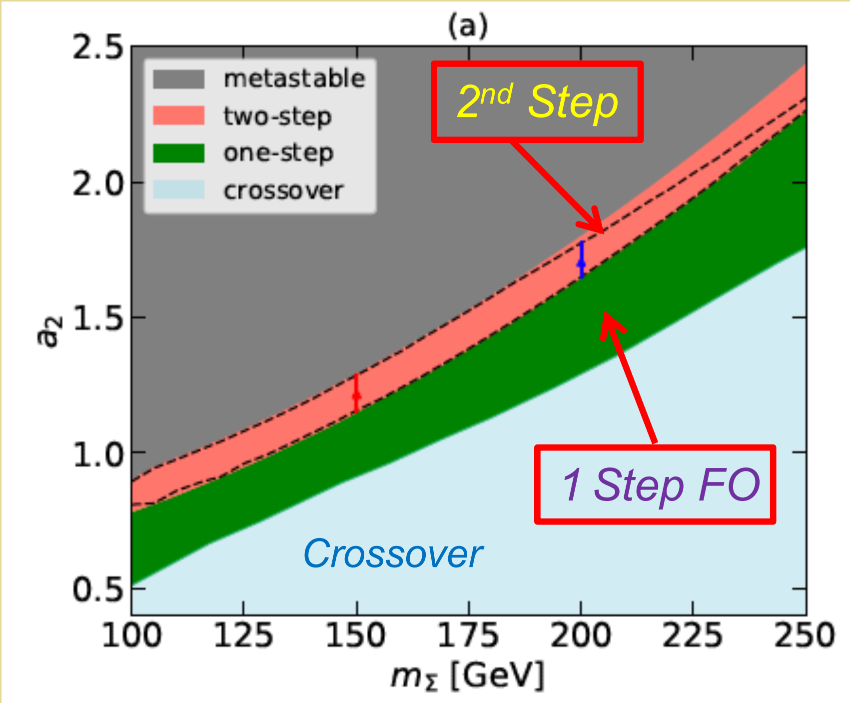


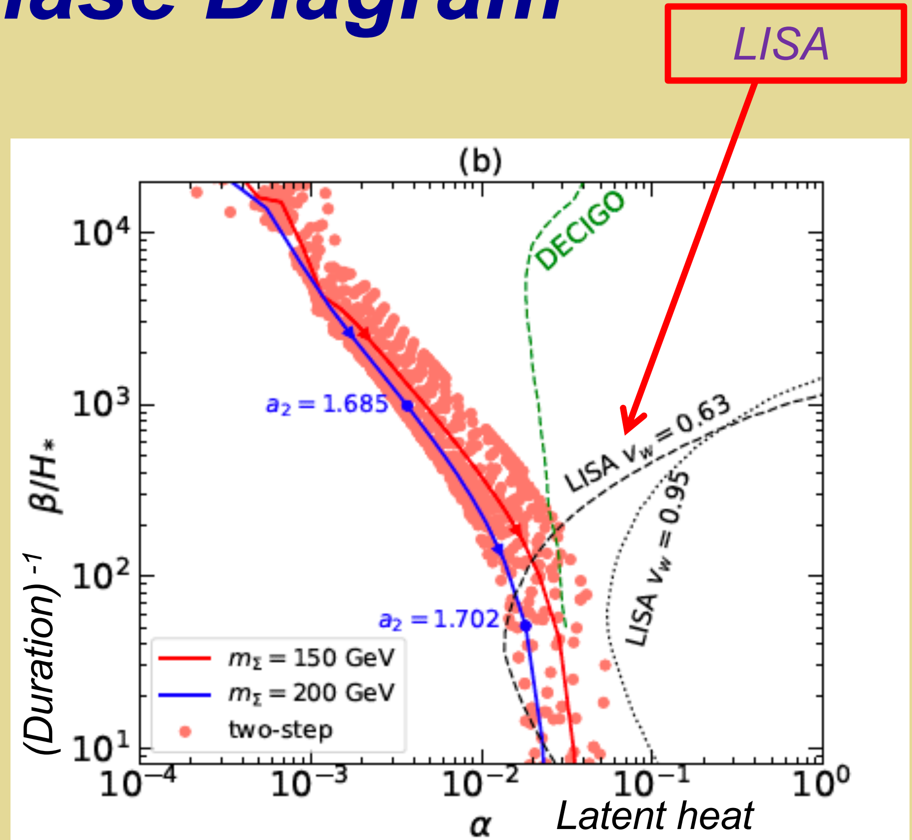
Niemi, R-M, Tenkanen, Weir 2005.11332

- *1 or 2 step*
- *Non-perturbative*

GW & EWPT Phase Diagram

Real triplet extension



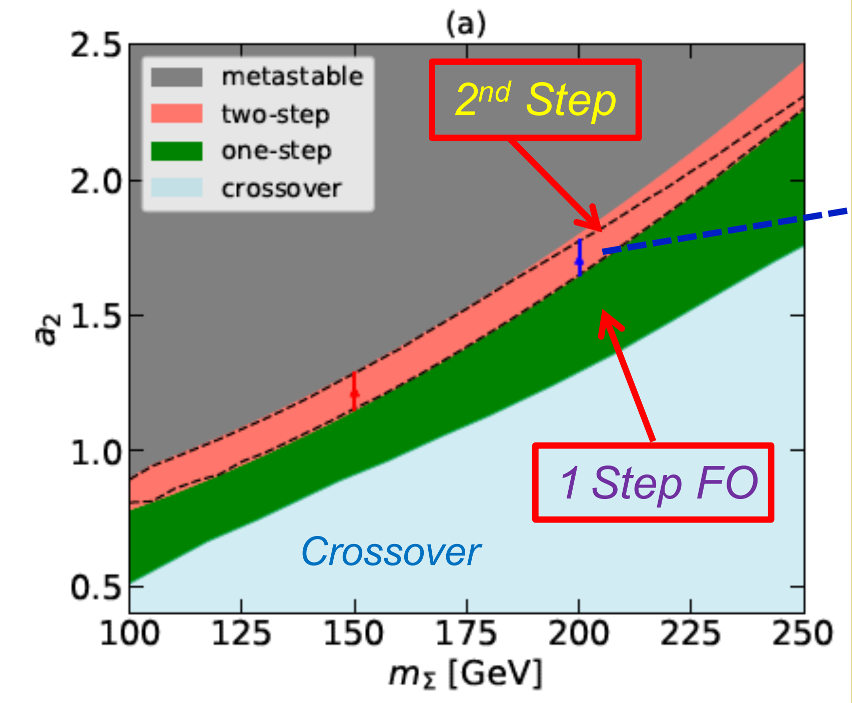


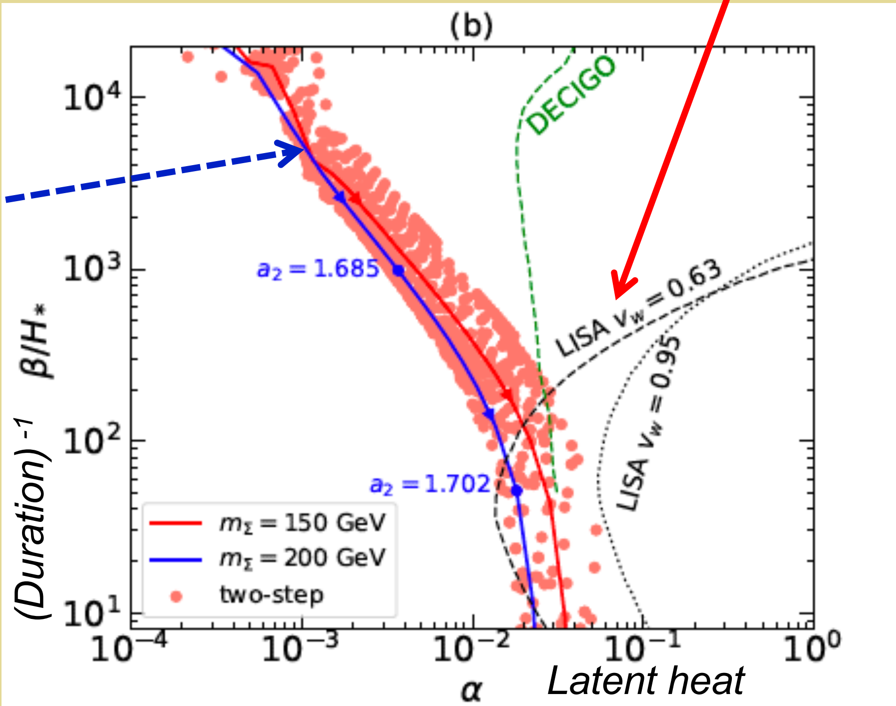
30.1

Friedrich, MJRM, Tenkanen, Tran 2203.05889

GW & EWPT Phase Diagram

Real triplet extension





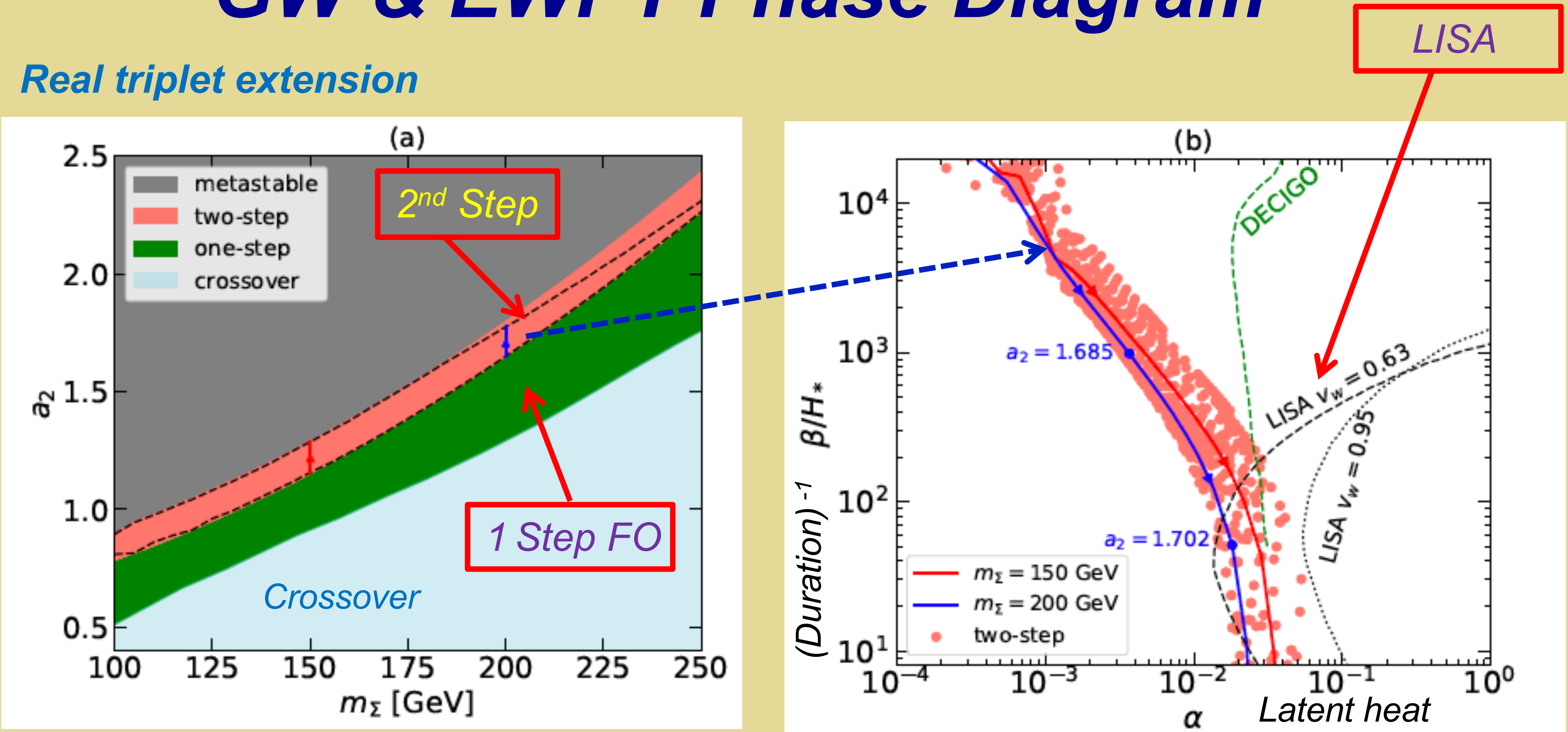
LISA

30.2

Friedrich, MJRM, Tenkanen, Tran 2203.05889

GW & EWPT Phase Diagram

Real triplet extension

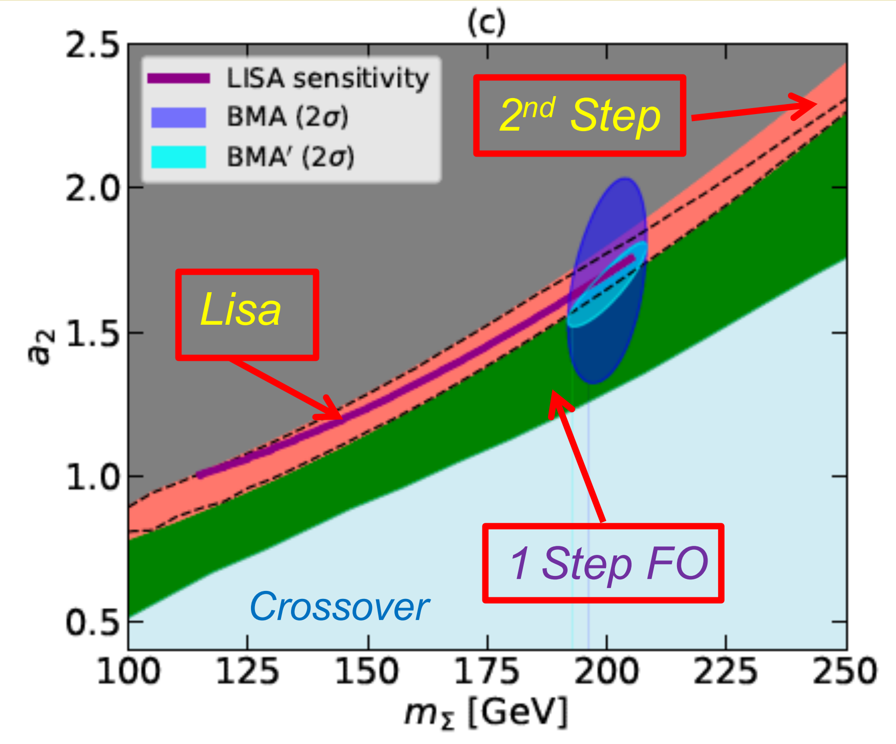


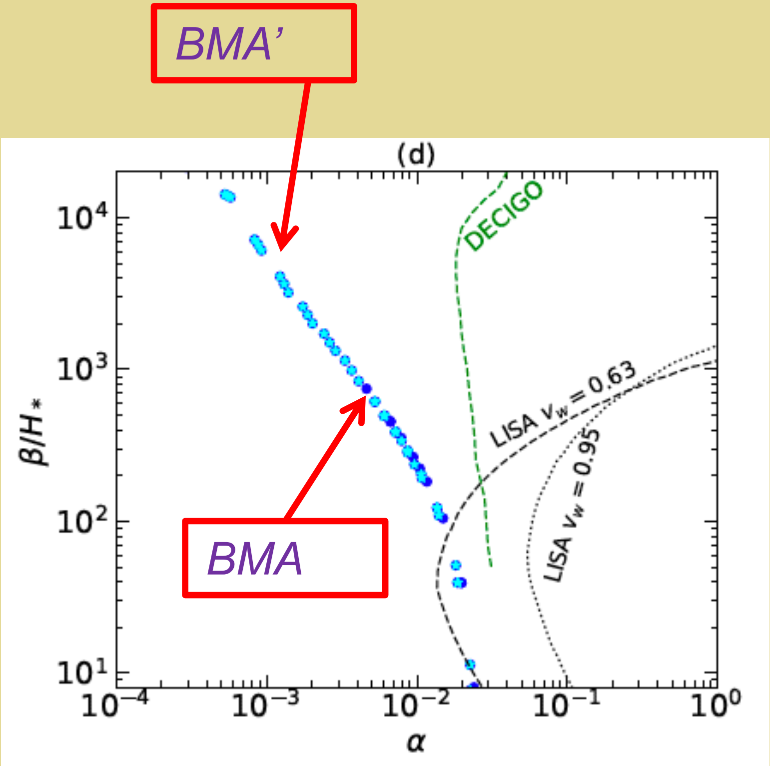
- *Single step transition: GW well outside LISA sensitivity*
- *Second step of 2-step transition can be observable*
- *Significant GW sensitivity to portal coupling*

Friedrich, MJRM, Tenkanen, Tran 2203.05889

GW & Collider : EWPT Phase Diagram

Real triplet extension





BMA: m_Σ + $h \rightarrow \gamma\gamma$

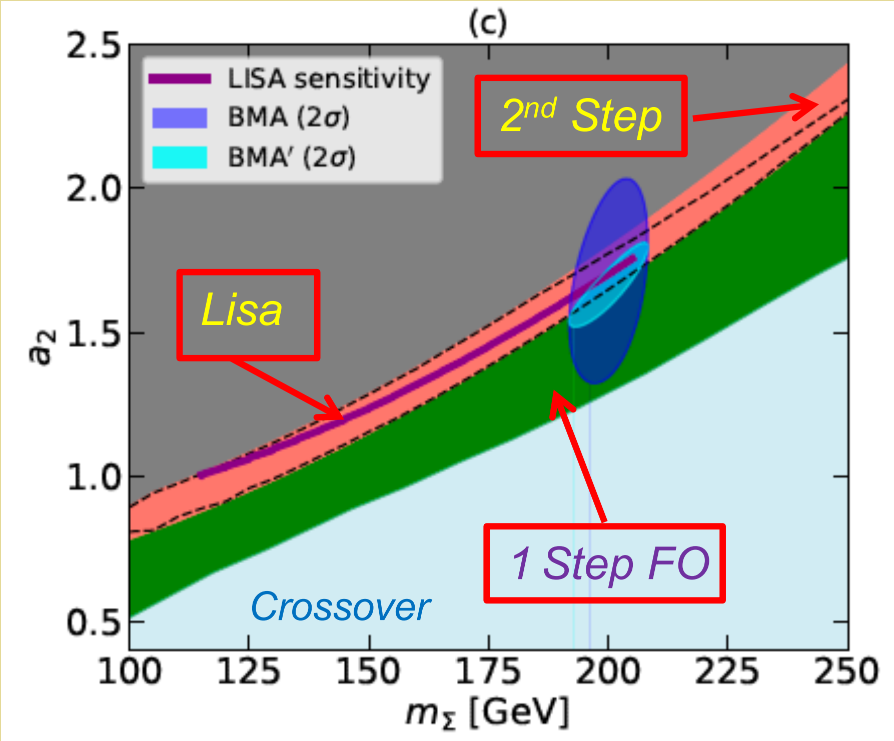
BMA' : BMA + $\Sigma^0 \rightarrow ZZ$

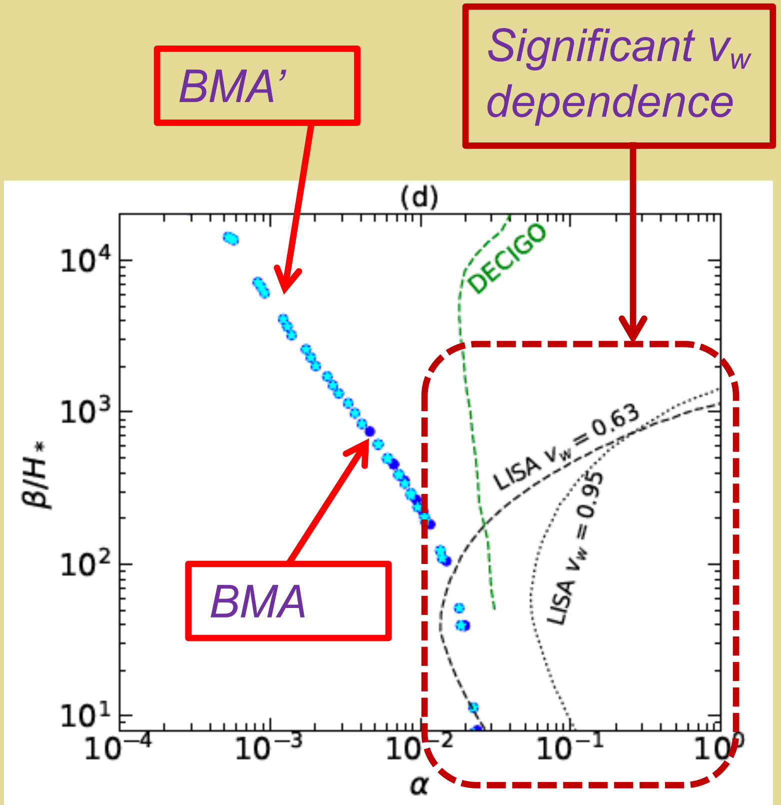
- Two-step
- EFT+ Non-perturbative

Friedrich, MJRM, Tenkanen, Tran 2203.05889

GW & Collider : EWPT Phase Diagram

Real triplet extension





BMA: m_Σ + h → $\gamma\gamma$

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Friedrich, MJRM, Tenkanen, Tran 2203.05889

GW & Collider : EWPT Phase Diagram

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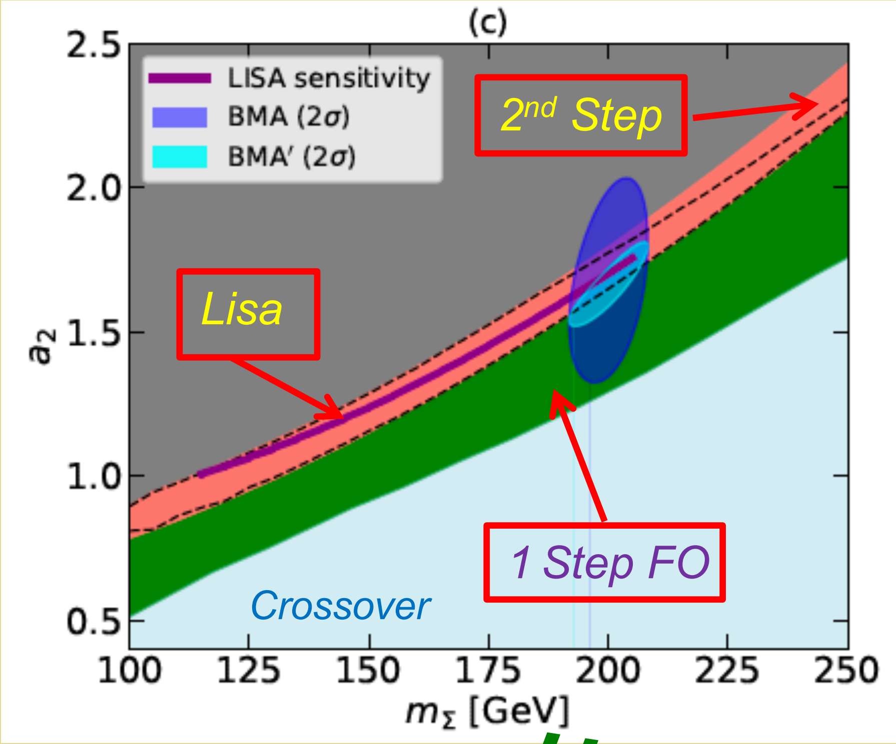

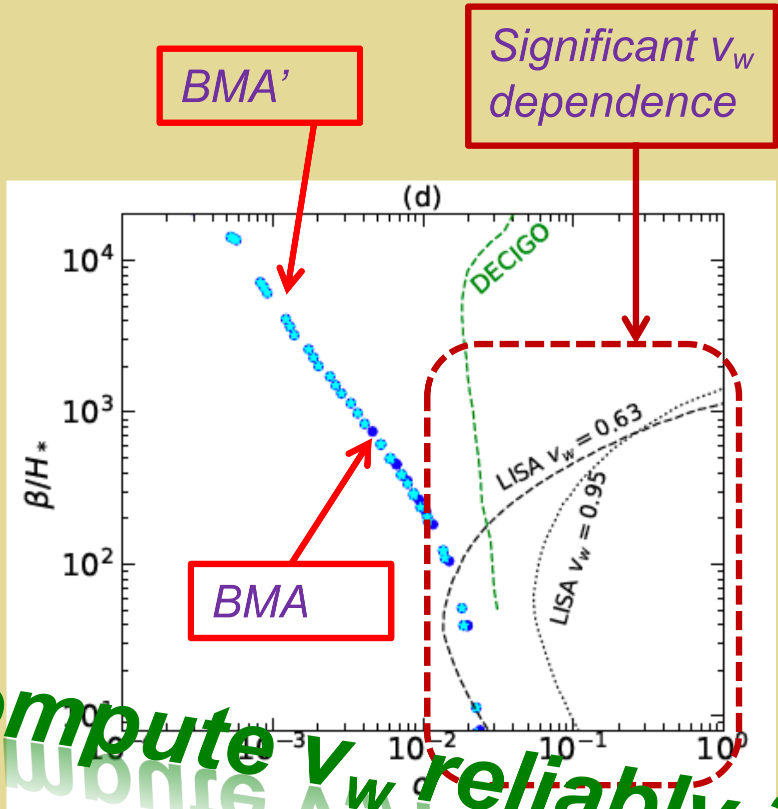

How to compute v_w reliably ?

BMA: m_Σ + $h \rightarrow \gamma\gamma$

BMA' : BMA + $\Sigma^0 \rightarrow ZZ$

- *Two-step*
- *EFT+ Non-perturbative*

Friedrich, MJRM, Tenkanen, Tran 2203.05889

Theory-Pheno Interface

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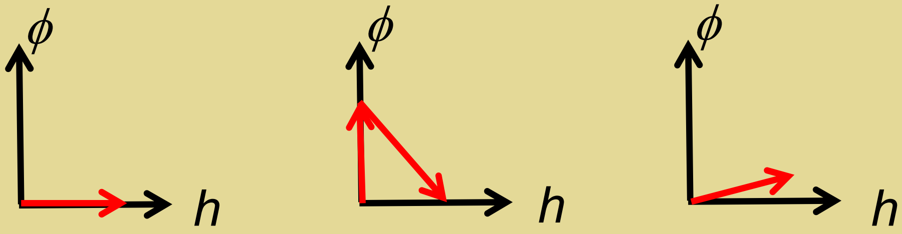

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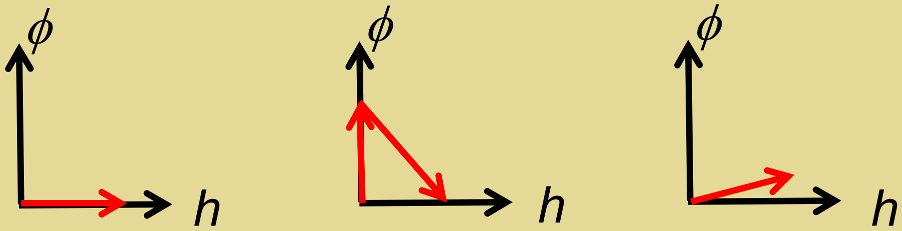
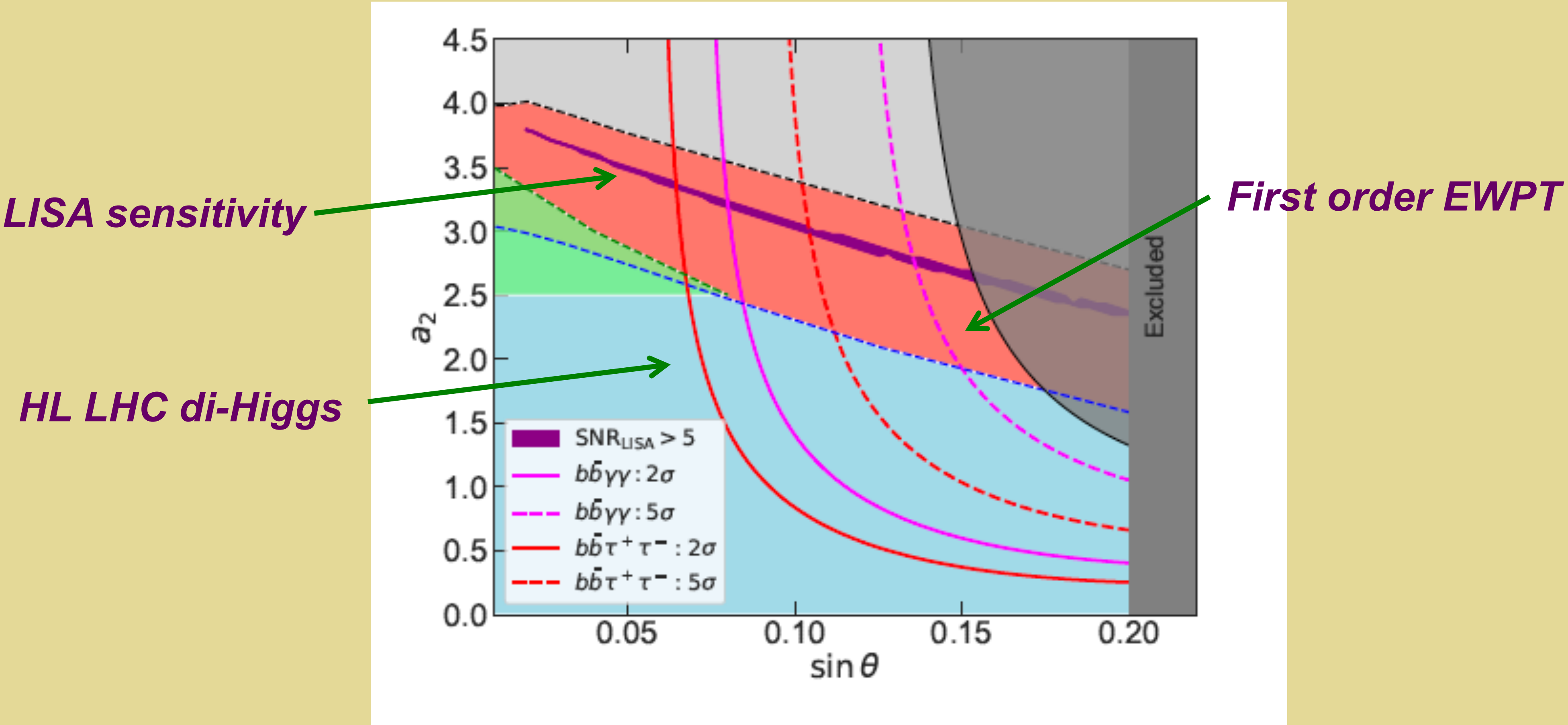
H-ϕ mixing



Gravitational Radiation & Colliders



MJRM, T.V.I. Tenkanen, V.Q. Tran, 2409.17554

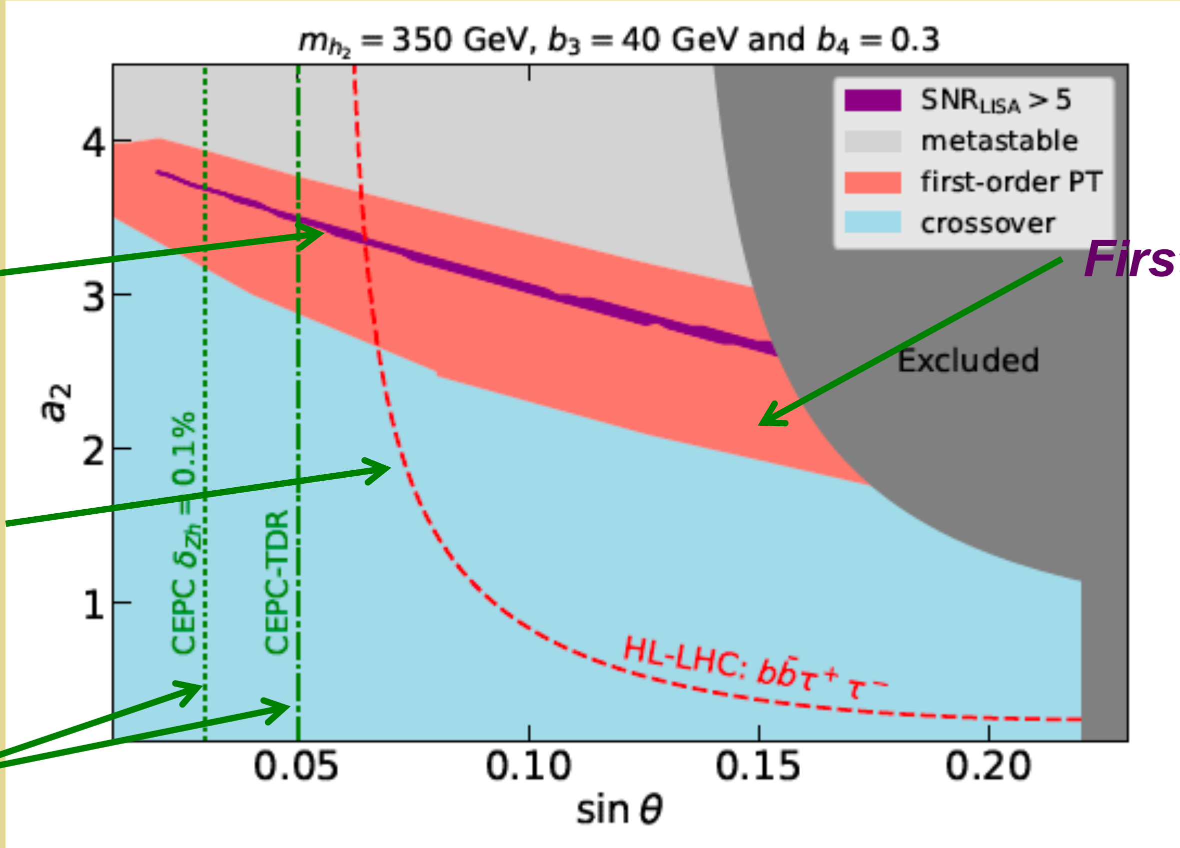
Gravitational Radiation & Colliders

LISA sensitivity

HL LHC di-Higgs

CEPC

First order EWPT



CEPC BSM white paper, 2505.24810 (V.Q. Tran…)

V1. Outlook

- *The possibility of primordial gravitational waves generated from various particle physics dynamics has become an exciting area of exploration*
- *There exist many creative ideas for novel phenomena and dynamics that could have generated GW*
- *Realizing which, if any, of these ideas was realized in nature requires input from additional observables and performing the most rigorous theoretical calculations*
- *The electroweak phase transition provides a unique "laboratory" for testing our theoretical methods and ideas, with LHC and Higgs factory measurements providing key input*

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- The electroweak phase transition provides a unique "laboratory" for testing our theoretical methods and ideas, with LHC and Higgs factory measurements providing key input

Recent progress with EFT, lattice, rigorous QFT…more challenges remain

谢谢！

Back Up Slides

VI. Quantum Field Theory Challenges

Theoretical Robustness

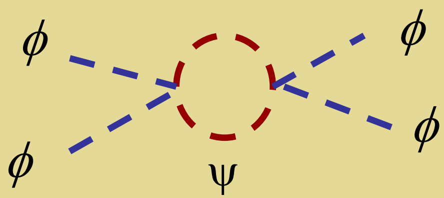
- ***IR Problem***
- ***Nucleation @ finite T: gauge invariance***
- ***Wall velocity***

EWPT & Perturbation Theory: IR Problem

Bosonic loop at T>0

Bose dist fn

$$I(T) = g^2 \int \frac{d^3p}{(2\pi)^3}\, f_B(E,T)\, \frac{1}{(p^2+m^2)^n}$$



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Small p regime

$$f_B(E,T) \longrightarrow \frac{T}{m}$$

EWPT & Perturbation Theory: IR Problem

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Bose dist fn

$$I(T) = g^2 \int \frac{d^3p}{(2\pi)^3} f_B(E,T) \frac{1}{(p^2+m^2)^n} \longrightarrow \frac{g^2 T}{m} \int_{\mathrm{I.R.}} \frac{d^3p}{(2\pi)^3} \frac{1}{(p^2+m^2)^n}$$

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$$f_B(E,T) \longrightarrow \frac{T}{m}$$

EWPT & Perturbation Theory: IR Problem

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Small p regime

Effective expansion parameter

$$f_B(E,T) \longrightarrow \frac{T}{m}$$

EWPT & Perturbation Theory: IR Problem

Bosonic loop at T>0

Bose dist fn

$$I(T) = g^2 \int \frac{d^3p}{(2\pi)^3} f_B(E,T) \frac{1}{(p^2+m^2)^n} \longrightarrow \boxed{\frac{g^2 T}{m}} \int_{\text{I.R.}} \frac{d^3p}{(2\pi)^3} \frac{1}{(p^2+m^2)^n}$$

Effective expansion parameter

Small p regime

$$f_B(E,T) \longrightarrow \frac{T}{m}$$

Field-dependent thermal mass

$$m^2(\varphi,T) \sim C_1\, g^2\varphi^2 + C_2\, g^2 T^2 \equiv m_T^2(\varphi)$$

- *Near phase transition: $\varphi \sim 0$*
- *$m_T(\varphi) < g\, T$*

EWPT & Perturbation Theory

Expansion parameter

$$g_{\text{eff}} \equiv \frac{g^2 T}{\pi m_T(\varphi)}$$

Infrared sensitive near phase trans

SM lattice studies: *g_{eff} ~ 0.8 in vicinity of EWPT for m_H ~ 70 GeV* *

* *Kajantie et al, NPB 466 (1996) 189; hep/lat 9510020 [see sec 10.1]*

Challenges for Theory

Perturbation theory

- I.R. problem: poor convergence
- Thermal resummations
- Gauge Invariance (radiative barriers)
- RG invariance at T>0

Non-perturbative (I.R.)

- Computationally and labor intensive

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BSM proposals

Non-perturbative (I.R.)

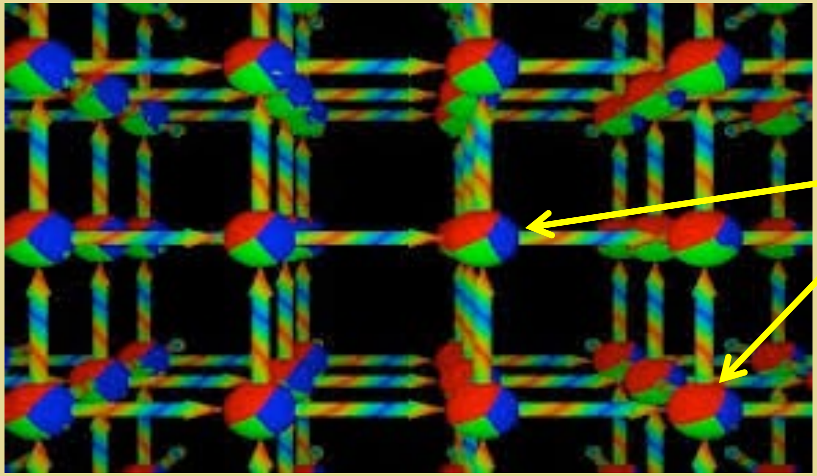
- Computationally and labor intensive

Simplest Model

SM Higgs + BSM Scalar

$$H \qquad\qquad\qquad \phi$$

"Non-perturbative" (lattice) methods



- *Discretize space*
- *H, ϕ take on different values at each site*
- *Vary field values using Monte Carlo methods*
- *Parameters depend on T*

Theory Meets Phenomenology

A. Non-perturbative

- *Most reliable determination of character of EWPT & dependence on parameters*
- *Broad survey of scenarios & parameter space not viable*

B. Perturbative

- *Most feasible approach to survey broad ranges of models, analyze parameter space, & predict experimental signatures*
- *Quantitative reliability needs to be verified*

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Benchmark pert theory

III. Theoretical Robustness

- *IR Problem*
- *Nucleation @ finite T: gauge invariance*
- *Wall velocity*

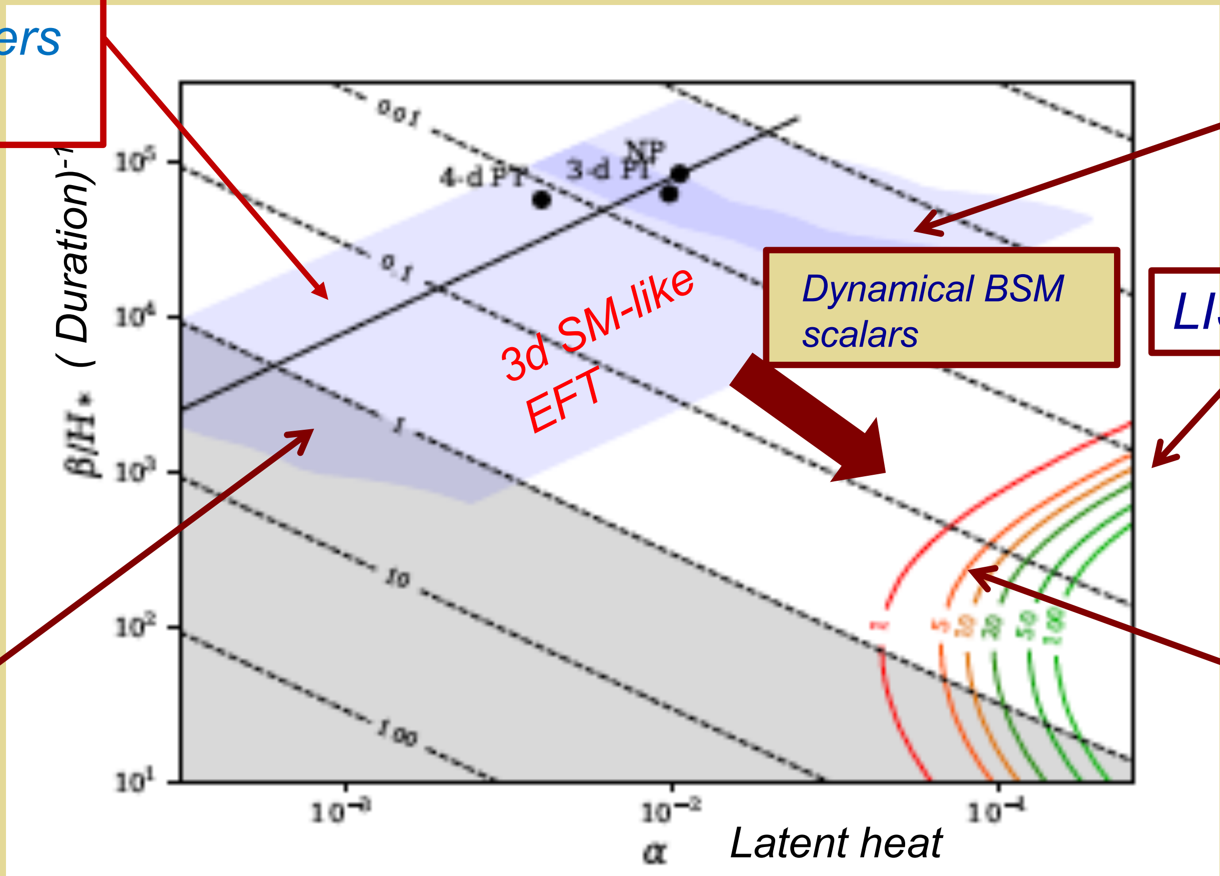
$$\boxed{\frac{\beta}{H_*} = T\frac{d}{dT}\frac{S_3}{T}}$$

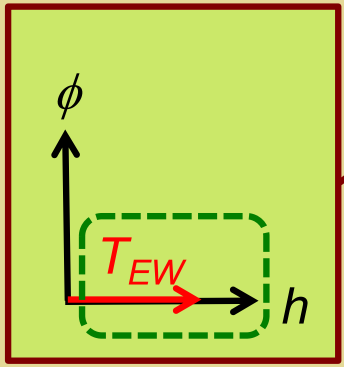
$+ \ T_N$

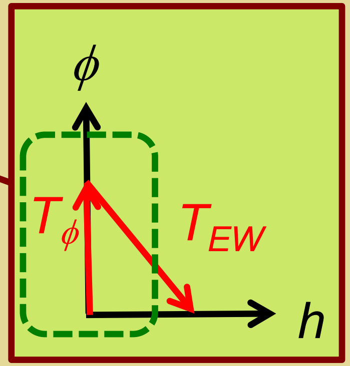
BSM Scalar: EWPT & GW

Collider probes of BSM parameters in $\mathcal{L}_{full}$

Integrate out BSM DOF





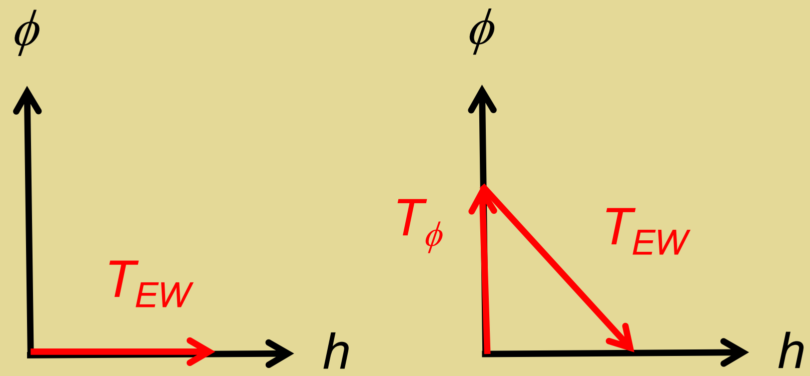


Radiative barrier from gauge sector

B9

Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

Tunneling @ T>0



Tunneling rate / unit volume:

$$\boxed{\Gamma = A e^{-\beta S_3/\hbar}\left[1 + \mathcal{O}(\hbar)\right]}$$

$$\boxed{A \sim \mathcal{O}(1) \times T^4}$$

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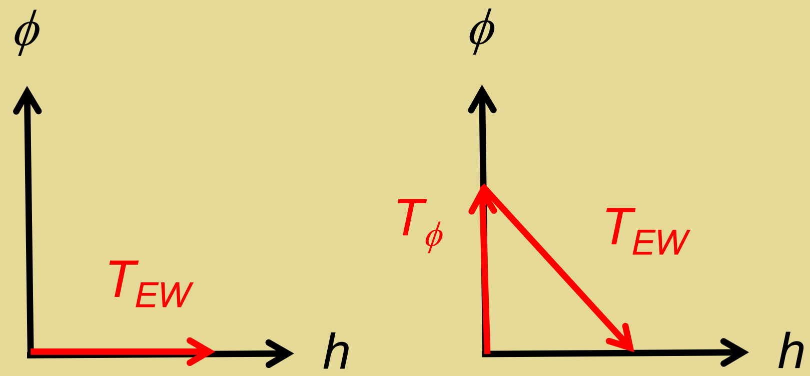


$$\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} = V'(\varphi, T)$$

Exponent in Γ

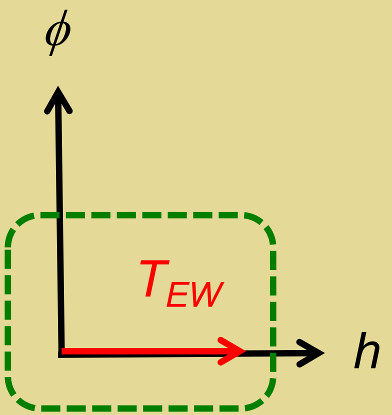
Path: minimize S_E

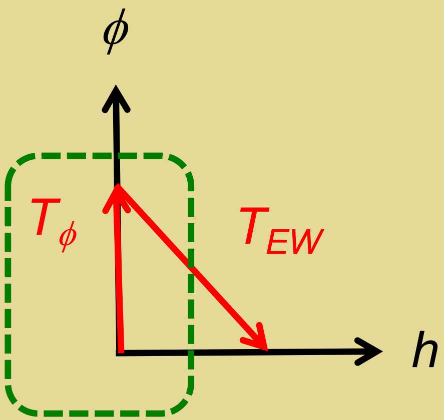
$$S_3 = \int d^3x \left\{ \frac{1}{2}(\vec{\nabla}\varphi)^2 + V(\varphi, T) \right\}$$

$$A \sim \mathcal{O}(1) \times T^4$$

Tunneling @ T>0

Radiative barriers → standard method gauge-dependent Γ





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SSB @ T>0 : Power Counting

Lofgren, MRM, Tenkanen, Schicho 2112.0752 → PRL

$$\mu_{\mathrm{eff}}^2 \equiv \boxed{\mu^2} + \boxed{(4\lambda + 3g^2)\frac{T^2}{12}}$$

T=0 parameter < 0

Thermal corrections > 0

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T=0 parameter < 0

Thermal corrections > 0

Near cancellation for $T \sim T_C$

For a range of $T \sim T_{nuc}$: N = 1

$$\boxed{\mu^2_{\rm eff} \sim \mathcal{O}(g^{2+N}\, T^2) < \mathcal{O}(g^2\, T^2)}$$

Power Counting

Lofgren, MRM, Tenkanen, Schicho 2112.0752 → PRL

$$\phi \sim T$$

$$\lambda \sim g^3$$

$$\mu^2 \sim g^2 T^2$$

$$\mu_{\mathrm{eff}}^2 \sim g^3 T^2$$

$$V_{\mathrm{LO}}^{\mathrm{eff}} = \frac{1}{2}\mu_{\mathrm{eff}}^2 \phi^2 + \frac{1}{4}\lambda\phi^4 \boxed{- \frac{g^3 T}{12\pi}\left[2\phi^3 + \left(\frac{1}{3}T^2 + \phi^2\right)^{\frac{3}{2}}\right]}$$

Radiative barrier:
ξ-independent

(Re) Organize the Perturbative Expansion

Illustrate w/ Abelian Higgs

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \Phi)^* (D_\mu \Phi) + \mu^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2 + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}$$

- *Lofgren, MRM, Tenkanen, Schicho 2112.0752 → PRL*
- *Hirvonen, Lofgren, MRM, Tenkanen, Schicho 2112.08912*

Full 3D effective action

$$S_3 = \int \mathrm{d}^3 x \Big[V^{\mathrm{eff}}(\phi, T) + \frac{1}{2} Z(\phi, T) \, (\partial_i \phi)^2 + \dots \Big]$$

Adopt appropriate power-counting in couplings

$$S_3 = a_0 g^{-\frac{3}{2}} + a_1 g^{-\frac{1}{2}} + \Delta$$

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- Hirvonen, Lofgren, MRM, Tenkanen, Schicho 2112.08912

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G.I. pertubative expansion only valid up to NLO → Δ: higher order contributions only via other methods

G.I. pertubative expansion

Tunneling @ T>0: G.I. & Nielsen Identities

Lofgren, MRM, Tenkanen, Schicho 2112.0752 → PRL

Adopt appropriate power-counting in couplings

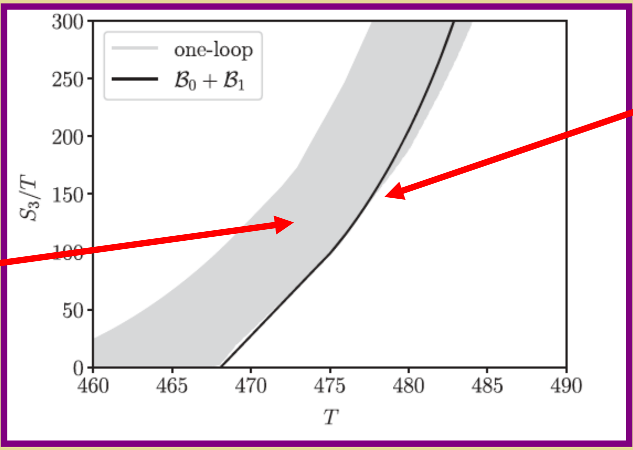
$$S_3 = a_0 g^{-\frac{3}{2}} + a_1 g^{-\frac{1}{2}} + \Delta$$

Order-by-order consistent with Nielsen Identities

$$\xi \frac{\partial S^{\text{eff}}}{\partial \xi} = -\int \mathrm{d}^d\mathbf{x} \frac{\delta S^{\text{eff}}}{\delta \phi(x)} \mathcal{C}(x)$$

$$\mathcal{C}(x) = \frac{ig}{2} \int \mathrm{d}^d\mathbf{y} \Big\langle \chi(x) c(x) \bar{c}(y) \times \Big[\partial_i B_i(y) + \sqrt{2} g \xi \phi \chi(y)\Big]\Big\rangle$$

Numerical comparison with conventional approach



Conventional: $0 < \xi < 4$

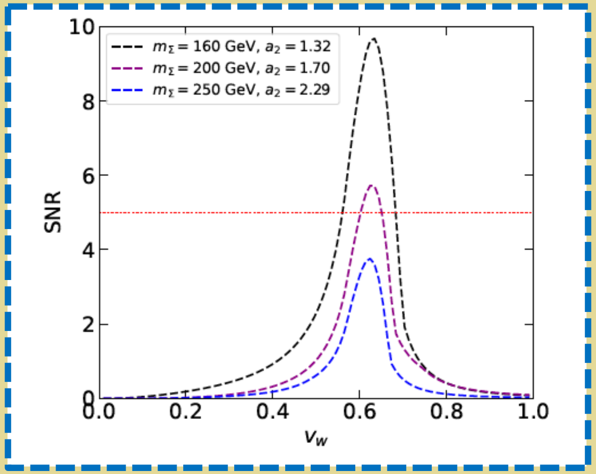
S_3 to $\mathcal{O}(g^{-1/2})$: $0 < \xi < 4$

III. Theoretical Robustness

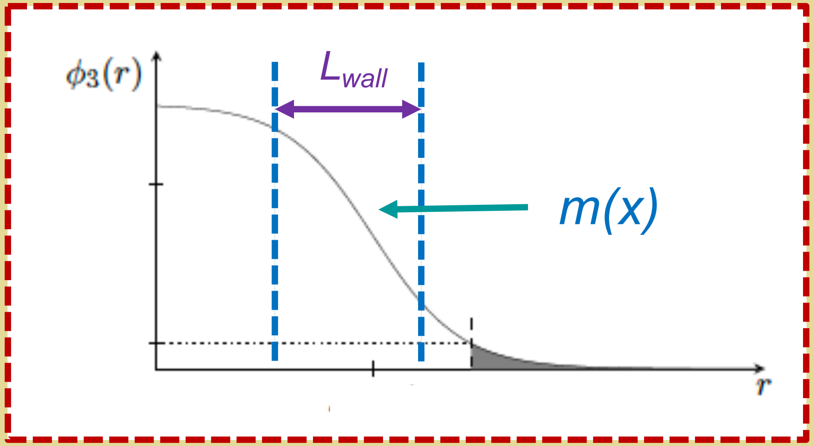
- *IR Problem*
- *Nucleation @ finite T: gauge invariance*
- *Wall velocity*

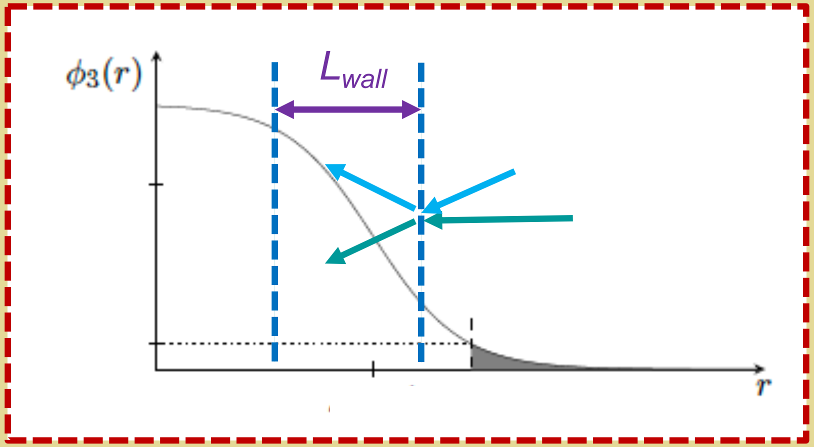
Computing Wall Velocity

LISA SNR

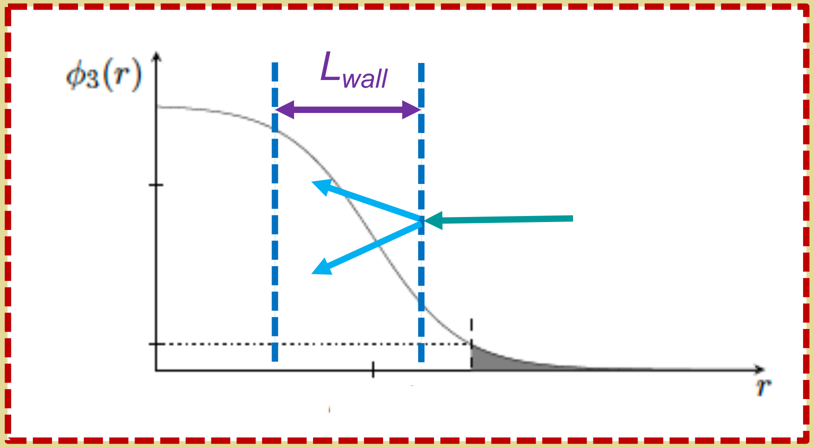


Mass variation





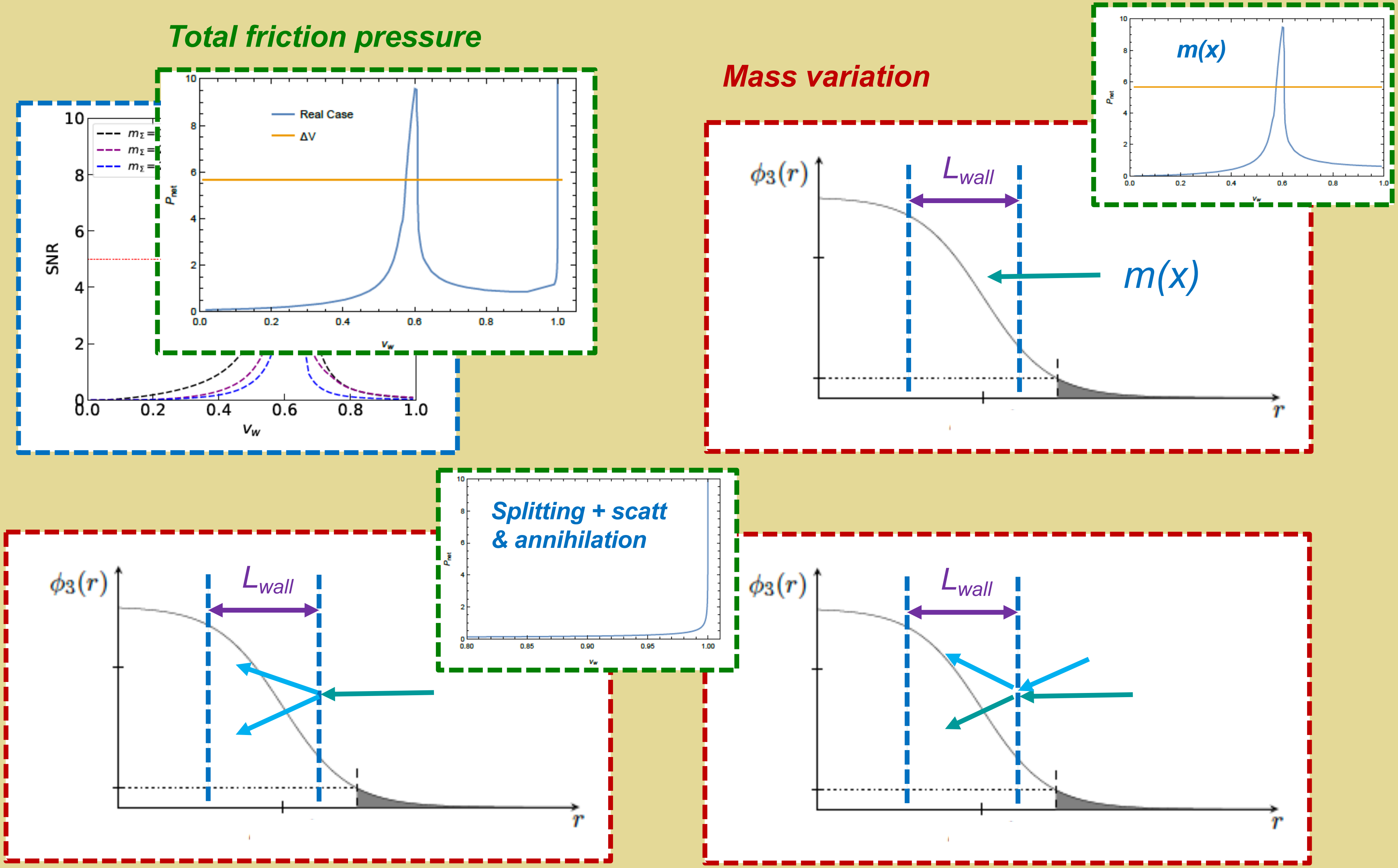
Splitting



Scattering & annihilation

MJRM, J. Zhu 2504.13724

Computing Wall Velocity



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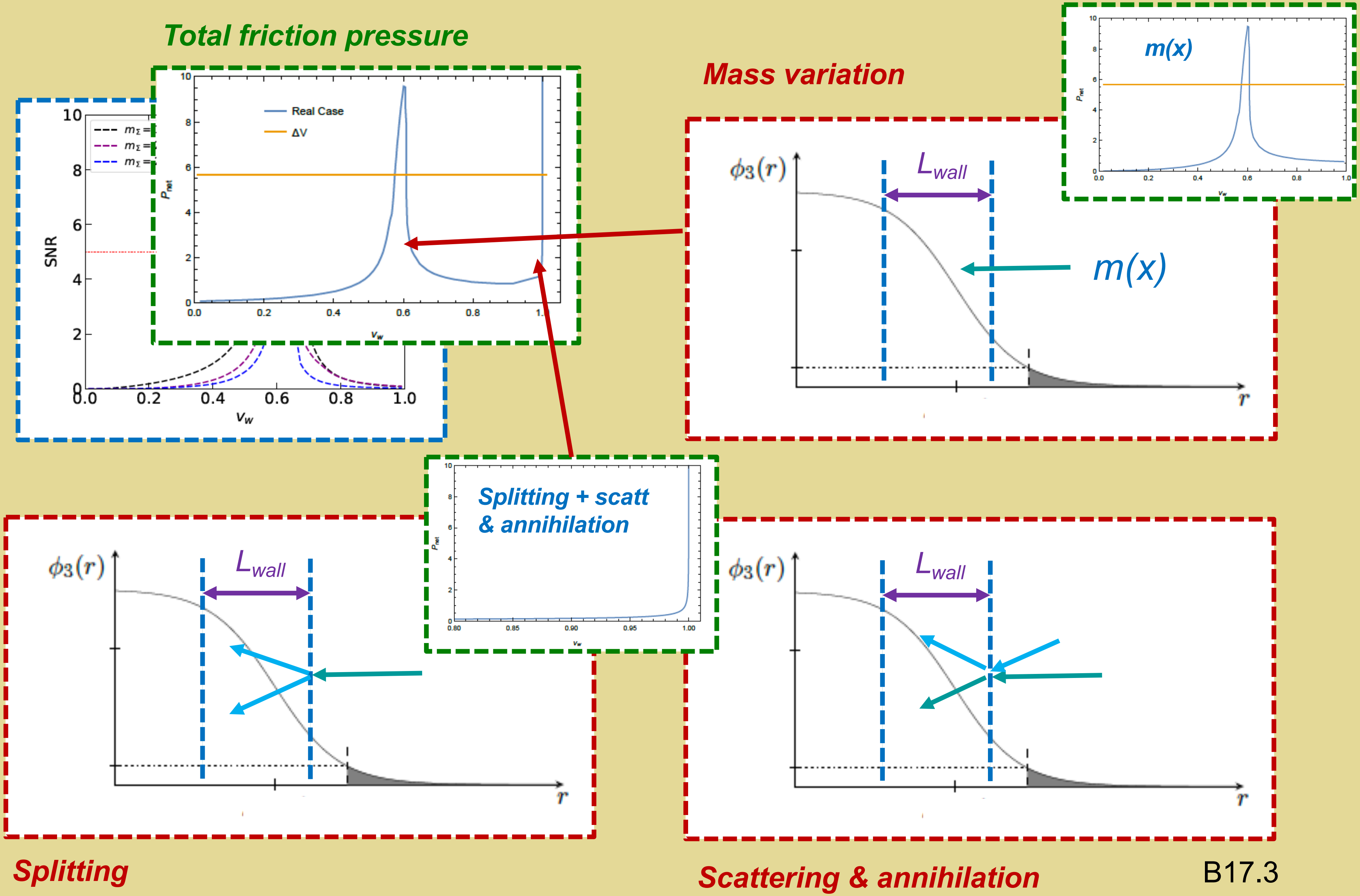

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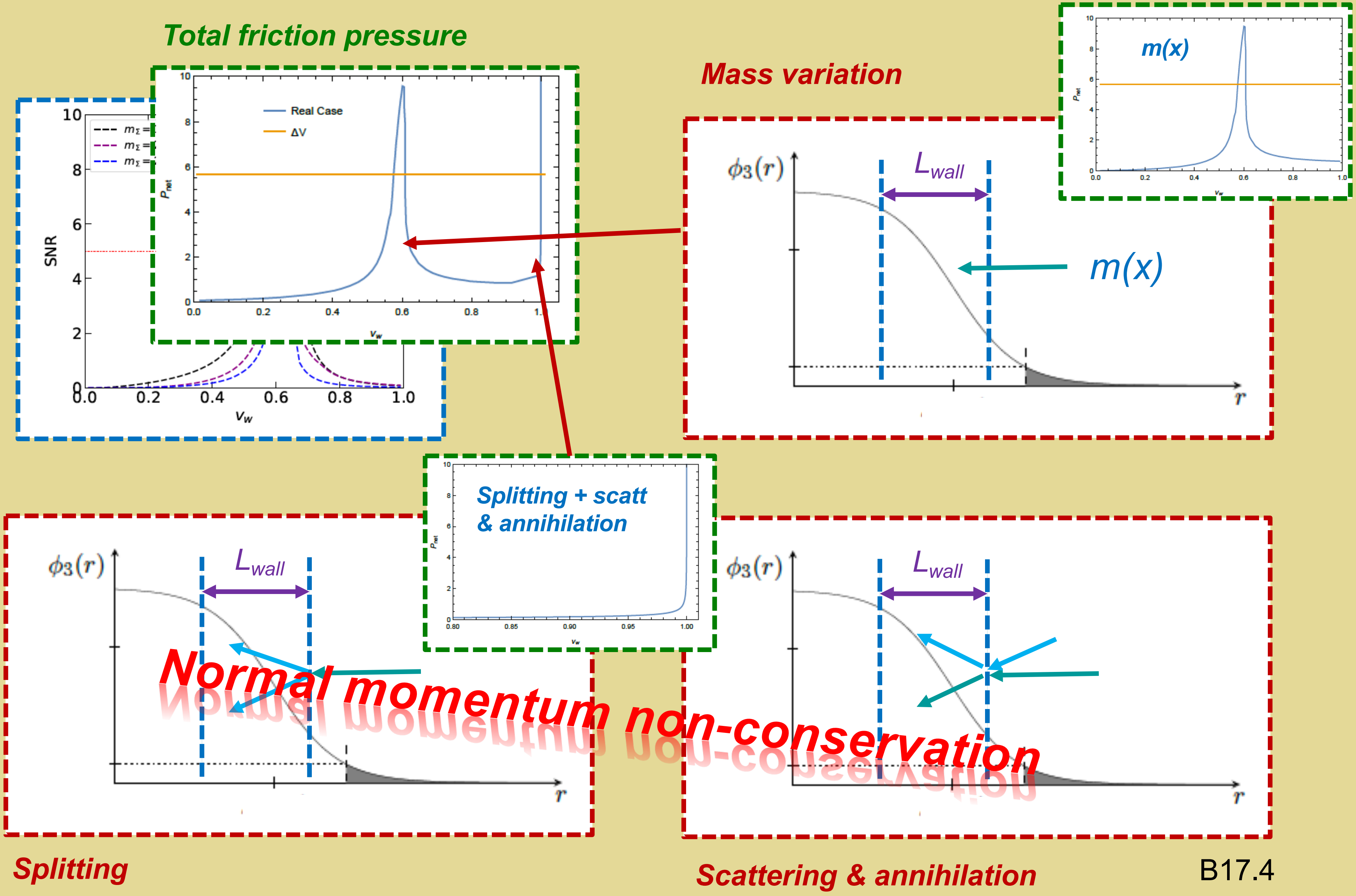
Computing Wall Velocity



Computing Wall Velocity

$$\partial_\mu T^{\mu\nu}_\phi + \partial_\mu T^{\mu\nu}_{\mathrm{plasma}} = 0$$

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EOM

Kadanoff-Baym → Boltzmann

$$\Delta V(\phi, T)$$

$$L[f_j] = \mathcal{C}[f_j]$$

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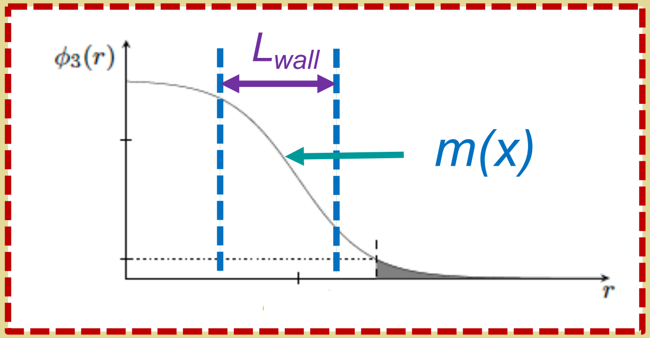

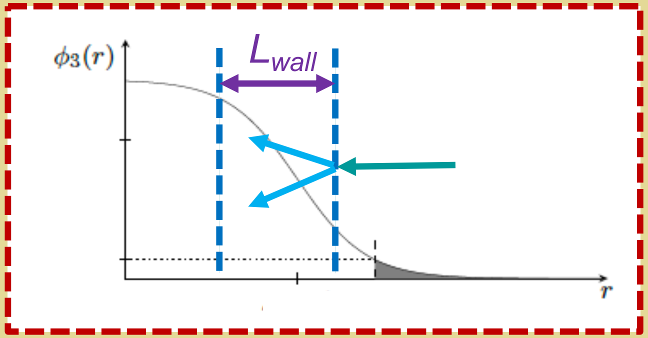

Computing Wall Velocity

Kadanoff-Baym "constraint eq"

$$-2ik\cdot\partial_X G^{\gtrless}+e^{-i\diamond}[m^2,G^{\gtrless}]=-ie^{-i\diamond}([\Pi^h,G^{\gtrless}]+[\Pi^{\gtrless},G^h]+\frac{1}{2}\{\Pi^>,G^<\}-\frac{1}{2}\{G^>,\Pi^<\}),$$

Project out distribution functions

$$\begin{aligned}\int_0^\infty \frac{dk^0}{2\pi}k_z\frac{d}{dz}G^<(k,z)+\frac{i}{2}\int_0^\infty\frac{dk^0}{2\pi}e^{-i\diamond}[m_a^2(z),G^<(k,z)]\\ +\frac{1}{4}\int_0^\infty\frac{dk^0}{2\pi}e^{-i\diamond}(\{\Pi_a^>,G_a^<\}-\{\Pi_a^<,G_a^>\})\\ =\frac{1}{2}\int_0^\infty\frac{k^0}{2\pi}e^{-i\diamond}([\Pi_a^h,G_a^<]+[\Pi_a^<,G_a^h]).\end{aligned}$$

Gradient expansion except on δ fns

$$\begin{aligned}&\left[2k_z\frac{\partial}{\partial z}-\frac{dm^2(z)}{dz}\frac{\partial}{\partial k_z}\right]\frac{f_\phi(k,z)}{E_k}\\ &\quad=-\int\frac{d^3\mathbf{p}}{(2\pi)^3}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}F(k,z)\frac{1+f_\Phi(p,z)}{2E_p}\frac{1+f_\Phi(p',z)}{2E_{p'}}\\ &\qquad\times(2\pi)^3\delta(E_k-E_p-E_{p'})\delta^2(\mathbf{k}_\perp-\mathbf{p}_\perp-\mathbf{p}'_\perp)+(\Delta p_z\leftrightarrow-\Delta p_z)\\ &\quad+\mathrm{InverseProcess},\end{aligned}$$

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Computing Wall Velocity

Kadanoff-Baym "constraint eq"

Wigner transformed Wightman functions

$$-2ik\cdot\partial_X G^{\gtrless} + e^{-i\diamond}[m^2, G^{\gtrless}] = -ie^{-i\diamond}([\Pi^h, G^{\gtrless}] + [\Pi^{\gtrless}, G^h] + \frac{1}{2}\{\Pi^>, G^<\} - \frac{1}{2}\{G^>, \Pi^<\}),$$

"Diamond operator"

$$\diamond\Big(A(k,x)B(k,x)\Big) = \frac{1}{2}\left(\frac{\partial A}{\partial x^\mu}\frac{\partial B}{\partial k_\mu} - \frac{\partial A}{\partial k_\mu}\frac{\partial B}{\partial x^\mu}\right)$$

Project out distribution functions

$$\begin{aligned}\int_0^\infty \frac{dk^0}{2\pi} k_z \frac{d}{dz} G^<(k,z) + \frac{i}{2}\int_0^\infty \frac{dk^0}{2\pi} e^{-i\diamond}[m_a^2(z), G^<(k,z)] \\ + \frac{1}{4}\int_0^\infty \frac{dk^0}{2\pi} e^{-i\diamond}(\{\Pi_a^>, G_a^<\} - \{\Pi_a^<, G_a^>\}) \\ = \frac{1}{2}\int_0^\infty \frac{k^0}{2\pi} e^{-i\diamond}([\Pi_a^h, G_a^<] + [\Pi_a^<, G_a^h]).\end{aligned}$$

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MJRM, J. Zhu 2504.13724

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Gradient expansion except on δ fns

mass variation

Collision term: p_z non-cons

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$$F(k,z)=\int dz' f_\phi(k,z')Y(z')Y(2z-z')e^{-2i\Delta p_z(z-z')}$$

Gravitational Radiation

$$\boxed{\frac{d\log\Omega_{\mathrm{GW}}}{d\log k} = K^2\,(kL_f)^3 \int \frac{dt_1}{t_1} \int \frac{dt_2}{t_2}\,\cos\left[k(t_1 - t_2)\right]\tilde{\Pi}(k, t_1, t_2)}$$

$$\boxed{K = \frac{\kappa\,\alpha_n}{1+\alpha_n}}$$

Latent heat → KE fraction

κ : "efficiency factor"

Mean bubble separation

Wall velocity

Sound speed

$$\boxed{L_f \sim R_*}$$

$$\boxed{R_* = \frac{(8\pi)^{1/3}}{\beta}\,\mathrm{Max}(v_{\mathrm{w}},\, c_s)}$$

Gravitational Radiation

Spectrum

$$h^2\Omega_{\rm env}(f) \;=\; 1.67\times10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{0.11\,v_w^3}{0.42+v_w^2}\right)S_{\rm env}(f)$$

Fluid

$$h^2\Omega_{\rm sw}(f) = 2.65\times10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w\,S_{\rm sw}(f)$$

Sound

$$h^2\Omega_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w\,S_{\rm turb}(f)$$

Turbulence

Peak frequency

$$f_{\rm env} = 16.5\times10^{-3}\,{\rm mHz}\left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

$$f_{\rm sw} = 1.9\times10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

$$f_{\rm turb} = 2.7\times10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

Caprini et al, 1512.06239 [astro-ph]

Gravitational Radiation

Spectrum

Thermal QFT

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Gravitational Radiation

Spectrum

Thermal QFT **Transport Theory**

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Fluid

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Peak frequency

$$f_{\rm env} = 16.5\times10^{-3}\,{\rm mHz}\left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

$$f_{\rm sw} = 1.9\times10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

$$f_{\rm turb} = 2.7\times10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

Caprini et al, 1512.06239 [astro-ph]

Gravitational Radiation

Spectrum — **Thermal QFT** — **Transport Theory**

$$h^2\Omega_{\rm env}(f) \;=\; 1.67\times 10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{0.11\,v_w^3}{0.42+v_w^2}\right)S_{\rm env}(f)$$

Fluid

$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w S_{\rm sw}(f)$$

Sound

$$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w S_{\rm turb}(f)$$

Turbulence

Peak frequency

$$f_{\rm env} = 16.5\times 10^{-3}\,{\rm mHz}\left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

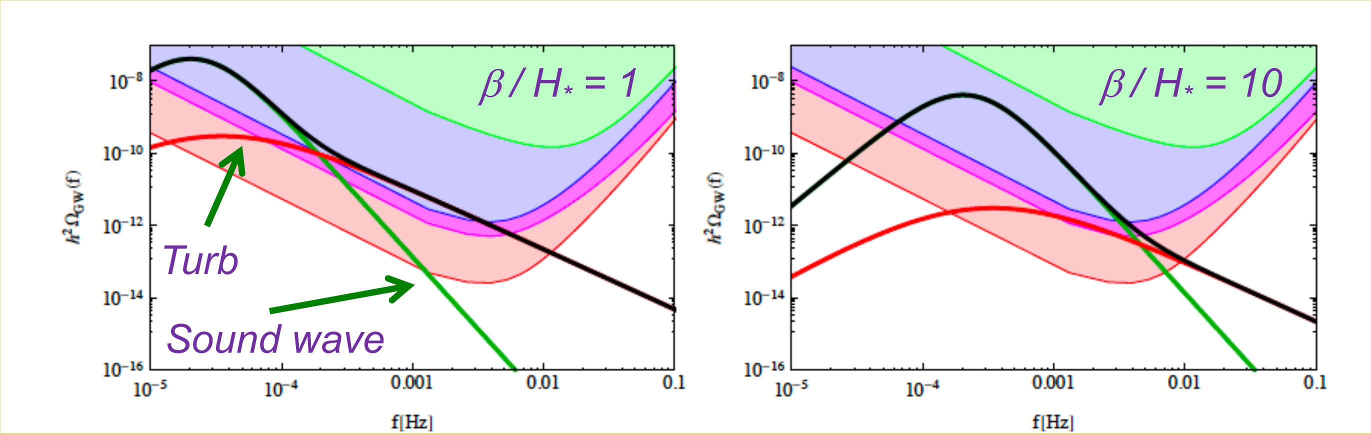
$$f_{\rm sw} = 1.9\times 10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

$$f_{\rm turb} = 2.7\times 10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

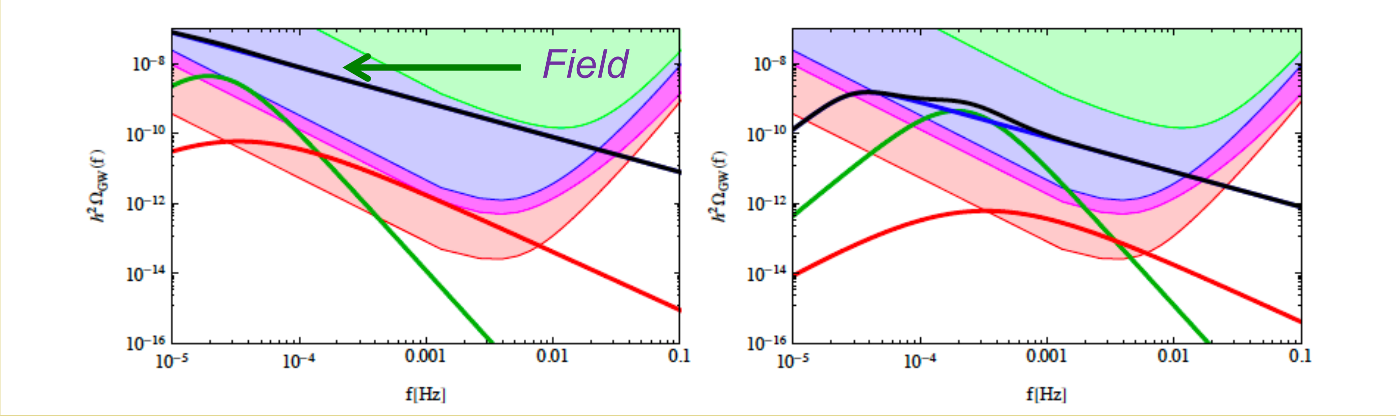
Caprini et al, 1512.06239 [astro-ph]

Gravitational Radiation

Non-run away bubbles



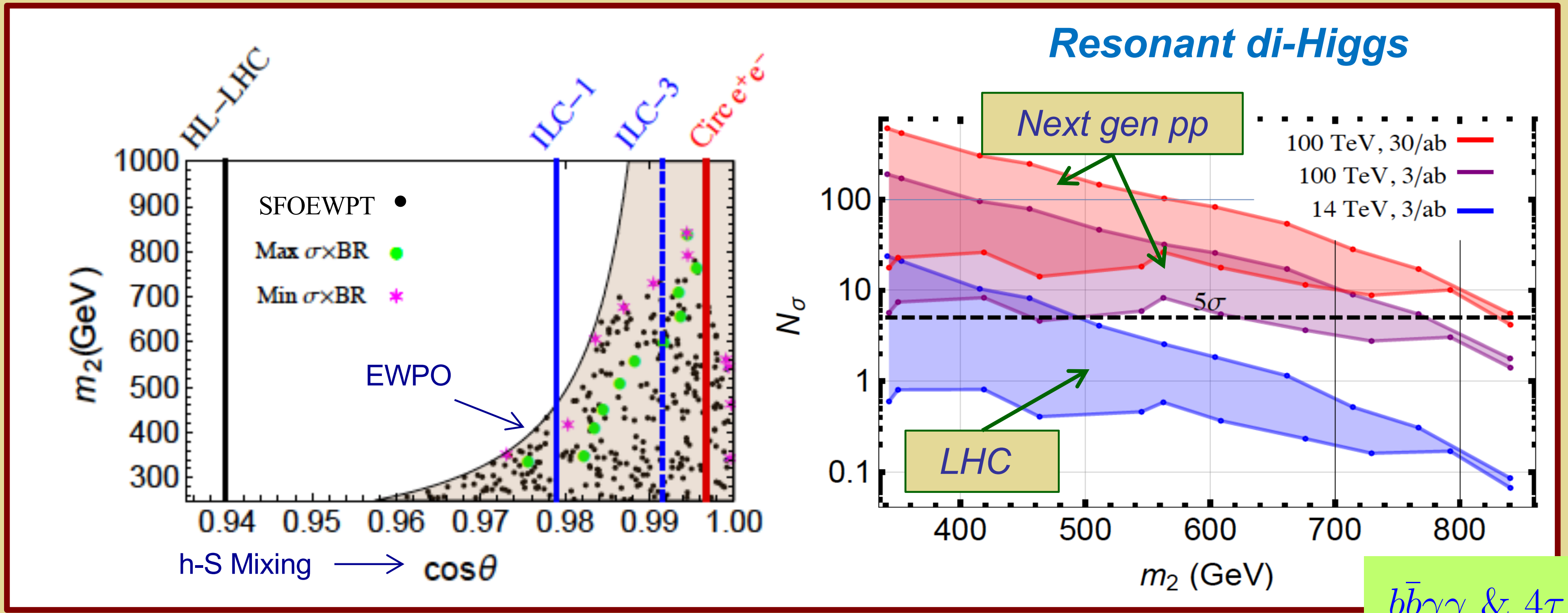
Run away bubbles



Caprini et al, 1512.06239 [astro-ph]

Singlets: Precision & Res Di-Higgs Prod

*SFOEWPT Benchmarks**: Resonant di-Higgs & precision Higgs studies*



$b\bar{b}\gamma\gamma$ & 4τ

Kotwal, No, R-M, Winslow 1605.06123

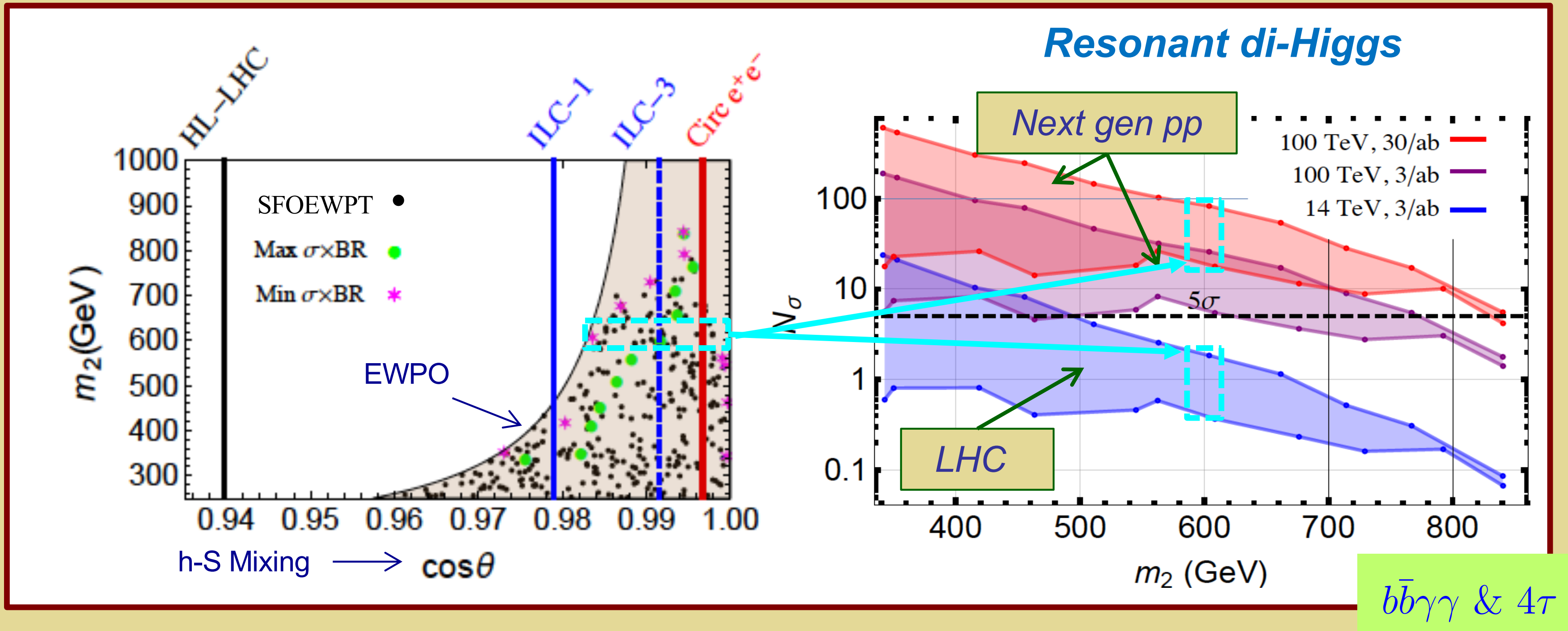
** ***Perturbative thermal QFT***

See also: Huang et al, 1701.04442;
Li et al, 1906.05289

B26.1

Singlets: Precision & Res Di-Higgs Prod

*SFOEWPT Benchmarks**: Resonant di-Higgs & precision Higgs studies*



Kotwal, No, R-M, Winslow 1605.06123

** ***Perturbative thermal QFT***

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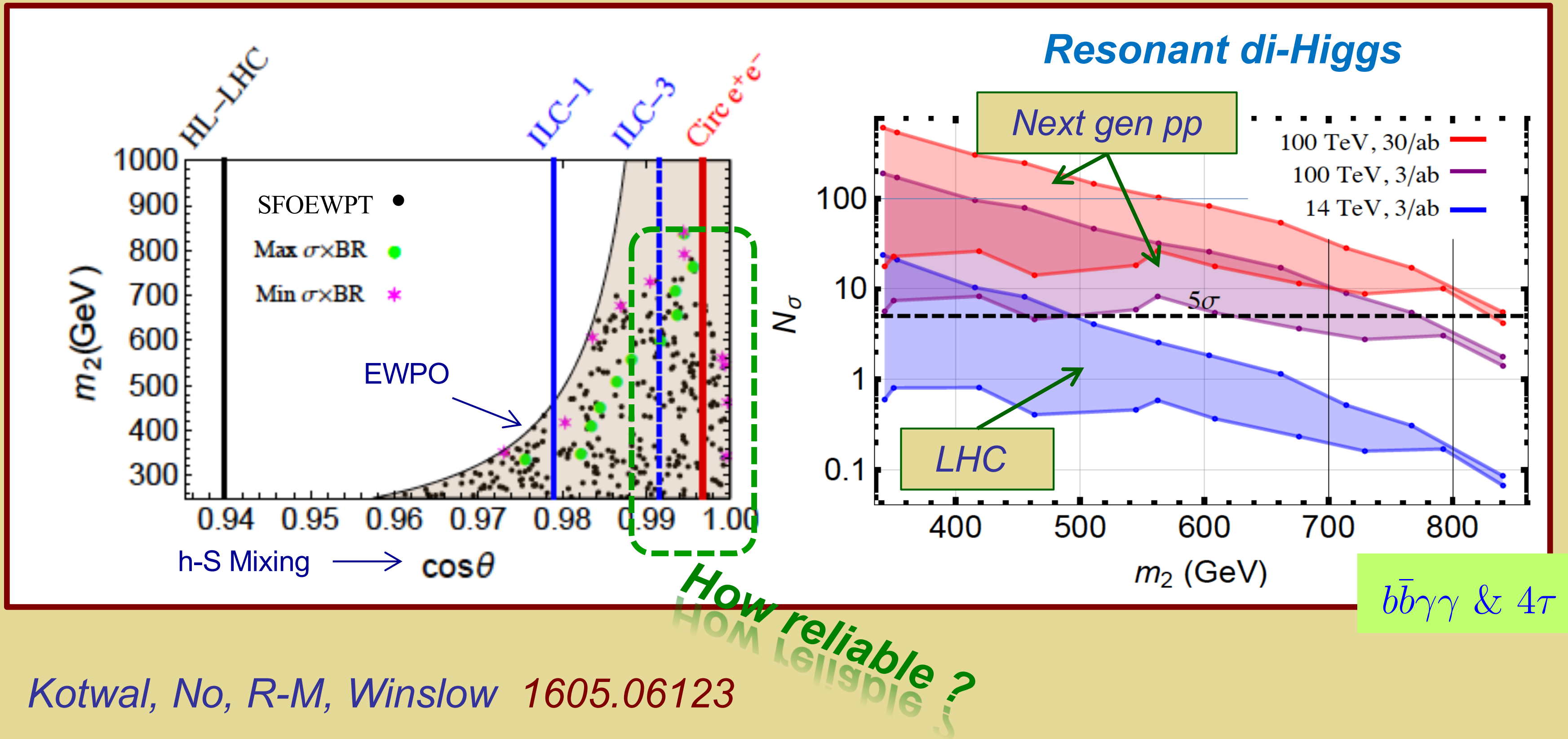


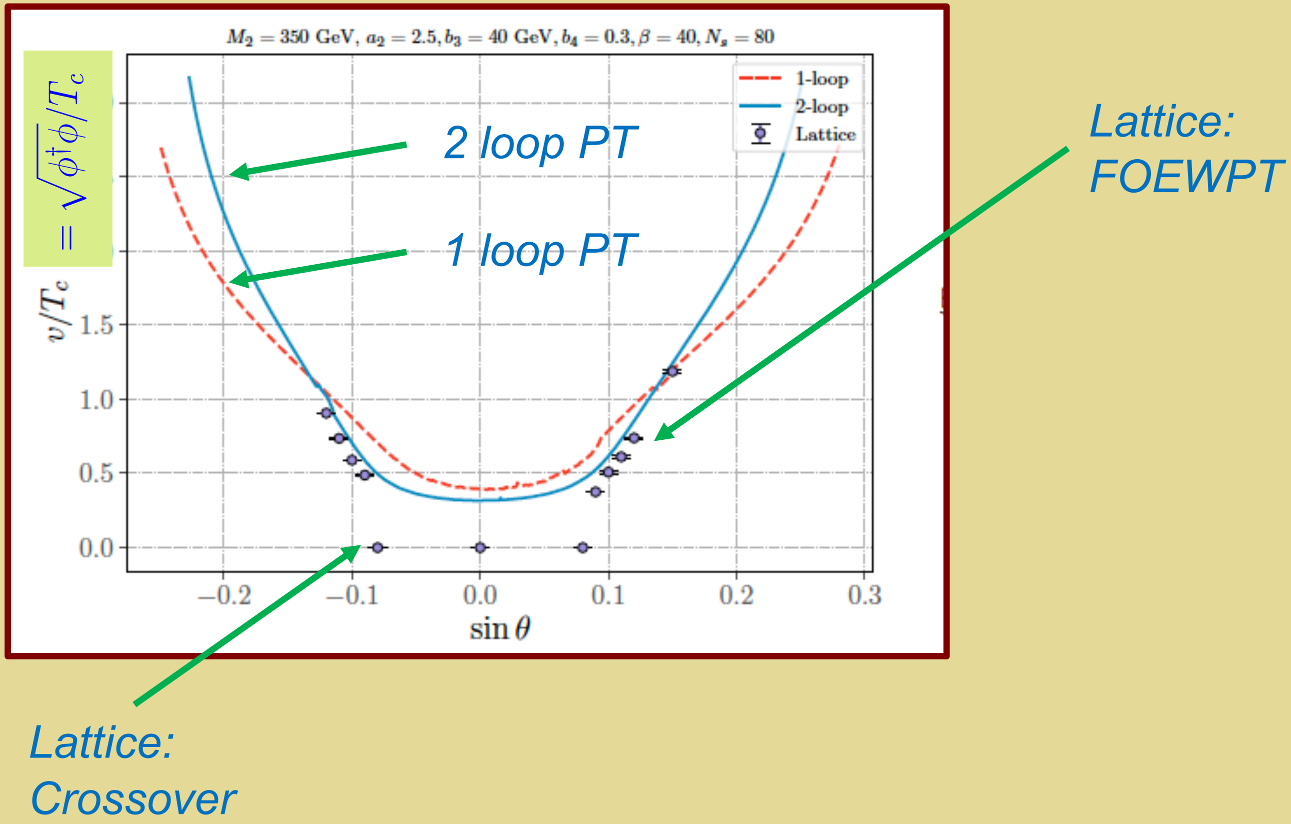
Kotwal, No, R-M, Winslow 1605.06123

*** **Perturbative thermal QFT***

See also: Huang et al, 1701.04442;
Li et al, 1906.05289

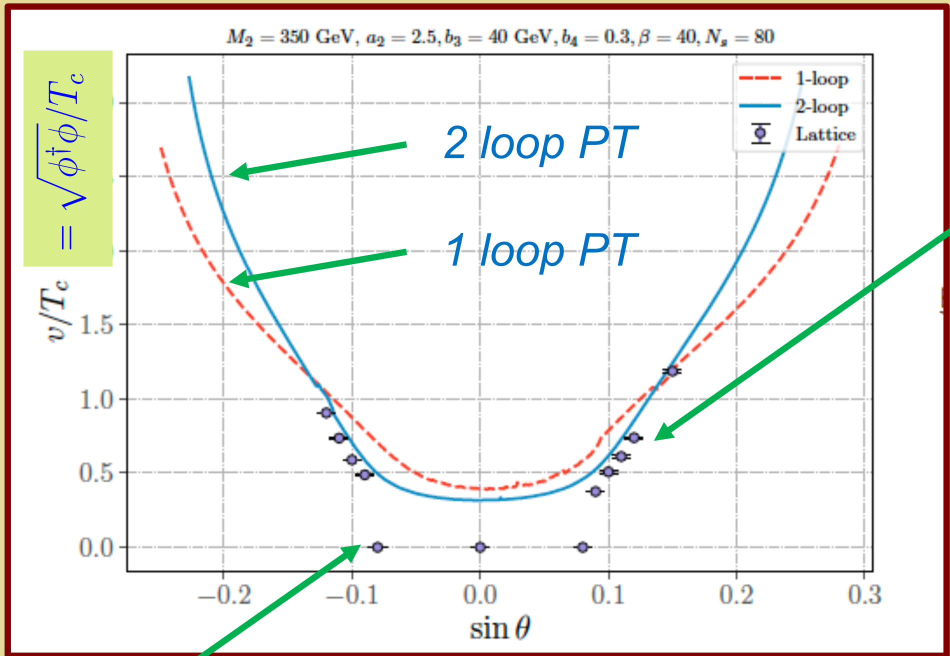
Lauri Niemi, MJRM, Gutao Xia, 2405.01191

Singlets: Lattice vs. Pert Theory



Lauri Niemi, MJRM, Gutao Xia, 2405.01191

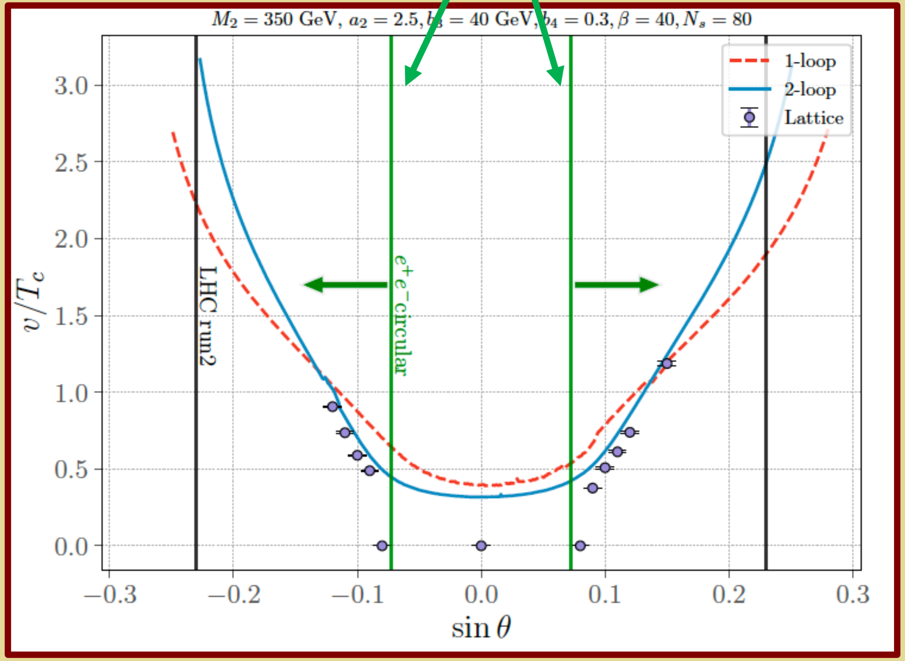
Singlets: Lattice vs. Pert Theory



Lattice: FOEWPT

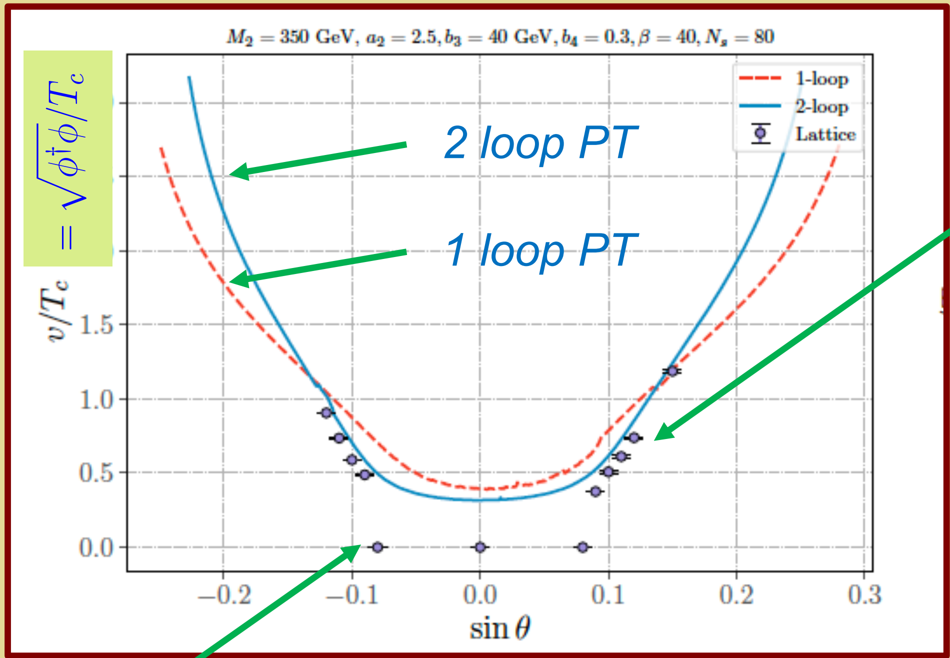
Future e⁺e⁻

Lattice: Crossover



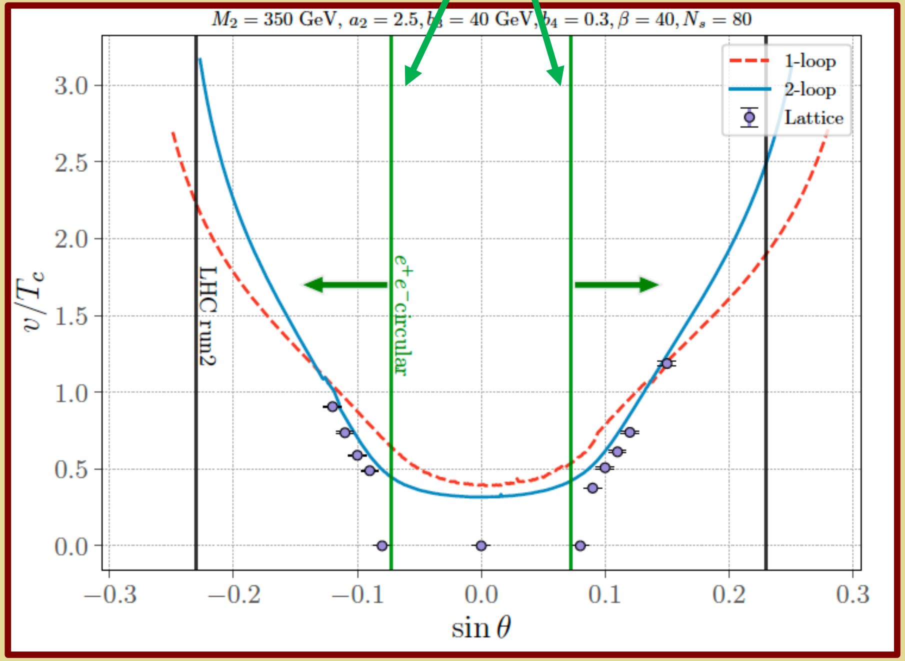
Lauri Niemi, MJRM, Gutao Xia, 2405.01191

Singlets: Lattice vs. Pert Theory



Lattice: FOEWPT

Future e^+e^-

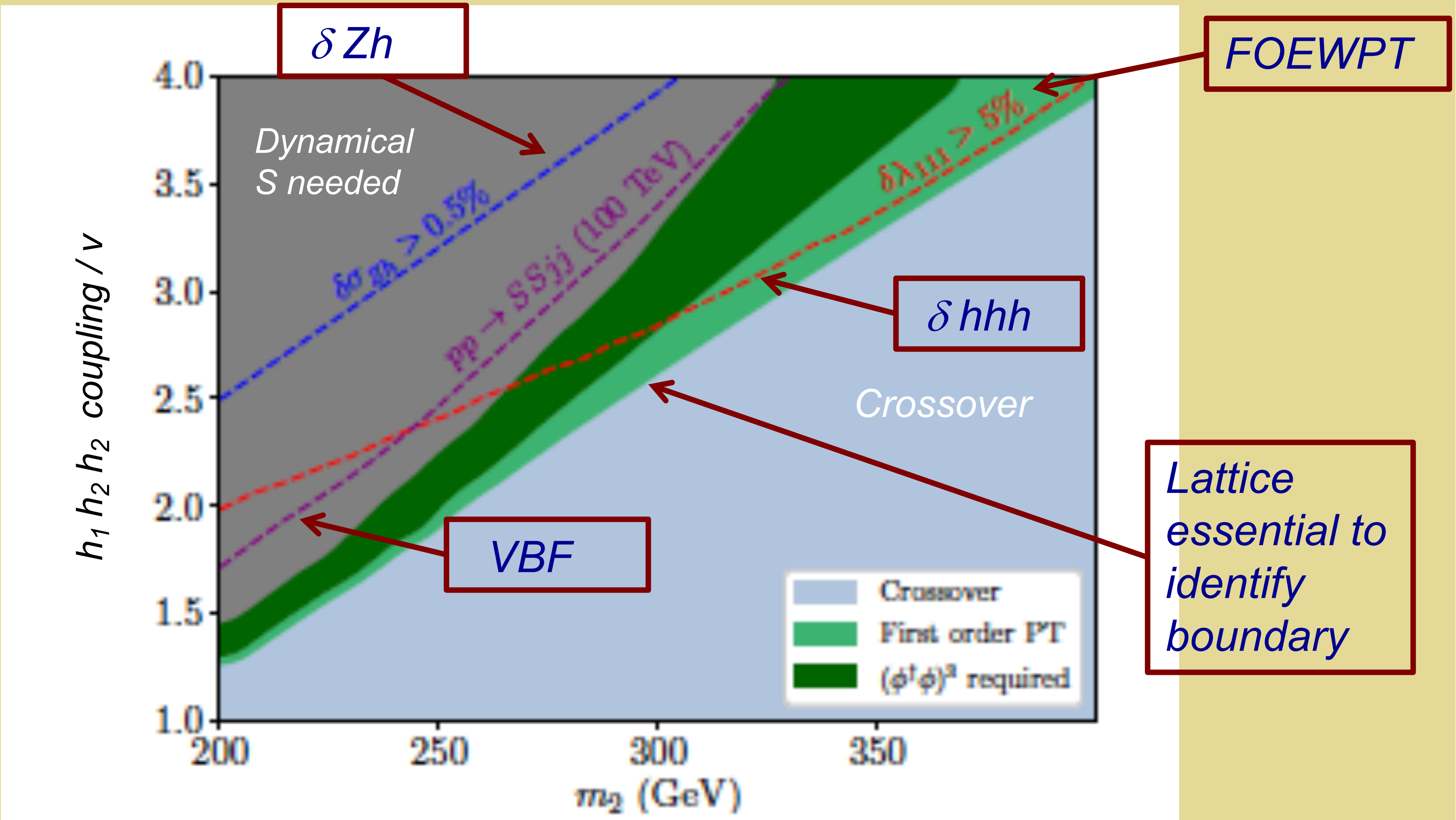


Lattice: Crossover

- Lattice: crossover-FOEWPT boundary
- FOEWPT region: PT-lattice agreement
- Pheno: precision Higgs studies may be sensitive to a greater portion of FOEWPT-viable param space than earlier realized

B27.3

Non-Dynamcial Real Singlet & EWPT: Probes

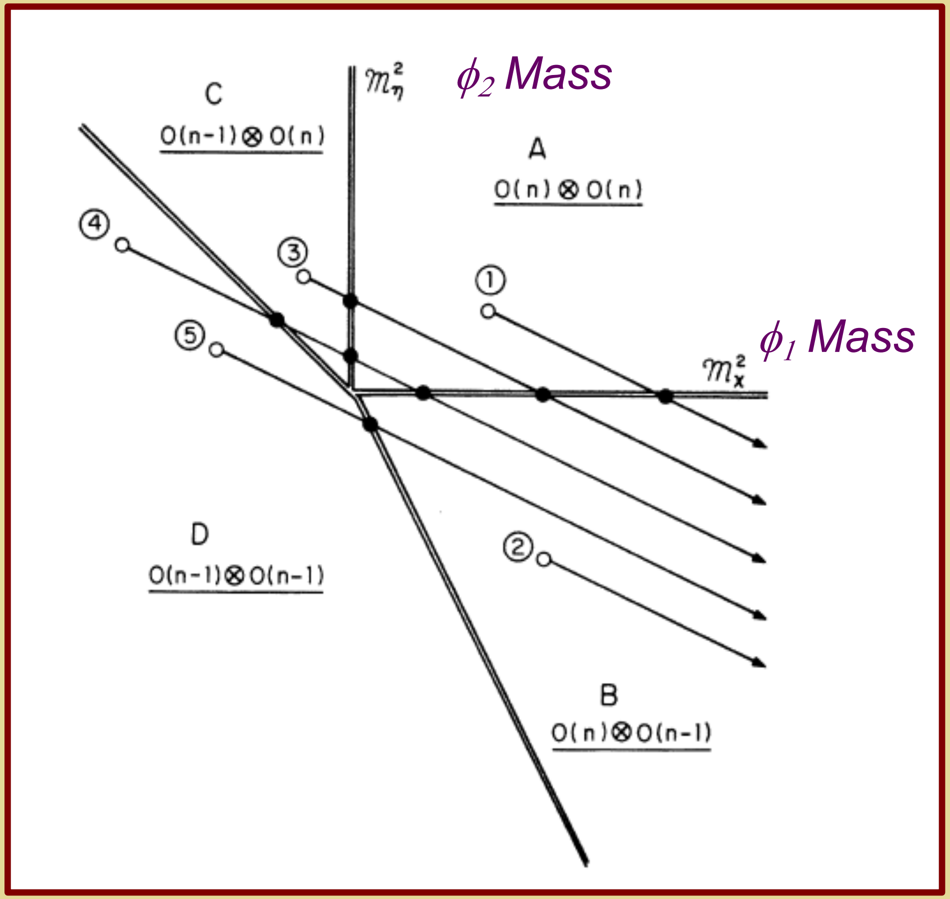


Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

- One-step
- Non-perturbative

B28.1

Patterns of Symmetry Breaking



S. Weinberg, PRD 9 (1974) 3357

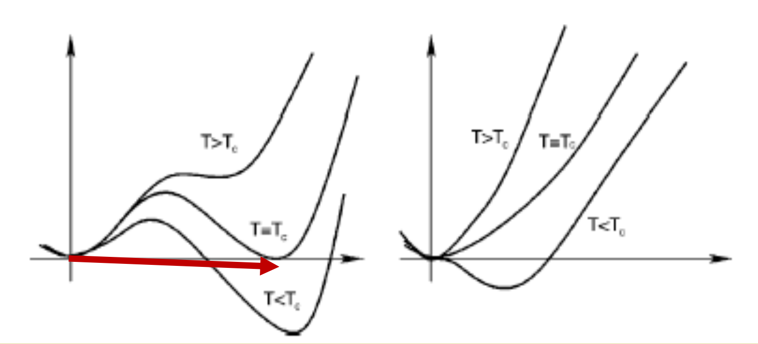
Phase Transitions

Ehrenfest classification:

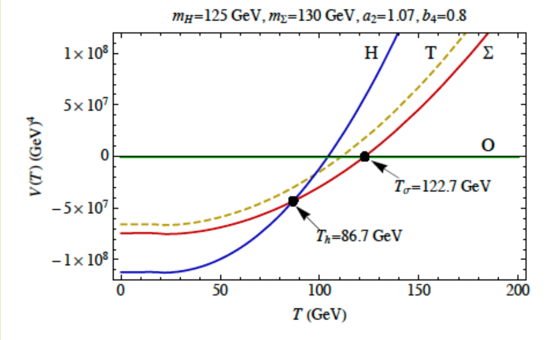
1st order PT: discontinuous first derivative of free energy

$$\left(\frac{\partial F}{\partial T}\right)_V = -S \qquad \left(\frac{\partial G}{\partial T}\right)_P = -S$$

Jump in entropy (i.e., entropy injection)



Real triplet model



H. Patel & MJRM '12 [1201.0809]