Two-Loop Electroweak Radiative Corrections to Low-Energy Electron-Electron Scattering

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The 2025 International Workshop on the High Energy Circular Electron Positron Collider

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Based on

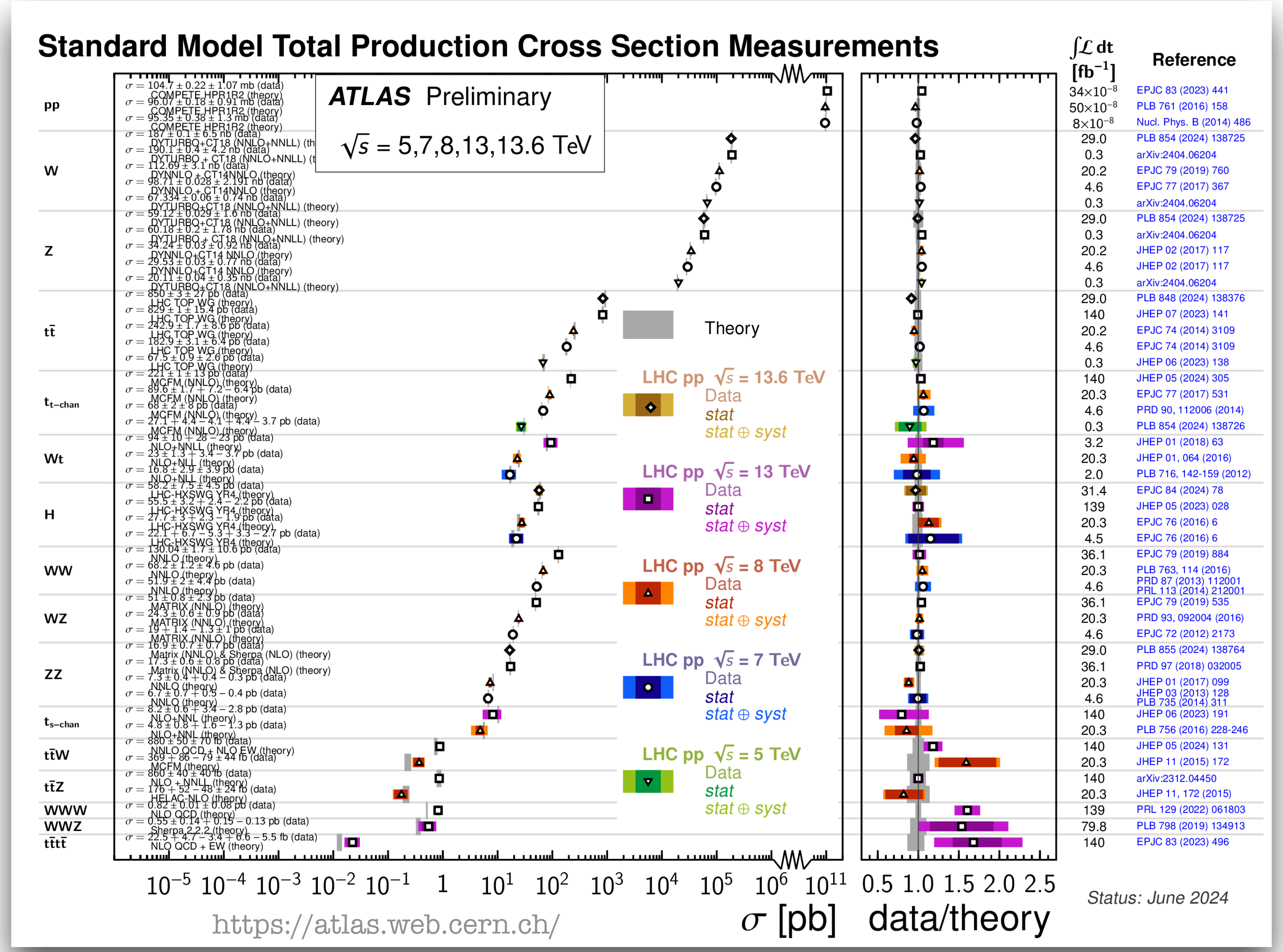
PRL 126 (2021) 13, 131801, with Ayres Freitas, Hiren Patel, Michael Ramsey-Musolf

IMP 中国科学院近代物理研究所

Institute of Modern Physics, Chinese Academy of Sciences

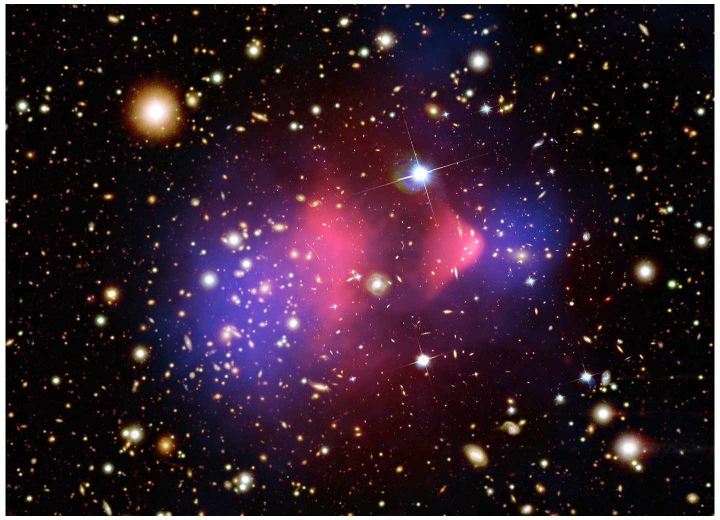
Introduction

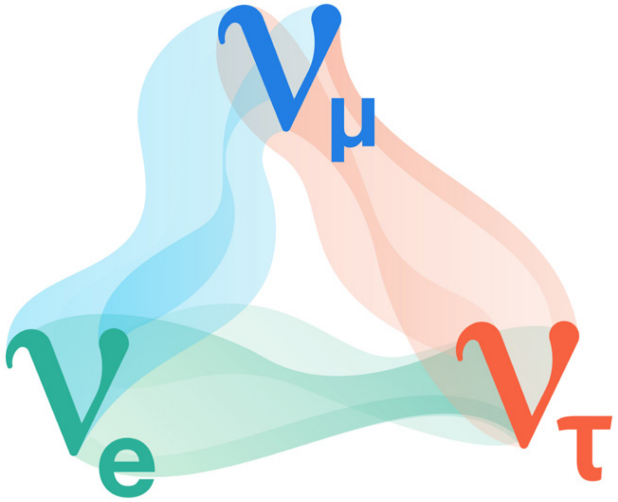
The standard model of particle physics is quite *successful*



Introduction

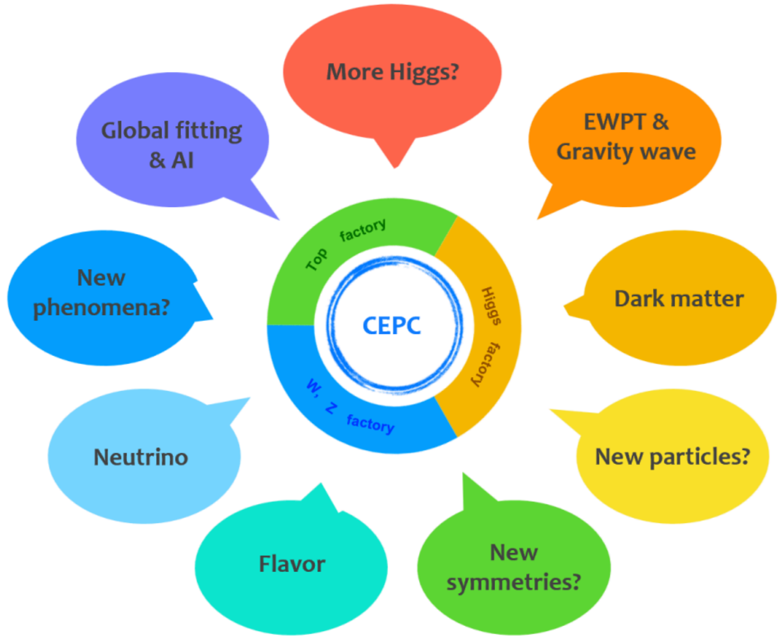
yet we know new physics beyond the SM is needed



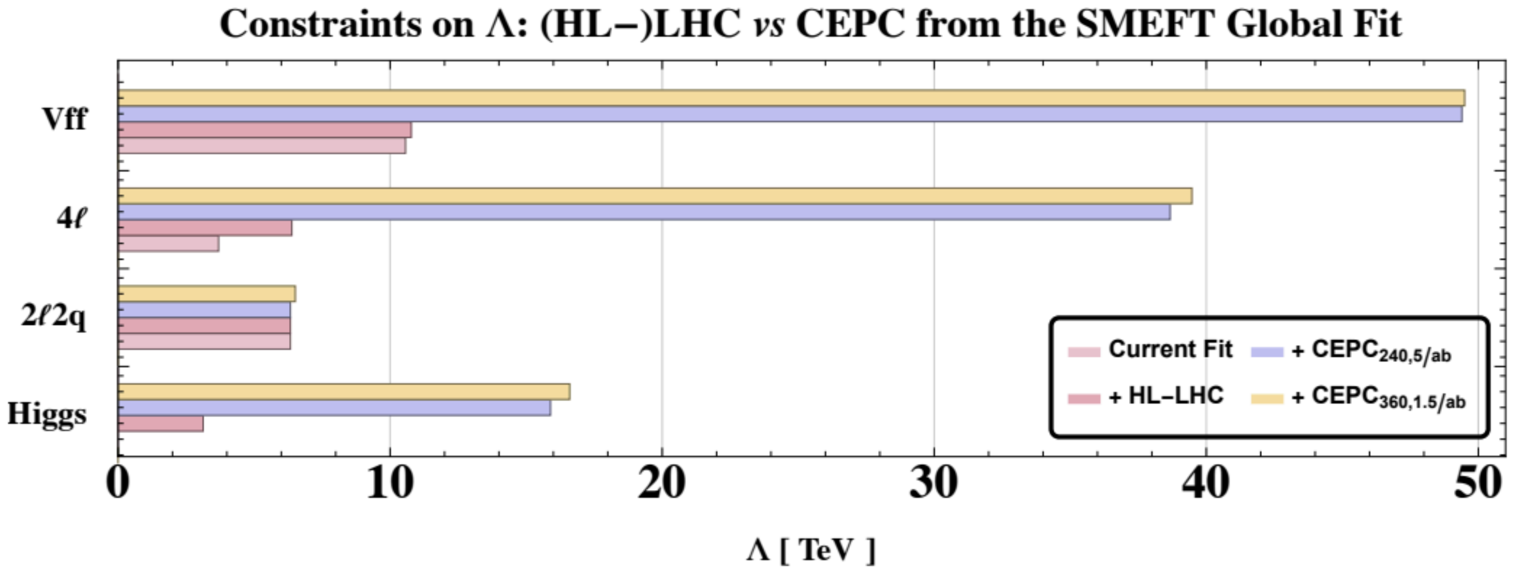


+ …

CEPC: a powerful machine — broad reach + precision



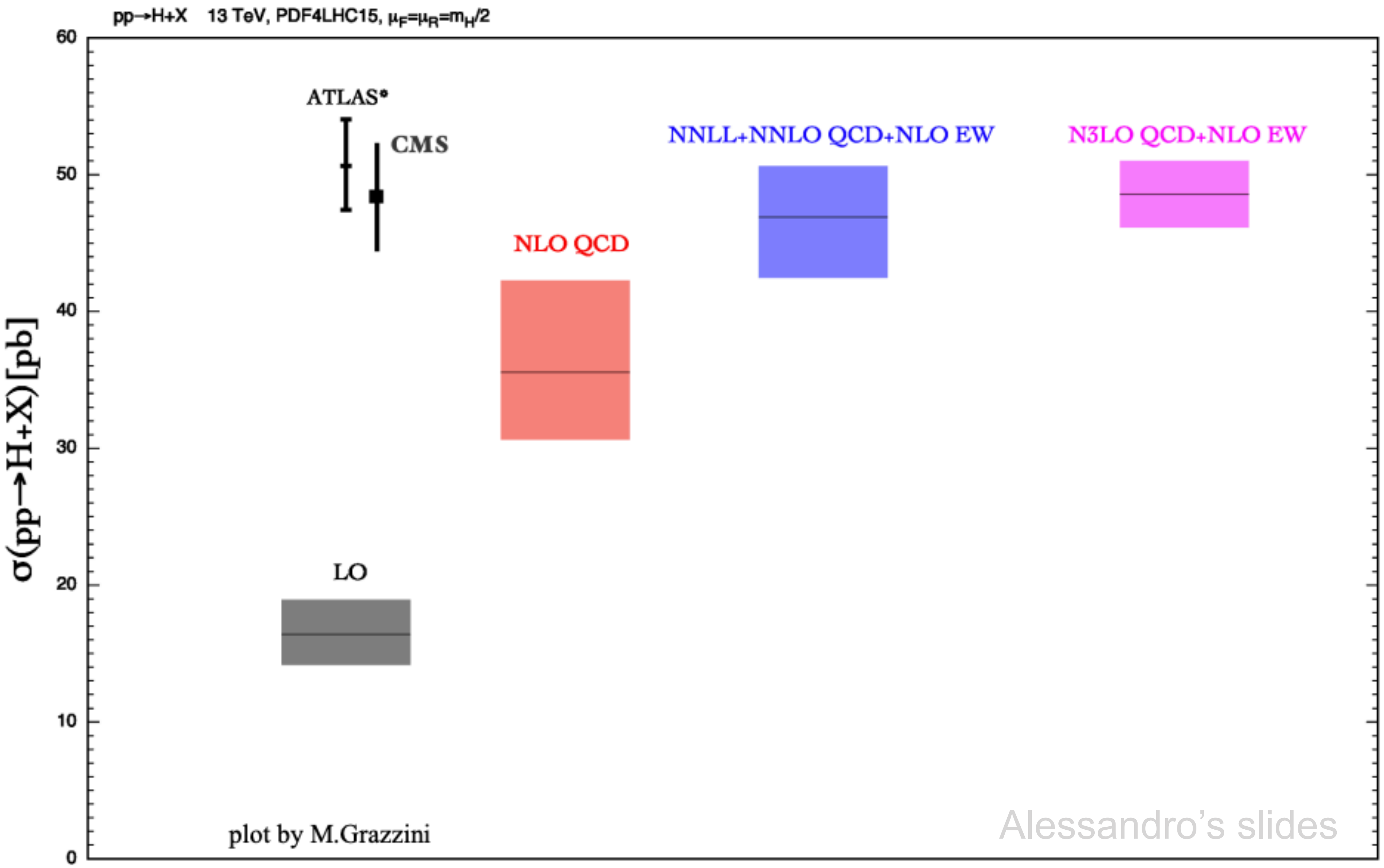
CEPC study group, 2505.24810



See Jorge's talk on Thursday for more

Introduction

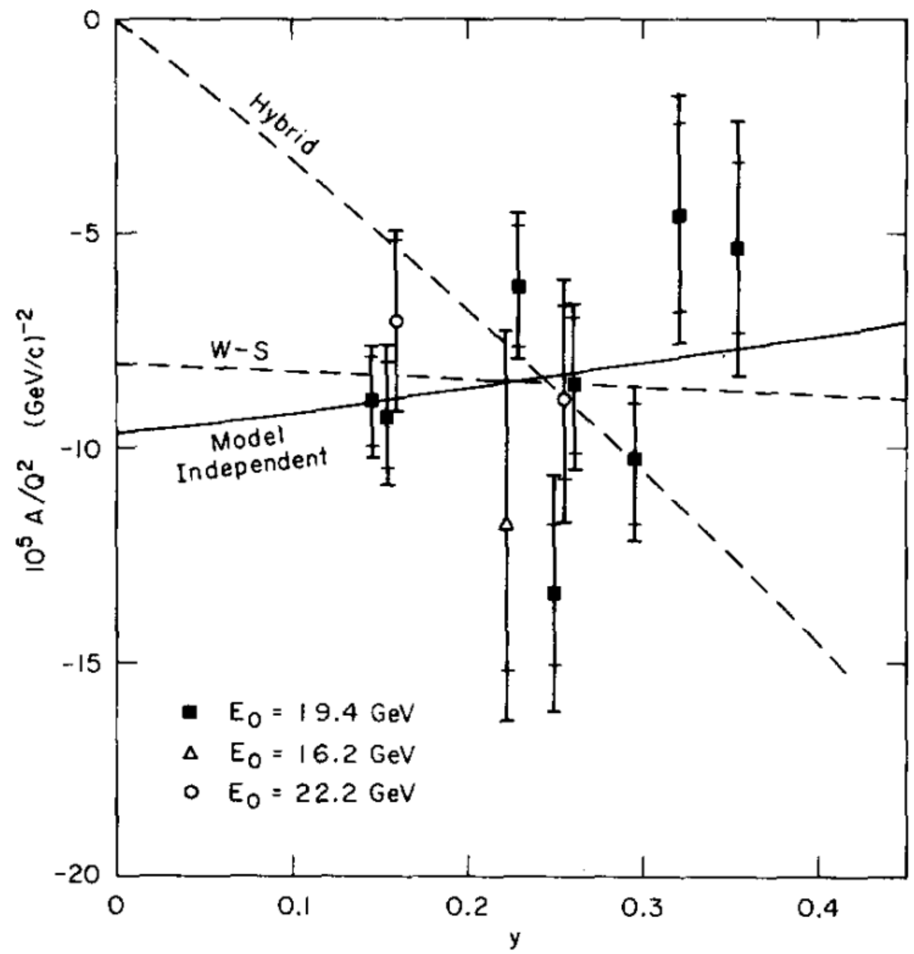
the statement on new physics needs carefulness — precision match + convergence (pert)



See Alessandro's talk on Thursday

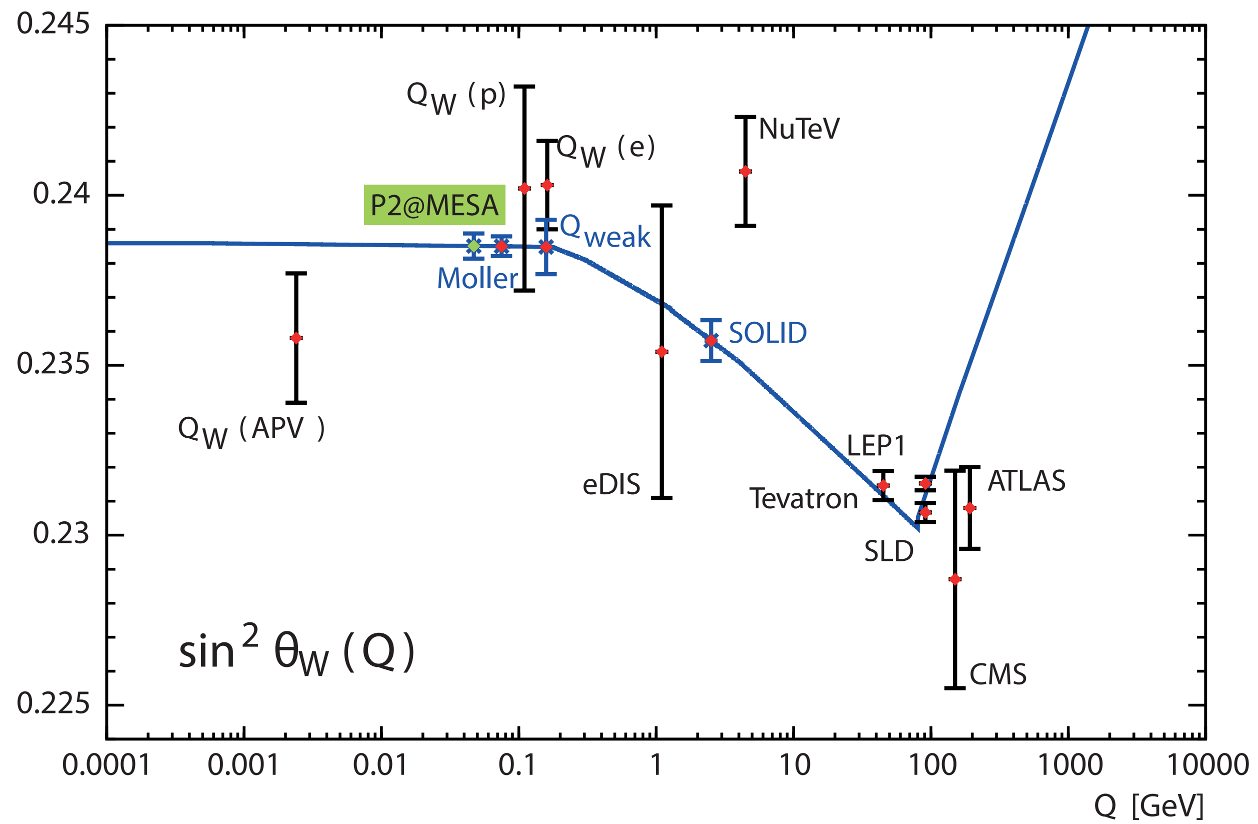
Introduction

Precision $\sin^2\theta_W$ from parity-violating electron scattering is my focus today

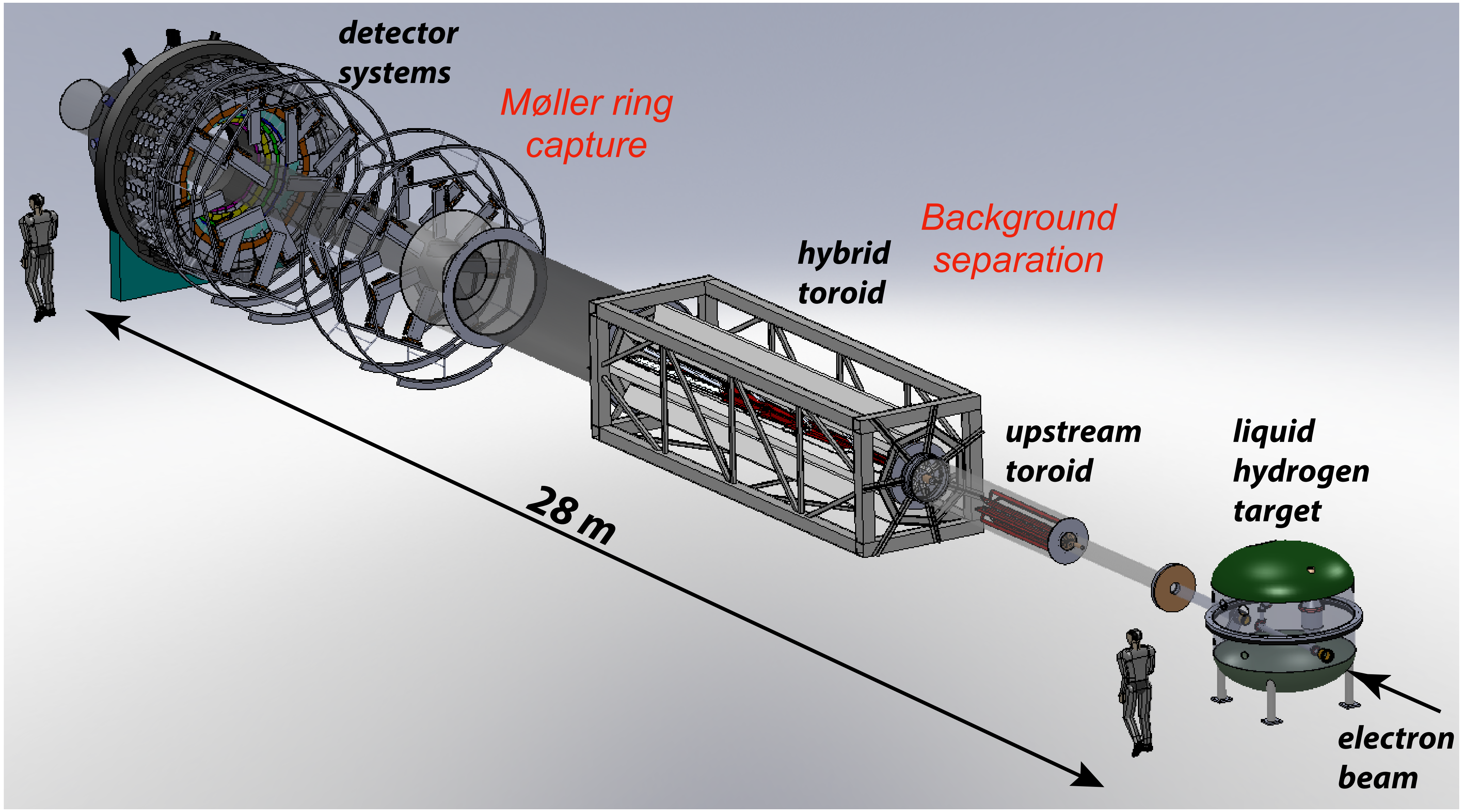


Prescott et al, 1978, 1979

(past) model selection



(future) precision test + ?



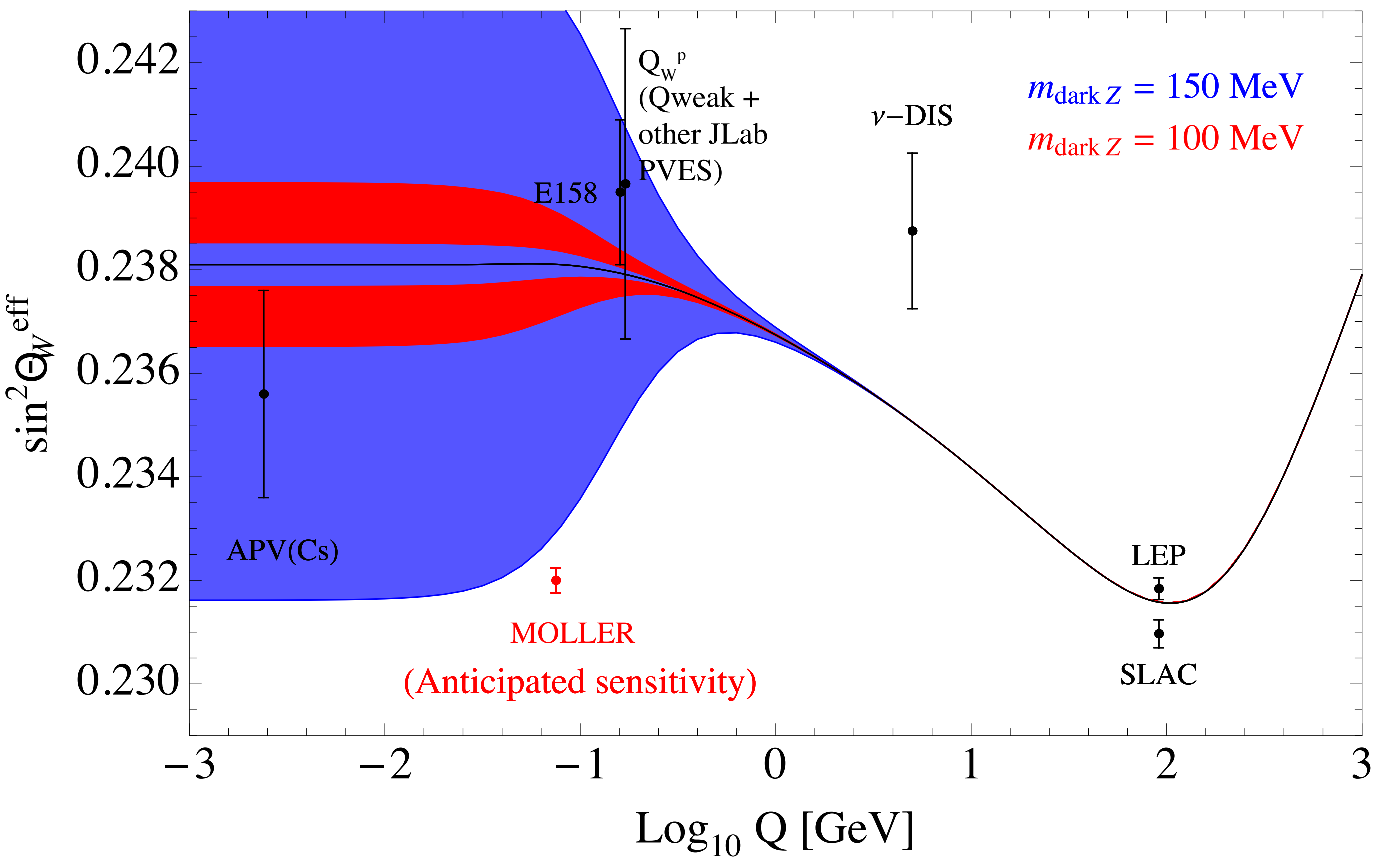
11GeV
Longitudinal
polarized
electrons

MOLLER Collaboration, 1411.4088

target precision: $\Delta Q_W^e = 1.1 \times 10^{-3}$

Introduction

Also good opportunity for new physics searches + complementary to CEPC!



MOLLER Collaboration, 1411.4088

$$\mathscr{L}^{\rm PV}_{\rm SM} = \frac{G_F}{2\sqrt{2}} g^e_A \bar{e}\gamma_\mu\gamma_5 e \sum_f g^f_V \bar{f}\gamma^\mu f$$

$$\mathscr{L}^{\rm PV}_{\rm NP} = \frac{4\pi\kappa^2}{\Lambda^2} \bar{e}\gamma_\mu\gamma_5 e \sum_f h^f_V \bar{f}\gamma^\mu f$$

1% relative precision in the nucleon/electron weak charge leads to

$$\frac{\Delta Q_w}{Q_w} \leq 1\,\% \iff \Lambda \gtrsim \left(\frac{8\sqrt{2}\pi\kappa^2}{0.01 G_F}\right)^{1/2} \approx 20\,\kappa\ \mathrm{TeV}$$

Ramsey-Musolf, hep-ph/9903264

Similarly, we can ask: theoretical prediction precise enough and convergent?

Theoretical status: LO

LO calculation:

$\left[Q_W^e\right]_{\rm LO}$ Derman and Marciano, 1979

$$A_{\rm PV} = \frac{G_\mu Q^2}{\sqrt{2}\pi\alpha_{\rm EM}} \frac{1-y}{1+y^4+(1-y)^4} Q_W^e$$

$$\left[Q_W^e\right]_{\rm LO} = 1 - 4\sin^2\theta_W$$

Radiative corrections are parameterized as

$$Q_W^e = \left[Q_W^e\right]_{\rm LO} + \left[Q_W^e\right]_{\rm NLO} + \left[Q_W^e\right]_{\rm NNLO} + \cdots = \sum_n \left(\frac{\alpha_{\rm EM}}{\pi}\right)^n c_n$$

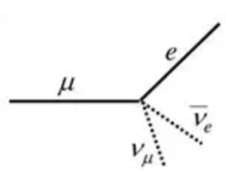
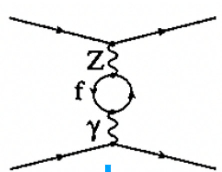
where terms proportional to G_μ in c_n are ignored. Plugging in $\sin^2\theta_W(m_Z) = 0.2314$:

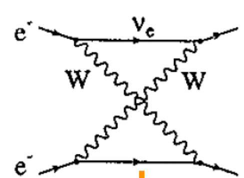
$$\left[Q_W^e\right]_{\rm LO} = 1 - 4\sin^2\theta_W = 0.0744$$

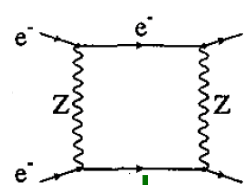
Theoretical status: NLO

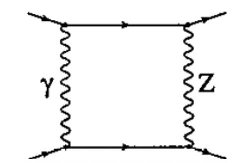
NLO calculation:

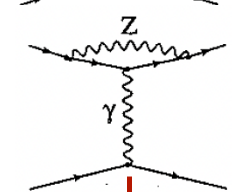
$[Q_W^e]_{\rm NLO}$ Czarnecki and Marciano, 1996; Denner and Pozzorini, 1998; Petriello, 2003; Zykunov, 2004; Kolomensky et al, 2005; Zykunov et al, 2005; Zykunov, 2009; Aleksejevs et al, 2010, 2011, 2012







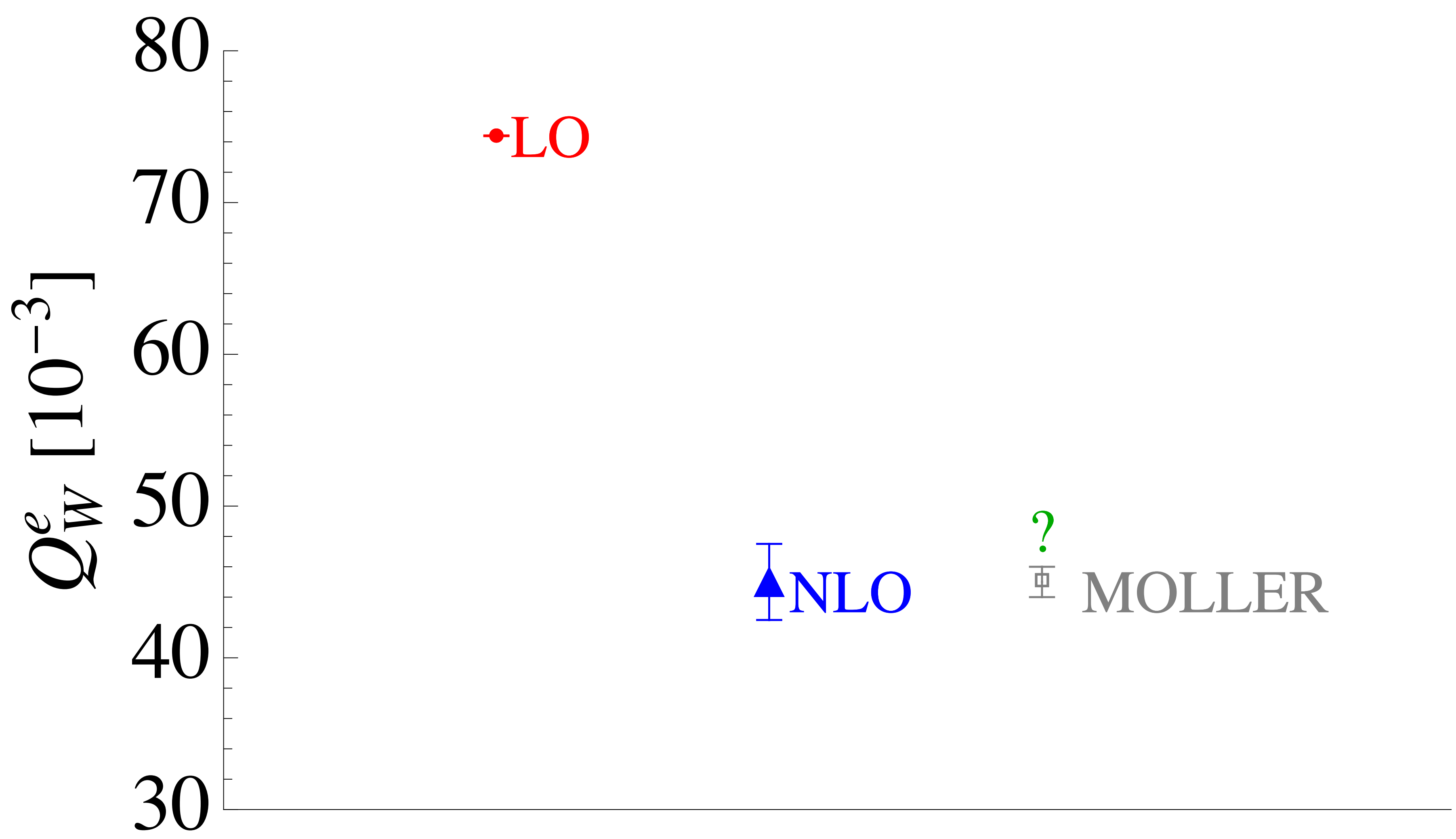


$$[Q_W^e]_{\rm LO+NLO} = \rho\left\{1 - 4\kappa(0)s_W^2 + \frac{\alpha_{\rm EM}}{4\pi s_W^2} - \frac{3\alpha_{\rm EM}}{32\pi s_W^2 c_W^2}(1-4s_W^2)[1+(1-4s_W^2)^2] + F_1(y,Q^2) + F_2(y,Q^2)\right\}$$

$$= 0.0450 \pm 0.0023 \pm 0.0010$$

Theoretical status: NLO

NLO calculation:



40% reduction! Not precise enough for future experiment, neither good convergence.

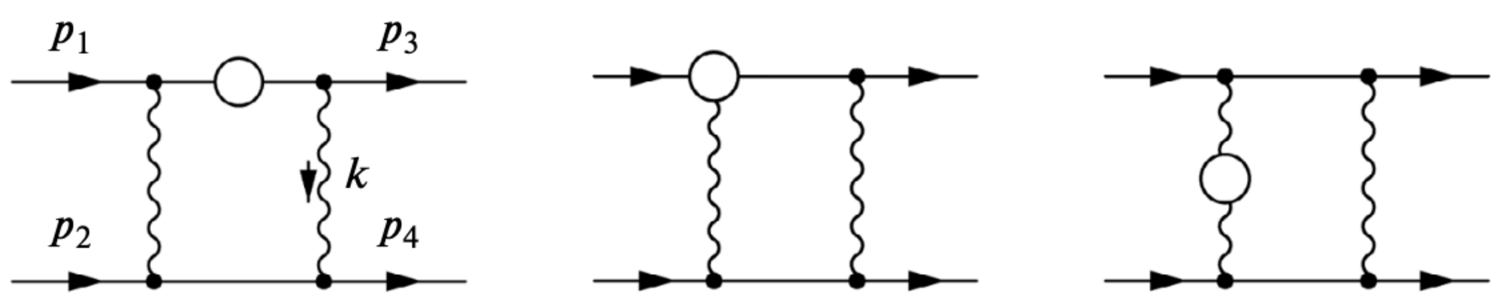
Theoretical status: NNLO

NNLO calculation: *earlier efforts*

$\left[Q_W^e\right]_{\mathrm{NNLO}}$ Aleksejevs et al, 2011, 2012, 2015 (partial estimation)

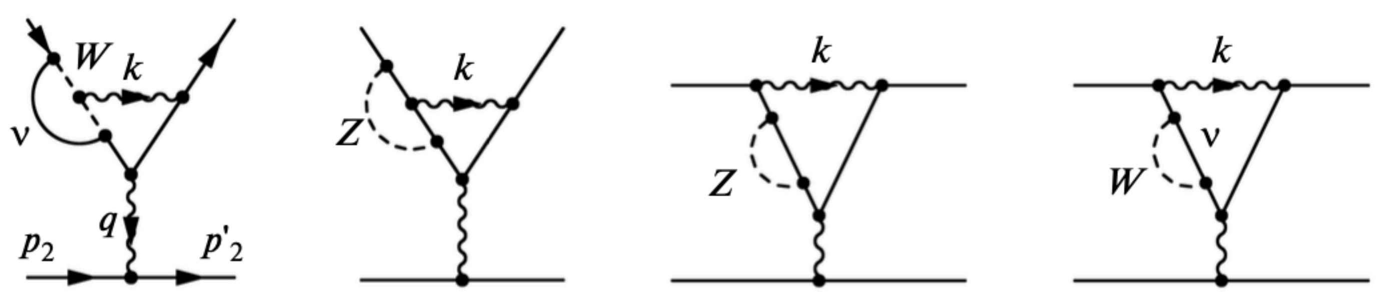
NNLO Electroweak Corrections for Polarized Møller Scattering: One-Loop Insertions to Boxes[1]

A. G. Aleksejevs[a], S. G. Barkanova[b], Yu. M. Bystritskiy[c], E. A. Kuraev[c, †], and V. A. Zykunov[d]



Two-Loop Electroweak Vertex Corrections for Polarized Møller Scattering[1]

A. G. Aleksejevs[a], S. G. Barkanova[b], Yu. M. Bystritskiy[c], E. A. Kuraev[c, †], and V. A. Zykunov[d]



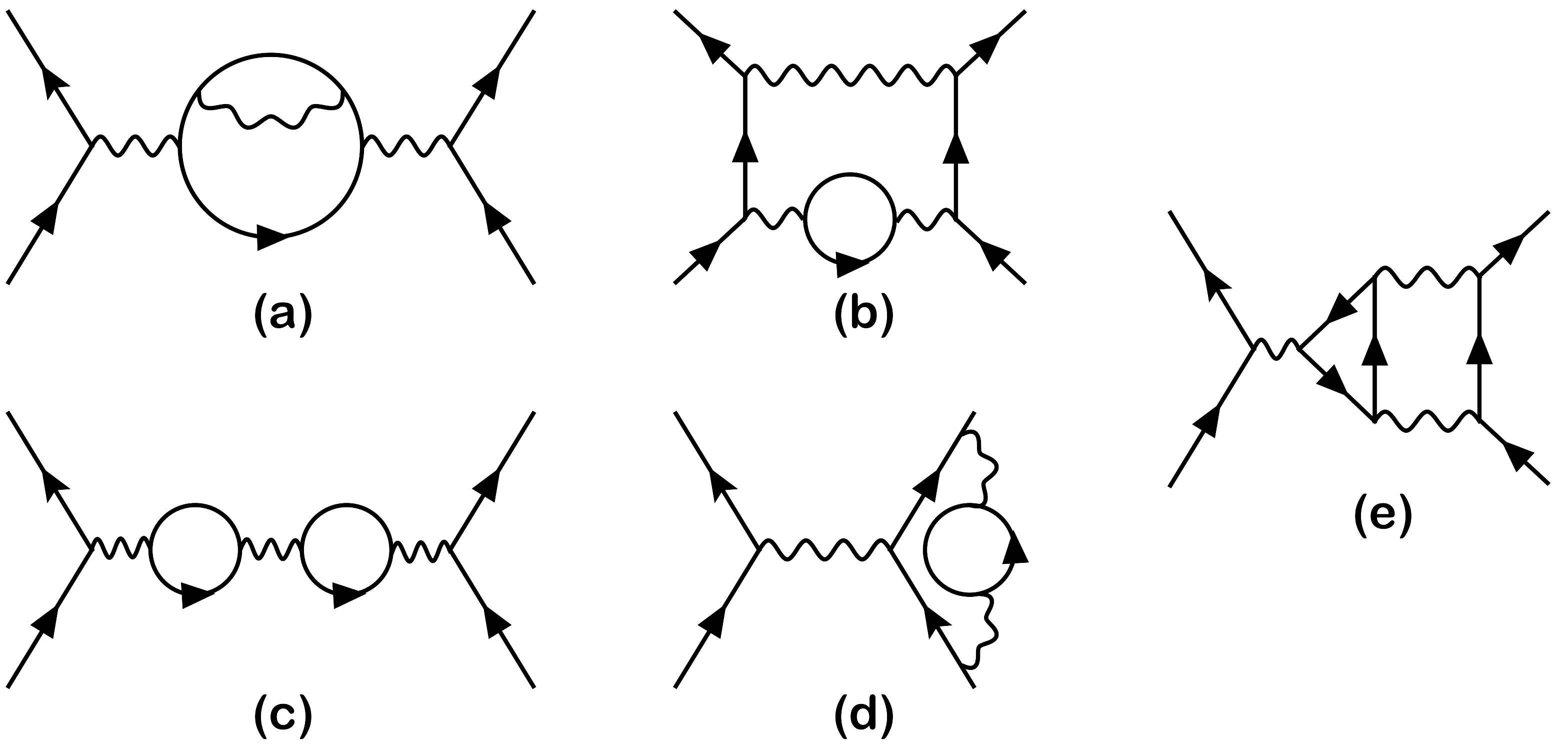
$+\cdots$

Take-aways: A full NNLO computation is needed for MOLLER.

Theoretical status: Our work @ NNLO

Our strategy: firstly focus on NNLO topologies with closed fermion loops

- *Gauge-invariant subset*
- *Dominant by fermion numbers/flavors vs bosonic loops more EW scale suppression*

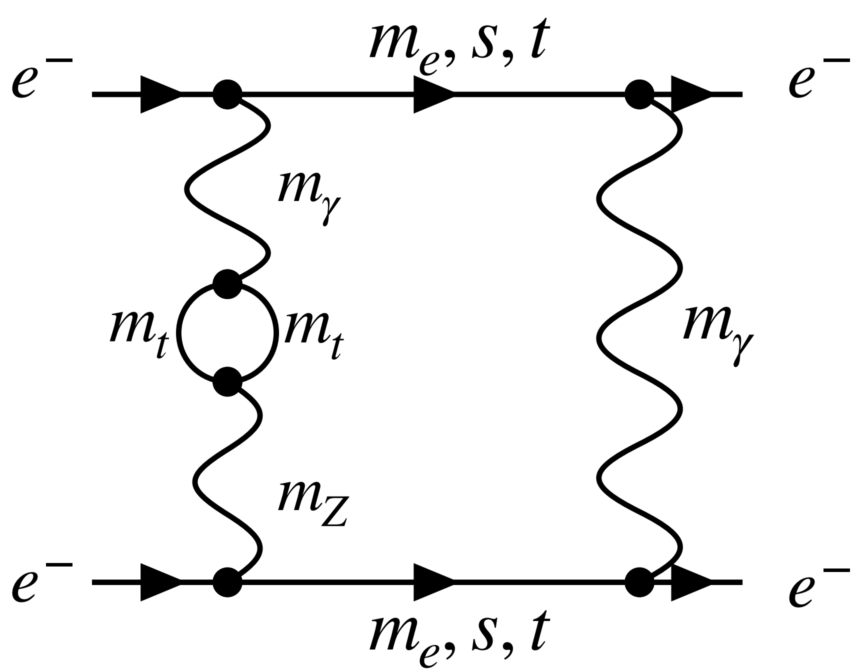


Work flow: FeynArts → FeynCalc/Package-X → code in-house → FIRE/Kira

See Qian's talk on yesterday

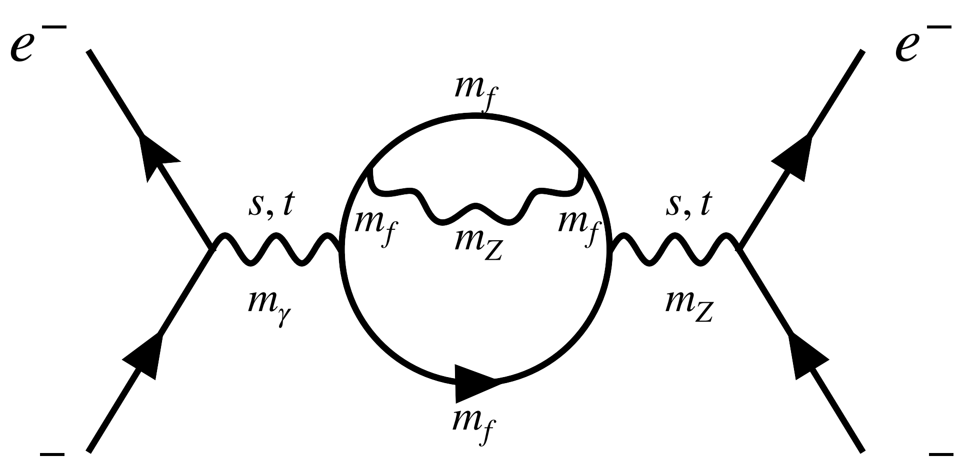
NNLO Methods: ***EBRs***

Generically speaking, integrals at NNLO and beyond contain *multiple scales*, which complicate the analytic behavior of the loop integrand



$$I = \int \tilde{d}q_1 \tilde{d}q_2\, f(q_1, q_2, m_t, m_Z, m_\gamma, m_e, s, t)$$

6 scales



$$I = \int \tilde{d}q_1 \tilde{d}q_2\, f(q_1, q_2, m_Z, m_f, m_e, s, t)$$

5 scales

NNLO Methods: ***EBRs***

For us, we aim for analytical results:

to gain stability through avoiding large number cancellation

to provide instantaneous evaluation with arbitrary precision

to track individual UV/IR divergence

NNLO Methods: ***EBRs***

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to gain stability through avoiding large number cancellation

to provide instantaneous evaluation with arbitrary precision

to track individual UV/IR divergence

Is this (technically) doable to systematically handle $\mathcal{O}(10^3 \sim 10^4)$ Feynman diagrams ?

Yes! MOLLER is a low-energy scattering process, we have a natural hierarchy:

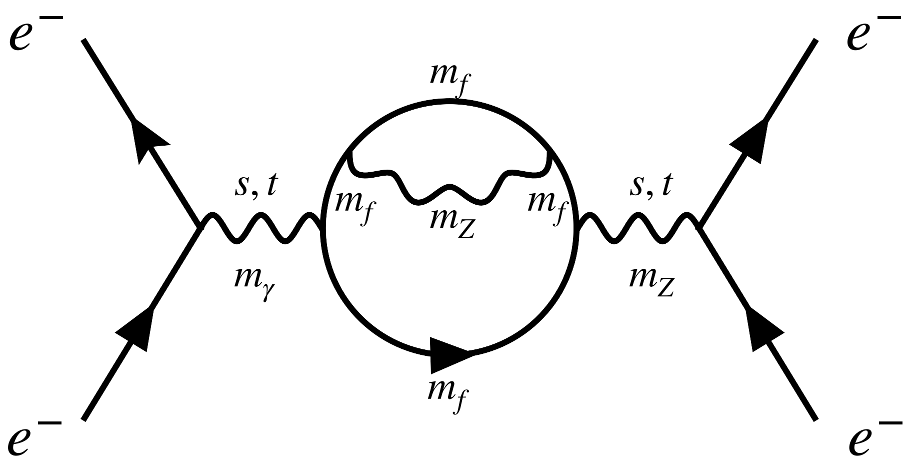
$$\begin{array}{ccccc} m_f & \ll & s, t & \ll & m_{W,Z,H,t} \\ \mathcal{O}(10^{-4})\,\mathrm{GeV} & & \mathcal{O}(10^{-1})\,\mathrm{GeV} & & \mathcal{O}(10^{2})\,\mathrm{GeV} \end{array}$$

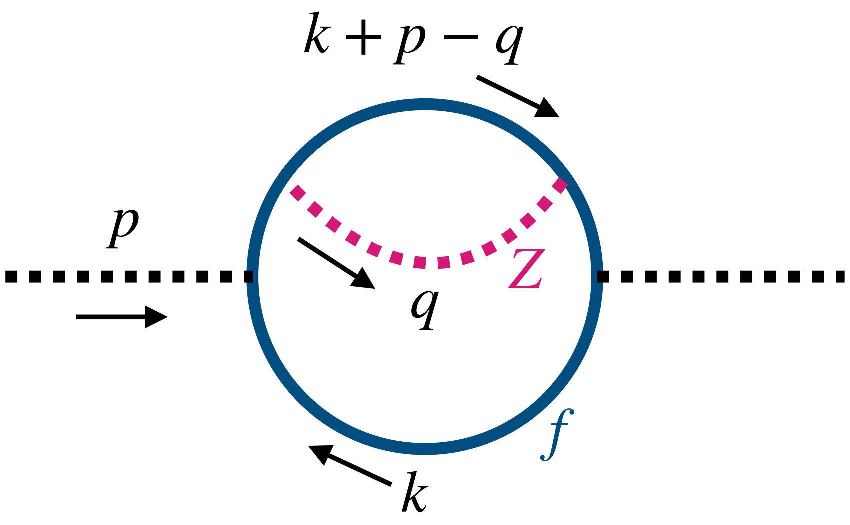
Asymptotic expansion based on this hierarchy renders it possible to systematically compute the integrals and to have analytical results in the end.

Another gain from this hierarchy, as we'll see shortly, is to oftentimes reform the two-loop integral into a one-loop one.

NNLO Methods: ***EBRs***

This asymptotic strategy for *multiple scale* integrals is based on the method of regions (recall that we are only interested in $A_{PV} \propto G_\mu$)





Let's focus on the scalar integral (always achievable through Chisholm expansion)

$$I_{\bigcirc}\left(p, m_f, m_Z\right) \equiv \int d\tilde{q} d\tilde{k} \frac{1}{\left(k^2 - m_f^2\right)\left[(k+p)^2 - m_f^2\right]^2 \left(q^2 - m_Z^2\right)\left[\left(k+p-q\right)^2 - m_f^2\right]}$$

NNLO Methods: ***EBRs***

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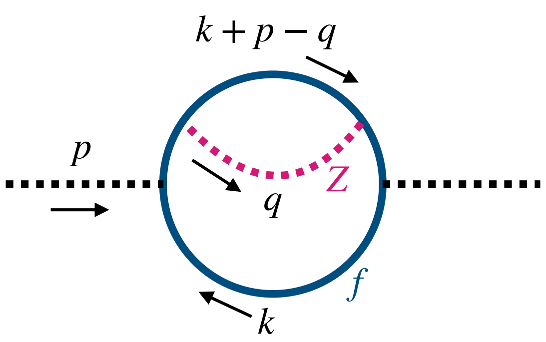

$$I_{\bigcirc}\left(p, m_f, m_Z\right) \equiv \int d\tilde{q} d\tilde{k} \frac{1}{\left(k^2 - m_f^2\right)\left[(k+p)^2 - m_f^2\right]^2 \left(q^2 - m_Z^2\right)\left[\left(k+p-q\right)^2 - m_f^2\right]}$$

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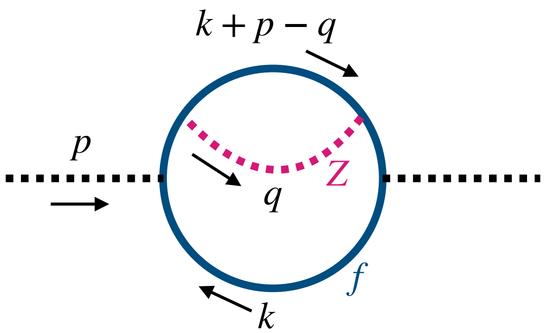
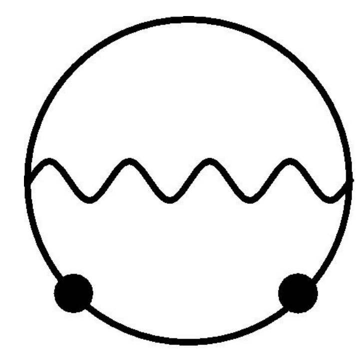


$$I_{\bigcirc}\left(p, m_f, m_Z\right) \equiv \int d\tilde{q} d\tilde{k} \frac{1}{\left(k^2-m_f^2\right)\left[(k+p)^2-m_f^2\right]^2\left(q^2-m_Z^2\right)\left[(k+p-q)^2-m_f^2\right]}$$

hard-hard region: $\{m_f^2, p^2, s, t, u\} \ll k^2 \simeq q^2 \simeq (k-q)^2 \simeq m_Z^2$

$$I_{\bigcirc}^{hh} = \int d\tilde{q} d\tilde{k} \frac{1}{\left(k^2\right)^3\left(q^2-m_Z^2\right)(k-q)^2} + \cdots$$

Power counting: $4 \times 2 - 2 \times 3 - 2 - 2 = -2$



single-scale vacuum

NNLO Methods: ***EBRs***

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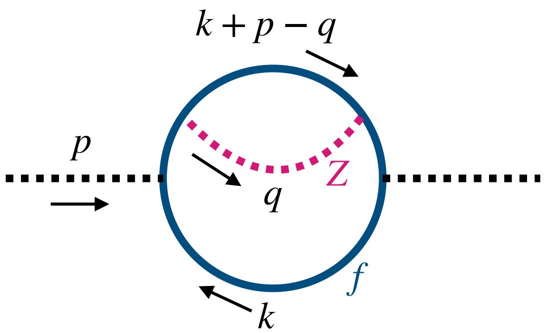
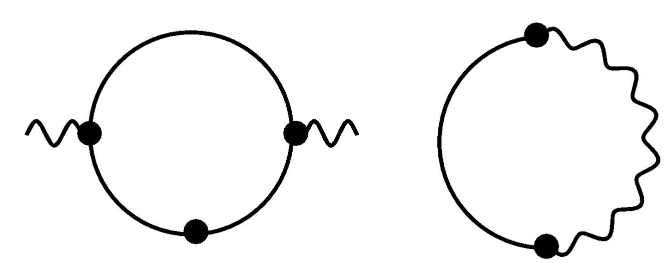


$$I_{\bigcirc}\left(p, m_f, m_Z\right) \equiv \int d\tilde{q} d\tilde{k} \frac{1}{\left(k^2-m_f^2\right)\left[(k+p)^2-m_f^2\right]^2\left(q^2-m_Z^2\right)\left[\left(k+p-q\right)^2-m_f^2\right]}$$

soft-hard region: $\{m_f^2, p^2, k^2, s, t, u\} \ll q^2 \simeq m_Z^2$

$$I_{\bigcirc}^{sh}\left(p, m_f, m_Z\right) = \int d\tilde{q} d\tilde{k} \frac{1}{\left(k^2-m_f^2\right)\left[(k+p)^2-m_f^2\right]^2\left(q^2-m_Z^2\right) q^2} + \cdots$$

Power counting: $4-2-2=0$

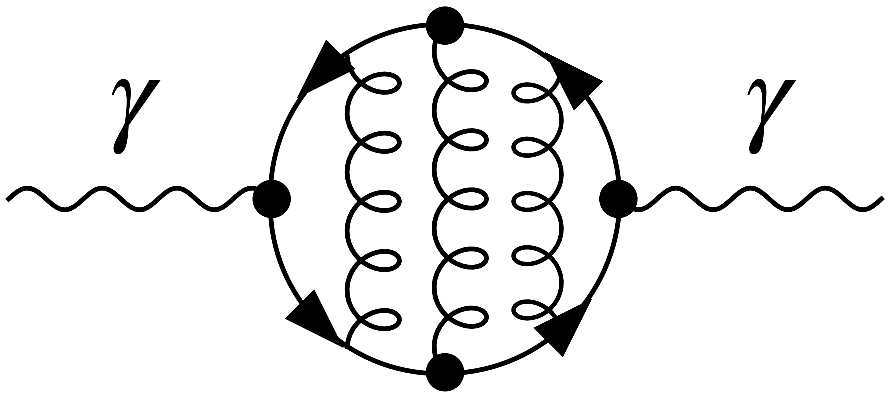


decoupled

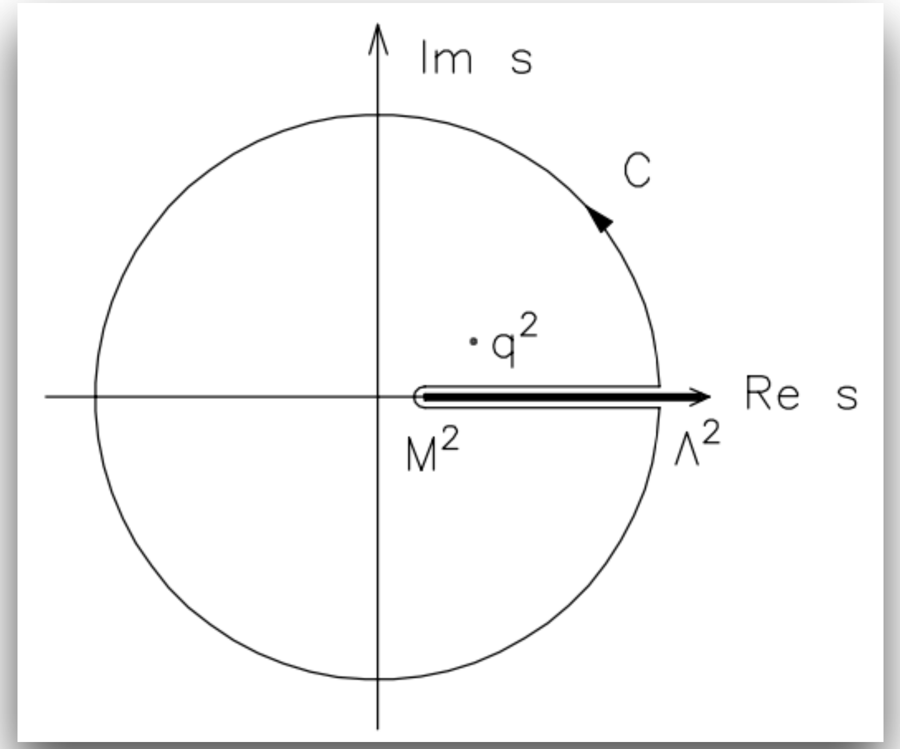
NNLO Methods: *DRs*

This method of regions similarly applies to the other topologies, while for box diagrams, *dispersion relation* is another trick to gain control over hadronic uncertainties

This idea is mature:



The amplitude develops a non-vanishing absorptive part above some threshold, allowing the reconstruction of the full amplitude by integrating along the branch cut



$$F\left(q^2\right) = \frac{1}{2\pi i}\oint_{\infty} ds \frac{F(s)}{s-q^2}$$

$$= \frac{1}{\pi}\int_{M^2}^{\infty^2} ds \frac{\mathrm{Im}\, F(s)}{s-q^2-i\epsilon} + \cancelto{0}{\frac{1}{2\pi i}\oint_{|s|=\infty} ds \frac{F(s)}{s-q^2}}$$

as long as analytic is guaranteed at infinity

NNLO Methods: ***DRs***

Otherwise, we reconstruct the full result from the *n_{th}-subtracted dispersion relations*:

$$F(q^2) = \sum_{j=0}^{n-1} \frac{\left(q^2 - q_0^2\right)^j}{j!} F^{(j)}(q_0^2) + \frac{\left(q^2 - q_0^2\right)^n}{\pi} \int_{M^2}^{\infty} ds \, \frac{\mathrm{Im}F(s)}{\left(s - q_0^2\right)^n \left(s - q^2 - i\epsilon\right)}$$

with n (typically) the *minimal* number that guarantees analyticity at infinity

$$\oint_{|s|=\infty} ds \, \frac{F(s)}{\left(s - q_0^2\right)^n \left(s - q^2\right)} = 0$$

NNLO Methods: ***DRs***

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$$\oint_{|s|=\infty} ds \, \frac{F(s)}{\left(s - q_0^2\right)^n \left(s - q^2\right)} = 0$$

For one-loop $\gamma\gamma$ and γZ self energies, the physical condition is $\Pi_T^{\gamma\gamma,\gamma Z}(0) = 0$ and $n = 1$ is enough since $\Pi_T^{\gamma\gamma,\gamma Z}(s) \to \log s$ for $s \to \infty$. Thus the once-subtracted DR is given by

$$\Pi_T^{\gamma\gamma,\gamma Z}\left(q^2\right) = \frac{q^2 - q_0^2}{\pi} \int_{M^2}^{\infty^2} \frac{ds}{s - q_0^2} \frac{\mathrm{Im}\, \Pi_T^{\gamma\gamma,\gamma Z}(s)}{s - q^2 - i\epsilon}$$

where the UV divergence in $\Pi_T^{\gamma\gamma,\gamma Z}\left(q^2\right)$ is treated in the standard way

NNLO Methods: ***DRs***

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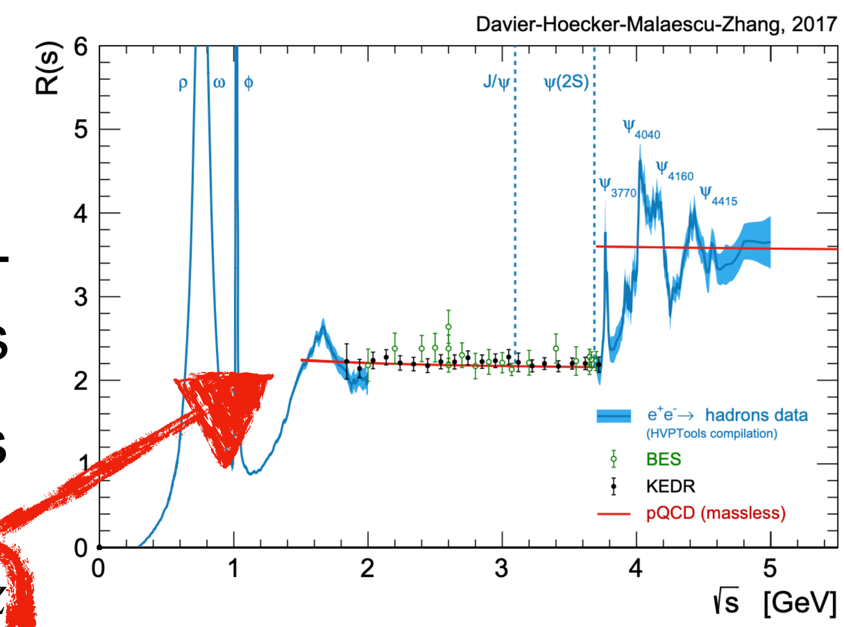

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NNLO Methods: *DRs*

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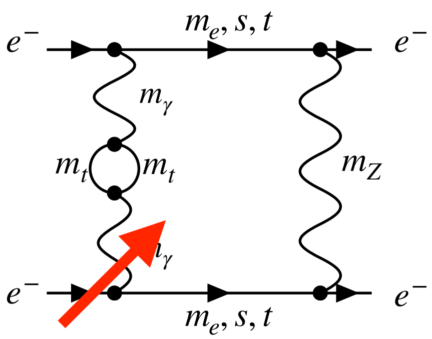

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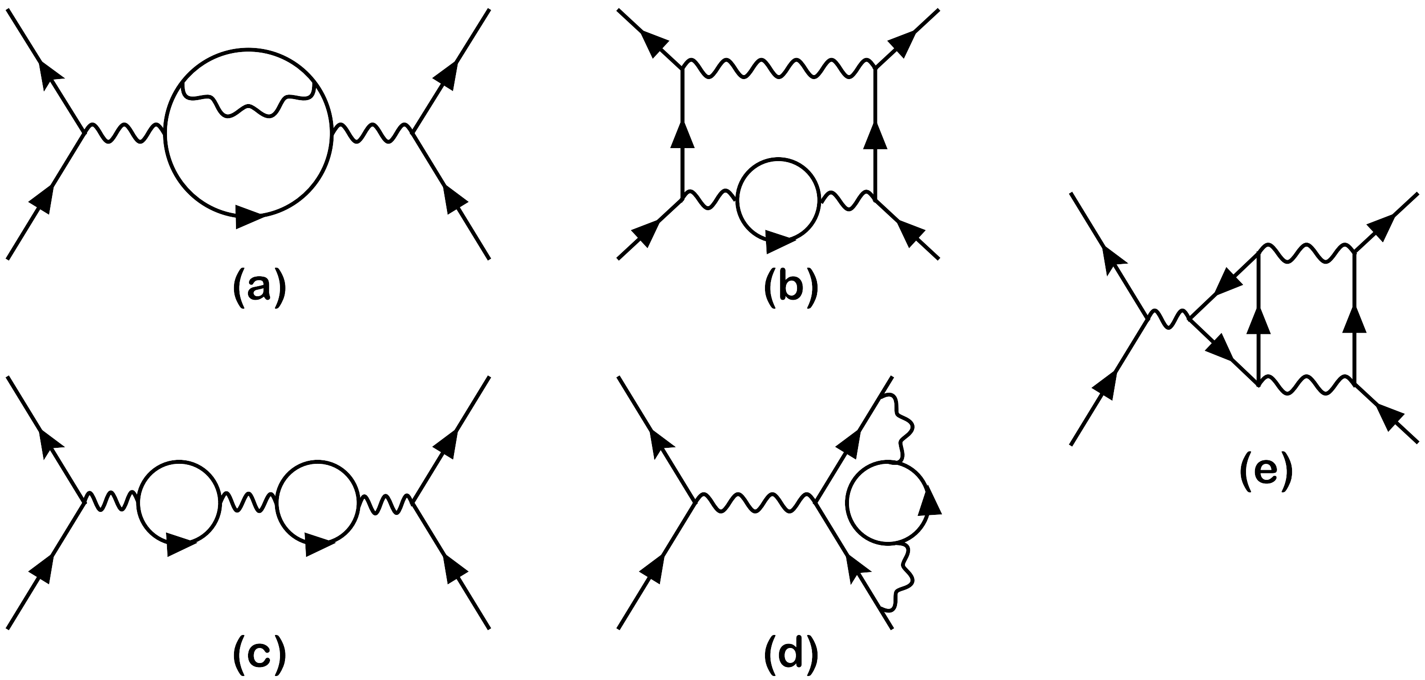
NNLO Results

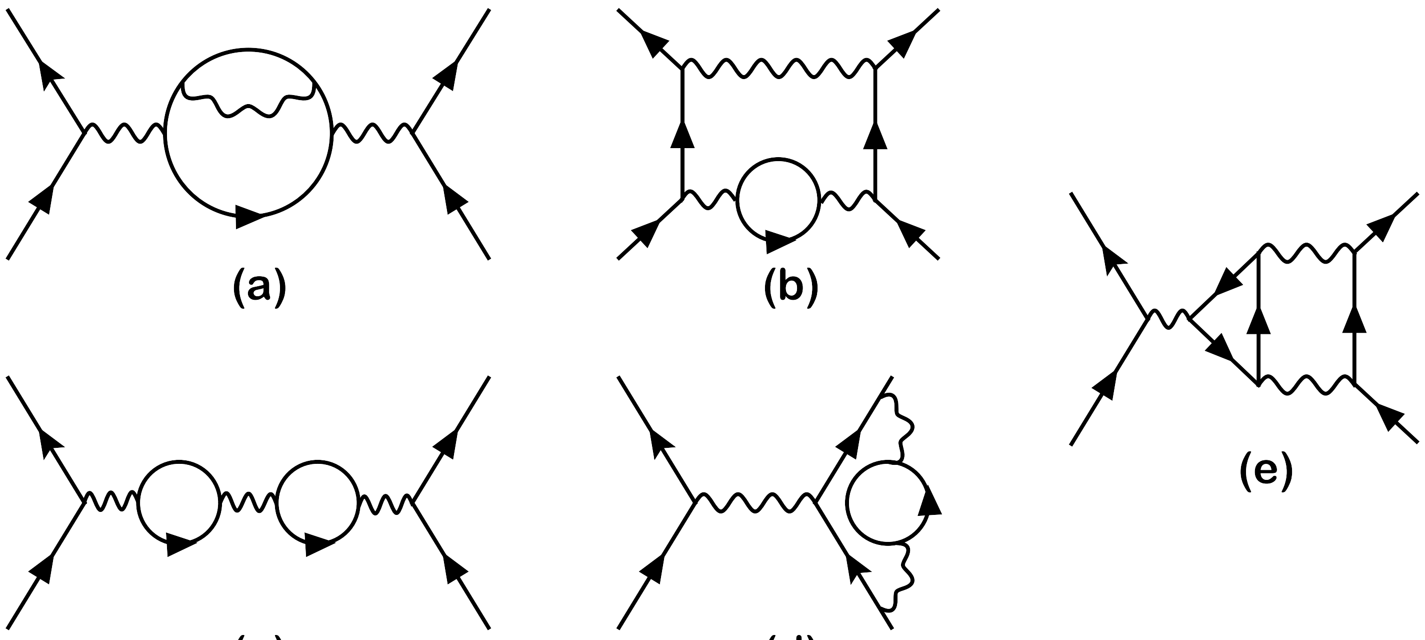


$$m_Z = 91.1876 \text{ GeV}, \qquad s_W^2 = 0.2314,$$
$$m_H = 125.1 \text{ GeV}, \qquad m_t = 173.0 \text{ GeV},$$
$$m_\tau = 1.777 \text{ GeV}, \qquad m_b = 3.99 \text{ GeV},$$
$$m_\mu = 105.7 \text{ MeV}, \qquad m_c = 1.185 \text{ GeV},$$
$$m_e = 0.511 \text{ MeV}, \qquad m_s = 0.342^{+0.048}_{-0.053} \text{ GeV},$$
$$m_{u,d} = 0.246^{+0.054}_{-0.057} \text{ GeV},$$
$$\Delta\alpha = 0.02761_{\text{had}} + 0.0314976_{\text{lep}},$$

Quantity	Contribution ($\times 10^{-3}$)
1–4 $\sin^2\theta_W$	$+74.4$
$\Delta Q^e_{W(1,1)}$	-29.0
$\Delta Q^e_{W(1,0)}$	$+3.1$
$\Delta Q^e_{W(2,2)}$	$-0.18^{+0.0024}_{-0.0040}$
$\Delta Q^e_{W(2,1)}$	$+1.18^{+0.015}_{-0.010}$
$\Delta Q^e_{W(2,0)}$	± 0.13 (estimate)

YD, Freitas, Patel, Ramsey-Musolf, PRL 2021

NNLO Results



(a) (b) (c) (d) (e)

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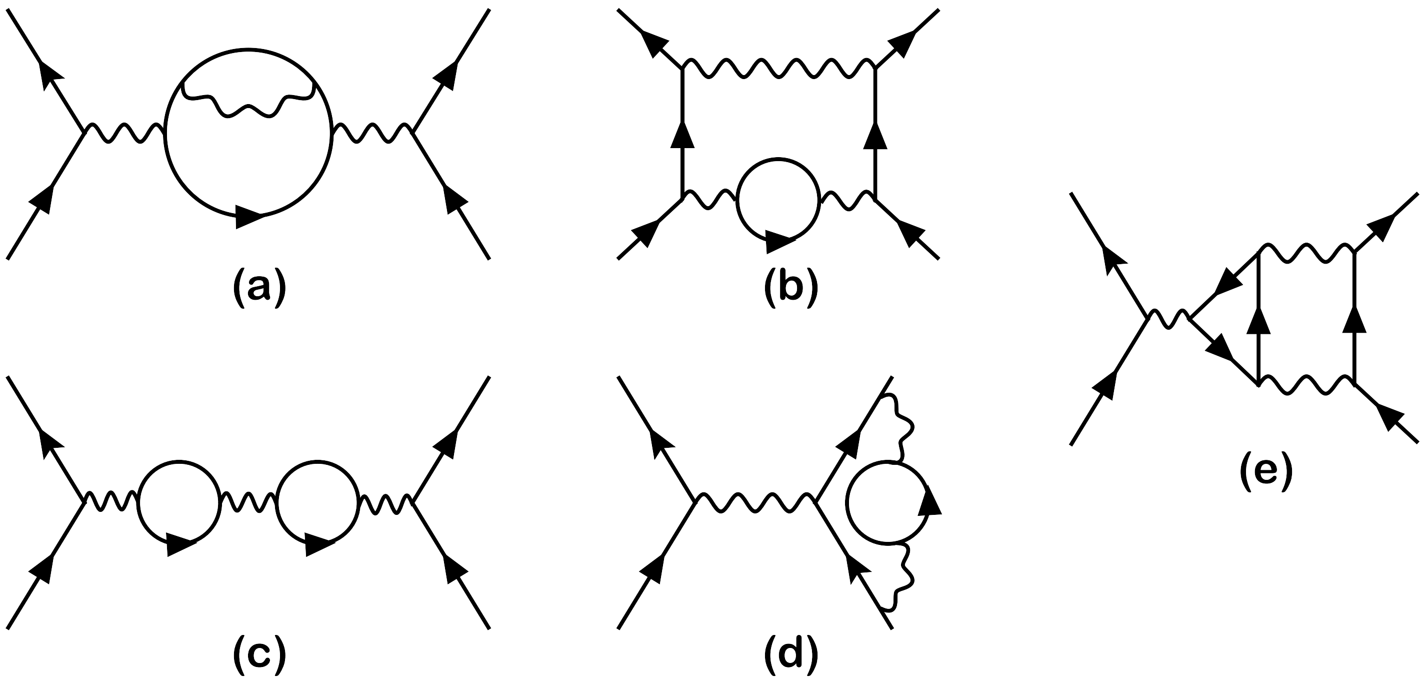
Q: Uncertainties?

NNLO Results



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YD, Freitas, Patel, Ramsey-Musolf, PRL 2021

Q: Uncertainties?

NNLO Results

Our input of $\sin^2\theta_W(m_Z) = 0.2314$ only takes into account LO large logs like $\log m_f^2/m_Z^2$

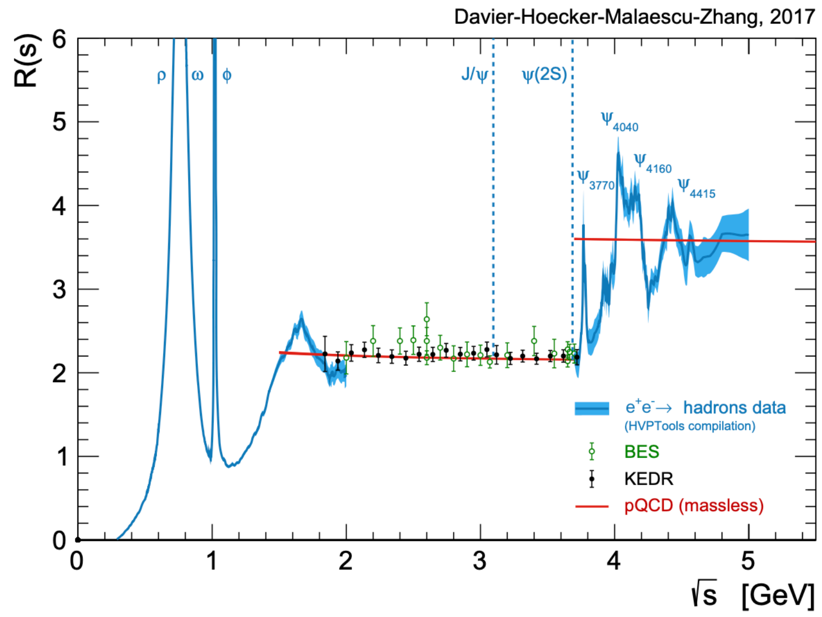
To resume all the large logs, one uses $\sin^2\theta_W(0)$ through the RGE ($\overline{\mathrm{MS}}$ scheme):

Jens & Michael, 2005

$$\hat{s}^2(\mu) = \hat{s}^2(\mu_0) + \frac{\hat{\alpha}(\mu_0)}{\pi}\left(\beta_0\lambda_1 + \frac{\lambda_2}{3} - \beta_0\hat{s}^2(\mu_0)\right)\ln\frac{\mu^2}{\mu_0^2} + \frac{\hat{\alpha}^2(\mu_0)}{\pi^2}[\cdots]$$

Perturbative regime: straightforward computation

Non-Perturbative regime: included in $\hat{\alpha}_{\mathrm{had}}$ from the dispersion relation



Reevaluating A_{PV} with $\sin^2\theta_W(0) = 0.23861$ leads to an error of $\Delta Q_W^e = 0.08\times 10^{-3}$

Erler, Ferro-Hernández, Freitas, 2023

NNLO Results

Parametric uncertainties in quark masses:

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This introduces another error of $\Delta Q_W^e = 0.03 \times 10^{-3}$ when evaluating A_{PV} at $\sin^2\theta_W(0)$. This is further enlarged by a factor 2 to be conservative

Erler, Ferro-Hernández, Freitas, 2023

NNLO Results

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This introduces another error of $\Delta Q_W^e = 0.03 \times 10^{-3}$ when evaluating A_{PV} at $\sin^2\theta_W(0)$. This is further enlarged by a factor 2 to be conservative —- *could be improved by dispersive method and lattice*

Source	$\delta\sin^2\hat{\theta}_W(0) \times 10^5$
LQCD (Mainz)	2.3
pQCD	0.1
Condensates	0.2
Total	2.3

Erler, Ferro-Hernández, Freitas, 2023

Ce et al, 2203.08676

Erler, Ferro-Hernández, Kuberski, PRL, 2024

NNLO Results

Scheme dependence: *trading $\alpha(0)$ with G_μ*, one can estimate the order of NNNLO

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{2\hat{s}_Z^2\hat{c}_Z^2 m_Z^2}(1+\Delta r)$$

Freitas, Hollik, Walter, Weiglein, 2002

Freitas, Hollik, Walter, Weiglein, 2003

Awramik, Czakon, 2003

This uncertainty is then computed by comparing A_{PV} obtained in these two schemes, leading to an error of $\Delta Q_W^e = 0.23 \times 10^{-3}$ (*dominant!*)

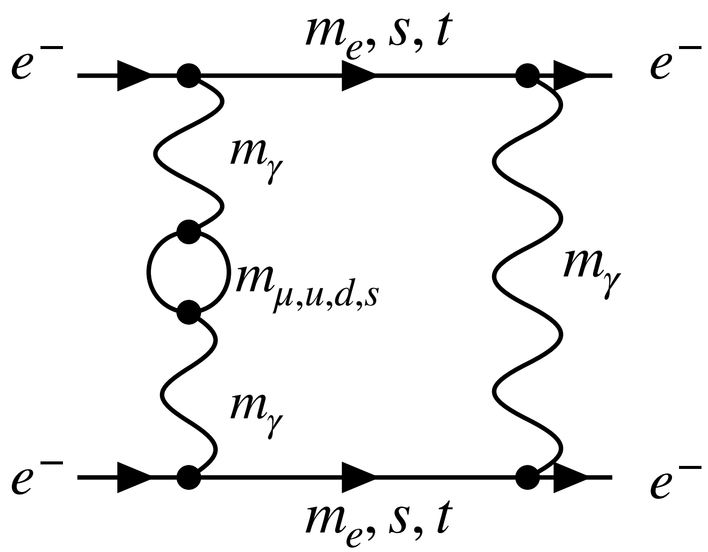
$$\begin{aligned}\Delta Q_W^e &= \left(\pm 0.08_{\hat{s}(0)} \pm 0.06_{\Delta Q^{e,X}_{W(2,1)}(\text{had})} \pm 0.13_{\Delta Q^{e,X}_{W(2,0)}(\text{missing})} \pm 0.23_{\text{scheme}}\right) \times 10^{-3} \\ &= \pm\, 0.28_{\text{theory}} \times 10^{-3}\end{aligned}$$

5 times better than the MOLLER precision goal

Erler, Ferro-Hernández, Freitas, 2023

Finite Q^2 effects?

Is the hierarchical expansion based on $m_f \ll s, t$ justified?



In particular, here $|Q| \simeq 0.1\,\mathrm{GeV} \simeq \mathrm{m}_{\mu,\mathrm{u,d,s}}$

such that $\dfrac{m^2_{\mu,u,d,s}}{s, t}$ *is not small!*

$$A_{\mathrm{PV}} = \frac{G_\mu Q^2}{\sqrt{2}\pi\alpha_{\mathrm{EM}}} \frac{1-y}{1+y^4+(1-y)^4}\left[1 - 4\sin^2\theta_W + F_2(Q^2, y)\right]$$

At NLO, accidental cancellation leads to

$$F_2(Q^2, 0.5) \approx 2 \times 10^{-5}$$

Czarnecki, Marciano, 1996

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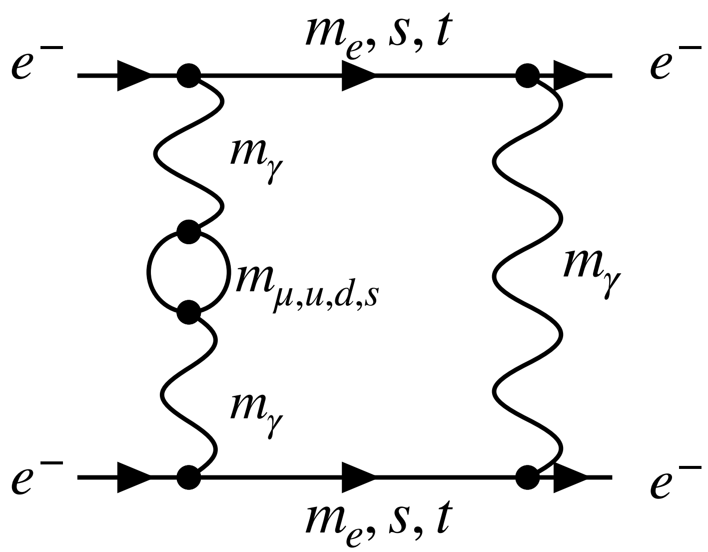

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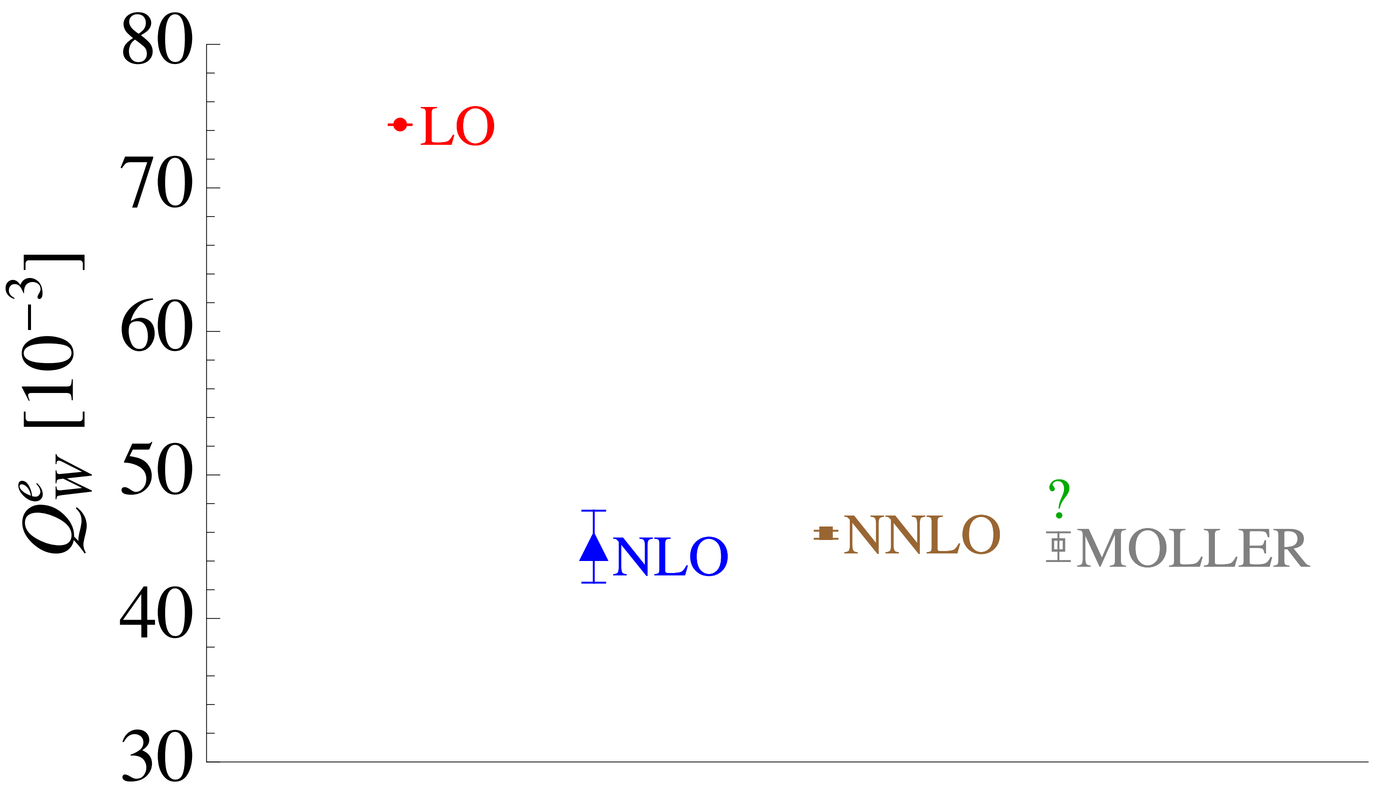
y	$F_2^{(1,1)}$ $[10^{-5}]$	$F_2^{(2,2)}$ $[10^{-5}]$	$F_2^{(2,1)}$ $[10^{-5}]$
0.25 (0.75)	5.01	−0.54	0.00
0.30 (0.70)	4.11	−0.25	−0.18
0.35 (0.65)	3.39	−0.02	−0.29
0.40 (0.60)	2.86	0.15	−0.36
0.45 (0.55)	2.55	0.25	−0.40
0.50	2.44	0.29	−0.41

Erler, Ferro-Hernández, Freitas, 2023

Results summary

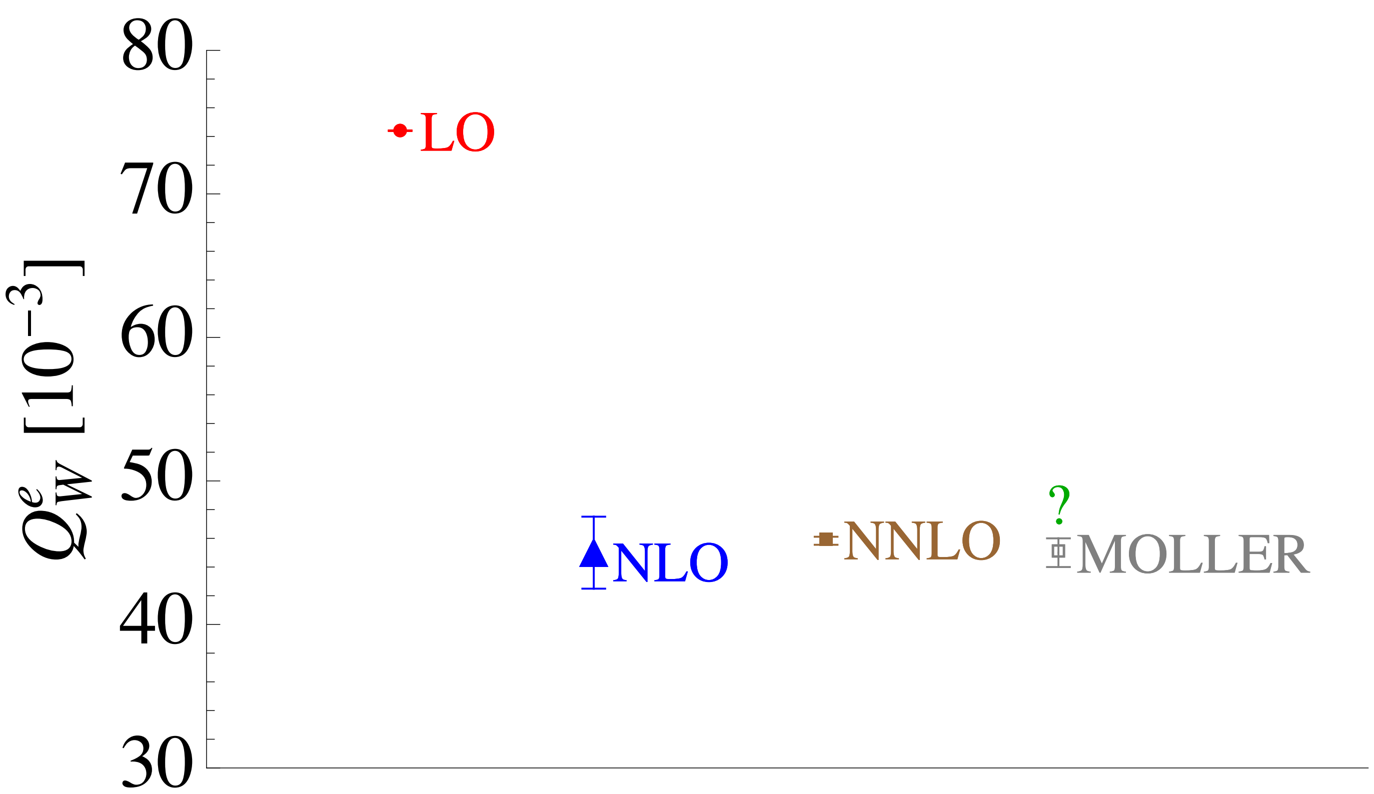


prediction stabilized

theoretical uncertainty under control

catalyzed approval of MOLLER from DOE

Results summary



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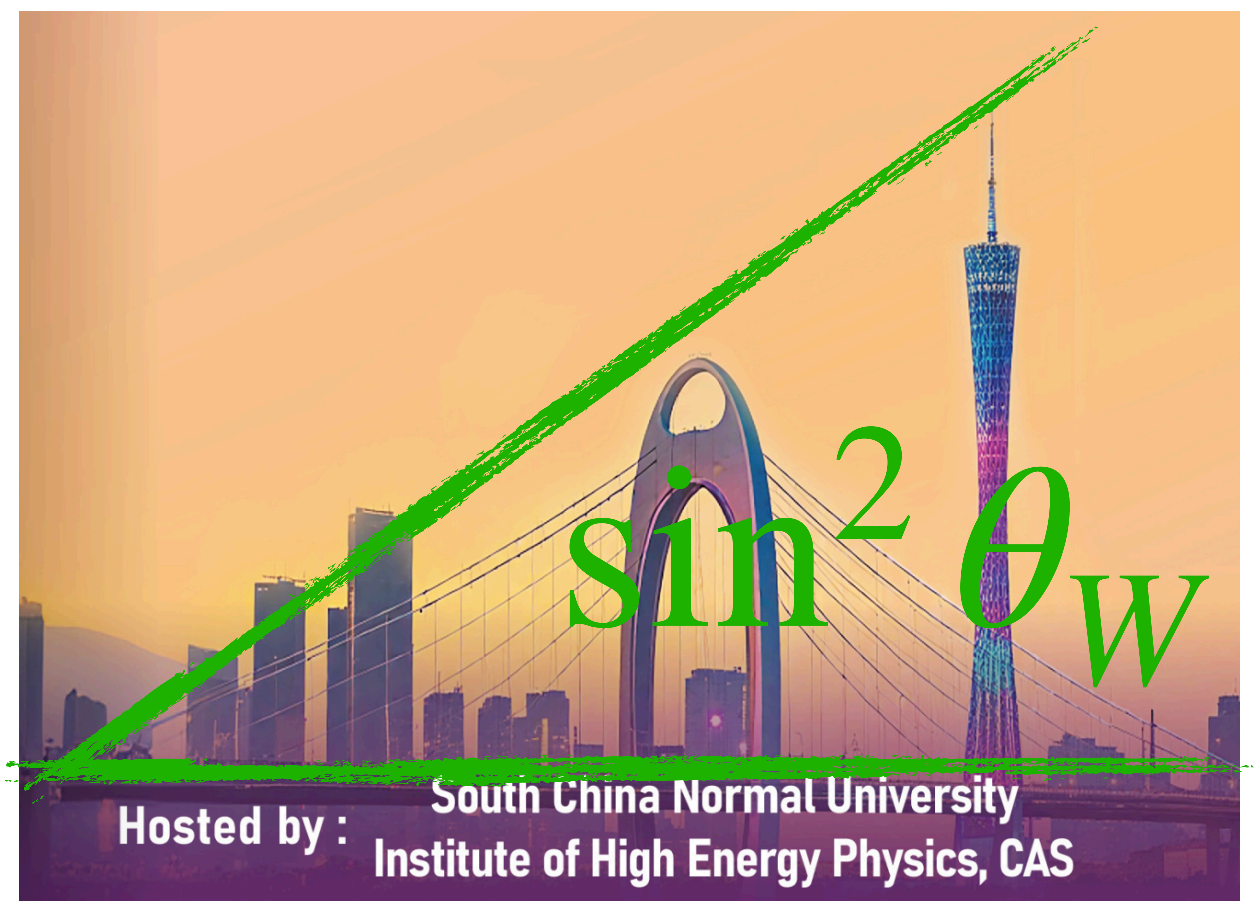
Now time for MOLLER to fit the central value of the electron weak charge

- Engineering Design complete: final technical design report is public
- Acquisitions ongoing and Construction has begun
- Installation: 2025-2026
- Commissioning: Early 2027
- Physics Analysis through 2029 and beyond

Huong Nguyen, SPIN2025

Summary

- MOLLER will measure $\sin^2\theta_W$ at low scale with a precision comparable to $\sin^2\theta_W(m_Z)$
    - *NNLO corrections with closed fermion loop: comparable to exp precision target*
    - *Theoretical uncertainties ~ 1/5 of exp precision target*
    - *Construction at JLab underway, data analysis expected in 2029*
- *Room for surprise from NNLO bosonic corrections? Ongoing efforts, stay tuned.*



Loop calculations are fun

CEPC is another machine besides the Canton Tower (+ more than measuring θ_W)

Postdoc hiring (EFT + precision) ongoing, check out here and here. Very welcome to get in touch or please help spread the news

Backup