# Semi-automatic Calculations of Multi-loop Feynman Amplitudes with **AmpRed**

Based on:

WC, Comput.Phys.Commun. 312 (2025) 109607, arXiv: 2505.13540

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# Outline

1. Feynman amplitudes
2. **AmpRed**
3. An example: four-photon scattering
4. Usages of **AmpRed**
   - Generate amplitudes
   - Tensor algebras
   - Integral reduction
   - Numerical evaluation of master integrals
   - Other functionalities
5. Summary

# Feynman amplitudes

## General structure of Feynman amplitudes

dimensional regularization

$$T_{\alpha\beta\gamma\cdots} \times \int \mathrm{d}^{d} l_1 \mathrm{d}^{d} l_2 \cdots \frac{l_1^{\alpha} l_1^{\beta} \cdots l_2^{\gamma} \cdots}{\left(q_1^2 - m_1^2\right)^{i_1} \left(q_2^2 - m_2^2\right)^{i_2} \cdots}. \tag{1}$$

## Workflow of loop calculations

- Amplitude generation
- Tensor algebras
- Integral reduction (to master integrals)
- Calculation of master integrals

*All implemented in* ***AmpRed****.*

# AmpRed

## Foundations of **AmpRed**

Reduce Feynman integrals through the parametric representation:

- arXiv: 1902.10387
- arXiv: 1912.08606
- arXiv: 2007.00507
- arXiv: 2508.18419

Recursive calculation of parametric integrals:

- arXiv: 2309.03832
- arXiv: 2505.13540

Present **AmpRed**

- arXiv: 2408.06426

Download from:
https://gitlab.com/chenwenphy/ampred

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# An example: four-photon scattering

Kinematics

```
SetKinematics[{p[1], p[2]} → {k[1], k[2]}, {0, 0} → {0, 0}];
s = 1; t = -1 / 3; m = 2;
```

Generate the amplitude

```
amplitudes = FA2AR[Get["./FourPhotonAmplitudes.m"], ToFeynmanInt → True] /.
    {Incoming → p, Outgoing → k, Internal → l, FA["ME"] → m, FA["EL"] → 1} // Total;
```

Tensor reduction:

```
amplitudes1 = amplitudes // SpinorChainSimplify;
```

Integral reduction and numerical evaluation of master integrals:

```
amplitudes2 = AlphaIntEvaluateN[amplitudes1, 6, PrecisionGoal → 20] // First // epsSeries;
```

Square the total amplitude:

```
SquareAmplitude[amplitudes2]
```

$2.732504612406 \times 10^{-10}$

# Generate amplitudes

FeynArts interface:

`FA2AR`

QGRAF interace:

`ARQgraf`

```
graphs = ARQgraf[{"F", "-F"} → {"V", "V"}, 0, "Model" → "QCD", Style → "Diagram"];
```

```
graphs // ARTikzFeynman
```

```
amplitudes = ARQgraf[{"F", "-F"} → {"V", "V"}, 0, "Model" → "QCD", Style → "Amplitude"];
```

# Tensor algebras

- Metric $g^{ab}$: `Pair[LorentzIndex[a], LorentzIndex[b]]` or `MT[a, b]`
- Four momentum $p^a$: `Pair[Momentum[p], LorentzIndex[a]]` or `FV[p, a]`
- Scalar product $p \cdot q$: `Pair[Momentum[p], Momentum[q]]` or `SP[p, q]`
- Spinor $u(p)$: `DiracSpinor[p, 1, I]`
- Gamma matrix $\gamma^a$: `DiracGamma[LorentzIndex[a]]` or `GA[a]`
- Slash $\not{p}$: `DiracGamma[Momentum[p]]` or `GS[p]`
- Chain of spinors $\bar{v}_q \not{p} \gamma^a u_p$: `SpinorChain[DiracSpinor[q, -1, -I], GS[p], GA[a], DiracSpinor[p, 1, I]]`
- Trace of gamma matrices $\langle \gamma^a \gamma^b \rangle$: `SpinorChain[Bra,GA[a,b],Ket]`
- Contract repeated Lorenta indices: `ContractLI[exp]`
- Simplify chains of spinors and gamma matrices: `SpinorChainSimplify[exp]`

# Integral reduction

Tensor integral:

```
exp = TI[{l}, {{l, m, 2}, {l - p, m}}, FV[l, a]]
```

$$\pi^{-D/2} \int d^D l \, \frac{l^a}{\left(l^2 - m^2\right)^2 \left((l-p)^2 - m^2\right)}$$

Reduce with built-in function `AlphaReduce`:

```
AlphaReduce[exp, AlphaToFeynman → True] // Simplify
```

$$\frac{p^a \left(\left((D-4)\, p^2 + 4\, m^2\right) \pi^{-D/2} \int d^D l_1 \, \frac{1}{\left(l_1{}^2 - m^2\right)\left((l_1 - p)^2 - m^2\right)} - 2\,(D-2)\, \pi^{-D/2} \int d^D l_1 \, \frac{1}{l_1{}^2 - m^2}\right)}{2\, p^2 \left(4\, m^2 - p^2\right)}$$

Reduce with KIRA interface `DoKira`:

```
DoKira[exp] // AlphaToFeynman // Simplify
```

$$\frac{p^a \left(\left((D-4)\, p^2 + 4\, m^2\right) \pi^{-D/2} \int d^D l_1 \, \frac{1}{\left(l_1{}^2 - m^2\right)\left((l_1 - p)^2 - m^2\right)} - 2\,(D-2)\, \pi^{-D/2} \int d^D l_1 \, \frac{1}{l_1{}^2 - m^2}\right)}{2\, p^2 \left(4\, m^2 - p^2\right)}$$

# Numerical evaluation of master integrals

```
AlphaIntEvaluateN[exp, 4, {m → 1, SP[p] → 5}, PrecisionGoal → 20][[1]]
```

$$(-1.6928141744892050565 + 0.0055985302884311632\, i)\, \epsilon^2\, p^a - (0.60537923770464950 50 - 1.5115223721311336819\, i)\, \epsilon\, p^a + (0.56198517848325811145 + 0.37216357638560161556\, i)\, p^a$$

# Other functionalities

- Construct and solve differential equations for Feynman integrals
- Algebras in non-relativistic theories
- ...

# Summary

**AmpRed** is a powerful tool for the automatic calculation of multi-loop Feynman amplitudes. The usage of **AmpRed** is briefly introduced with detailed examples in this talk.

*Please contact the speaker for any questions and suggestions (chenwenphy@gmail.com).*
*Thanks for your attention.*