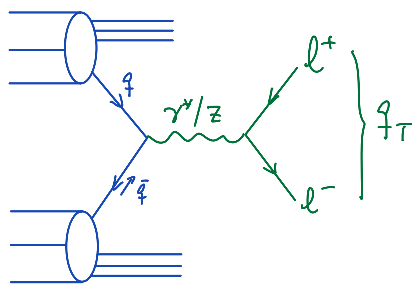
1. Introduction

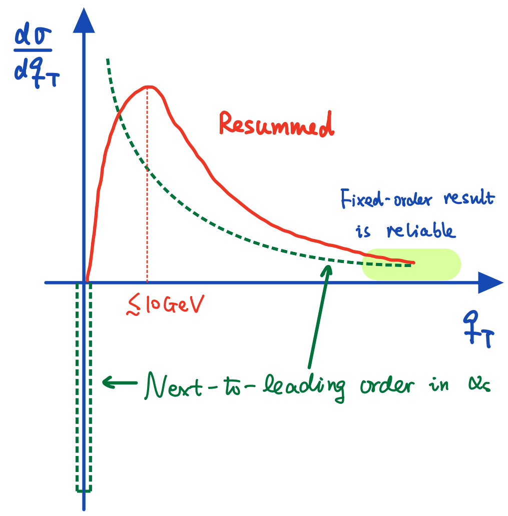
- Multi scales are involved in measurements of collider observables

Example:

transverse momentum distribution

for Drell-Yan process





Fixed-Order result

$$\frac{d\sigma}{dq_T} \sim A_0\,\delta(q_T) + \sum_{n=1}^{\infty} \alpha_s^n(\mu)\left[C_1^n \left(\frac{1}{q_T} \ln^{2n-1}\frac{q_T}{Q}\right)_* + C_2^n \left(\frac{1}{q_T} \ln^{2n-2}\frac{q_T}{Q}\right)_* + \cdots \right] + \mathcal{O}(\alpha_i^2)$$

- Most of the "global" observables studied so far have the property of exponentiation.

$$\sigma(\omega) \sim \left[1 + \sum_{n=1}^{\infty} c_n \left(\frac{\alpha_s}{2\pi}\right)^n\right] e^{\underbrace{L g_1(\alpha_s L)}_{LL} + \underbrace{g_2(\alpha_s L)}_{NLL} + \underbrace{\alpha_s g_3(\alpha_s L)}_{NNLL} + \cdots} + \mathcal{O}(\omega), \qquad \omega \ll 1 \,.\; L = \ln\omega \sim \alpha_s^{-1}$$

distribution is given by $\frac{d\sigma}{d\omega}$

The logrithms originates from infrared (IR) divergences.

higher scale.

IR divergences

Collinear matrix elements.

universal,

$$\sigma \sim H \otimes \underbrace{\prod_i J \otimes S}_{\text{lower scales}}$$

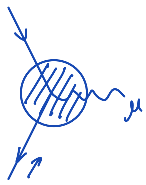
← Exponentiation Theorem,

rescaling invariant.

lower scales

UV divergences

Quark form factor up to two loops:



$$= |\mathcal{M}_2(\epsilon)\rangle$$

0507039 . Moch et al

$$e_0\, \bar{v}(p_2)\gamma^\mu u(p_1)$$

$$\times \Big\{ 1 + \frac{\alpha_{s,0}}{4\pi}(-Q^2-i0)^{-\epsilon}\, C_F\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}+\frac{\pi^2}{6}-8+O(\epsilon)\right)$$

$$+\left(\frac{\alpha_{s,0}}{4\pi}\right)^2(-Q^2-i0)^{-2\epsilon}\, C_F\left[C_F\left(\frac{2}{\epsilon^4}+\frac{6}{\epsilon^3}+\cdots\right)+C_A\left(-\frac{11}{6\epsilon^3}+\cdots\right)+n_f T_F\left(\frac{1}{3\epsilon^3}+\cdots\right)\right]$$

$$+ O(\alpha_s^3) \Big\}$$

with $\alpha_{s,0} = Z_\alpha\, \alpha_s(\mu)\, \mu^{2\epsilon}$

$$Z_\alpha = 1 + \frac{\alpha_s}{4\pi}\left(-\frac{\beta_0}{\epsilon}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{\beta_0^2}{\epsilon^2}-\frac{\beta_1}{2\epsilon}\right) + O(\alpha_s^3)$$

$$\frac{d\alpha_s}{d\ln\mu} = \beta(\alpha_s) - 2\epsilon\,\alpha_s$$

$$0 = \frac{d}{d\ln\mu}\ln Z_\alpha + \frac{d}{d\ln\mu}\ln\alpha_s + 2\epsilon$$

$$\Rightarrow \frac{d\alpha_s}{d\ln\mu} = \left(-Z_\alpha^{-1}\frac{d}{d\ln\mu}Z_\alpha - 2\epsilon\right)\alpha_s$$

$$= \beta(\alpha_s) - 2\epsilon\,\alpha_s$$

Q: Can we predict $1/\epsilon^{2n}$ and $1/\epsilon^{2n-1}$ at $O(\alpha_s^n)$ from one-loop result?

Yes!!!

The IR poles can be predicted by anomalous dimensions

In $\overline{MS}$ scheme

$$|\mathcal{M}_n(\epsilon)\rangle = Z(\epsilon,\mu)\, |\mathcal{M}_n(\mu)\rangle$$

Generally is a vector in color space

$$\frac{d}{d\ln\mu}|\mathcal{M}_n(\epsilon)\rangle = 0$$

$$\Rightarrow Z(\epsilon,\mu)\frac{d}{d\ln\mu}|\mathcal{M}_n(\mu)\rangle + \left(\frac{d}{d\ln\mu}Z(\epsilon,\mu)\right)|\mathcal{M}_n(\mu)\rangle = 0$$

$$\Rightarrow \frac{d}{d\ln\mu}|\mathcal{M}_n(\mu)\rangle = \underbrace{-Z^{-1}\left(\frac{d}{d\ln\mu}Z\right)}_{\Gamma(\mu)}|\mathcal{M}_n(\mu)\rangle$$

$\Gamma(\mu) \leftarrow$ can be determined from Z loop-by-loop

Generally. $\Gamma(\alpha_s, \mu) = -\Gamma_{cusp}^F(\alpha_s) \ln \frac{\mu^2}{-Q^2-i0} + 2\gamma^q(\alpha_s)$

For the one-loop form factor:

$$Z(\varepsilon, \mu) = 1 + \frac{\alpha_s}{4\pi} C_F \left[-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-3 - 2\ln \frac{\mu^2}{-Q^2-i0}\right) \right]$$

We can get the anomalous dimension at one loop

$$\Gamma(\mu) = \frac{\alpha_s}{4\pi} C_F \left(-4 \ln \frac{\mu^2}{-Q^2-i0} - 6 \right)$$

By solving RGE $\frac{d}{d\ln\mu} Z = Z\Gamma$ with boundary condition $\lim_{\mu\to\infty} Z(\varepsilon,\mu) = 1$

we know Z is an exponential, and at one loop:

$$Z(\varepsilon, \mu) = e^{\frac{\alpha_s}{4\pi} C_F \left[-\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-3 - 2\ln \frac{\mu^2}{-Q^2-i0}\right) \right]}$$

Next, we can use $\Gamma(\mu)$ to predict IR poles to all order in α_s

$$\ln Z(\varepsilon,\mu) = \int_\mu^\infty \frac{d\mu'}{\mu'} \Gamma(\alpha_s(\mu'), \mu')$$

$$= \int_0^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha) - 2\varepsilon\alpha} \left(-\Gamma_{cusp}(\alpha) \ln \frac{\mu^2}{-Q^2-i0} + 2\gamma^q(\alpha) \right) - \underline{2\int_\mu^\infty \frac{d\mu'}{\mu'} \Gamma_{cusp}(\alpha) \int_\mu^{\mu'} \frac{d\mu''}{\mu''}}$$

$$= 2\int_\mu^\infty \frac{d\mu''}{\mu''} \int_{\mu''}^\infty \frac{d\mu'}{\mu'} \Gamma_{cusp}(\alpha)$$

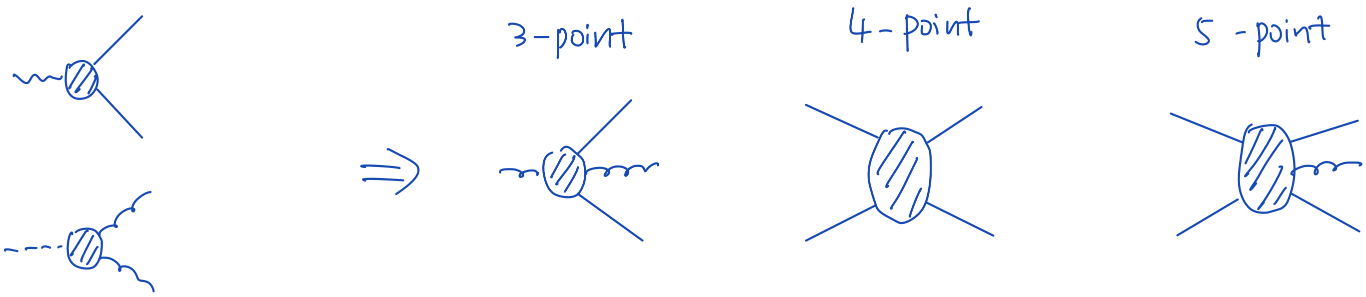
$$= 2\int_0^{\alpha_s} \frac{d\bar\alpha}{\beta(\bar\alpha) - 2\varepsilon\bar\alpha} \int_0^{\bar\alpha} \frac{d\alpha}{\beta(\alpha) - 2\varepsilon\alpha} \Gamma_{cusp}(\alpha)$$

$$\Rightarrow \ln Z(\varepsilon,\mu) = \frac{\alpha_s}{4\pi}\left(\frac{\Gamma_0'}{4\varepsilon^2} + \frac{\Gamma_0}{2\varepsilon} \right)$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^2 \left(-\frac{3\beta_0}{16\varepsilon^3}\Gamma_0' + \frac{\Gamma_1' - 4\beta_0\Gamma_0}{16\varepsilon^2} + \frac{\Gamma_1}{4\varepsilon} \right)$$
$$+ \mathcal{O}(\alpha_s^3)$$

0903.1126

How to generalize the anomalous dimension to multi leg scattering ?

What can we know from quark / gluon form factors ?



For soft k^μ

- Eikonal approximation

$$p^\mu = \bar{n}\cdot p \, \frac{n^\mu}{2}, \qquad \bar{n}\cdot p \sim Q, \qquad k^\mu \sim Q(\tau,\tau,\tau)$$

Incoming quark　　Incoming anti-quark　　Outgoing quark　　Outgoing anti-quark

$$\frac{i\not{p}}{2p\cdot k + i0} \simeq \frac{i(\not{p}-\not{k})}{(p-k)^2+i0}\, ig_s\gamma^\mu t^a_{ji}\, u(p) \qquad \bar{v}(p)\, ig_s\gamma^\mu t^a_{ij}\, \frac{i(-\not{p}+\not{k})}{(p-k)^2+i0} \qquad \bar{u}(p)\, ig_s\gamma^\mu t^a_{ij}\, \frac{i(\not{p}+\not{k})}{(p+k)^2+i0} \qquad \frac{i(-\not{p}-\not{k})}{(p+k)^2+i0}\, ig_s\gamma^\mu t^a_{ji}\, u(p)$$

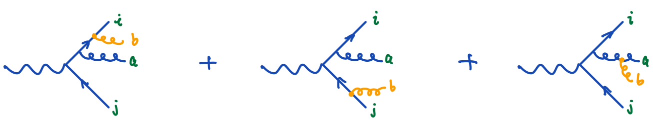
$$= u(p)(-t^a_{ji})\, ig_s n^\mu \frac{i}{n\cdot k - i0} \qquad = \bar{v}(p)\, ig_s n^\mu \frac{i}{n\cdot k - i0}\, t^a_{ij} \qquad = \bar{u}(p)\, ig_s n^\mu \frac{i}{n\cdot k + i0}\, t^a_{ij} \qquad = v(p)\, ig_s n^\mu \frac{i}{n\cdot k + i0}(-t^a_{ji})$$

So it has "uniform" kinematic part : $ig_s n^\mu \dfrac{i}{n\cdot k \pm i0}$

but distinct for color: $$T^a|c\rangle = \begin{cases} t^a_{ij} & \text{for incoming anti-quark / outgoing quark} \\ -t^a_{ji} & \text{for incoming quark / outgoing anti-quark} \\ if^{abc} & \text{for gluon} \end{cases}$$

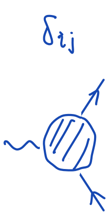
color conservation : $\sum_{\text{over ext. legs}} T^a_i |c\rangle = 0$　　　Catani-Seymour formalism

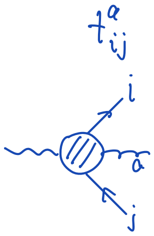
Example:

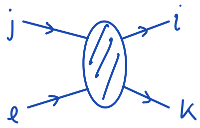


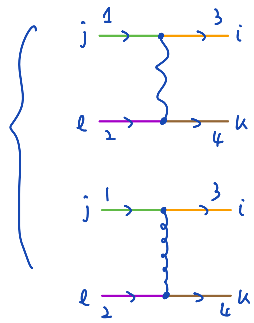
$$t^b_{ik} t^a_{kj} + t^a_{ik}(-t^b_{kj}) + if^{abc} t^c_{ij}$$

$$= -[t^a, t^b]_{ij} + if^{abc} t^c_{ij} = 0$$









$$|C_1\rangle = \frac{1}{N_c}\,\delta_{ij}\,\delta_{kl}$$

$$|C_2\rangle = \frac{1}{N_c}\,t^a_{ij}\,t^a_{kl}$$

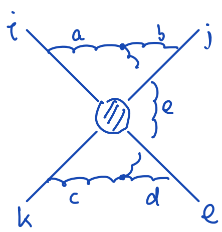
General Formula for massless scattering up to three loops : 0903.1126

$$\Gamma(\alpha_s,\mu) = \underbrace{\sum_{(i,j)} \frac{T_i\cdot T_j}{2}\,\gamma_{cusp}(\alpha_s)\,\ln\frac{\mu^2}{-S_{ij}} + \sum_i \gamma^i}_{\text{Dipole structure}}$$

$$\underbrace{+\ \underbrace{f(\alpha_s)\sum_{(i,j,k)} T_{iijk}}_{\text{tripole}} + \underbrace{\sum_{(i,j,k,l)} T_{ijkl}\,F(\beta_{ijlk},\beta_{iklj})}_{\text{quadrupole}}}_{\text{start at three loops}}$$

$$S_{ij} = -\sigma_{ij}\,2\,p_i\cdot p_j \quad \text{with} \quad \begin{array}{ll}\sigma_{ij} = 1 & \text{timelike}\\ \sigma_{ij} = -1 & \text{spacelike}\end{array}$$

$$T_{ijkl} = f^{abe} f^{cde}\,(T_i^a T_j^b T_k^c T_l^d)_+ \quad \leftarrow \text{permutation}$$

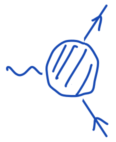


The permutation is trivial if (i,j,k,l) are fully different.

$$\beta_{ijkl} = \beta_{ij} + \beta_{kl} - \beta_{il} - \beta_{jk}$$

$$= \ln\frac{(p_i\cdot p_l)(p_j\cdot p_k)}{(p_i\cdot p_j)(p_k\cdot p_l)}$$

Example 1: Form Factor



δ_{ij}

$$T_1 \cdot T_2 = -T_1 \cdot T_1 = -C_{R1} = -C_{R2}$$

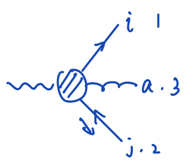
$$\Gamma_{ij} = -C_F\, \gamma_{cusp}(\alpha_s) \ln \frac{\mu^2}{-Q^2 - i0} + 2\gamma^q(\alpha_s)$$

from above section, we know $\gamma_{cusp}(\alpha_s)$ and $\gamma^q(\alpha_s)$ at one loop.

$$\gamma_{cusp}^{(1)} = 4 \quad , \quad \gamma^{q(0)} = -3C_F$$

An important observation is that γ_{cusp} and $\gamma^{q/g}$ can be determined by quark & gluon form factors at every high loop orders!

Example 2: two-loop amplitude for 3 jets



$$T_1 \cdot T_2 = T_1 \cdot (-T_1 - T_3) = -T_1^2 - (-T_2 - T_3) \cdot T_3 = -T_1^2 + T_3^2 + T_2 \cdot T_3$$

$$= -T_1^2 + T_3^2 + T_2(-T_1 - T_2)$$

$$= -T_1^2 - T_2^2 + T_3^2 - T_1 \cdot T_2$$

$$\Rightarrow \quad T_1 \cdot T_2 = \frac{1}{2}(-T_1^2 - T_2^2 + T_3^2) = \frac{1}{2}(-C_{R_1} - C_{R_2} + C_{R_3})$$

$C_R = C_F$ quark

$C_R = C_A$ gluon

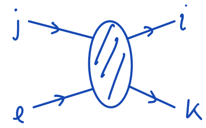
similarly :

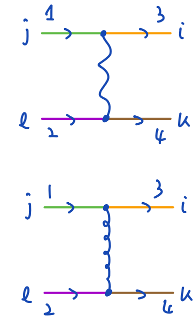
$$T_2 \cdot T_3 = \frac{1}{2}(+T_1^2 - T_2^2 - T_3^2)$$

$$T_1 \cdot T_3 = \frac{1}{2}(-T_1^2 + T_2^2 - T_3^2)$$

using γ_{cusp} and $\gamma^{q/g}$ determined from the form factors. we can derive the anomalous dimension

Example 3: 4-parton scattering:





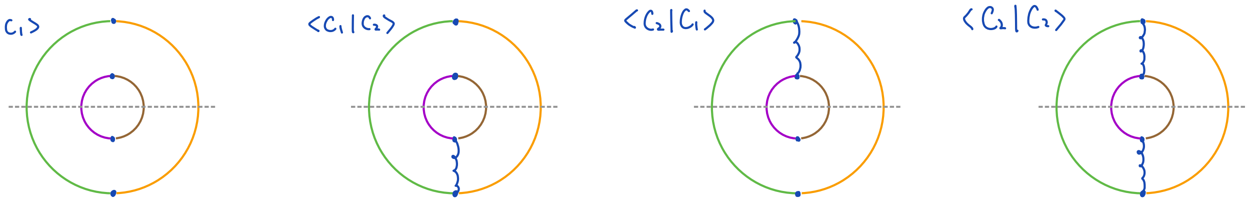
$$|C_1\rangle = \frac{1}{N_c}\,\delta_{ij}\,\delta_{kl}$$

$$|C_2\rangle = \frac{1}{N_c}\,t^a_{ij}\,t^a_{kl}$$

Examples:

$$\langle C_I | C_J \rangle = \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-1}{4N_c^2} \end{pmatrix}$$

$\langle C_1|C_1\rangle$ $\langle C_1|C_2\rangle$ $\langle C_2|C_1\rangle$ $\langle C_2|C_2\rangle$

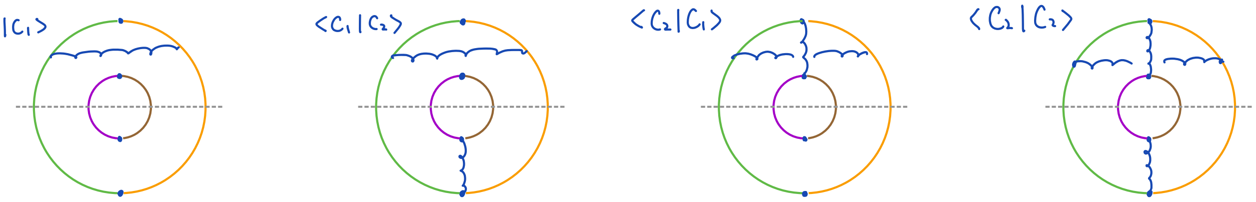


$\sim \mathrm{tr}(t^a)\,\mathrm{tr}(t^a) = 0$

$\frac{1}{N_c^2}\,\mathrm{tr}(t^a t^b)\,\mathrm{tr}(t^a t^b)$

$$\langle C_I | T_1 \cdot T_3 | C_J \rangle = \begin{pmatrix} -\frac{N_c^2-1}{2N_c} & 0 \\ 0 & \frac{N_c^2-1}{8N_c^3} \end{pmatrix}$$

$\langle C_1|C_1\rangle$ $\langle C_1|C_2\rangle$ $\langle C_2|C_1\rangle$ $\langle C_2|C_2\rangle$

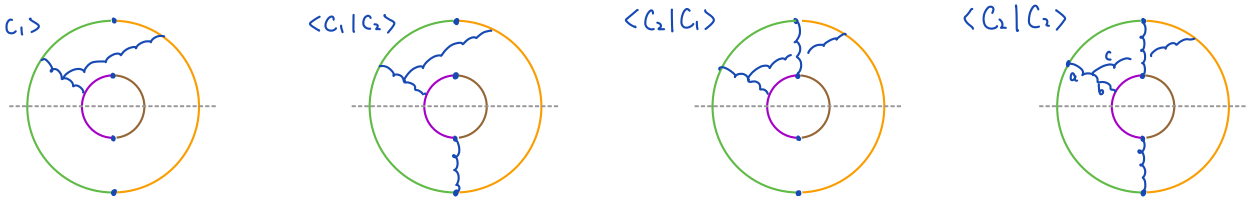


$\frac{1}{N_c^2}\,\delta_{ii}\,\mathrm{tr}(-t^a t^a) = -C_F$

$\frac{1}{N_c^2}\,\mathrm{tr}(-t^a t^b t^a t^c)\,\mathrm{tr}(t^b t^c)$

$$\langle C_I | \, i f^{abc}\, T_1^a\, T_2^b\, T_3^c \, | C_J \rangle = \frac{N_c^2-1}{8N_c} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$\langle C_1|C_1\rangle$ $\langle C_1|C_2\rangle$ $\langle C_2|C_1\rangle$ $\langle C_2|C_2\rangle$



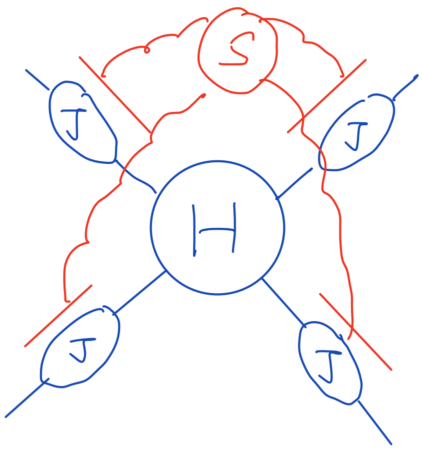
$\frac{1}{N_c^2}\, i f^{abc}\,\mathrm{tr}(-t^c t^e t^a t^f)\,\mathrm{tr}(-t^e t^b t^f)$

Fierz identity: $\sum_a t^a_{ij}\, t^a_{kl} = \frac{1}{2}\left(\delta_{il}\,\delta_{kj} - \frac{1}{N}\,\delta_{ij}\,\delta_{kl}\right)$

IR Structures for multileg scattering

How to determine the IR singularities ? — kinematics / color

1. Soft-collinear factorization :



Hard	$\lvert \mathcal{M}_n(\{p\})\rangle$	$s_{ij} = \pm 2 p_i \cdot p_j$	$p_i^2 = 0$
Jet	$J(p_i)$	p_i^2	
Soft	$S(\{p\})$	$\frac{p_i^2\, p_j^2}{s_{ij}}$	

$$G(\{p\}) = \prod_i J(p_i^2, \varepsilon_{UV})\, S(\{\underline{p}\}, \varepsilon_{UV})\, \lvert \mathcal{M}_n(\{\underline{p}\}, \varepsilon_{IR})\rangle + O(p_i^2/s_{ij})$$

For the renormalized $\lvert \mathcal{M}_n(\mu)\rangle$ $S(\{p\},\mu)$ and $J(p_i^2,\mu)$, we build RGEs

$$\frac{d}{d\ln\mu} \lvert \mathcal{M}_n(\mu)\rangle = \Gamma(\mu)\, \lvert \mathcal{M}_n(\mu)\rangle$$
$$\sim \left(\Gamma_{cusp} \ln \frac{\mu^2}{s_{12}} + \gamma^H\right) \lvert \mathcal{M}_n(\mu)\rangle$$

$$\frac{d}{d\ln\mu} J(p_i^2,\mu) = -\Gamma_J(\mu)\, J(\mu)$$
$$\sim \left(\Gamma_{cusp} \ln \frac{\mu^2}{p_i^2} + \gamma^J\right) J(\mu)$$

$$\frac{d}{d\ln\mu} S(\{p\},\mu) = -\Gamma_S\, S(\mu)$$
$$\sim \left(\Gamma_{cusp} \ln \frac{\mu^2 s_{12}}{p_1^2 \cdot p_2^2} + \gamma^S\right) S(\mu)$$

$\beta_{12} \to$ Cusp angle

RG invariance / cancellation of poles gives

$$\underset{\text{independent on } p_i^2}{\underline{\Gamma}} = \underbrace{\Gamma_S + \Gamma_j}$$

Γ_S linearly depends on the cusp angle.

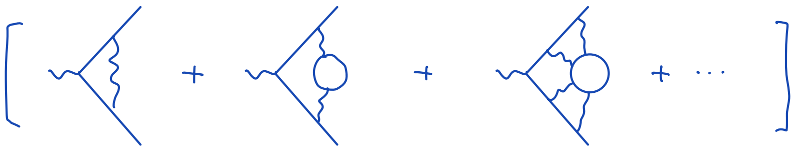
$\Rightarrow$ p_i^2 cancel out in $\Gamma_S + \Gamma_j$

2. Non-Abelian Exponentiation Theorem

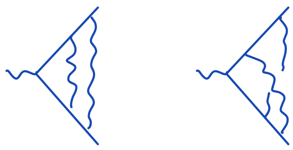
$$S_n(x) = e^{ig_s\int_0^\infty dt\, n\cdot A_s^a(x^\mu+tn^\mu)\,T^a}$$

QED :

$$\langle 0| S_{n_1} S_{n_2} |0\rangle\Big|_{\substack{\text{uv}\\ \text{poles}}} = \exp$$



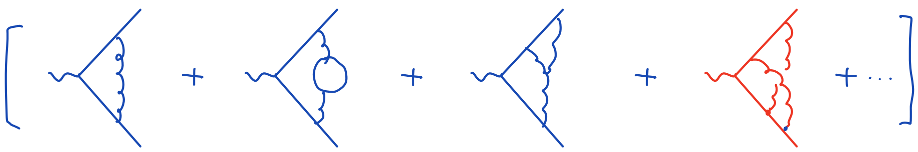
Fully connected diagrams



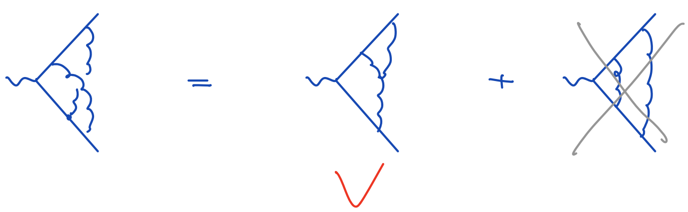
don't contribute to exponents.

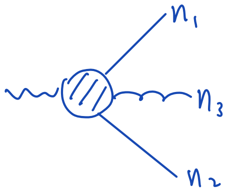
QCD

$$\langle 0| S_{n_1} S_{n_2} |0\rangle\Big|_{\substack{\text{uv}\\ \text{poles}}} = \exp$$



Fully color connected diagrams





$$
\begin{aligned}
S_{12} + S_{13} + S_{23} &= 2p_1\cdot p_2 + 2p_1\cdot p_3 + 2p_2\cdot p_3 \\
&= (p_1+p_2+p_3)^2 \\
&= Q^2
\end{aligned}
$$

$$x = S_{12}/Q^2 \ , \quad y = S_{13}/Q^2 \ , \quad z = S_{23}/Q^2$$

Two independent kinematic variables.

How about soft function?

$$S_n(x) = e^{ig_s \int_0^\infty dt \, n\cdot A_s^a(x^\mu + t n^\mu) T^a}$$

S_n is invariant under $n \to \lambda$.

so the kinematic variables have to be cross ratios

For 3 jet, impossible to construct cross ratio by $n_1 . n_2 . n_3$

$$\sum_{(i,j,k)} i f^{abc} T_i^a T_j^b T_k^c \ f(\alpha_s)$$

Collinear Limit:

$$\Gamma_{Sp}(\{p_1,p_2\},\mu) = T_1\cdot T_2\,\gamma_{cusp}(\alpha_s)\ln\frac{\mu^2}{-S_{12}} + \sum_{j\neq 1,2}\gamma_{cusp}(\alpha_s)\left[T_1\cdot T_j\left(\ln\frac{\mu^2}{-2z\,P_{1+2}\cdot P_j} - \ln\frac{\mu^2}{-2P_{1+2}\cdot P_j}\right) + T_2\cdot T_j\left(\ln\frac{\mu^2}{-2(1-z)P_{1+2}\cdot P_j} - \ln\frac{\mu^2}{-2P_{1+2}\cdot P_j}\right)\right]$$

$$+ \sum_I \gamma_{cusp}(\alpha_s)\left[T_1\cdot T_I\left(\ln\frac{m_I\mu}{-2z\,P_{1+2}\cdot P_j} - \ln\frac{m_I\mu}{-2P_{1+2}\cdot P_j}\right) + T_2\cdot T_I\left(\ln\frac{\mu^2}{-2(1-z)P_{1+2}\cdot P_j} - \ln\frac{m_I\mu}{-2P_{1+2}\cdot P_j}\right)\right] + \left[\gamma^1(\alpha_s) + \gamma^2(\alpha_s) - \gamma^{1+2}(\alpha_s)\right]\mathbb{1}$$

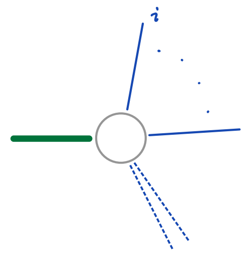
$$+ \boxed{f(\alpha_s)\left[\sum_{(j,k)}^{j,k\neq 1,2}(T_{11jk} + T_{22jk}) + \sum_{(j,k)}^{j,k\neq 1} 2T_{jj1k} + \sum_{(j,k)}^{j,k\neq 2} 2T_{jj2k} - 2\sum_{j\neq 1,2} T_{jj12} - \sum_{(j,k)}^{j,k\neq P}(T_{PPjk} + 2T_{jjPk})\right]}$$

$$+ \cancel{2\sum_{i,I}^{i\neq 1,2} F_{hel}(\rho_{i1I})(T_{1iII} + T_{2iII})} + 2F_{hel}(0)\sum_I T_{12II} \cancel{- 2\sum_{i,I}^{i\neq P} F(\rho_{PiI})\,T_{PiII}}$$

$$+ \boxed{\lim_{S_{12}\to 0} 8\sum_{i<j}^{i,j\neq 1,2}\left[T_{12ij}F(\beta_{12ij},\lambda_{12ij}) + T_{12ji}F(\beta_{12ji},\lambda_{12ji}) + T_{1ji2}F(\beta_{1ji2},\lambda_{1ji2})\right]}$$

$$+ \underbrace{\lim_{S_{12}\to 0}\sum_{2<i<j<k}\left[T_{(1ijk)}F(\beta_{(1ijk)},\lambda_{(1ijk)}) + T_{(2ijk)}F(\beta_{(2ijk)},\lambda_{(2ijk)}) - T_{(Pijk)}F(\beta_{(Pijk)},\lambda_{(Pijk)})\right]}_{}$$

Vanish



$$\rho_{1ia} = \rho_{2ia} = \rho_{Pia}$$

Cancel

$$+ \sum_a\sum_{2<i<j}\left[2T_{1ija}F_{hel}(\rho_{1ia},\rho_{1ja},\rho_{ija}) + 2T_{i1ja}F_{hel}(\rho_{1ia},\rho_{ija},\rho_{1ja}) + 2T_{j1ia}F_{hel}(\rho_{1ja},\rho_{ija},\rho_{1ia})\right]$$

$$+ \sum_a\sum_{2<i<j}\left[2T_{2ija}F_{hel}(\rho_{2ia},\rho_{2ja},\rho_{ija}) + 2T_{i2ja}F_{hel}(\rho_{2ia},\rho_{ija},\rho_{2ja}) + 2T_{j2ia}F_{hel}(\rho_{2ja},\rho_{ija},\rho_{2ia})\right]$$

$$- \sum_a\sum_{2<i<j}\left[2T_{Pija}F_{hel}(\rho_{Pia},\rho_{Pja},\rho_{ija}) + 2T_{iPja}F_{hel}(\rho_{Pia},\rho_{ija},\rho_{Pja}) + 2T_{jPia}F_{hel}(\rho_{Pja},\rho_{ija},\rho_{Pia})\right]$$

$$+ \sum_a\sum_i^{i\neq 1,2}\left[2T_{12ia}F_{hel}(\rho_{12a},\rho_{1ia},\rho_{2ia}) + 2T_{21ia}F_{hel}(\rho_{12a},\rho_{2ia},\rho_{1ia}) + \underbrace{\cancel{2T_{i12a}F_{hel}(\rho_{i1a},\rho_{i2a},\rho_{12a})}}_{F_{hel}(\rho,\rho,0)=0}\right]$$

+ non-dipole terms involving two or more massive legs