NNLO Electroweak Corrections to ZH Production at Future Lepton Colliders

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in collaboration with Ayres Freitas and Keping Xie
based on: 2308.13030, 2305.16547, 2209.07612, 2101.00308

2025 International Workshop on the High Energy Circular Electron Positron Collider, Guangzhou, November 6-10

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Outline

CEPC physics Overview I

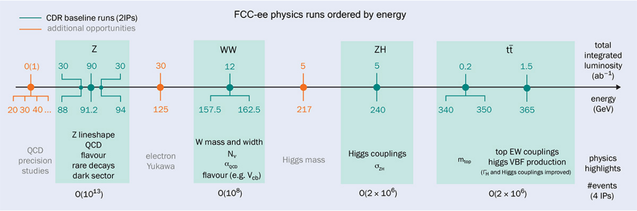
- Good agreement of SM predictions and no direct sign of new physics.
- High precision measurements of Higgs boson properties can hint for scale of new physics. In particular, new physics beyond the SM can lead to observable deviations in Higgs couplings relative to SM expectations.

$$\delta \propto \frac{\nu^2}{M_{\mathrm{NP}}^2}$$

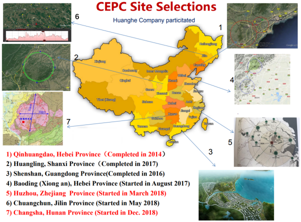
[J. B. Guimarães da Costa *et al.*]
[CEPC Study Group] [arXiv:1811.10545 [hep-ex]]

- Probing new physics lies at TeV level requires measuring Higgs couplings with sub-percent-level accuracy.
- Due to complicated background, it is hard to measure higgs boson properties to the very high precision. Achieving such a level of precision will require new facilities, for which a lepton collider operating as a Higgs factory is a natural candidate.

CEPC physics Overview II



[https://cerncourier.com/a/fcc-the-physics-case/]



[http://cepc.ihep.ac.cn/intro.html]

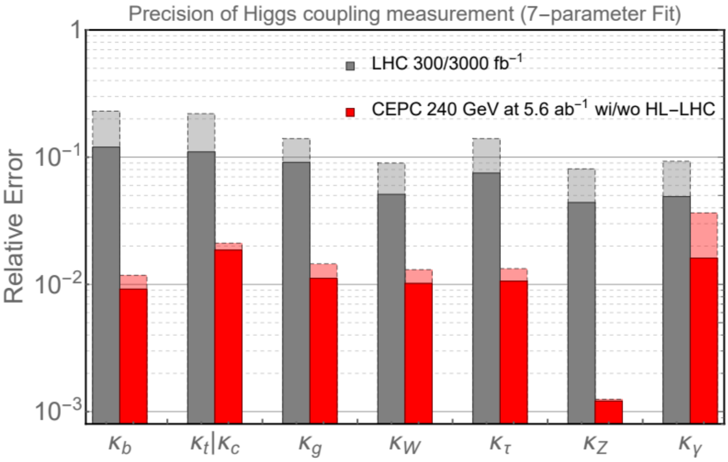
☞ *Jorge de Blas*'s talk

The Circular Electron Positron Collider (CEPC) is a large international scientific facility proposed by the Chinese particle physics community in 2012 to explore the aforementioned physics program. The CEPC, to be hosted in China in a circular underground tunnel of approximately 100 km in circumference, is designed to operate at around 91.2 GeV as a Z factory, at around 160 GeV of the WW production threshold, and at 240 GeV as a Higgs factory. The CEPC will produce close to one trillion Z bosons, 100 million W bosons and over one million Higgs bosons. The vast amount of bottom quarks, charm quarks and τ-leptons produced in the decays of the Z bosons also makes the CEPC an effective B-factory and τ-charm factory. The CEPC offers an unmatched opportunity for precision measurements and searches for BSM physics.

[J. B. Guimarães da Costa *et al.*]
[CEPC Study Group] [arXiv:1811.10545 [hep-ex]]

CEPC physics Overview III

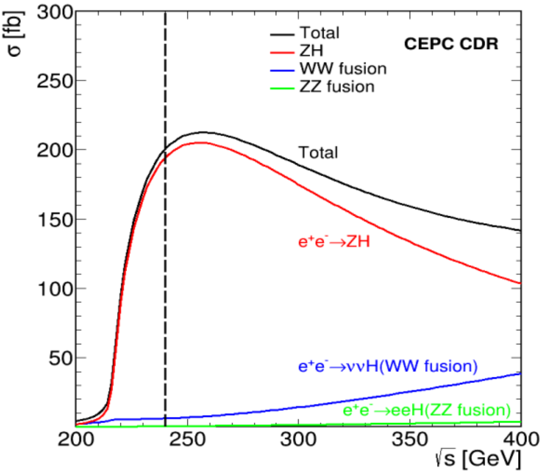
[J. B. Guimarães da Costa *et al.*]
[CEPC Study Group] [arXiv:1811.10545 [hep-ex]]

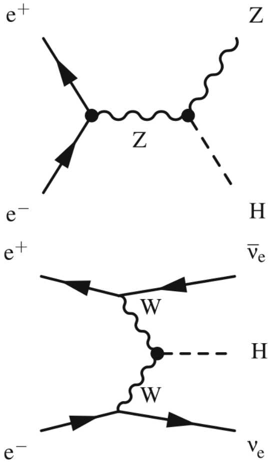


$$\kappa_X = \frac{g_{HXX}}{g_{HXX}^{\mathrm{SM}}}$$

[S. Dawson, A. Gritsan, H. Logan, J. Qian, C. Tully, R. Van Kooten, A. Ajaib, A. Anastassov, I. Anderson and D. Asner, *et al.* [arXiv:1310.8361 [hep-ex]].]

Future ee collider: ZH production I





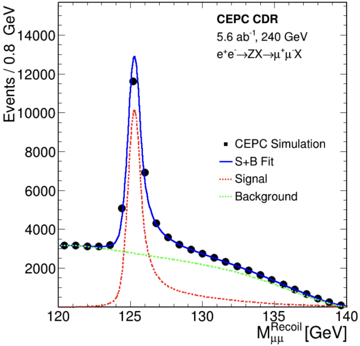
Maximum events at $\sqrt{s} = 240\text{GeV}$

[J. B. Guimarães da Costa *et al.*]
[CEPC Study Group] [arXiv:1811.10545 [hep-ex]]

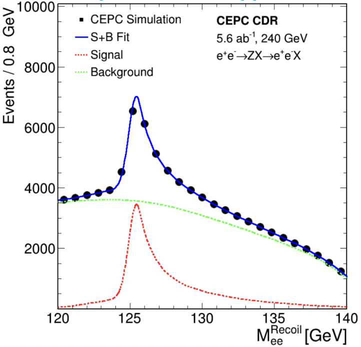
Future ee collider: ZH production II

- inclusive measurement: $\delta\sigma_{\mathrm{ZH}} = 0.5\%,\ \delta m_H = 5.9\mathrm{MeV}$
- combine with exclusive: $\delta\Gamma_H = 3.1\%, \delta\mathrm{Br}(H \to cc) = 3.3\%, \cdots$
- Higgs couplings(shown before)
- ...

$M_{\mathrm{ff}}^{\mathrm{Recoil}} = s - 2E_{\mathrm{ff}}\sqrt{s} + m_{\mathrm{ff}}^2$



[J. B. Guimarães da Costa *et al.*]
[CEPC Study Group] [arXiv:1811.10545 [hep-ex]]



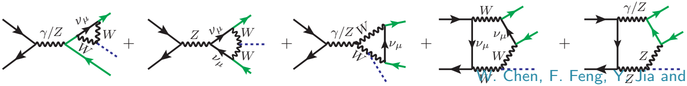
Theoretical Prediction for ZH I

Theoretical prediction for $\mu^+\mu^- H$, the matrix element = signal + background (without Z resonance)

A. Freitas and Q. Song, Phys. Rev. Lett. **130**, no.3, 031801 (2023)

$$\mathcal{M}_{e^+e^-\to\mu^+\mu^- H} = \Gamma_{\rm prod}\frac{1}{p_z^2 - m_Z^2 + \Sigma_Z(p_z^2)}\Gamma_{\rm dec} + \mathcal{M}_{\rm bkgd}$$
$$= \frac{1}{p_z^2 - s_0}A(s_0) + B(s_0) + (p_z^2 - s_0)C(s_0) + \cdots$$
$$\overset{\Gamma_Z \ll m_Z}{=} \frac{1}{p_z^2 - s_0}A(m_z^2) + \frac{p_z^2 - m_z^2}{p_z^2 - s_0}B(m_Z^2) + \mathcal{O}(p_z^2 - m_Z^2, \Gamma_Z)$$

The experimental analysis will select $\mu^+\mu^-$ pairs with $p_z^2 \sim m_Z^2$, series decrease in numerical magnitude. If one seeks NNLO for leading A term, NLO precision is sufficient for B term.



nonresonant $\mu^+\mu^-$ production

W. Chen, F. Feng, Y. Jia and W. L. Sang, Chin. Phys. C **43**, no.1, 013108 (2019)

Theoretical Prediction for ZH II

The differential cross-section for leading A term can be expressed as

$$\frac{d^2\sigma_{\mu\mu H}}{d\cos\theta\, dp_z^2} = \frac{d\sigma_{\mathrm{ZH}}}{d\cos\theta}\frac{\pi^{-1} m_Z \Gamma_{Z\to\mu\mu}}{(p_z^2 - m_z^2)^2 + m_z^2\Gamma_Z^2}$$

As a first step, calculate the differential cross section for ZH production. Expand matrix element perturbatively with respect to EW coupling α, and QCD coupling α_s

$$\begin{aligned}\frac{d\sigma_{\mathrm{ZH}}}{d\cos\theta} &= \frac{1}{32\pi s}\beta(1, \frac{m_Z^2}{s}, \frac{m_H^2}{s})\left((M^{(0)} + M^{(\alpha)} + M^{(\alpha\alpha_s)} + M^{(\alpha^2)}) \times \mathrm{C.C.}\right) \\ &= \frac{1}{32\pi s}\beta(1, \frac{m_Z^2}{s}, \frac{m_H^2}{s})\Bigg(\underbrace{|M^{(0)}|^2}_{\text{LO}} + \underbrace{2\mathrm{Re}(M^{(0)*}M^{(\alpha)})}_{\text{NLO}} \\ &\quad + \underbrace{2\mathrm{Re}(M^{(0)*}M^{(\alpha\alpha_s)})}_{\text{NNLO(EW+QCD)}} + \underbrace{|M^{(\alpha)}|^2 + 2\mathrm{Re}(M^{(0)*}M^{(\alpha^2)})}_{\text{NNLO(EW+EW)}}\Bigg) + \cdots\end{aligned}$$

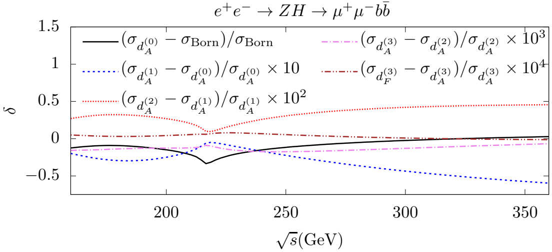
[B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. D **16** (1977), 1519]

Theoretical Prediction for ZH III

Contribution of ISR of soft and collinear photons can be evaluated using

M. Greco, G. Montagna, O. Nicrosini, F. Piccinini and G. Volpi, Phys. Lett. B **777**, 294-297 (2018)

$$d\sigma(s) = \int dx_1 dx_2 D(x_1, s) D(x_2, s) d\sigma_0(x_1 x_2 s) \Theta(\mathrm{cuts})$$

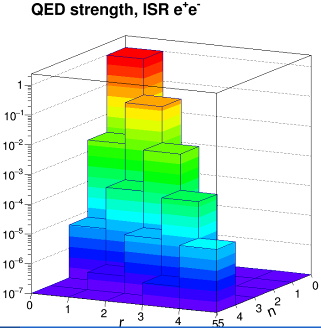


☞ *Andrej Arbuzov*'s talk

effective strength of QED corrections for $e^{\pm}$ beam at the Z peak:

$$\gamma_{nr} = \left(\frac{\alpha}{\pi}\right)^n \left(2 \ln \frac{M_Z^2}{m_f^2}\right)^r, \; 0 \le r \le n$$

S. Jadach and M. Skrzypek, Eur. Phys. J. C **79**, no.9, 756 (2019)



Theoretical Prediction for ZH IV

S. Jadach and M. Skrzypek,
Eur. Phys. J. C **79**, no.9, 756 (2019)

The important message to theorists specialising in QED+EW multiloop calculations is the following: *do not add soft real emissions to multiloop results in order to eliminate infrared singularities á la Bloch-Nordsieck*, if you want these results to be used in the MC generators with IR resummation. Instead, you should subtract IR parts (YFS virtual formfactor) from the amplitudes, before squaring and spin summing[3]. Why? Because combining IR soft and real contributios and the differential cross section level is already done in the Monte Carlo.

Theoretical Prediction for ZH V

NLO:

unpolarized beam 5-10%;

J. Fleischer and F. Jegerlehner,
Nucl. Phys. B **216**, 469-492 (1983);
B. A. Kniehl, Z. Phys. C **55**, 605-618 (1992);
A. Denner, J. Kublbeck, R. Mertig and M. Bohm,
Z. Phys. C **56**, 261-272 (1992)

polarized beam 10-20%

S. Bondarenko, Y. Dydyshka, L. Kalinovskaya,
L. Rumyantsev, R. Sadykov and V. Yermolchyk,
Phys. Rev. D **100**, no.7, 073002 (2019);

NNLO(EW+QCD):

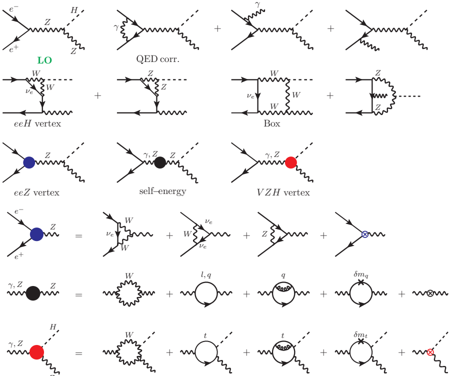
0.4-1.3% with $\{\alpha(0), \alpha(m_Z), G_\mu\}$

Y. Gong, Z. Li, X. Xu, L. L. Yang and X. Zhao
Phys. Rev. D **95**, no.9, 093003 (2017)

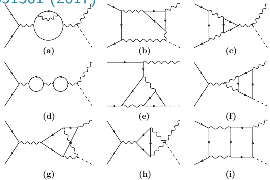
1.3% with $\{\overline{\mathrm{MS}}, \alpha(m_Z)\}$

Q. F. Sun, F. Feng, Y. Jia and W. L. Sang,
Phys. Rev. D **96**, no.5, 051301 (2017)

largest missing correction:
NNLO(EW+EW),
fermionic corrections dominate

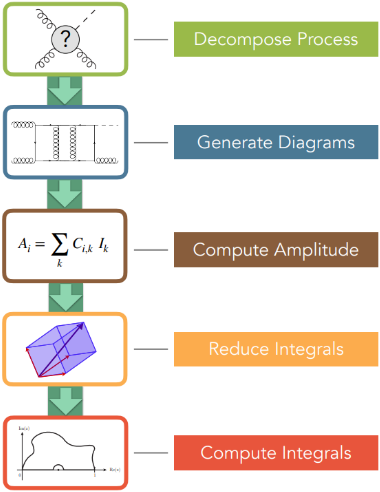


Q. F. Sun, F. Feng, Y. Jia and W. L. Sang, Phys. Rev. D **96**, no
051301 (2017)



A. Freitas and Q. Song, Phys. Rev. Lett. **130**, no.3, 031801 (202

Theoretical Calculation: Overview I



[Stephen Jones @ DESY Theory Workshop 2025]

FeynArts, QGRAF ...

[T. Hahn, hep-ph/0012260;
J. Küblbeck and M. Böhm and A. Denner, Comput. Phys. Commun. **60**, 165-180 (1990); ...]

FeynRules, LanHEP, CalcHEP ...

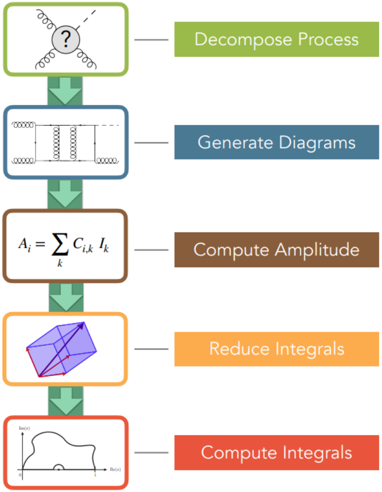
[N. D. Christensen and C. Duhr, Comput. Phys. Commun. **180**, 1614-1641 (2009)
A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, Comput. Phys. Commun. **185**, 2250-2300 (2014)
A. Semenov, Comput. Phys. Commun. **201**, 167-170 (2016)
A. Belyaev, N. D. Christensen and A. Pukhov, Comput. Phys. Commun. **184**, 1729-1769 (2013); ...]

⚠The number of diagrams grows fast with loop numbers. Be aware of memory issue!

LO	NLO	NNLO	N^3LO
1	<100	25377	$\mathcal{O}(10^6)$?

[Z. Li, Y. Wang and Q. f. Wu, Chin. Phys. C **45** (2021) no.5, 053102]

Theoretical Calculation: Overview II



[Stephen Jones @ DESY Theory Workshop 2025]

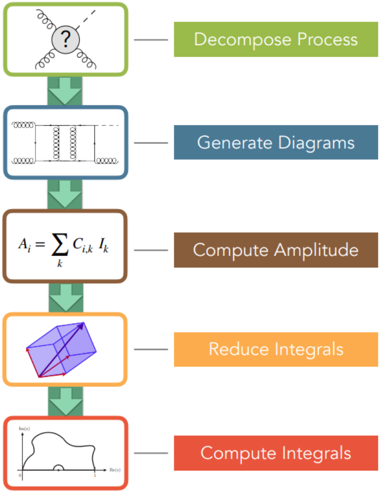
Reduce Integrals with *Laporta algorithm* [S. Laporta]

- generate a large set of IBP relations through total derivative(trivial)
- solve for MI coefficients with Gaussian elimination(bottleneck)

This method has been very successful and implemented into many public packages, such as *Kira*, *FIRE*, *LiteRed*, *Blade* ...

[P. Maierhöfer, J. Usovitsch and P. Uwer]
[J. Klappert, F. Lange, P. Maierhöfer and J. Usovitsch]
[A. V. Smirnov,]
[A. V. Smirnov and F. S. Chukharev]
[A. V. Smirnov and M. Zeng]
[R. N. Lee]
[X. Guan, X. Liu, Y. Q. Ma and W. H. Wu]

Theoretical Calculation: Overview III



[Stephen Jones @ DESY Theory Workshop 2025]

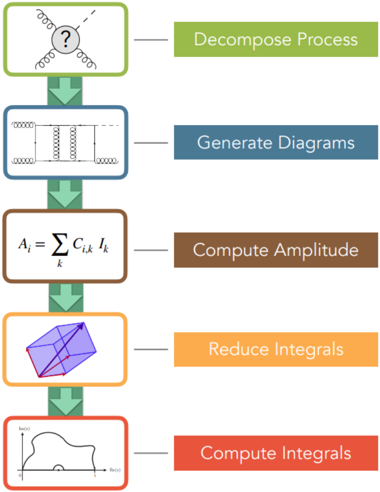
The main bottleneck of IBP reduction is the memory usage and time-consuming due to huge size of IBP relations. For multiloop calculations up to 2022, the start-of-art calculation on IBP reduction took a few hundred thousand CPU core hours and 2TB memory.

One way to improve IBP reduction is to target on smaller but complete set of IBP relations, which can be realized by searching for block-triangular form of IBPs, machine-learning technique, syzygy-based approach ...

[F. Febres Cordero, A. von Manteuffel and T. Neumann, Comput. Softw. Big Sci. **6** (2022) no.1, 14]

[X. Guan,X. Liu, Y. Q. Ma and W. H. Wu] [Z. Y. Song, T. Z. Yang, Q. H. Cao, M. X. Luo and H. X. Zhu] [M. von Hippel and M. Wilhelm] [M. Zeng] [J. Gluza, K. Kajda, D. A. Kosower] [K. J. Larsen, Y. Zhang] [R. Schabinger] [D. Cabarcas, J. Ding] [B. Agarwal, S. P. Jones, A. von Manteuffel] [S. Abreu, G. De Laurentis, H. Ita, M. Klinkert, B. Page, V. Sotnikov] [Z. Wu, J. Boehm, R. Ma, H. Xu and Y. Zhang] [B. Page and Q. Song,]...

Theoretical Calculation: Overview IV



[Stephen Jones @ DESY Theory Workshop 2025]

Feynman diagram is expressed as a linear combination of MIs, whose coefficients are often rational functions in kinematic variables and space-time dimension. To approach analytical solution of amplitude, we need to

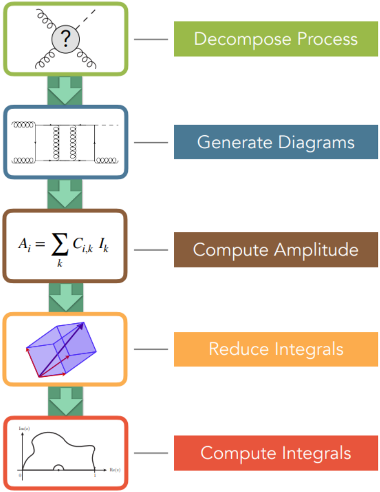
- reconstruct these coefficients (ansatz)

R^7	#unknowns	memory	time
deg = 10	$8k$	$512Mb$	$90s$
deg = 30	$2m$	$32Tb$	$40y$
deg = 61	$1b$	$20^6 Tb$	$10^6 y$

[calculated in finite field with 32-bit prime in 32-core machine]

- identify special functions of MIs
- implement numerical algorithms to evaluate special functions
- fast and stable numerical algorithms for analytical result ⚠numerical cancellations

Theoretical Calculation: Overview V



[Stephen Jones @ DESY Theory Workshop 2025]

For phenomenological usage, numerical approach is more reasonable,. In this framework, approaching numerical result requires

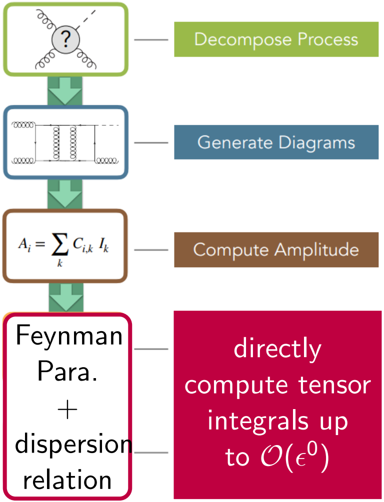
- evaluation of MI coefficients at numerical phase-space points

  which is straight forward, but remember memory usage and time cost

- numerical evaluation of MIs

  Differential equation technique for ZH production
  [X. Chen, X. Guan, C. Q. He, Z. Li, X. Liu and Y. Q. Ma, [arXiv:2209.14953 [hep-ph]].]

Theoretical Calculation: Overview VI



[Stephen Jones @ DESY Theory Workshop 2025; Modified]

In the framework of Feynman parametrization, dispersion relation (and suitable subtraction terms), IBP reduction on two-loop integrals is no longer needed.

The basic idea is to convert two-loop Feynman integral into at-most three fold numerical integral, where the integration function is **(derivative of) one-loop bubble integral times general one-loop tensor integral**(or one-loop MIs if IBP reduction is performed). The numerical integration can be computed with Gauss-Kronrod quadrature method.

$\Rightarrow$ low requirement on PC; use single core CPU.

[N. Agrawal *et al.*, "Boost Math Toolkit 2.13.0," www.boost.org/doc/libs/master/libs/math/doc/html/index.
R. Piessens, E. de Doncker-Kapenga, C. W. Überhuber, D. K. Ka
"QUADPACK, A Subroutine Package for Automatic Integration,
Springer," Berlin (1983).]

Theoretical Calculation: Overview VII

Before going into calculation details, it is worth mentioning that dispersion relation has been implemented into many two and three loop examples:

- analytical Two-loop self-energies
  [S. Bauberger, F. A. Berends, M. Bohm and M. Buza; Nucl. Phys. B **434**, 383-407 (1995)]
  [S. Bauberger and M. Bohm, Nucl. Phys. B **445**, 25-48 (1995)]
- three-loop self-energies
  S. Bauberger and A. Freitas; [arXiv:1702.02996 [hep-ph]].
  S. Bauberger, A. Freitas and D. Wiegand, JHEP **01**, 024 (2020)
- two-loop corrections to electroweak mixing angle
  M. Awramik, M. Czakon and A. Freitas, JHEP **11**, 048 (2006)
  M. Awramik, M. Czakon, A. Freitas and B. A. Kniehl, Nucl. Phys. B **813**, 174-187 (2009) [arXiv:0811.1364 [hep-ph]].
  I. Dubovyk, A. Freitas, J. Gluza, T. Riemann and J. Usovitsch, Phys. Lett. B **762**, 184-189 (2016) [arXiv:1607.08375 [hep-ph]].
- three-loop corrections to electroweak precision observables
  [L. Chen and A. Freitas, JHEP **07**, 210 (2020)]
  [L. Chen and A. Freitas, JHEP **03** (2021), 215]
- Two-loop EW corrections to Parity-Violating Møller Scattering
  [Y. Du, A. Freitas, H. H. Patel and M. J. Ramsey-Musolf, Phys. Rev. Lett. **126**, no.13, 131801 (2021)].
- Two-loop EW corrections to ZH production
  [Q. Song and A. Freitas, JHEP **04**, 179 (2021)
  A. Freitas and Q. Song, Phys. Rev. Lett. **130**, no.3, 031801 (2023)
  A. Freitas, Q. Song and K. Xie, Phys. Rev. D **108**, no.5, 053006 (2023)]
- ...

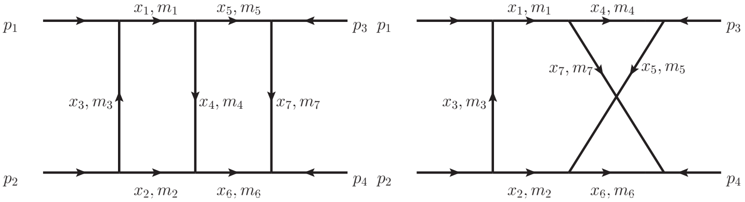
Calculation Details I

This planar double-box integral is converted into 6-fold numerical integral if purely using Feynman parametrization

$$I_{\text{plan}} = -\int_0^1 d\rho \int_0^1 d\xi \int_0^1 du_1 \int_0^{1-u_1} du_2 \int_0^1 du_3 \int_0^{1-u_3} du_4 \frac{\mathcal{C}}{(\mathcal{D} - i\epsilon\mathcal{C})^3} \rho^3 \xi^2 (1-\xi)^2$$

iterated Gaussian quadrature; 0.01% accuracy for integral with 4 parameters, $\{s, t, m, M\}$; It takes few days because integrand converges slowly

[F. Yuasa, E. de Doncker, N. Hamaguchi, T. Ishikawa, K. Kato, Y. Kurihara, J. Fujimoto and Y. Shimizu, Comput. Phys. Commun. **183** (2012), 2136-2144]

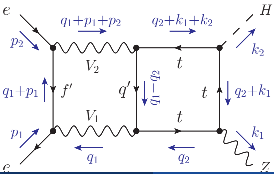


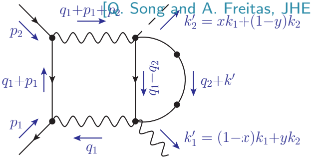
Calculation Details II

$$
\begin{aligned}
I_{\text{plan}} &= \int d^D q_1 d^D q_2 \frac{1}{[q_1^2 - m_{V_1}^2][(q_1+p_1)^2 - m_{f'}^2][(q_1+p_1+p_2)^2 - m_{V_2}^2]} \\
&\times \frac{1}{[(q_1-q_2)^2 - m_{q'}^2][q_2^2 - m_t^2][(q_2+k_1)^2 - m_t^2][(q_2+k_1+k_2)^2 - m_t^2]} . \\
&= -\int dx\, dy \left\{ \int_{\sigma_0}^{\infty} d\sigma\, \partial^2_{m'} \Delta B_0(\sigma, m'^2, m_{q'}^2) \times \Big[D_0(p_1^2, p_2^2, k_2'^2, k_1'^2, s, t', m_{V_1}^2, m_{f'}^2, m_{V_2}^2, \sigma) \right. \\
&\quad - \frac{\sigma_0}{\sigma} D_0(p_1^2, p_2^2, k_2'^2, k_1'^2, s, t', m_{V_1}^2, m_{f'}^2, m_{V_2}^2, \sigma_0) \Big] \\
&\quad \left. + \sigma_0\, \partial^2_{m'} B_0(0, m'^2, m_{q'}^2)\, D_0(p_1^2, p_2^2, k_2'^2, k_1'^2, s, t', m_{V_1}^2, m_{f'}^2, m_{V_2}^2, \sigma_0) \right\},
\end{aligned}
$$

iterated Gaussian quadrature; 0.1% precision for amplitude(numerical cancellation); most complicated integral contains 7 parameters, $\{s, t, m_Z, m_H, m_{V_1}, m_{V_2}, m_t\}$; takes a few minutes.

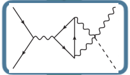
[Q. Song and A. Freitas, JHEP **04** (2021), 179]





Calculation Details III

framework-1



Generate Diagrams

$$A_i = \sum_k C_{i,k}\, I_k$$

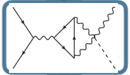
Compute Amplitude

Feynman Para.
+
dispersion relation

directly compute tensor integrals up to $\mathcal{O}(\epsilon^0)$

[Stephen Jones @ DESY Theory Workshop 2025: Modified]

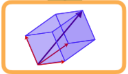
framework-2



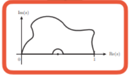
Generate Diagrams

$$A_i = \sum_k C_{i,k}\, I_k$$

Compute Amplitude



Reduce Integrals



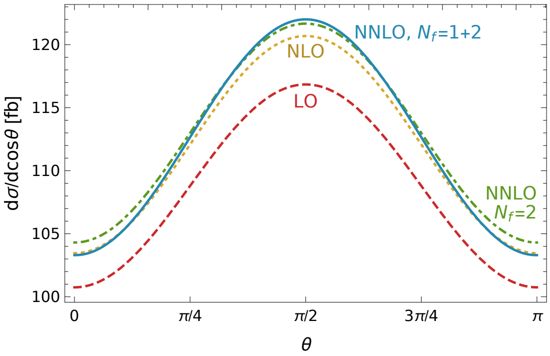
Compute Integrals

[Stephen Jones @ DESY Theory Workshop 2025]

Numerical Result I

The following input parameters are used for the numerical evaluation:

$m_W = 80.352$ GeV, $m_Z = 91.1535$ GeV, $m_H = 125.1$ GeV, $m_t = 172.76$ GeV,
$\alpha^{-1} = 137.036, \Delta\alpha = 0.059, \sqrt{s} = 240$ GeV.



[A. Freitas and Q. Song, Phys. Rev. Lett. **130** (2023) no.3, 031801]

Numerical Result II

Table: Numerical results for the integrated cross section at LO, NLO and NNLO. Electroweak one-loop and two-loop corrections are also provided and divided according to the number of fermion loops symbolized as N_f.

	(fb)	Contribution	(fb)
σ^{LO}	222.958		
σ^{NLO}	229.893		
		$\mathcal{O}(\alpha_{N_f=1})$	21.130
		$\mathcal{O}(\alpha_{N_f=0})$	-14.195
σ^{NNLO}	231.546		
		$\mathcal{O}(\alpha^2_{N_f=2})$	1.881
		$\mathcal{O}(\alpha^2_{N_f=1})$	-0.226

[A. Freitas and Q. Song, Phys. Rev. Lett. **130** (2023) no.3, 031801]

Numerical Result III

NNLO(EW+EW) corrections are found to increase the NLO cross-section by 0.7%. Missing bosonic corrections $0.1 - 0.3\%$ are lower than the anticipated experimental precision (0.4–1%), but a direct calculation of these missing contributions is still desirable.

Table: Numerical results for the unpolarized integrated ZH production cross section, in fb, for two different renormalization schemes.

	$\alpha(0)$ scheme	G_μ scheme	scheme dependence
σ^{LO} [fb]	223.14	239.64	16.50
σ^{NLO} [fb]	229.78	232.46	2.68
$\sigma^{\mathrm{NNLO,EW\times QCD}}$ [fb]	232.21	233.29	1.08
$\sigma^{\mathrm{NNLO,EW}}$ [fb]	233.86	233.98	0.12

[A. Freitas, Q. Song and K. Xie, Phys. Rev. D **108** (2023) no.5, 053006]

Probing Dark Sector Fermions I

goal: consider the impact of physics beyond SM on σ_{ZH}. The high precision measurement can be used in turn to constrain physics beyond the SM.

Our focus is on the Higgs portal with fermionic dark matter, which is one of the simplified dark matter models: two UV complete models that extend the SM with vector fermions: the singlet-doublet model and the doublet-triplet model. Both models contains two fermion multiplets to form a Yukawa interaction term.

Probing Dark Sector Fermions II

models	gauge states	mass states	free parameters
DSDM	$\chi_S = \chi_S^0, \chi_D = \begin{pmatrix} \chi_D^+ \\ \chi_D^0 \end{pmatrix}$	$\chi_{h,l}^0, \chi^+$	$y, m_l^0, \Delta m_{\rm hl} = m_h^0 - m_l^0$
MSDM	$\chi_S = \chi_S^0, \chi_D = \begin{pmatrix} \chi_D^+ \\ \chi_D^0 \end{pmatrix}$	$\chi_{h,l}^0, \chi_D^0, \chi_D^+$	$y, m_l^0, \Delta m_{\rm hl} = m_h^0 - m_l^0$
DDTM1	$\chi_D = \begin{pmatrix} \chi_D^0 \\ \chi_D^- \end{pmatrix}, \chi_T = \begin{pmatrix} \chi_T^0 \\ \chi_T^- \\ \chi_T^{--} \end{pmatrix}$	$\chi_{h,l}^0, \chi_{h,l}^-, \chi^{--}$	$y, m_l^0, \Delta m_{\rm ll} = m_l^+ - m_l^0$
DDTM0	$\chi_D = \begin{pmatrix} \chi_D^+ \\ \chi_D^0 \end{pmatrix}, \chi_T = \begin{pmatrix} \chi_T^+ \\ \chi_T^0 \\ \chi_T^{\prime -} \end{pmatrix}$	$\chi_{h,l}^0, \chi_{h,l}^+, \chi^-$	$y, m_l^0, \Delta m_{\rm hl} = m_h^0 - m_l^0$
MDTM	$\chi_D = \begin{pmatrix} \chi_D^+ \\ \chi_D^0 \end{pmatrix}, \chi_T = \begin{pmatrix} \chi_T^+ \\ \chi_T^0 \\ \chi_T^- \end{pmatrix}$	$\chi_l^{0,\pm}, \chi_h^{0,\pm}, \chi_m^0$	$y, m_l^0, \Delta m_{\rm hl} = m_h^0 - m_l^0$

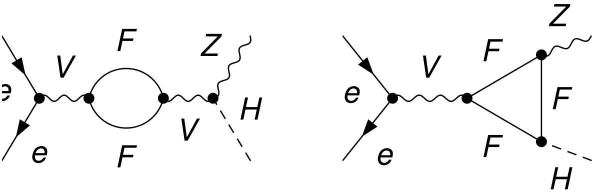
[Q. Song, "Radiative Corrections for ZH Production at Electron-Positron Collider,"]

Probing Dark Sector Fermions III

New fermions contributes to σ_{ZH} through self-energy and vertex contributions, thus causes deviation

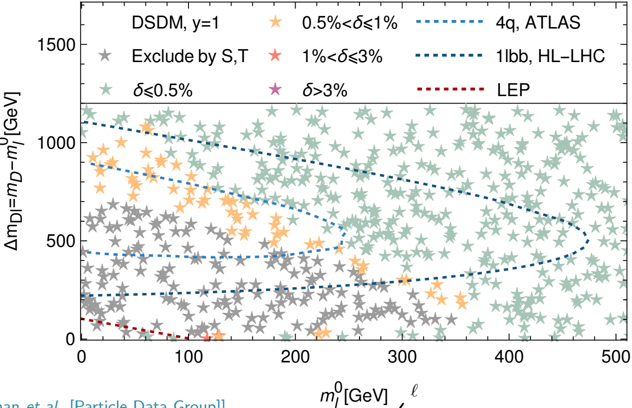
$$\delta = \frac{\sigma^{\mathrm{FDM}}(e^+e^- \to ZH)}{\sigma^{\mathrm{SM}}(e^+e^- \to ZH)},$$

where σ^{FDM} considers the contribution from new fermions only. The Standard Model contribution, $\sigma^{\mathrm{SM}}(e^+e^- \to ZH)$, takes into account one-loop EW as well as fermionic two-loop electroweak corrections.

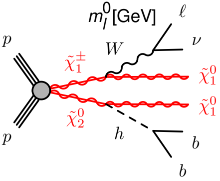


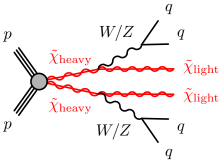
Probing Dark Sector Fermions IV

[A. Freitas and Q. Song, JHEP **01** (2024), 137]



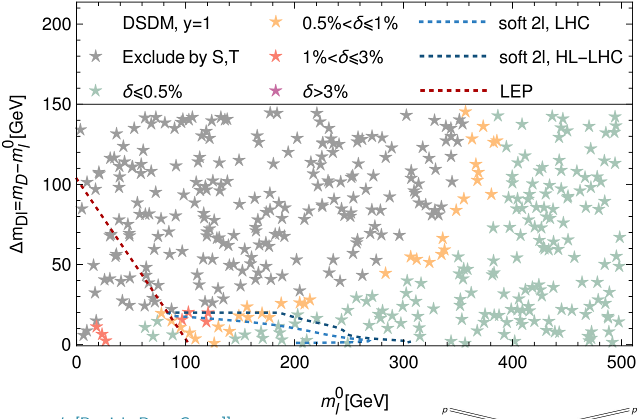
[R. L. Workman *et al.* [Particle Data Group]]
[ATL-PHYS-PUB-2018-048]
[Phys. Rev. D **104**, no.11, 112010 (2021)]
[$m_{\chi^+} > 103.5\,\mathrm{GeV}$ from LEP
J. Abdallah *et al.* DELPHI]



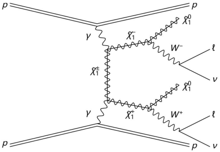


Probing Dark Sector Fermions V

[A. Freitas and Q. Song, JHEP **01** (2024), 137]

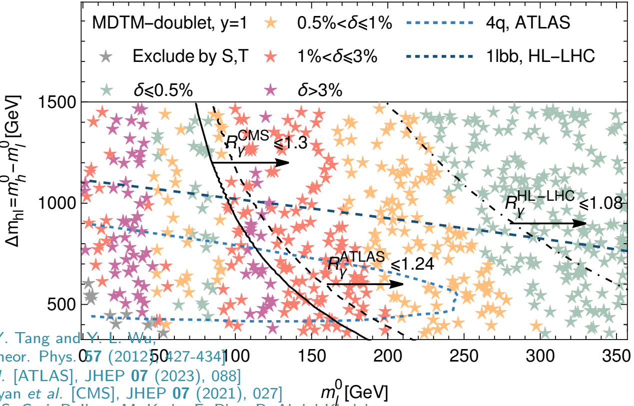


[R. L. Workman *et al.* [Particle Data Group]]
[H. Zhou and N. Liu, JHEP **10**, 092 (2022)]]
[$m_{\chi^+} > 103.5\mathrm{GeV}$ from LEP:
J. Abdallah *et al.* DELPHI]



Probing Dark Sector Fermions VI

[A. Freitas and Q. Song, JHEP **01** (2024), 137]

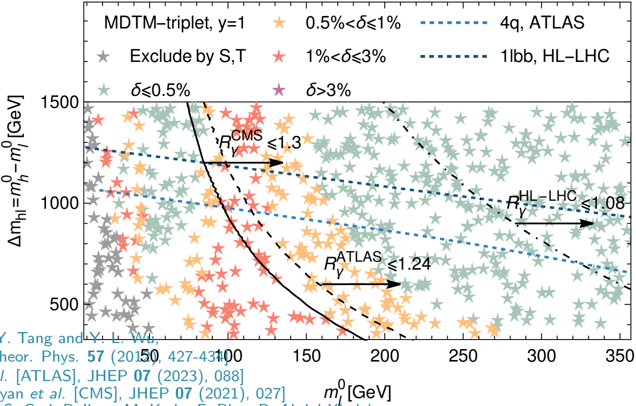


[D. Huang, Y. Tang and Y. L. Wu, Commun. Theor. Phys. **57** (2012), 427-434]
[G. Aad *et al.* [ATLAS], JHEP **07** (2023), 088]
[A. M. Sirunyan *et al.* [CMS], JHEP **07** (2021), 027]
[M. Cepeda, S. Gori, P. Ilten, M. Kado, F. Riva, R. Abdul Khalek, A. Aboubrahim, J. Alimena, S. Alioli and A. Alves, *et al.* CERN Yellow Rep. Monogr. **7** (2019), 221-584]
[ATL-PHYS-PUB-2018-048]
[Phys. Rev. D **104**, no.11, 112010 (2021)]
[m_{χ^+} > 103.5GeV from LEP
J. Abdallah *et al.* DELPHI]

$$\begin{pmatrix} \chi_h^0 \\ \chi_m^0 \\ \chi_l^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & \sin\theta_2/\sqrt{2} & -\sin\theta_2/\sqrt{2} \\ 0 & i/\sqrt{2} & i/\sqrt{2} \\ \sin\theta_2 & -\cos\theta_2/\sqrt{2} & \cos\theta_2/\sqrt{2} \end{pmatrix} \begin{pmatrix} \chi_T^0 \\ \chi_D^{c0} \\ \chi_D^0 \end{pmatrix}, \; R_\gamma = \frac{\Gamma(h\to\gamma\gamma)}{\Gamma_{SM}(h\to\gamma\gamma)}$$

Probing Dark Sector Fermions VII

[A. Freitas and Q. Song, JHEP **01** (2024), 137]



[D. Huang, Y. Tang and Y. L. Wu, Commun. Theor. Phys. **57** (2012), 427-434]
[G. Aad *et al.* [ATLAS], JHEP **07** (2023), 088]
[A. M. Sirunyan *et al.* [CMS], JHEP **07** (2021), 027]
[M. Cepeda, S. Gori, P. Ilten, M. Kado, F. Riva, R. Abdul Khalek, A. Aboubrahim, J. Alimena, S. Alioli and A. Alves, *et al.* CERN Yellow Rep. Monogr. **7** (2019), 221-584]
[ATL-PHYS-PUB-2018-048]
[Phys. Rev. D **104**, no.11, 112010 (2021)]
[$m_{\chi^+} > 103.5$ GeV from LEP
J. Abdallah *et al.* DELPHI]

$$\begin{pmatrix}\chi_h^0\\ \chi_m^0\\ \chi_l^0\end{pmatrix} = \begin{pmatrix}\cos\theta_2 & \sin\theta_2/\sqrt{2} & -\sin\theta_2/\sqrt{2}\\ 0 & i/\sqrt{2} & i/\sqrt{2}\\ \sin\theta_2 & -\cos\theta_2/\sqrt{2} & \cos\theta_2/\sqrt{2}\end{pmatrix}\begin{pmatrix}\chi_T^0\\ \chi_D^{c0}\\ \chi_D^0\end{pmatrix},\ R_\gamma = \frac{\Gamma(h\to\gamma\gamma)}{\Gamma_{\rm SM}(h\to\gamma\gamma)}$$

Conclusion

- Due to the expected high precision for $\sigma_{\rm ZH}$ at future lepton colliders, NNLO EW corrections must be calculated, in which fermion corrections dominate.
- With the approach of Feynman parametrization, dispersion relation and subtraction method, all relevant two-loop diagrams are reduced to at most three-dimensional numerical integrals that can be evaluated with typically 3–4 digits precision within minutes on a single CPU core.
- These theory error estimates, $0.1 - 0.3\%$ are lower than the anticipated experimental precision (0.4–1%).
- These deviations, if observed, would need to be explained by new physics beyond the Standard Model. We explore the Higgs portal models with fermion multiplets as a potential source of such new physics. At certain parameter region, there exist some parameter regions where the Higgs precision measurements can provide complementary information to direct LHC searches,

Thanks for your attention!

[http://xhslink.com/o/5wO4Z2bkWmt]

[http://xhslink.com/o/9OYgyWsFszw]

Backup I

Feynman Para. + dispersion relation — directly compute tensor integrals up to $\mathcal{O}(\epsilon^0)$

[G. Passarino and M. J. G. Veltman, Nucl. Phys. B **160** (1979), 151-207]

$\Rightarrow$

1. introduce subtraction terms for integrals with divergence
2. Feynman para. to convert q_2 subloop to bubble
3. Passrino-Veltman reduction for q_2 subloop
4. insert dispersion relation for one-loop bubble
5. integrate q_1 subloop (w/o IBP)
6. expand one-loop integrals according to ϵ
7. integrate UV finite part numerically; UV divergent part is known analytically

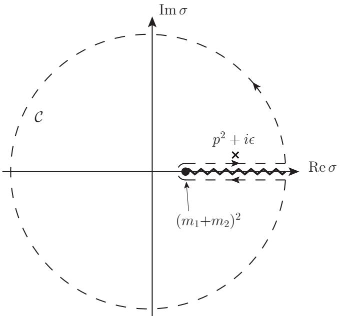
Backup II

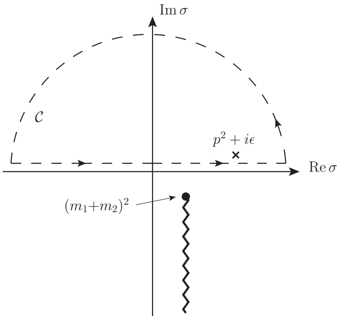
discontinuity for C_0:[S. Bauberger, "Two-loop contributions to muon decay,"]

1. dispersion relation for 1-loop triangle cannot disentangle two-subloops
The basic dispersion relation formula for the one-loop bubble

$$
\begin{aligned}
B_{ijk} &= \int_{(m_1+m_2)^2}^{\infty} d\sigma \frac{1}{\pi} \frac{\mathrm{Im} B_{ijk}(\sigma, m_1^2, m_2^2)}{\sigma - q_1^2 - i\epsilon} \ , \ \mathrm{Im}((m_1+m_2)^2) = 0 \\
&= \int_{-\infty}^{+\infty} d\sigma \frac{1}{2\pi i} \frac{B_{ijk}(\sigma, m_1^2, m_2^2)}{\sigma - q_1^2 - i\epsilon} \ , \ \mathrm{Im}((m_1+m_2)^2) \neq 0
\end{aligned}
$$

[Q. Song and A. Freitas, JHEP **04** (2021), 179]





2. Subtraction terms are used to cancel UV divergences (cancels with corrections from counter-term diagrams). Subtraction terms are usually related to vacuum/self-energy integrals, thus are known analytically.

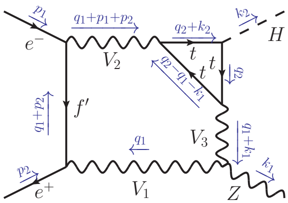
$$I_{\text{UV-div}} = \underbrace{I_{\text{UV-div}} - I_{\text{subtra}}}_{\text{UV-finite}} + \underbrace{I_{\text{subtra}}}_{\text{UV-div}}$$

Taking $V_1 = V_2 = V_3 = W, f' = b$ as an example, the UV divergent part for loop diagram and its corresponding CT diagram is

$$\text{loop diagram} : -2.808566335 \times 10^{-6} + 1.176922426 \times 10^{-6} i$$

$$\text{CT diagram} : +2.808566335 \times 10^{-6} - 1.176922426 \times 10^{-6} i$$

Clearly, the sum of them equals 0.



[Q. Song, "Radiative Corrections for ZH Production at Electron-Positron Collider,"]

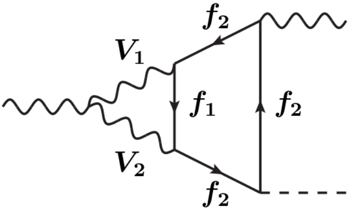
Backup IV

3. Systematic ways of building surface terms: identify tensor terms that generate UV divergences, and then simplify denominator by taking limits
The general tensor integral for VZH vertex can be written as

$$I = \int \frac{d^D q_2}{i\pi^2} \frac{d^D q_1}{i\pi^2} \sum_{n_0,n_1,n_2,i,j} c_{ij}^{n_0,n_1,n_2} \times \{p_i^{n_0}, q_1^{n_1}, q_2^{n_2}\}_j \times \frac{1}{((q_2+q_1)^2 - m_{f_1}^2)}$$

$$\times \frac{1}{(q_2^2 - m_{V_2}^2)((q_2+p)^2 - m_{V_1}^2)(q_1^2 - m_{f_2}^2)((q_1-p_h)^2 - m_{f_2}^2)((q_1-p)^2 - m_{f_2}^2)}$$

The SM Feynman rules require that $n_1 \leq 4, n_2 \leq 2, n_0 + n_1 + n_2 \leq 6$. $c_{ij}^{n_0,n_1,n_2}$ is the coefficient of a dot product, and it is a function of masses and dimension D. $I \to \infty$ when $\{q_{1,2} \to \infty\}, \{q_1 \to \infty\}, \{q_2 \to \infty\}$



Backup V

To make the q_1 integral UV finite, the following subtraction term is constructed:

$$I_{\text{subtr}}^{q_1} = \int \frac{d^D q_2}{i\pi^2} \frac{d^D q_1}{i\pi^2} \sum_{i,j} \Big[c_{ij}^{2,4,0} \times \{p_i^2, q_1^4, q_2^0\}_j + c_{ij}^{1,4,1} \times \{p_i^1, q_1^4, q_2^1\}_j + c_{ij}^{0,4,2} \times \{p_i^0, q_1^4, q_2^2\}_j + c_{ij}^{1,4,0} \times \{p_i^1, q_1^4, q_2^0\}_j + c_{ij}^{0,4,1} \times \{p_i^0, q_1^4, q_2^1\}_j + c_{ij}^{0,4,0} \times \{p_i^0, q_1^4, q_2^0\}_j \Big] \times \frac{1}{(q_2^2 - m_{V_2}^2)((q_2+p)^2 - m_{V_1}^2)} \times \frac{1}{(q_1^2 - m_{f_1}^2)(q_1^2 - m_{f_2}^2)(q_1^2 - m_{f_2}^2)(q_1^2 - m_{f_2}^2)}$$

Similar for q_2 subloop.