

Measurement of the Forward-Backward Asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ at CEPC Z Pole with the TDR Reference Detector

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Introduction

- The $ee \rightarrow \mu\mu$ channel at Z pole is the simplest channel at CEPC, and can be utilized to verify the basic performance of CEPC software.
- θ_w (electroweak mixing angle) determines the value of A_{FB} (forward-backward asymmetry).
- The measurement A_{FB} of $ee \rightarrow Z/\gamma^* \rightarrow \mu\mu$ provides a precise verification of the θ_w .
- The A_{FB} is defined by the angle θ_{CM} between the final-state μ^- and the initial-state e^- in the dilepton center-of-mass frame.

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta} = \frac{3}{8} (1 + \cos^2 \theta) + A_{FB} \cos\theta$$

➤ LEP Result

The combination of LEP measurements yields a value of $A_{\text{FB}} = 0.0169 \pm 0.0013$ in the $\mu^+ \mu^-$ channel.

([arXiv:0509008v3](https://arxiv.org/abs/0509008v3) figure2.13)

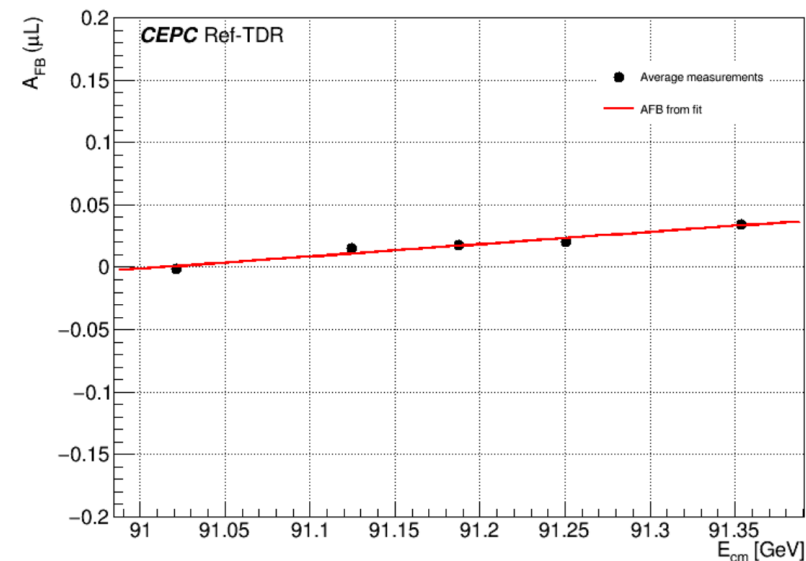
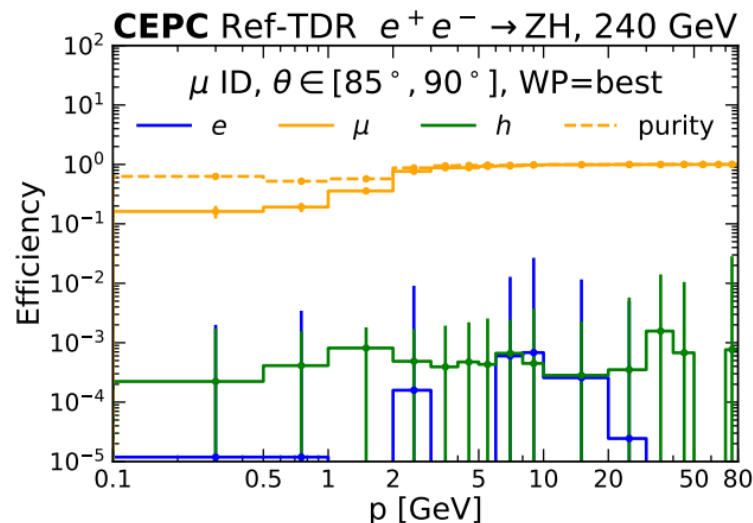
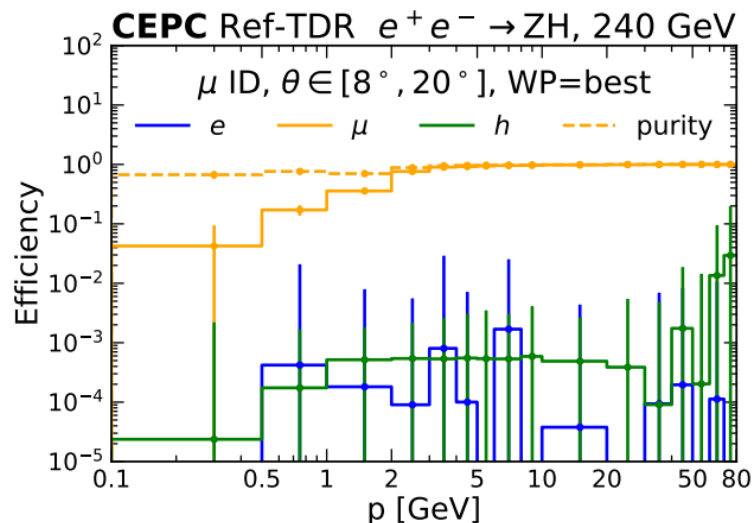
➤ FCC-ee Prediction

- Statistical uncertainty : 3×10^{-6} (Base on 10^{11} muon pairs)
- Energy spread uncertainty : 9.2×10^{-6} (Assume $\Delta E_{\text{cm}} = 0.1 \text{ MeV}$)
- Total uncertainty : 1×10^{-5}

([arXiv:2106.13885v2](https://arxiv.org/abs/2106.13885v2) table3)

Sample production

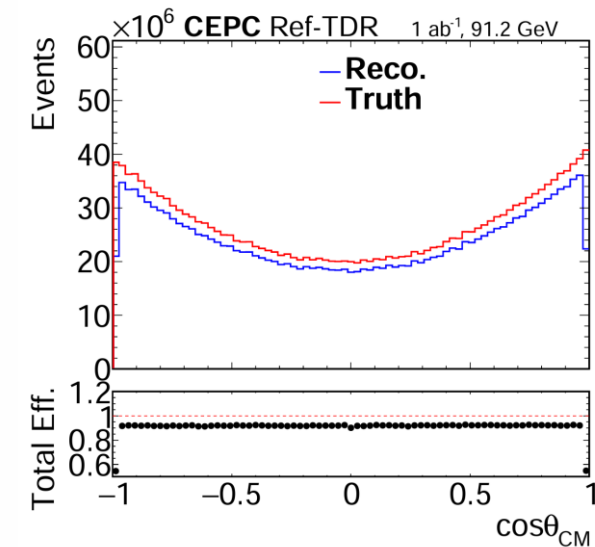
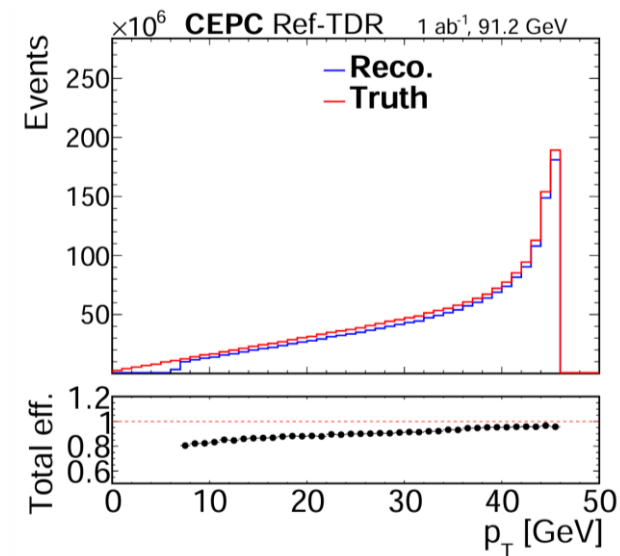
- $ee \rightarrow \mu\mu$ events are simulated with Whizard + Pythia at LO and Z pole energy.
- The simulation includes the Z/γ^* interference along with ISR and FSR effects, and incorporates the beam energy spread of 0.13% at the Z pole.
- Process the sample into with the detector ref-TDR version of the CEPC software.
- A XGBoost classifier was developed for muon ID, with excellent performance.



The event selection and cutflow

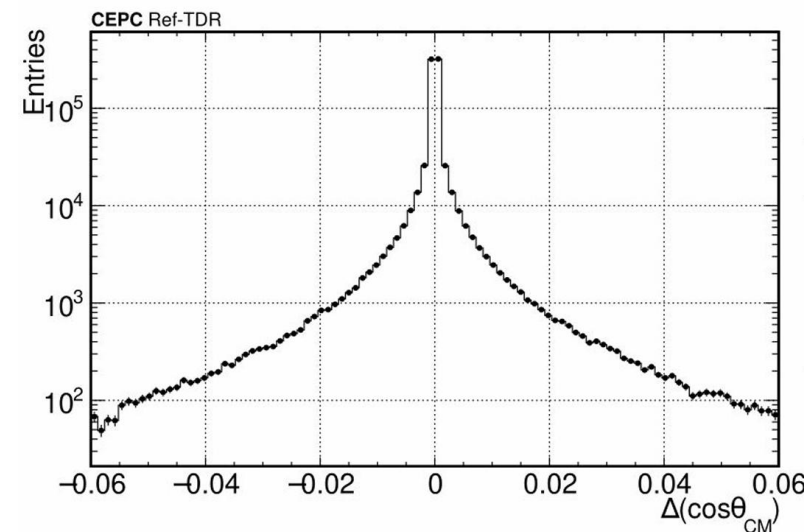
- $P_T > 1\text{ GeV}$ and $|\cos\theta| < 0.99$
- Reco-level selection for a pair of opposite charge muons from Z with muon ID.
- Selection in a $\pm 10\text{ GeV}$ Z mass window.
- A $|\cos\theta| > 0.05$ cut to reduce muon misidentification.

	$e^+e^- \rightarrow \mu^+\mu^-$	$e^+e^- \rightarrow \tau^+\tau^-$	$e^+e^- \rightarrow b\bar{b}$	$e^+e^- \rightarrow e^+e^-$
Cross section	1.5 nb	1.5 nb	6.7 nb	6.6 nb
Simulated events	982476	185855	44550	32397
A pair of muons	967262	5135	1035	0
Z mass window	903640	5	0	0
Muon $ \cos(\theta) > 0.05$	869450 (88.5%)	5 (0.003%)	0 (<0.002%)	0 (<0.003%)



The Calculation of $A_{FB}(\mu)$ counting Method

- Forward and backward events are classified by the angle θ_{CM} of the μ^- , which is reconstructed in the CM frame.
- $\Delta\theta_{CM}$ is a function of both the energy and angular resolution of the PFO.
- The observed $A_{FB}(\mu)$ measured using PFO is corrected back to the full phase space.



	No selection	After selections	Using PFO
Forward ($\cos(\theta_{CM}) > 0$)	499136	442727	442723
Backward ($\cos(\theta_{CM}) < 0$)	483340	426723	426727
A_{FB}^{μ} or A_{FB}^{obs}	0.016078	0.018407	0.018398
A_{FB}^{μ} after a phase space correction	N/A	0.016078	0.016070

➤ Statistical uncertainties

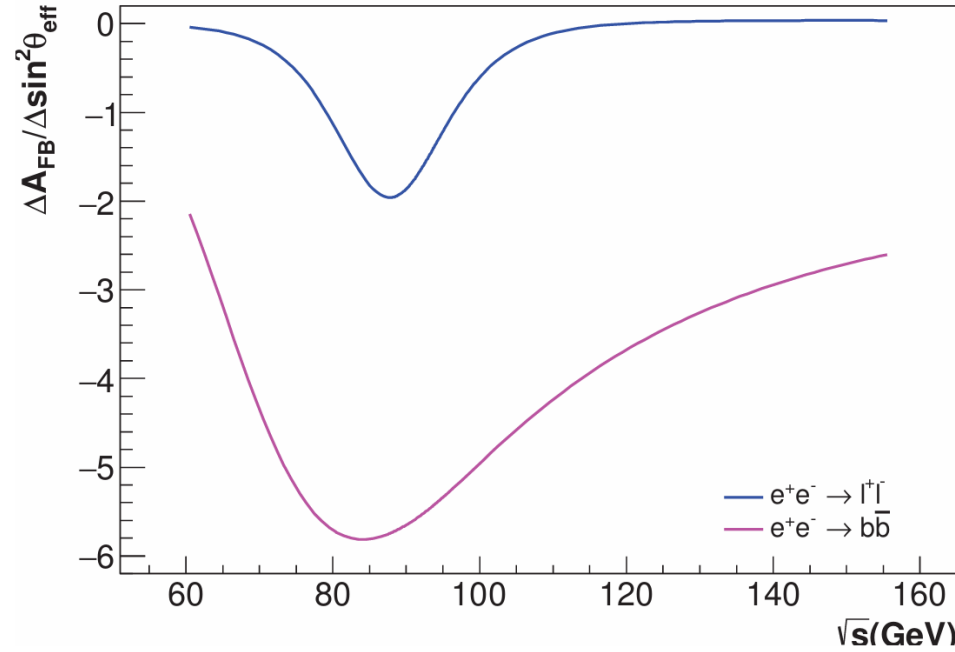
One-month run: 1.34 billion muon pairs (4×10^{10} Z bosons), 3.1×10^{-5} statistical uncertainty.

➤ Systematic uncertainties

- Muon mis-identification: Impact of selecting incorrect PFO pairs is negligible.
- Background: Uncertainty measured as 1×10^{-6} from background contamination.
- Detector acceptance & resolution: Uncertainty estimated to be 9×10^{-6} for $|\cos\theta_{\mu^-}|$ and $p_T^{\mu^-}$.
- Beam energy spread: Uncertainty of 1×10^{-6} from 0.13%-0.14% Gaussian spread.
- Beam energy calibration: Dominant uncertainty of 2.7×10^{-5} from 300 keV calibration uncertainty.

Summary

- $\Delta A_{\text{FB}}(\mu) : \pm 0.000031 \text{ (stat.) } \pm 0.000028 \text{ (syst.) } @ 1 \text{ ab}^{-1}$
- Orders of magnitude improvement with respect to LEP (± 0.0014)
- Comparable with FCC-ee (total uncertainty $\pm 0.00001 @ 60 \text{ ab}^{-1}$)
- A 300 keV assumption for the beam energy calibration, compared with 100 keV at FCC-ee.
- This is the first time that $A_{\text{FB}}(\mu)$ measurements have been conducted based on the TDR software and detailed systematic error analysis.
- We will conduct more measurements on the properties of Z boson and the final states of Z decays.



[arXiv:2204.09921v2](https://arxiv.org/abs/2204.09921v2)

FIG. 3: The sensitivity of $S = \partial A_{FB} / \partial \sin^2 \theta_{\text{eff}}^{\ell}$ as a function of \sqrt{s} for $e^+e^- \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$ and $e^+e^- \rightarrow Z/\gamma^* \rightarrow b\bar{b}$ productions.

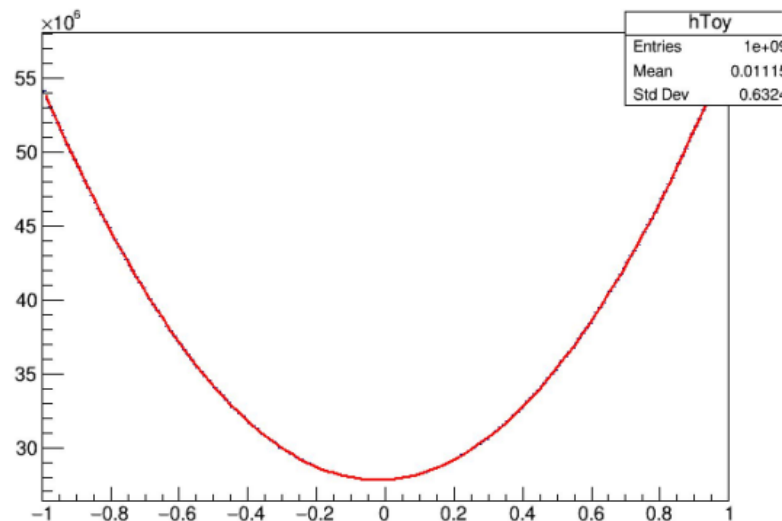
Fitting the costheta distribution

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta \quad \sigma_B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta \quad (9)$$

and $\cos\theta$ is the angle of the outgoing fermion measured relative to the incident electron direction. The experiments determine A_{FB} from fits to the angular distribution which can be written as

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta} = \frac{3}{8} (1 + \cos^2\theta) + A_{FB} \cos\theta \quad (10)$$

- Set costheta function = $[0] * (1 + [1] * x + x * x)$, where $[1] = 8/3 * A_{FB}$
- Tested with a toy with 10^9 events, based on the 1M MC sample in analysis
 - Input AFB = 0.016736
 - Fitted AFB = 0.016732 ± 0.0000296
 - Counting AFB = 0.016736 ± 0.0000316
- The results are consistent, however, the statistic error didn't significantly reduced and it's hard to estimate systematics with fitting method, so it's only a verification



$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

where

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta \quad \sigma_B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta \quad (9)$$

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At the Z^0 pole, ignoring the γ and γZ interference terms, we can write

$$A_{FB}^{0,l} = \frac{3}{4} A_e A_l \quad (11)$$

where the lepton asymmetries are defined by

$$A_l \equiv \frac{2g_v^l g_a^l}{((g_v^l)^2 + (g_a^l)^2)} \quad (12)$$

with g_v^l and g_a^l being the vector and axial-vector coupling constants.

$$\sin^2\theta_w^{eff} = \frac{1}{4} \left(1 - \frac{g_v}{g_a} \right). \quad (6)$$

$$A_{FB}^{0,l} = \frac{3}{4} (A_l)^2 \propto \left(\frac{1 - 4 \sin^2\theta_w^{eff}}{(1 - 4 \sin^2\theta_w^{eff})^2 + 1} \right)^2$$

[https://doi.org/10.1016/S0920-5632\(99\)00392-8](https://doi.org/10.1016/S0920-5632(99)00392-8)