Institute of Particle Physics
粒子物理研究所

b-hadron FCNC decays as a probe of light Dark Matter

Xing–Bo Yuan (袁兴博)

Central China Normal University (华中师范大学)

Biao-Feng Hou, Xin-Qiang Li, Meng Shen, Ya-Dong Yang, XBY, JHEP06(2024)172
Meng-Chao Gao, Xin-Qiang Li, Ya-Dong Yang, XBY, Xin Zhang, work in progress

Physics beyond the Standard Model

Problems of the SM $\Longrightarrow$ Physics beyond the SM

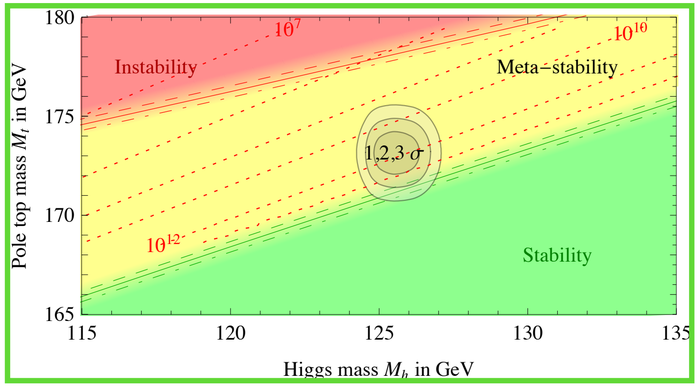
experimental

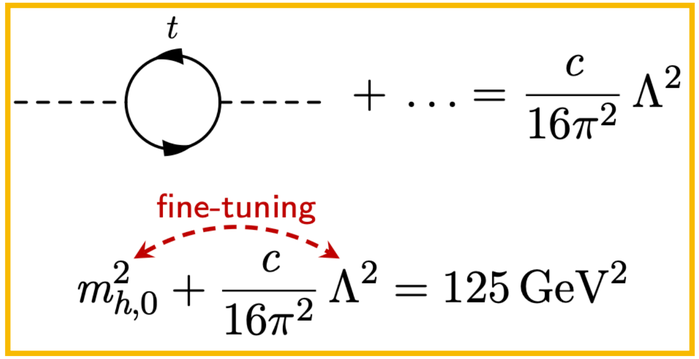
- Dark Matter (26%)
- Dark Energy (69%)
- Matter-antimatter asymmetry
- Anomalies in experimental data

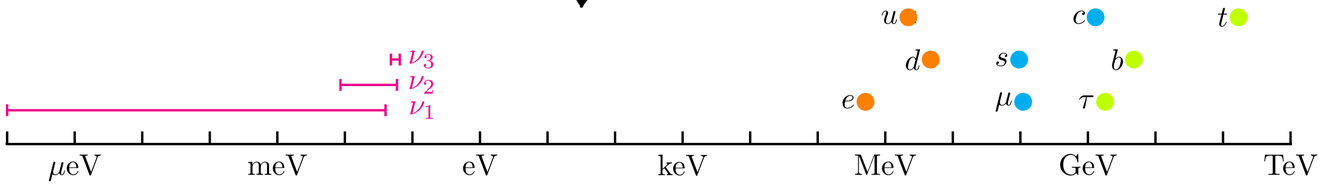
- Neutrino masses
- Strong CP problem
- Vacuum metastability
- NP flavour puzzle

conceptual

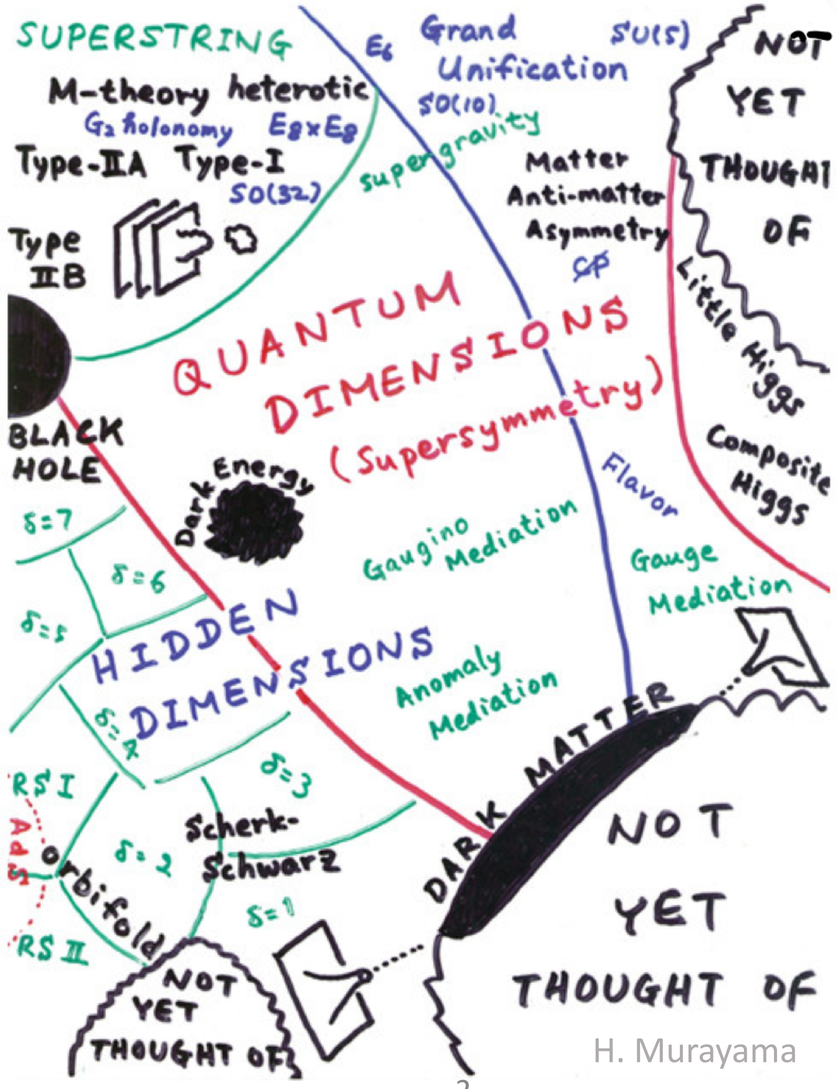
- Gravity
- Hierarchy problem
- SM flavour puzzle
- Family replication …







Theories beyond the SM

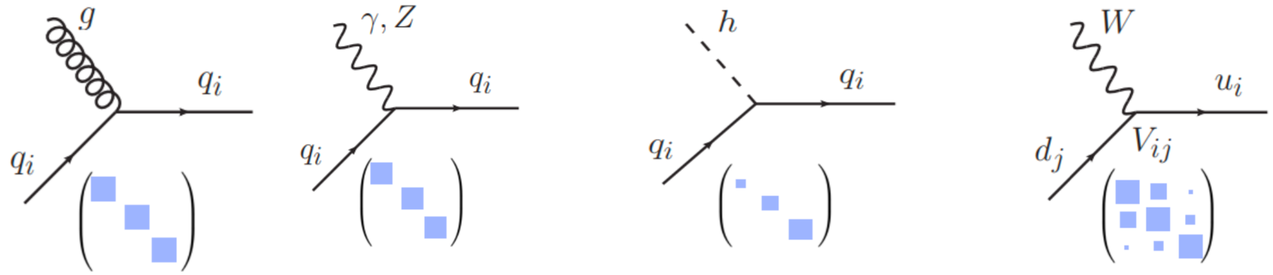


H. Murayama

Flavour Physics as a Probe of New Physics

- **Flavour universal**
  - couplings $\propto \delta_{ij}$ in flavour space
  - example: strong and electromagnetic interactions
  - consequence of gauge invariance
- **Flavour diagonal**
  - couplings $\propto \lambda_i \delta_{ij}$ (diagonal, but not necessarily universal)
  - example: Yukawa interactions
- **Flavour violation (changing)**
  - couplings involve different quarks
  - no flavour violation in lepton sector ($m_\nu = 0$)
  - example: $W^\pm$ interactions in quark section
- **Flavour Changing Neutral Current (FCNC)**
  - absent at the tree-level
  - arise at the one-loop, but suppressed by GIM mechanism
- **Why flavour physics**
  - New physics $\Longleftarrow \mathcal{O}(10^9)$ $B\bar{B}$ events at BaBar and Belle
  - structure of CKM and mass
  - CP violation
  - strong interaction

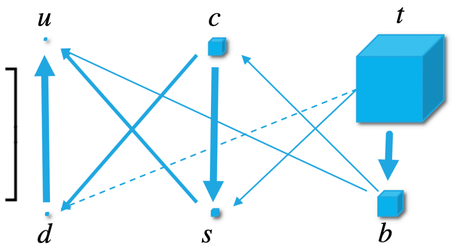
flavour universal	flavour diagonal	flavour violation



CKM matrix

- Cabibbo–Kobayashi–Maskawa matrix

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$



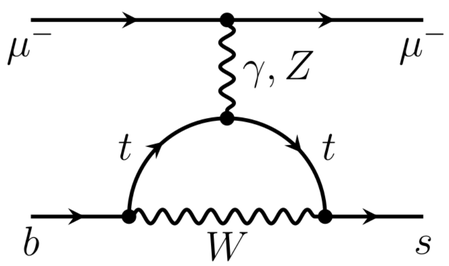
- 3 mixing angules and 1 CP phase
- CP violation in the Standard Model

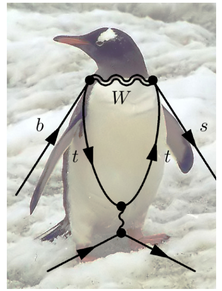
↓

not enough to explain the baryon asymmetry in our universe

↓

new CP violation sources





penguin diagram

experimental status

no evidence of NP	$> 5\,\sigma$
but, anomalies	$2 \sim 4\,\sigma$

Theoretically cleanest processes in heavy flavour physics

$B_{s,d} - \bar{B}_{s,d}$ mixing

$b \to u(c)\ell\bar{\nu}$: $B \to D\ell\bar{\nu}$, $B \to D^*\ell\bar{\nu}$, ...

$b \to s(d)\gamma$: $B \to X_s\gamma$, $B \to K^*\gamma$, ...

$b \to s(d)\ell^+\ell^-$: $B_s \to \ell^+\ell^-$, $B \to K^*\ell^+\ell^-$, ...

$b \to s(d)\nu\bar{\nu}$: $B \to K\nu\bar{\nu}$, $K \to \pi\nu\bar{\nu}$, ...

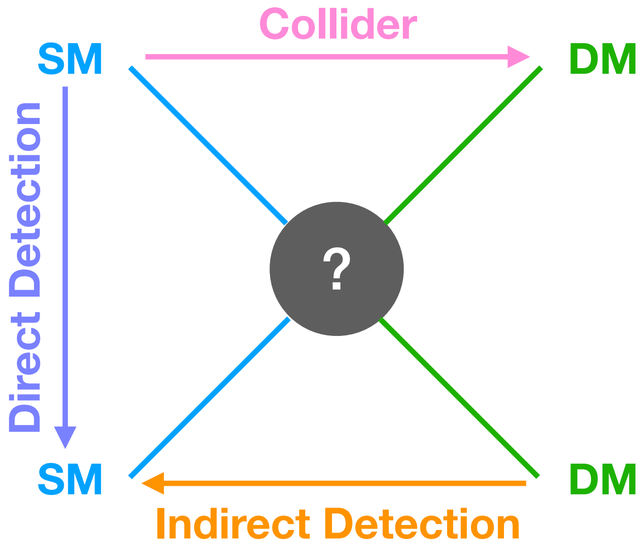
NP Effects $< \mathcal{O}(20\%)$

Signal: $B \to K + \text{inv}$ NOT measured yet !

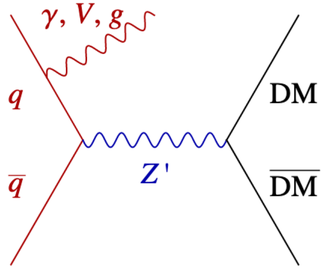
$B \to K + \nu + \bar{\nu}$

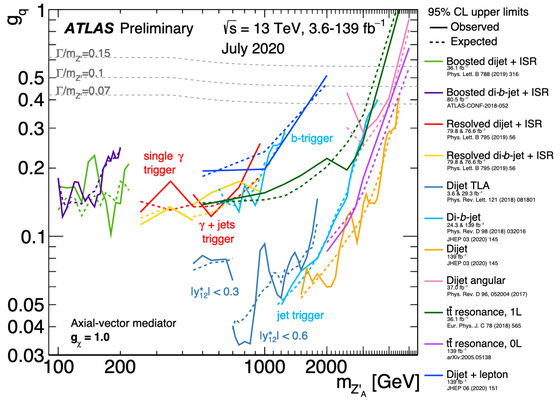
$B \to K +$ DM+DM

Dark Matter Detection

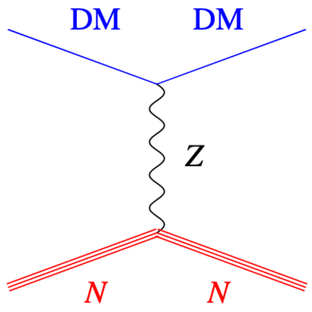


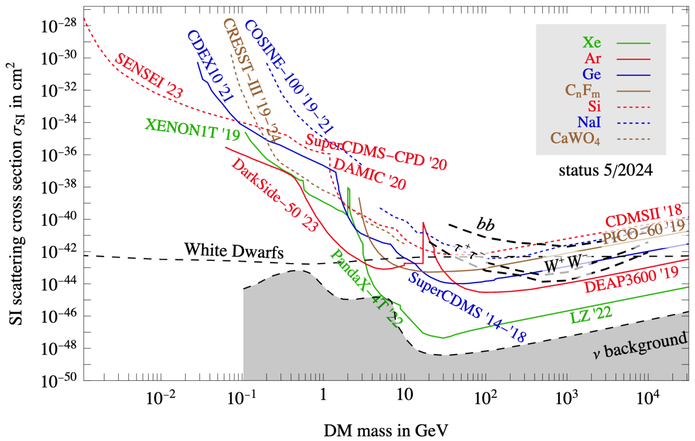
Collider



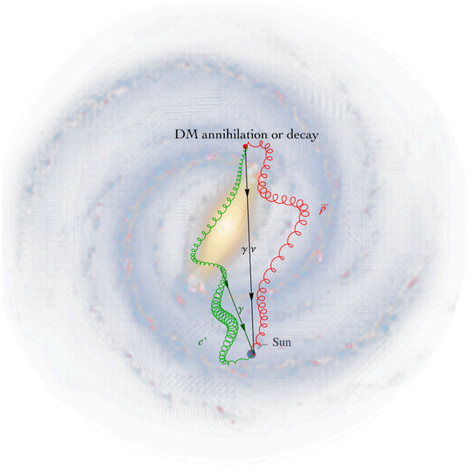


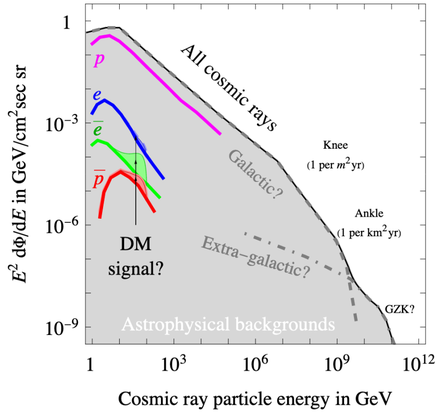
Direct Detection



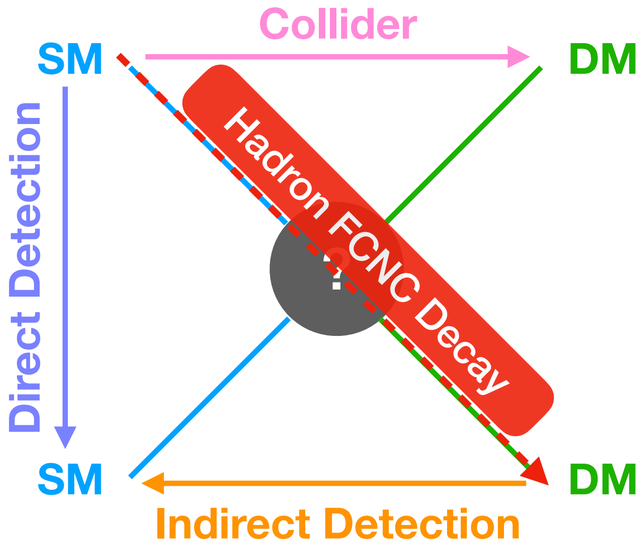


Indirect Detection

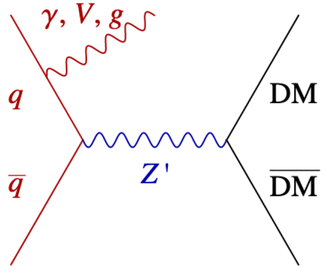


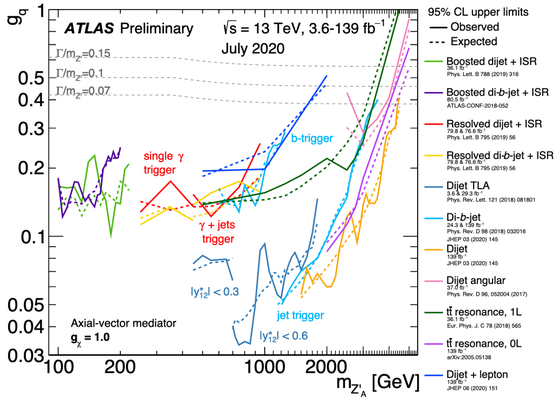


Dark Matter Detection

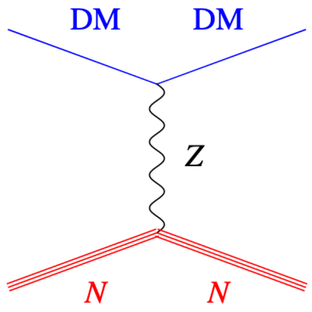


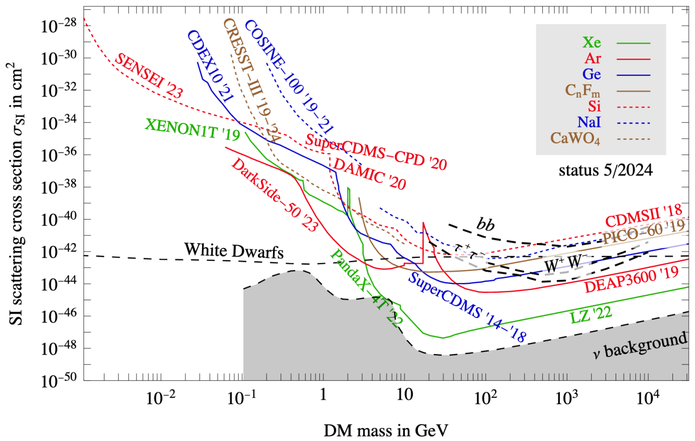
Collider





Direct Detection

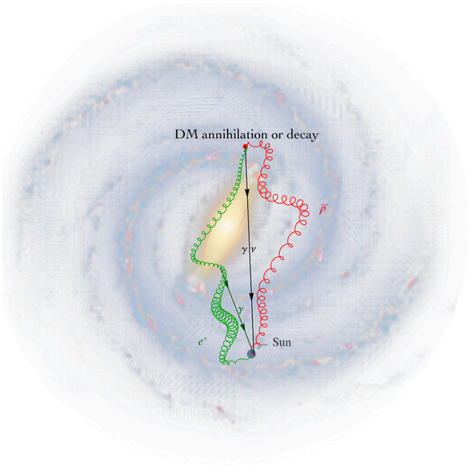


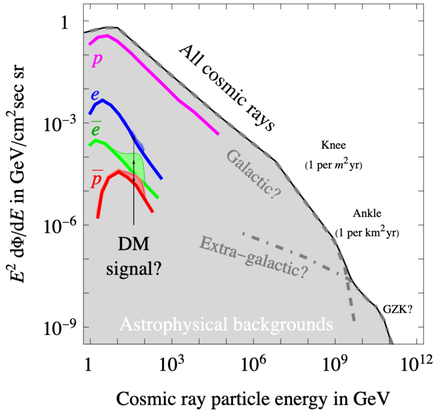


Hadron FCNC Decay

$$B^+ \to K^+ + \text{DM} + \text{DM}$$
$$\Lambda_b \to \Lambda + \text{DM} + \text{DM}$$
$$K^+ \to \pi^+ + \text{DM} + \text{DM}$$
$$D^0 \to \pi^0 + \text{DM} + \text{DM}$$

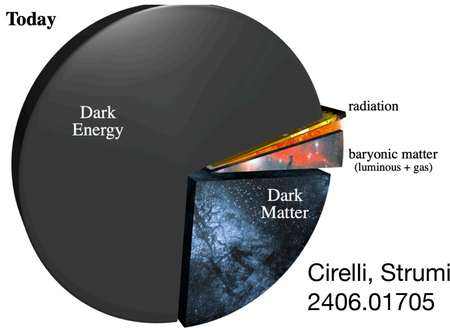
Indirect Detection





Light DM: a bottom-up view

DM is electrically neutral ! $\Longrightarrow$ DM only have FC or FCNC couplings to fermions !



Cirelli, Strumia, Zupan
2406.01705

	d	s	b
d	DD	NA62/KOTO	Belle II/CEPC
s		DD	Belle II/CEPC
b			Belle II/LHC

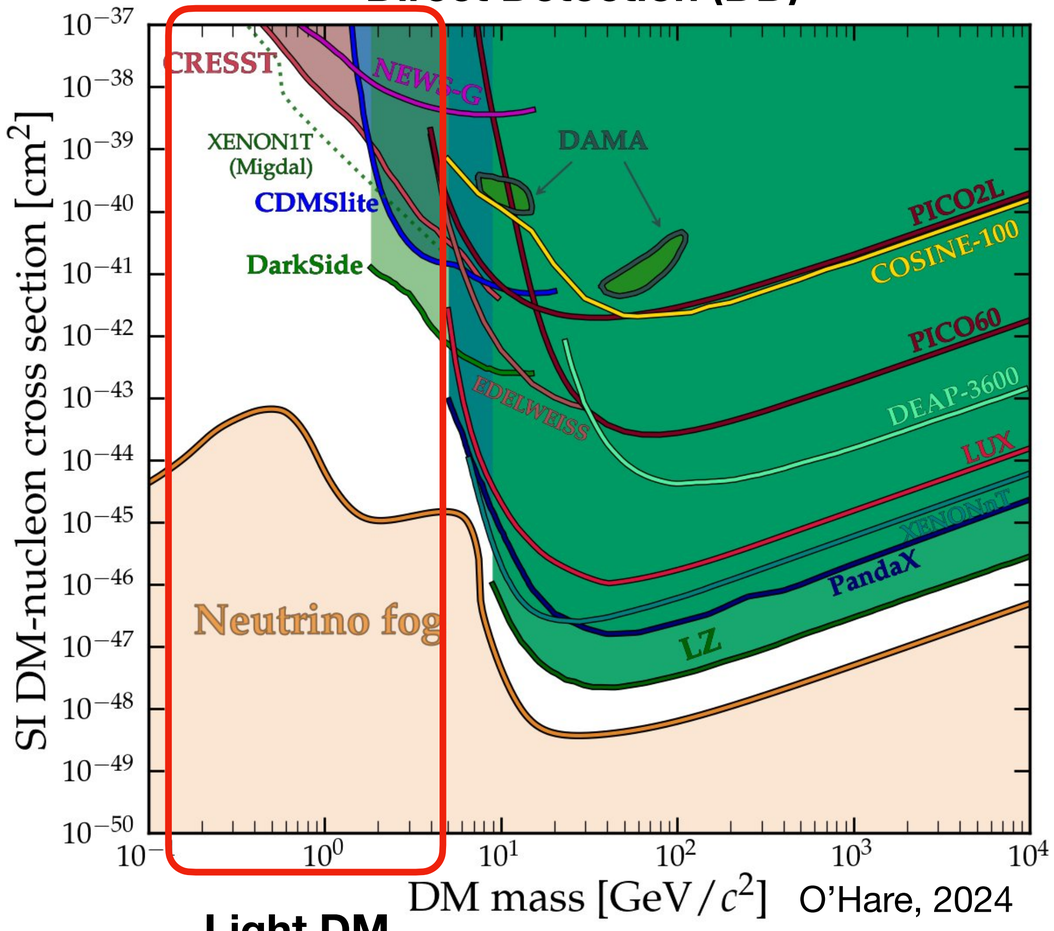
	u	c	t
u	DD	BES/STCF	LHC/CEPC
c		BES/STCF	LHC/CEPC
t			LHC/CEPC

DD = Direct Detection

means related to the DM relic density

example:
$B^+ \to K^+ + \text{DM} + \text{DM}$
$K^+ \to \pi^+ + \text{DM} + \text{DM}$
$D^0 \to \pi^0 + \text{DM} + \text{DM}$

Direct Detection (DD)



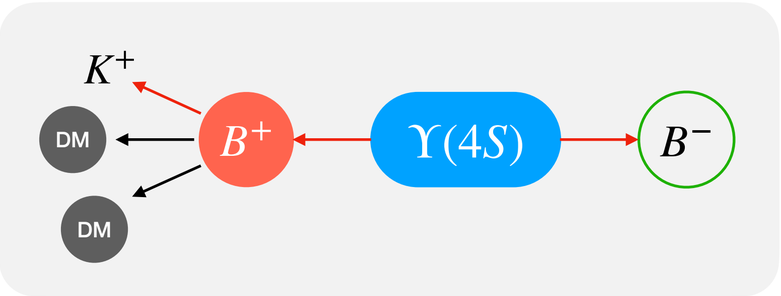
O'Hare, 2024

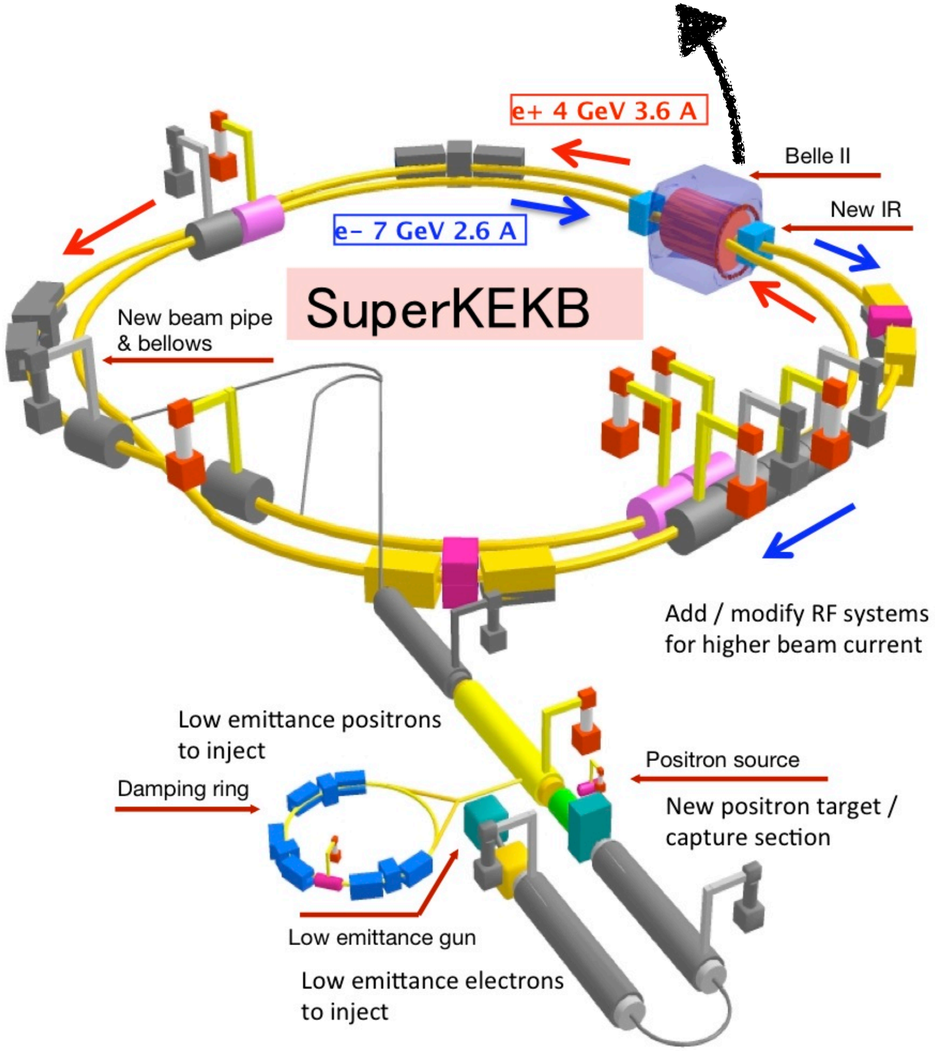
Light DM

Experimental Search

	Observable	SM	Exp	Unit	
$b \to s$	$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})$	4.16 ± 0.57	$23 \pm 5^{+5}_{-4}$	10^{-6}	Belle II CEPC FCC-ee
	$\mathcal{B}(B^0 \to K^0\nu\bar{\nu})$	3.85 ± 0.52	< 26	10^{-6}	
	$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})$	9.70 ± 0.94	< 61	10^{-6}	
	$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$	9.00 ± 0.87	< 18	10^{-6}	
	$\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	9.93 ± 0.72	< 5400	10^{-6}	
	$\mathcal{B}(B_s \to \nu\bar{\nu})$	≈ 0	< 5.9	10^{-4}	
$b \to d$	$\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})$	1.40 ± 0.18	< 140	10^{-7}	
	$\mathcal{B}(B^0 \to \pi^0\nu\bar{\nu})$	6.52 ± 0.85	< 900	10^{-8}	
	$\mathcal{B}(B^+ \to \rho^+\nu\bar{\nu})$	4.06 ± 0.79	< 300	10^{-7}	
	$\mathcal{B}(B^0 \to \rho^0\nu\bar{\nu})$	1.89 ± 0.36	< 400	10^{-7}	
	$\mathcal{B}(B^0 \to \nu\bar{\nu})$	≈ 0	< 1.4	10^{-4}	
$s \to d$	$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$	8.42 ± 0.61	$10.6^{+4.0}_{-3.4} \pm 0.9$	10^{-11}	NA62 KOTO
	$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	3.41 ± 0.45	< 300	10^{-11}	

$c \to u$ $D^+ \to \pi^+$ + inv, $D^0 \to \rho^0$ + inv, … … BESIII STCF





Experimental Search

	Observable	SM	Exp	Unit	
$b \to s$	$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})$	4.16 ± 0.57	$23 \pm 5^{+5}_{-4}$	10^{-6}	Belle II CEPC FCC-ee
	$\mathcal{B}(B^0 \to K^0\nu\bar{\nu})$	3.85 ± 0.52	< 26	10^{-6}	
	$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})$	9.70 ± 0.94	< 61	10^{-6}	
	$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$	9.00 ± 0.87	< 18	10^{-6}	
	$\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	9.93 ± 0.72	< 5400	10^{-6}	
	$\mathcal{B}(B_s \to \nu\bar{\nu})$	≈ 0	< 5.9	10^{-4}	
$b \to d$	$\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})$	1.40 ± 0.18	< 140	10^{-7}	
	$\mathcal{B}(B^0 \to \pi^0\nu\bar{\nu})$	6.52 ± 0.85	< 900	10^{-8}	
	$\mathcal{B}(B^+ \to \rho^+\nu\bar{\nu})$	4.06 ± 0.79	< 300	10^{-7}	
	$\mathcal{B}(B^0 \to \rho^0\nu\bar{\nu})$	1.89 ± 0.36	< 400	10^{-7}	
	$\mathcal{B}(B^0 \to \nu\bar{\nu})$	≈ 0	< 1.4	10^{-4}	
$s \to d$	$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$	8.42 ± 0.61	$10.6^{+4.0}_{-3.4} \pm 0.9$	10^{-11}	NA62
	$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	3.41 ± 0.45	< 300	10^{-11}	KOTO

$c \to u$ $D^+ \to \pi^+$ + inv, $D^0 \to \rho^0$ + inv, … …

BESIII
STCF

CEPC, FCC-ee

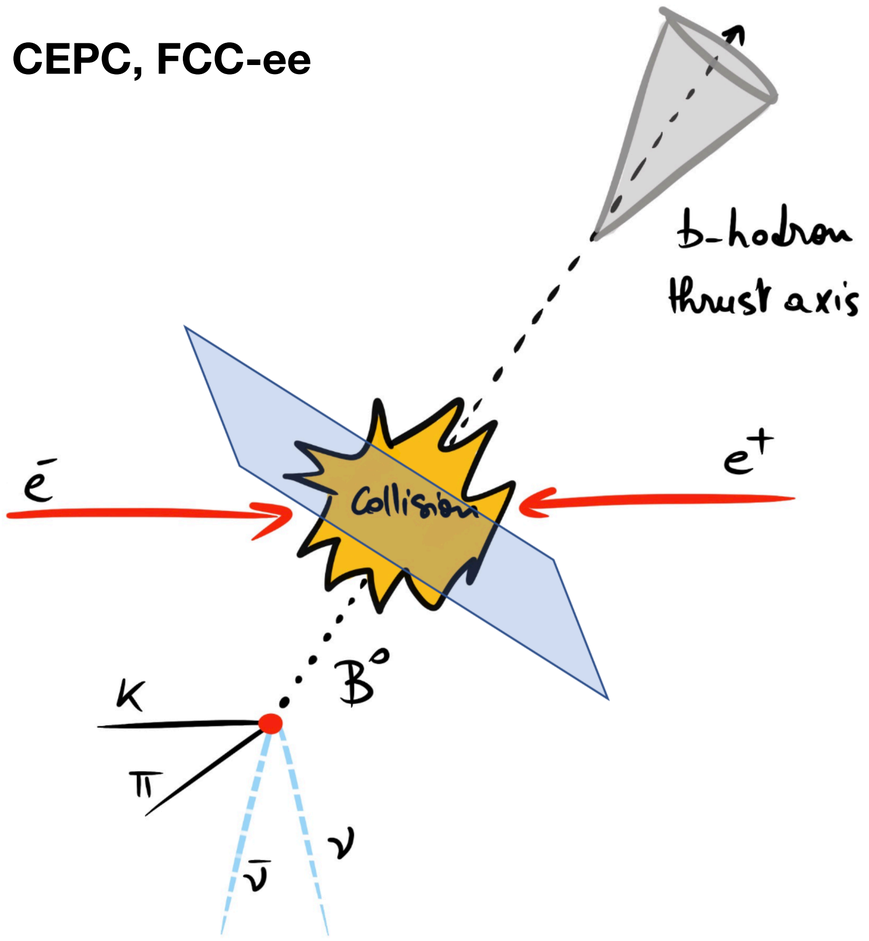


figure from Amhis, Kenzie, Reboud, Wiederhold, 2309.11353

Experimental Search

	Observable	SM	Exp	Unit
$b \to s$	$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})$	4.16 ± 0.57	$23 \pm 5^{+5}_{-4}$	10^{-6}
	$\mathcal{B}(B^0 \to K^0\nu\bar{\nu})$	3.85 ± 0.52	< 26	10^{-6}
	$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})$	9.70 ± 0.94	< 61	10^{-6}
	$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$	9.00 ± 0.87	< 18	10^{-6}
	$\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	9.93 ± 0.72	< 5400	10^{-6}
	$\mathcal{B}(B_s \to \nu\bar{\nu})$	≈ 0	< 5.9	10^{-4}
$b \to d$	$\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})$	1.40 ± 0.18	< 140	10^{-7}
	$\mathcal{B}(B^0 \to \pi^0\nu\bar{\nu})$	6.52 ± 0.85	< 900	10^{-8}
	$\mathcal{B}(B^+ \to \rho^+\nu\bar{\nu})$	4.06 ± 0.79	< 300	10^{-7}
	$\mathcal{B}(B^0 \to \rho^0\nu\bar{\nu})$	1.89 ± 0.36	< 400	10^{-7}
	$\mathcal{B}(B^0 \to \nu\bar{\nu})$	≈ 0	< 1.4	10^{-4}
$s \to d$	$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$	8.42 ± 0.61	$10.6^{+4.0}_{-3.4} \pm 0.9$	10^{-11}
	$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	3.41 ± 0.45	< 300	10^{-11}

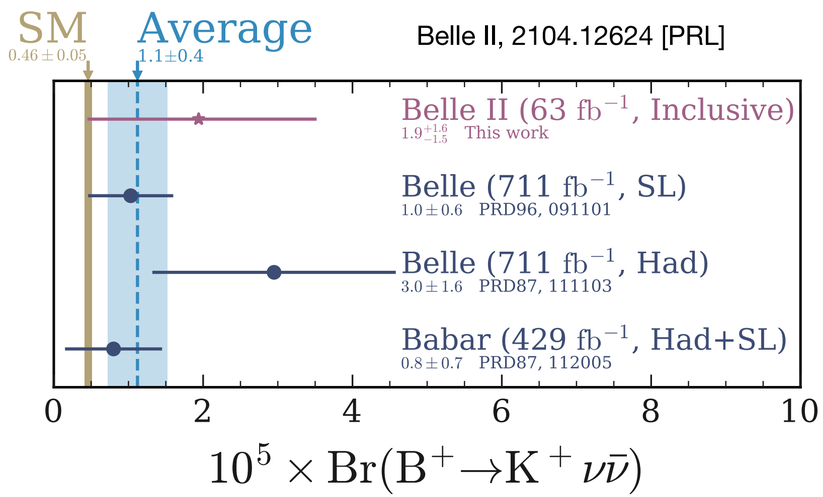
Belle II
CEPC
FCC-ee

NA62
KOTO

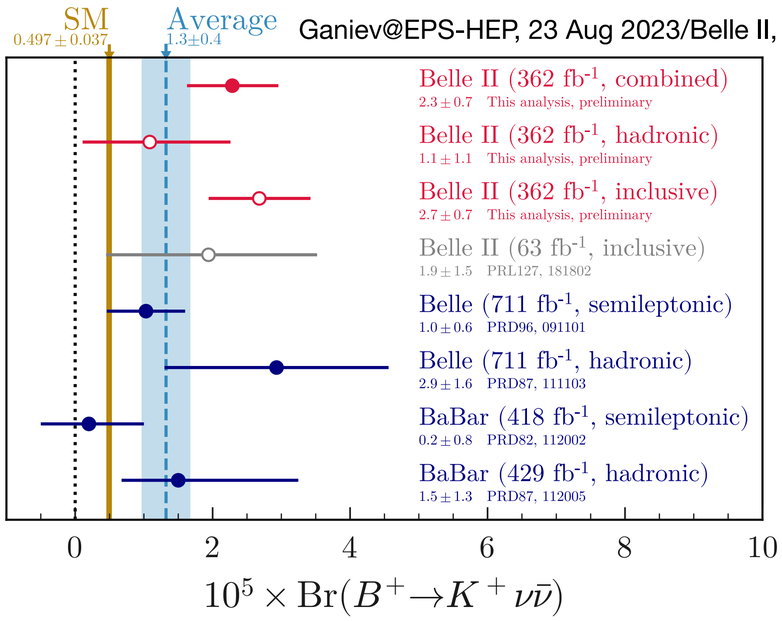
$c \to u$ $D^+ \to \pi^+$ + inv, $D^0 \to \rho^0$ + inv, … …

BESIII
STCF

- **2021 Apr**



- **2023 Aug: first evidence**



Experimental Search

	Observable	SM	Exp	Unit	Expected sensitivities at future Belle II, CEPC, FCC-ee
$b \to s$	$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})$	4.16 ± 0.57	$23 \pm 5^{+5}_{-4}$	10^{-6}	19%, Belle II, 5 fb^{-1}
	$\mathcal{B}(B^0 \to K^0\nu\bar{\nu})$	3.85 ± 0.52	< 26	10^{-6}	87%, Belle II, 5 fb^{-1}; 3.37%, FCC-ee, Tera-Z
	$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})$	9.70 ± 0.94	< 61	10^{-6}	75%, Belle II, 5 fb^{-1}; 0.53%, FCC-ee, Tera-Z
	$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$	9.00 ± 0.87	< 18	10^{-6}	40%, Belle II, 5 fb^{-1}
	$\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	9.93 ± 0.72	< 5400	10^{-6}	2%, CEPC, Tera-Z; 1.2% FCC-ee, Tera-Z
	$\mathcal{B}(B_s \to \nu\bar{\nu})$	≈ 0	< 5.9	10^{-4}	$< 1.1 \times 10^{-5}$, Belle II, 5 fb^{-1}, $\Upsilon(5S)$; $< 7.2 \times 10^{-9}$, FCC-ee, Tera-Z
$b \to d$	$\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})$	1.40 ± 0.18	< 140	10^{-7}	
	$\mathcal{B}(B^0 \to \pi^0\nu\bar{\nu})$	6.52 ± 0.85	< 900	10^{-8}	
	$\mathcal{B}(B^+ \to \rho^+\nu\bar{\nu})$	4.06 ± 0.79	< 300	10^{-7}	
	$\mathcal{B}(B^0 \to \rho^0\nu\bar{\nu})$	1.89 ± 0.36	< 400	10^{-7}	
	$\mathcal{B}(B^0 \to \nu\bar{\nu})$	≈ 0	< 1.4	10^{-4}	
$s \to d$	$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$	8.42 ± 0.61	$10.6^{+4.0}_{-3.4} \pm 0.9$	10^{-11}	
	$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	3.41 ± 0.45	< 300	10^{-11}	

Belle II snowmass white paper, 2207.06307

CEPC flavor white paper, 2412.19743

Amhis, Kenzie, Reboud, Wiederhold, 2309.11353

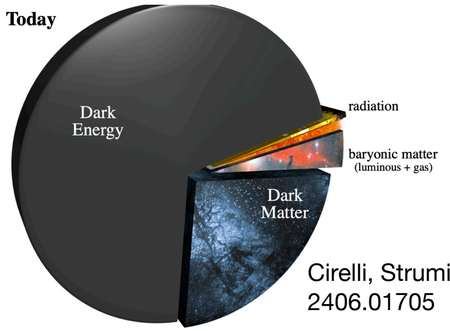
Cartelle, Kenzie, Mangrulkar, Wiederhold, Wood, 2508.04471

see also talks by Lorenzo Calibbi, Xunwu Zuo, Sen Jia

$c \to u$ $D^+ \to \pi^+$ + inv, $D^0 \to \rho^0$ + inv, … …

Light DM: a bottom-up view

DM is electrically neutral ! $\Longrightarrow$ DM only have FC or FCNC couplings to fermions !



Cirelli, Strumia, Zupan 2406.01705

	d	s	b
d	DD	NA62/KOTO	Belle II
s		DD	Belle II
b			Belle II/LHC

	u	c	t
u	DD	BES/STCF	LHC/CEPC
c		BES/STCF	LHC/CEPC
t			LHC/CEPC

DD = Direct Detection

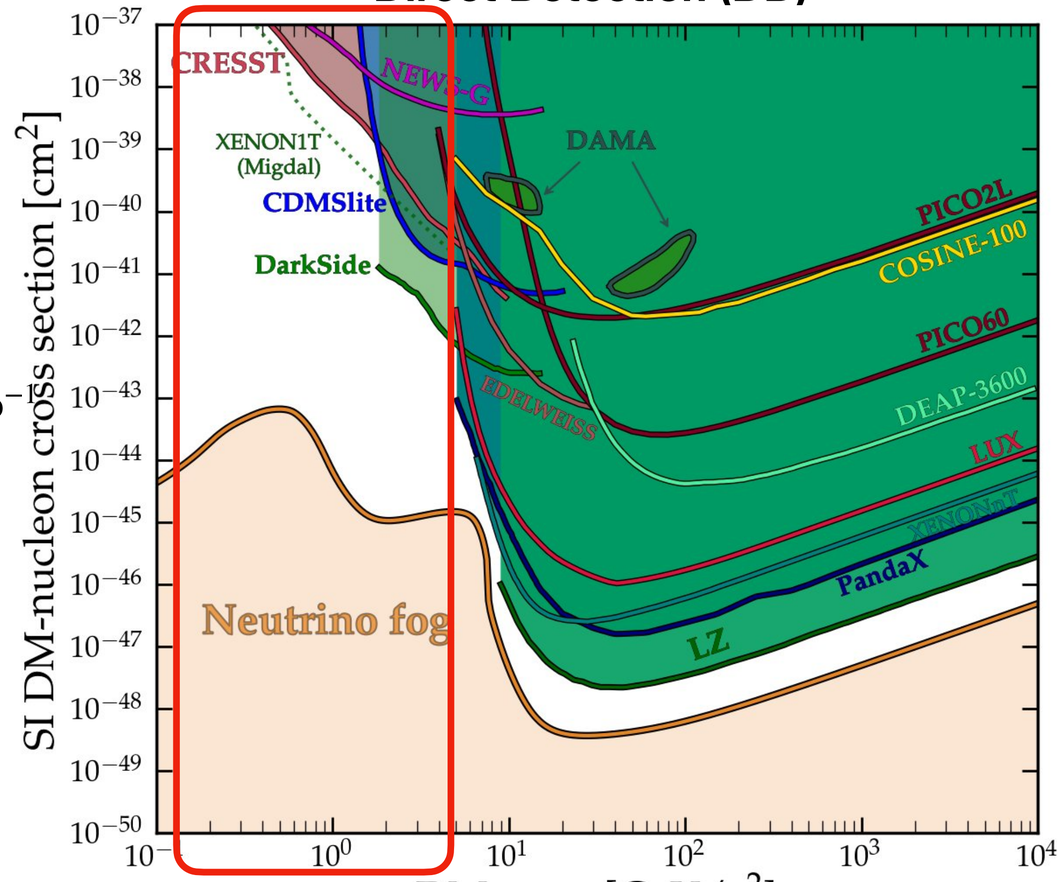
means related to the DM relic density

example:
$B^+ \to K^+ + \text{DM} + \text{DM}$
$K^+ \to \pi^+ + \text{DM} + \text{DM}$
$D^0 \to \pi^0 + \text{DM} + \text{DM}$

future exp uncertainties:
$\mathcal{O}(20 \sim 40\%)$@Belle II with 5ab^{-1}
$\mathcal{O}(2 \sim 4\%)$@CEPC Tera-Z

future theo uncertainties:
less than $10\,\%$

Direct Detection (DD)

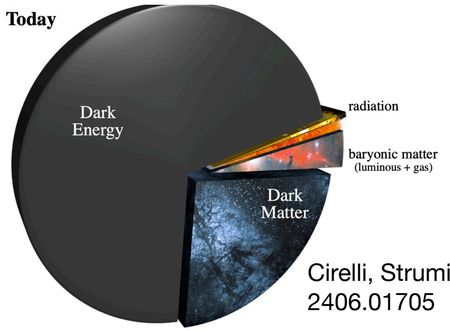


O'Hare, 2024

Light DM

Light DM: a bottom-up view

DM is electrically neutral ! $\Longrightarrow$ DM only have FC or FCNC couplings to fermions !

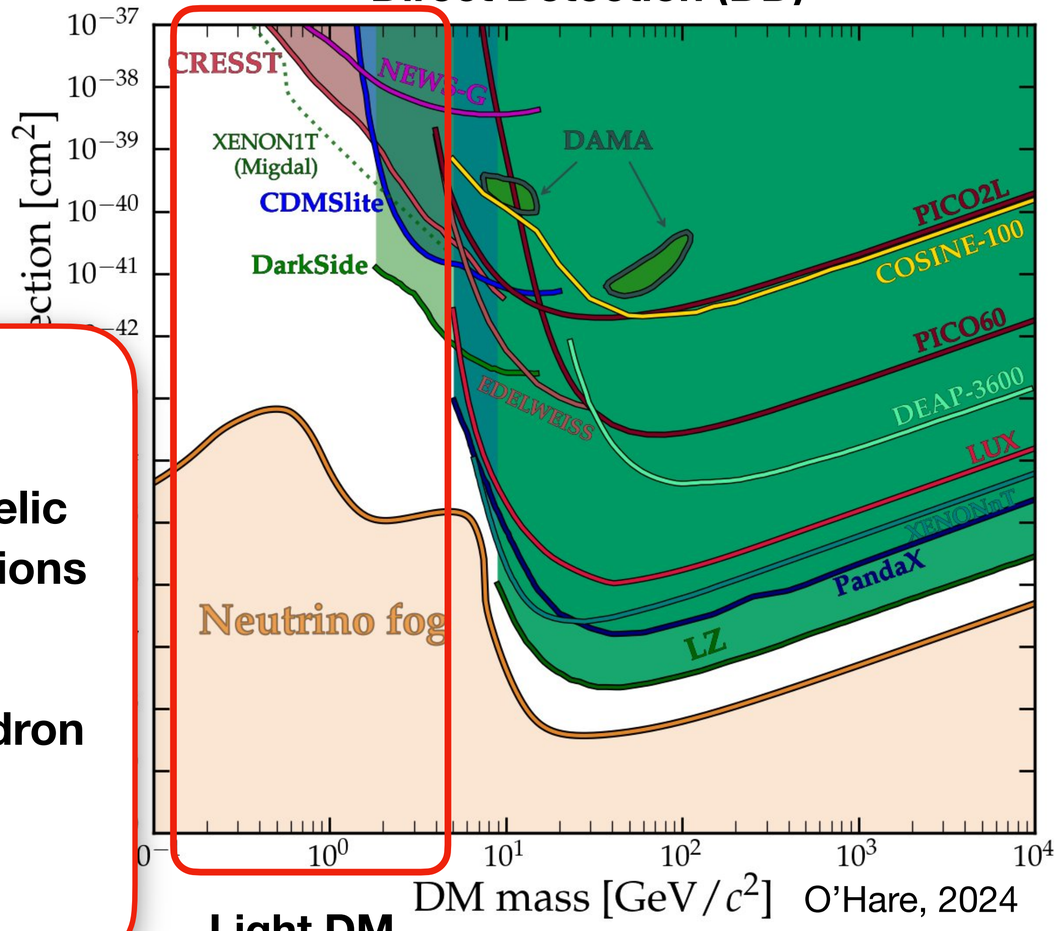


Cirelli, Strumia, Zupan
2406.01705

	d	s	b
d	DD	NA62/KOTO	Belle II
s		DD	Belle II
b			Belle II/LHC

example:
$B^+ \to K^+ + \text{DM} + \text{DM}$
$K^+ \to \pi^+ + \text{DM} + \text{DM}$
$D^0 \to \pi^0 + \text{DM} + \text{DM}$

Direct Detection (DD)



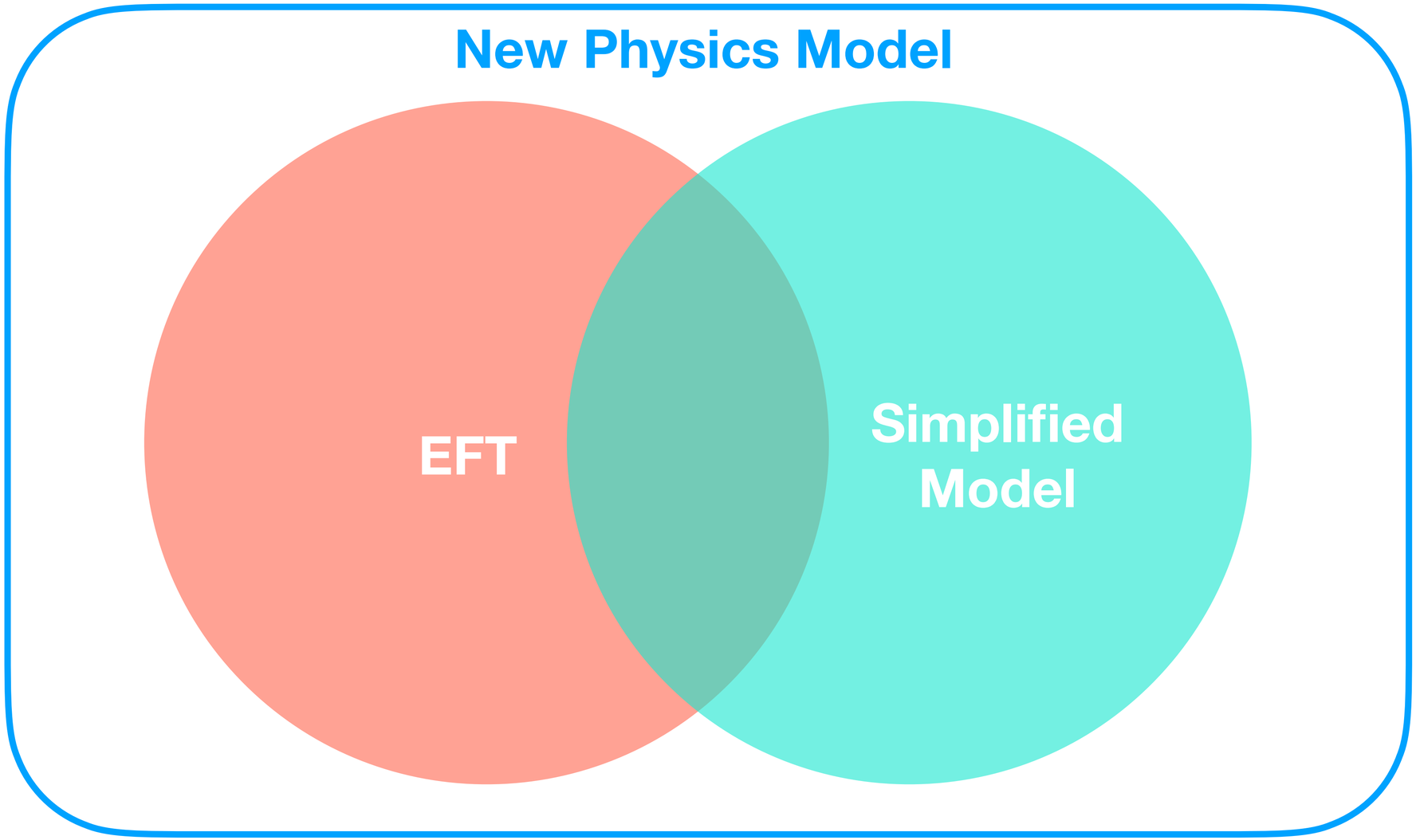
O'Hare, 2024

Light DM

It's natural to ask:

1. **By combing the direct detection, flavour measurements, and relic density, is it possible to obtain the flavour structure of interactions between light DM and SM particles ?**
2. **Compared to DM direct detection, what's the advantage of hadron decays ? which DM scenario is more sensitive to ? Can they provide other information ?**

Theoretical Framework



Effective Field Theory approach to combine the various experimental searches

In EFT, DM is just a singlet under the SM gauge group.

for light DM

connect to UV model

relic density

hadron decay

direct detection

$\mu_{\rm EW}$

μ_b

μ_c

1 GeV

Dark SMEFT $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, all the SM particles

example

$$\mathcal{Q}_{d\phi^2} = (\bar{q}_p d_r H)\phi^2$$
$$\mathcal{Q}_{d\chi} = (\bar{d}_p \gamma_\mu d_r)(\bar{\chi}\gamma^\mu \chi)$$
$$\mathcal{Q}_{dX^2} = (\bar{q}_p d_r H) X_\mu X^\mu$$
$$\mathcal{Q}_{qa} = (\bar{q}_p \gamma_\mu q_r)\partial^\mu a$$

2009 H.Zhang, Q.H.Cao, C.R.Chen, C.S.Li
2011 Kamenik, Smith
2014 Duch, Grzadkowski, Wudka
2017 Brod, Gootjes-Dreesbach, Tammaro, Zupan
2021 Criado, Djouadi, Perez-Victoria, Santiago
2022 Aebischer, Altmannshofer, Jenkins, Manohar (basis@dim-6)
2023 Song, Sun, Yu (basis@dim-8)

Axion-like particle, see also H.Y.Cheng, Phys.Rept 1988

2020 Bauer, Neubert, Renner, Schnubel, Thamm
2023 Song, Sun, Yu (basis@dim-8)

Dark LEFT $SU(3)_C \otimes U(1)_{\rm em}$, W, Z, h, t have been integrated out

$$\mathcal{O}_{d\phi^2} = (\bar{d}_{Lp} d_{Rr})\phi^2$$
$$\mathcal{O}^{V,LR}_{d\chi} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})(\bar{\chi}_a \gamma^\mu \chi_b)$$
$$\mathcal{O}^{L}_{dXX} = (\bar{d}_{Lp}\gamma_\mu d_{Lr}) X^{\mu\nu} X_\nu$$
$$\mathcal{O}^{L}_{da} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})\partial^\mu a$$

example

2022 Aebischer, Altmannshofer, Jenkins, Manohar (basis@dim-6)
2022 He, Ma, Valencia (basis@dim-6)
2023 Liang, Liao, Ma, Wang (basis@dim-8)

NRET/HDMET/Chiral EFT

$$\chi(x) = e^{-im_\chi v\cdot x}(\chi_v(x) + X_v(x))$$

2010 J.J.Fan, Reece, L.T.Wang
2011 Hill, Solon
2012 Fitzpatrick, Haxton, Katz, Lubbers, Xu
2013 Hill, Solon
2013 Anand, Fitzpatrick, Haxton
2016 Bishara, Brod, Grinstein, Zupan

$H_1 \to H_2 + \mathrm{DM}$ theoretical calculation and experimental searches

- $d_i \to d_j + \phi + \phi$

2011 Kamenik, Smith
2014 Bird, Jackson, Kowalewski, Pospelov
2019 G.Li, J.Y. Su, Tandean
$\Lambda \to n + \phi\phi$, $\Sigma^+ \to p + \phi\phi$, $\Xi^0 \to \Lambda + \phi\phi$,
$\Xi^- \to \Sigma^- \phi\phi$, $\Omega^- \to \Sigma^- + \phi\phi$
2020 X.G. He, X.D. Ma, Tandean, Valencia
2020 C.Q.Geng, Tandean, $K \to \pi\pi + \phi\phi$
2021 G. Li, T. Wang, J.B. Zhang, G.L. Wang
2022 Kling, S. Li, H. Song, S. Su, W. Su

- $d_i \to d_j + \chi + \chi$

2011 Kamenik, Smith
2019 J.Y. Su, Tandean
2020 G. Li, T. Wang, Y. Jiang, J.B. Zhang, G.L. Wang
2021 Felkl, S. L. Li, Schmidt

- $d_i \to d_j + X + X$

2011 Kamenik, Smith
2021 G. Li, T. Wang, J.B. Zhang, G.L. Wang
2022 X.G. He, X.D. Ma, Valencia

- $d_i \to d_j + a$

2020 Camalich, Pospelov, Vuong, Ziegler, Zupan,
2021 Bauer, Neubert, Renner, Schnubel, Thamm
2022 Guerrera and S. Rigolin

- theoretically clean: $A \propto C \cdot \langle H_2 | O | H_1 \rangle \cdot$ DM current (form factor)
- no GIM suppression
- possibly two-body decay

} **enhancement**

Observable
$\mathcal{B}(B^+ \to K^+ \nu\bar{\nu})$
$\mathcal{B}(B^0 \to K^0 \nu\bar{\nu})$
$\mathcal{B}(B^+ \to K^{*+} \nu\bar{\nu})$
$\mathcal{B}(B^0 \to K^{*0} \nu\bar{\nu})$
$\mathcal{B}(B_s \to \phi \nu\bar{\nu})$
$\mathcal{B}(B^+ \to \pi^+ \nu\bar{\nu})$
$\mathcal{B}(B^0 \to \pi^0 \nu\bar{\nu})$
$\mathcal{B}(B^+ \to \rho^+ \nu\bar{\nu})$
$\mathcal{B}(B^0 \to \rho^0 \nu\bar{\nu})$
$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$
$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$
$\mathcal{B}(B_s \to \nu\bar{\nu})$
$\mathcal{B}(B^0 \to \nu\bar{\nu})$
$\nu \to \mathrm{DM}$

longitudinal polarization F_L

$\Lambda_b \to \Lambda + \mathrm{DM} + \mathrm{DM}$

- $c \to u + \mathrm{DM}$

2022 C.Q.Geng, G.Li
2023 G.Li, Tandean

$D^0 \to s\bar{s}'$
$D^0 \to \gamma s\bar{s}'$
$D^0 \to \pi^0 s\bar{s}'$
$D^+ \to \pi^+ s\bar{s}'$
$D_s^+ \to K^+ s\bar{s}'$
$D^0 \to \rho^0 s\bar{s}'$
$D^+ \to \rho^+ s\bar{s}'$
$D_s^+ \to K^{*+} s\bar{s}'$
$\Lambda_c^+ \to p s\bar{s}'$
$\Xi_c^+ \to \Sigma^+ s\bar{s}'$
$\Xi_c^0 \to \Sigma^0 s\bar{s}'$
$\Xi_c^0 \to \Lambda s\bar{s}'$

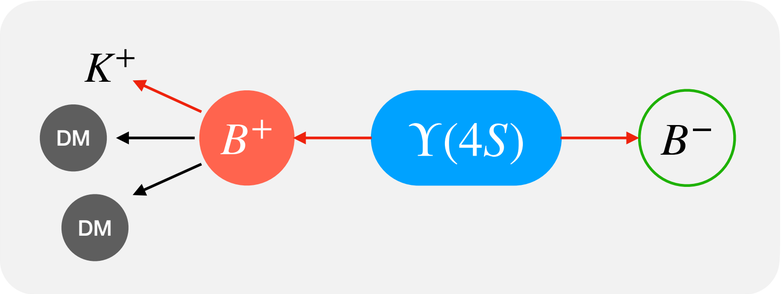
based on complete EFT basis (Dark LEFT)

`HadronToNP`: a package to calculate decay of hadron to new particles
B.F. Hou, X.Q.Li, H.Yan, Y.D.Yang, **XBY** *to be finished*

$b \to s\nu\bar{\nu}$: DSMEFT

Can DSMEFT operators explain the Belle II excess, while satisfy other $b \to s$ bounds ?

Observable	SM	Exp	Unit
$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})$	4.16 ± 0.57	$23 \pm 5^{+5}_{-4}$	10^{-6}
$\mathcal{B}(B^0 \to K^0\nu\bar{\nu})$	3.85 ± 0.52	< 26	10^{-6}
$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})$	9.70 ± 0.94	< 61	10^{-6}
$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$	9.00 ± 0.87	< 18	10^{-6}
$\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	9.93 ± 0.72	< 5400	10^{-6}
$\mathcal{B}(B_s \to \nu\bar{\nu})$	≈ 0	< 5.9	10^{-4}
$\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})$	1.40 ± 0.18	< 140	10^{-7}
$\mathcal{B}(B^0 \to \pi^0\nu\bar{\nu})$	6.52 ± 0.85	< 900	10^{-8}
$\mathcal{B}(B^+ \to \rho^+\nu\bar{\nu})$	4.06 ± 0.79	< 300	10^{-7}
$\mathcal{B}(B^0 \to \rho^0\nu\bar{\nu})$	1.89 ± 0.36	< 400	10^{-7}
$\mathcal{B}(B^0 \to \nu\bar{\nu})$	≈ 0	< 1.4	10^{-4}
$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$	8.42 ± 0.61	$10.6^{+4.0}_{-3.4} \pm 0.9$	10^{-11}
$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	3.41 ± 0.45	< 300	10^{-11}



Dark SMEFT

$$\mathcal{Q}_{d\phi} = (\bar{q}_p d_r H)\phi + \text{h.c.}, \qquad \mathcal{Q}_{d\phi^2} = (\bar{q}_p d_r H)\phi^2 + \text{h.c.},$$
$$\mathcal{Q}_{\phi q} = (\bar{q}_p \gamma_\mu q_r)(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2), \qquad \mathcal{Q}_{\phi d} = (\bar{d}_p \gamma_\mu d_r)(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2),$$

scalar: 4

$$\mathcal{Q}_{q\chi} = (\bar{q}_p \gamma_\mu q_r)(\bar{\chi}\gamma^\mu \chi), \qquad \mathcal{Q}_{d\chi} = (\bar{d}_p \gamma_\mu d_r)(\bar{\chi}\gamma^\mu \chi),$$

fermion: 2

$$\mathcal{Q}_{dHX} = (\bar{q}_p \sigma_{\mu\nu} d_r) H X^{\mu\nu} \qquad \mathcal{Q}_{dX^2} = (\bar{q}_p d_r H) X_\mu X^\mu$$

vector: 1+13

$$\mathcal{Q}_{qa} = (\bar{q}_p \gamma_\mu q_r)\partial^\mu a \qquad \mathcal{Q}_{da} = (\bar{d}_p \gamma_\mu d_r)\partial^\mu a$$

ALP: 2

μ_{EW}

Dark LEFT

$$\mathcal{O}_{d\phi} = (\bar{d}_{Lp} d_{Rr})\phi + \text{h.c.}, \qquad \mathcal{O}^L_{\phi d} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2),$$

scalar: 4

$$\mathcal{O}^{V,LR}_{d\chi} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})(\bar{\chi}_a \gamma^\mu \chi_b), \mathcal{O}^{V,RR}_{d\chi} = (\bar{d}_{Rp}\gamma_\mu d_{Rr})(\bar{\chi}_a \gamma^\mu \chi_b),$$

fermion: 5

$$\mathcal{O}^T_{dX} = (\bar{d}_{Lp}\sigma_{\mu\nu} d_{Rr}) X^{\mu\nu}_a \qquad \mathcal{O}^L_{dXX} = (\bar{d}_{Lp}\gamma_\mu d_{Lr}) X^{\mu\nu} X_\nu$$

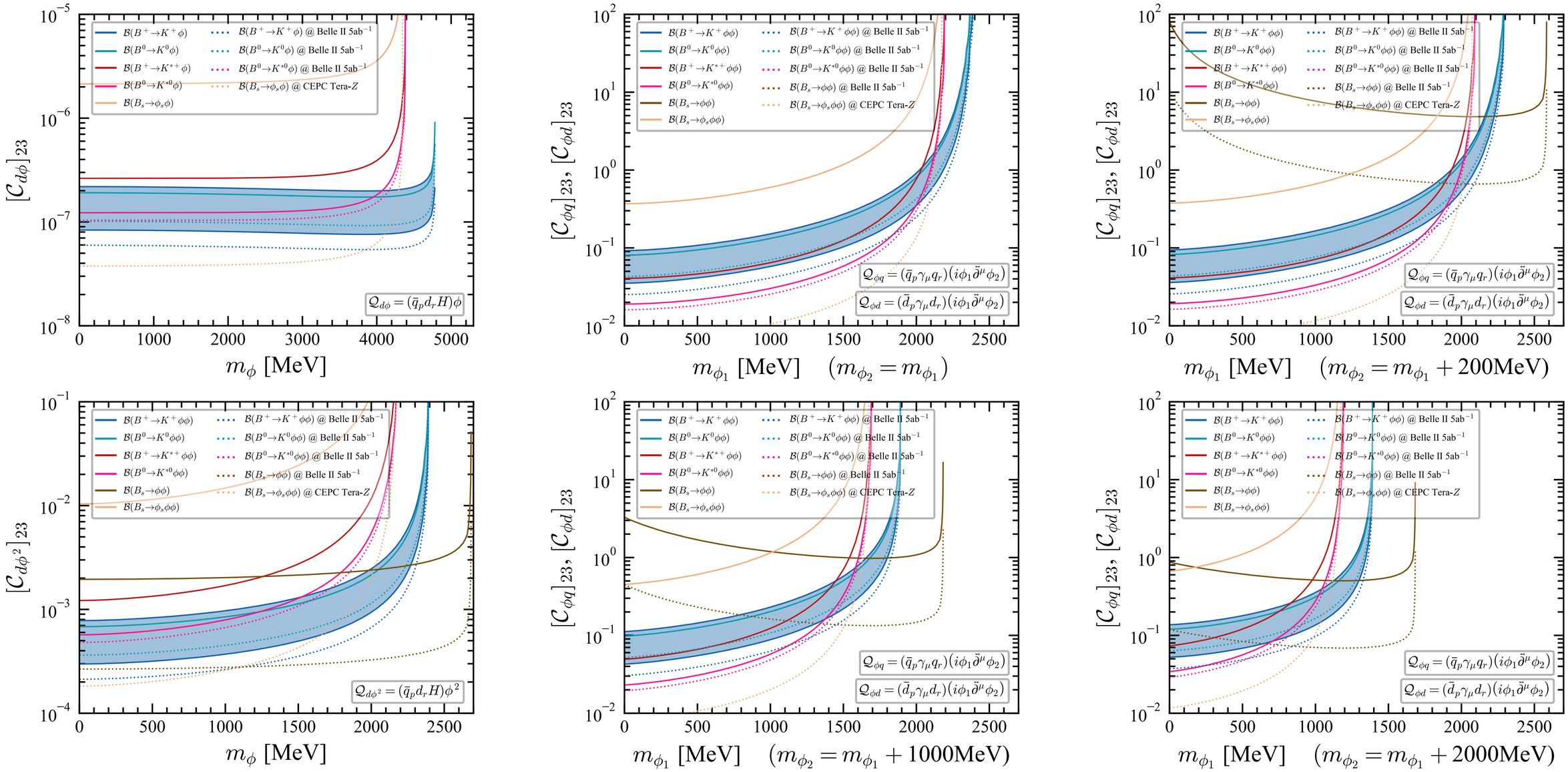
vector: 1+10

μ_b

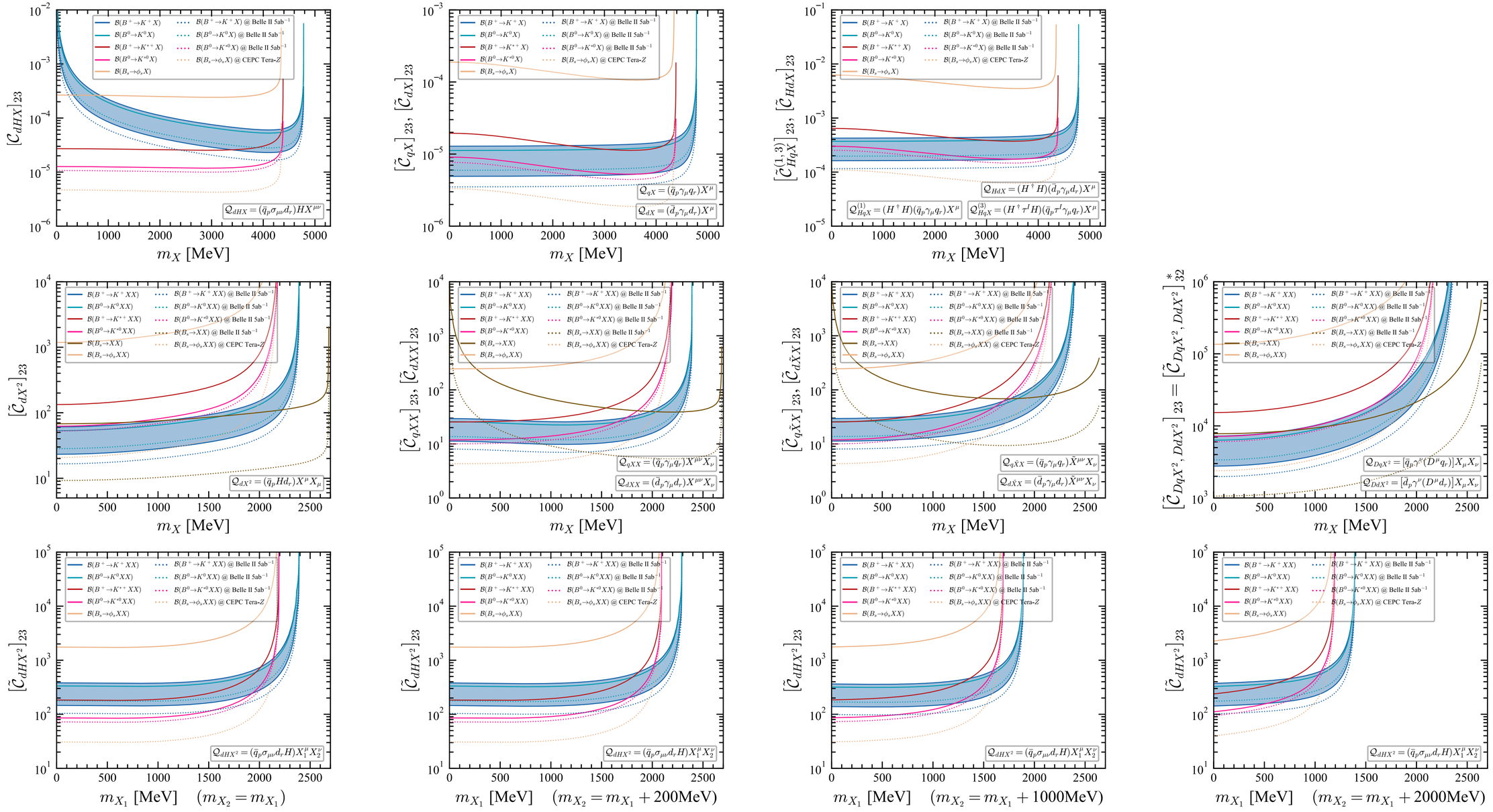
$$\mathcal{O}^L_{da} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})\partial^\mu a, \qquad \mathcal{O}^R_{da} = (\bar{d}_{Rp}\gamma_\mu d_{Rr})\partial^\mu a$$

ALP: 2

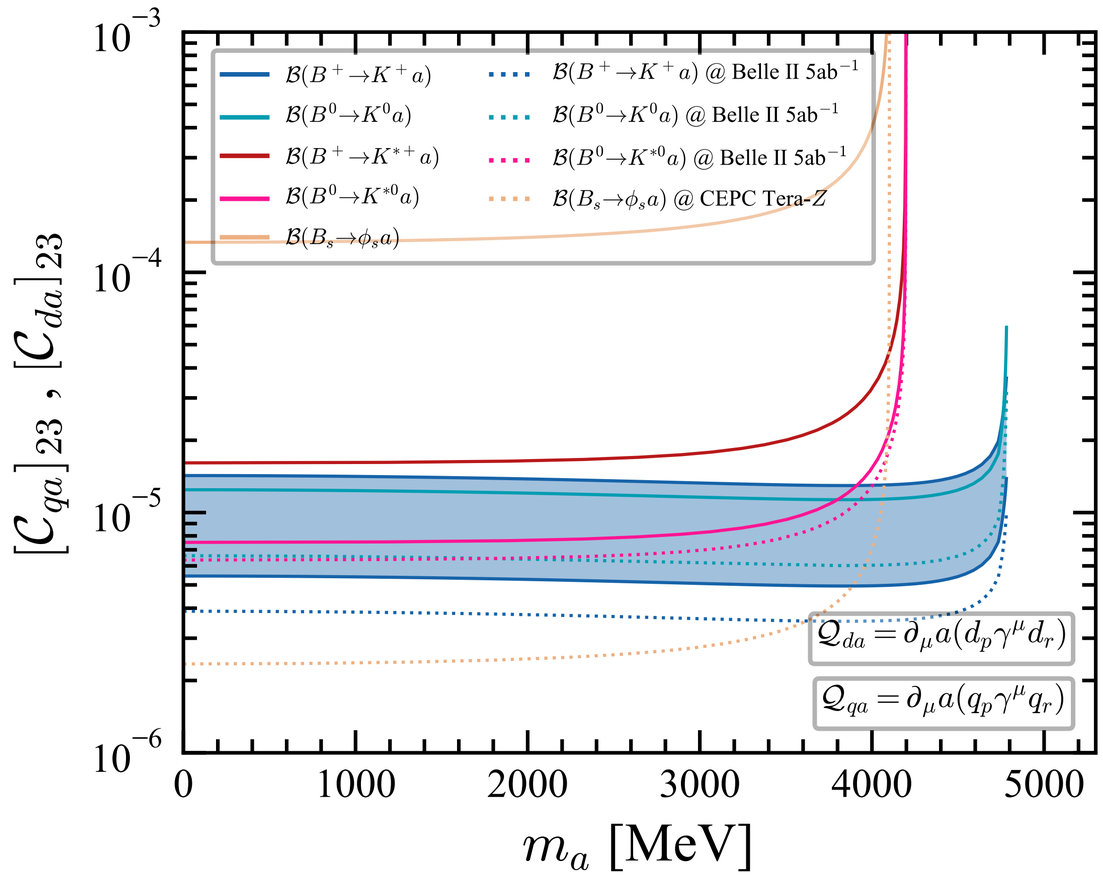
Dark SMEFT: Scalar

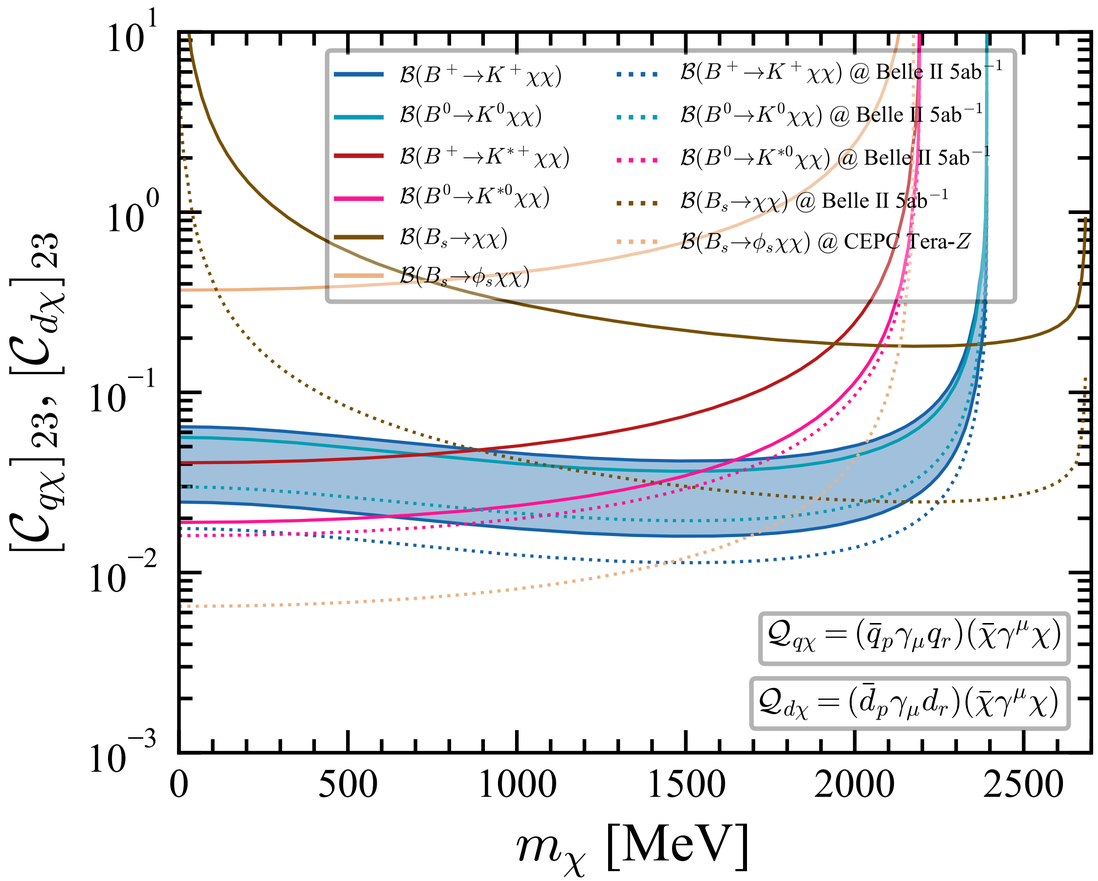


Dark SMEFT: Vector

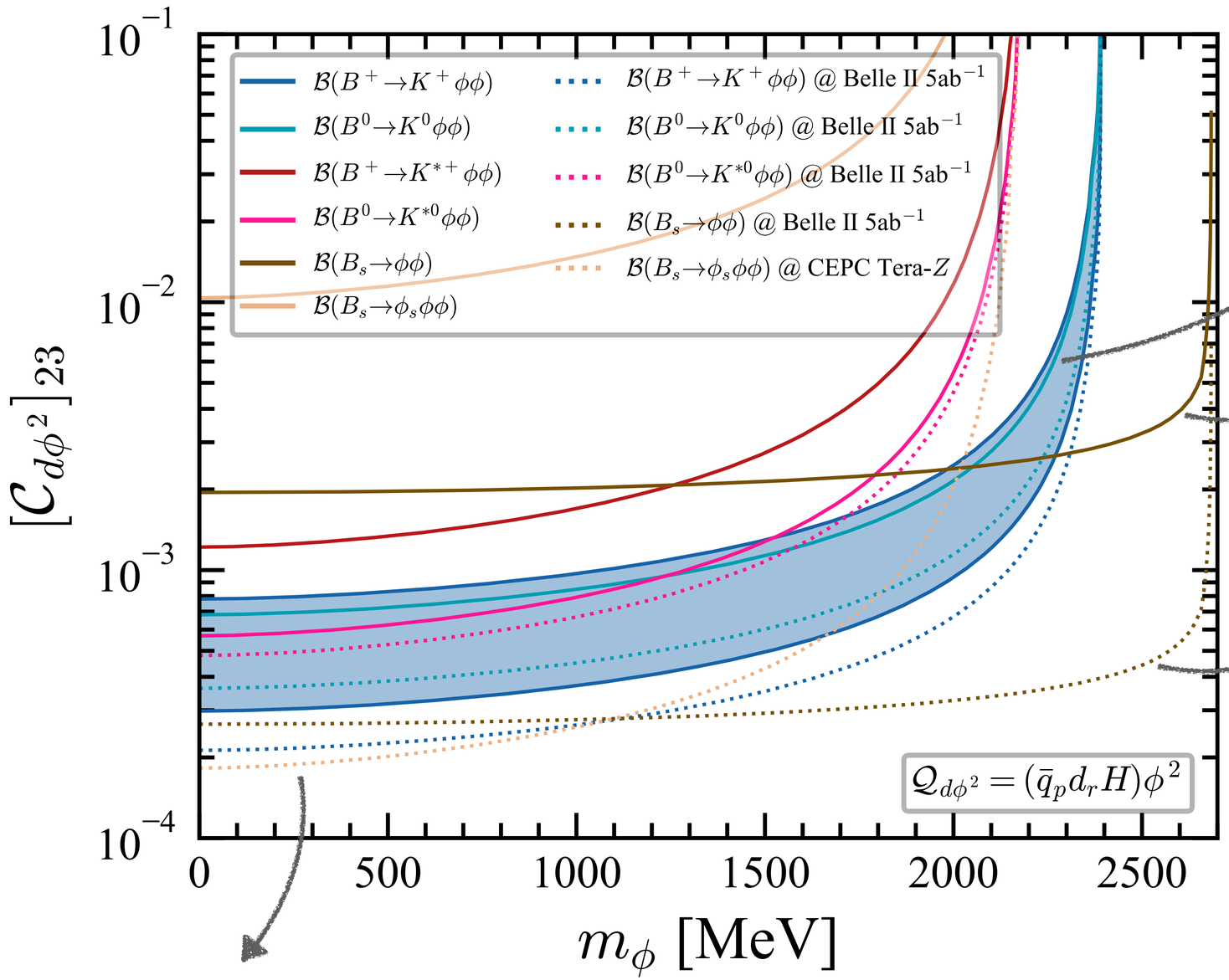


Dark SMEFT: Fermion, ALP





One example



parameter space explaining Belle II excess

$\mathscr{B}(B_s \to$ **inv**$) < 5.9 \times 10^{-4}$, **LEP**

Alonso-Alvarez, Abenza, 2310.13043

$\mathscr{B}(B_s \to$ **inv**$) < 1.1 \times 10^{-5}$, **Belle II** $5\text{ab}^{-1}(\Upsilon(5S))$

$\mathscr{B}(B_s \to \phi +$ **inv**$)@2\,\%$, **CEPC (Tera-Z)**

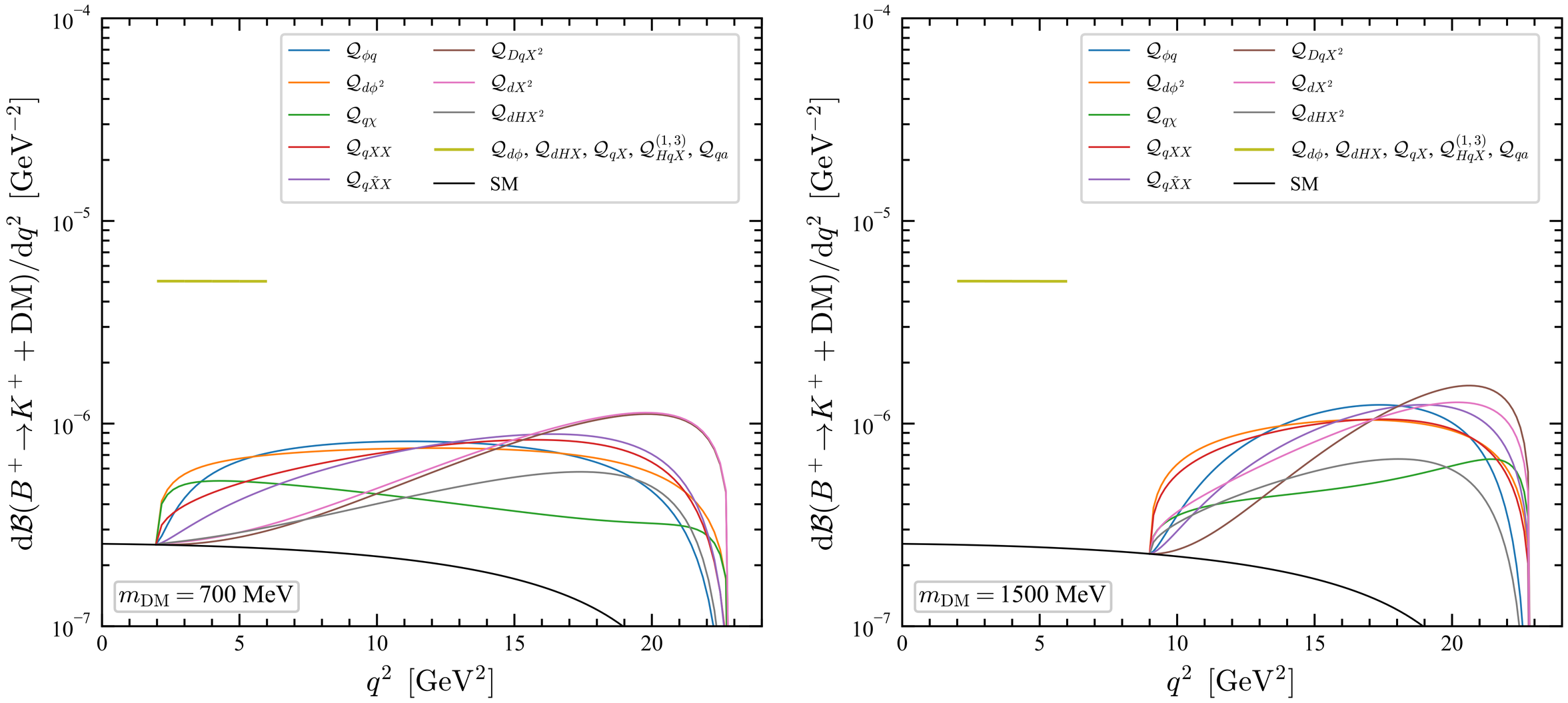
Summary of the numerical analysis

All the operators survive from the constraints of the various FCNC decays.

In the future, all the parameter space to explain the Belle II excess can be covered by combing

- $B_s \to \text{inv}$
- $b \to s + \text{inv}$ ($B_s \to \phi + \text{inv}$, $B^0 \to K^0 + \text{inv}$, ...)
- Belle II, CEPC, FCC-ee

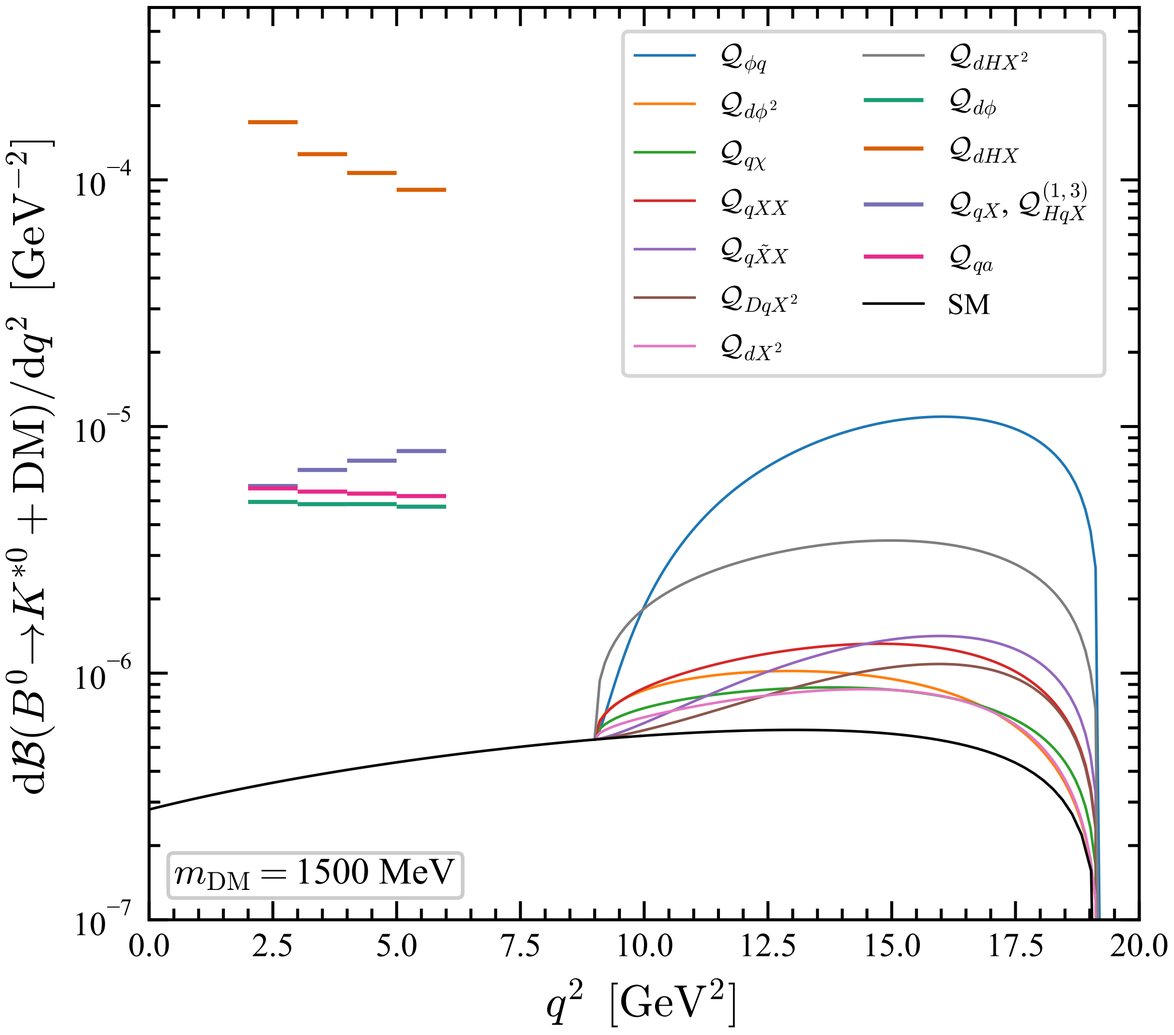
Dark SMEFT: $\mathrm{d}B/\mathrm{d}q^2$

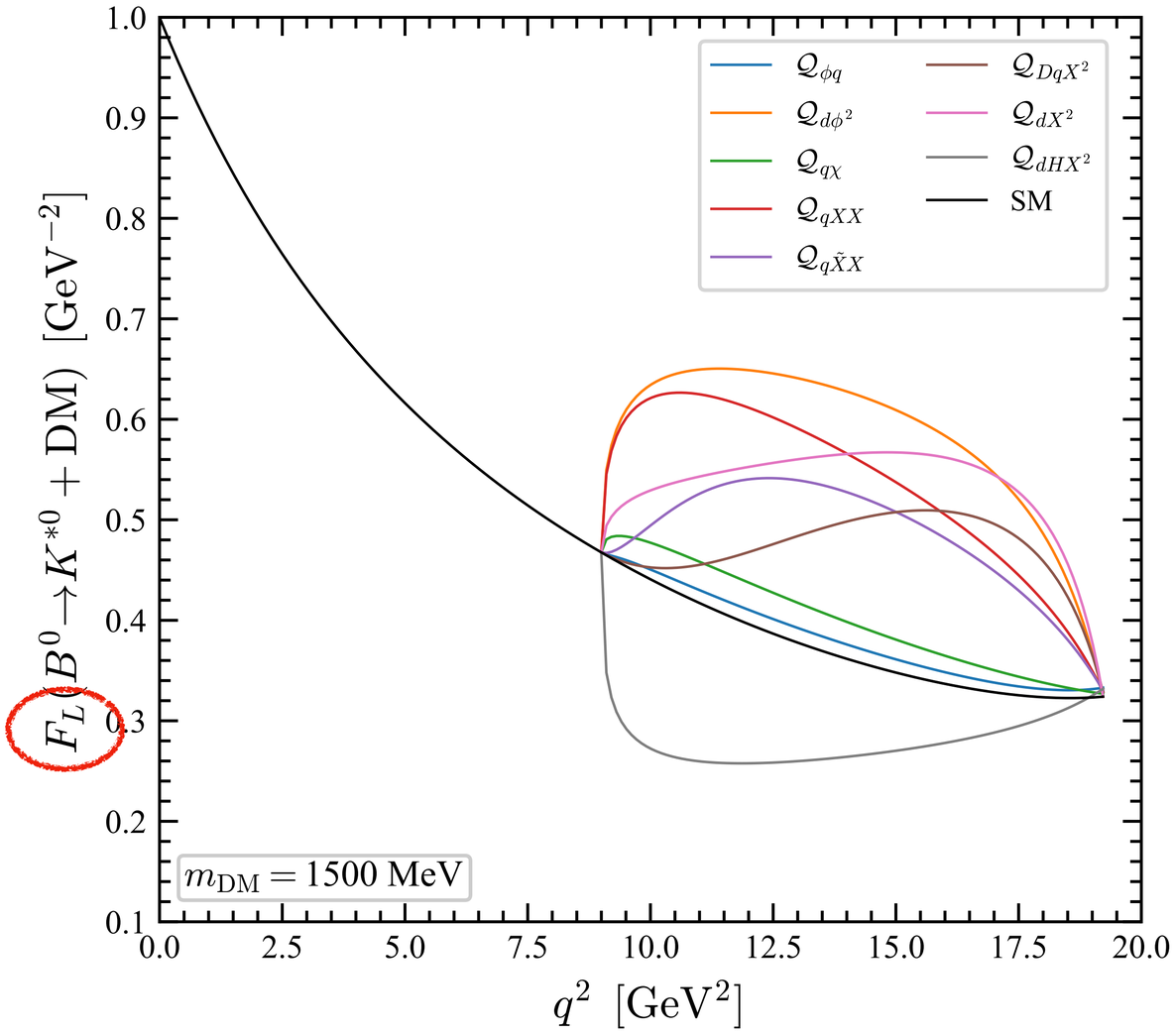


Difficult to distinguish the DSMEFT operators by considering only the $B^+ \to K^+\nu\bar{\nu}$ decay. However,

Dark SMEFT: $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$, F_L

$m_{\mathrm{DM}} = 1500\,\mathrm{MeV}$





All the operators are distinguishable from each other by combing these observables

Effective Field Theory approach to combine the various experimental searches

In EFT, DM is just a singlet under the SM gauge group.

for light DM

connect to UV model

relic density

hadron decay

direct detection

$\mu_{\rm EW}$

μ_b

μ_c

1 GeV

Dark SMEFT $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, all the SM particles

example

$$\mathcal{Q}_{d\phi^2} = (\bar{q}_p d_r H)\phi^2$$
$$\mathcal{Q}_{d\chi} = (\bar{d}_p \gamma_\mu d_r)(\bar{\chi}\gamma^\mu \chi)$$
$$\mathcal{Q}_{dX^2} = (\bar{q}_p d_r H) X_\mu X^\mu$$
$$\mathcal{Q}_{qa} = (\bar{q}_p \gamma_\mu q_r)\partial^\mu a$$

2009 H.Zhang, Q.H.Cao, C.R.Chen, C.S.Li
2011 Kamenik, Smith
2014 Duch, Grzadkowski, Wudka
2017 Brod, Gootjes-Dreesbach, Tammaro, Zupan
2021 Criado, Djouadi, Perez-Victoria, Santiago
2022 Aebischer, Altmannshofer, Jenkins, Manohar (basis@dim-6)
2023 Song, Sun, Yu (basis@dim-8)

Axion-like particle, see also H.Y.Cheng, Phys.Rept 1988

2020 Bauer, Neubert, Renner, Schnubel, Thamm
2023 Song, Sun, Yu (basis@dim-8)

Dark LEFT $SU(3)_C \otimes U(1)_{\rm em}$, W, Z, h, t have been integrated out

$$\mathcal{O}_{d\phi^2} = (\bar{d}_{Lp} d_{Rr})\phi^2$$
$$\mathcal{O}^{V,LR}_{d\chi} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})(\bar{\chi}_a \gamma^\mu \chi_b)$$
$$\mathcal{O}^{L}_{dXX} = (\bar{d}_{Lp}\gamma_\mu d_{Lr}) X^{\mu\nu} X_\nu$$
$$\mathcal{O}^{L}_{da} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})\partial^\mu a$$

example

2022 Aebischer, Altmannshofer, Jenkins, Manohar (basis@dim-6)
2022 He, Ma, Valencia (basis@dim-6)
2023 Liang, Liao, Ma, Wang (basis@dim-8)

NRET/HDMET/Chiral EFT

$$\chi(x) = e^{-im_\chi v\cdot x}(\chi_v(x) + X_v(x))$$

2010 J.J.Fan, Reece, L.T.Wang
2011 Hill, Solon
2012 Fitzpatrick, Haxton, Katz, Lubbers, Xu
2013 Hill, Solon
2013 Anand, Fitzpatrick, Haxton
2016 Bishara, Brod, Grinstein, Zupan

Top-flavored DM

- Dark SMEFT with 3rd generation@$\mu_{\rm EW}$

$$\mathcal{Q}_{u\chi} = (\bar{u}_p\gamma_\mu u_r)(\bar{\chi}\gamma^\mu\chi), \Longrightarrow (\bar{t}_R\gamma_\mu t_R)(\bar{\chi}\gamma^\mu\chi)$$

$$\mathcal{Q}_{q\chi} = (\bar{q}_p\gamma_\mu q_r)(\bar{\chi}\gamma^\mu\chi),$$

$$\mathcal{Q}_{u\chi^2} = (\bar{q}_p u_r \widetilde{H})(\bar{\chi}\chi), \qquad \mathcal{C} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{C}_{33}$$

2013 Tongyan Lin, Kolb, Lian-Tao Wang
2015 Kilic, Klimek, Jiang-Hao Yu
2015 Haisch, Re
2015 Boucheneb, Cacciapaglia, Deandrea, Fuks
2017 Blanke, Kast
2021 Blanke, Pani, Polesello, Rovelli
2021 Haisch, Polesello, Schulte
2021 Hermanna, Worek
2022 Yandong Liu, Bin Yan, Rui Zhang
… …

2019 ATLAS [JHEP05(2019)142]

$\mu_{\rm EW}$ — **Dark SMEFT** one-loop matching

μ_b — **Dark LEFT**

1 GeV

hadron decay

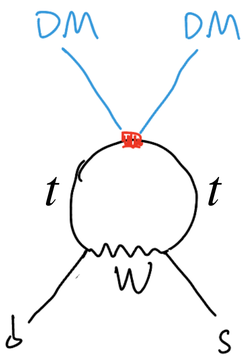
$$\mathcal{O}_{d\phi^2} = (\bar{d}_p P_R d_r)\phi^2$$

$$\mathcal{O}^L_{dXX} = (\bar{d}_p\gamma^\mu P_L d_r)X_{\mu\nu}X^\nu$$

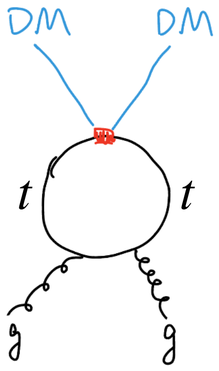
$$\mathcal{O}^{V,LR}_{d\chi} = (\bar{d}_p\gamma_\mu P_L d_r)(\bar{\chi}\gamma^\mu\chi),$$

+ $\mathcal{O}(20)$ operators

$$\langle H_2 | \mathcal{O}_i | H_1 \rangle$$



direct detection



$$\mathcal{O}_{g\phi} = G^a_{\mu\nu}G^{\mu\nu,a}\phi^2,$$

$$\mathcal{O}_{gX} = G^a_{\mu\nu}G^{\mu\nu,a}X_\rho X^\rho,$$

$$\mathcal{O}_{g\chi} = G^a_{\mu\nu}G^{\mu\nu,a}\bar{\chi}\chi$$

+ operators with u, d, s

$$\langle N | \mathcal{O}_j | N \rangle$$

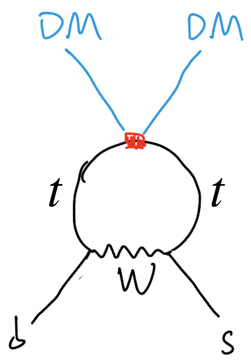
One-Loop Matching between Dark SMEFT and Dark LEFT

- Dark SMEFT

$$\mathcal{L}_{\rm DSMEFT} \supset \sum_i \mathcal{C}_i \mathcal{Q}_i^{(4)} + \boxed{\frac{1}{\Lambda}\sum_j \mathcal{C}_j \mathcal{Q}_j^{(5)}} + \boxed{\frac{1}{\Lambda^2}\sum_k \mathcal{C}_k \mathcal{Q}_k^{(6)}} + \boxed{\frac{1}{\Lambda^3}\sum_l \mathcal{C}_l \mathcal{Q}_l^{(7)}}$$

$\mathcal{Q}_{u\phi^2} = (\bar{q}_p u_r \widetilde{H})\phi^2 + \text{h.c.}$	$\mathcal{Q}_{\phi u} = (\bar{u}_p \gamma_\mu u_r)(i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2)$	$\mathcal{Q}_{\phi q} = (\bar{q}_p \gamma_\mu q_r)(i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2)$	$\cdots\cdots$	scalar
$\mathcal{Q}_{u\chi^2} = (\bar{q}_p u_r \widetilde{H})(\chi^T C \chi)$	$\mathcal{Q}_{u\chi} = (\bar{u}_p \gamma_\mu u_r)(\bar{\chi}\gamma^\mu \chi)$	$\mathcal{Q}_{q\chi} = (\bar{q}_p \gamma_\mu q_r)(\bar{\chi}\gamma^\mu \chi)$	$\cdots\cdots$	fermion
$\mathcal{Q}_{qXX} = (\bar{q}_p \gamma_\mu q_r) X^{\mu\nu} X_\nu$	$\mathcal{Q}_{u\widetilde{X}X} = (\bar{u}_p \gamma_\mu u_r) \widetilde{X}^{\mu\nu} X_\nu$	$\mathcal{Q}_{uX^2} = (\bar{q}_p u_r \widetilde{H}) X_\mu X^\mu + \text{h.c.}$	$\cdots\cdots$	vector

- Matching



- Dark LEFT: **hadron detection**

$$\mathcal{L}_{\rm DLEFT} \supset \sum_k L_k \mathcal{O}_i^{(4)} + \boxed{\frac{1}{v}\sum_i L_i \mathcal{O}_i^{(5)}} + \boxed{\frac{1}{v^2}\sum_j L_j \mathcal{O}_j^{(6)}} + \boxed{\frac{1}{v^3}\sum_l L_l \mathcal{O}_l^{(7)}}$$

$\mathcal{O}_{\phi d}^R = (\bar{d}_p \gamma_\mu P_R d_r)(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2)$	$\mathcal{O}_{d\phi^2} = (\bar{d}_p P_R d_r)\phi^2$	$\cdots\cdots$	scalar
$\mathcal{O}_{d\chi}^{V,RR} = (\bar{d}_p \gamma_\mu P_R d_r)(\bar{\chi}\gamma^\mu \chi)$	$\mathcal{O}_{d\chi}^{V,LR} = (\bar{d}_p \gamma_\mu P_L d_r)(\bar{\chi}\gamma^\mu \chi)$	$\cdots\cdots$	fermion
$\mathcal{O}_{dXX}^R = (\bar{d}_p \gamma^\mu P_R d_r) X_{\mu\nu} X^\nu$	$\mathcal{O}_{dXX}^L = (\bar{d}_p \gamma^\mu P_L d_r) X_{\mu\nu} X^\nu$	$\cdots\cdots$	vector

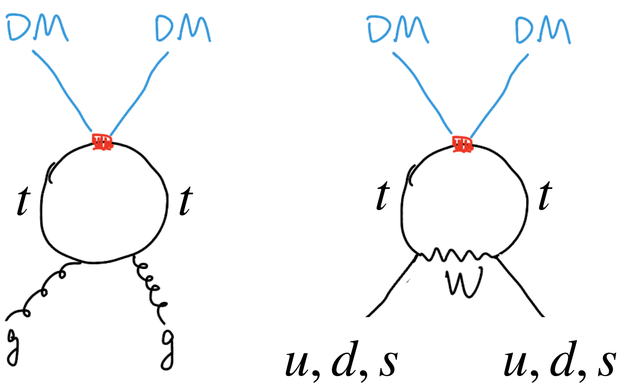
One-Loop Matching between Dark SMEFT and Dark LEFT

- Dark SMEFT

$$\mathcal{L}_{\rm DSMEFT} \supset \sum_i \mathcal{C}_i \mathcal{Q}_i^{(4)} + \frac{1}{\Lambda}\sum_j \mathcal{C}_j \mathcal{Q}_j^{(5)} + \frac{1}{\Lambda^2}\sum_k \mathcal{C}_k \mathcal{Q}_k^{(6)} + \frac{1}{\Lambda^3}\sum_l \mathcal{C}_l \mathcal{Q}_l^{(7)}$$

$\mathcal{Q}_{u\phi^2} = (\bar{q}_p u_r \widetilde{H})\phi^2 + \text{h.c.}$	$\mathcal{Q}_{\phi u} = (\bar{u}_p \gamma_\mu u_r)(i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2)$	$\mathcal{Q}_{\phi q} = (\bar{q}_p \gamma_\mu q_r)(i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2)$	$\cdots\cdots$	scalar
$\mathcal{Q}_{u\chi^2} = (\bar{q}_p u_r \widetilde{H})(\chi^T C \chi)$	$\mathcal{Q}_{u\chi} = (\bar{u}_p \gamma_\mu u_r)(\bar{\chi}\gamma^\mu \chi)$	$\mathcal{Q}_{q\chi} = (\bar{q}_p \gamma_\mu q_r)(\bar{\chi}\gamma^\mu \chi)$	$\cdots\cdots$	fermion
$\mathcal{Q}_{qXX} = (\bar{q}_p \gamma_\mu q_r) X^{\mu\nu} X_\nu$	$\mathcal{Q}_{u\widetilde{X}X} = (\bar{u}_p \gamma_\mu u_r) \widetilde{X}^{\mu\nu} X_\nu$	$\mathcal{Q}_{uX^2} = (\bar{q}_p u_r \widetilde{H}) X_\mu X^\mu + \text{h.c.}$	$\cdots\cdots$	vector

- Matching

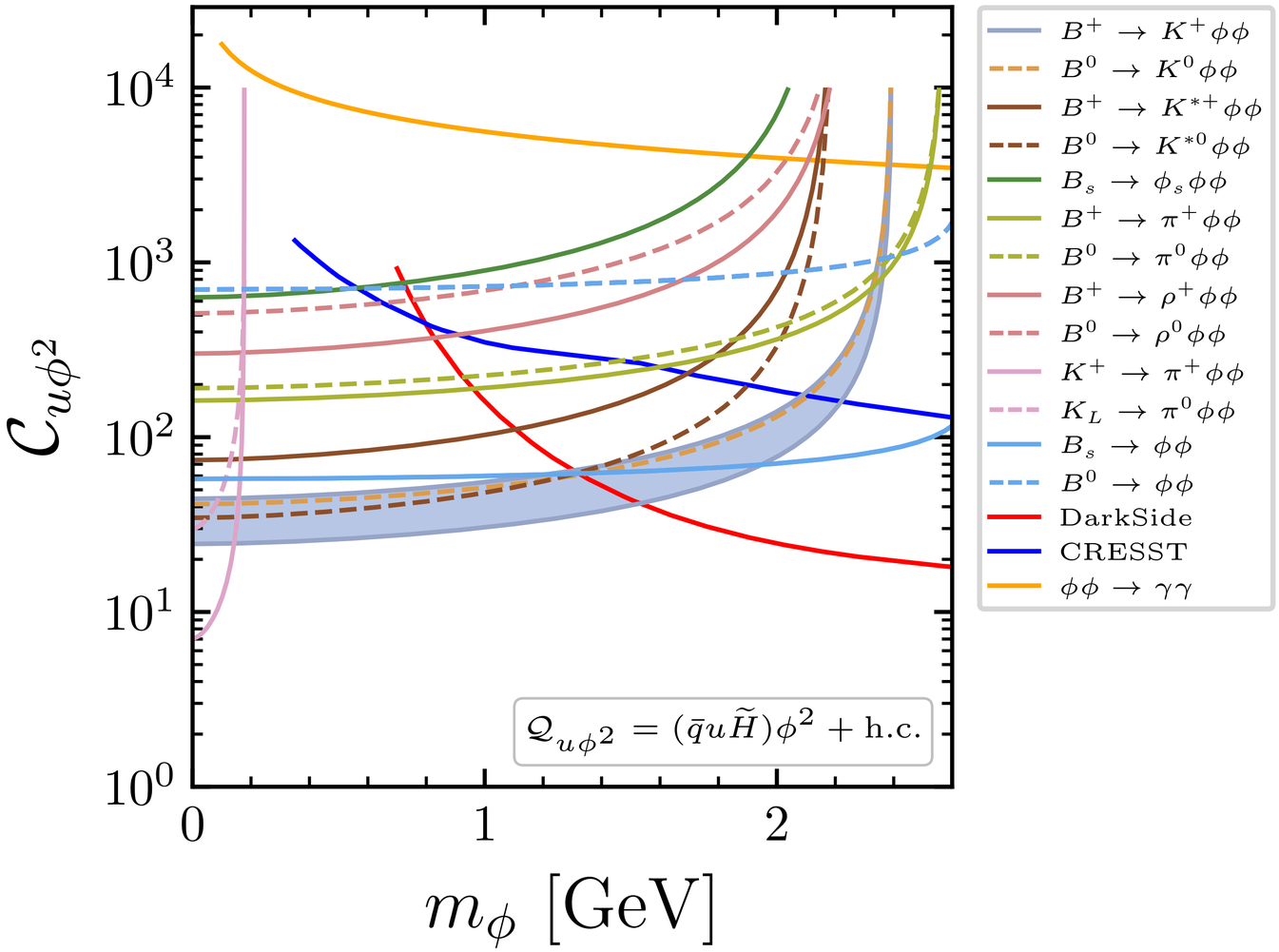


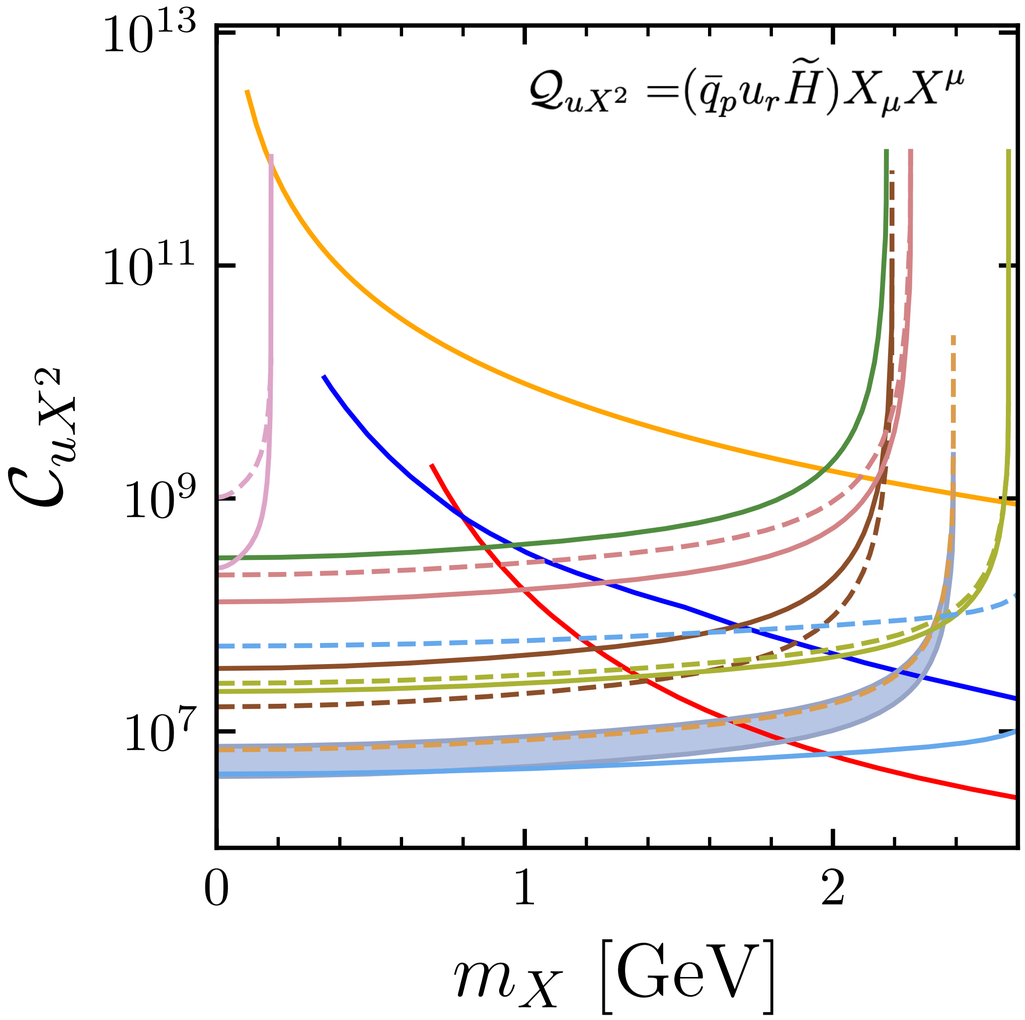
- Dark LEFT: **direct detection**

$$\mathcal{L}_{\rm DLEFT} \supset \sum_k L_k \mathcal{O}_i^{(4)} + \frac{1}{v}\sum_i L_i \mathcal{O}_i^{(5)} + \frac{1}{v^2}\sum_j L_j \mathcal{O}_j^{(6)} + \frac{1}{v^3}\sum_l L_l \mathcal{O}_l^{(7)}$$

$\mathcal{O}_{g\phi} = G^a_{\mu\nu} G^{\mu\nu,a} \phi^2$	$\mathcal{O}_{d\phi^2} = (\bar{d}_p P_R d_r)\phi^2$	$\cdots\cdots$	scalar
$\mathcal{O}_{g\chi} = G^a_{\mu\nu} G^{\mu\nu,a} \bar{\chi}\chi$	$\mathcal{O}^{V,LR}_{d\chi} = (\bar{d}_p \gamma_\mu P_L d_r)(\bar{\chi}\gamma^\mu \chi)$	$\cdots\cdots$	fermion
$\mathcal{O}_{gX} = G^a_{\mu\nu} G^{\mu\nu,a} X_\rho X^\rho$	$\mathcal{O}^L_{dXX} = (\bar{d}_p \gamma^\mu P_L d_r) X_{\mu\nu} X^\nu$	$\cdots\cdots$	vector

Hadron decay vs Direct detection



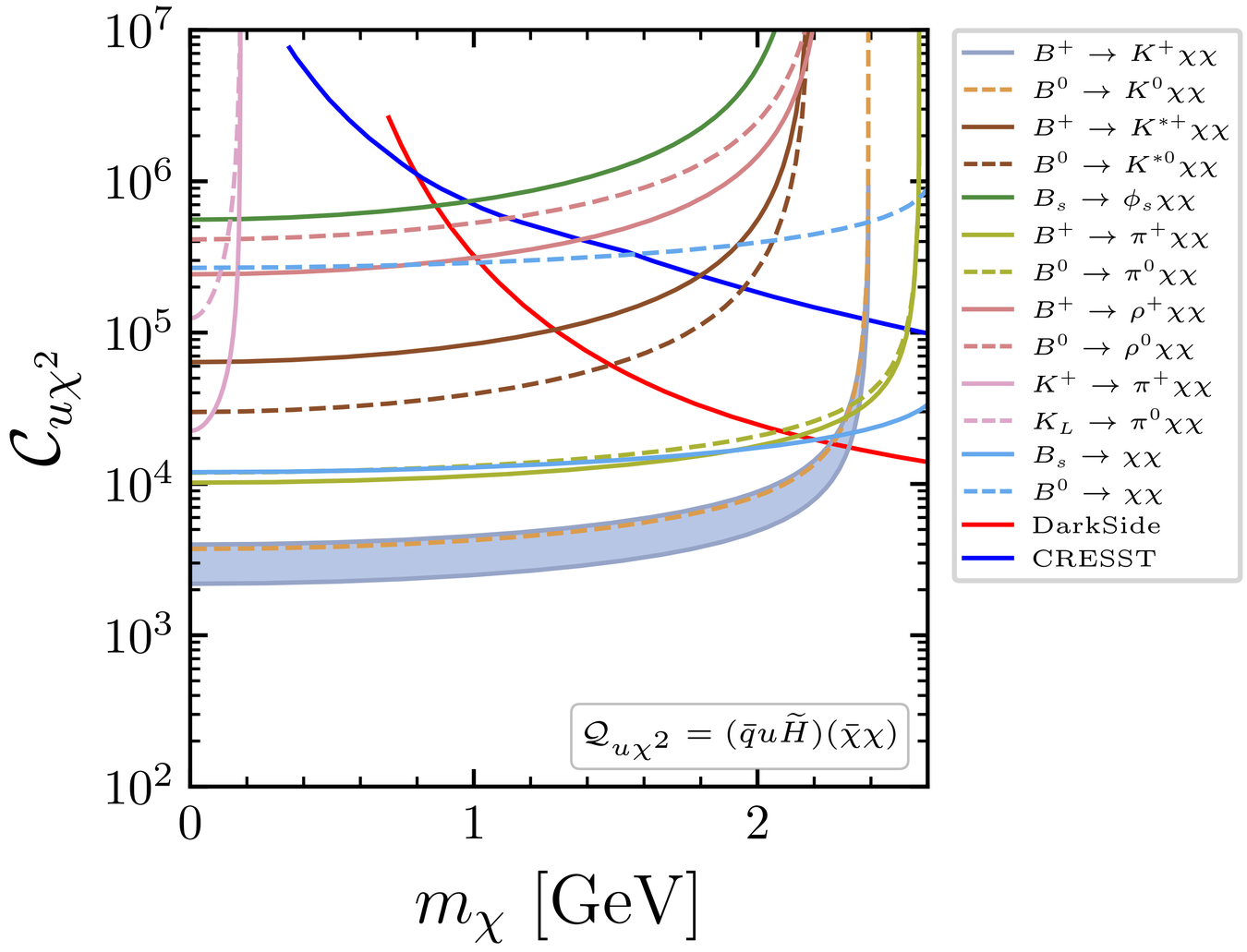


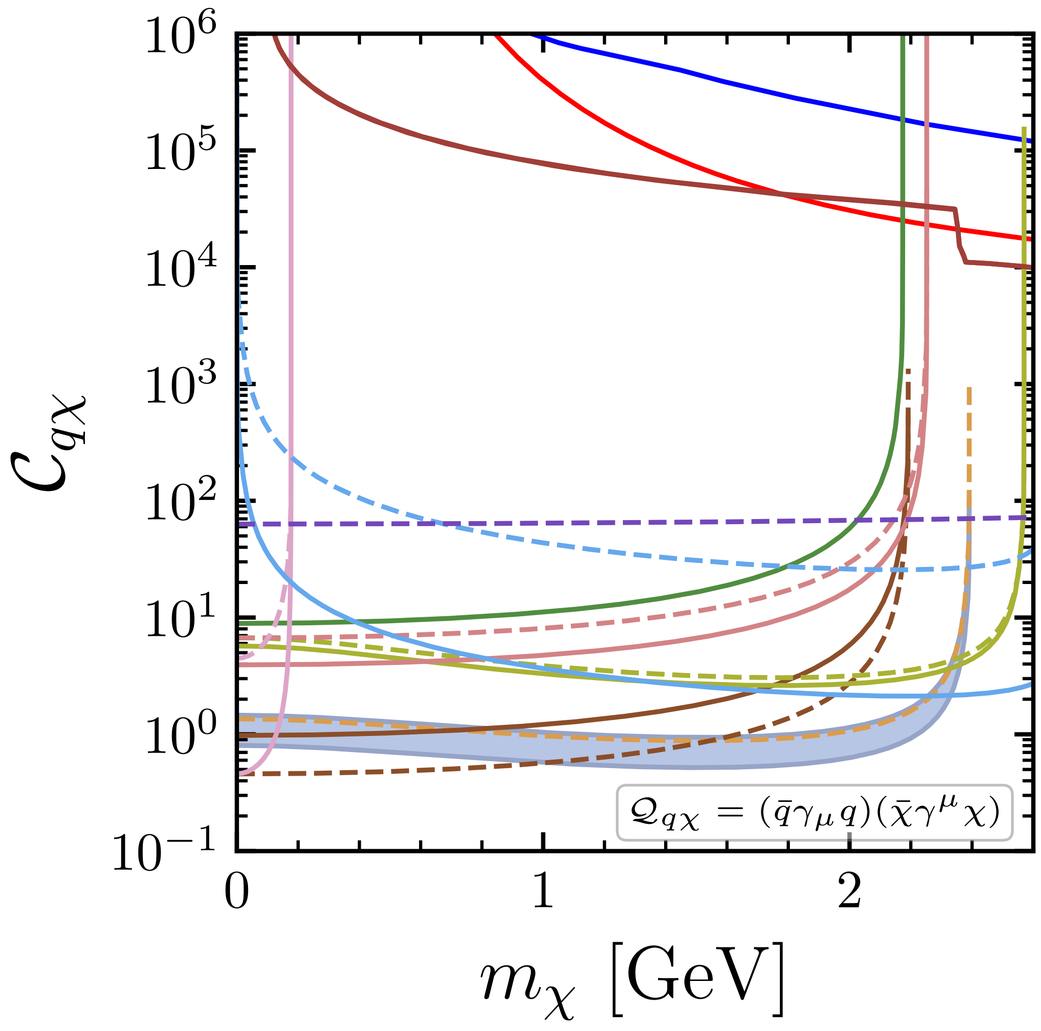
$$\mathscr{L} \supset \frac{C_{u\phi^2}}{\Lambda^2} O_{u\phi^2}$$

$\Lambda = 1$ TeV

$$\mathscr{L} \supset \frac{C_{uX^2}}{\Lambda^2}\frac{m_X^2}{\Lambda^2} O_{uX^2}$$

Hadron decay vs Direct detection





$$\mathscr{L} \supset \frac{C_{u\chi^2}}{\Lambda^3} O_{u\chi^2}$$

$\Lambda = 1$ TeV

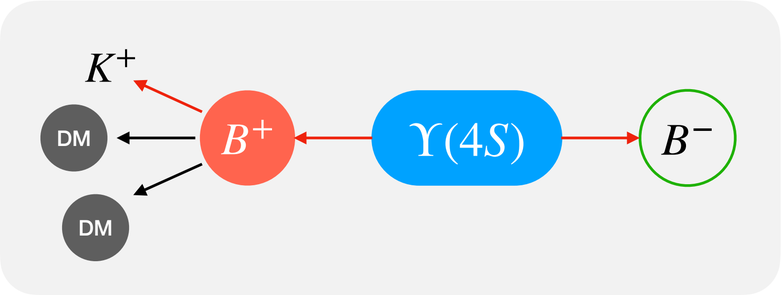
$$\mathscr{L} \supset \frac{C_{q\chi^2}}{\Lambda^2} O_{q\chi^2}$$

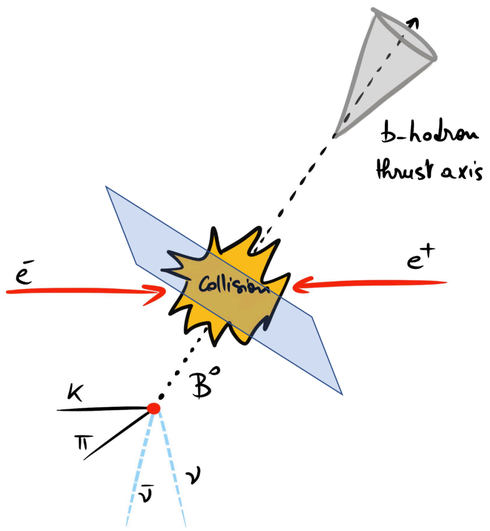
Conclusion

A Possible Flavour Path to Dark Matter

	d	s	b
d	DD	NA62/KOTO	Belle II/CEPC
s		DD	Belle II/CEPC
b			Belle II/LHC

	u	c	t
u	DD	BES/STCF	LHC/CEPC
c		BES/STCF	LHC/CEPC
t			LHC/CEPC





b-hadron FCNC decays with invisible final states is a possible path to New Physics, e.g., dark matter.

Many promising decay modes can be searched at CEPC, FCC-ee and Belle II

- $B_s \to \text{inv}$
- $\Lambda_b \to \Lambda + \text{inv}$
- $B_s \to \phi + \text{inv}$
- $B \to K^{(*)} + \text{inv}$

… …

Thank You !

Backup

Summary

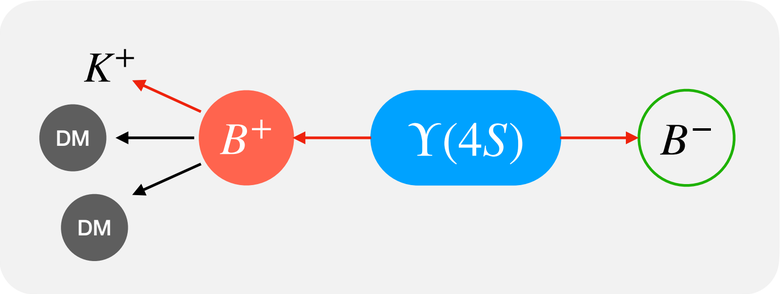
It's natural to ask:

1. **By combing the direct detection, flavour measurements, and relic density, is it possible to obtain the flavour structure of interactions between light DM and SM particles ?**

   top-flavoured DM

2. **Compared to DM direct detection, what's the advantage of hadron decays? which DM scenario is more sensitive to ? Can they provide other information ?**

   angular distribution (F_L) ;



A Possible Flavour Path to Dark Matter

	d	s	b
d	DD	NA62/KOTO	Belle II
s		DD	Belle II
b			Belle II/LHC

	u	c	t
u	DD	BES/STCF	LHC/CEPC
c		BES/STCF	LHC/CEPC
t			LHC/CEPC

`HadronToNP`: a package to calculate decay of hadron to new particles

$B \to K + \mathrm{DM},\ B \to \rho + \mathrm{DM},\ \Lambda_b \to \Lambda + \mathrm{DM},\ \Upsilon \to \mathrm{DM}, \ldots$

$D \to \pi + \mathrm{DM},\ D \to \rho + \mathrm{DM},\ \Xi_c \to \Xi + \mathrm{DM},\ J/\psi \to \mathrm{DM}, \ldots$

to be finished

many things to do, e.g., other flavour structures (MFV, $U(2)^5$), simplified model (light mediator), operator basis (SPVA), UV completion, full one-loop matching between DLEFT and DSMEFT, astro-constraints, …

Dark SMEFT Operator

operator	definition	tree	loop	eq
$\mathcal{Q}_{u\phi}$	$(\bar{q}_p u_r \widetilde{H})\phi + \text{h.c.}$			
$\mathcal{Q}_{u\phi^2}$	$(\bar{q}_p u_r \widetilde{H})\phi^2 + \text{h.c.}$			
$\mathcal{Q}_{\phi q}$	$(\bar{q}_p \gamma_\mu q_r)(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2)$			
$\mathcal{Q}_{\phi u}$	$(\bar{u}_p \gamma_\mu u_r)(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2)$			
$\mathcal{Q}_{q\chi}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{\chi}\gamma^\mu \chi)$			
$\mathcal{Q}_{u\chi}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{\chi}\gamma^\mu \chi)$			
$\mathcal{Q}_{u\chi^2}$	$(\bar{q}_p u_r \widetilde{H})(\chi^T C \chi)$			

$\mathcal{Q}_{qX}$	$(\bar{q}_p \gamma^\mu q_r) X_\mu$			
$\mathcal{Q}_{uX}$	$(\bar{u}_p \gamma^\mu u_r) X_\mu$			
$\mathcal{Q}_{HqX}^{(1)}$	$(H^\dagger H)(\bar{q}_p \gamma^\mu q_r) X_\mu$			
$\mathcal{Q}_{HqX}^{(3)}$	$(H^\dagger \tau^I H)(\bar{q}_p \tau^I \gamma^\mu q_r) X_\mu$			
$\mathcal{Q}_{uX^2}$	$(\bar{q}_p u_r \widetilde{H}) X_\mu X^\mu + \text{h.c.}$			
$\mathcal{Q}_{HuX}$	$(H^\dagger H)(\bar{u}_p \gamma^\mu u_r) X_\mu$			
$\mathcal{Q}_{qXX}$	$(\bar{q}_p \gamma_\mu q_r) X^{\mu\nu} X_\nu$			
$\mathcal{Q}_{q\widetilde{X}X}$	$(\bar{q}_p \gamma_\mu q_r) \widetilde{X}^{\mu\nu} X_\nu$			
$\mathcal{Q}_{uXX}$	$(\bar{u}_p \gamma_\mu u_r) X^{\mu\nu} X_\nu$			
$\mathcal{Q}_{u\widetilde{X}X}$	$(\bar{u}_p \gamma_\mu u_r) \widetilde{X}^{\mu\nu} X_\nu$			
$\mathcal{Q}_{DqX^2}$	$(\bar{q}_p \gamma^\nu i D^\mu q_r)\, X_\mu X_\nu + \text{h.c.}$			
$\mathcal{Q}_{DuX^2}$	$(\bar{u}_p \gamma^\nu i D^\mu u_r)\, X_\mu X_\nu + \text{h.c.}$			
$\mathcal{Q}_{uHX^2}$	$(\bar{q}_p \sigma_{\mu\nu} u_r \widetilde{H}) X_1^\mu X_2^\nu + \text{h.c.}$			
$\mathcal{Q}_{uHX}$	$(\bar{q}_p \sigma_{\mu\nu} u_r) \widetilde{H} X^{\mu\nu} + \text{h.c.}$			

Dark LEFT Operator

operator	definition	tree	loop
$\mathcal{O}_{d\phi}$	$(\bar{d}_{Lp}d_{Rr})\phi + \text{h.c.}$	–	$\mathcal{Q}_{u\phi}$
$\mathcal{O}_{d\phi^2}$	$(\bar{d}_{Lp}d_{Rr})\phi^2 + \text{h.c.}$	–	$\mathcal{Q}_{u\phi^2}$
$\mathcal{O}^L_{\phi d}$	$(\bar{d}_{Lp}\gamma_\mu d_{Lr})(i\phi_1\overleftrightarrow{\partial^\mu}\phi_2)$	$\mathcal{Q}_{\phi q}$	$\mathcal{Q}_{\phi u}$
$\mathcal{O}^R_{\phi d}$	$(\bar{d}_{Rp}\gamma_\mu d_{Rr})(i\phi_1\overleftrightarrow{\partial^\mu}\phi_2)$	–	–
$\mathcal{O}^{S,LR}_{d\chi}$	$(\bar{d}_{Rp}d_{Lr})(\chi^T C\chi) + \text{h.c.}$	–	$\mathcal{Q}_{u\chi^2}$
$\mathcal{O}^{S,RR}_{d\chi}$	$(\bar{d}_{Lp}d_{Rr})(\chi^T C\chi) + \text{h.c.}$	–	$\mathcal{Q}_{u\chi^2}$
$\mathcal{O}^{V,LR}_{d\chi}$	$(\bar{d}_{Lp}\gamma_\mu d_{Lr})(\bar{\chi}\gamma^\mu\chi)$	$\mathcal{Q}_{q\chi}$	$\mathcal{Q}_{u\chi}$
$\mathcal{O}^{V,RR}_{d\chi}$	$(\bar{d}_{Rp}\gamma_\mu d_{Rr})(\bar{\chi}\gamma^\mu\chi)$	–	–
$\mathcal{O}^{T,RR}_{d\chi}$	$(\bar{d}_{Lp}\sigma^{\mu\nu}d_{Rr})(\chi^T C\sigma_{\mu\nu}\chi) + \text{h.c.}$	–	–
$\mathcal{O}^T_{dX}$	$(\bar{d}_{Lp}\sigma^{\mu\nu}d_{Rr})X_{\mu\nu} + \text{h.c.}$	–	$\mathcal{Q}_{uX}, \mathcal{Q}_{qX}, \mathcal{Q}_{uHX}, \mathcal{Q}_{HuX}, \mathcal{Q}_{HqX}$
$\mathcal{O}^L_{dX}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})X_\mu$	$\mathcal{Q}_{qX}, \mathcal{Q}_{HqX}$	$\mathcal{Q}_{uX}, \mathcal{Q}_{HuX}$
$\mathcal{O}^R_{dX}$	$(\bar{d}_{Rp}\gamma^\mu d_{Rr})X_\mu$	–	–
$\mathcal{O}_{dX^2}$	$(\bar{d}_{Lp}d_{Rr})X_\mu X^\mu + \text{h.c.}$	–	$\mathcal{Q}_{uX^2}, \mathcal{Q}_{DuX^2}, \mathcal{Q}_{DqX^2}$
$\mathcal{O}^T_{dX^2}$	$(\bar{d}_{Lp}\sigma_{\mu\nu}d_{Rr})X_1^\mu X_2^\nu + \text{h.c.}$	–	$\mathcal{Q}_{uHX^2}$
$\mathcal{O}^L_{dXX}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})X_{\mu\nu}X^\nu$	$\mathcal{Q}_{qXX}$	$\mathcal{Q}_{uXX}$
$\mathcal{O}^R_{dXX}$	$(\bar{d}_{Rp}\gamma^\mu d_{Rr})X_{\mu\nu}X^\nu$	–	–
$\mathcal{O}^L_{d\widetilde{X}X}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})\widetilde{X}_{\mu\nu}X^\nu$		
$\mathcal{O}^R_{d\widetilde{X}X}$	$(\bar{d}_{Rp}\gamma^\mu d_{Rr})\widetilde{X}_{\mu\nu}X^\nu$		
$\mathcal{O}^L_{DdX^2}$	$(\bar{d}_{Lp}\gamma^\mu iD^\nu d_{Lr})X_\mu X_\nu + \text{h.c.}$	$\mathcal{Q}_{DqX^2}$	$\mathcal{Q}_{DuX^2}$
$\mathcal{O}^R_{DdX^2}$	$(\bar{d}_{Rp}\gamma^\mu iD^\nu d_{Rr})X_\mu X_\nu + \text{h.c.}$	–	–
$\mathcal{O}^L_{da}$	$(\bar{d}_{Lp}\gamma_\mu d_{Lr})\partial^\mu a$	$\mathcal{Q}_{qa}$	$\mathcal{Q}_{ua}$
$\mathcal{O}^R_{da}$	$(\bar{d}_{Rp}\gamma_\mu d_{Rr})\partial^\mu a$	–	–
$\mathcal{O}_{GG\phi^2}$	$G^a_{\mu\nu}G^{\mu\nu,a}\phi^2\,(\mathcal{O}_{g\phi})$	–	$\mathcal{Q}_{u\phi^2}$
$\mathcal{O}_{G\widetilde{G}\phi^2}$	$G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}\phi^2\,(\mathcal{O}_{g\phi})$	–	$\mathcal{Q}_{u\phi^2}$
$\mathcal{O}_{GG\chi}$	$G^a_{\mu\nu}G^{\mu\nu,a}\chi^T C\chi\,(\mathcal{O}_{g\chi})$	–	$\mathcal{Q}_{u\chi^2}$
$\mathcal{O}_{G\widetilde{G}\chi}$	$G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}\chi^T C\chi\,(\mathcal{O}_{g\chi})$	–	$\mathcal{Q}_{u\chi^2}$
$\mathcal{O}_{GGXX}$	$G^a_{\mu\nu}G^{\mu\nu,a}X_\rho X^\rho\,(\mathcal{O}_{gX})$	–	$\mathcal{Q}_{uX^2}$
$\mathcal{O}_{G\widetilde{G}XX}$	$G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a}X_\rho X^\rho\,(\mathcal{O}_{gX})$	–	$\mathcal{Q}_{uX^2}$
$\mathcal{O}_{GXGX}$	$G^a_{\lambda\mu}G^{\lambda,a}_{\ \ \nu}X^\mu X^\nu\,(\mathcal{O}_{gXX})$	–	$\mathcal{Q}_{DqX^2}, \mathcal{Q}_{DuX^2}$
$\mathcal{O}_{GX\widetilde{G}X}$	$G^a_{\lambda\mu}\widetilde{G}^{\lambda,a}_{\ \ \nu}X^\mu X^\nu\,(\mathcal{O}_{gXX})$		
$\mathcal{O}_{FF\phi^2}$	$F_{\mu\nu}F^{\mu\nu}\phi^2\,(\mathcal{O}_{\phi\gamma})$	–	$\mathcal{Q}_{u\phi^2}$
$\mathcal{O}_{F\widetilde{F}\phi^2}$	$F_{\mu\nu}\widetilde{F}^{\mu\nu}\phi^2\,(\mathcal{O}_{\phi\gamma})$		
$\mathcal{O}_{FF\chi}$	$F_{\mu\nu}F^{\mu\nu}\chi^T C\chi\,(\mathcal{O}_{\chi\gamma})$	–	$\mathcal{Q}_{u\chi}$
$\mathcal{O}_{F\widetilde{F}\chi}$	$F_{\mu\nu}\widetilde{F}^{\mu\nu}\chi^T C\chi\,(\mathcal{O}_{\chi\gamma})$		
$\mathcal{O}_{FFXX}$	$F_{\mu\nu}F^{\mu\nu}X_\rho X^\rho\,(\mathcal{O}_{X\gamma})$	–	$\mathcal{Q}_{uX^2}$
$\mathcal{O}_{F\widetilde{F}XX}$	$F_{\mu\nu}\widetilde{F}^{\mu\nu}X_\rho X^\rho\,(\mathcal{O}_{X\gamma})$		
$\mathcal{O}_{FXFX}$	$F_{\lambda\mu}F^\lambda_{\ \ \nu}X^\mu X^\nu\,(\mathcal{O}_{XX\gamma})$	–	$\mathcal{Q}_{DqX^2}, \mathcal{Q}_{DuX^2}$
$\mathcal{O}_{FX\widetilde{F}X}$	$F_{\lambda\mu}\widetilde{F}^\lambda_{\ \ \nu}X^\mu X^\nu\,(\mathcal{O}_{XX\gamma})$		

$$\mathcal{C} = \begin{cases} \mathcal{C}_t & \text{for } \bar{u}_L\gamma^\mu\mathcal{C}u_L,\ u_R\gamma^\mu\mathcal{C}u_R,\ \bar{u}_L\mathcal{C}u_R,\ \bar{u}_L\sigma^{\mu\nu}\mathcal{C}u_R, \\ V^\dagger\mathcal{C}_tV & \text{for } \bar{d}_L\gamma^\mu\mathcal{C}d_L, \\ V^\dagger\mathcal{C}_t & \text{for } \bar{d}_L\mathcal{C}u_R,\ \bar{d}_L\sigma^{\mu\nu}\mathcal{C}u_R, \end{cases}$$

$$V^\dagger\mathcal{C}_tV = \mathcal{C}_{33}\begin{pmatrix} V_{td}^*V_{td} & V_{td}^*V_{ts} & V_{td}^*V_{tb} \\ V_{ts}^*V_{td} & V_{ts}^*V_{ts} & V_{ts}^*V_{tb} \\ V_{tb}^*V_{td} & V_{tb}^*V_{ts} & V_{tb}^*V_{tb} \end{pmatrix} \qquad \mathcal{C}_t \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathcal{C}_{33} \end{pmatrix}$$

$$= \mathcal{C}_{33}\begin{pmatrix} 0.71 & -3.09-1.45i & 76.81+34.29i \\ -3.09+1.45i & 16.43 & -404.93+7.87i \\ 76.81-34.29i & -404.93-7.87i & 9982.86 \end{pmatrix} \times 10^{-4}$$

$$V^\dagger\mathcal{C}_t = \mathcal{C}_{33}\begin{pmatrix} 0 & 0 & V_{td}^* \\ 0 & 0 & V_{ts}^* \\ 0 & 0 & V_{tb}^* \end{pmatrix} = \mathcal{C}_{33}\begin{pmatrix} 0 & 0 & 0.77+0.34i \\ 0 & 0 & -4.05+0.08i \\ 0 & 0 & 99.91 \end{pmatrix} \times 10^{-2}$$

$$\mathcal{C}_i = \begin{cases} \mathcal{C}_t^i & \text{for } \mathcal{Q}_i = \mathcal{Q}_{\phi u},\ \mathcal{Q}_{u\chi},\ \mathcal{Q}_{uX},\ \mathcal{Q}_{HuX},\ \mathcal{Q}_{uXX},\ \mathcal{Q}_{u\tilde{X}X},\ \mathcal{Q}_{DuX^2},\ \mathcal{Q}_{ua}, \\ (\mathcal{C}_t^i,\ V^\dagger\mathcal{C}_t^i) & \text{for } \mathcal{Q}_i = \mathcal{Q}_{u\phi},\ \mathcal{Q}_{u\phi^2},\ \mathcal{Q}_{u\chi^2},\ \mathcal{Q}_{qu\chi l},\ \mathcal{Q}_{uHX},\ \mathcal{Q}_{uX^2},\ \mathcal{Q}_{uHX^2}, \\ (\mathcal{C}_t^i,\ V^\dagger\mathcal{C}_t^iV) & \text{for } \mathcal{Q}_i = \mathcal{Q}_{\phi q},\ \mathcal{Q}_{q\chi},\ \mathcal{Q}_{qX},\ \mathcal{Q}_{HqX}^{(1,3)},\ \mathcal{Q}_{qXX},\ \mathcal{Q}_{q\tilde{X}X},\ \mathcal{Q}_{DqX^2},\ \mathcal{Q}_{qa}, \end{cases}$$

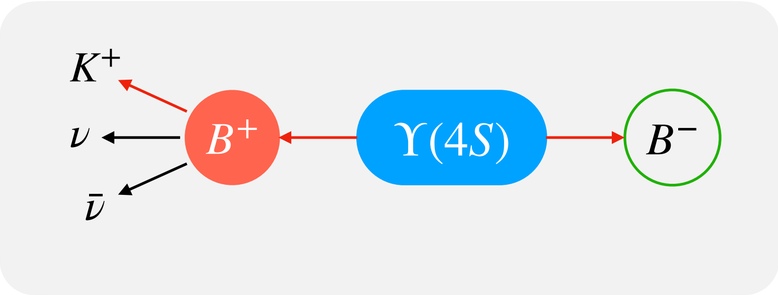
Conclusion

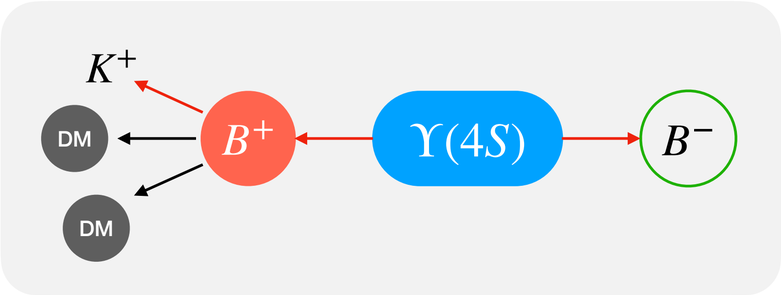
HadronToNP: a package to calculate decay of hadron to new particles

$B \to K + \mathrm{DM},\ B \to \rho + \mathrm{DM},\ \Lambda_b \to \Lambda + \mathrm{DM},\ \Upsilon \to \mathrm{DM}, \ldots$

$D \to \pi + \mathrm{DM},\ D \to \rho + \mathrm{DM},\ \Xi_c \to \Xi + \mathrm{DM},\ J/\psi \to \mathrm{DM}, \ldots$

to be finished

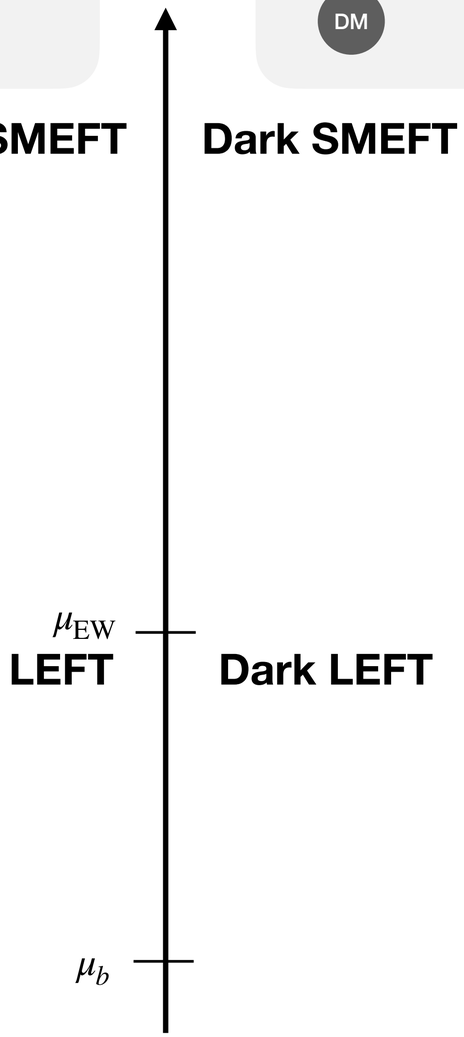




$$\frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\mathrm{SM}}}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\mathrm{SM}}} = 0.46 \pm 0.07$$

$$\frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})}{\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\mathrm{SM}}}{\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}}} = 29.7 \pm 5.6$$

Belle II excess (if confirmed in the future) implies:
- impossible to explain in SMEFT with MFV
- NP flavour structure is highly non-trivial
- **NP structure in quark sector is beyond MFV**
- **flavour violation is beyond Yukawa coupling**



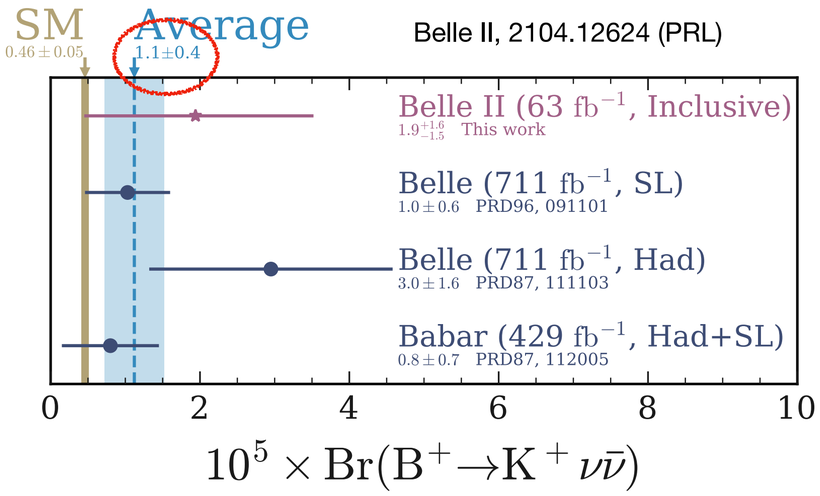
A Possible Flavour Path to Dark Matter

	d	s	b
d	DD	NA62/KOTO	Belle II
s		DD	Belle II
b			Belle II/LHC

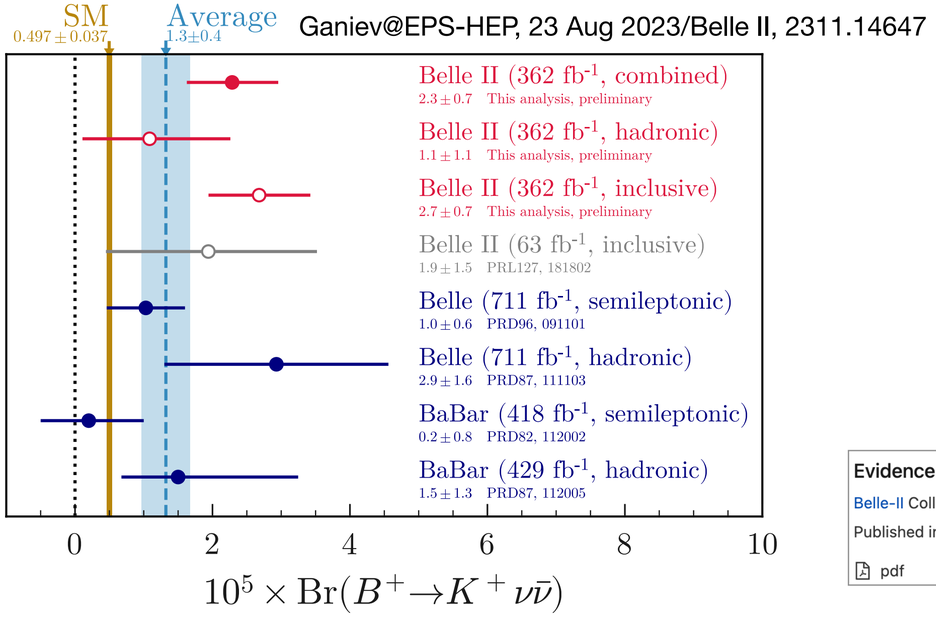
	u	c	t
u	DD	BES/STCF	LHC/CEPC
c		BES/STCF	LHC/CEPC
t			LHC/CEPC

$b \to s\nu\bar{\nu}$: exp & theory

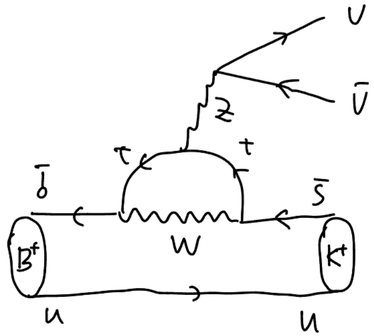
- **2021 Apr**



- **2023 Aug**



- **Exp vs SM** $[10^{-6}]$



$$\mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\mathrm{SM}} = 4.16 \pm 0.57$$
$$\mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\mathrm{exp}} = 23 \pm 7$$

2.7σ difference

NP/SM $\gtrsim 2$

$$\mathscr{B}(B^+ \to K^+\nu\bar{\nu})_{\mathrm{exp}} \gtrsim 10 \ (2\sigma \text{ lower bound})$$

- **Theoretical prediction**

Factorization

$$\mathscr{A} \propto C_L \cdot \langle K | \bar{s}\gamma^\mu b | \bar{B} \rangle \cdot \bar{\nu}\gamma_\mu \nu$$

Wilson coef **quark current** **neutrino current**

theoretically, simple and clean
one of the cleanest channels in flavour physics

$\mathcal{O}_L = (\bar{s}\gamma_\mu P_L b)(\bar{\nu}\gamma^\mu P_L \nu)$ **in the SM**

$\mathcal{O}_R = (\bar{s}\gamma_\mu P_R b)(\bar{\nu}\gamma^\mu P_L \nu)$ **possible in BSM**

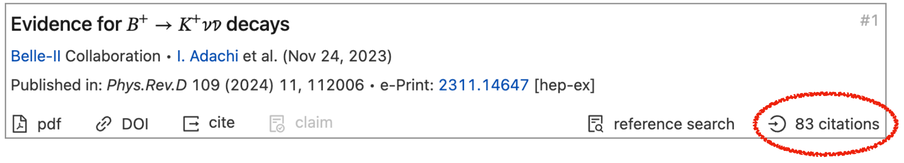
$\mathcal{O}_L = (\bar{s}P_L b)(\bar{\nu}P_L \nu)$ ✘

$\mathcal{O}_R = (\bar{s}P_R b)(\bar{\nu}P_R \nu)$ ✘

$\mathcal{O}_T = (\bar{s}\sigma_{\mu\nu} b)(\bar{\nu}\sigma^{\mu\nu} \nu)$ ✘

$\mathcal{O}_{T5} = (\bar{s}\sigma_{\mu\nu}\gamma_5 b)(\bar{\nu}\sigma^{\mu\nu} \nu)$ ✘

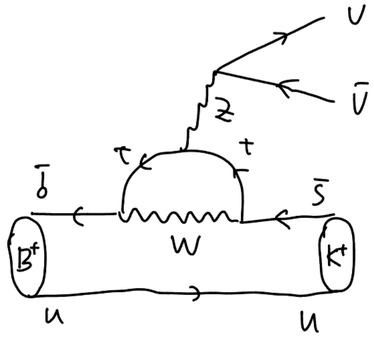
operator structure highly constrained by LH neutrino



$b \to s\nu\bar{\nu}$: exp & theory

	Observable	SM	Exp	Unit
$b \to s$	$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})$	4.16 ± 0.57	$23 \pm 5^{+5}_{-4}$	10^{-6}
	$\mathcal{B}(B^0 \to K^0\nu\bar{\nu})$	3.85 ± 0.52	< 26	10^{-6}
	$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})$	9.70 ± 0.94	< 61	10^{-6}
	$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$	9.00 ± 0.87	< 18	10^{-6}
	$\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	9.93 ± 0.72	< 5400	10^{-6}
	$\mathcal{B}(B_s \to \nu\bar{\nu})$	≈ 0	< 5.9	10^{-4}
$b \to d$	$\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})$	1.40 ± 0.18	< 140	10^{-7}
	$\mathcal{B}(B^0 \to \pi^0\nu\bar{\nu})$	6.52 ± 0.85	< 900	10^{-8}
	$\mathcal{B}(B^+ \to \rho^+\nu\bar{\nu})$	4.06 ± 0.79	< 300	10^{-7}
	$\mathcal{B}(B^0 \to \rho^0\nu\bar{\nu})$	1.89 ± 0.36	< 400	10^{-7}
	$\mathcal{B}(B^0 \to \nu\bar{\nu})$	≈ 0	< 1.4	10^{-4}
$s \to d$	$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$	8.42 ± 0.61	$10.6^{+4.0}_{-3.4} \pm 0.9$	10^{-11}
	$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	3.41 ± 0.45	< 300	10^{-11}

Why such a large NP effect has not shown up in other $b \to s$ decays ?
in $b \to d, s \to d$ decays ?



▶ Exp vs SM $[10^{-6}]$

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm SM} = 4.16 \pm 0.57$$
$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm exp} = 23 \pm 7$$

2.7σ difference
NP/SM $\gtrsim 2$

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm exp} \gtrsim 10 \ (2\sigma \text{ lower bound})$$

▶ Theoretical prediction

Factorization

$$\mathcal{A} \propto C_L \cdot \langle K | \bar{s}\gamma^\mu b | \bar{B} \rangle \cdot \bar{\nu}\gamma_\mu \nu$$

Wilson coef — quark current — neutrino current

theoretically, simple and clean
one of the cleanest channels in flavour physics

$\mathcal{O}_L = (\bar{s}\gamma_\mu P_L b)(\bar{\nu}\gamma^\mu P_L \nu)$ **in the SM**

$\mathcal{O}_R = (\bar{s}\gamma_\mu P_R b)(\bar{\nu}\gamma^\mu P_L \nu)$ **possible in BSM**

$\mathcal{O}_L = (\bar{s}P_L b)(\bar{\nu}P_L \nu)$ ✘

$\mathcal{O}_R = (\bar{s}P_R b)(\bar{\nu}P_R \nu)$ ✘

$\mathcal{O}_T = (\bar{s}\sigma_{\mu\nu} b)(\bar{\nu}\sigma^{\mu\nu} \nu)$ ✘

$\mathcal{O}_{T5} = (\bar{s}\sigma_{\mu\nu}\gamma_5 b)(\bar{\nu}\sigma^{\mu\nu} \nu)$ ✘

operator structure highly constrained by LH neutrino

$b \to s\nu\bar{\nu}$: exp & theory

	Observable	SM	Exp	Unit
$b \to s$	$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})$	4.16 ± 0.57	$23 \pm 5^{+5}_{-4}$	10^{-6}
	$\mathcal{B}(B^0 \to K^0\nu\bar{\nu})$	3.85 ± 0.52	< 26	10^{-6}
	$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})$	9.70 ± 0.94	< 61	10^{-6}
	$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$	9.00 ± 0.87	< 18	10^{-6}
	$\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	9.93 ± 0.72	< 5400	10^{-6}
	$\mathcal{B}(B_s \to \nu\bar{\nu})$	≈ 0	< 5.9	10^{-4}
$b \to d$	$\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})$	1.40 ± 0.18	< 140	10^{-7}
	$\mathcal{B}(B^0 \to \pi^0\nu\bar{\nu})$	6.52 ± 0.85	< 900	10^{-8}
	$\mathcal{B}(B^+ \to \rho^+\nu\bar{\nu})$	4.06 ± 0.79	< 300	10^{-7}
	$\mathcal{B}(B^0 \to \rho^0\nu\bar{\nu})$	1.89 ± 0.36	< 400	10^{-7}
	$\mathcal{B}(B^0 \to \nu\bar{\nu})$	≈ 0	< 1.4	10^{-4}
$s \to d$	$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$	8.42 ± 0.61	$10.6^{+4.0}_{-3.4} \pm 0.9$	10^{-11}
	$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	3.41 ± 0.45	< 300	10^{-11}

Why such a large NP effect has not shown up in other $b \to s$ decays ?
in $b \to d, s \to d$ decays ? **NP flavour structure**

SMEFT

$$\mathcal{Q}_{Hq}^{(1)} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{q}_p \gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hq}^{(3)} = \left(H^\dagger i \overleftrightarrow{D}^I_\mu H\right)\left(\bar{q}_p \tau^I \gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hd} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{d}_p \gamma^\mu d_r\right),$$
$$\mathcal{Q}_{ld} = \left(\bar{l}_p \gamma^\mu l_r\right)\left(\bar{d}_s \gamma_\mu d_t\right),$$
$$\mathcal{Q}_{lq}^{(1)} = \left(\bar{l}_p \gamma^\mu l_r\right)\left(\bar{q}_s \gamma_\mu q_t\right),$$
$$\mathcal{Q}_{lq}^{(3)} = \left(\bar{l}_p \gamma^\mu \tau^I l_r\right)\left(\bar{q}_s \tau^I \gamma_\mu q_t\right),$$

μ_{EW}

LEFT

$$\mathcal{O}_L^{\nu_i\nu_j} = \left(\bar{s}\gamma_\mu P_L b\right)\left(\bar{\nu}_i \gamma^\mu P_L \nu_j\right)$$
$$\mathcal{O}_R^{\nu_i\nu_j} = \left(\bar{s}\gamma_\mu P_R b\right)\left(\bar{\nu}_i \gamma^\mu P_L \nu_j\right)$$

μ_b

operator structure highly constrained by Left-handed neutrino

Minimal Flavour Violation

- Flavour symmetry without Yukawa

$$G_{\mathrm{QF}} = SU(3)_q \otimes SU(3)_u \otimes SU(3)_d$$

- Flavour symmetry breaking only from SM Yukawa

$$-\mathcal{L}_Y = \bar{q}\, Y_d H d + \bar{q}\, Y_u \tilde{H} u + \mathrm{h.c.}$$

- Flavour symmetry recovering: Yukawa coupling $\Longrightarrow$ spurion field

$$Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \qquad\qquad Y_d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$$

D'Ambrosio, Giudice, Isidori, Strumia, 2009

- EFT with MFV: operators, constructed from SM and Yukawa spurion fields, are invariant under CP and G_{QF}

$$\mathcal{C}^{\mathrm{MFV}} = \begin{cases} f(\mathsf{A},\mathsf{B}) & \text{for } \bar{q}\gamma^\mu \mathcal{C} q, \\ f(\mathsf{A},\mathsf{B}) Y_d & \text{for } \bar{q}\mathcal{C} d,\ \bar{q}\sigma^{\mu\nu}\mathcal{C} d, \\ \epsilon_0 \mathbb{1} + Y_d^\dagger g(\mathsf{A},\mathsf{B}) Y_d & \text{for } \bar{d}\gamma^\mu \mathcal{C} d, \end{cases}$$

$$f(\mathsf{A},\mathsf{B}) = \epsilon_0 1 + \epsilon_1 \mathsf{A} + \epsilon_2 \mathsf{B} + \epsilon_3 \mathsf{A}^2 + \epsilon_4 \mathsf{B}^2 + \epsilon_5 \mathsf{AB} + \ldots$$

$$\mathsf{A} = Y_u Y_u^\dagger$$

$$\mathsf{B} = Y_d Y_d^\dagger$$

Minimal Flavour Violation

- Spurion function

$$f(\mathsf{A},\mathsf{B}) = \epsilon_0 1 + \epsilon_1 \mathsf{A} + \epsilon_2 \mathsf{B} + \epsilon_3 \mathsf{A}^2 + \epsilon_4 \mathsf{B}^2 + \epsilon_5 \mathsf{AB} + \ldots\ldots$$

- Cayley-Hamilton identity for 3×3 invertible matrix X

$$X^3 = \mathrm{Det}X \cdot \mathbb{1} + \frac{1}{2}[\mathrm{Tr}X^2 - (\mathrm{Tr}X)^2] \cdot X + \mathrm{Tr}X \cdot X^2$$

- Spurion function after resummation

Colangelo, Nikolidakis, Smith, 2009
Mercolli, Smith, 2009

$$\begin{aligned} f(\mathsf{A},\mathsf{B}) = \epsilon_0 \mathbb{1} &+ \epsilon_1 \mathsf{A} + \epsilon_3 \mathsf{A}^2 + \epsilon_5 \mathsf{AB} + \epsilon_7 \mathsf{ABA} + \epsilon_{10} \mathsf{AB}^2 + \epsilon_{12} \mathsf{A}^2\mathsf{B}^2 + \epsilon_{14} \mathsf{B}^2\mathsf{AB} + \epsilon_{15} \mathsf{AB}^2\mathsf{A}^2 \\ &+ \epsilon_2 \mathsf{B} + \epsilon_4 \mathsf{B}^2 + \epsilon_6 \mathsf{BA} + \epsilon_9 \mathsf{BAB} + \epsilon_8 \mathsf{BA}^2 + \epsilon_{13} \mathsf{B}^2\mathsf{A}^2 + \epsilon_{11} \mathsf{ABA}^2 + \epsilon_{16} \mathsf{B}^2\mathsf{A}^2\mathsf{B}. \end{aligned}$$

- assumption #1: neglect tiny imaginary parts of ϵ_i
- assumption #2: neglect spurion B (suppressed by $\mathcal{O}(\lambda_d^2)$)

$$f(\mathsf{A},\mathsf{B}) \approx \epsilon_0 \mathbb{1} + \epsilon_1 \mathsf{A} + \epsilon_2 \mathsf{A}^2$$

Minimal Flavour Violation

- MFV coupling **FCNC controlled by CKM**

$$C^{\mathrm{MFV}} = \begin{cases} \epsilon_0 1 + \epsilon_1 \Delta_q & \text{for } \bar{d}_L \gamma^\mu C d_L \\ \epsilon_0 \hat{\lambda}_d + \epsilon_1 \Delta_q \hat{\lambda}_d & \text{for } \bar{d}_L C d_R,\ \bar{d}_L \sigma^{\mu\nu} C d_R \\ \epsilon_0 1 & \text{for } \bar{d}_R \gamma^\mu C d_R \end{cases} \qquad \Delta_q = V^\dagger \hat{\lambda}_u^2 V$$

No Right-handed down-type FCNC !

- Numerics

$$\Delta_q = \begin{pmatrix} 0.8 & -3.3 - 1.5i & 79.3 + 35.4i \\ -3.3 + 1.5i & 16.6 & -397.5 + 8.1i \\ 79.3 - 35.4i & -397.5 - 8.1i & 9839.0 \end{pmatrix} \times 10^{-4}$$

$$\Delta_q \hat{\lambda}_d = \begin{pmatrix} 0.0021 & -0.18 - 0.08i & 191.3 + 85.4i \\ -0.009 + 0.004i & 0.88 & -958.7 + 19.6i \\ 0.21 - 0.10i & -21.1 - 0.4i & 23728.1 \end{pmatrix} \times 10^{-6}$$

$b \to s\nu\bar{\nu}$: SMEFT with MFV

- **Prediction**

$$\frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm SM}}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm SM}} = 0.46 \pm 0.07$$

$$\frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})}{\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm SM}}{\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})_{\rm SM}} = 29.7 \pm 5.6$$

- **prediction**

$$\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm SM} = (9.00 \pm 0.87) \times 10^{-6}$$
$$\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm MFV} = \left(50^{+17}_{-16}\right) \times 10^{-6}$$
$$\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm exp} < 18 \times 10^{-6}$$

$$\mathscr{B}(B^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (1.40 \pm 0.18) \times 10^{-7}$$
$$\mathscr{B}(B^+ \to \pi^+\nu\bar{\nu})_{\rm MFV} = \left(7.8^{+2.8}_{-2.6}\right) \times 10^{-7}$$
$$\mathscr{B}(B^+ \to \pi^+\nu\bar{\nu})_{\rm exp} < 140 \times 10^{-7}$$

SMEFT

$$\mathcal{Q}_{Hq}^{(1)} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{q}_p \gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hq}^{(3)} = \left(H^\dagger i \overleftrightarrow{D}^I_\mu H\right)\left(\bar{q}_p \tau^I \gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hd} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{d}_p \gamma^\mu d_r\right),$$

induce $\bar{s}bZ$ interaction,
Thus, universally affect
$b \to s e^+e^-, \mu^+\mu^-, \tau^+\tau^-$

$$\mathcal{Q}_{ld} = \left(\bar{l}_p \gamma^\mu l_r\right)\left(\bar{d}_s \gamma_\mu d_t\right),$$

forbidden by MFV

$$\mathcal{Q}_{lq}^{(1)} = \left(\bar{l}_p \gamma^\mu l_r\right)\left(\bar{q}_s \gamma_\mu q_t\right),$$
$$\mathcal{Q}_{lq}^{(3)} = \left(\bar{l}_p \gamma^\mu \tau^I l_r\right)\left(\bar{q}_s \tau^I \gamma_\mu q_t\right),$$

$\mu_{\rm EW}$

LEFT

$$\mathcal{O}_L^{\nu_i\nu_j} = \left(\bar{s}\gamma_\mu P_L b\right)\left(\bar{\nu}_i \gamma^\mu P_L \nu_j\right)$$
$$\mathcal{O}_R^{\nu_i\nu_j} = \left(\bar{s}\gamma_\mu P_R b\right)\left(\bar{\nu}_i \gamma^\mu P_L \nu_j\right)$$

one LEFT operator !
just the SM operator

μ_b

$b \to s\nu\bar{\nu}$: SMEFT with MFV

- **Prediction**

$$\frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm SM}}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm SM}} = 0.46 \pm 0.07$$

$$\frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})}{\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm SM}}{\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})_{\rm SM}} = 29.7 \pm 5.6$$

- **prediction**

$$\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm SM} = (9.00 \pm 0.87) \times 10^{-6}$$

$$\left.\begin{aligned} &\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm MFV} = \left(50^{+17}_{-16}\right) \times 10^{-6} \\ &\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm exp} < 18 \times 10^{-6} \end{aligned}\right\}$$ **Inconsistent** →

Belle II excess (if confirmed in the future) implies:
- impossible to explain in SMEFT with MFV
- NP flavour structure is highly non-trivial
- **NP structure in quark sector is beyond MFV**
- **flavour violation is beyond Yukawa coupling**

$$\mathscr{B}(B^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (1.40 \pm 0.18) \times 10^{-7}$$

$$\mathscr{B}(B^+ \to \pi^+\nu\bar{\nu})_{\rm MFV} = \left(7.8^{+2.8}_{-2.6}\right) \times 10^{-7}$$

$$\mathscr{B}(B^+ \to \pi^+\nu\bar{\nu})_{\rm exp} < 140 \times 10^{-7}$$

This conclusion only assumes the quark MFV.
No lepton flavour structure is assumed.

$b \to s\nu\bar{\nu}$: SMEFT

SMEFT

$$\mathcal{Q}_{Hq}^{(1)} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{q}_p \gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hq}^{(3)} = \left(H^\dagger i \overleftrightarrow{D}^I_\mu H\right)\left(\bar{q}_p \tau^I \gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hd} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{d}_p \gamma^\mu d_r\right),$$
$$\mathcal{Q}_{ld} = \left(\bar{l}_p \gamma^\mu l_r\right)\left(\bar{d}_s \gamma_\mu d_t\right),$$
$$\mathcal{Q}_{lq}^{(1)} = \left(\bar{l}_p \gamma^\mu l_r\right)\left(\bar{q}_s \gamma_\mu q_t\right),$$
$$\mathcal{Q}_{lq}^{(3)} = \left(\bar{l}_p \gamma^\mu \tau^I l_r\right)\left(\bar{q}_s \tau^I \gamma_\mu q_t\right),$$

μ_{EW}

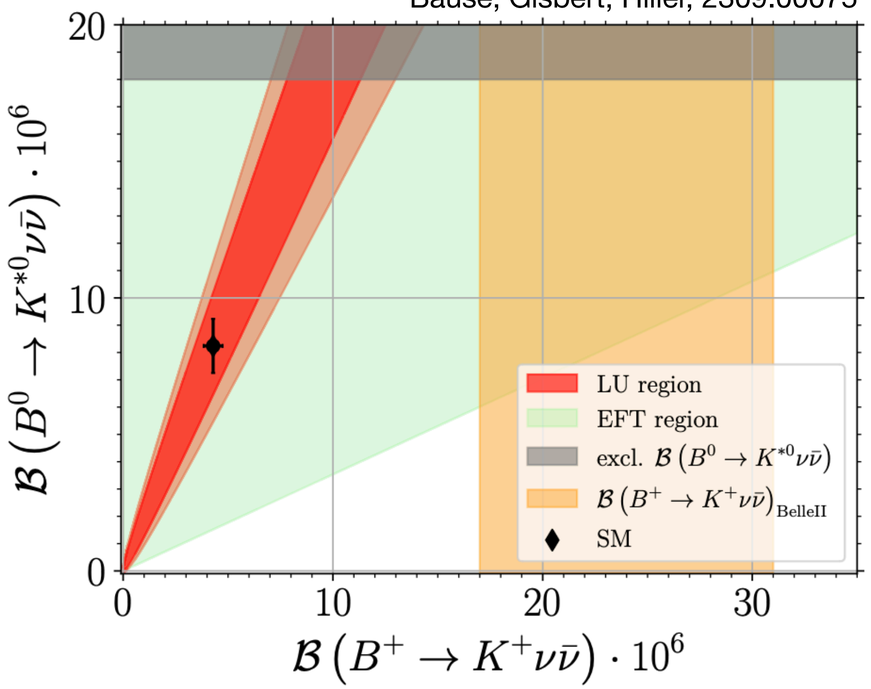
LEFT

$$\mathcal{O}_L^{\nu_i \nu_j} = \left(\bar{s}\gamma_\mu P_L b\right)\left(\bar{\nu}_i \gamma^\mu P_L \nu_j\right)$$
$$\mathcal{O}_R^{\nu_i \nu_j} = \left(\bar{s}\gamma_\mu P_R b\right)\left(\bar{\nu}_i \gamma^\mu P_L \nu_j\right)$$

μ_b

operator structure highly constrained by Left-handed neutrino

Bause, Gisbert, Hiller, 2309.00075



$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) = A_+^{BK}\, x^+ ,$$
$$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu}) = A_+^{BK^*}\, x^+ + A_-^{BK^*}\, x^- ,$$
$$x^\pm = \sum_{\nu,\nu'} |C_L^{\nu\nu'} \pm C_R^{\nu\nu'}|^2 ,$$

Bause, Gisbert, Hiller, 2309.00075
Allwicher, Becirevic, Piazza, Rosauro-Alcaraz, Sumensari, 2309.02246
Chen, Wen, Xu, 2401.11552

$b \to s\nu\bar{\nu}$: SMEFT

SMEFT

$$\mathcal{Q}_{Hq}^{(1)} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{q}_p \gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hq}^{(3)} = \left(H^\dagger i \overleftrightarrow{D}^I_\mu H\right)\left(\bar{q}_p \tau^I \gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hd} = \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{d}_p \gamma^\mu d_r\right),$$
$$\mathcal{Q}_{ld} = \left(\bar{l}_p \gamma^\mu l_r\right)\left(\bar{d}_s \gamma_\mu d_t\right),$$
$$\mathcal{Q}_{lq}^{(1)} = \left(\bar{l}_p \gamma^\mu l_r\right)\left(\bar{q}_s \gamma_\mu q_t\right),$$
$$\mathcal{Q}_{lq}^{(3)} = \left(\bar{l}_p \gamma^\mu \tau^I l_r\right)\left(\bar{q}_s \tau^I \gamma_\mu q_t\right),$$

$\mu_{\rm EW}$

LEFT

$$\mathcal{O}_L^{\nu_i\nu_j} = \left(\bar{s}\gamma_\mu P_L b\right)\left(\bar{\nu}_i \gamma^\mu P_L \nu_j\right)$$
$$\mathcal{O}_R^{\nu_i\nu_j} = \left(\bar{s}\gamma_\mu P_R b\right)\left(\bar{\nu}_i \gamma^\mu P_L \nu_j\right)$$

μ_b

operator structure highly constrained by Left-handed neutrino

	Observable	SM	Exp	Unit
$b \to s$	$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})$	4.16 ± 0.57	$23 \pm 5^{+5}_{-4}$	10^{-6}
	$\mathcal{B}(B^0 \to K^0\nu\bar{\nu})$	3.85 ± 0.52	< 26	10^{-6}
	$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})$	9.70 ± 0.94	< 61	10^{-6}
	$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})$	9.00 ± 0.87	< 18	10^{-6}
	$\mathcal{B}(B_s \to \phi\nu\bar{\nu})$	9.93 ± 0.72	< 5400	10^{-6}
	$\mathcal{B}(B_s \to \nu\bar{\nu})$	≈ 0	< 5.9	10^{-4}
$b \to d$	$\mathcal{B}(B^+ \to \pi^+\nu\bar{\nu})$	1.40 ± 0.18	< 140	10^{-7}
	$\mathcal{B}(B^0 \to \pi^0\nu\bar{\nu})$	6.52 ± 0.85	< 900	10^{-8}
	$\mathcal{B}(B^+ \to \rho^+\nu\bar{\nu})$	4.06 ± 0.79	< 300	10^{-7}
	$\mathcal{B}(B^0 \to \rho^0\nu\bar{\nu})$	1.89 ± 0.36	< 400	10^{-7}
	$\mathcal{B}(B^0 \to \nu\bar{\nu})$	≈ 0	< 1.4	10^{-4}
$s \to d$	$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$	8.42 ± 0.61	$10.6^{+4.0}_{-3.4} \pm 0.9$	10^{-11}
	$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	3.41 ± 0.45	< 300	10^{-11}

Why such a large NP effect has not shown up in other $b \to s$ decays ?
in $b \to d, s \to d$ decays ? **NP flavour structure**

$b \to s\nu\bar{\nu}$ and $b \to s\ell\ell$

SMEFT notation: $l = \begin{pmatrix}\nu\\ e\end{pmatrix}_L, q = \begin{pmatrix}u\\ d\end{pmatrix}_L, d = d_R$

B.F.Hou, X.Q.Li, M.Shen, Y.D.Yang, **XBY**, 2402.19208

- **Prediction**

$$\frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})} = \frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\rm SM}}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm SM}} = 0.46 \pm 0.07$$

- **prediction**

$$\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm SM} = (9.00 \pm 0.87) \times 10^{-6}$$

$$\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm SMEFT} = \left(50^{+17}_{-16}\right) \times 10^{-6}$$

$$\mathscr{B}(B^0 \to K^{*0}\nu\bar{\nu})_{\rm exp} < 18 \times 10^{-6}$$

conflict

- **Only $\mathcal{O}_{lq}^{(3)}$ is relevant with $R_{D^{(*)}}$**

- **$\mathcal{O}_{ld}$ can explain the $B^+ \to K^+\nu\bar{\nu}$ data**
- **$\mathcal{O}_{ld}$ also induce $O'_{9,ij}$ and $O'_{10,ij}$**
- **They can't improve the $b \to s\ell\ell$ fit**
  - O'_{9e} and $O'_{10\mu}$ worsen the fit. **weird** (LFV, $\tau\tau \gg ee, \mu\mu$)
  - $O'_{9,ij}$ and $O'_{10,ij}$ with $i = j = \tau$ has no effect.
  - $O'_{9,ij}$ and $O'_{10,ij}$ with $i \neq j$ (i.e. LFV) has no effect.

SMEFT

$$\mathcal{Q}_{Hq}^{(1)} = \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{q}_p\gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hq}^{(3)} = \left(H^\dagger i\overleftrightarrow{D}^I_\mu H\right)\left(\bar{q}_p\tau^I\gamma^\mu q_r\right),$$
$$\mathcal{Q}_{Hd} = \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{d}_p\gamma^\mu d_r\right),$$

induce $\bar{s}bZ$ interaction,
Thus, universally affect
$b \to se^+e^-, \mu^+\mu^-, \tau^+\tau^-$

$$\mathcal{Q}_{ld} = \left(\bar{l}_p\gamma^\mu l_r\right)\left(\bar{d}_s\gamma_\mu d_t\right),$$

$$\mathcal{Q}_{lq}^{(1)} = \left(\bar{l}_p\gamma^\mu l_r\right)\left(\bar{q}_s\gamma_\mu q_t\right),$$
$$\mathcal{Q}_{lq}^{(3)} = \left(\bar{l}_p\gamma^\mu\tau^I l_r\right)\left(\bar{q}_s\tau^I\gamma_\mu q_t\right),$$

$\mu_{\rm EW}$

LEFT

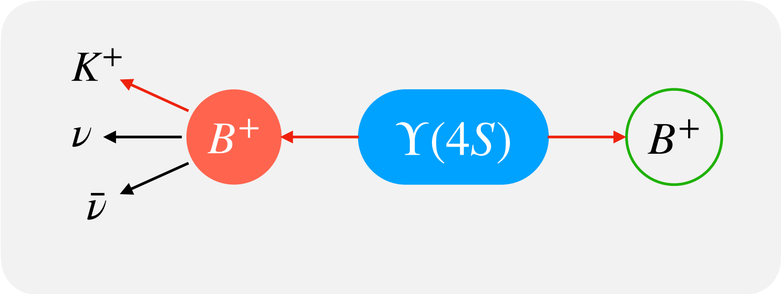
$$\mathcal{O}_L^{\nu_i\nu_j} = \left(\bar{s}\gamma_\mu P_L b\right)\left(\bar{\nu}_i\gamma^\mu P_L\nu_j\right)$$
$$\mathcal{O}_R^{\nu_i\nu_j} = \left(\bar{s}\gamma_\mu P_R b\right)\left(\bar{\nu}_i\gamma^\mu P_L\nu_j\right)$$

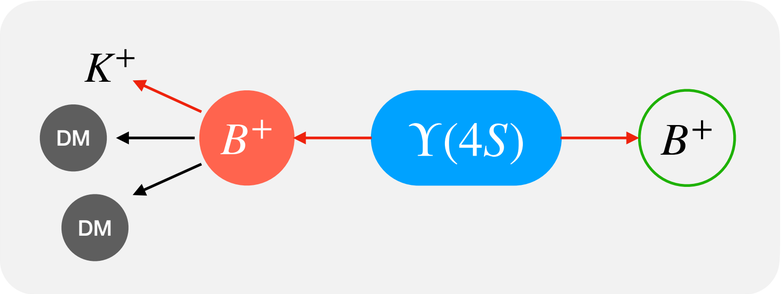
one LEFT operator !
just the SM operator

μ_b

$$O'_{9,ij} = (\bar{b}\gamma^\mu P_R s)(\bar{\ell}_i\gamma_\mu\ell_j)$$
$$O'_{10,ij} = (\bar{b}\gamma^\mu P_R s)(\bar{\ell}_i\gamma_\mu\gamma_5\ell_j)$$

$b \to s\nu\bar{\nu}$: exp picture





SMEFT

$$\mathcal{Q}_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)\,,$$
$$\mathcal{Q}_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r),$$
$$\mathcal{Q}_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r),$$

$$\mathcal{Q}_{ld} = (\bar{l}_p \gamma^\mu l_r)(\bar{d}_s \gamma_\mu d_t),$$

$$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma^\mu l_r)(\bar{q}_s \gamma_\mu q_t)\,,$$
$$\mathcal{Q}_{lq}^{(3)} = (\bar{l}_p \gamma^\mu \tau^I l_r)(\bar{q}_s \tau^I \gamma_\mu q_t)\,,$$

$$\mathcal{O}_L^{\nu_i \nu_j} = (\bar{s}\gamma_\mu P_L b)(\bar{\nu}_i \gamma^\mu P_L \nu_j)$$
$$\mathcal{O}_R^{\nu_i \nu_j} = (\bar{s}\gamma_\mu P_R b)(\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

Dark SMEFT

example

$$\mathcal{Q}_{d\phi^2} = (\bar{q}_p d_r H)\phi^2$$
$$\mathcal{Q}_{d\chi} = (\bar{d}_p \gamma_\mu d_r)(\bar{\chi}\gamma^\mu \chi)$$
$$\mathcal{Q}_{dX^2} = (\bar{q}_p d_r H) X_\mu X^\mu$$
$$\mathcal{Q}_{qa} = (\bar{q}_p \gamma_\mu q_r)\partial^\mu a$$

2011 Kamenik, Smith
2014 Duch, Grzadkowski, Wudka
2017 Brod, Gootjes-Dreesbach, Tammaro, Zupan
2021 Criado, Djouadi, Perez-Victoria, Santiago
2022 Aebischer, Altmannshofer, Jenkins, Manohar (basis@dim-6)
2023 Song, Sun, Yu (basis@dim-8)

Axion-like particle, see also H.Y.Cheng, Phys.Rept 1988
2020 Bauer, Neubert, Renner, Schnubel, Thamm
2023 Song, Sun, Yu (basis@dim-8)

$\mu_{\rm EW}$

LEFT

Dark LEFT

$$\mathcal{O}_{d\phi^2} = (\bar{d}_{Lp} d_{Rr})\phi^2$$
$$\mathcal{O}_{d\chi}^{V,LR} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})(\bar{\chi}_a \gamma^\mu \chi_b)$$
$$\mathcal{O}_{dXX}^{L} = (\bar{d}_{Lp}\gamma_\mu d_{Lr}) X^{\mu\nu} X_\nu$$
$$\mathcal{O}_{da}^{L} = (\bar{d}_{Lp}\gamma_\mu d_{Lr})\partial^\mu a$$

2022 Aebischer, Altmannshofer, Jenkins, Manohar (basis@dim-6)
2022 He, Ma, Valencia (basis@dim-6)
2023 Liang, Liao, Ma, Wang (basis@dim-8)

μ_b

example

Dark SMEFT with MFV

- MFV coupling $b \to s, b \to d, s \to d$ **are connected with each other.**

$$\mathcal{C}_i^{\rm MFV} = \begin{cases} \epsilon_0^i \hat{\lambda}_d + \epsilon_1^i \Delta_q \hat{\lambda}_d & \text{for } \mathcal{Q}_i = \mathcal{Q}_{d\phi}, \mathcal{Q}_{d\phi^2}, \mathcal{Q}_{dHX}, \mathcal{Q}_{dHX^2}, \mathcal{Q}_{dX^2}, \\ \epsilon_0^i \mathbb{1} + \epsilon_1^i \Delta_q & \text{for } \mathcal{Q}_i = \mathcal{Q}_{\phi q}, \mathcal{Q}_{q\chi}, \mathcal{Q}_{qXX}, \mathcal{Q}_{q\tilde{X}X}, \mathcal{Q}_{DqX^2}, \mathcal{Q}_{qX}, \mathcal{Q}_{HqX}^{(1,3)}, \mathcal{Q}_{qa}, \\ \epsilon_0^i \mathbb{1} & \text{for } \mathcal{Q}_i = \mathcal{Q}_{\phi d}, \mathcal{Q}_{d\chi}, \mathcal{Q}_{dXX}, \mathcal{Q}_{d\tilde{X}X}, \mathcal{Q}_{DdX^2}, \mathcal{Q}_{dX}, \mathcal{Q}_{HdX}, \mathcal{Q}_{da}, \end{cases}$$

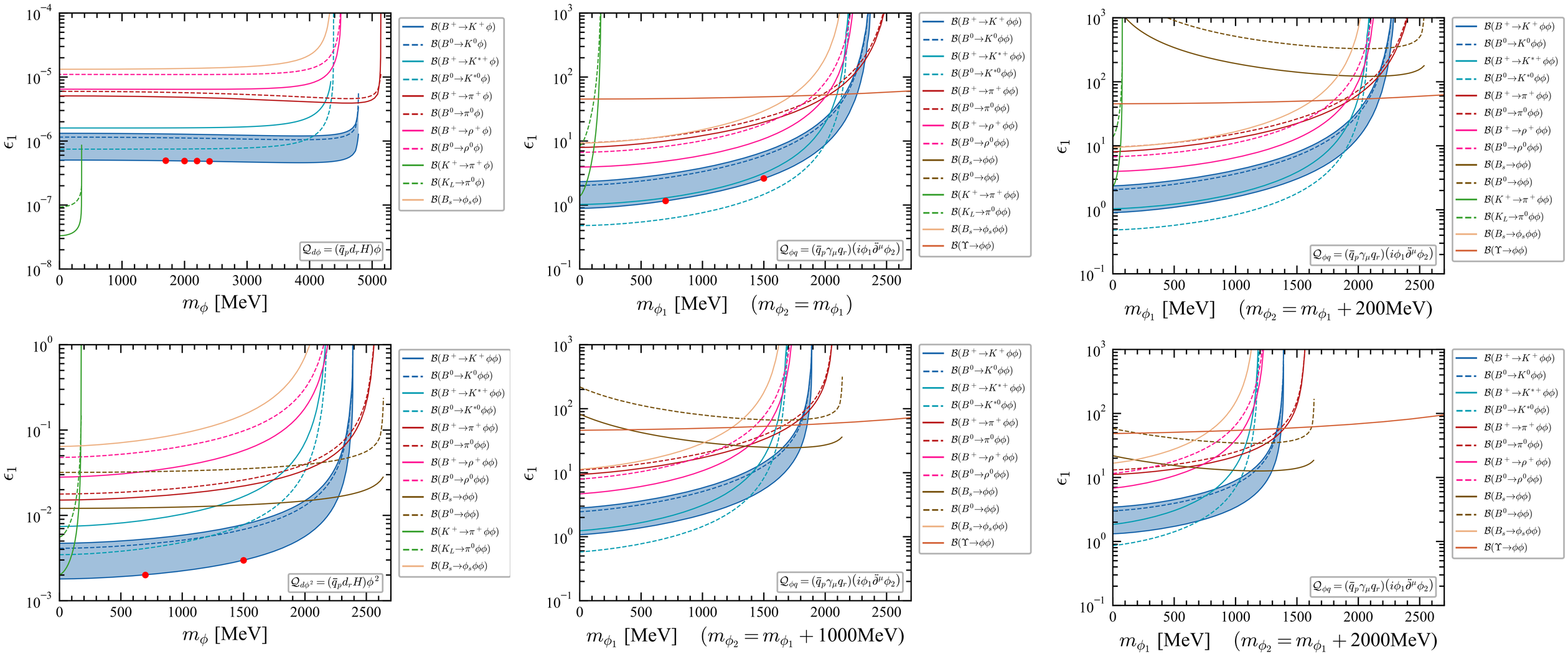
8 operators are eliminated

- Numerics

$$\Delta_q = \begin{pmatrix} 0.8 & -3.3 - 1.5i & 79.3 + 35.4i \\ -3.3 + 1.5i & 16.6 & -397.5 + 8.1i \\ 79.3 - 35.4i & -397.5 - 8.1i & 9839.0 \end{pmatrix} \times 10^{-4}$$

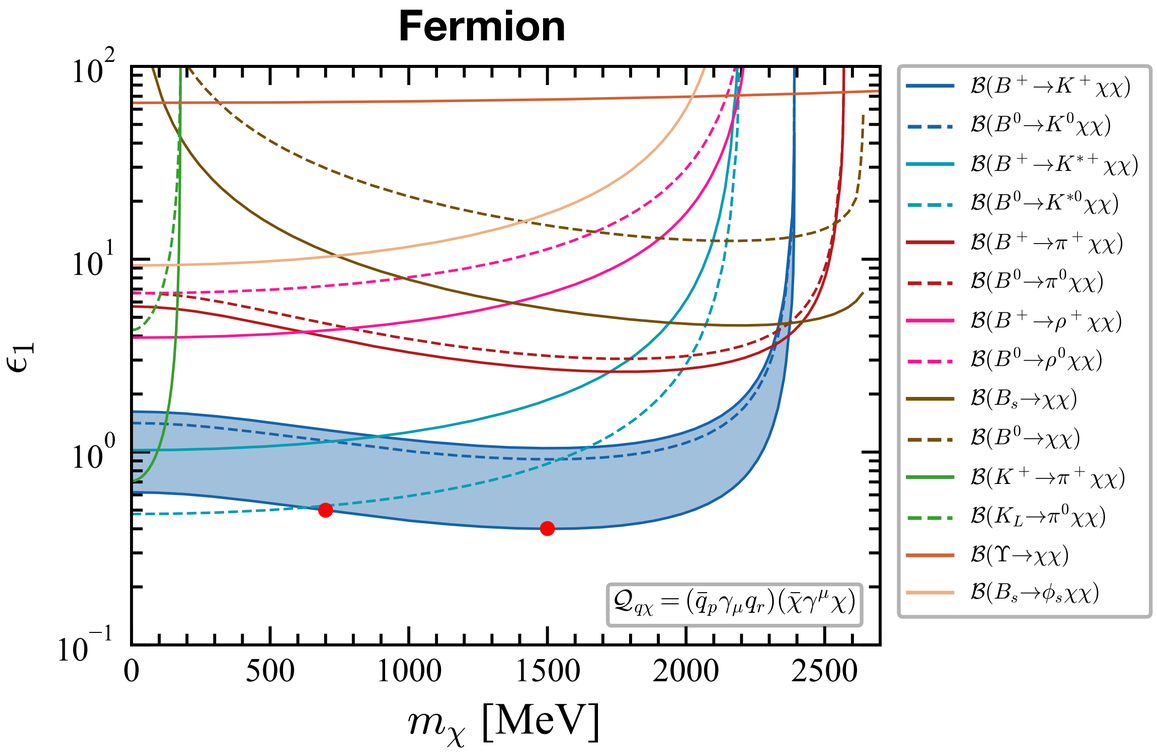
$$\Delta_q \hat{\lambda}_d = \begin{pmatrix} 0.0021 & -0.18 - 0.08i & 191.3 + 85.4i \\ -0.009 + 0.004i & 0.88 & -958.7 + 19.6i \\ 0.21 - 0.10i & -21.1 - 0.4i & 23728.1 \end{pmatrix} \times 10^{-6}$$

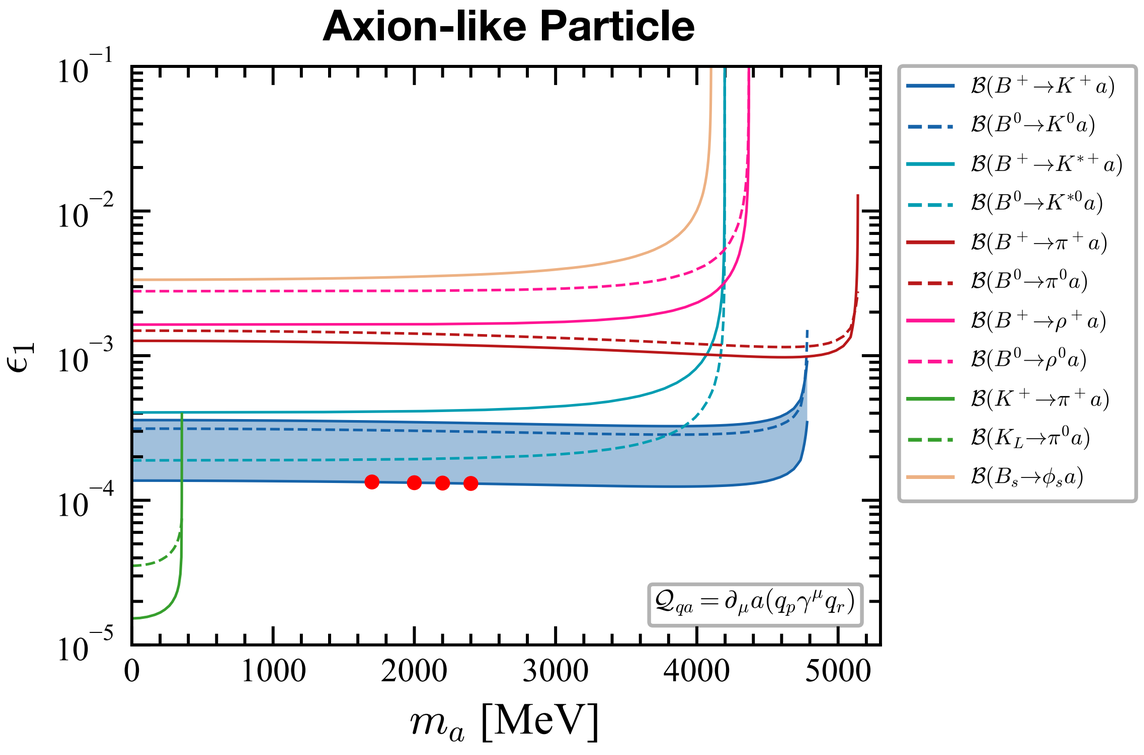
Dark SMEFT with MFV: Scalar



all the operators survive
some ones highly constrained

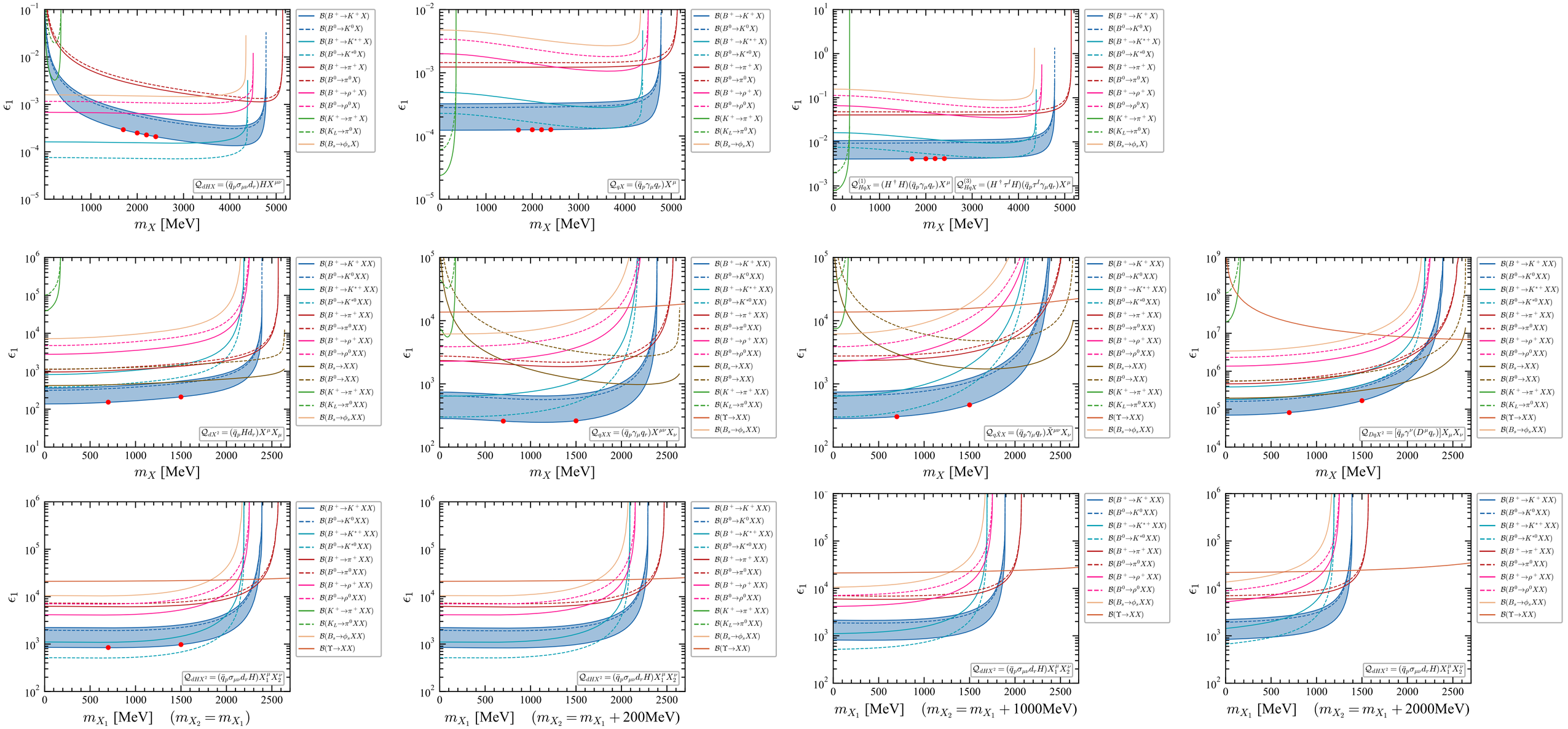
Dark SMEFT with MFV: Fermion, ALP





all the operators survive

Dark SMEFT with MFV: Vector



all the operators survive, some ones highly constrained

Backup

$$\mathcal{Q}_{d\phi} = \big(\bar{q}_p d_r H\big)\phi + \text{h.c.}, \qquad \mathcal{Q}_{d\phi^2} = \big(\bar{q}_p d_r H\big)\phi^2 + \text{h.c.},$$
$$\mathcal{Q}_{\phi q} = \big(\bar{q}_p \gamma_\mu q_r\big)\big(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2\big), \qquad \mathcal{Q}_{\phi d} = \big(\bar{d}_p \gamma_\mu d_r\big)\big(i\phi_1 \overleftrightarrow{\partial^\mu} \phi_2\big), \tag{4.2}$$

$$\mathcal{Q}_{q\chi} = \big(\bar{q}_p \gamma_\mu q_r\big)(\bar{\chi}\gamma^\mu \chi), \qquad \mathcal{Q}_{d\chi} = \big(\bar{d}_p \gamma_\mu d_r\big)(\bar{\chi}\gamma^\mu \chi), \tag{4.3}$$

$$\mathcal{Q}_{dHX} = \big(\bar{q}_p \sigma_{\mu\nu} d_r\big) H X^{\mu\nu} + \text{h.c.}, \tag{4.4}$$

$$\begin{aligned}
\mathcal{Q}_{dX} &= \big(\bar{d}_p \gamma_\mu d_r\big) X^\mu, & \mathcal{Q}_{HdX} &= \big(H^\dagger H\big)\big(\bar{d}_p \gamma^\mu d_r\big) X_\mu, \\
\mathcal{Q}_{qX} &= \big(\bar{q}_p \gamma_\mu q_r\big) X^\mu, & \mathcal{Q}^{(1)}_{HqX} &= \big(H^\dagger H\big)\big(\bar{q}_p \gamma^\mu q_r\big) X_\mu, \\
\mathcal{Q}_{dX^2} &= \big(\bar{q}_p d_r H\big) X_\mu X^\mu + \text{h.c.}, & \mathcal{Q}^{(3)}_{HqX} &= \big(H^\dagger \tau^I H\big)\big(\bar{q}_p \tau^I \gamma^\mu q_r\big) X_\mu, \\
\mathcal{Q}_{qXX} &= \big(\bar{q}_p \gamma_\mu q_r\big) X^{\mu\nu} X_\nu\,, & \mathcal{Q}_{dXX} &= \big(\bar{d}_p \gamma_\mu d_r\big) X^{\mu\nu} X_\nu, \\
\mathcal{Q}_{q\tilde{X}X} &= \big(\bar{q}_p \gamma_\mu q_r\big) \tilde{X}^{\mu\nu} X_\nu\,, & \mathcal{Q}_{d\tilde{X}X} &= \big(\bar{d}_p \gamma_\mu d_r\big) \tilde{X}^{\mu\nu} X_\nu\,, \\
\mathcal{Q}_{DqX^2} &= i\big(\bar{q}_p \gamma^\mu D^\nu q_r\big) X_\mu X_\nu + \text{h.c.}, & \mathcal{Q}_{DdX^2} &= i\big(\bar{d}_p \gamma^\mu D^\nu d_r\big) X_\mu X_\nu + \text{h.c.}, \\
\mathcal{Q}_{dHX^2} &= (\bar{q}_p \sigma_{\mu\nu} d_r H) X_1^\mu X_2^\nu + \text{h.c.},
\end{aligned} \tag{4.5}$$

$$\mathcal{C}_i = \tilde{\mathcal{C}}_i \cdot \begin{cases} (m_X/\Lambda)^2 & \text{for } \mathcal{Q}_i = \mathcal{Q}_{dX^2}, \mathcal{Q}_{DdX^2}, \mathcal{Q}_{DqX^2}, \mathcal{Q}_{dHX^2}, \\ (m_X/\Lambda) & \text{for } \mathcal{Q}_i = \text{others}. \end{cases}$$

$$\mathcal{Q}_{qa} = \big(\bar{q}_p \gamma_\mu q_r\big)\partial^\mu a, \qquad \mathcal{Q}_{da} = \big(\bar{d}_p \gamma_\mu d_r\big)\partial^\mu a, \tag{4.7}$$

Backup

One can also apply the MFV hypothesis to the lepton sector. However, since the mechanism of neutrino mass generation is still unknown, there are different approaches to formulate the leptonic MFV [73–79]. Here, we consider the realization of leptonic MFV within the so-called minimal field content [73, 74], in which the neutrino masses are generated by the Weinberg operator. In this case, the Yukawa interactions in the lepton sector can be written as

$$-\Delta\mathcal{L} = \bar{e}\, Y_e H^\dagger l + \frac{1}{2\Lambda_{\rm LN}}\big(\bar{l}^c \tau_2 H\big) Y_\nu \big(H^T \tau_2 l\big) + \text{h.c.}, \tag{2.18}$$

where l denotes the left-handed lepton doublet with the charge conjugated field given by $l^c = -i\gamma_2 l^*$, and e is the right-handed charged lepton singlet. $\Lambda_{\rm LN}$ denotes the breaking scale of the lepton number symmetry $U(1)_{\rm LN}$. Y_e and Y_ν stand for the 3×3 Yukawa coupling matrices in flavour space. In the absence of these Yukawa couplings, the lepton sector respects the flavour symmetry

$$G_{\rm LF} = SU(3)_l \otimes SU(3)_e. \tag{2.19}$$

finite polynomial of A_ℓ and B_ℓ. After neglecting all the terms involving B_ℓ, which are suppressed by the small lepton Yukawa couplings Y_e, we obtain

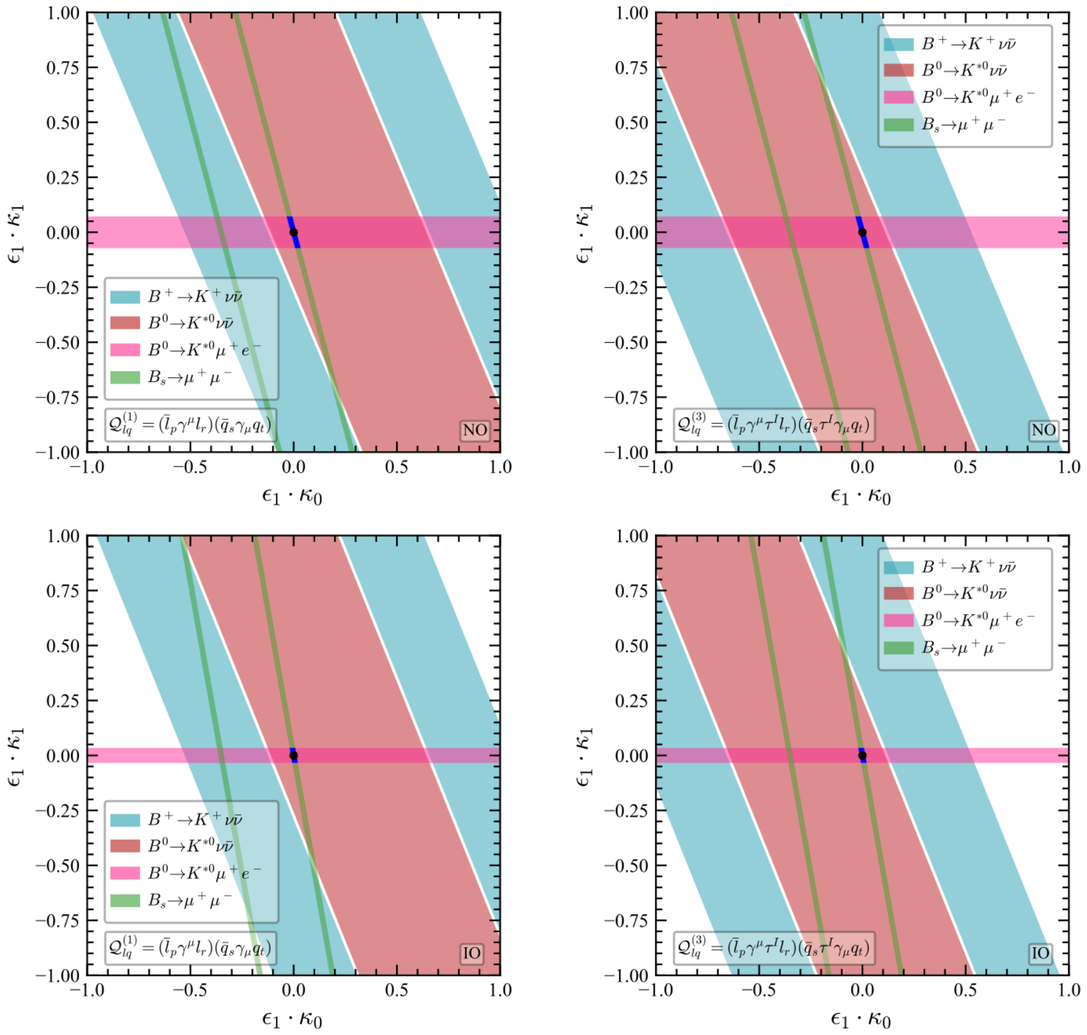
$$\mathcal{C}_{\rm MFV} \approx \kappa_0 + \kappa_1 \mathsf{A}_\ell + \kappa_2 \mathsf{A}_\ell^2, \tag{2.21}$$

where the coefficients $\kappa_{0,1,2}$ are free real parameters. In the numerical analysis, we keep only the leading lepton flavour violation term A_ℓ for simplicity, i.e., $\kappa_2 = 0$. Turning to the lepton mass eigenbasis, the current $\bar{l}\gamma^\mu \mathcal{C} l$ gives in the MFV hypothesis the following interactions:

$$\bar{e}_L \gamma^\mu (\kappa_0 \mathbb{1} + \kappa_0 \Delta_\ell) e_L + \bar{\nu}_L \gamma^\mu (\kappa_0 \mathbb{1} + \kappa_0 \hat{\lambda}_\nu^2) \nu_L, \tag{2.22}$$

where the basic LFV coupling Δ_ℓ can be obtained from A_ℓ and takes the form

$$\Delta_\ell = U \hat{\lambda}_\nu^2 U^\dagger, \tag{2.23}$$

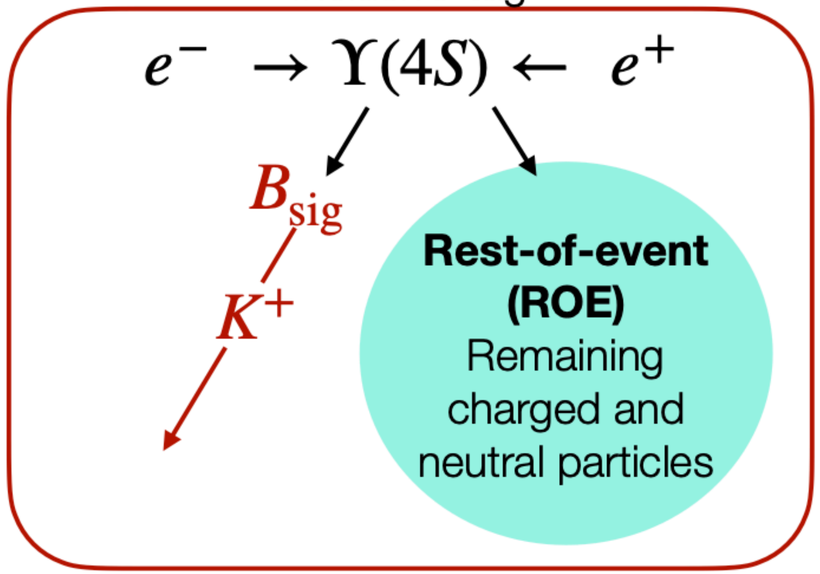


$$\Delta_\ell^{\rm NO} = \begin{pmatrix} -0.19-0.01i & -0.25-0.02i & 0.31-0.04i \\ 0.12+0.01i & 0.28-0.00i & 0.29+0.04i \\ -0.37-0.01i & 0.21-0.05i & -0.03+0.01i \end{pmatrix}, \quad \Delta_\ell^{\rm IO} = \begin{pmatrix} 0.21+0.09i & -0.34+0.05i & 0.03+0.11i \\ 0.31+0.12i & 0.19+0.00i & -0.15-0.14i \\ 0.12-0.02i & 0.04-0.19i & 0.34-0.10i \end{pmatrix}$$

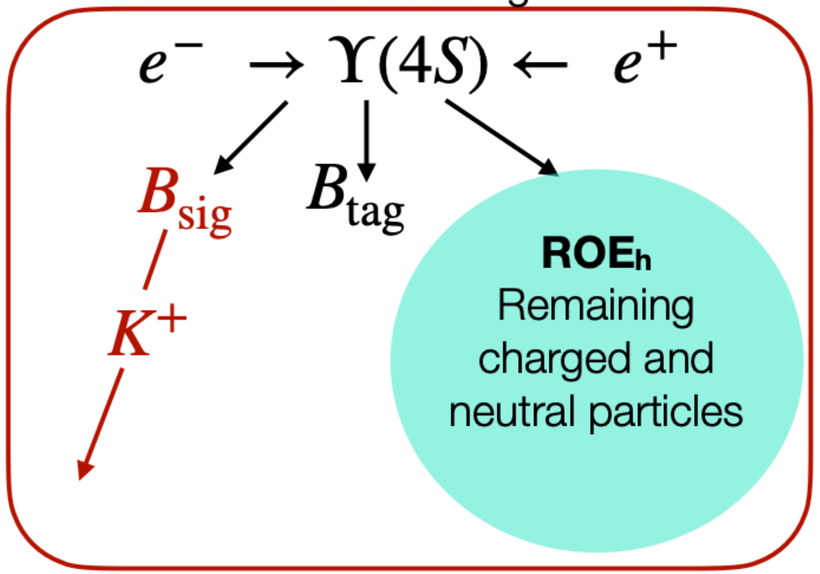
RECONSTRUCTION AND SELECTION

- Charged particles: p_T> 100 MeV/*c*, close to collision point, in the central part of the detector
- Neutral particles: *E* > 100 MeV, in the central part of the detector
- Signal kaon candidates reconstructed applying kaon-enriching selection

Inclusive-tag



Hadronic-tag

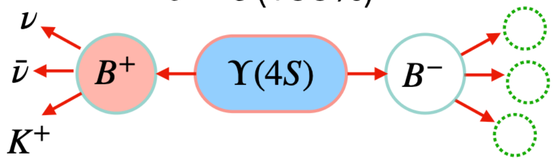


In following, fo

Inclusive tag

$B_{tag} \rightarrow$ anything

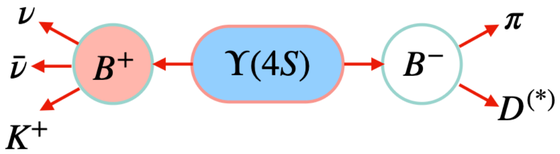
$\epsilon = \mathcal{O}(100\%)$



Hadronic tag

$B_{tag} \rightarrow$ hadrons, e . g $B \rightarrow D^{(*)}n\pi$

$\epsilon = \mathcal{O}(2\%)$



Particle	BESIII	STCF (1 ab^{-1})	Belle II (50 ab^{-1} on $\Upsilon(4S)$)	LHCb (300 fb^{-1})	CEPC (TDR)
B^0, $\bar{B}^0$	-	-	5.4×10^{10}	3×10^{13}	4.8×10^{11}
$B^{\pm}$	-	-	5.7×10^{10}	3×10^{13}	4.8×10^{11}
B_s^0, $\bar{B}_s^0$	-	-	6.0×10^{8} (5 ab^{-1} on $\Upsilon(5S)$)	1×10^{13}	1.2×10^{11}
$B_c^{\pm}$	-	-	-	1×10^{11}	7.2×10^{8}
Λ_b^0, $\bar{\Lambda}_b^0$	-	-	-	2×10^{13}	1×10^{11}