The Electroweak precision observables of the 2HDM+S

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Introduction

- The 2HDM+S (complex singlet extension of 2HDM) could be motivated by Dark Matter candidate, etc..
- We need to explore some possible way to distinguish different BSM models.
  (2HDM or 2HDM+S?)
- The 2HDM+S can differ from 2HDM, where the W and Z bosons self energy (i.e. STU observables) receive the difference. $(STU \leftrightarrow \Delta m_i, \alpha_i)$

The 2HDM+S

The 2HDM + Singlet is an extension of 2HDM [S. Baum et al. 18'], with the following scalars:

$$\Phi_1 = \begin{pmatrix} \chi_1^+ \\ \frac{v_1+\rho_1+i\eta_1}{\sqrt{2}} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \chi_2^+ \\ \frac{v_2+\rho_2+i\eta_2}{\sqrt{2}} \end{pmatrix}, \qquad S = v_S + \rho_S + i\eta_S, \tag{1}$$

Where Φ_1, Φ_2 are the $SU(2)_L$ doublets, and S is the gauge singlet.

- The mass spectrum of the 2HDM+S includes three neutral CP-even Higgs (h, H and h_S), two neutral CP-odd Higgs (A and A_S), and one pair of charged Higgs $H^\pm$.
- The *STU* observables only depend on the $h_i VV$ couplings, which are independent on the explicit symmetry structures of the Higgs potential and the Yukawa type.
- We only focus on the general *STU* effect despite of other constraints.

The 2HDM+S in the mass eigenstate

The CP-even fields mix and generate three scalar Higgs

$$R \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_S \end{pmatrix} = \begin{pmatrix} H \\ h \\ h_S \end{pmatrix}, \qquad RM_S^2 R^T = \text{diag}\{m_H^2, m_h^2, m_{h_S}^2\}. \tag{2}$$

We fix the order of eigenvalues and the R matrix is given by the following configuration

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\alpha_{hS}} & s_{\alpha_{hS}} \\ 0 & -s_{\alpha_{hS}} & c_{\alpha_{hS}} \end{pmatrix} \begin{pmatrix} c_{\alpha_{HS}} & 0 & s_{\alpha_{HS}} \\ 0 & 1 & 0 \\ -s_{\alpha_{HS}} & 0 & c_{\alpha_{HS}} \end{pmatrix} \begin{pmatrix} c_\alpha & s_\alpha & 0 \\ -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

The CP-odd fields mix and generate one goldstone boson and two pseudoscalar Higgs

$$\begin{pmatrix} 1 & 0 \quad 0 \\ 0 & \\ 0 & R^A \end{pmatrix} \begin{pmatrix} c_\beta & s_\beta & 0 \\ -s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_S \end{pmatrix} = \begin{pmatrix} G^0 \\ A \\ A_S \end{pmatrix}, \qquad R^A = \begin{pmatrix} c_{\alpha_{AS}} & s_{\alpha_{AS}} \\ -s_{\alpha_{AS}} & c_{\alpha_{AS}} \end{pmatrix}, \tag{4}$$

The 2HDM+S in the mass eigenstate

The input parameters of the mass eigenstate

$$\underbrace{\tan\beta,\ m_h,\ m_H,\ m_A,\ m_{H^\pm},\ c_{\beta-\alpha}}_{\text{2HDM parameters}}\ \underbrace{v_S,\ m_{h_S},\ m_{A_S},\ \alpha_{HS},\ \alpha_{hS},\ \alpha_{AS}}_{\text{singlet parameters}}.$$

We disentangled the 3×3 mixing scenarios into the following fundamental scenarios

Case 0	$c_{\beta-\alpha}=\alpha_{HS}=\alpha_{hS}=\alpha_{AS}=0$	
Case I	$\alpha_{HS}=\alpha_{hS}=\alpha_{AS}=0$	$c_{\beta-\alpha}\neq 0$
Case II	$c_{\beta-\alpha}=\alpha_{HS}=\alpha_{AS}=0$	$\alpha_{hS}\neq 0$
Case III	$c_{\beta-\alpha}=\alpha_{hS}=\alpha_{AS}=0$	$\alpha_{HS}\neq 0$
Case IV	$c_{\beta-\alpha}=\alpha_{hS}=\alpha_{HS}=0$	$\alpha_{AS}\neq 0$

- Case-0 is the alignment limit of 2HDM, h_{125} couplings are the same to SM
- Case-I is on the 2HDM case, the singlet fields are decoupled
- Case-II is the case where h_{125} mix with h_S
- Case-III is the case where doublet H mix with h_S
- Case-IV is the case where doublet A mix with A_S

The STU observables

The electroweak precision observables STU are defined by the self-energy of the W and Z bosons. [E. Peskin et al, 92']

$$\alpha(m_Z)T = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}, \quad (5)$$

$$\frac{\alpha(m_Z)}{4s_W^2 c_W^2} S = \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} - \frac{c_W^2 - s_W^2}{s_W c_W}\frac{\Pi_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}, \quad (6)$$

$$\frac{\alpha(m_Z)}{4s_W^2}(S+U) = \frac{\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2} - \frac{c_W}{s_W}\frac{\Pi_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}, \quad (7)$$

The observables and corresponding precision used in STU fitting at CEPC are as follows:[CEPCStudyGroup 18']

Observables	CEPC
δm_h [GeV]	5.9×10^{-3}
$\delta\alpha_{\rm had}$	4.7×10^{-5}
δm_Z [GeV]	5.0×10^{-4}
δm_t [GeV]	6.0×10^{-1}
δm_W [GeV]	1.0×10^{-3}
$\delta\Gamma_W$ [GeV]	2.8×10^{-3}
$\delta\Gamma_Z$ [GeV]	5.0×10^{-4}
$\delta A_b^{\rm FB}$	1.0×10^{-4}
$\delta A_c^{\rm FB}$	2.2×10^{-4}
$\delta A_\ell^{\rm FB}$	5.0×10^{-5}
δR_b	4.3×10^{-5}
δR_c	1.7×10^{-4}
δR_ℓ	2.1×10^{-3}
$\delta\sigma_{\rm had}$ [nb]	5.0×10^{-3}

The STU observables

The estimated ranges of STU and their corresponding correlation matrix were obtained using the Gfitter package [Gfitter 14']. The results are shown below:

$$S^{\rm exp} = 0 \pm 1.82 \times 10^{-2}, \quad T^{\rm exp} = 0 \pm 2.56 \times 10^{-2}, \quad U^{\rm exp} = 0 \pm 1.83 \times 10^{-2},$$
$$\text{corr}(S,T) = +0.9963, \quad \text{corr}(S,U) = -0.9745, \quad \text{corr}(T,U) = -0.9844.$$

Perform a global fit between theoretical and experimental values using χ^2 to determine the model's feasible region.

$$\chi^2_{STU} = \begin{pmatrix} S - S^{\rm exp} & T - T^{\rm exp} & U - U^{\rm exp} \end{pmatrix} \cdot \mathbf{cov}^{-1} \cdot \begin{pmatrix} S - S^{\rm exp} \\ T - T^{\rm exp} \\ U - U^{\rm exp} \end{pmatrix} < 5.99, \tag{8}$$

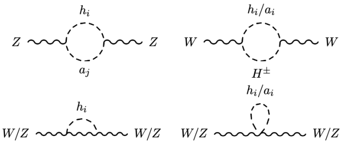
where

$$\mathbf{cov} = \begin{pmatrix} \Delta_S^2 & \text{corr}(S,T)\Delta_S\Delta_T & \text{corr}(S,U)\Delta_S\Delta_U \\ \text{corr}(S,T)\Delta_S\Delta_T & \Delta_T^2 & \text{corr}(T,U)\Delta_T\Delta_U \\ \text{corr}(S,U)\Delta_S\Delta_U & \text{corr}(T,U)\Delta_T\Delta_U & \Delta_U^2 \end{pmatrix}, \tag{9}$$

The two-dimensional fit to the STU parameters at 95% C.L. corresponds to $\Delta\chi^2 = \chi^2_{STU} - \chi^2_{STU}|_{\rm minimal} < 5.99$.

The STU observables in 2HDM+S

Feynman diagrams that contribute to the self energy of the SM gauge bosons.



The contributions to the STU parameters from various Higgses can be found in Ref.[W. Grimus et al. 08']. Using those expressions, the STU parameters in 2HDM+S are given by the following equation:

$$T = \frac{1}{16\pi s_W^2 m_W^2}\Bigg[\sum_i^3 |c_{h_i H^\pm W^\mp}|^2 F(m_{H^\pm}^2, m_{h_i}^2) + \sum_i^2 |c_{a_i H^\pm W^\mp}|^2 F(m_{H^\pm}^2, m_{a_i}^2) - \sum_{i,j} |c_{a_i h_j Z}|^2 F(m_{a_i}^2, m_{h_j}^2)$$
$$+ 3\sum_i^3 |c_{h_i VV}|^2 \left(F(m_Z^2, m_{h_i}^2) - F(m_W^2, m_{h_i}^2)\right) - 3\left(F(m_Z^2, m_h^2) - F(m_W^2, m_h^2)\right)\Bigg], \quad (10)$$

$$S = \frac{1}{24\pi}\Bigg[(2s_W^2 - 1)^2 G(m_{H^\pm}^2, m_{H^\pm}^2, m_Z^2) + \sum_{i,j} |c_{a_i h_j Z}|^2 G(m_{a_i}^2, m_{h_j}^2, m_Z^2) - 2\ln(m_{H^\pm}^2) - \ln(m_h^2)$$
$$+ \sum_i^3 c_{h_i h_i VV}\ln(m_{h_i}^2) + \sum_i^2 c_{a_i a_i VV}\ln(m_{a_i}^2) + \sum_i |c_{h_i VV}|^2 \hat{G}(m_{h_i}^2, m_Z^2) - \hat{G}(m_h^2, m_Z^2)\Bigg], \quad (11)$$

- The U observable does not get significant effect from this model
- The T observable usually play the dominant role of STU constraint

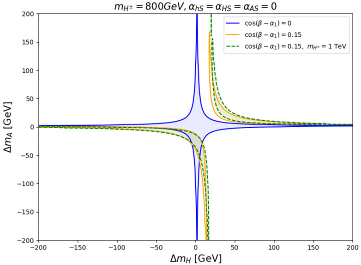
2HDM limit (Case-0 & Case-I)

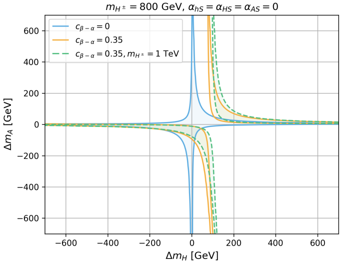
$\alpha_{hS} = \alpha_{HS} = \alpha_{AS} = 0$

For convenience, we define the following mass splittings:

$$\Delta m_H = m_H - m_H^{\pm}, \qquad \Delta m_A = m_A - m_H^{\pm},$$

$$\Delta m_{HS} = m_{h_S} - m_H^{\pm}, \qquad \Delta m_{AS} = m_{A_S} - m_H^{\pm},$$





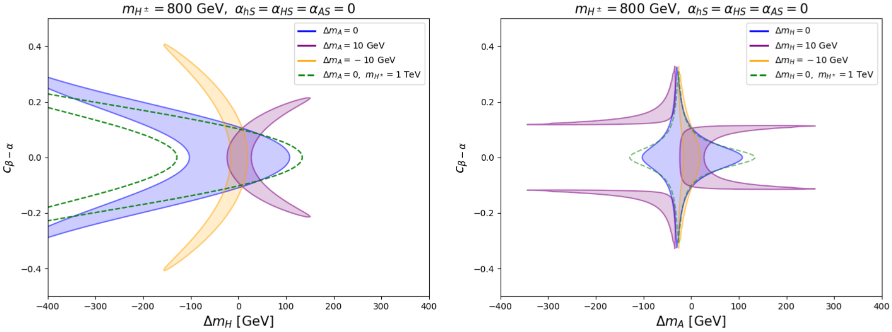
- Blue region: the Case-0, which centers around $\Delta m_H = 0$ or $\Delta m_A = 0$ (left CEPC and right LHC).
- The orange region in the left figure corresponds to $c_{\beta-\alpha} = 0.15$, with the allowed regions shift to the right. In particular, the $\Delta m_A = 0$ point with $\Delta m_H \sim$20 GeV and $c_{\beta-\alpha} = 0.15$ would be excluded.
- The orange region in the right figure, the $\Delta m_A = 0$ point with $\Delta m_H \sim$100 GeV and $c_{\beta-\alpha} = 0.35$ would be excluded.
- The *STU* observables behave basically the same as the 2HDM in this case.

2HDM limit (Case-0 & Case-I)

$\alpha_{hS} = \alpha_{HS} = \alpha_{AS} = 0$

$$T_0 = \frac{1}{16\pi s_W^2 m_W^2}[F(m_{H\pm}^2, m_H^2) - F(m_A^2, m_H^2) + F(m_{H\pm}^2, m_A^2)] \tag{12}$$

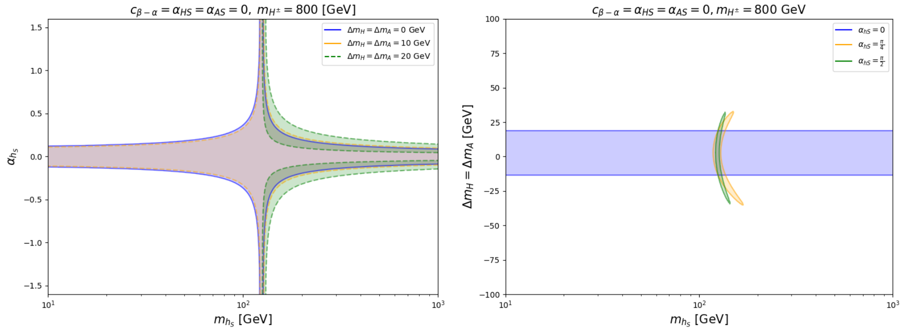
$$S_0 = \frac{1}{24\pi}[(2s_W^2 - 1)^2 G(m_{H\pm}^2, m_{H\pm}^2, m_Z^2) + G(m_A^2, m_H^2, m_Z^2) + \ln\left(\frac{m_H^2}{m_{H\pm}^2}\right) + \ln\left(\frac{m_A^2}{m_{H\pm}^2}\right)] \tag{13}$$



- These two figures demonstrate that the *STU* measurement results provide the maximum upper limit of $\Delta m_H \sim 100$ GeV or $\Delta m_A \sim 100$ GeV.
- Compared to the LHC, the upper limit on Δm_H or Δm_A has been reduced.

Case-II

$\alpha_{hS} \neq 0,\ c_{\beta-\alpha} = \alpha_{HS} = \alpha_{AS} = 0$

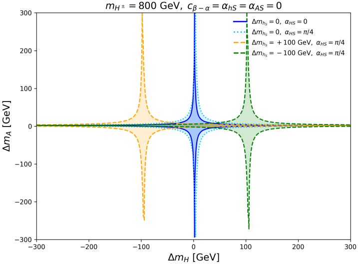


$$T = \frac{1}{16\pi s_W^2 m_W^2} 3 s_{\alpha_{hS}}^2 \left[F(m_Z^2, m_{h_S}^2) - F(m_W^2, m_{h_S}^2) - F(m_Z^2, m_{h_{125}}^2) + F(m_W^2, m_{h_{125}}^2) \right] + T_0 \tag{14}$$

- In the two-dimensional parameter space of m_{h_S} vs. α_{h_S} shown in the left panel, it can be seen that the STU constraints would be weak when m_{h_S} close to 125 GeV.
- For larger α_{hS}, larger values of $\Delta m_H = \Delta m_A$ can be reached, where 2HDM exclude, when m_{h_S} is heavier.

Case-III

$\alpha_{HS} \neq 0,\ c_{\beta-\alpha} = \alpha_{hS} = \alpha_{AS} = 0$

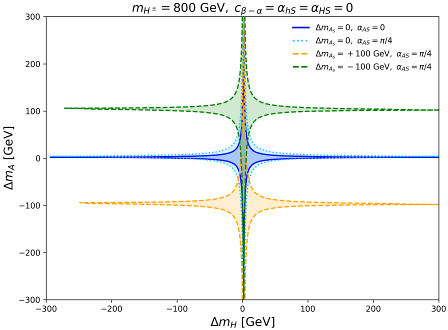


$$c^2_{\alpha_{HS}} m_H + s^2_{\alpha_{HS}} m_{h_S} = m_{H^\pm} \tag{15}$$

- Dark blue region: the Case-0. For $\alpha_{HS} = \pi/4$ and $\Delta m_{hS} = 0$, the *STU* allowed region would be slightly enlarged compared to the Case-0.
- The *STU* allowed region would be shifted by α_{HS} and Δm_{HS}, while the constraints can be always allowed when $\Delta m_A = 0$.
- The mass relation Eq. (15) ensures that the *STU* constraints can be fulfilled for arbitrary α_{HS}, when $\Delta m_A = 0$.

Case-IV

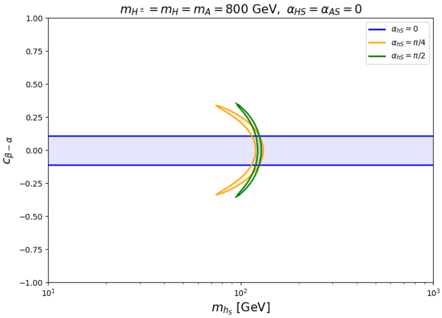
$\alpha_{AS} \neq 0,\ c_{\beta-\alpha} = \alpha_{hS} = \alpha_{HS} = 0$

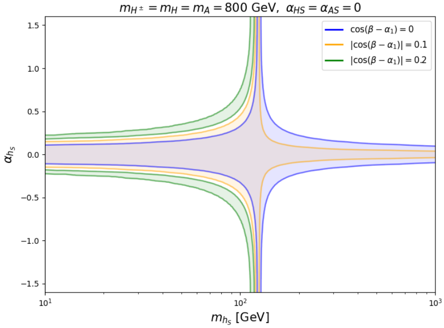


$$c^2_{\alpha_{AS}} m_A + s^2_{\alpha_{AS}} m_{A_S} = m_{H^\pm} \,. \tag{16}$$

- The *STU* allowed region would be shifted by α_{AS} and Δm_{AS}, while the constraints can be always allowed when $\Delta m_H = 0$.
- The figure is almost identical to the above, except that the H and A have been interchanged.

STU in 2HDM+S beyond the alignment limit





- We observe that the singlet mixing angles of 2HDMS can make the parameter space allow, where 2HDM is excluded.

Summary

Conclusions

- We disentangle and extract the effect of each mixing angles in the 2HDM+S, and set up four fundamental cases. wherem the case-0 and case-I are on the 2HDM limit.
- The *STU* properties of case II ($\alpha_{hS} \neq 0$) mainly depends on the mass difference between m_{h_S} and $m_{h_{125}}$.
- In case III or IV ($\alpha_{HS} \neq 0$ or $\alpha_{AS} \neq 0$), the *STU* constraints can be fulfilled when the mass relation $c^2_{\alpha_{HS}} m_H + s^2_{\alpha_{HS}} m_{h_S} = m_{H^\pm}$ or $c^2_{\alpha_{AS}} m_A + s^2_{\alpha_{AS}} m_{A_S} = m_{H^\pm}$ is satisfied
- 2HDMS can have the allowed parameter space where 2HDM is excluded. In this region, the two models can potentially be distinguished.
- At the CEPC, the *STU* observables are provided with higher precision.

Outlook

- Interplay with collider searches constraints as well as the cosmological effect. (In proceeding)

Thank you!

The Higgs to gauge bosons couplings

		0	I	II	III	IV
	$c_{h_i VV} = R_{i1} c_\beta + R_{i2} s_\beta$					
c_{HVV}	$c_{\beta-\alpha} c_{\alpha_{HS}}$	0	$c_{\beta-\alpha}$	0	0	0
c_{hVV}	$s_{\beta-\alpha} c_{\alpha_{hS}} - c_{\beta-\alpha} s_{\alpha_{HS}} s_{\alpha_{hS}}$	1	$s_{\beta-\alpha}$	$c_{\alpha_{hS}}$	1	1
$c_{h_S VV}$	$-s_{\beta-\alpha} s_{\alpha_{hS}} - c_{\beta-\alpha} s_{\alpha_{HS}} c_{\alpha_{hS}}$	0	0	$-s_{\alpha_{hS}}$	0	0
	$c_{A_i H_j Z} = R^A_{i1} R_{j1} + R^A_{i2} R_{j2}$					
c_{AHZ}	$-c_{\alpha_{AS}} c_{\alpha_{HS}} s_{\beta-\alpha}$	-1	$-s_{\beta-\alpha}$	-1	$-c_{\alpha_{HS}}$	$-c_{\alpha_{AS}}$
c_{AhZ}	$c_{\alpha_{AS}} \left(c_{\beta-\alpha} c_{\alpha_{hS}} + s_{\beta-\alpha} s_{\alpha_{HS}} s_{\alpha_{hS}} \right)$	0	$c_{\beta-\alpha}$	0	0	0
$c_{A h_S Z}$	$-c_{\alpha_{AS}} \left(c_{\beta-\alpha} s_{\alpha_{hS}} - s_{\beta-\alpha} s_{\alpha_{HS}} c_{\alpha_{hS}} \right)$	0	0	0	$s_{\alpha_{HS}}$	0
$c_{A_S HZ}$	$s_{\alpha_{AS}} c_{\alpha_{HS}} s_{\beta-\alpha}$	0	0	0	0	$s_{\alpha_{AS}}$
$c_{A_S hZ}$	$-s_{\alpha_{AS}} \left(c_{\beta-\alpha} c_{\alpha_{hS}} + s_{\beta-\alpha} s_{\alpha_{HS}} s_{\alpha_{hS}} \right)$	0	0	0	0	0
$c_{A_S h_S Z}$	$s_{\alpha_{AS}} \left(c_{\beta-\alpha} s_{\alpha_{hS}} - s_{\beta-\alpha} s_{\alpha_{HS}} c_{\alpha_{hS}} \right)$	0	0	0	0	0
	$c_{\phi_i H^\pm W^\mp} = R^\phi_{i2} c_\beta - R^\phi_{i1} s_\beta$					
$c_{HH^\pm W^\mp}$	$-i c_{\alpha_{HS}} s_{\beta-\alpha}$	-i	$-i s_{\beta-\alpha}$	-i	$-i c_{\alpha_{HS}}$	-i
$c_{hH^\pm W^\mp}$	$i \left(c_{\beta-\alpha} c_{\alpha_{hS}} + s_{\beta-\alpha} s_{\alpha_{HS}} s_{\alpha_{hS}} \right)$	0	$i c_{\beta-\alpha}$	0	0	0
$c_{h_S H^\pm W^\mp}$	$-i \left(c_{\beta-\alpha} s_{\alpha_{hS}} - s_{\beta-\alpha} s_{\alpha_{HS}} c_{\alpha_{hS}} \right)$	0	0	0	$-i s_{\alpha_{HS}}$	0
$c_{AH^\pm W^\mp}$	$c_{\alpha_{AS}}$	1	1	1	1	$c_{\alpha_{AS}}$
$c_{A_S H^\pm W^\mp}$	$-s_{\alpha_{AS}}$	0	0	0	0	$-s_{\alpha_{AS}}$

Back up

The general form of the Higgs potential

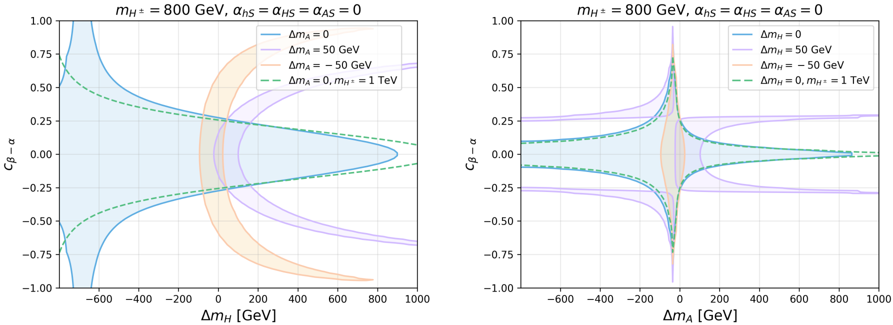
$$V_{\rm 2HDM} = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + {\rm h.c.}\right) + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left(\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + {\rm h.c.}\right). \quad (17)$$

$$V_{\rm S} = m_S^2 S^\dagger S + \frac{m_S'^2}{2}(S^2 + {\rm h.c.}) + \left(\frac{\lambda_1''}{4!}S^4 + \frac{\lambda_2''}{3!}S^2(S^\dagger S) + {\rm h.c.}\right) + \frac{\lambda_3''}{4}(S^\dagger S)^2 + \Big[S^\dagger S\left(\lambda_1'\Phi_1^\dagger\Phi_1 + \lambda_2'\Phi_2^\dagger\Phi_2 + \lambda_3'\Phi_1^\dagger\Phi_2\right) + S^2\left(\lambda_4'\Phi_1^\dagger\Phi_1 + \lambda_5'\Phi_2^\dagger\Phi_2 + \lambda_6'\Phi_1^\dagger\Phi_2 + \lambda_7'\Phi_2^\dagger\Phi_1\right) + \frac{\mu_{S1}}{3!}S^3 + \frac{\mu_{S2}}{2}S(S^\dagger S) + S\left(\mu_{11}\Phi_1^\dagger\Phi_1 + \mu_{22}\Phi_2^\dagger\Phi_2 + \mu_{12}\Phi_1^\dagger\Phi_2 + \mu_{21}\Phi_2^\dagger\Phi_1\right) + {\rm h.c.}\Big] \quad (18)$$

2HDM limit (Case-0 & Case-I)

$\alpha_{hS} = \alpha_{HS} = \alpha_{AS} = 0$

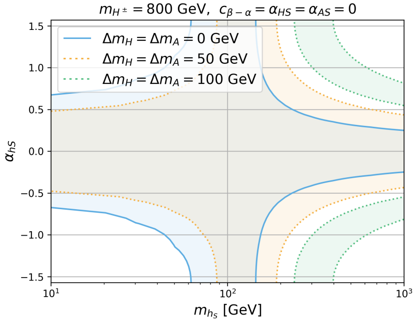
$$S_0 = \frac{1}{24\pi}[(2s_W^2 - 1)^2 G(m_{H\pm}^2, m_{H\pm}^2, m_Z^2) + G(m_A^2, m_H^2, m_Z^2) + \ln\left(\frac{m_H^2}{m_{H\pm}^2}\right) + \ln\left(\frac{m_A^2}{m_{H\pm}^2}\right)] \quad (19)$$

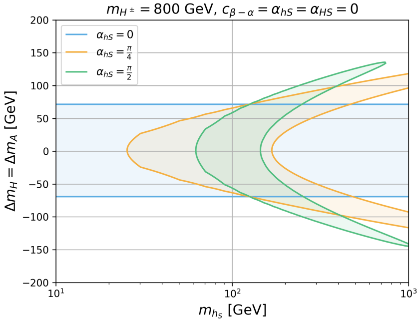


- The updated STU measurement results provide the maximum upper limit of Δm_H or Δm_A, when Δm_A or Δm_H is 0.
- When m_H is around 125 GeV, $c_{\beta-\alpha}$ has no limit for $\Delta m_A = 0$

Case-II

$\alpha_{hS} \neq 0, \ c_{\beta-\alpha} = \alpha_{HS} = \alpha_{AS} = 0$



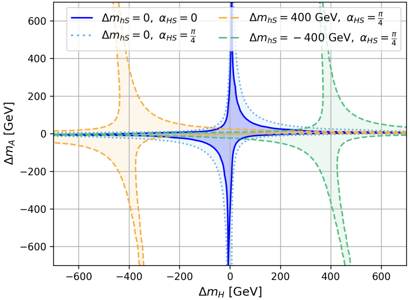


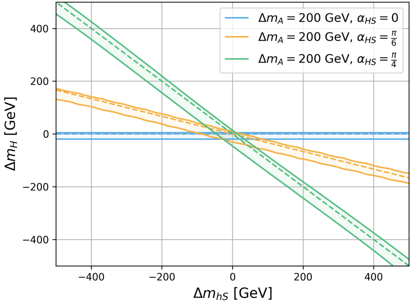
$$T = \frac{1}{16\pi s_W^2 m_W^2} 3 s_{\alpha_{hS}}^2 \left[F(m_Z^2, m_{h_S}^2) - F(m_W^2, m_{h_S}^2) - F(m_Z^2, m_{h_{125}}^2) + F(m_W^2, m_{h_{125}}^2) \right] + T_0 \quad (20)$$

- The *STU* constraints would be weak when m_{h_S} close to 125 GeV
- For larger α_{hS}, the larger $\Delta m_H = \Delta m_A$ can be reached, where 2HDM exclude, when m_{h_S} is heavier.

Case-III

$\alpha_{HS} \neq 0,\ c_{\beta-\alpha} = \alpha_{hS} = \alpha_{AS} = 0$



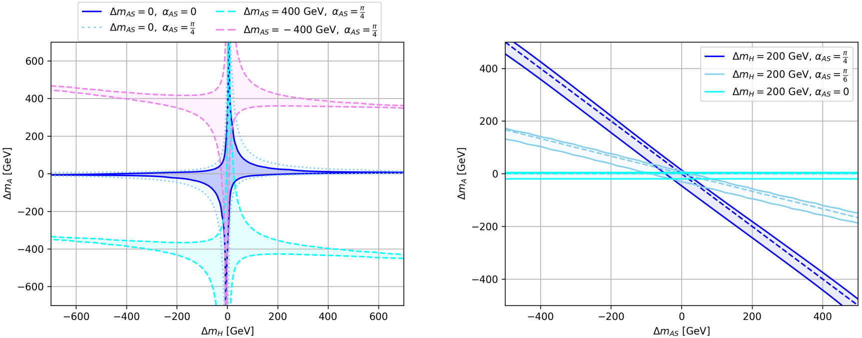


$$c_{\alpha_{HS}}^2 m_H + s_{\alpha_{HS}}^2 m_{h_S} = m_{H^\pm} \tag{21}$$

- The *STU* allowed region would be shifted by α_{HS} and Δm_{HS}, while the constraints can be always allowed when $\Delta m_A = 0$.
- The mass relation Eq. (15) ensures that the *STU* constraints can be fulfilled for arbitrary α_{HS}, when $\Delta m_A = 0$.

Case-IV

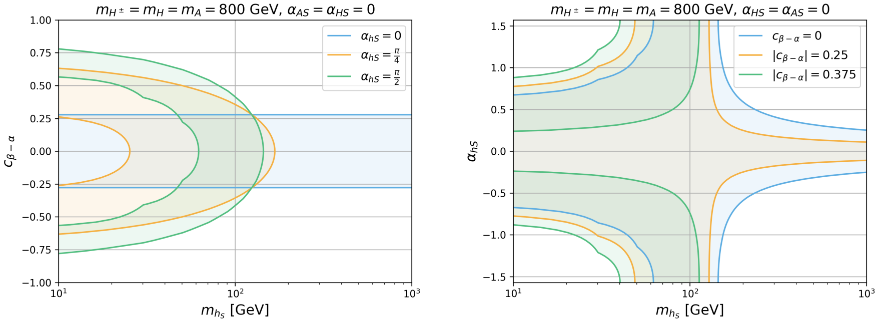
$\alpha_{AS} \neq 0,\ c_{\beta-\alpha} = \alpha_{hS} = \alpha_{HS} = 0$



$$c^2_{\alpha_{AS}} m_A + s^2_{\alpha_{AS}} m_{A_S} = m_{H^\pm} \,. \tag{22}$$

- The *STU* allowed region would be shifted by α_{AS} and Δm_{AS}, while the constraints can be always allowed when $\Delta m_H = 0$.
- The mass relation Eq. (16) ensures that the *STU* constraints can be fulfilled for arbitrary α_{AS}, when $\Delta m_H \neq 0$.

STU in 2HDM+S beyond the alignment limit



- The singlet mixing angles of 2HDMS can make the parameter space allow, where 2HDM is excluded.