暗物质的粒子候选者介绍之一
WIMP and Light DM

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2025 年“微扰量子场论及其应用”前沿讲习班暨前沿研讨会
2025-07-11 @ 山东大学中心校区

广义相对论

- 广义相对论
  - 时空具有自己的动力学
  - 度规局域化 $g_{\mu\nu}(x, y, z, t)$
  - 爱因斯坦方程 $G_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$
  - 物质影响时空，时空影响物质的运动

Friedmann–Robertson–Walker (FRW)度规

- 一个度规的严格解 $$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$$
  - 假设空间是同质的和各向同性的
  - 空间度规与时间无关
  - FRW度规： $$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$
  - 能够很好的描述我们的宇宙

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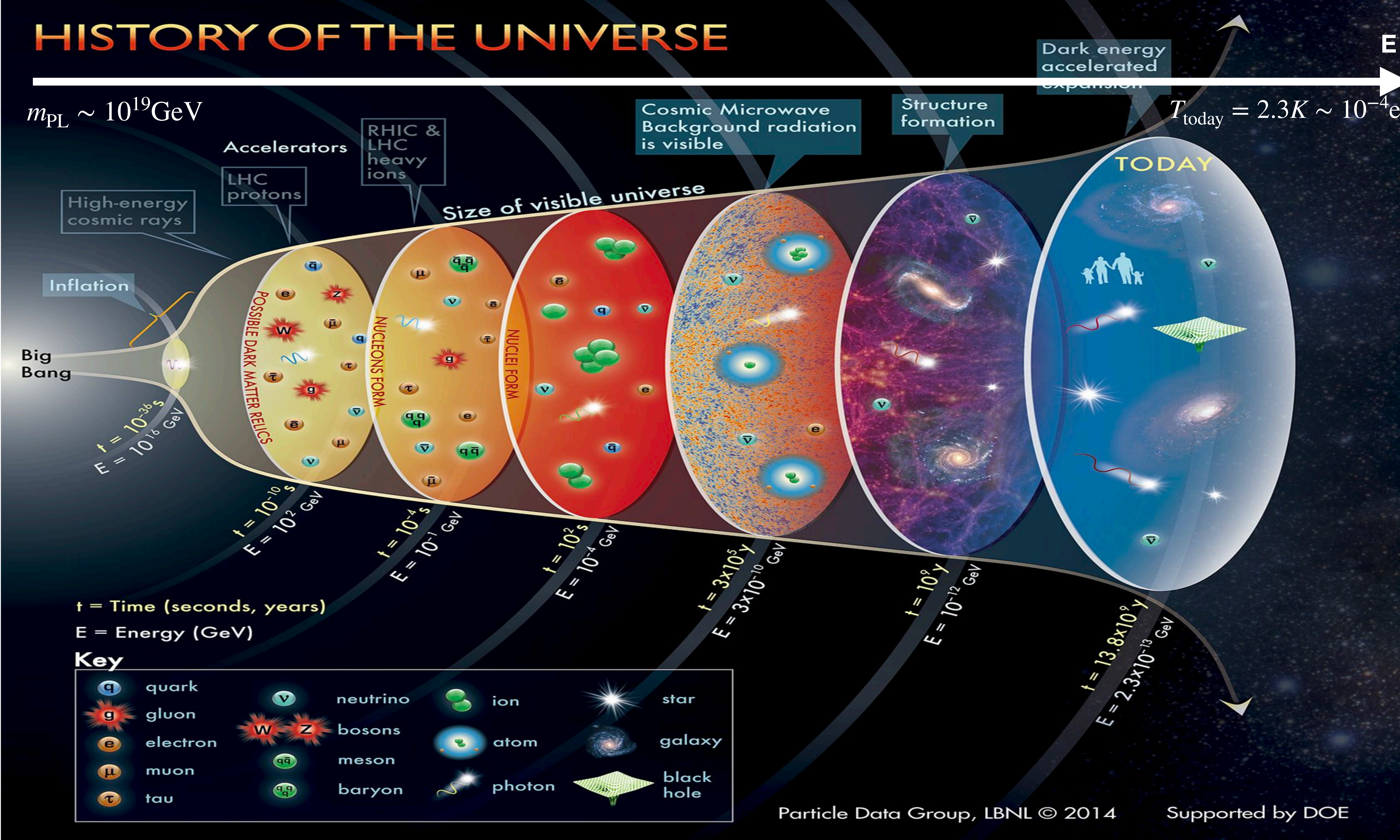
  - 能够很好的描述我们的宇宙

- 我们的宇宙
  - 平坦宇宙 $k = 0$
  - 哈勃常数 $H(t) = \dot{a}/a$
  - 能量密度来源
    - 物质$\rho_m \propto a^{-3}$, 辐射$\rho_r \propto a^{-4}$,
    - 暗能量$\rho_{cc} \propto a^0$

E

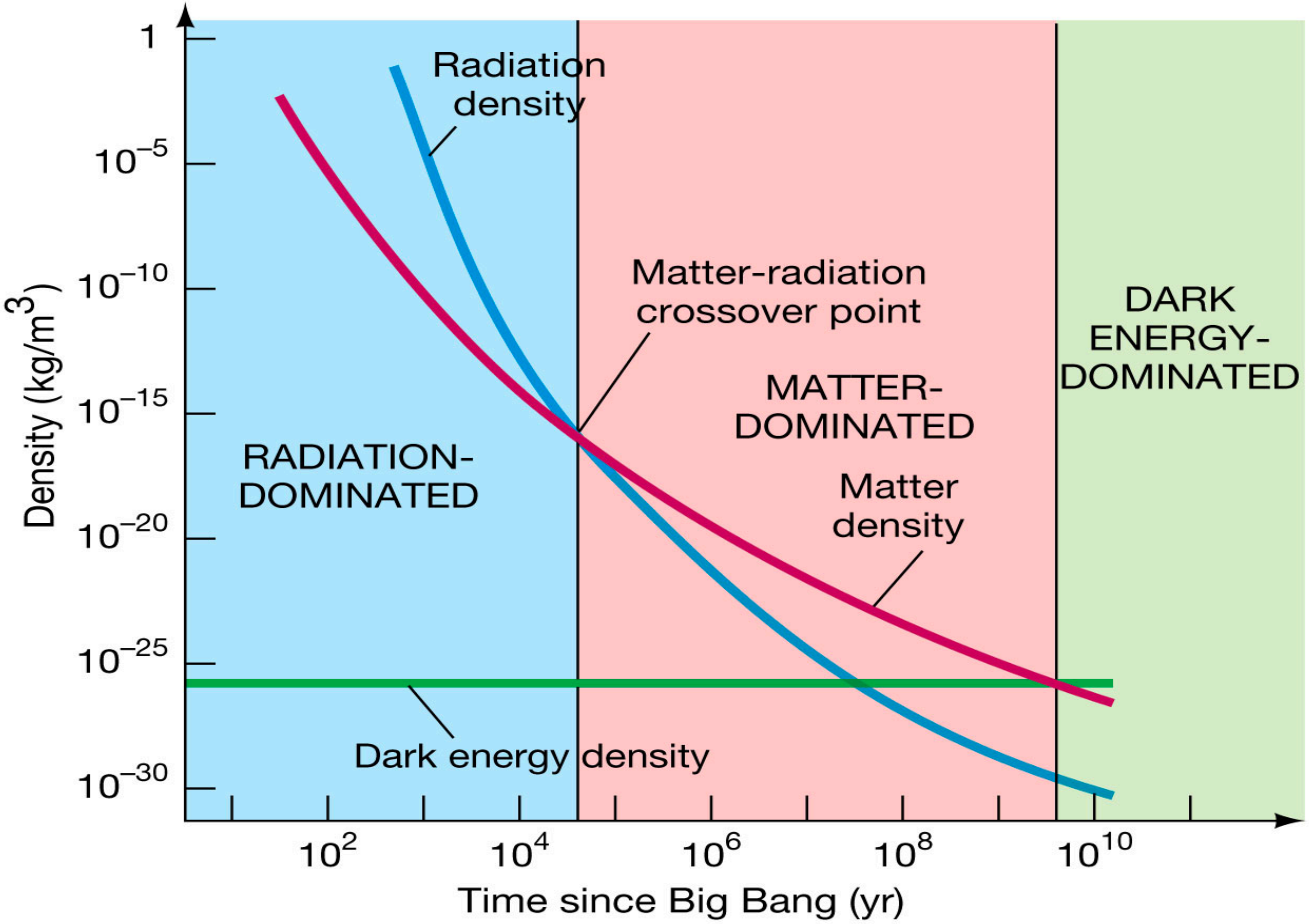
$m_{\mathrm{PL}} \sim 10^{19}\mathrm{GeV}$

$T_{\mathrm{today}} = 2.3K \sim 10^{-4}\mathrm{eV}$



对最大客体的已知：宇宙的演化

宇宙各组分能量的演化



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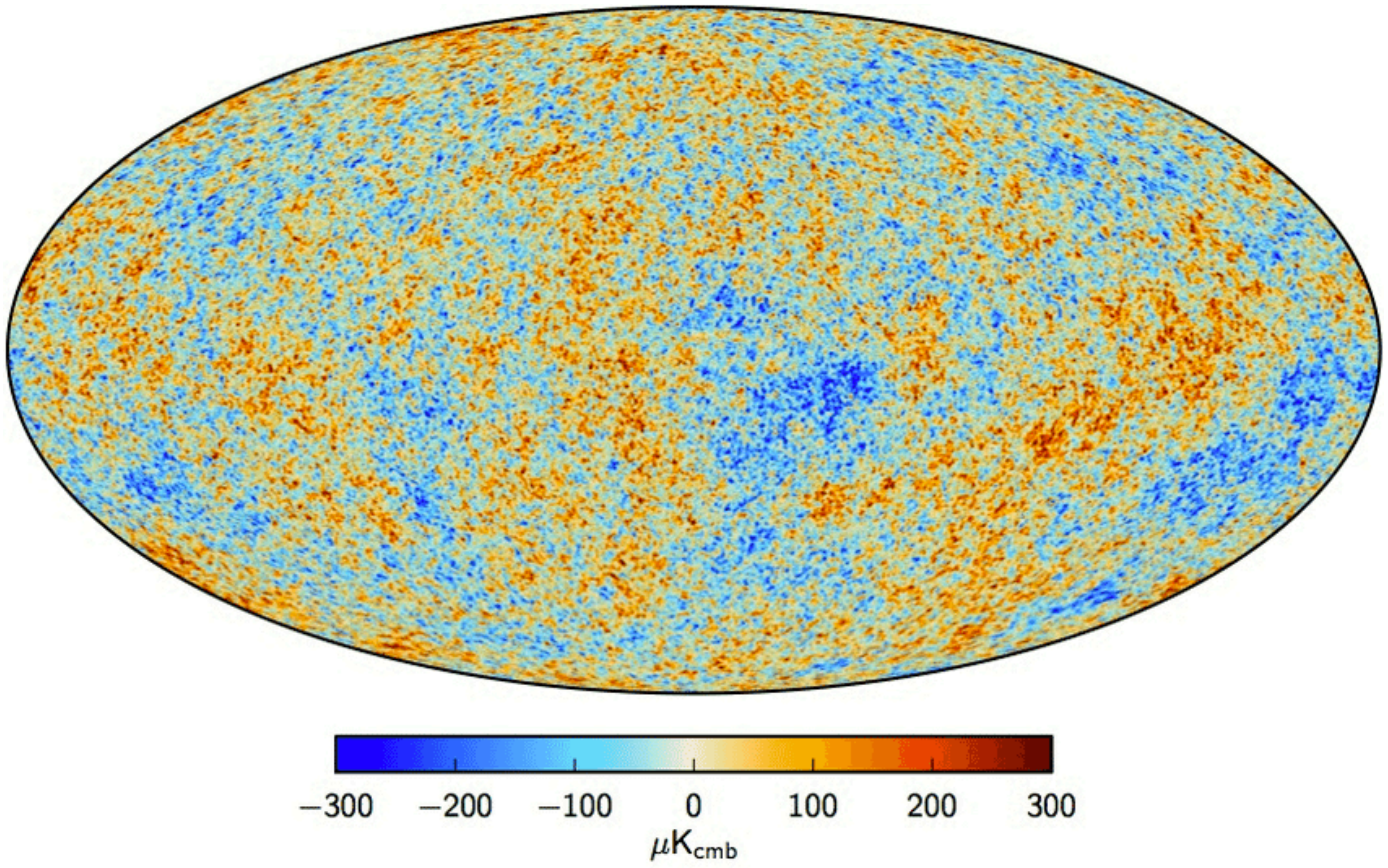
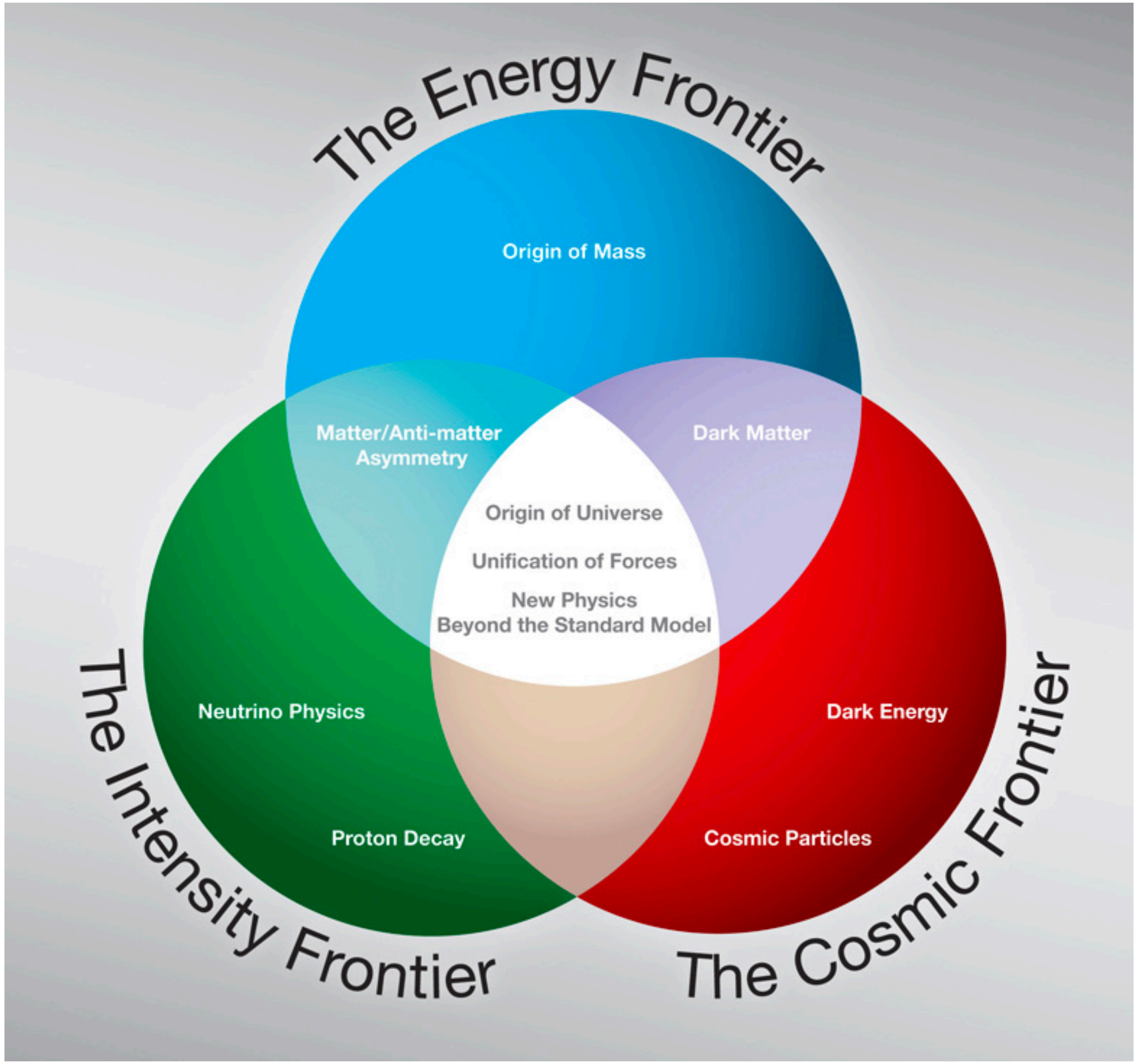
宇宙学标准模型：Lambda Cold Dark Matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles
- 6 parameter for the Universe: Baryon matter density, DM density, lifetime of the Universe …
  - Explain the structure of the CMB
  - Large-scale structure in the distribution of the galaxies
  - The observed abundance of H, D, He and Li
  - Accelerating expansion of the Universe

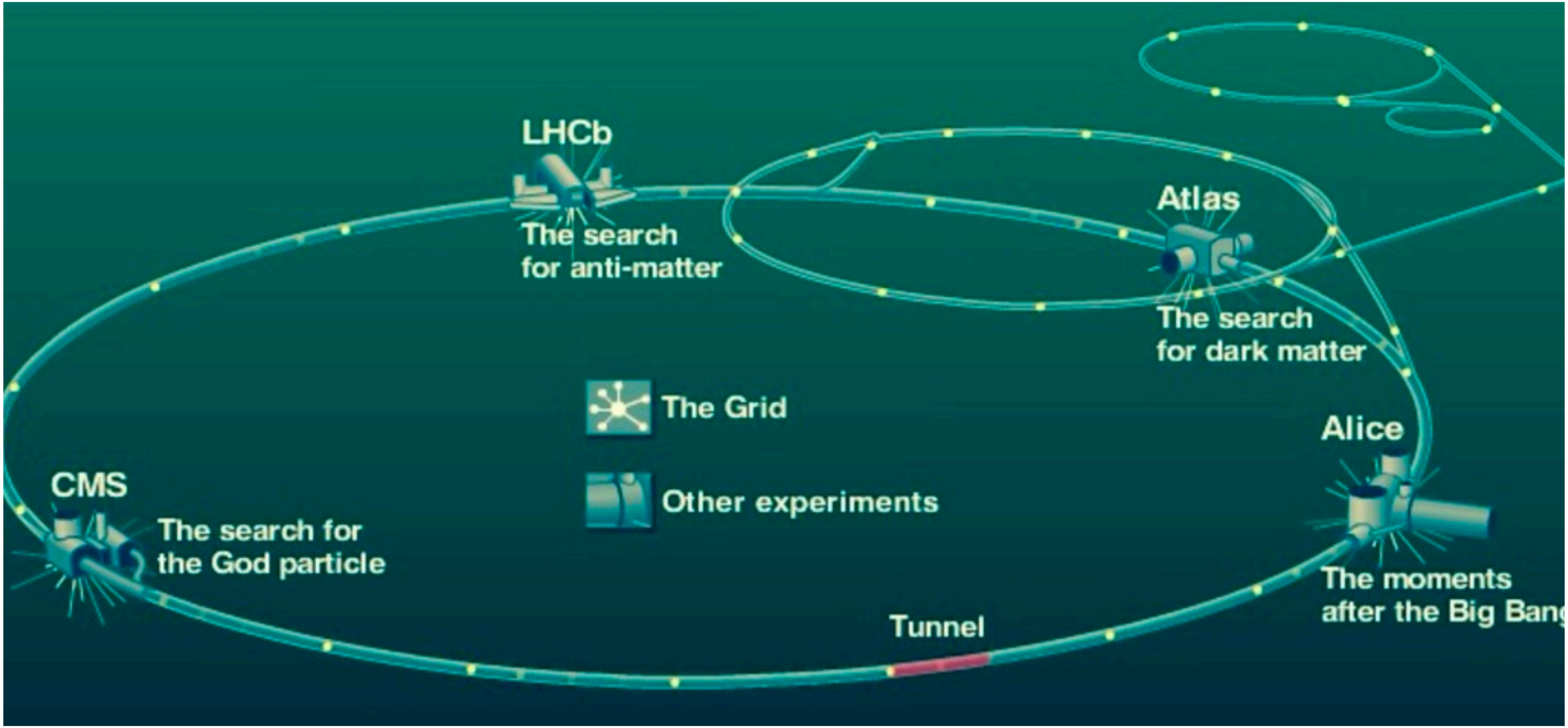


粒子物理的三个前沿

- 以研究方式分类
  - 能量前沿
  - 亮度前沿
  - 宇宙前沿



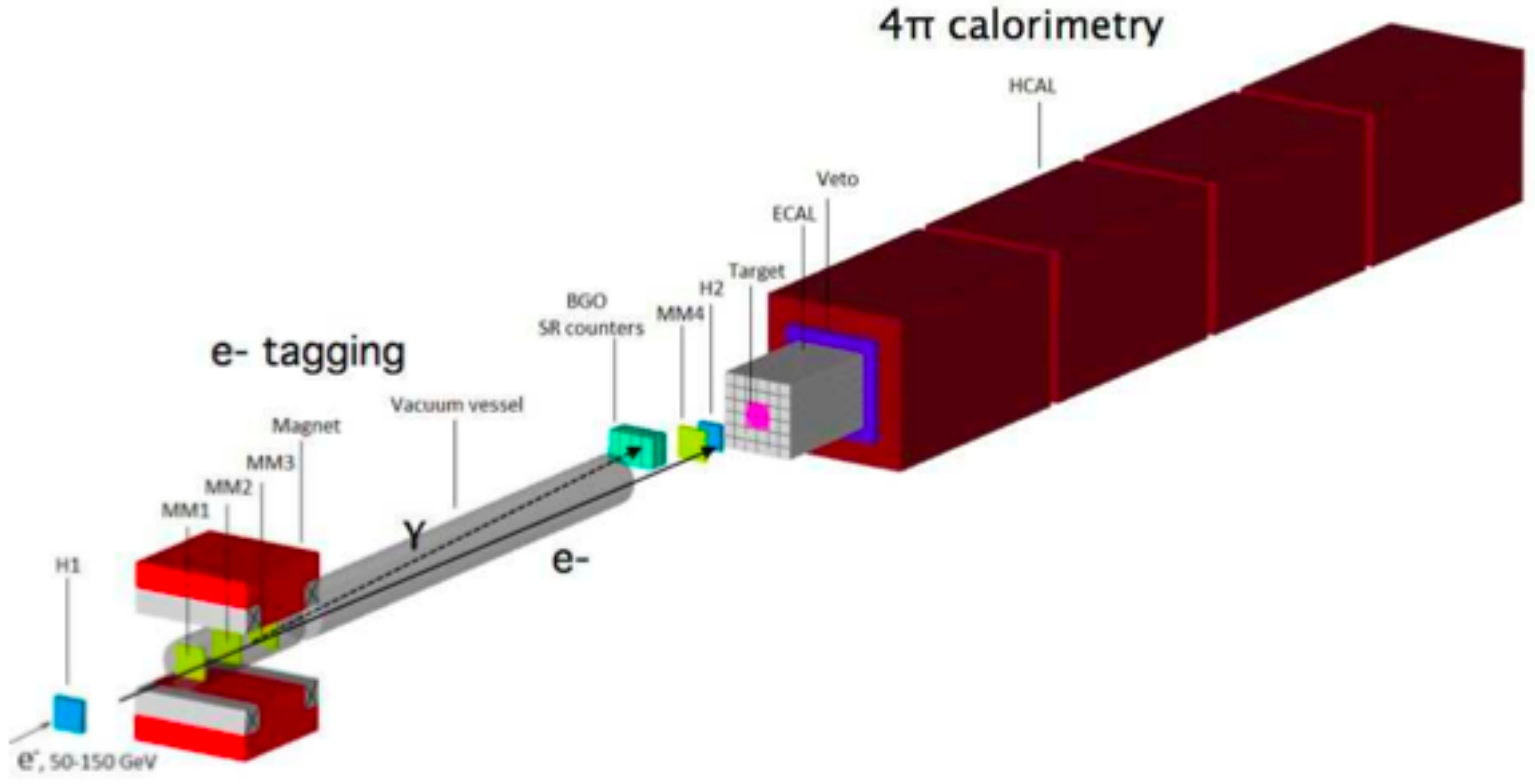
能量前沿：高能粒子对撞机物理



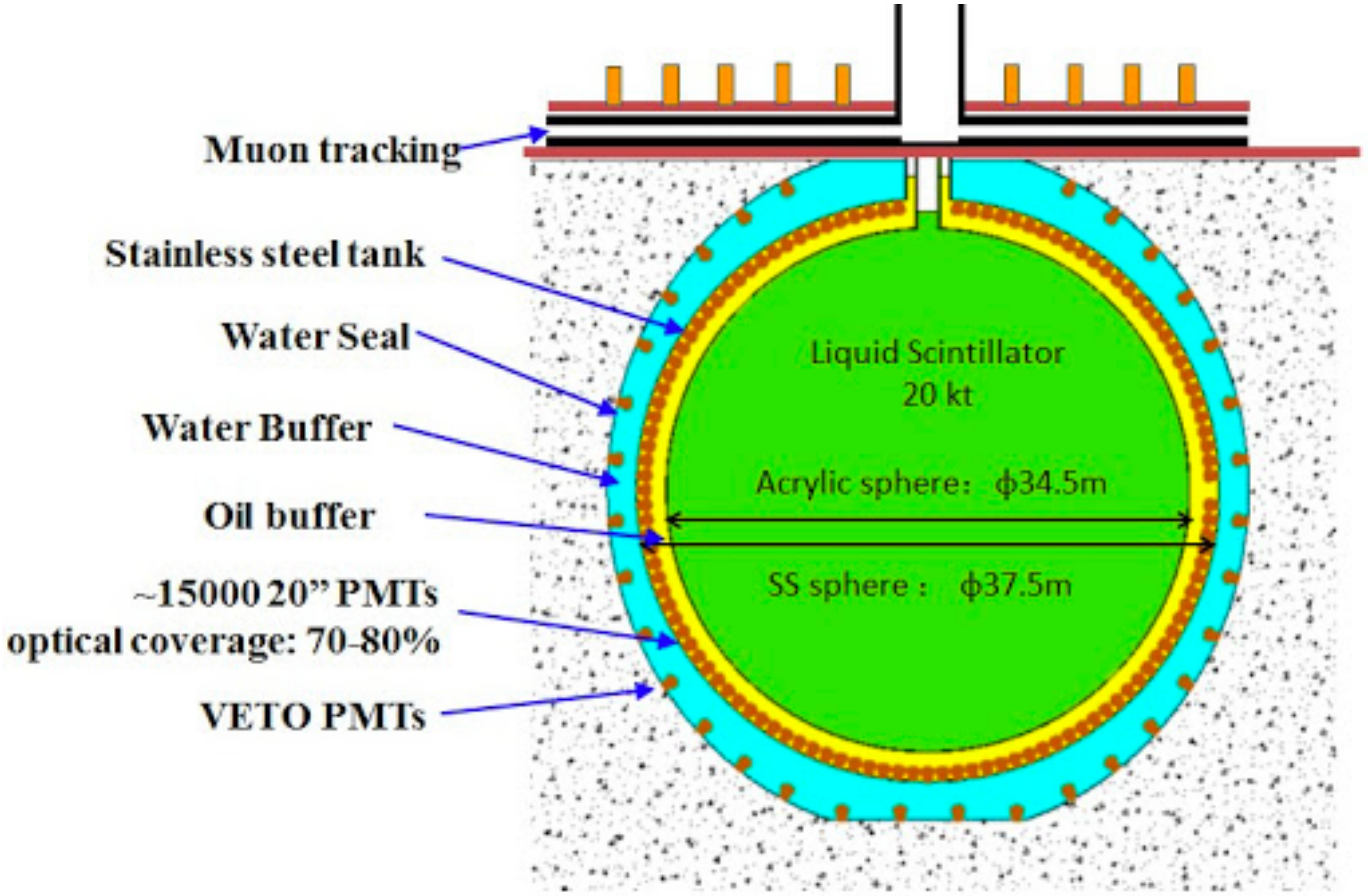
亮度前沿

- 低能量高亮度实验：实现精确测量

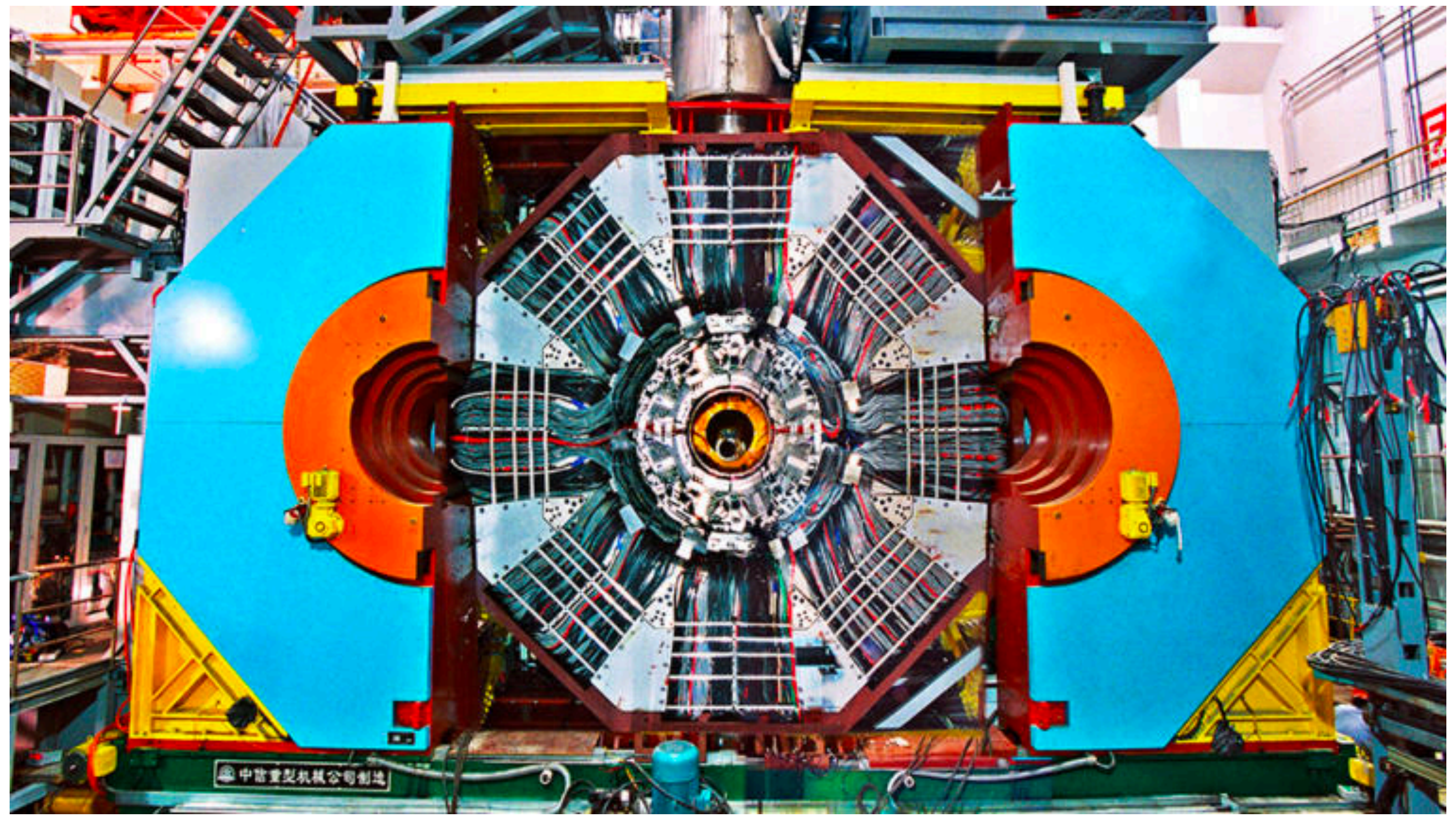
质子或电子固定靶实验

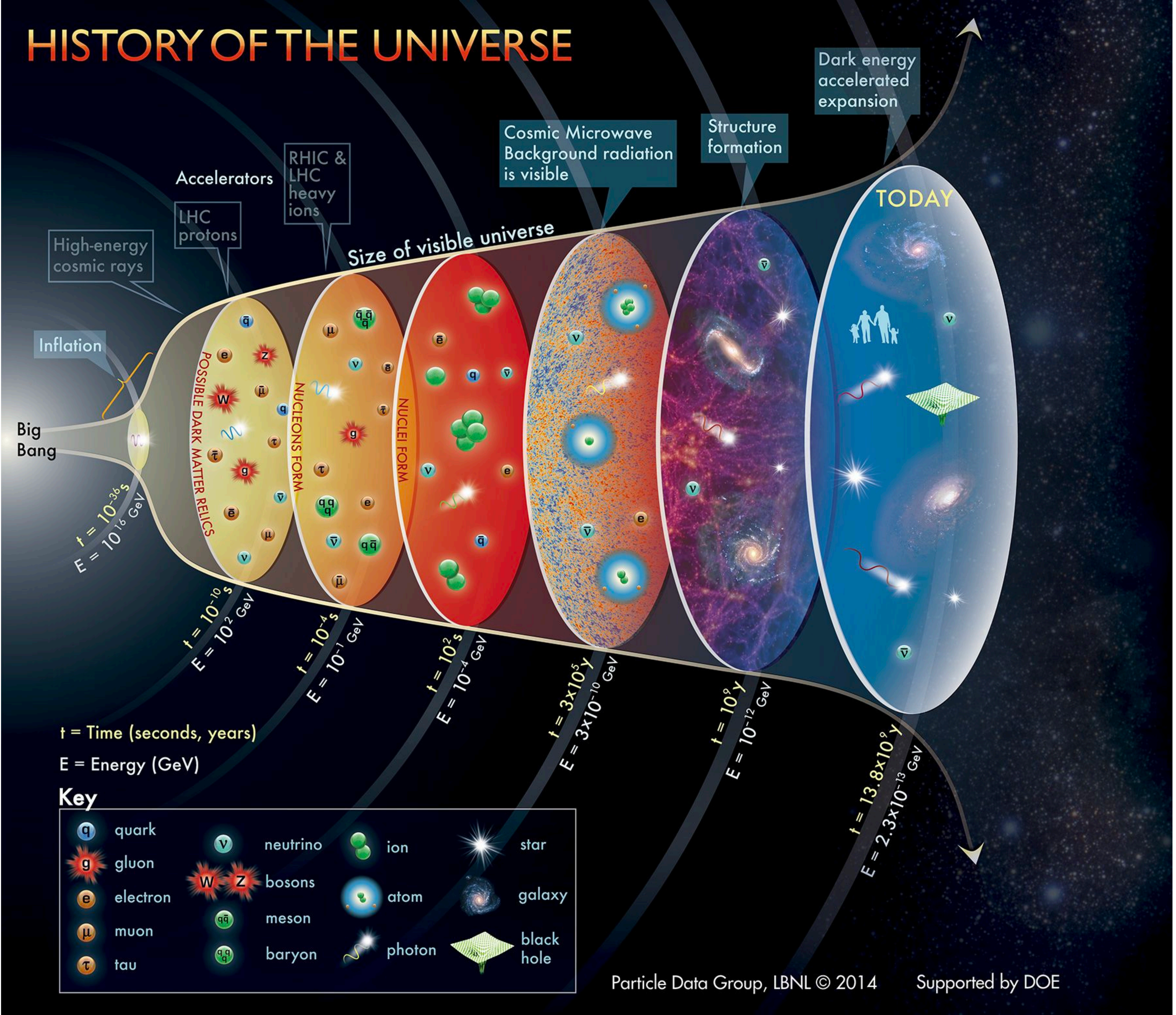


核反应堆中微子实验



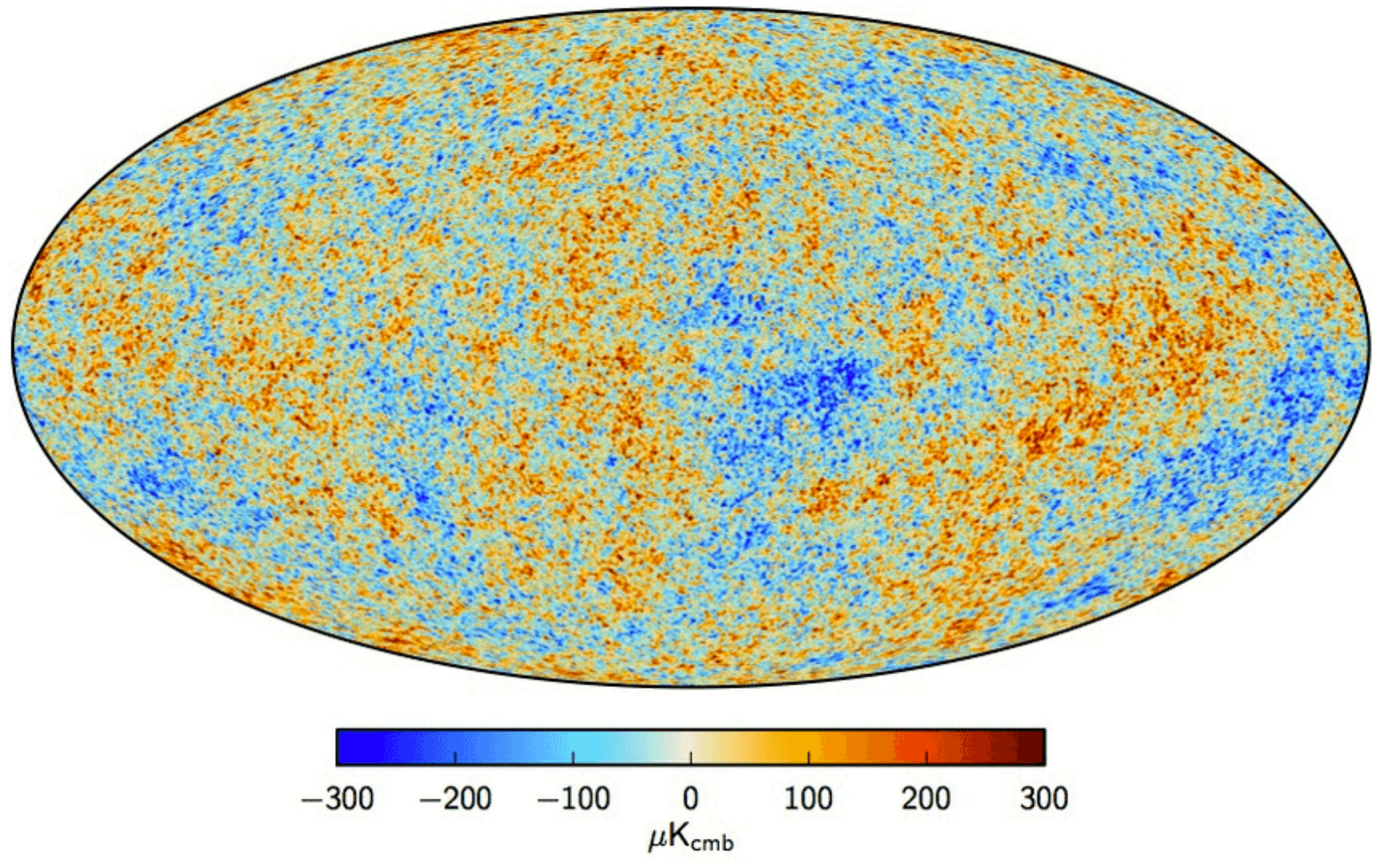
低能量高亮度正负电子对撞机



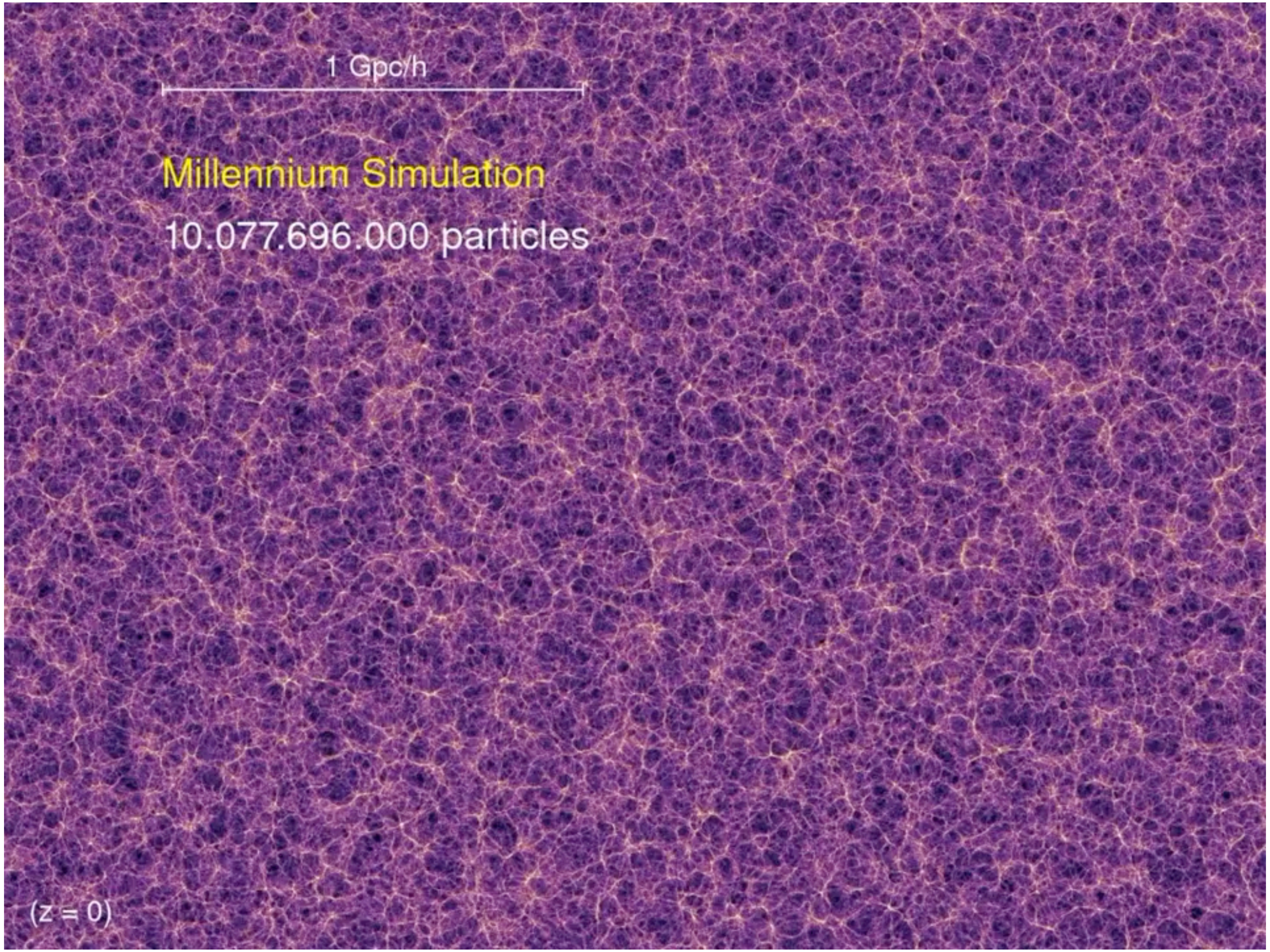


宇宙前沿

- 宇宙的6个参数：重子物质密度、暗物质密度、宇宙年龄…



宇宙前沿



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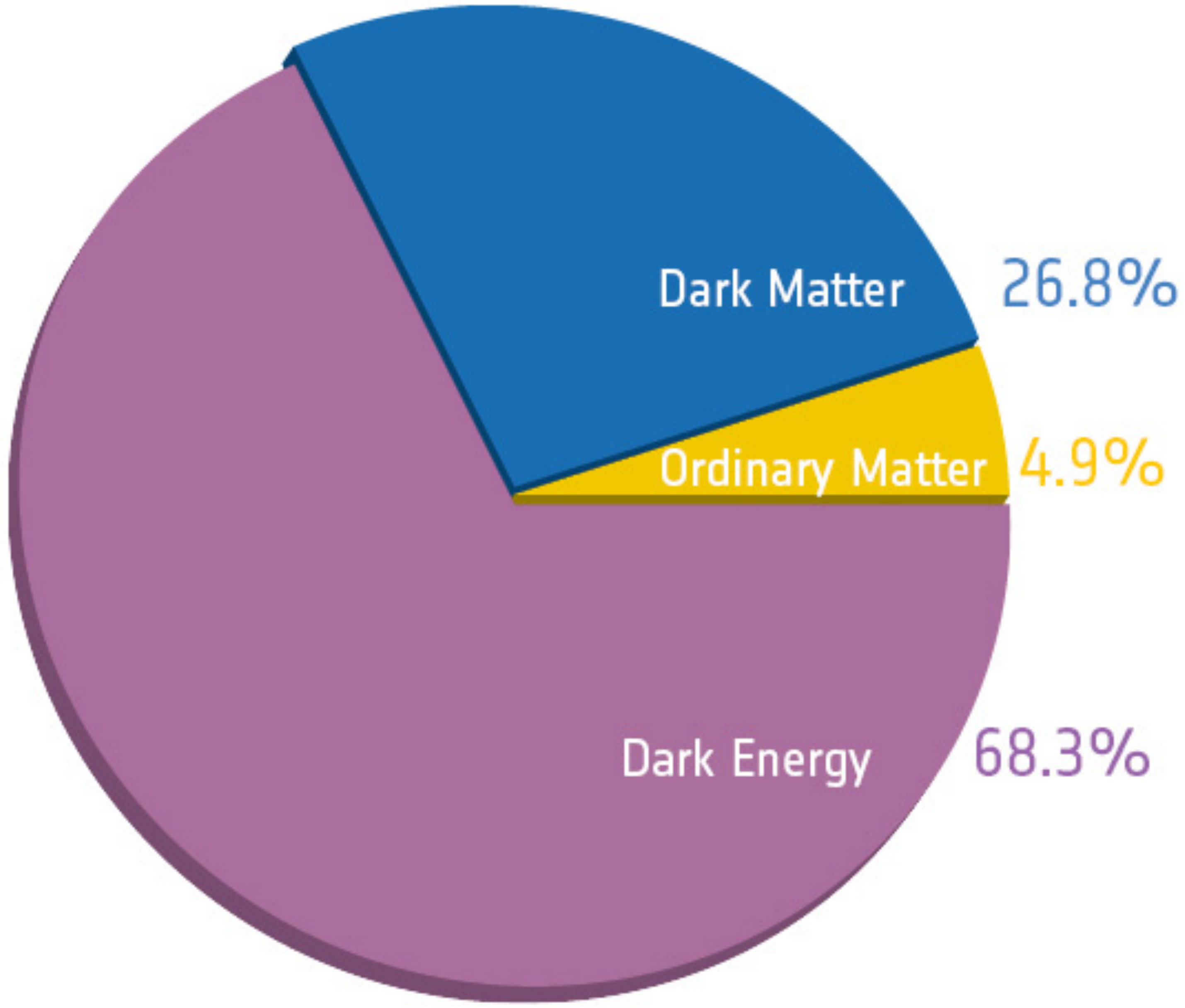
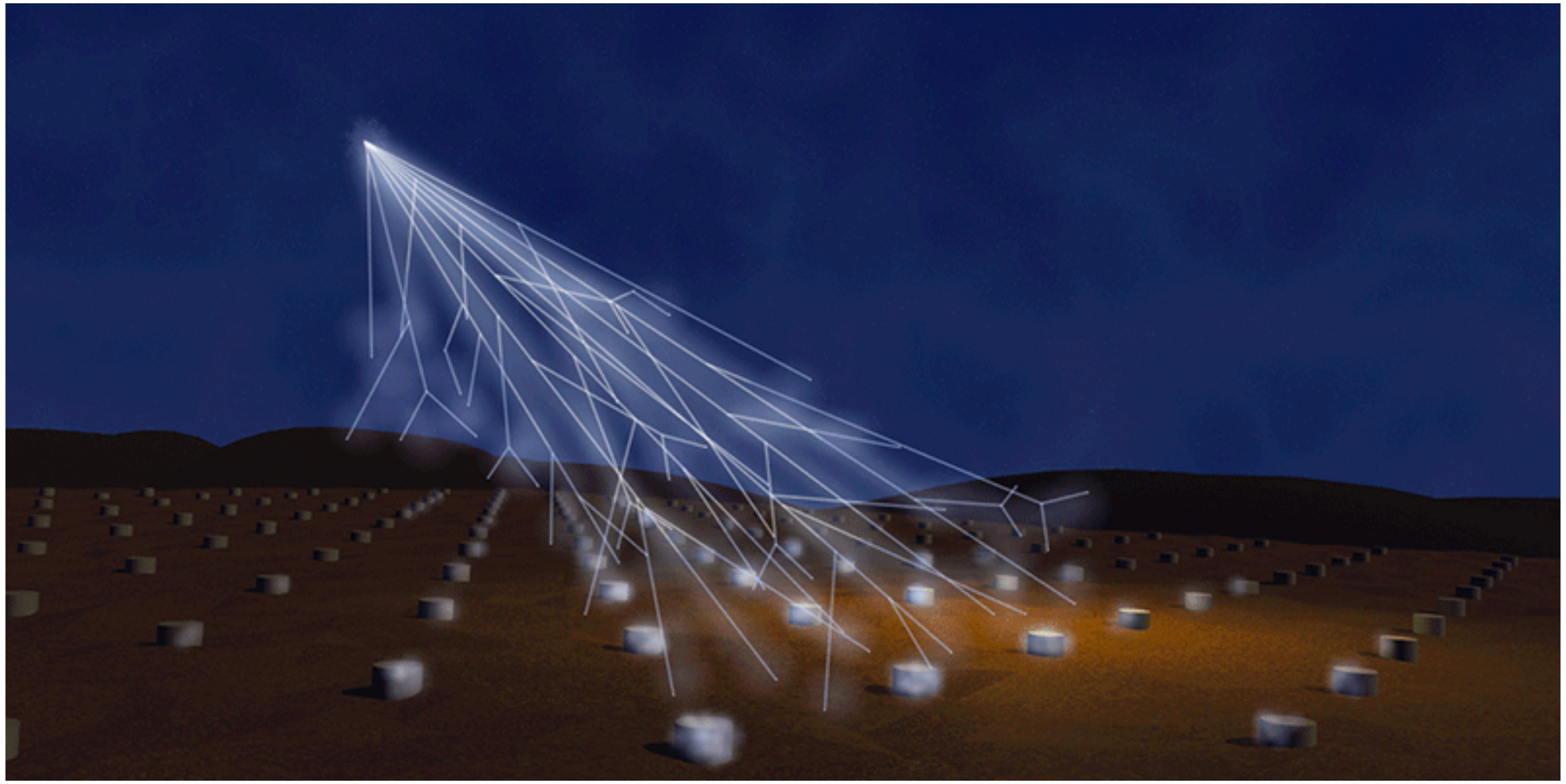


宇宙前沿

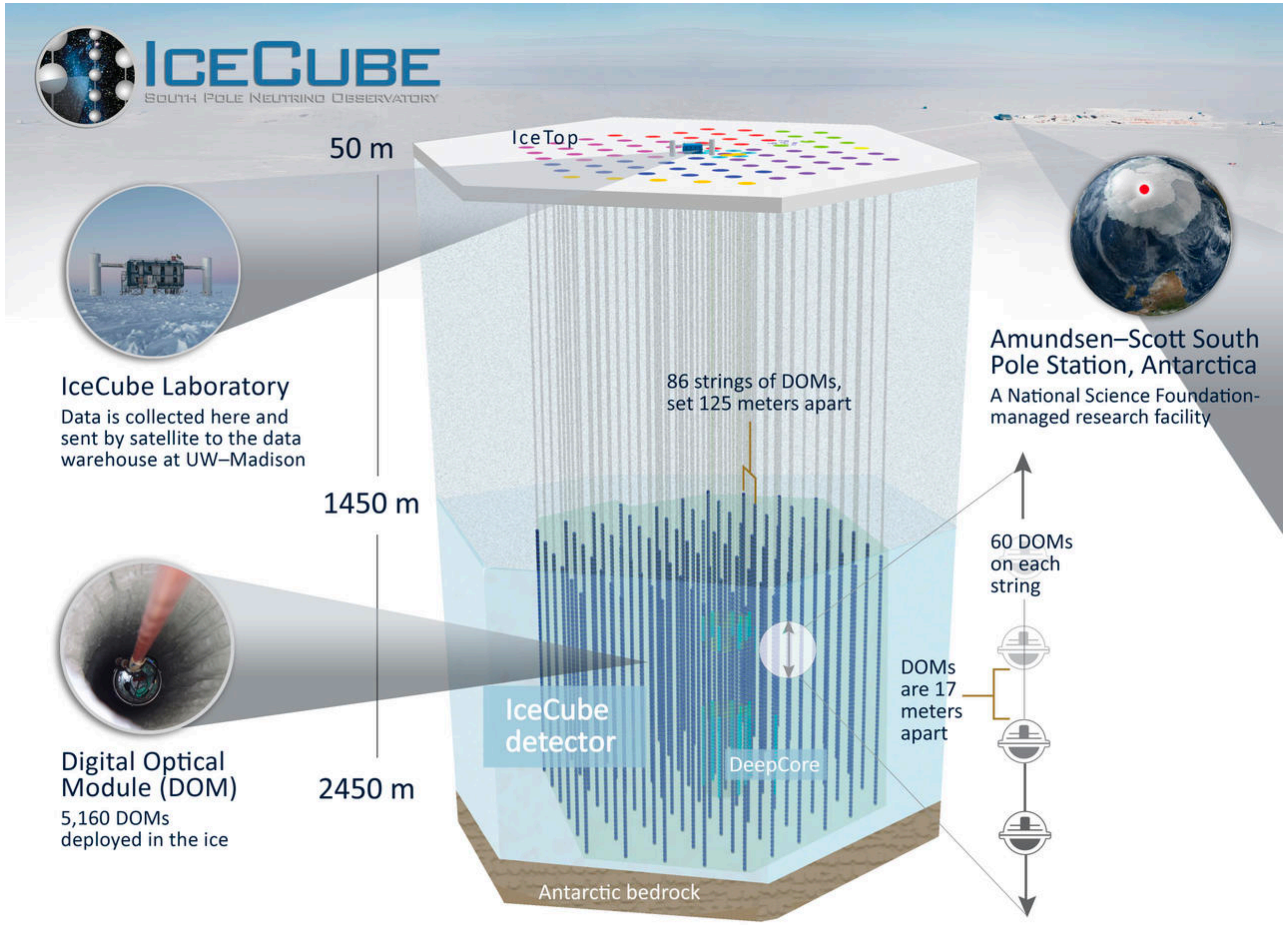


- 高能宇宙射线探测

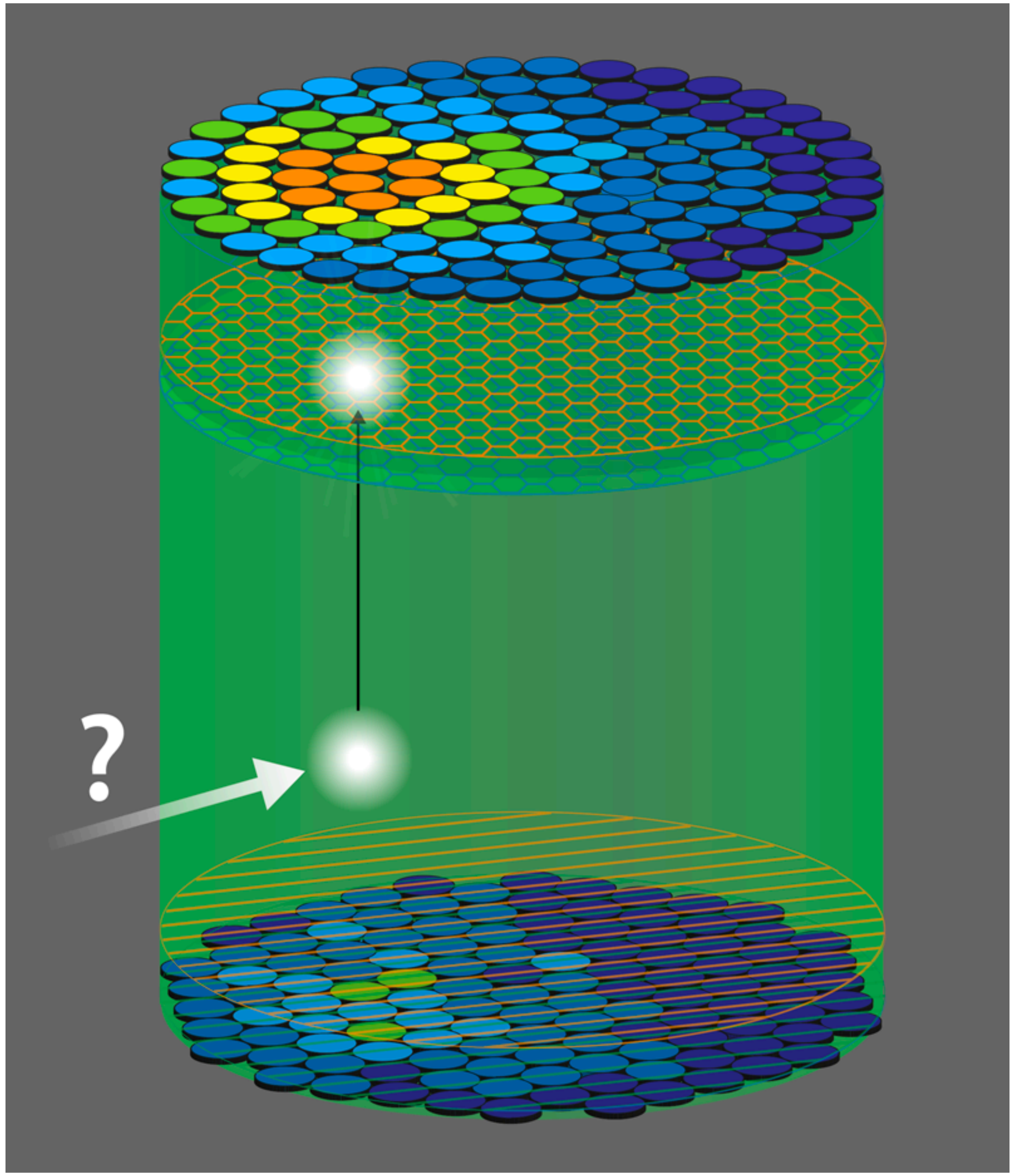
- 高能光子、正负电子探测

宇宙前沿

- 极高能中微子探测



- 暗物质直接探测



以问题导向分类：超出标准模型的新物理?

提纲

- 不知道什么 （已知的未知）
  - 粒子物理的三个前沿
  - 暗物质问题
    - 暗物质的天文观测证据
    - 暗物质分布
    - 暗物质的物理模型
      - 可能的暗物质候选者
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什么是暗物质?

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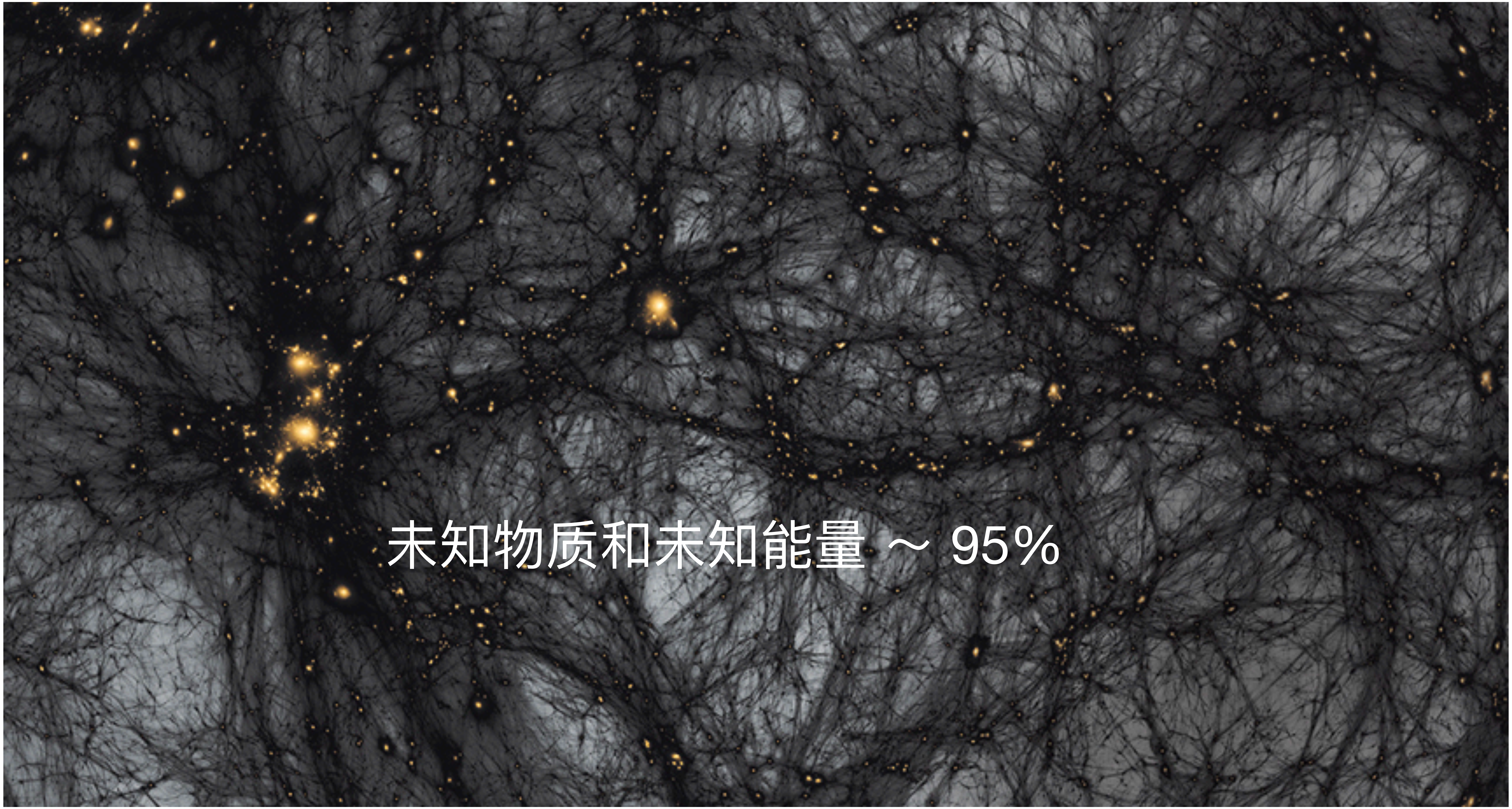
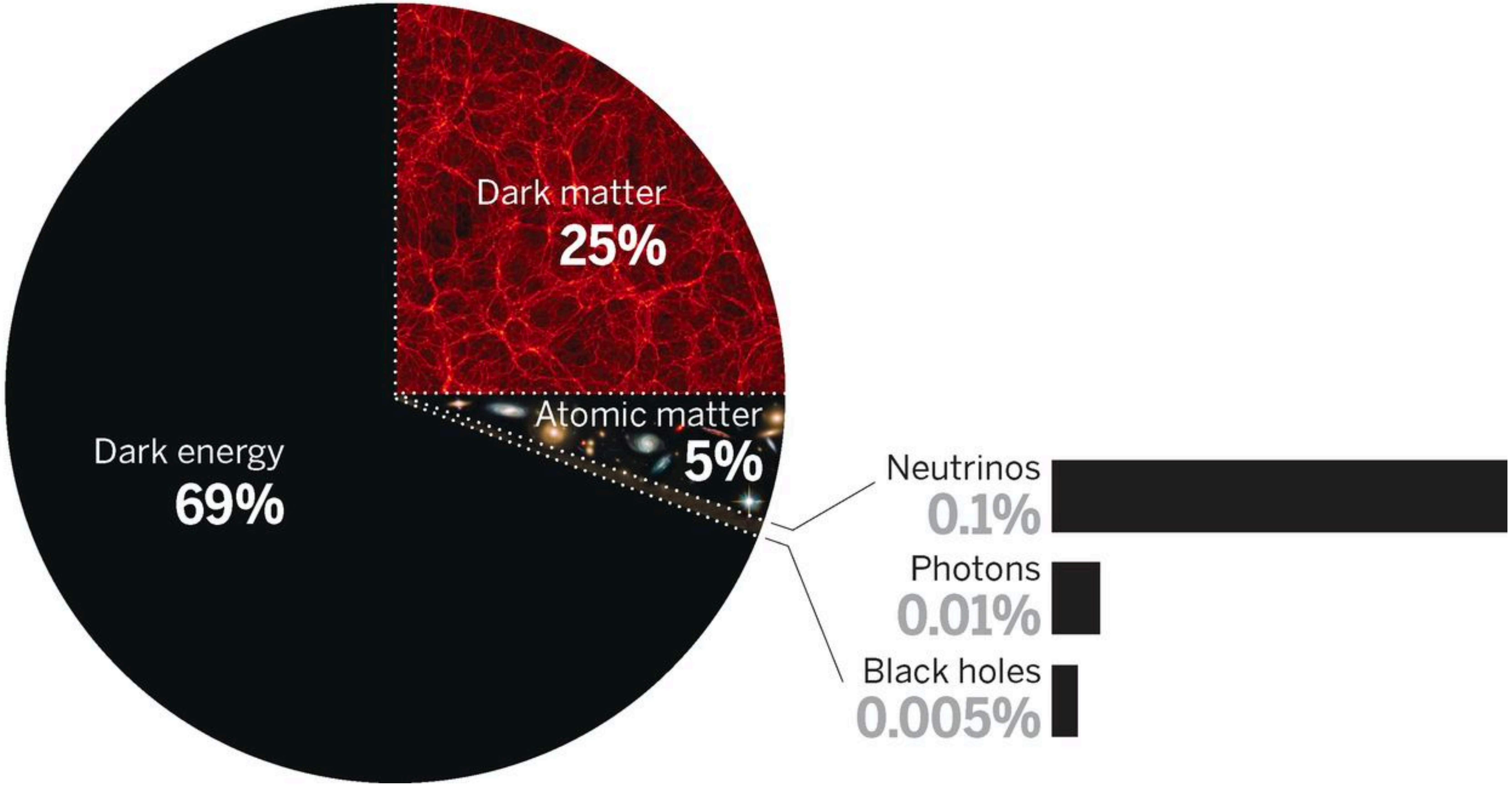


什么是暗物质?

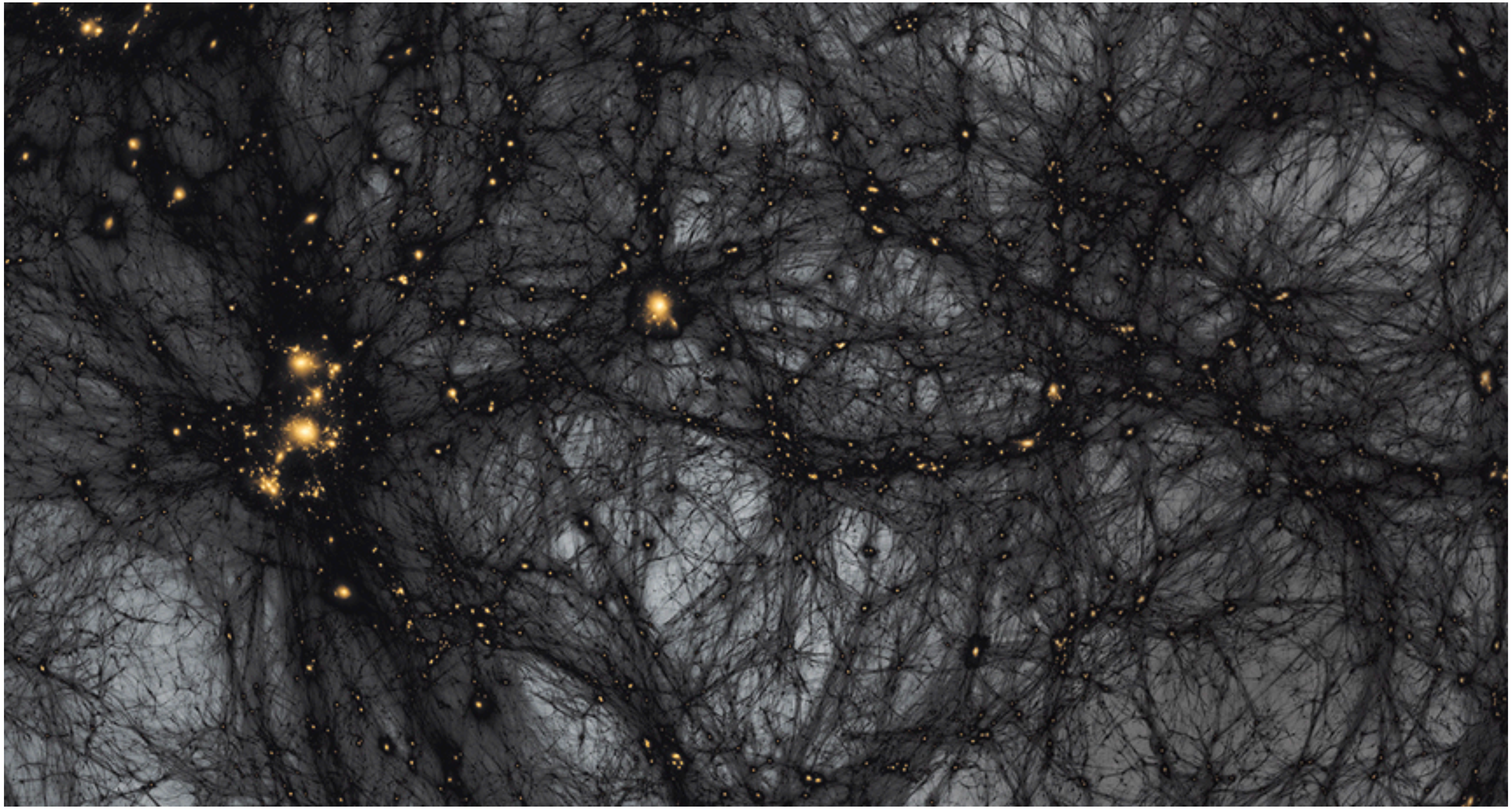
- 已知的可见物质 ~ 5%
- 未知物质和未知能量 ~ 95%



什么是暗物质?

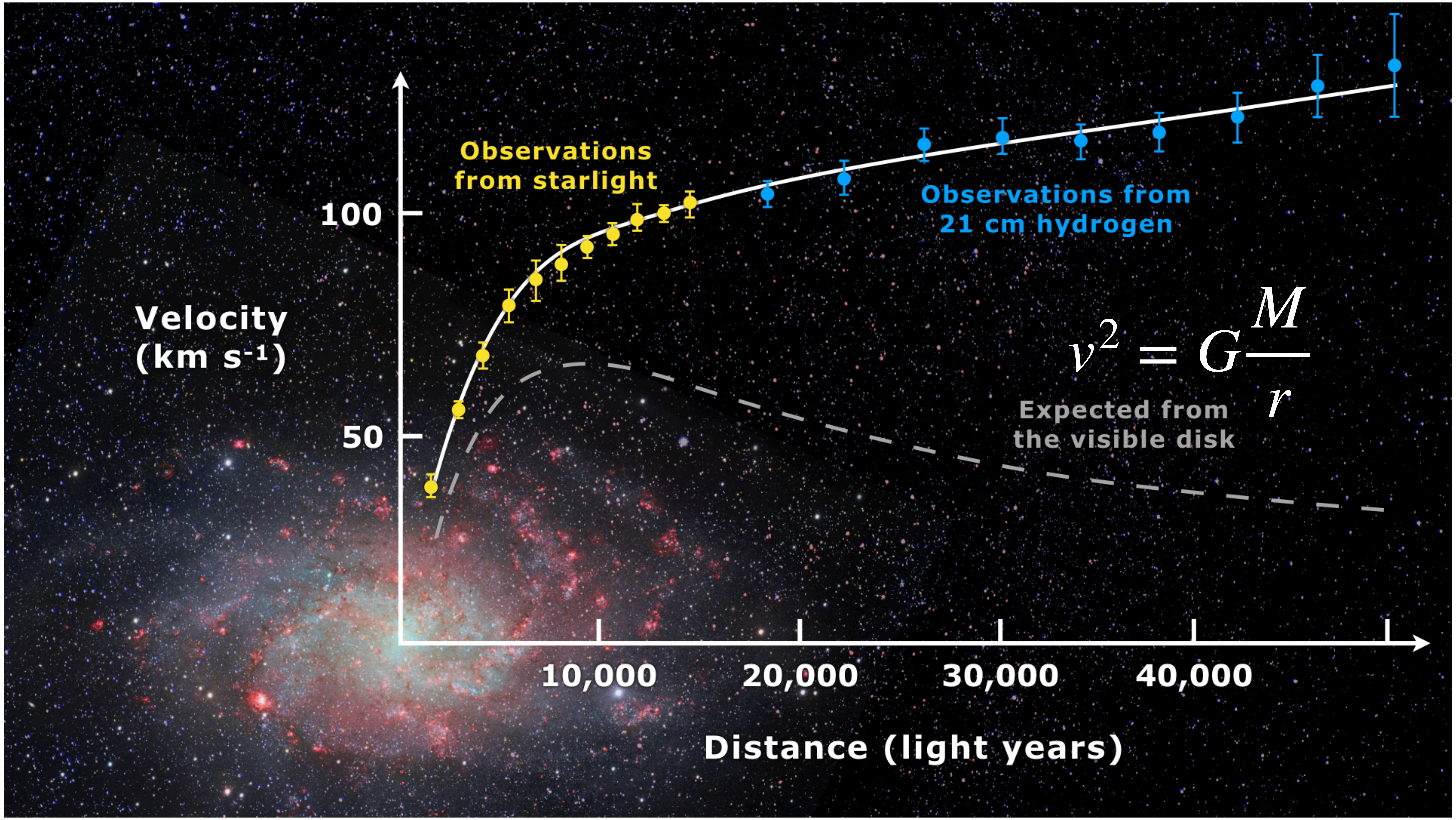
- 暗物质是理论上提出的可能存在于宇宙中的一种不可见的物质，它可能是宇宙物质的主要组成部分，但又不属于构成可见天体的任何一种已知的物质。

- 中性不带电
- 和可见物质相互作用小的
- 稳定
- 有质量的
- 冷的



Observational evidence for DM

- Galaxy rotation curves
- Bullet cluster
- Gravitational Lensing
- Structure formation
- Cosmic Microwave Background



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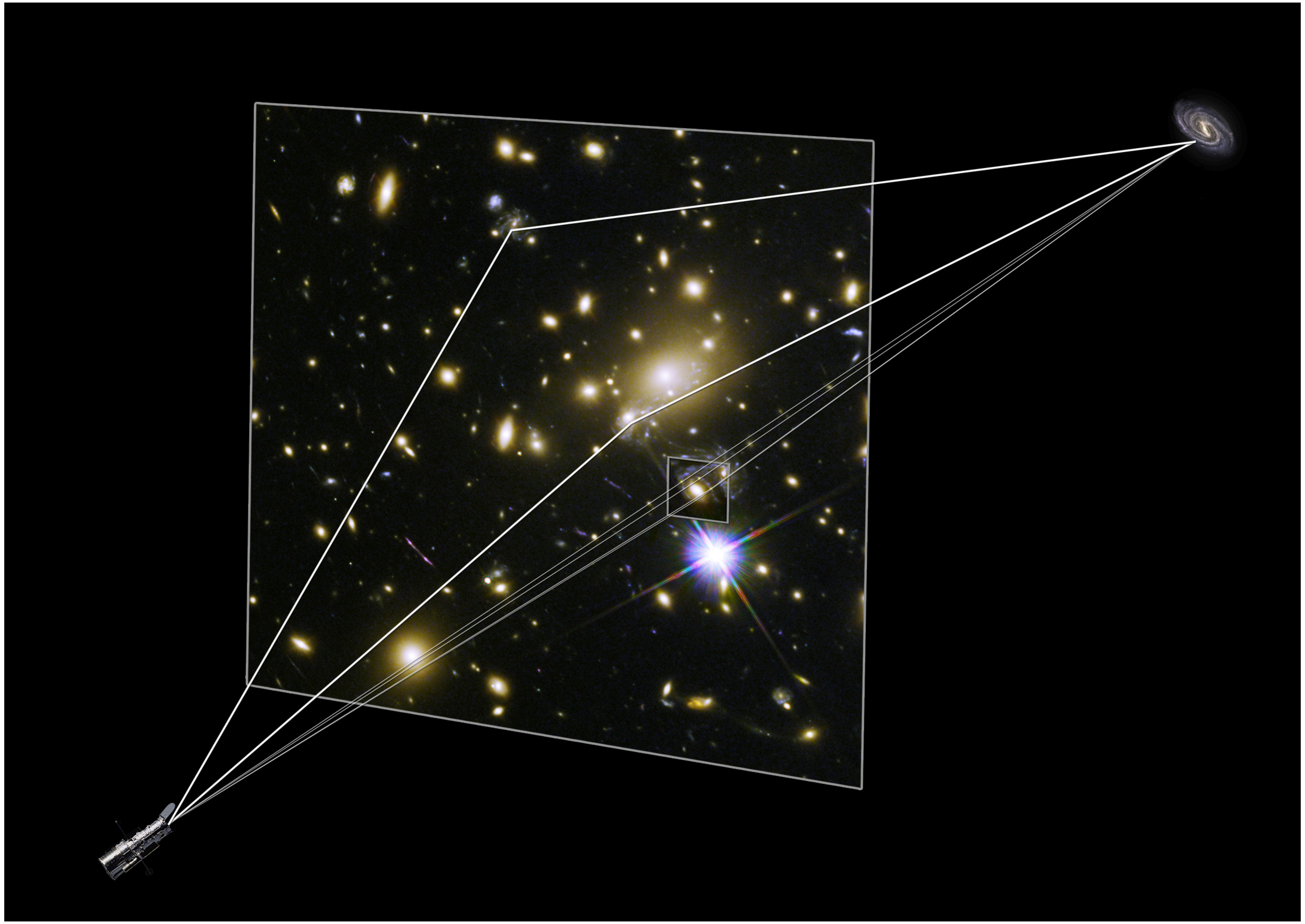

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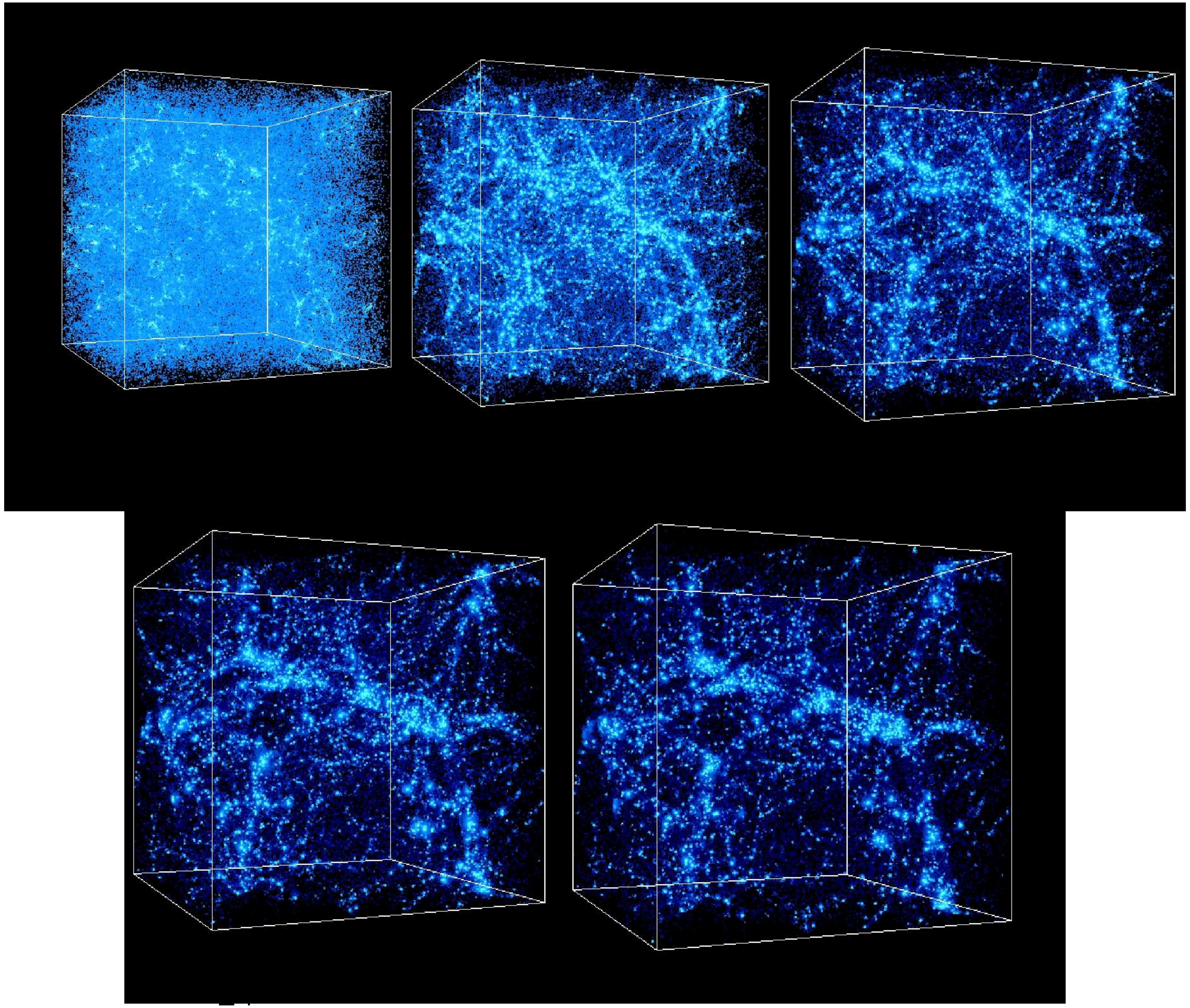

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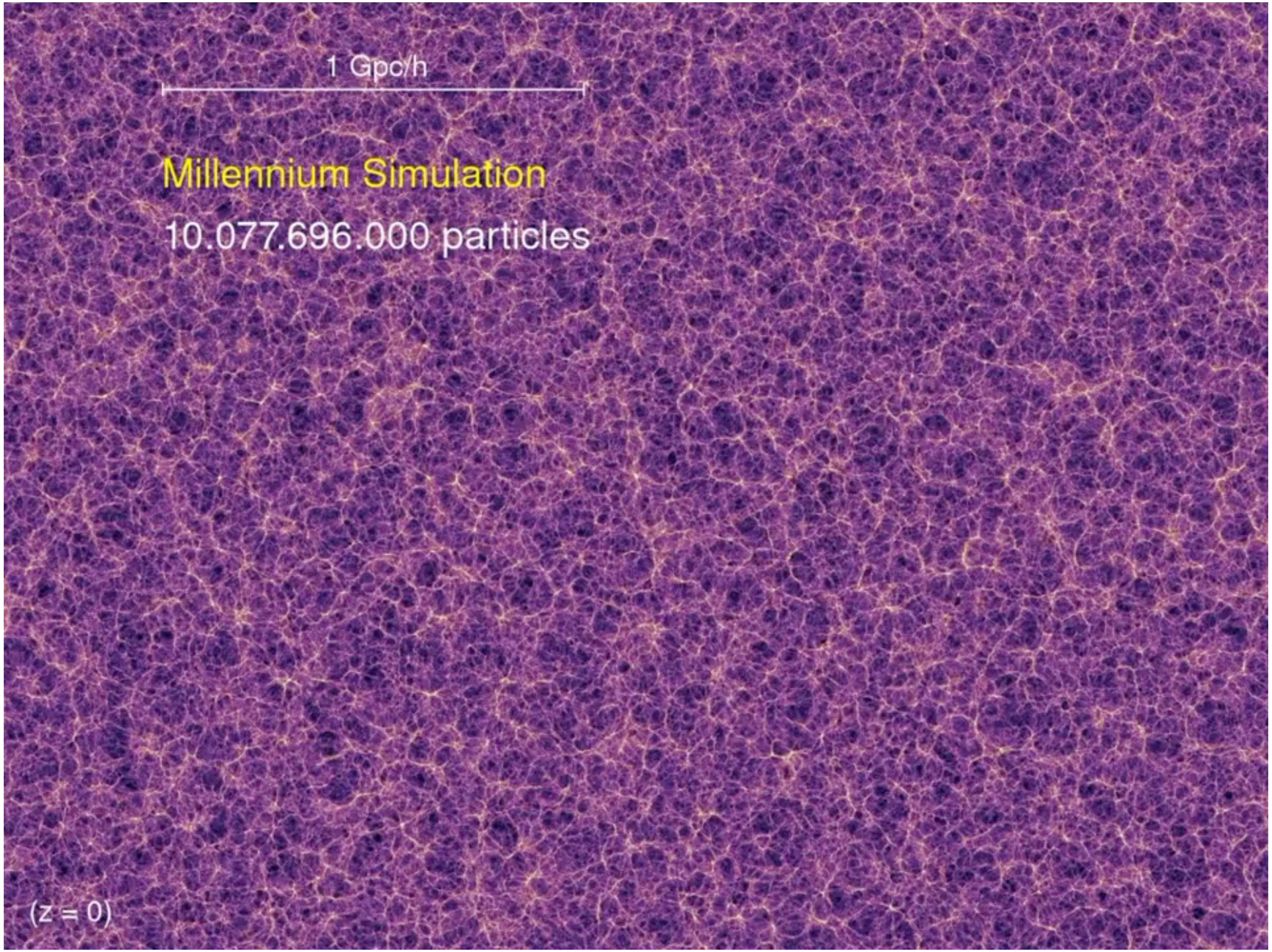
The *Millennium Run* used more than 10 billion particles to trace the evolution of the matter distribution in a cubic region of the Universe over 2 billion light-years on a side.



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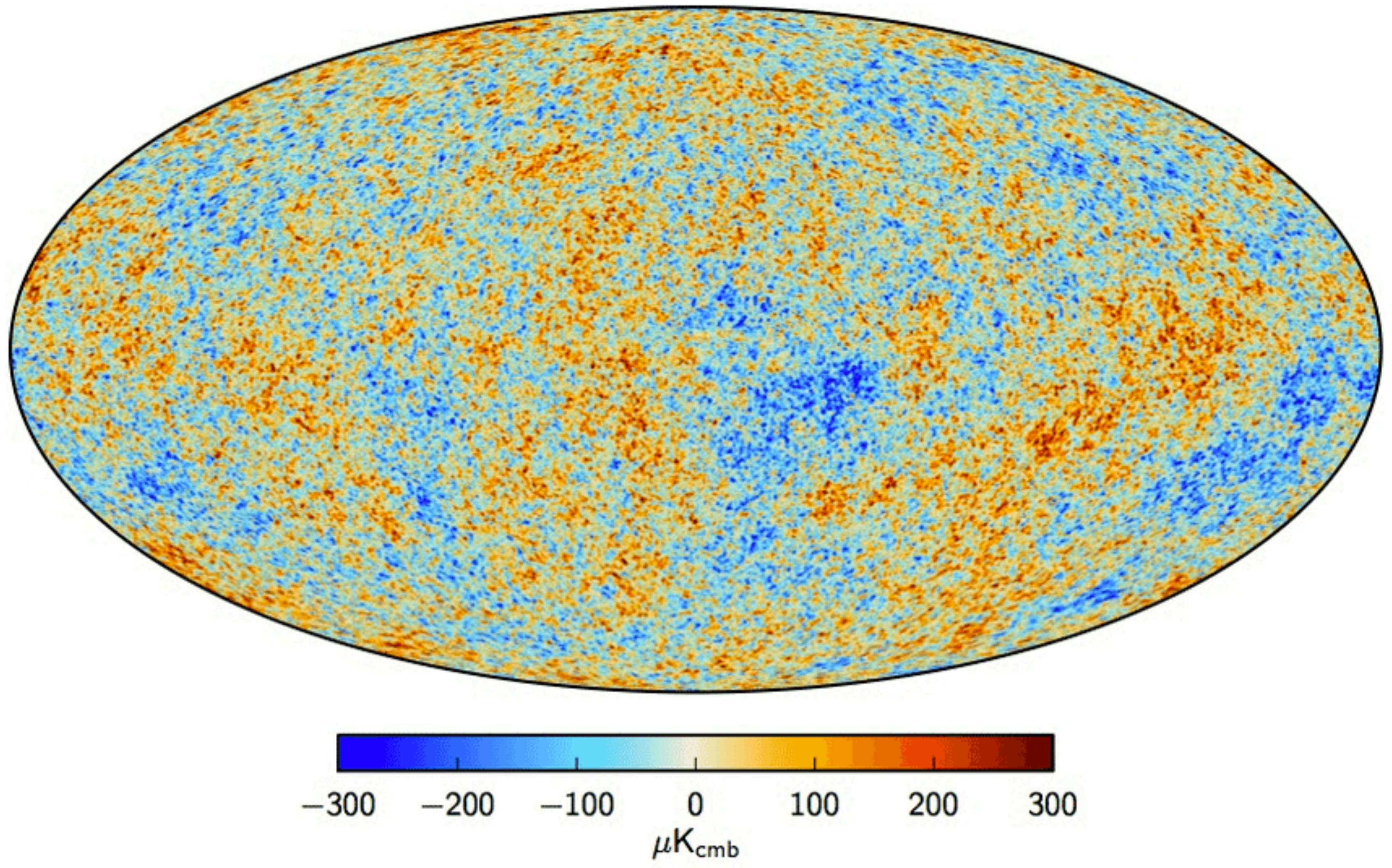
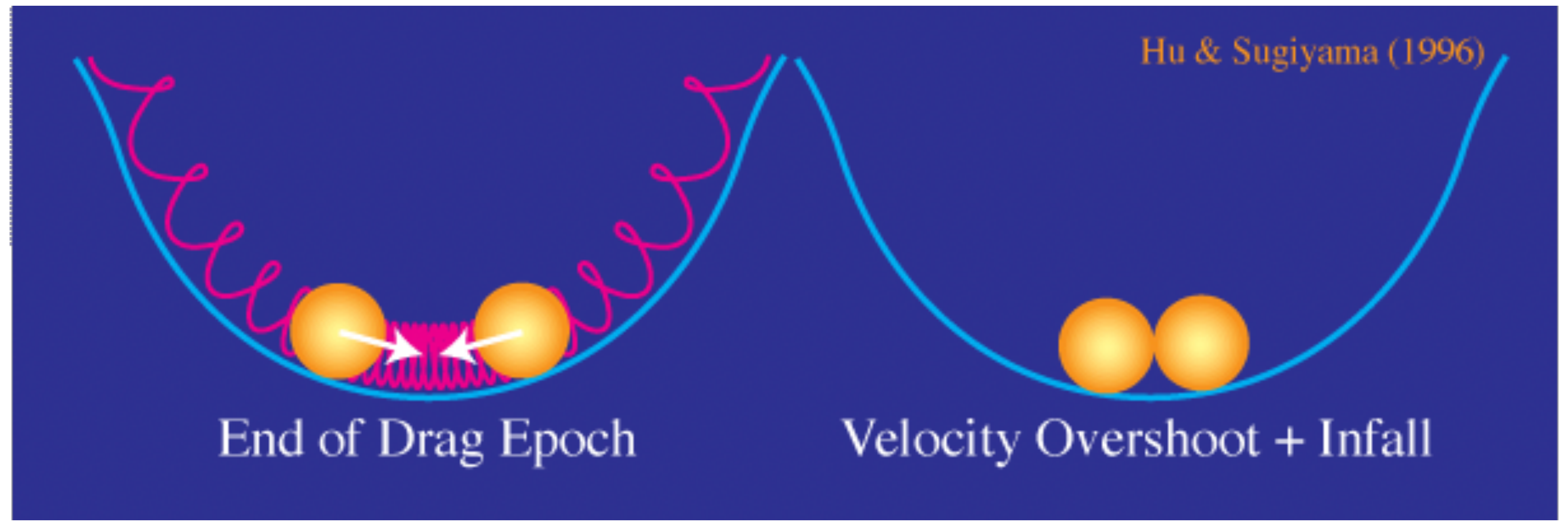
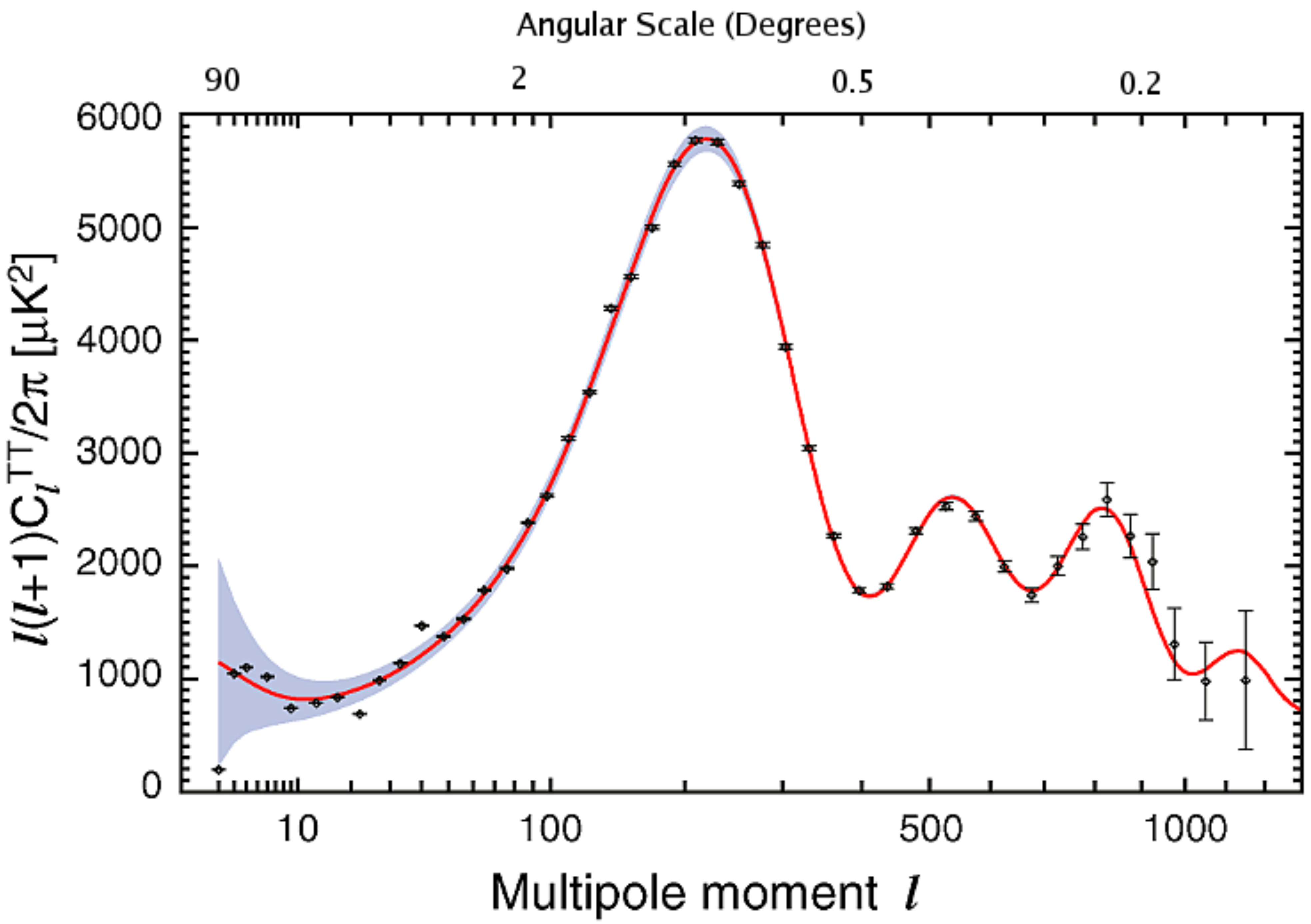
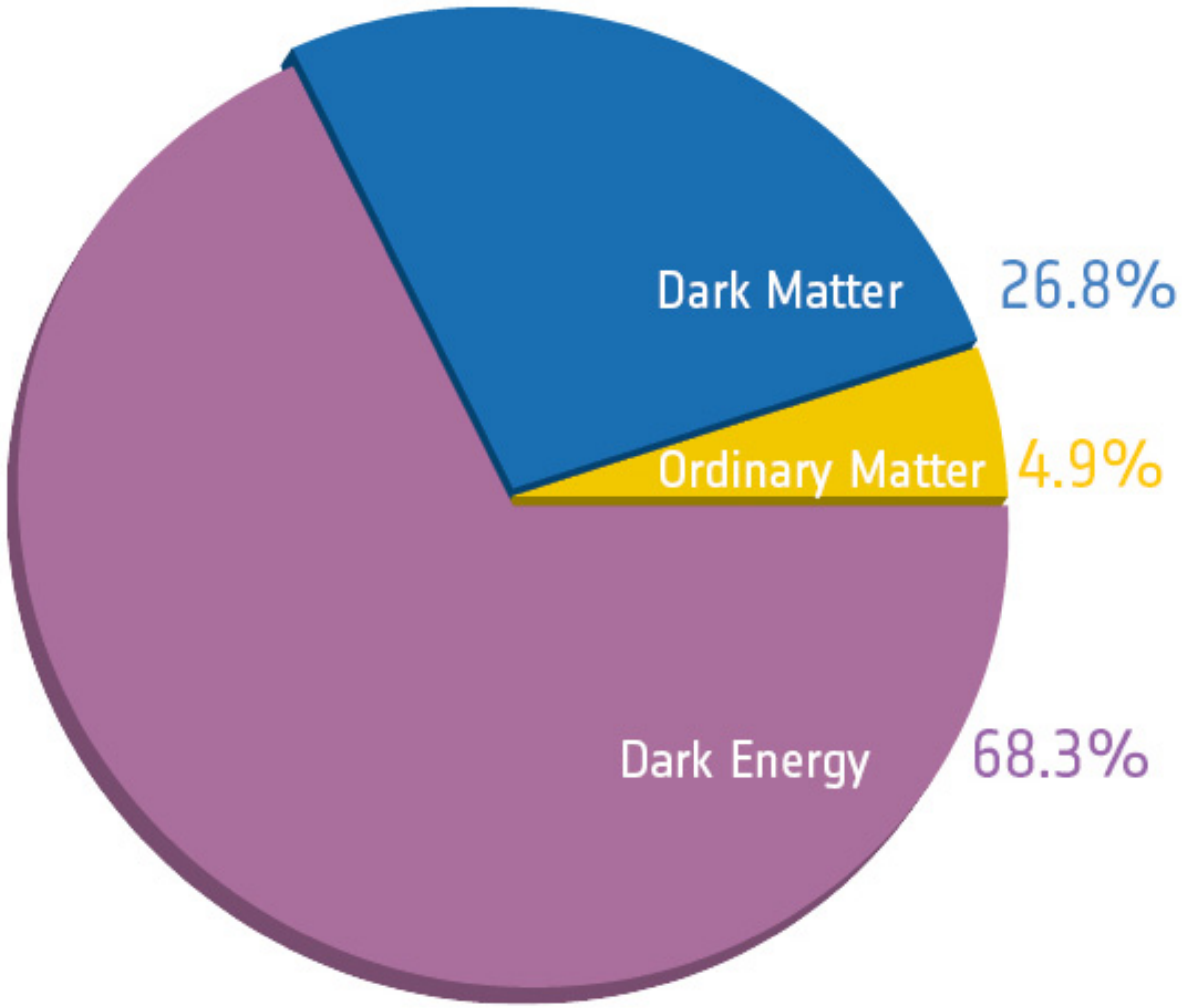






The success of the Lambda cold dark matter Model

- The standard model of Big Bang cosmology
  - Λ, dark energy; CDM, cold dark matter; Matter, SM particles



The dark matter in astrophysics/cosmology

- Energy density scales as $\rho \propto a^{-3}$, others $\rho_r \propto a^{-4}$, $\rho_{cc} \propto a^0$
- Massive, interacting gravitationally
- Neutral, not quite interacting with others, collision-less
- Stable
- Local DM energy density $\rho_{\rm DM} \sim 0.4\ \mathrm{GeV\ cm}^3$

HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Inflation
Big Bang
Size of visible universe
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
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STARGAZING LIVE

THE UNIVERSE THROUGH TIME

BBC TWO

The Open University

You can download the *Stargazing LIVE* Star Guide and find out more about free *Stargazing LIVE* events at bbc.co.uk/stargazing

UNOBSERVABLE UNIVERSE (PAST)

THE BEGINNING

The Universe begins 13.7 billion years ago with an event known as the Big Bang.

Both time and space are created in this event.

FRACTION OF A SECOND

Rapid expansion occurs during a billionth of a billionth of a billionth of a billionth of a second – the visible Universe is the size of a grapefruit.

1 SECOND

The Large Hadron Collider at CERN is recreating the conditions that prevailed a fraction of a second after the Big Bang.

100 – 1000 SECONDS

Nuclei of hydrogen, helium, lithium and other light elements form.

300,000 YEARS

We can detect radiation from the early formation of the Universe back as far as this point. Before this, the Universe is opaque: it's as if a veil has been pulled over it.

POTENTIALLY OBSERVABLE UNIVERSE (PAST)

A FEW HUNDRED MILLION YEARS

Matter clumps together under its own gravity forming the first protogalaxies and within them, the first stars.

Stars are nuclear furnaces in which heavier elements such as carbon, oxygen, silicon and iron are formed. Massive stars exploding as supernovae create even heavier elements. Such explosions send material into space ready to be incorporated into future generations of stars and planets.

A FEW BILLION YEARS

Initially, the expansion of the Universe decelerated – but a few billion years after the Big Bang, the expansion began to accelerate. The acceleration is caused by a mysterious force known as 'dark energy', the nature of which is completely unknown.

9 BILLION YEARS

The Sun, along with its eight planets, and all the asteroids, comets and Kuiper Belt objects, such as Pluto, form from the debris left behind by earlier generations of stars.

10 BILLION YEARS

The first life appears on Earth in the form of simple cells. Impacting comets and asteroids might have contributed organic molecules to Earth. Life spreads across the globe.

TODAY

13.7 BILLION YEARS

This is where we are today. Using our own ingenuity, humanity is probing the depths of the Universe and trying to unravel its mysteries, from our tiny, home planet, Earth. The visible Universe contains billions of galaxies, each comprising billions of stars. Within our own Galaxy, hundreds of exoplanets have been discovered orbiting other stars.

FUTURE

20 BILLION YEARS

In a few billion years the Sun's outer layers will expand as it turns into a Red Giant star. Life on Earth will become impossible.

Expansion of the Universe will continue to accelerate.

10^{100} YEARS

Stars no longer form; matter is trapped in black holes or dead stars. Protons decay and black holes evaporate, leaving the Universe to its ultimate fate as cold, dead, empty space, containing only radiation, which itself too will eventually disperse.

Stargazing LIVE is a BBC and Open University co-production. Credit: Photography sourced from NASA.

提纲

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    - 暗物质的天文观测证据
    - 暗物质分布
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        - 热退耦暗物质残余丰度计算

The dark matter distribution

- Astrophysicist knows the distribution of DM by simulation
- Navarro-Frenk-White profile: $$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$
- R_s is the "scale radius", $\{\rho_0, R_s\}$ varies from halo to halo
- Integrated mass: $$M = \int_0^{R_{\max}} 4\pi r^2 \rho(r)\, dr = 4\pi\rho_0 R_s^3 \left[\ln\left(\frac{R_s + R_{\max}}{R_s}\right) + \frac{R_s}{R_s + R_{\max}} - 1\right]$$
  - Virial radius R_{vir}: $R_{\text{vir}} = cR_s$, with c called "concentration parameter"
  - Typical c: Milky Way 10~15, others 4~40 for various size of halos
  - Total mass within R_{vir}: $$M = \int_0^{R_{\text{vir}}} 4\pi r^2 \rho(r)\, dr = 4\pi\rho_0 R_s^3 \left[\ln(1 + c) - \frac{c}{1 + c}\right]$$

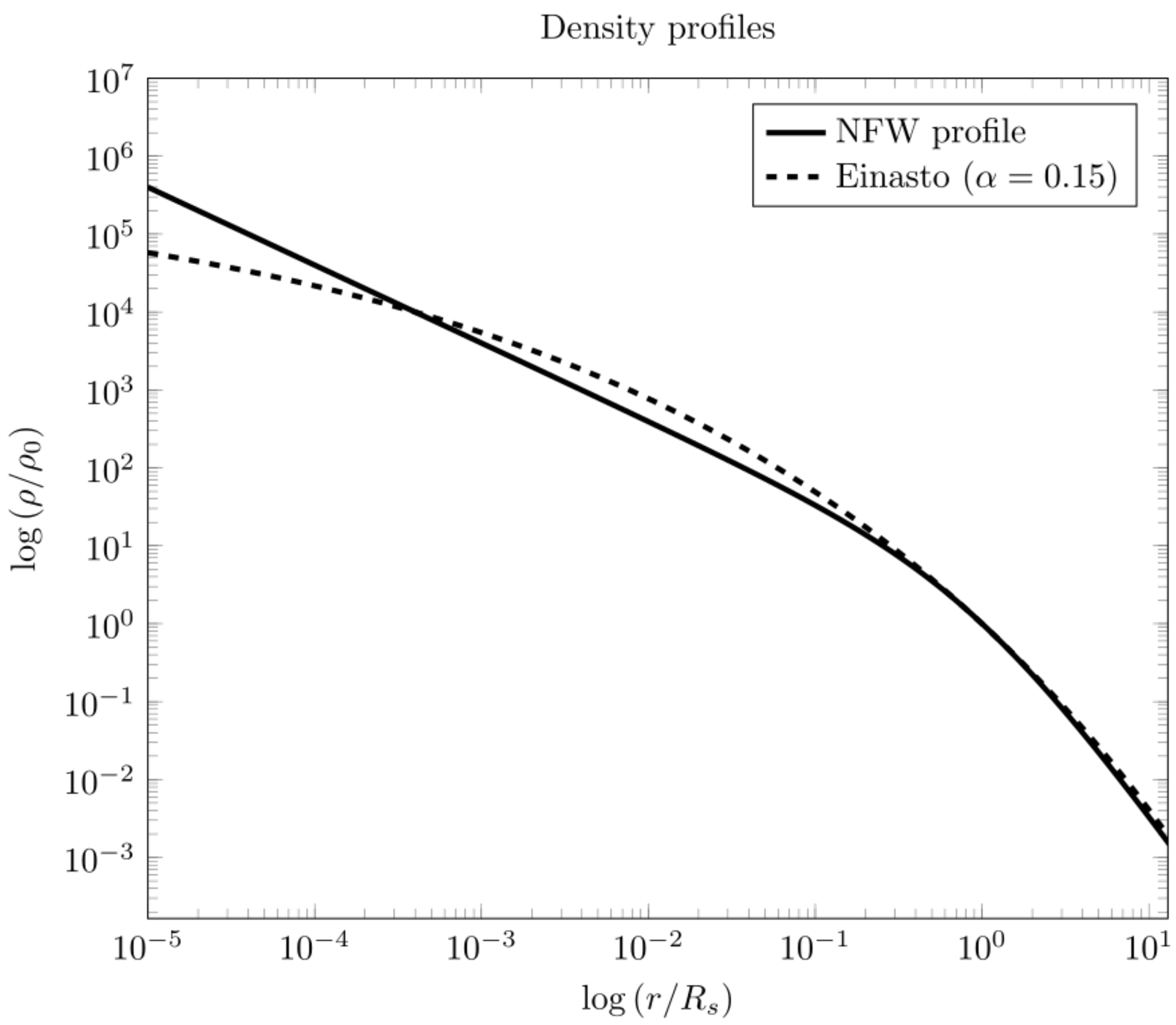
The dark matter distribution

- Astrophysicist knows the distribution of DM by N-body simulation

- Navarro-Frenk-White profile:

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$

- Other competing profile: Einasto



暗物质小尺度结构问题

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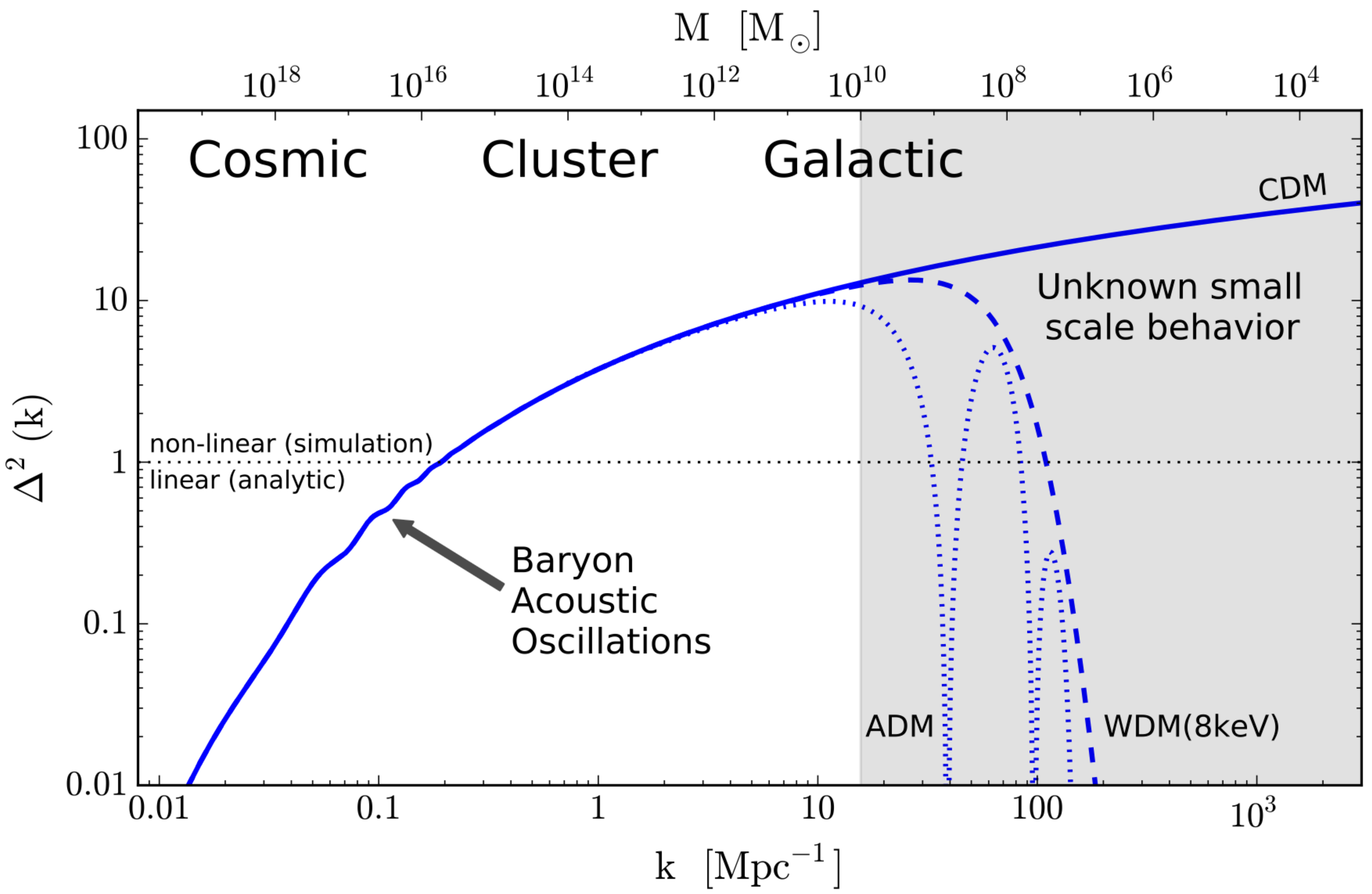
- CDM: very good for large scale, but problems at galactic scale



小尺度结构：尖峰-核分布问题

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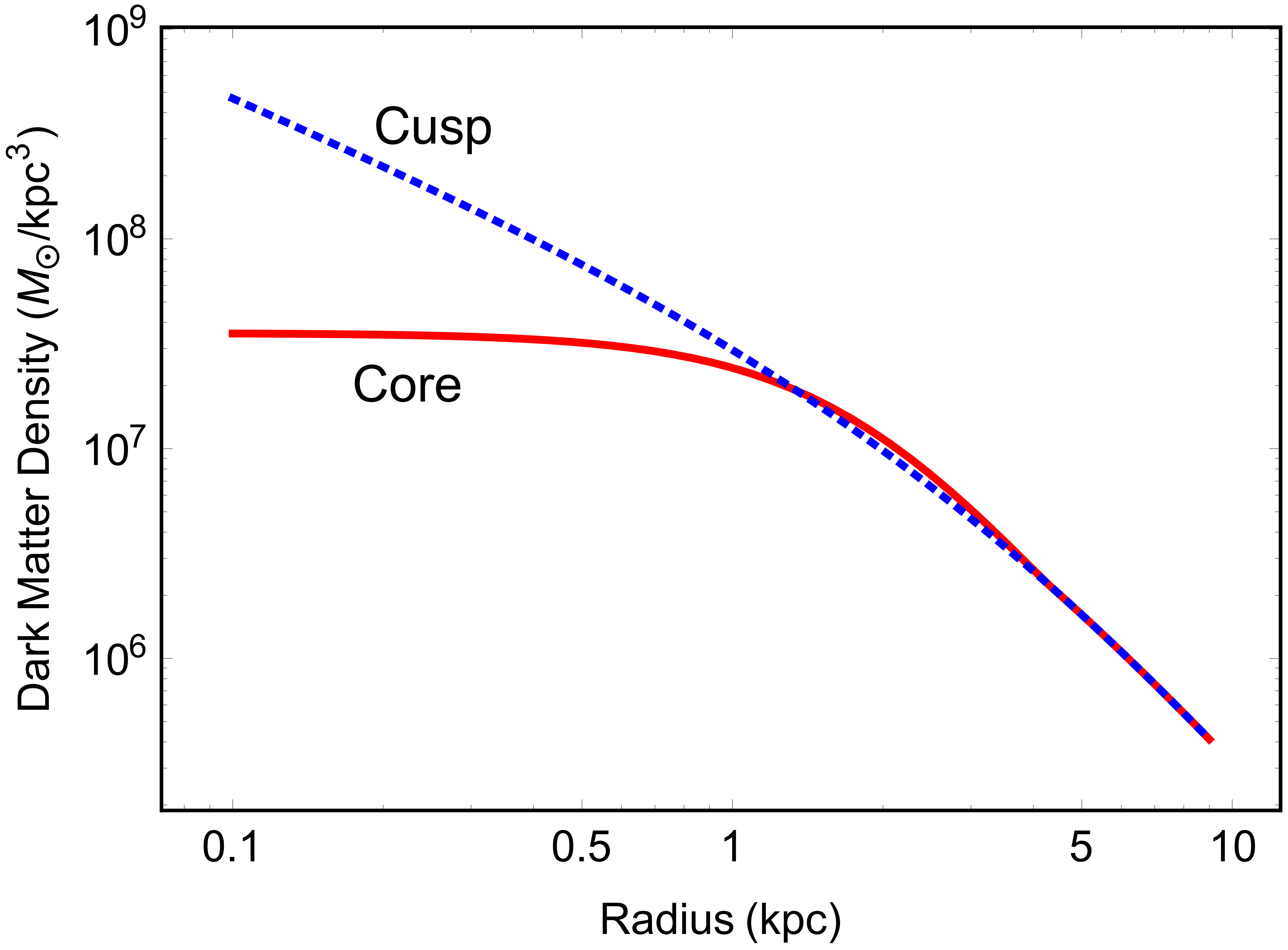
- Navarro-Frenk-White profile:
- $$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$
- CDM: very good for large scale, but problems at galactic scale
  - Core-Cusp problem of cold dark matter
  - Self-interacting DM as a possible solutions



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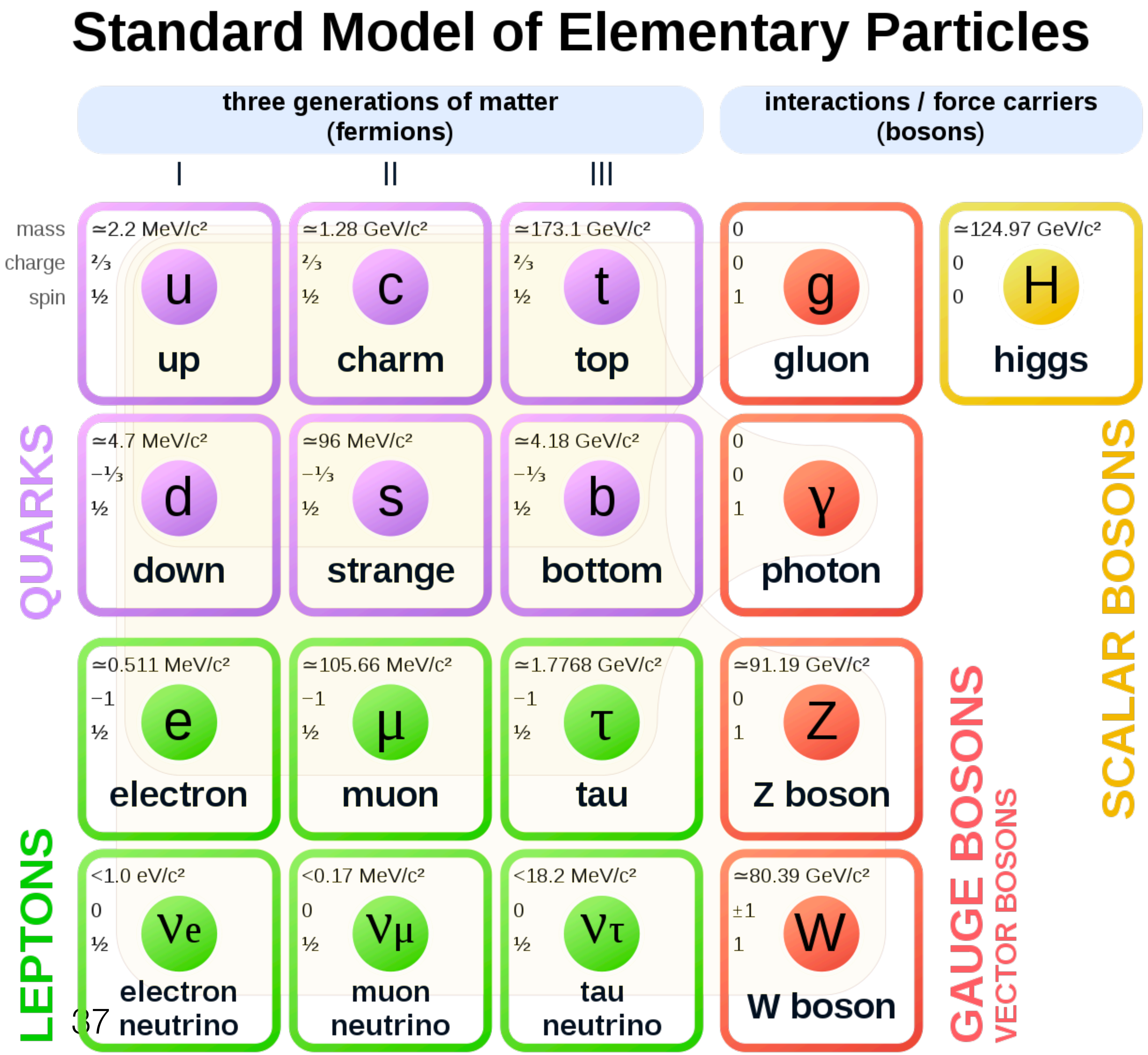
        - 热退耦暗物质残余丰度计算

粒子物理标准模型与暗物质

- No body knows what DM is
- Not in Standard Model
- There are good guesses

Not neutrinos ✘



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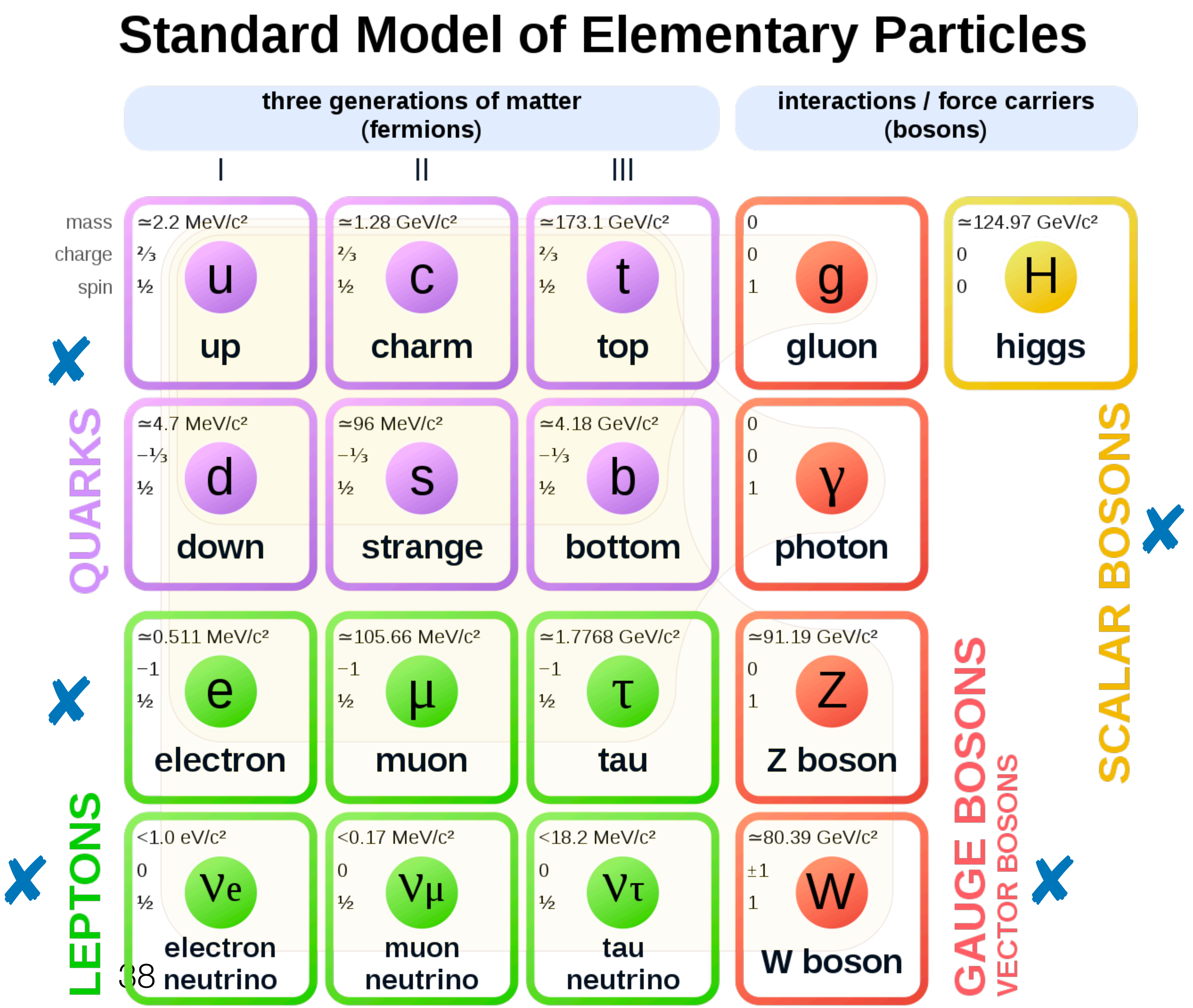
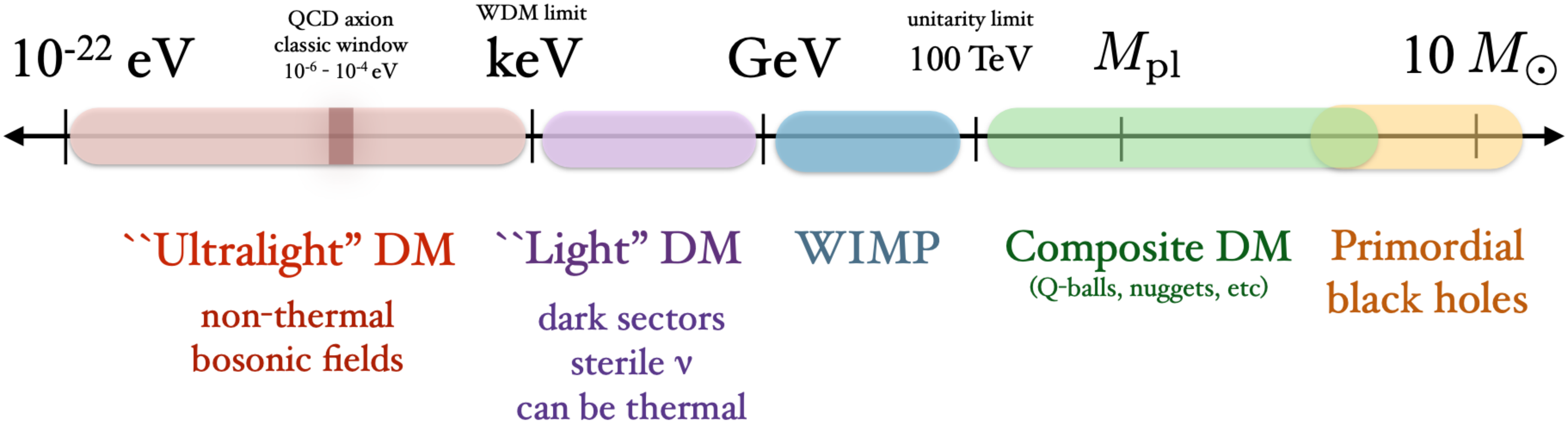


Not neutrinos ✗

暗物质的候选者模型

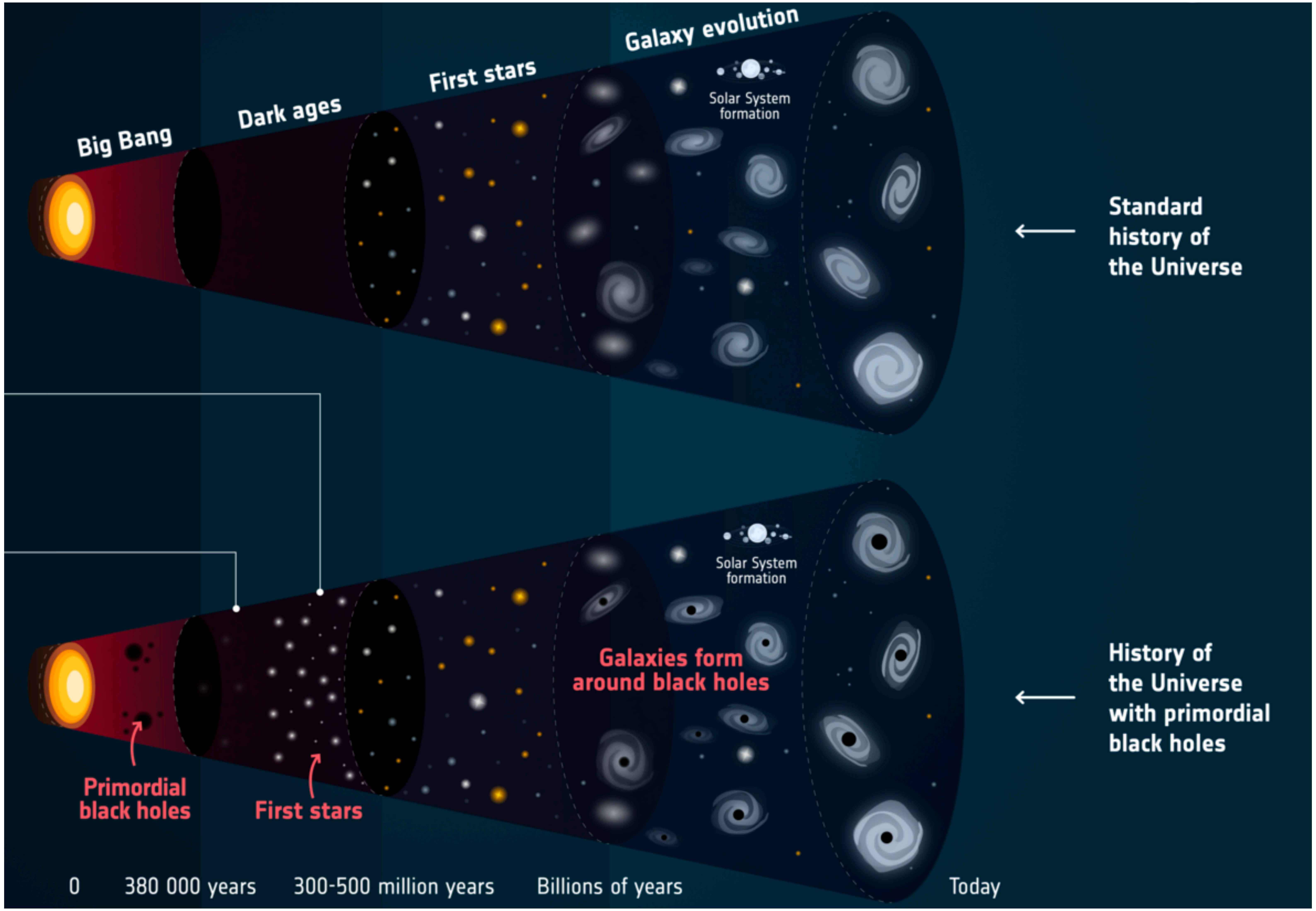
1904.07915，TASI lecture



- 原初黑洞 (Primordial Black Hole, PBH)
- 超轻波动型暗物质 (Ultralight Dark Matter)
- 具有弱相互作用的有质量粒子 (Weakly Interacting Massive Particle, WIMP)

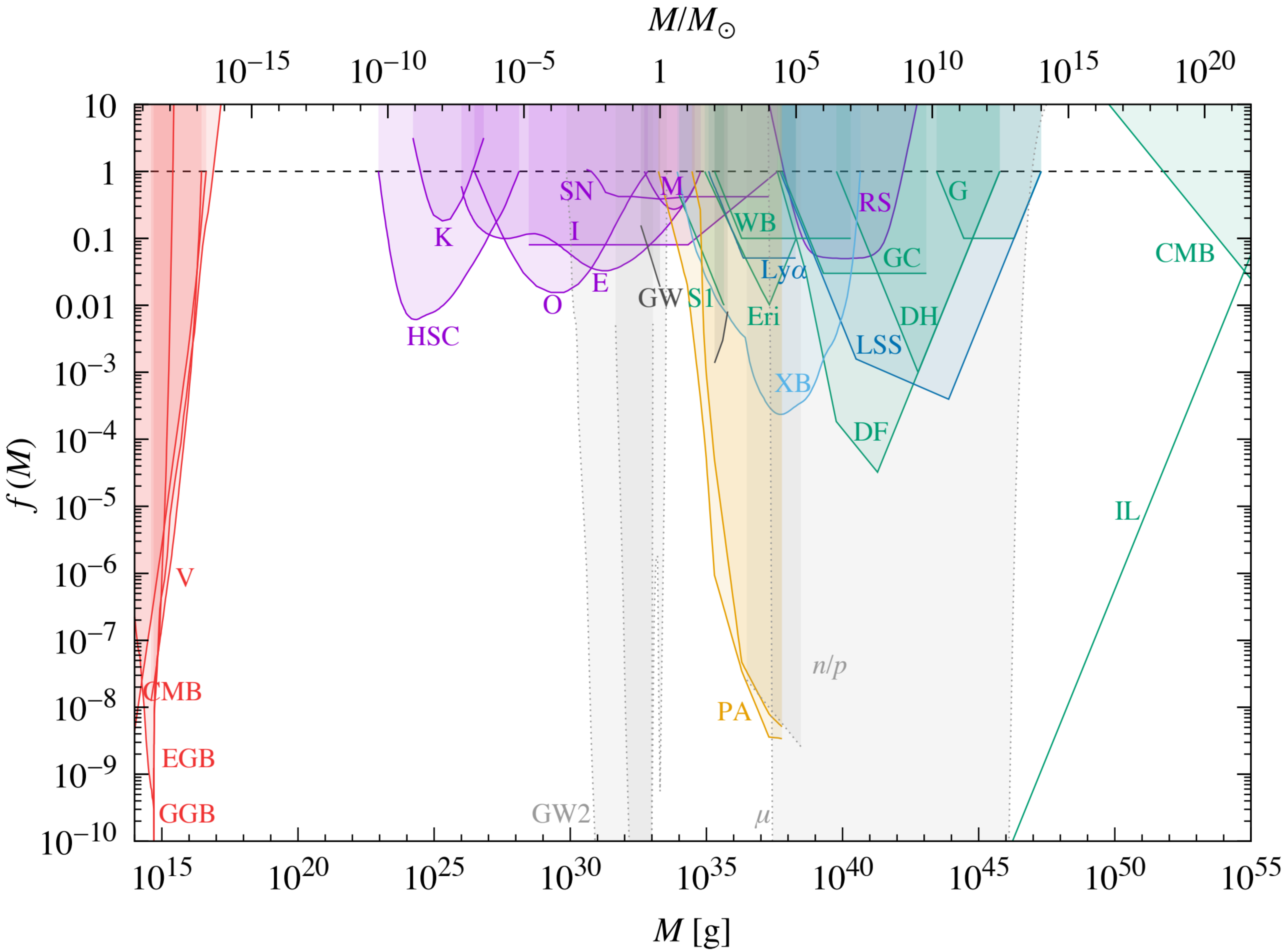
原初黑洞暗物质

- 宏观客体
- 先天黑洞
- 小行星质量大小的原初黑洞可以作为暗物质



原初黑洞暗物质

- 宏观客体
- 先天黑洞
- 小行星质量大小的原初黑洞可以作为暗物质
- 限制： evaporation (red), lensing (magenta), dynamical effects (green), gravitational waves (black), accretion (light blue), CMB distortions (orange), large-scale structure (dark blue) and background effects (grey).



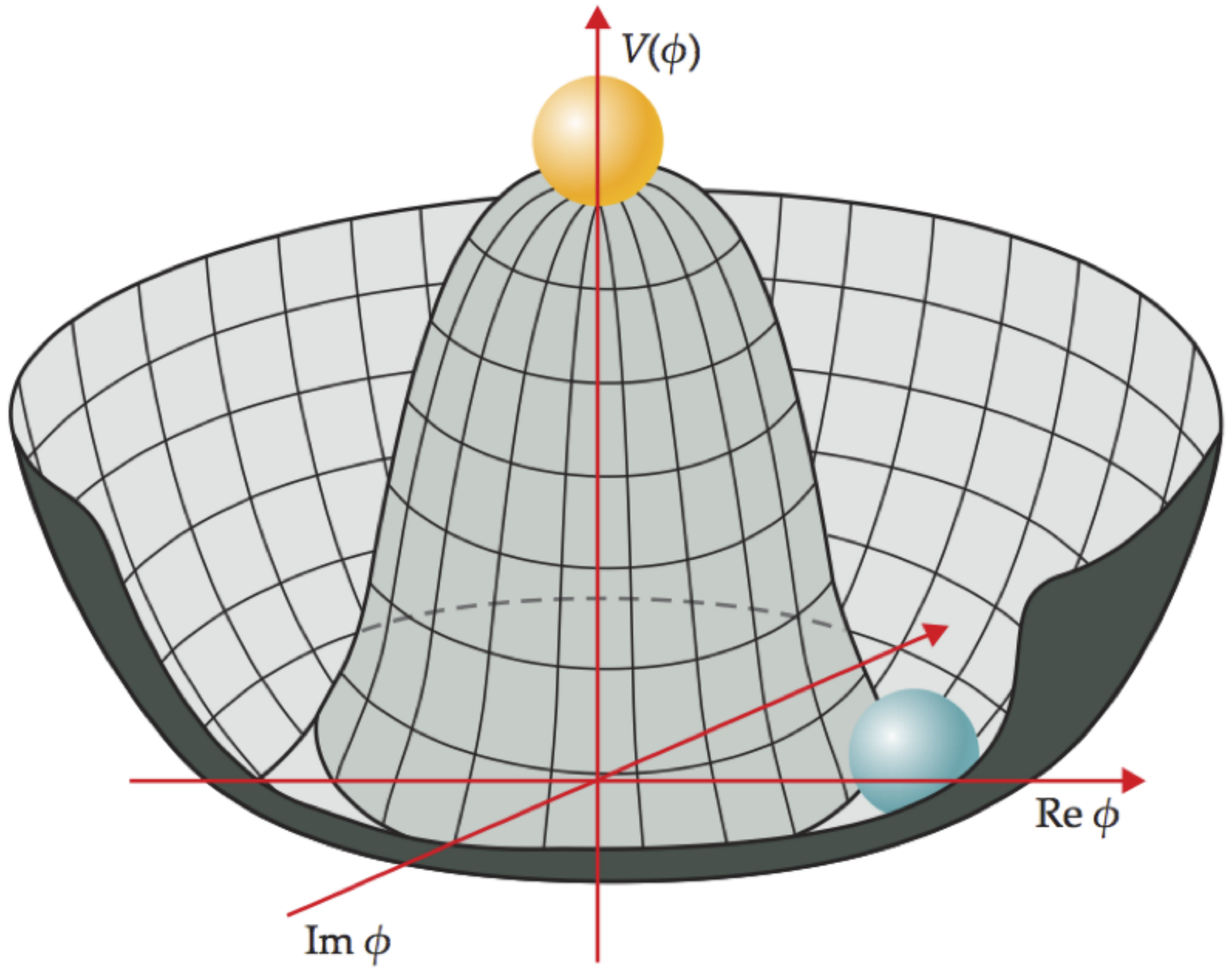
2002.12778 [Rept.Prog.Phys.]

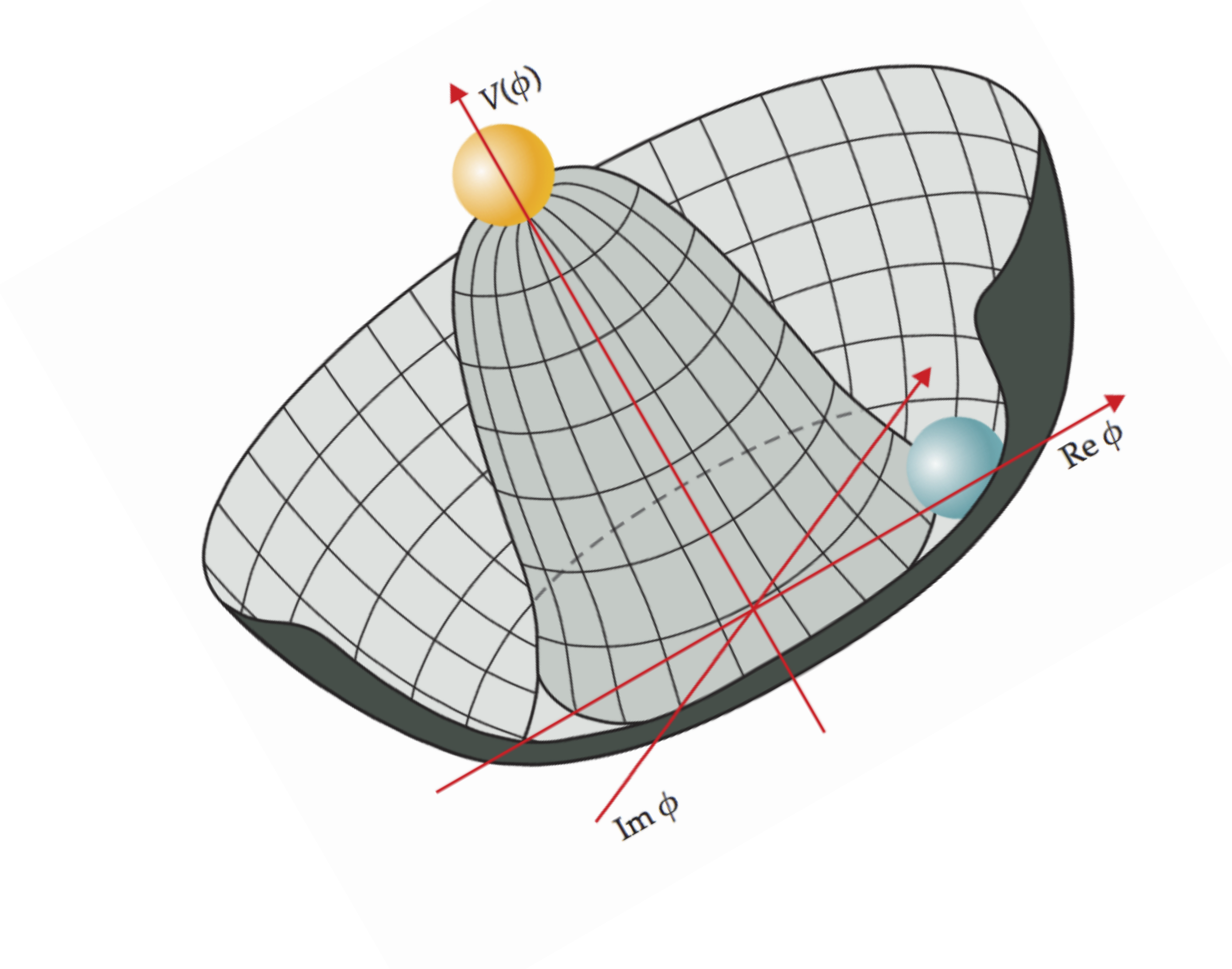
超轻波动型暗物质

- 超轻波动型暗物质 (Ultralight Dark Matter)
  - QCD轴子，类轴子，暗光子等等…
  - 产生机制：例如 Misalignment

超轻波动型暗物质

- 超轻波动型暗物质 (Ultralight Dark Matter)
  - QCD轴子，类轴子，暗光子等等…
  - 产生机制：例如 Misalignment
    - $\ddot{a} + 3H\dot{a} + m_a^2 a = 0$





超轻波动型暗物质的探测

- 超轻波动型暗物质 (Ultralight Dark Matter)
  - QCD轴子，类轴子，暗光子等等…
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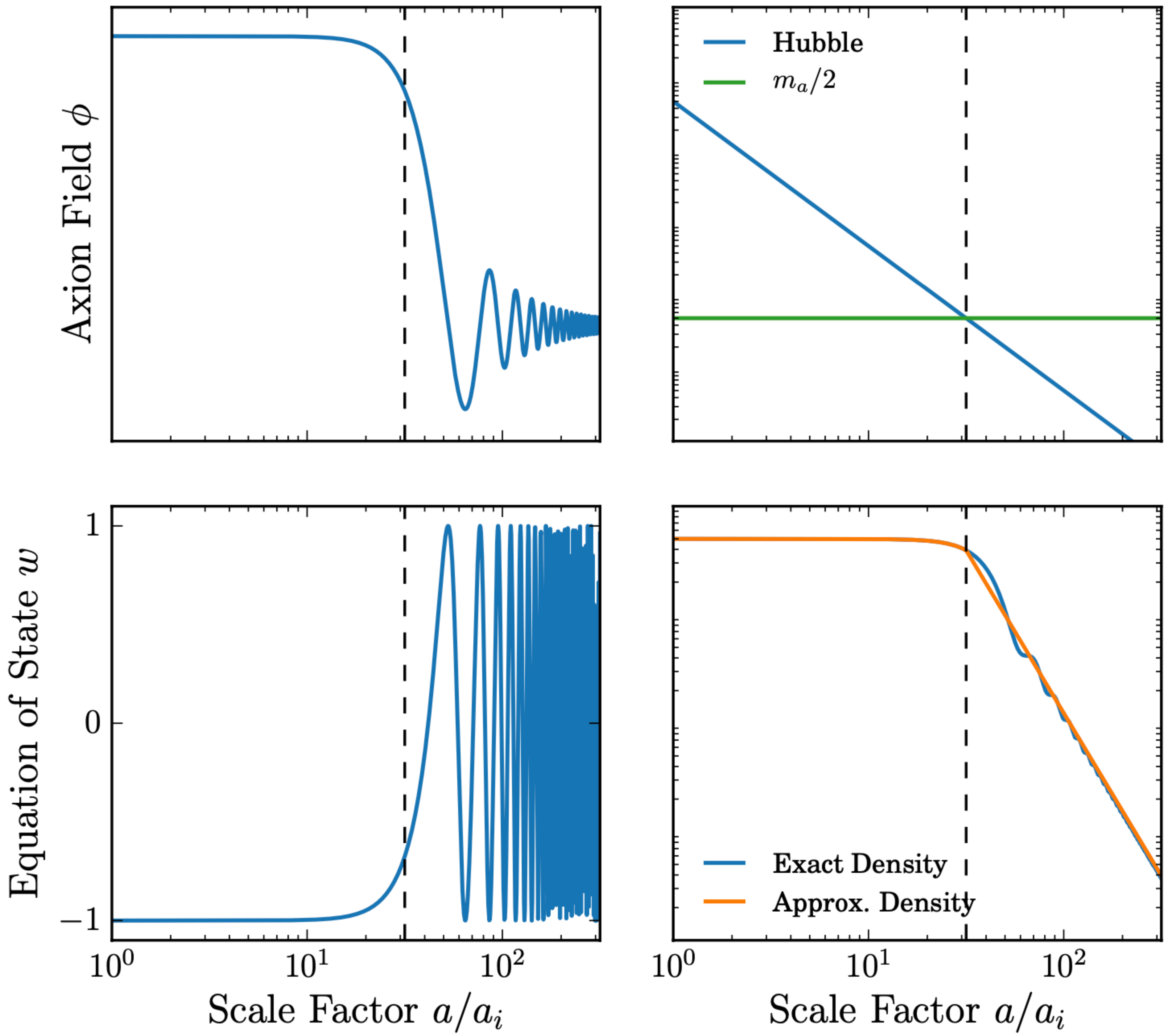


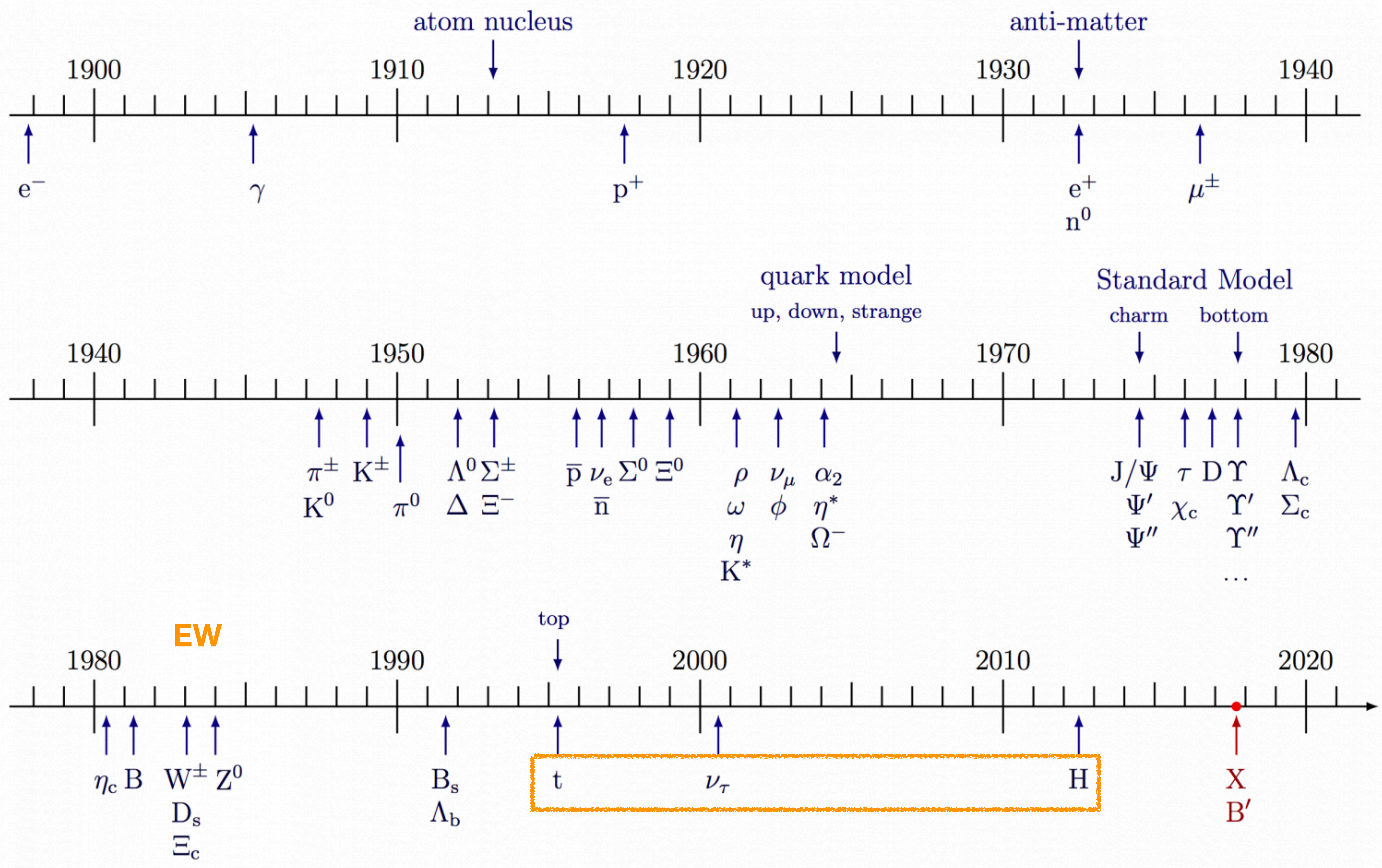
提纲

- 不知道什么　（已知的未知）
  - 粒子物理的三个前沿
  - 暗物质问题
    - 暗物质的天文观测证据
    - 暗物质分布
    - 暗物质的物理模型
      - 可能的暗物质候选者
      - WIMP暗物质
        - 热退耦暗物质残余丰度计算

今日主角：WIMP暗物质

- 为什么WIMP暗物质重要?
  - 理想的化身
  - 优秀的产生机制
  - 优秀的实验预期信号

粒子发现编年史



atom nucleus
anti-matter
1900
1910
1920
1930
1940
e⁻
γ
p⁺
e⁺
n⁰
μ±
quark model
up, down, strange
Standard Model
charm
bottom
1940
1950
1960
1970
1980
π± K± π⁰
K⁰
Λ⁰ Σ±
Δ Ξ⁻
p̄ νe Σ⁰ Ξ⁰
n̄
ρ ω η K*
νμ φ
α2 η* Ω⁻
J/Ψ Ψ′ Ψ″
τ χc
D
Υ Υ′ Υ″
...
Λc Σc
EW
top
1980
1990
2000
2010
2020
ηc B
W± Z⁰
Ds Ξc
Bs Λb
t
ντ
H
X
B′

等级问题

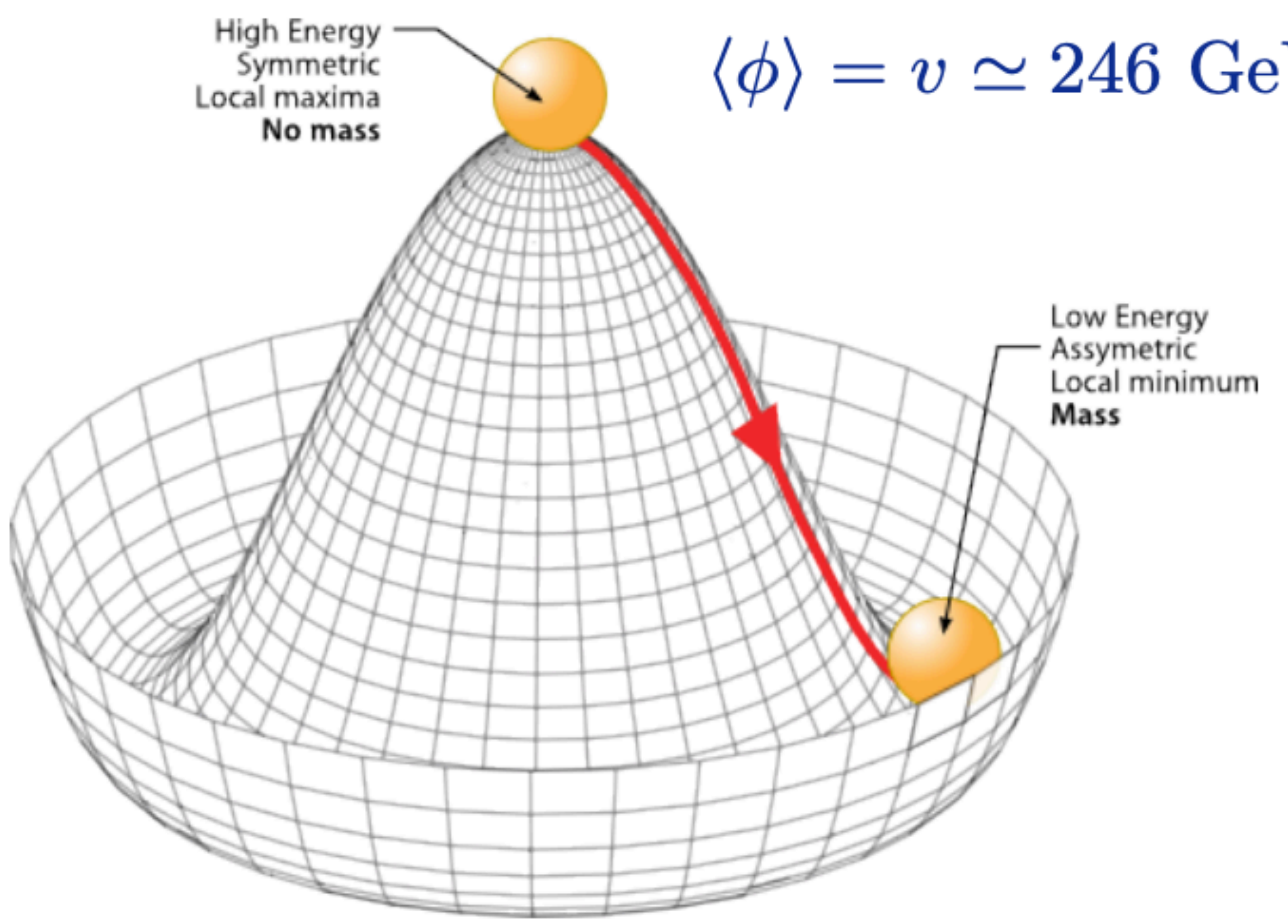
- 希格斯粒子的质量或电弱能标的来源

  - 迈斯纳效应：规范玻色子光子在超导体中是有质量的

$$F = \alpha|\phi|^2 + \frac{\beta}{2}|\phi^4| + \frac{1}{2m_e}|(-i\hbar\nabla - 2e\mathbf{A})\phi|^2$$

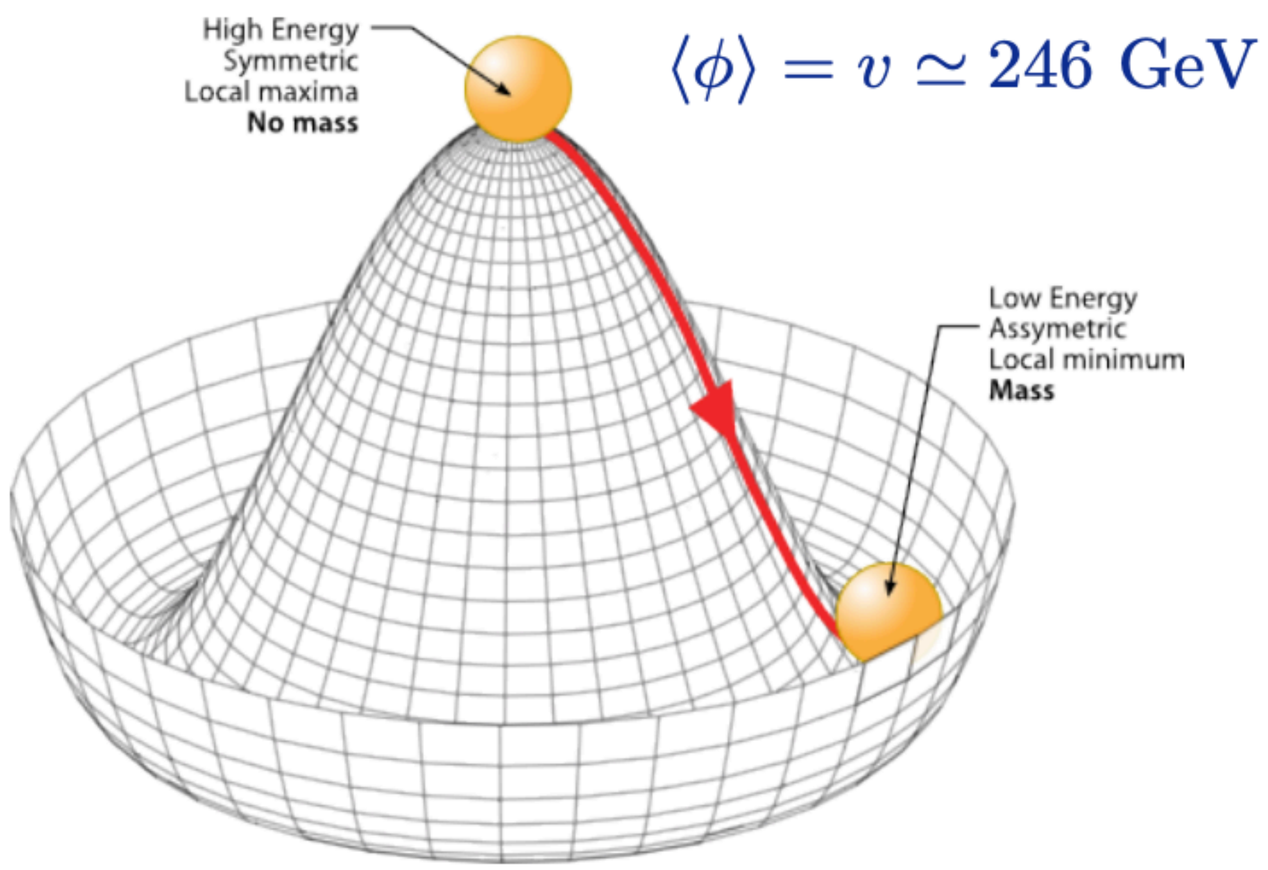
$$|\phi|^2 = -\frac{\alpha}{\beta} > 0 \qquad \textbf{BCS}:\ \phi \sim \psi^e_{\mathbf{k}}\psi^e_{-\mathbf{k}}$$

$$\langle\phi\rangle = v \simeq 246\ \mathrm{GeV}$$



等级问题

- 希格斯粒子的质量或电弱能标的来源



- 迈斯纳效应：规范玻色子光子在超导体中是有质量的

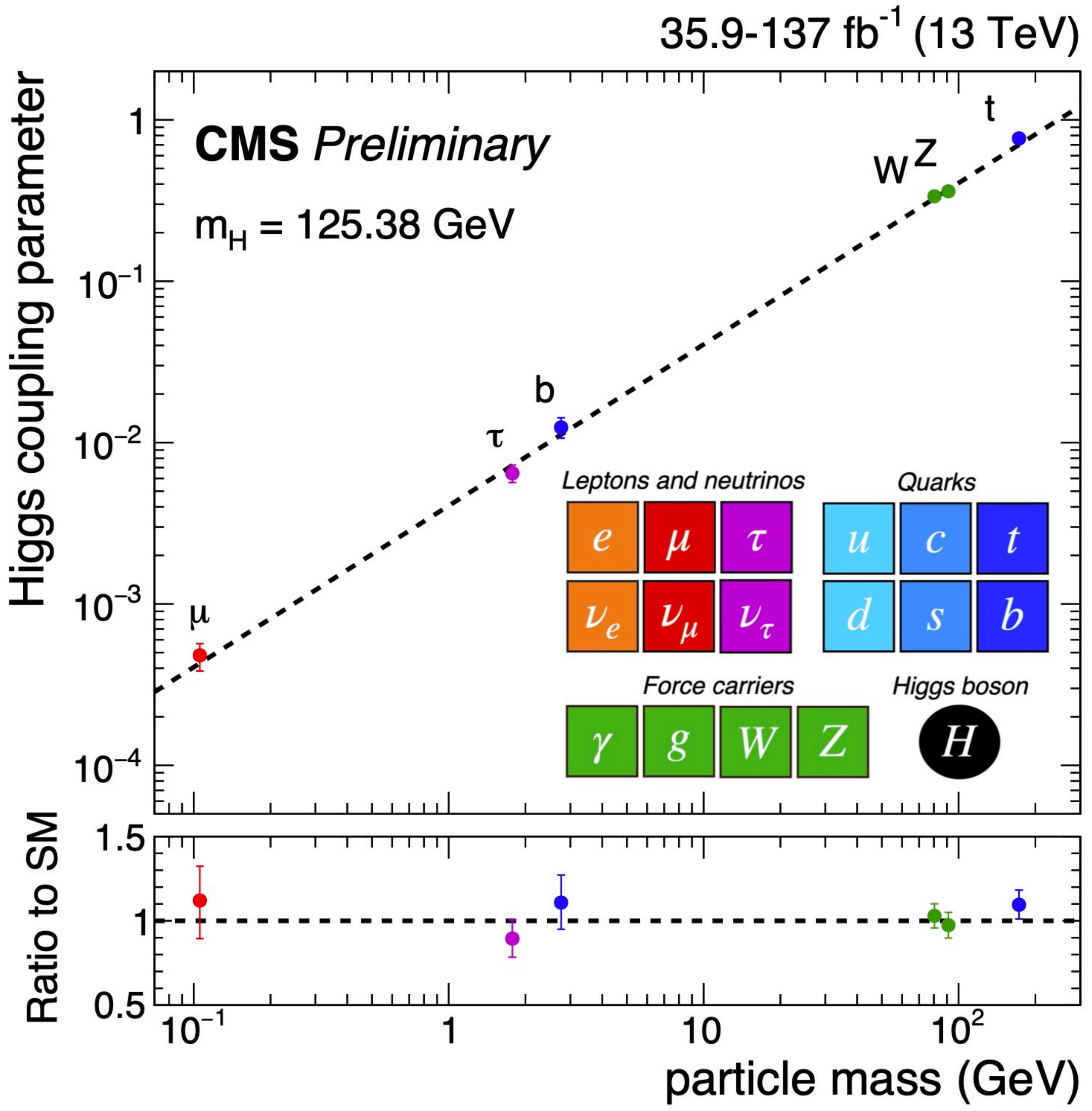
$$F = \alpha|\phi|^2 + \frac{\beta}{2}|\phi^4| + \frac{1}{2m_e}|(-i\hbar\nabla - 2e\mathbf{A})\phi|^2$$

$$|\phi|^2 = -\frac{\alpha}{\beta} > 0 \qquad \textbf{BCS}: \ \phi \sim \psi^e_{\mathbf{k}}\psi^e_{-\mathbf{k}}$$

- 现在的时空是电弱超导体：W和Z规范玻色子是有质量的

$$\mathscr{L} = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + (D^\mu\phi)^\dagger D_\mu\phi$$

$$\langle\phi^\dagger\phi\rangle \equiv v^2 = -\frac{\mu^2}{2\lambda} > 0$$



等级问题

- 希格斯粒子的质量或电弱能标的来源

$$\mathcal{L} = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + (D^\mu\phi)^\dagger D_\mu\phi$$

- 希格斯质量的量子修正

$$m^2_{Higgs} = m_0^2 + m^2_{quantum} = (125\ \mathbf{GeV})^2$$

$$m_0^2 \downarrow\ 10^{19}\text{GeV} \qquad m^2_{quantum} \searrow\ \Lambda^2 \sim (10^{19}\mathbf{GeV})^2$$

- 标准模型不能预言希格斯粒子质量或电弱能标

等级问题

- 希格斯粒子的质量或电弱能标的来源

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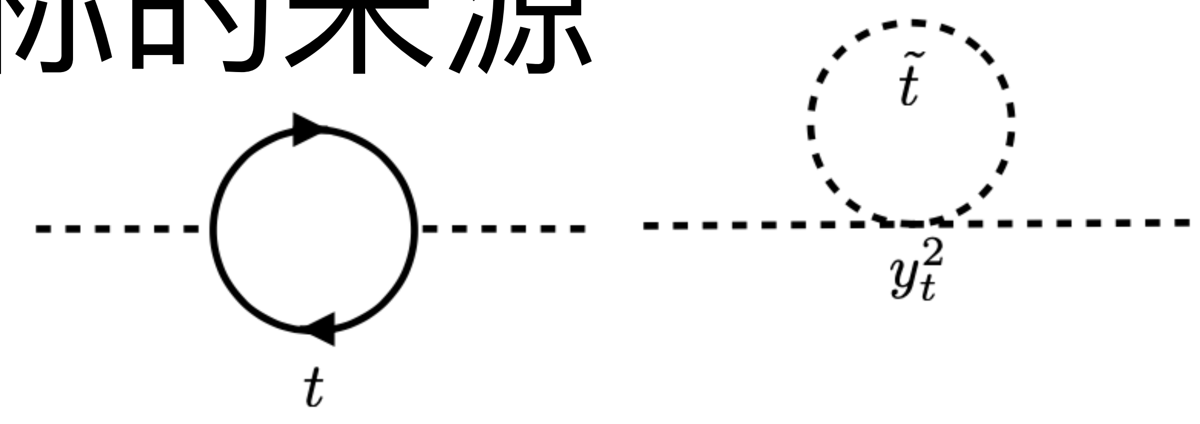

- 希格斯质量的量子修正

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- 标准模型不能预言希格斯粒子质量或电弱能标

- 可能的解决办法
    - 超对称模型
    - 复合希格斯粒子模型
    - 额外维模型
- 通常都预言希格斯粒子性质的偏离
- 希格斯精确测量的重要性

超出标准模型新物理菜单：最小超对称标准模型

粒子与超对称伴子
玻色子与费米子的对称
新的时空对称性
年份：？？？

希格斯粒子质量的问题
年份：？？？

Neutralino作为暗物质候选者
Weakly Interacting Massive Particle
年份：？？？

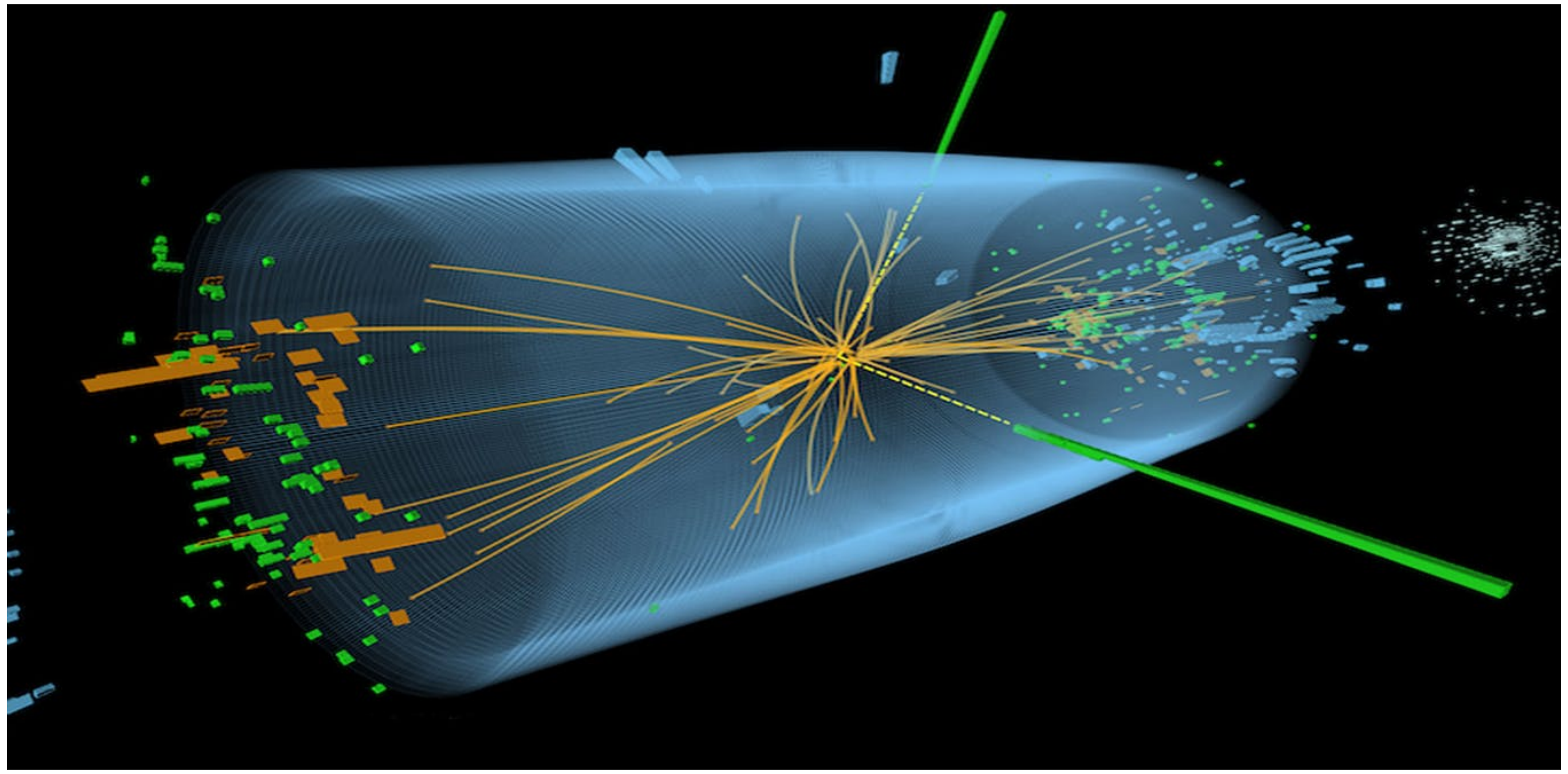
强、弱、电磁三种力的统一
年份：？？？

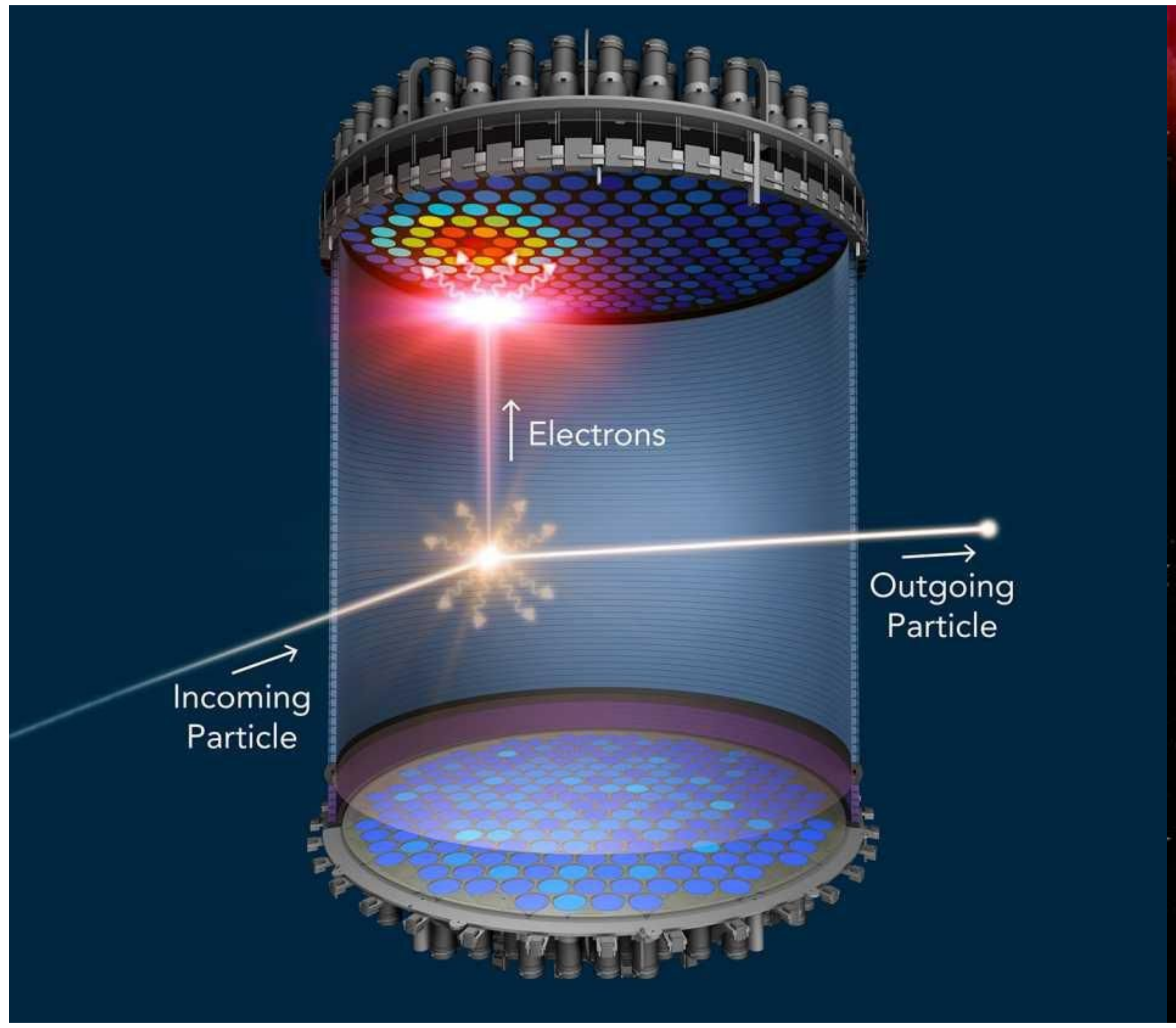
理想的化身

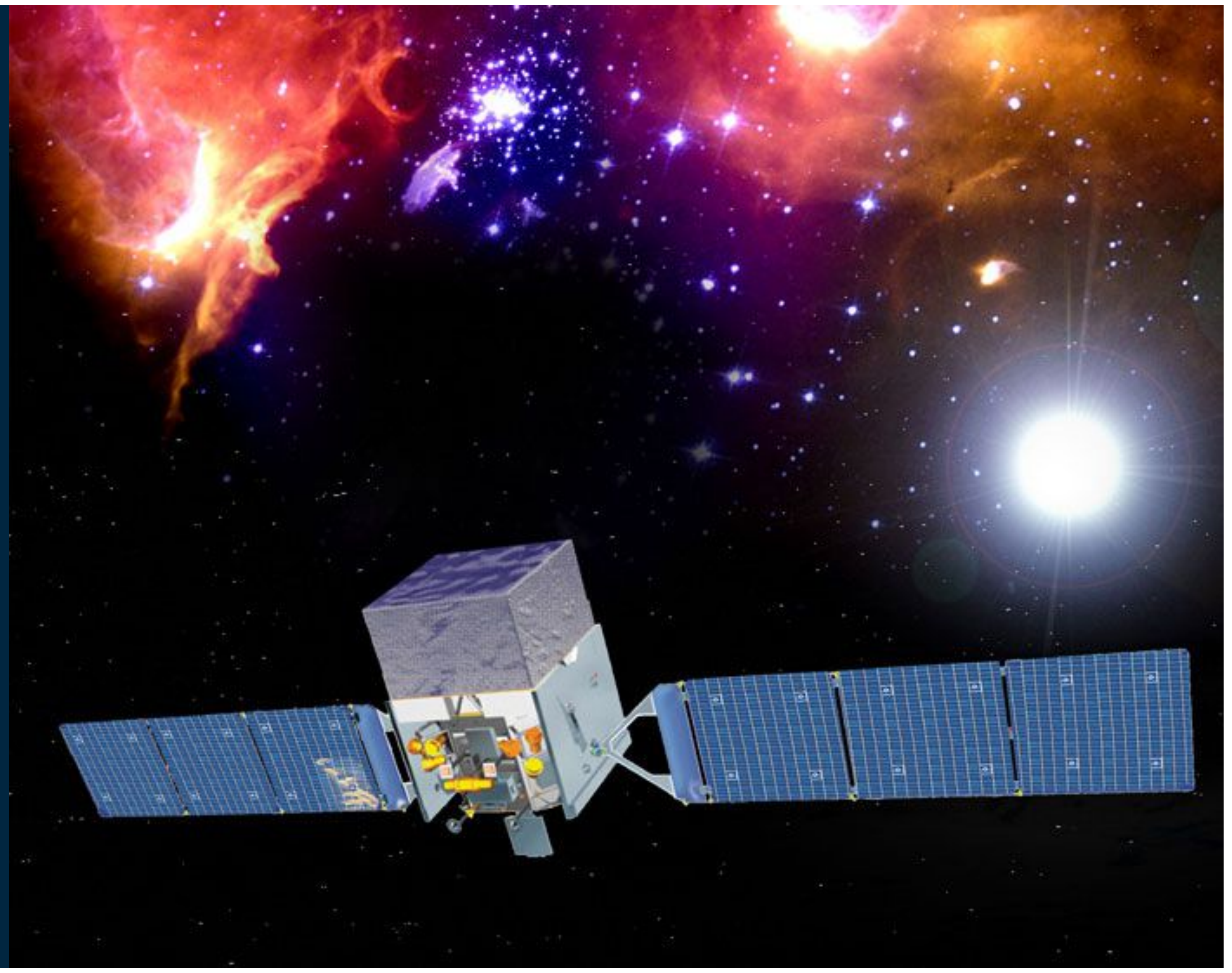
- 至尊私房菜：超对称
- 中性微子暗物质
- 超对称暗物质的理想化身：WIMP暗物质

优秀的实验信号预期

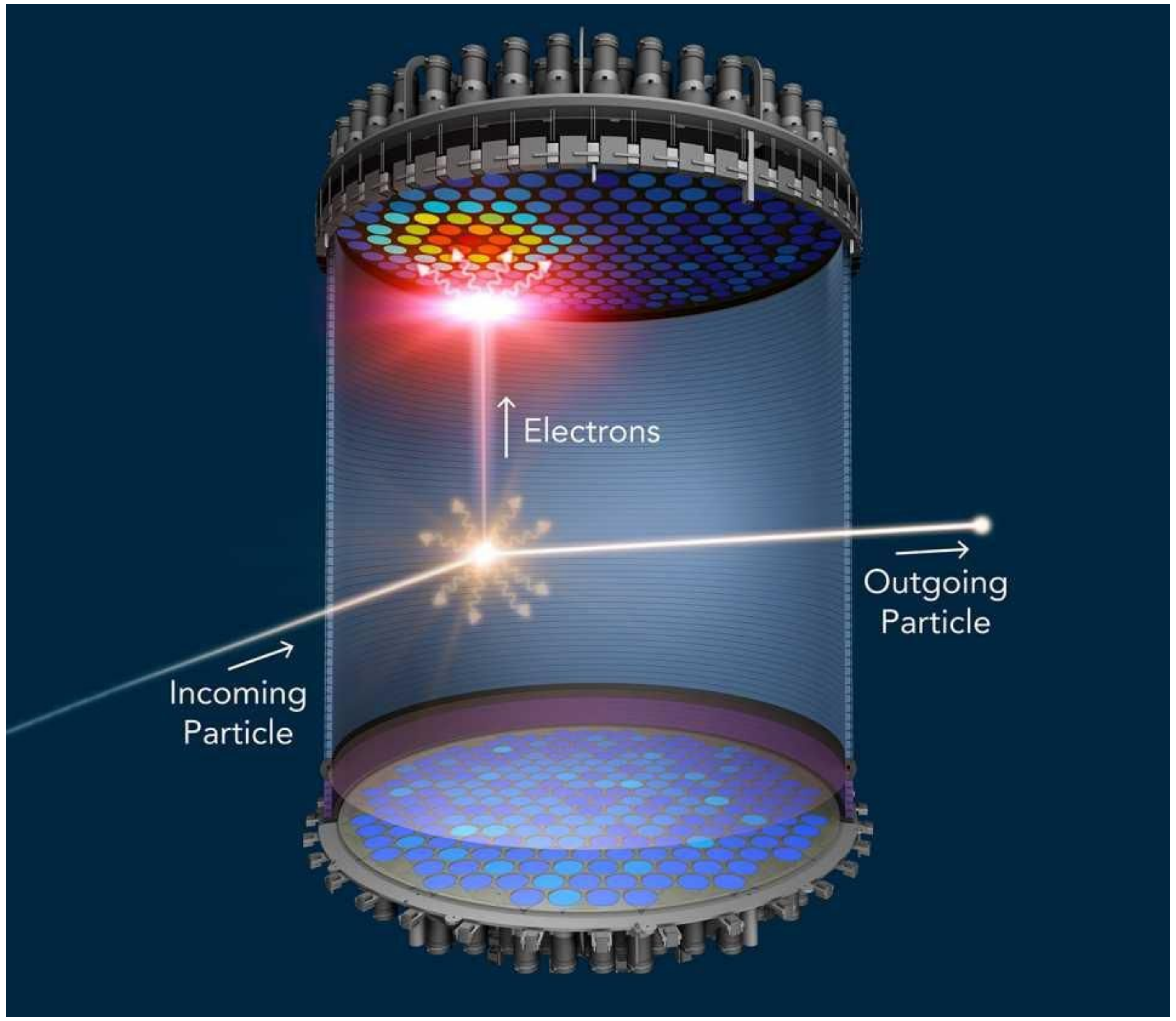
- WIMP暗物质和标准模型有较大的相互作用
  - 直接探测实验 SM + DM > SM + DM
  - 间接探测实验 DM + DM > SM + SM
  - 对撞机实验 SM + SM > DM + DM

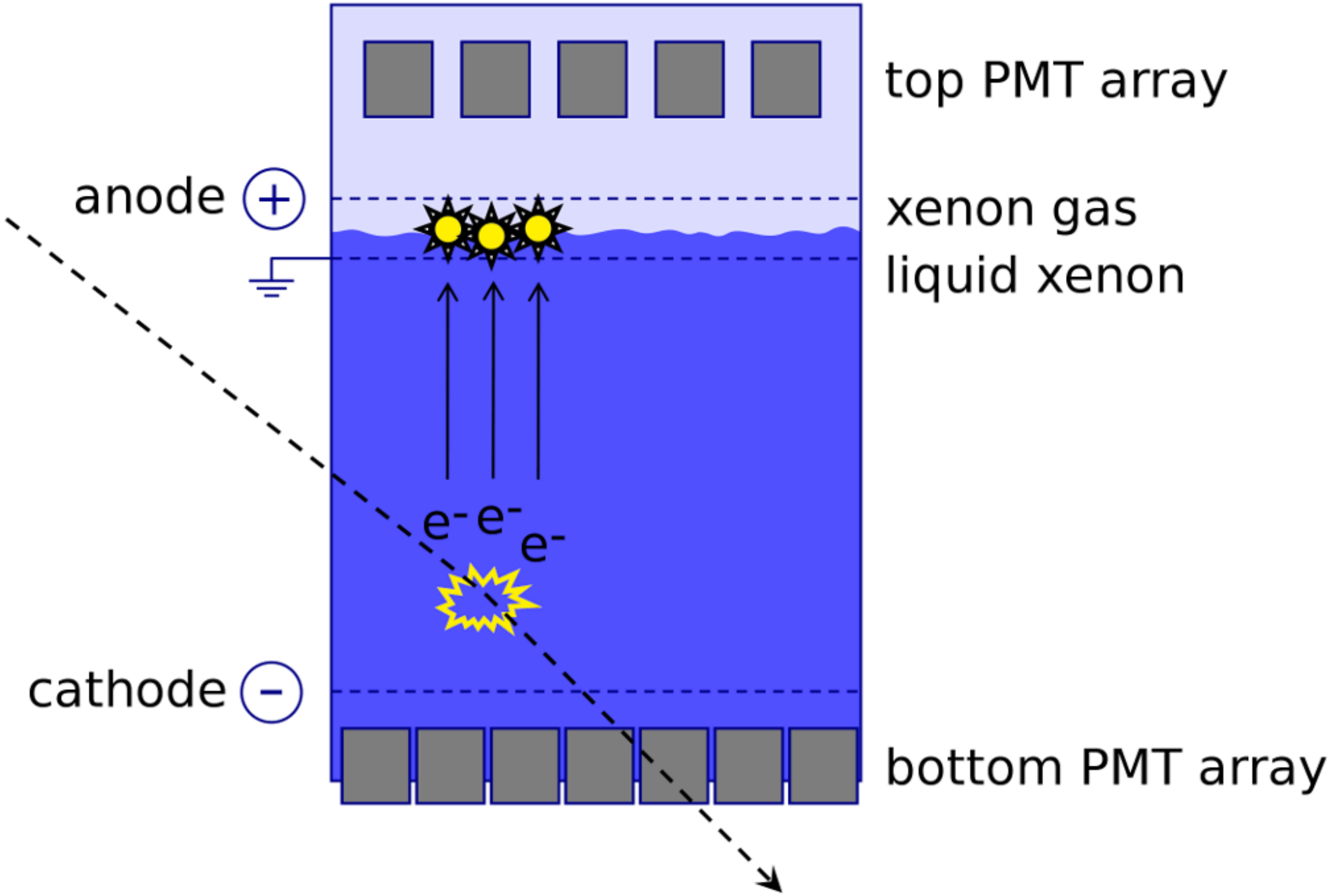






暗物质直接探测实验



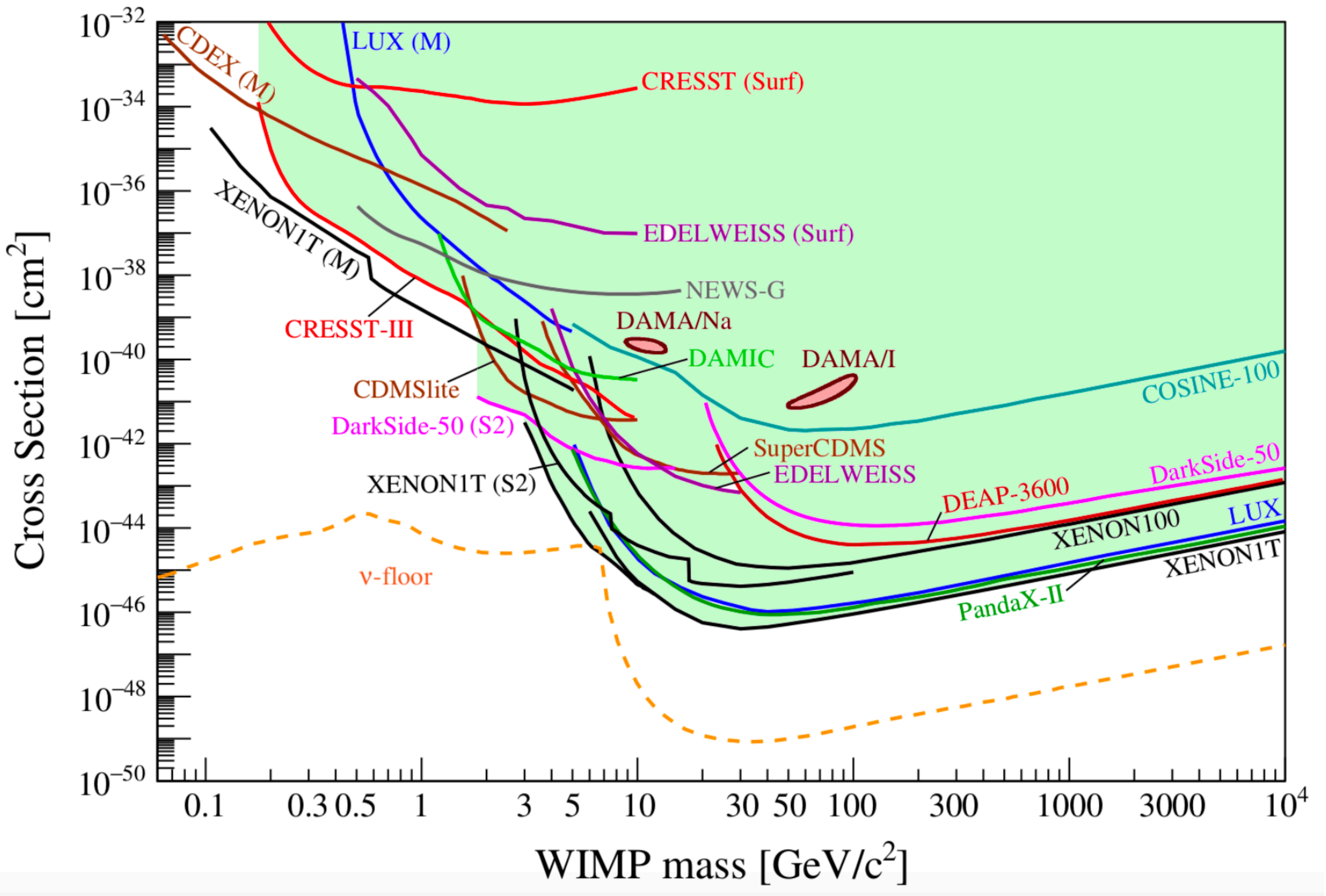


- DM + SM > DM + SM, 暗物质碰撞产生动能转移

暗物质直接探测实验

- 中国相关实验：CDEX, PANDA-X
- 实验探测到的事例数

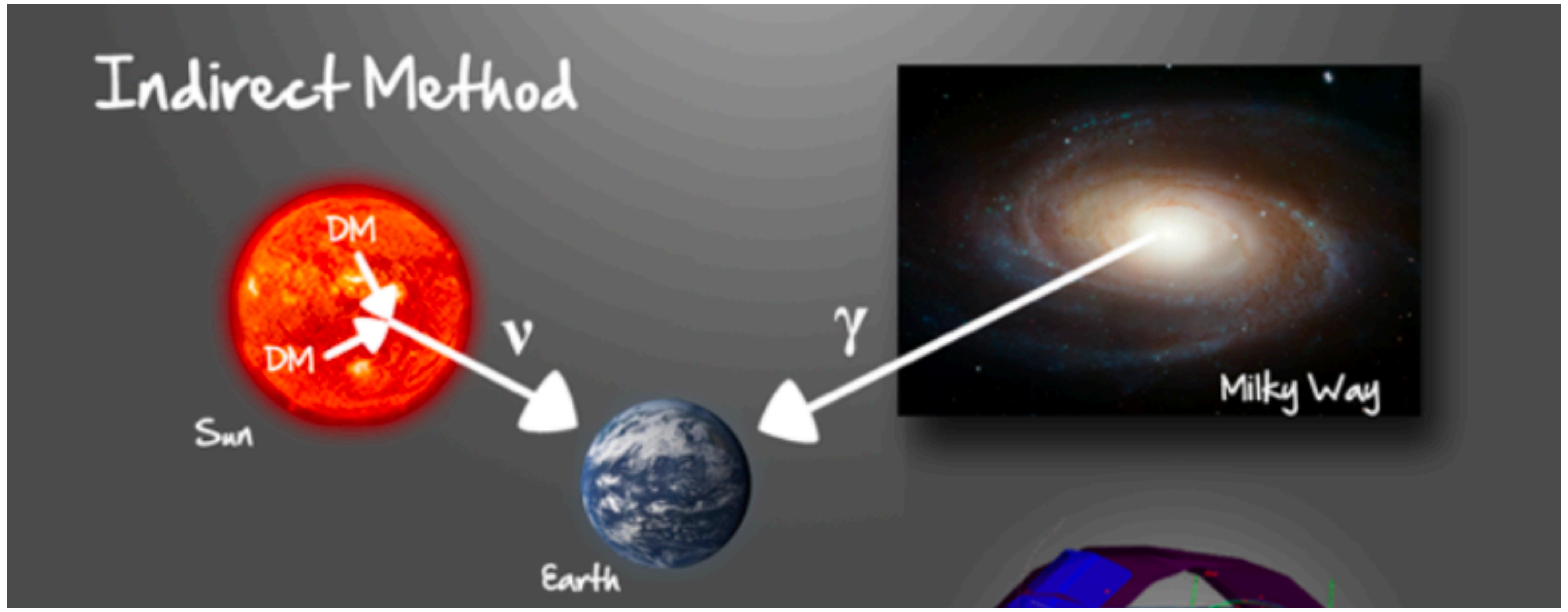
$$N = n_{\rm DM} N_{\rm target} \sigma v_{\rm DM} t_{\rm obs}$$

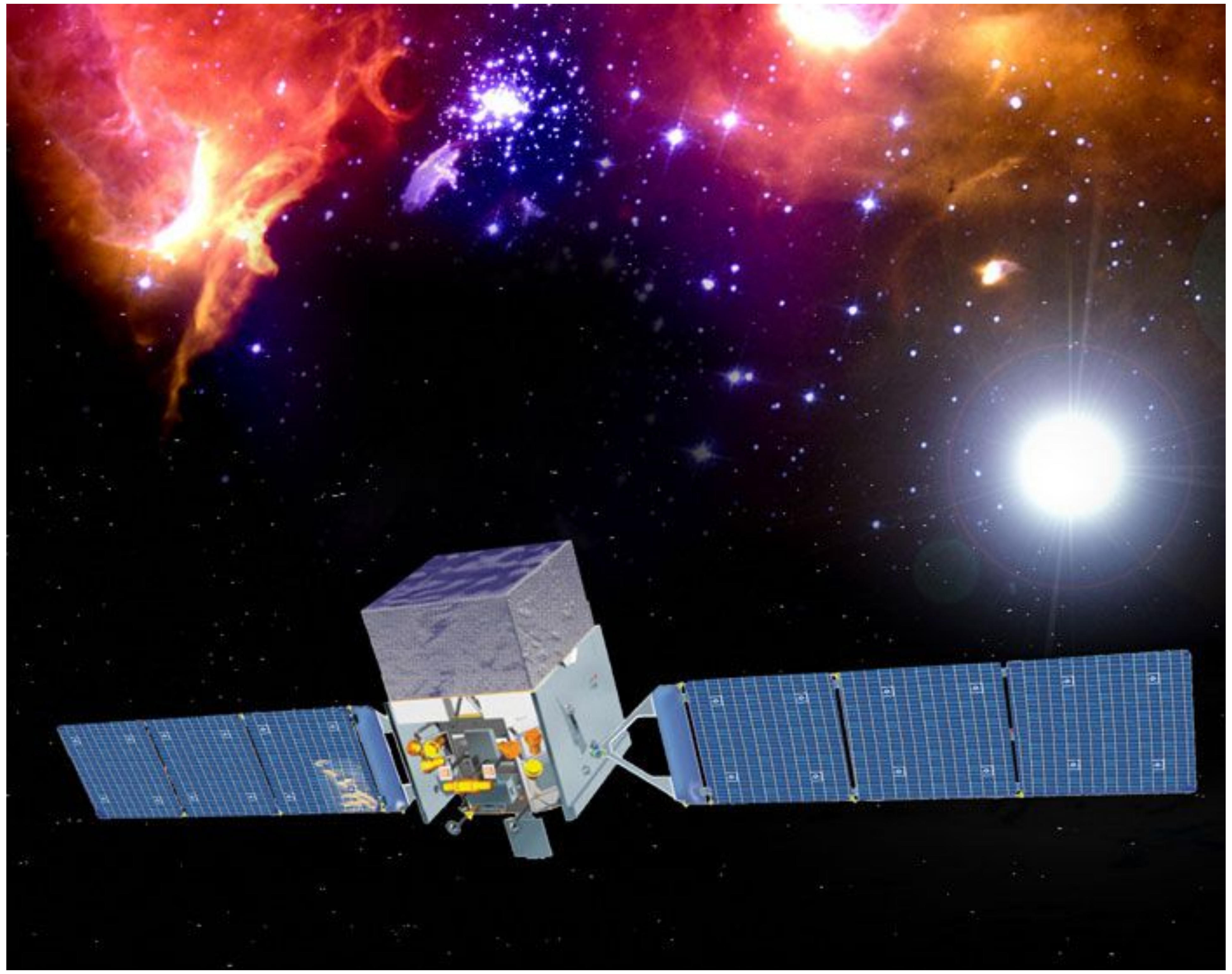


APPEC Committee Report: 2104.07634

暗物质间接探测实验

Credit: HAP/ A. Chantelauze





- 天体或者星系中心暗物质湮灭产生的次级粒子
  - DM + DM > SM + SM
- 中国相关实验：DAMPE, LHAASO
- 实验探测到的粒子流强

$$F = \frac{1}{4\pi} \frac{\langle \sigma v \rangle}{2m_{\rm dm}^2} \int_{\rm l.o.s} \rho_{\rm dm}^2 dl$$

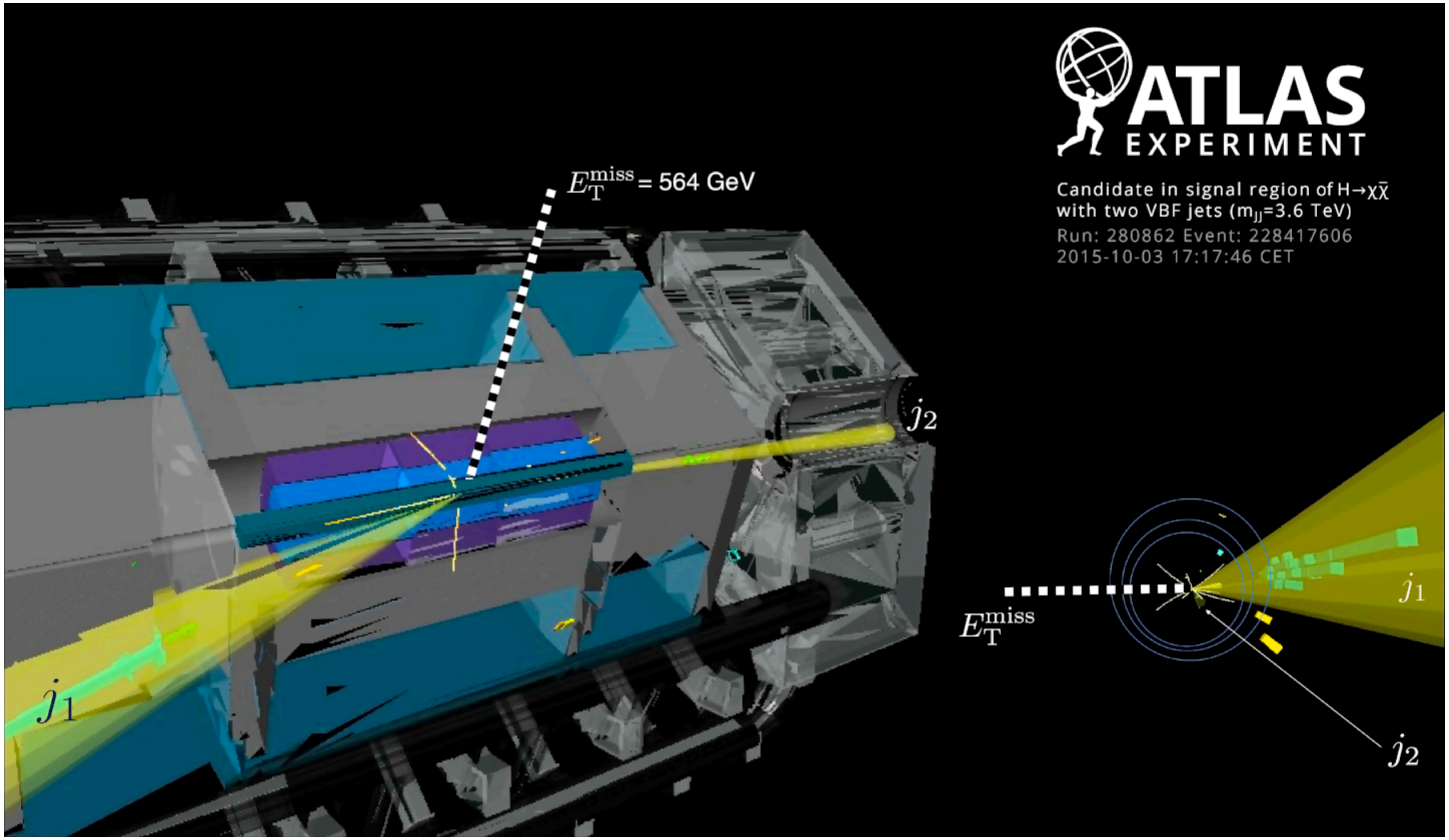
$$\left[{\rm cm}^2 \times \frac{\rm cm}{\rm sec} \times {\rm cm}^{-6} \times {\rm cm} \right] = \left[{\rm cm}^{-2} {\rm sec}^{-1} \right]$$

暗物质对撞机探测实验

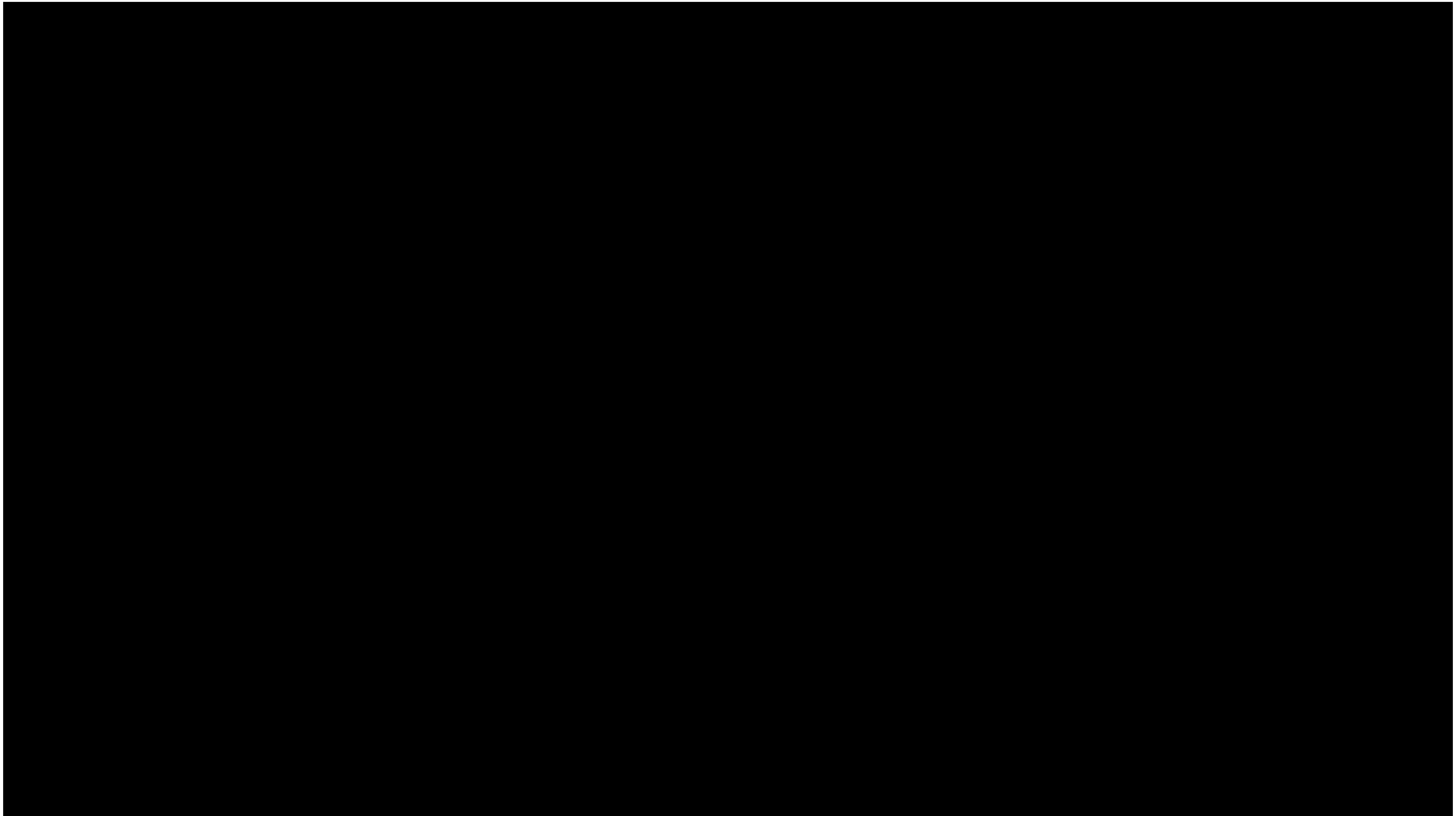
- SM + SM > DM pair + SM

- 暗物质在对撞机无法探测

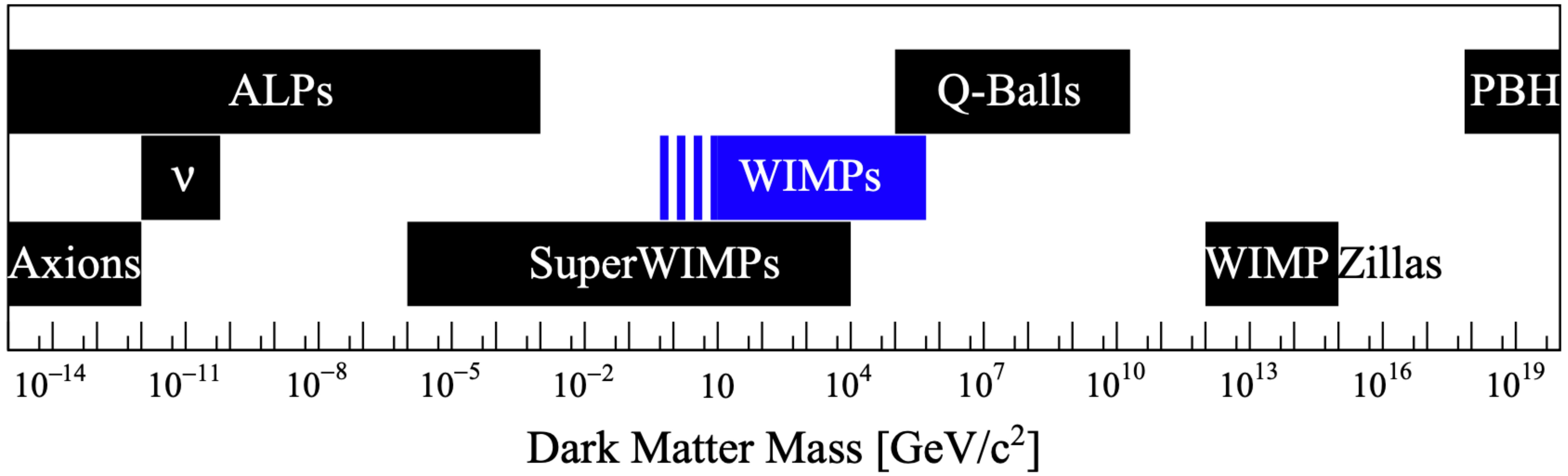
- 根据能动量守恒反推其存在，类似中微子之于beta衰变



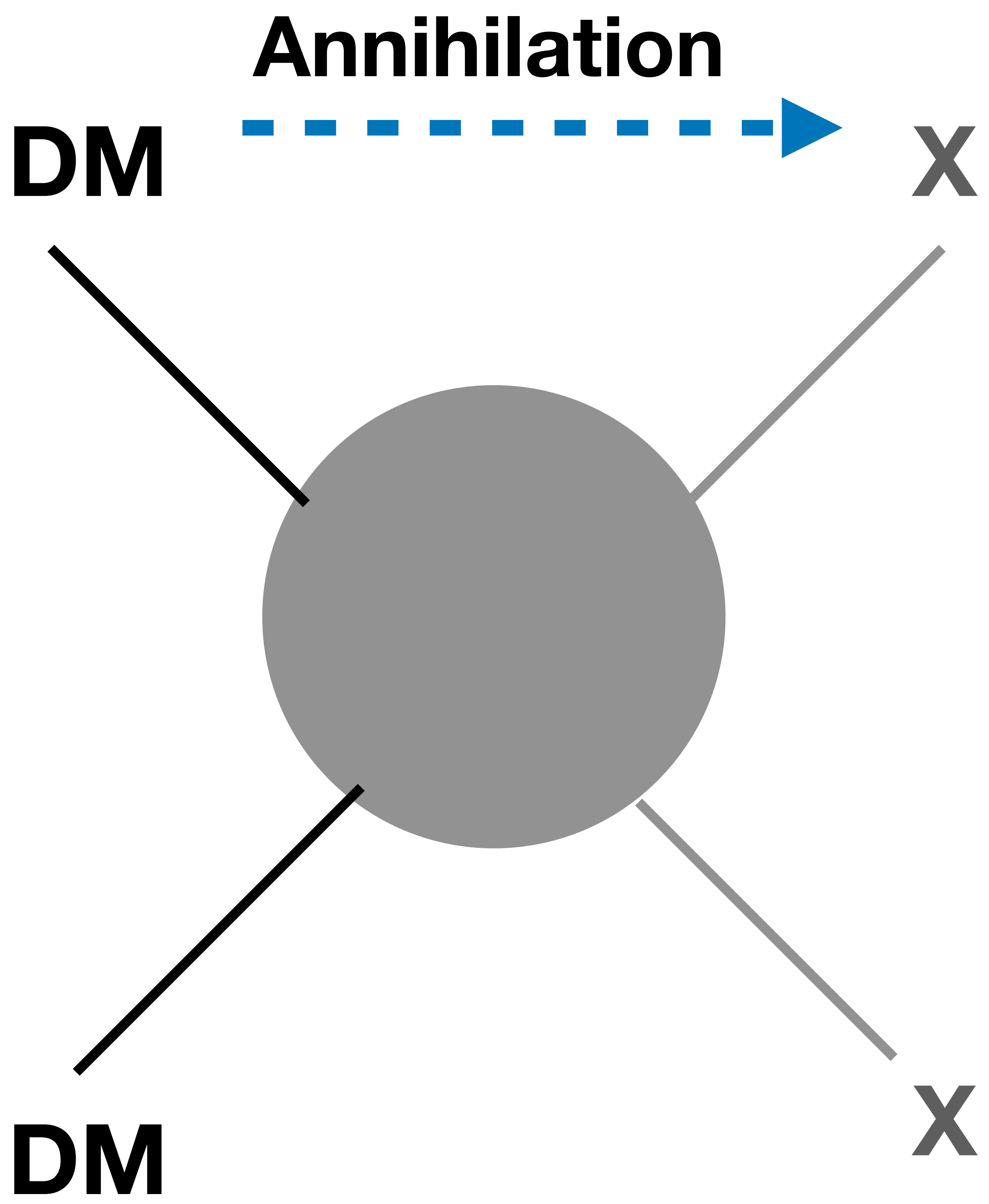
粒子物理的宇宙学前沿



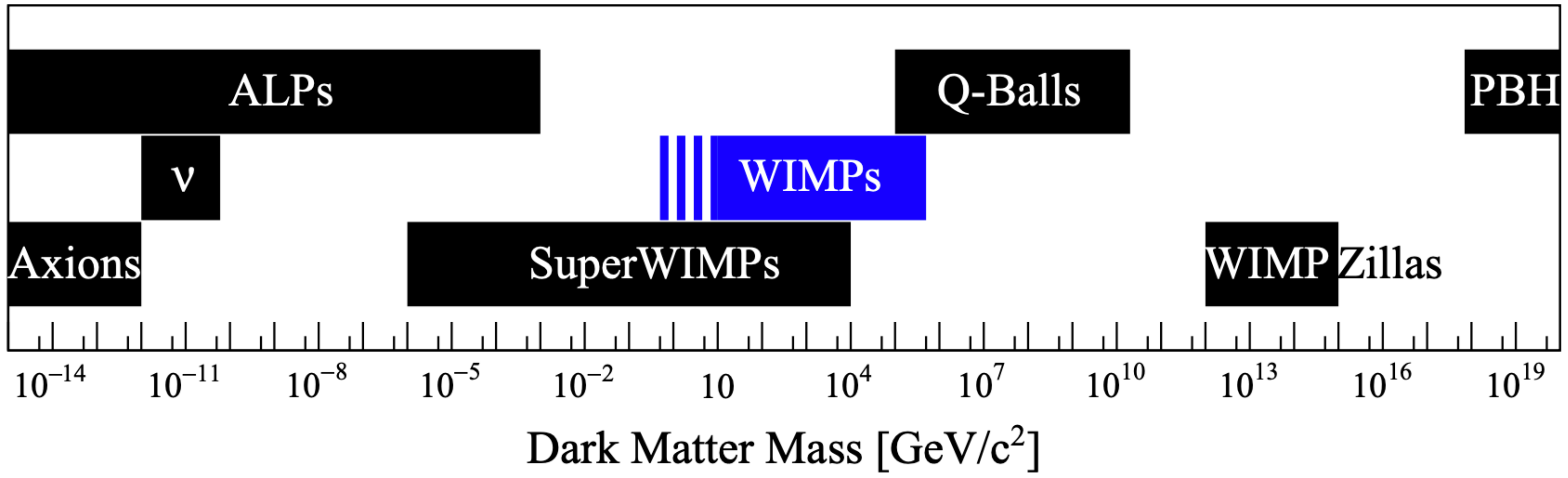
优秀的产生机制：热退耦合



- DM is a massive elementary particle
- DM has an electroweak-scale coupling
  - DM starts with thermal distribution
  - Relic abundance is determined by freeze-out mechanism
  - DM Annihilation into
    - X = Standard Model particles (direct coupling)



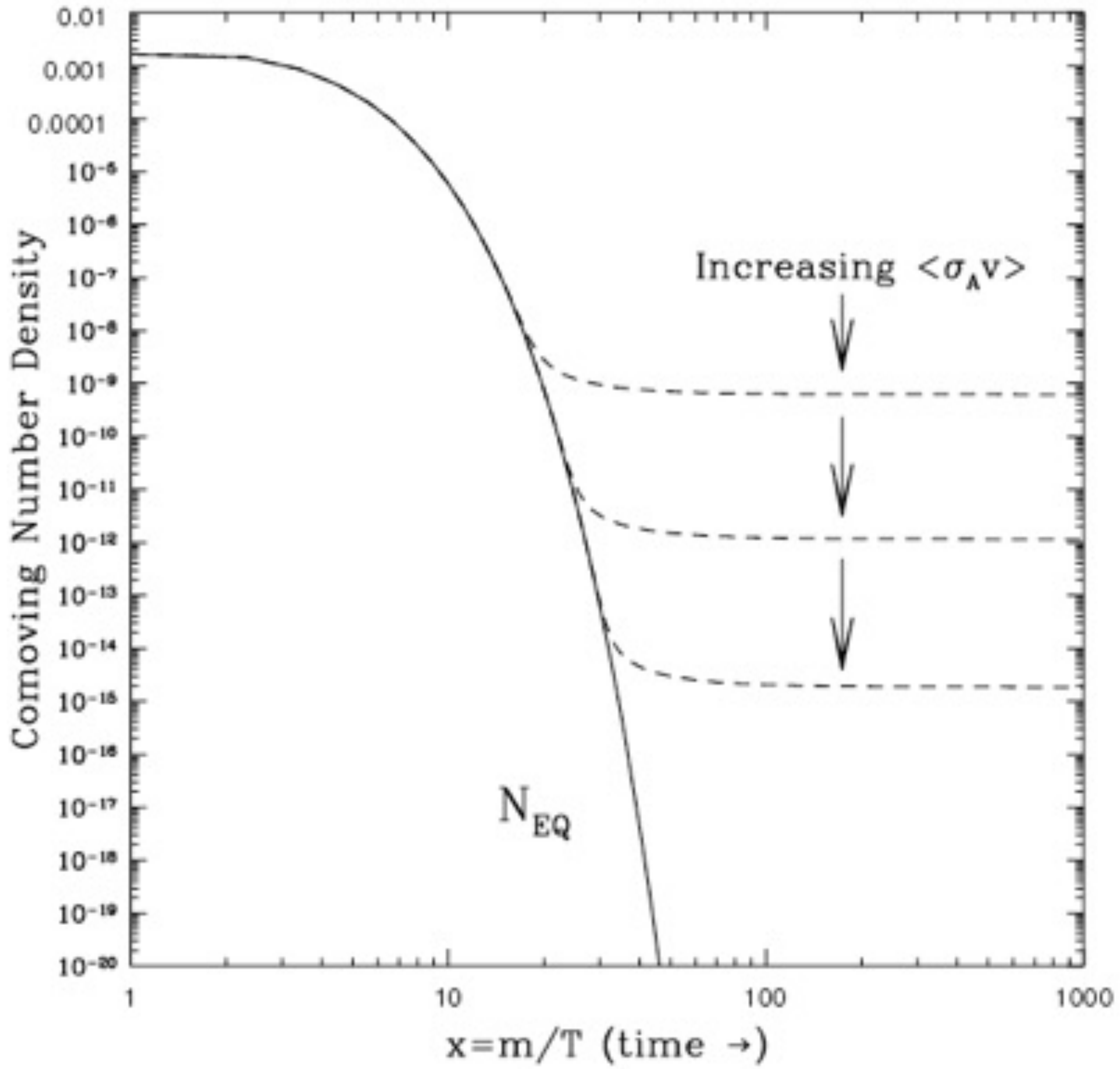
WIMP暗物质的热退耦合湮灭截面



- 热退耦合湮灭截面与电弱相互作用强度和能标吻合

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{m_W^2} \sim 3 \times 10^{-26} \mathrm{cm}^3 \mathrm{s}^{-1}$$

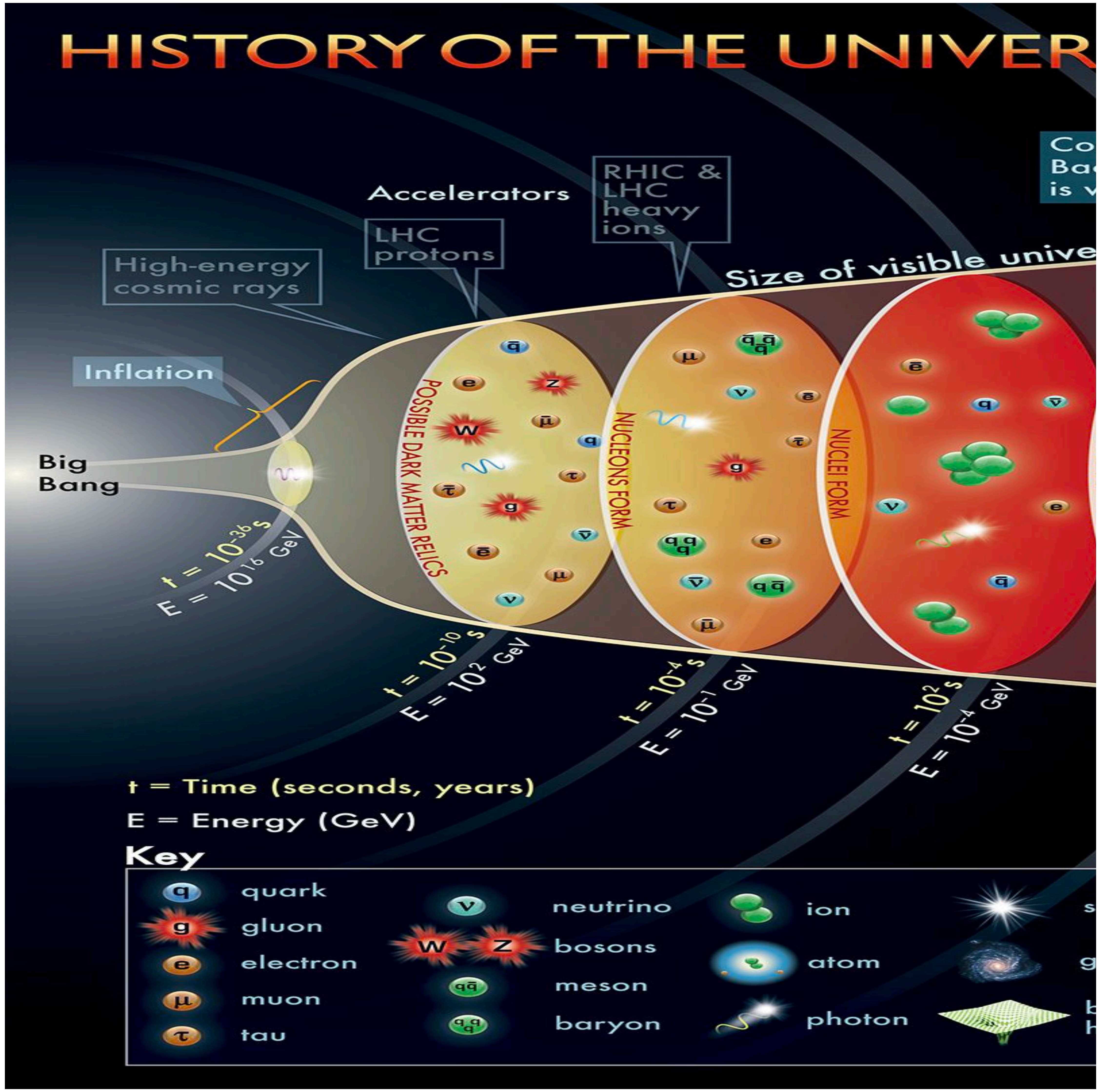
人们称该吻合为 WIMP miracle



Jungman *et al* hep-ph/9506380

热退耦合：优秀！

- 自然的得到暗物质残余丰度
  - 不需要UV信息 (以热平衡分布开局)
  - 电弱能标的湮灭截面
  - 与标准模型其他粒子相似的故事
    - (ν decoupling, n_p/n_n ratio, nuclear elements)
  - 预言了直接/间接/对撞机的实验信号



提纲

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  - 暗物质问题
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    - 暗物质分布
    - 暗物质的物理模型
      - 可能的暗物质候选者
      - WIMP暗物质
        - 热退耦暗物质残余丰度计算

背景知识1：狭义相对论与粒子

- 协变动量和逆变动量：

$$p^\mu = \{E, \vec{p}\} = \{E, p_x, p_y, p_z\}$$

$$p_\mu = g_{\mu\nu} p^\mu = \{E, -\vec{p}\} = \{E, -p_x, -p_y, -p_z\}$$

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- 质能关系和相位:

$$p^\mu p_\mu = E^2 - p^2 = m^2$$ Invariant mass

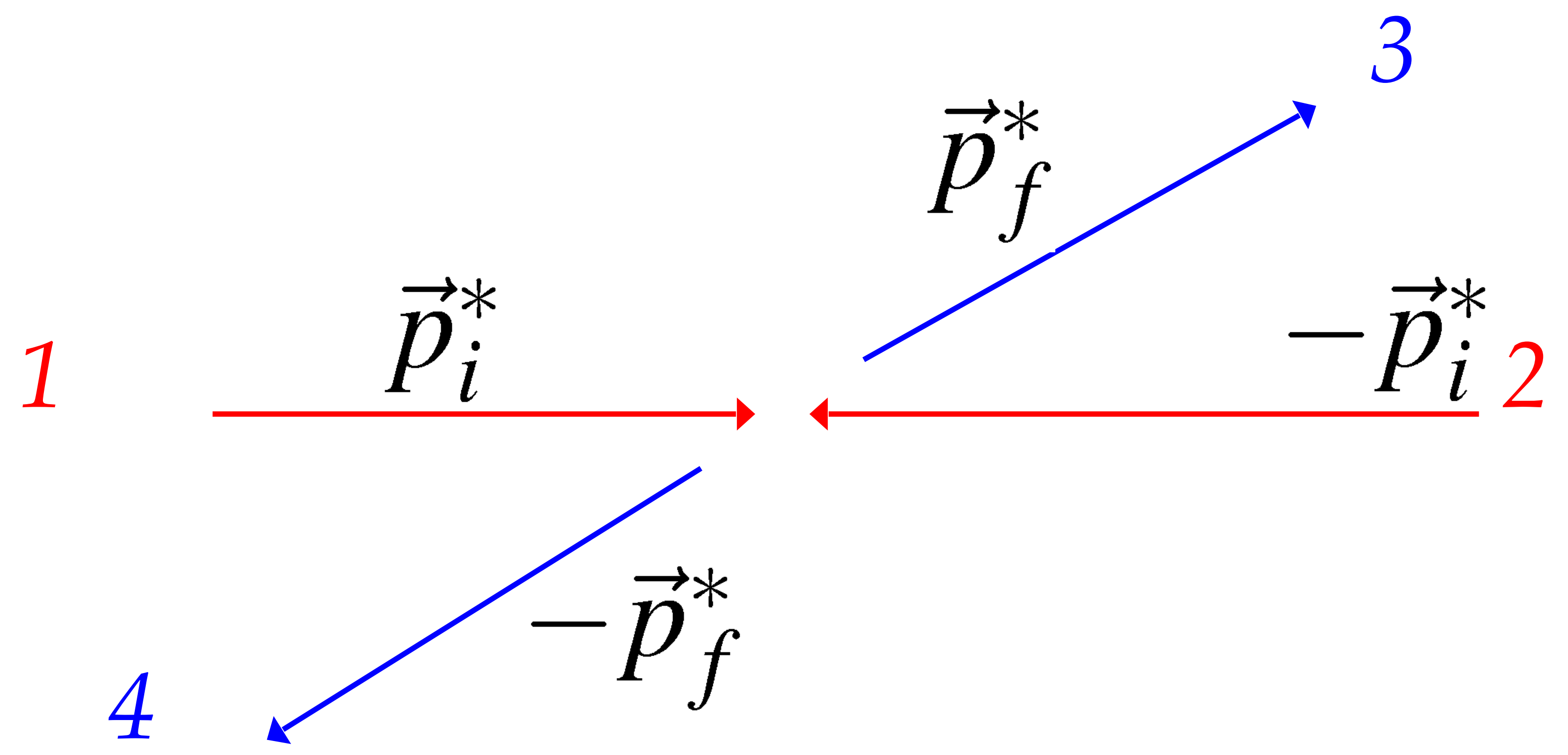
$$x^\mu p_\mu = Et - \vec{p}.\vec{r}$$ Phase

背景知识2：相空间与截面

- 粒子的相空间
  - 4D 洛伦兹不变的角度：单个on-shell 粒子的相空间

$$d\mathrm{PS} = \Theta(E)\delta(p\cdot p - m^2)d^4p = \frac{d^3\vec{p}}{(2\pi)^3 2E}$$

  - 为什么老是有$(2E)^{-1}$的因子?
  - Normalize to 2E particle in the volume



- 相互作用截面： $1+2\to 3+4$ (DM + DM > SM SM)

$$\sigma = \frac{1}{2E_1 2E_2\left|v_1 - v_2\right|}\int\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)\times\left|\mathscr{M}\left(p_1, p_2\to\left\{p_f\right\}\right)\right|^2(2\pi)^4\delta^{(4)}\left(p_1+p_2-\sum p_f\right)$$

背景知识3：粒子的态密度

- 粒子态相空间函数 $f(\vec{x}, \vec{p}, t)d\vec{x}d\vec{p}$
- 热平衡分布: $f_{\rm eq} = \dfrac{1}{e^{E/T} \pm 1} \approx e^{-E/T}$
- 粒子数密度 $n_{\rm eq} = \int d\vec{p} f_{\rm eq} = \int \dfrac{d\vec{p}}{(2\pi)^3} e^{-\frac{E}{T}}$
  - 高温$T \gg m$（相对论）极限 $n_{\rm eq} = T^3$
  - 低温$T \ll m$（非相对论）极限 $n_{\rm eq} = \left(\dfrac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}}$

背景知识4：宇宙度规与辐射为主的宇宙

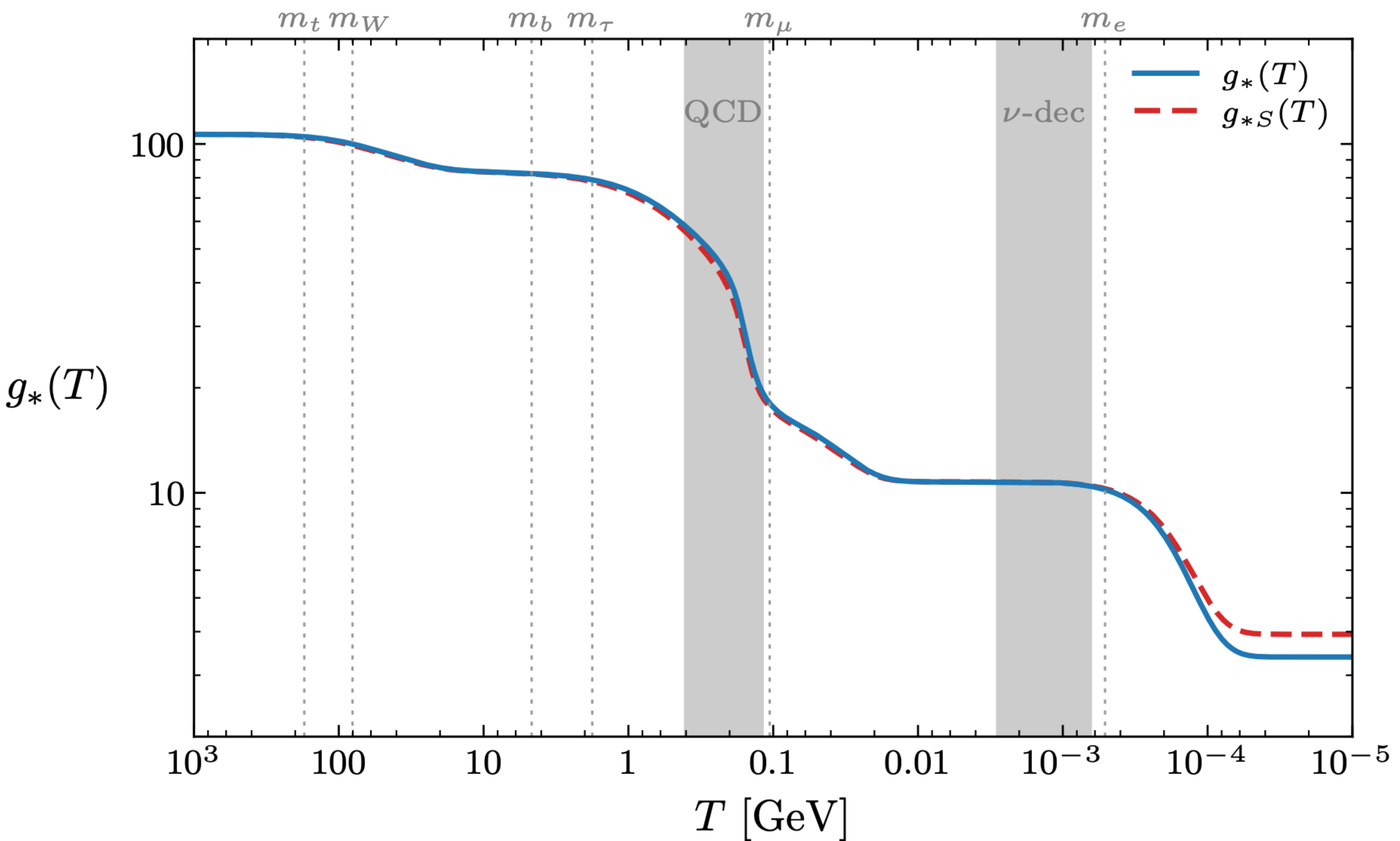
- 辐射为主的宇宙的膨胀速率

$$H_{\rm rad}^2 = \frac{8\pi^3}{90}\frac{g_* T^4}{m_{\rm PL}^2}$$

- 辐射为主的宇宙的温度红移

$$T \propto a(t)^{-1}$$

$$\rho_{\rm rad} \propto a(t)^{-4}$$



暗物质热退耦Boltzmann方程

- 直接从数密度出发

$$\mathrm{DM}_1 + \mathrm{DM}_2 \to \mathrm{SM}_3 + \mathrm{SM}_4$$

$$a^{-3}\frac{d(n_1 a^3)}{dt} = \int \Pi_{i=1}^{4} d\mathrm{PS}_i \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\mathscr{M}|^2$$

$$\times \left[f_3 f_4 (1 \pm f_1)(1 \pm f_2) - f_1 f_2 (1 \pm f_3)(1 \pm f_4)\right]$$

- 化简

暗物质热退耦Boltzmann方程

- 最终的暗物质演化Boltzmann方程

$$a^{-3}\frac{d(na^3)}{dt} = n_1^{\text{eq}} n_2^{\text{eq}} \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^{\text{eq}} n_4^{\text{eq}}} - \frac{n_1 n_2}{n_1^{\text{eq}} n_2^{\text{eq}}} \right)$$

$$\dot{n} + 3Hn = \langle \sigma v \rangle (n_{\text{eq}}^2 - n^2)$$

- 暗物质湮灭截面的热平均公式

$$\langle \sigma v \rangle \equiv \frac{1}{n_1^{\text{eq}} n_2^{\text{eq}}} \int \Pi_{i=1}^4 d\text{PS}_i \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\mathcal{M}|^2 \times e^{-\frac{E_1 + E_2}{T}}$$

求解—暗物质热退耦Boltzmann方程

- 粒子数密度的行为

$$n_{\rm eq}^{\rm rad} \sim T^3 \sim a^{-3}, \quad n_{\rm eq}^{\rm mat} \sim (mT)^{3/2} e^{-m/T}$$

$$n_{\rm freeze-out} \sim a^{-3}$$

- 一个实用的变量 DM Yield和采用温度变量x

$$Y_{\rm dm} \equiv n_{\rm dm}/s, \quad x \equiv m_{\rm dm}/T$$

- 暗物质演化Boltzmann方程 $dx/dt = (8\pi^3 g_\star/(90 m_{\rm Pl}^2))^{1/2} m^2/x$

$$\frac{dY}{dx} = \frac{\langle \sigma v \rangle x s}{\sqrt{\frac{8\pi^3 g_\star}{90 m_{\rm Pl}^2}} m^2} \left(Y_{\rm eq}^2 - Y^2\right)$$

求解—暗物质热退耦Boltzmann方程

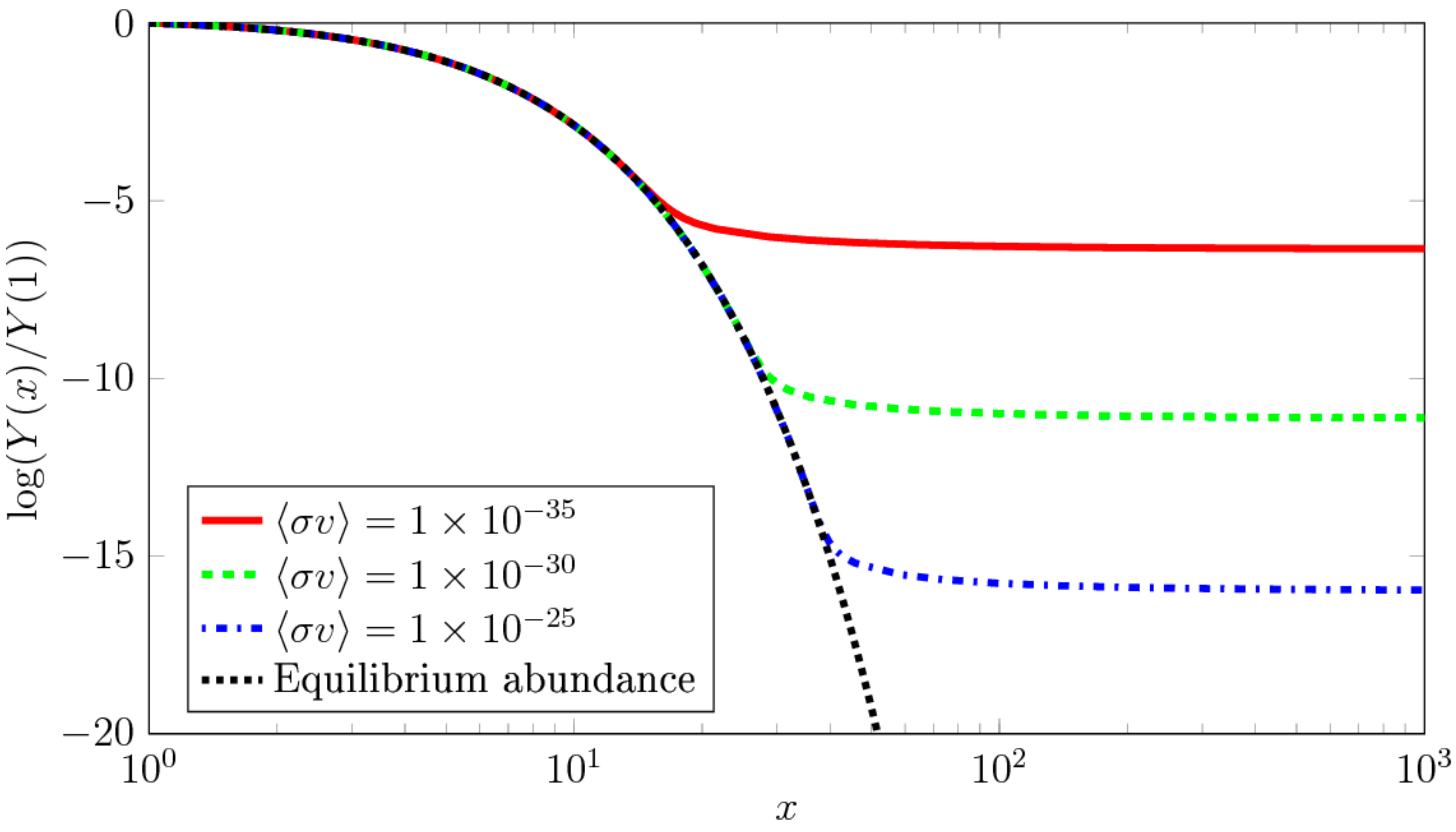
- 暗物质演化Boltzmann方程

$$\frac{dY}{dx} = \frac{\langle \sigma v \rangle x s}{\sqrt{\frac{8\pi^3 g_\star}{90 m_{\rm Pl}^2}}\, m^2} \left(Y_{\rm eq}^2 - Y^2\right)$$

- 暗物质热退耦温度

$$n_{\rm fo}\langle \sigma v \rangle \approx H_{\rm fo}$$

$$x_{\rm fo} \sim 25$$



近似求解—暗物质热退耦Boltzmann方程

- 近似求解Boltzmann方程

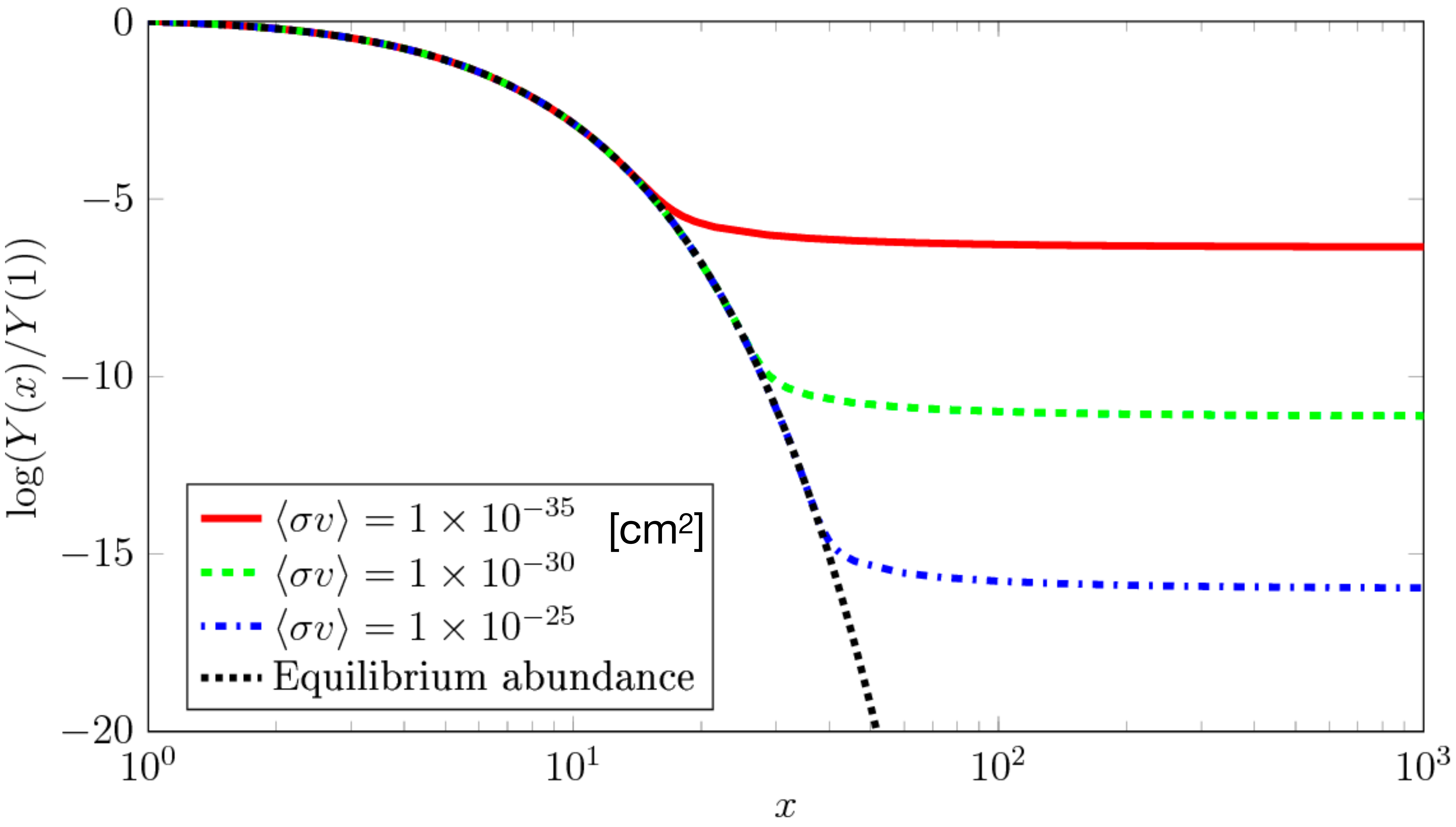
$$\frac{dY}{dx} = -\frac{\lambda}{x^2}(Y^2 - Y_{\rm eq}^2)$$

$$\frac{\lambda}{x^2} \equiv \langle \sigma v \rangle \frac{xs}{H_{\rm rad}(T = m_{\rm DM})}$$

- 对于 $x \gg 1$

$$\frac{dY}{dx} \approx -\frac{\lambda}{x^2} Y^2$$

$$\Longrightarrow Y_\infty^{-1} - Y_{\rm fo}^{-1} = \frac{\lambda}{x_{\rm fo}} \Longrightarrow Y_\infty^{-1} = \frac{\lambda}{x_{\rm fo}}$$



近似求解—暗物质热退耦Boltzmann方程

- 近似求解Boltzmann方程 $$Y_\infty^{-1} = \frac{\lambda}{x_{\rm fo}}$$
- 今天的暗物质能量密度占比 $\Omega_{\rm dm} = 26.8\,\%$

$$\Omega_{\rm dm} h^2 = \frac{Y_0 s_0 m_{\rm dm}}{\rho_{\rm cr}} h^2 \approx \frac{Y_\infty s_0 m_{\rm dm}}{\rho_{\rm cr}} h^2 \approx 0.3 \left(\frac{m_{\rm dm}}{\rm eV}\right) Y_\infty$$

$$\rho_{\rm cr} = 3H_0^2 m_{\rm Pl}^2/8\pi \approx 8\times 10^{-47} h^2\,{\rm GeV}^4 \text{ and } s_0 \approx 2970\,{\rm cm}^{-3}$$

- 暗物质热退耦湮灭截面大小

$$\Omega h^2 \approx 0.1 \left(\frac{x_f}{25}\right)\left(\frac{g_\star}{80}\right)^{-1}\left(\frac{3\times 10^{-26}{\rm cm}^3{\rm s}^{-1}}{\langle\sigma v\rangle}\right)$$

WIMP 暗物质奇迹

- 暗物质热退耦湮灭截面大小

$$\Omega h^2 \approx 0.1 \left(\frac{x_f}{25}\right)\left(\frac{g_\star}{80}\right)^{-1}\left(\frac{3\times 10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}}{\langle\sigma v\rangle}\right)$$

$$\langle \sigma v\rangle \sim 3\times 10^{-26} cm^3/s$$

$$\sim 10^{-8}\ \mathrm{GeV}^{-2} \sim \frac{\alpha^2}{m_W^2}$$

- 暗物质可能和弱相互作用能标相关联

TeV 新物理

- 至尊私房菜：超对称
- Neutralino WIMP 暗物质
    - 超对称暗物质的理想化身：WIMP暗物质
- 希格斯粒子级差问题

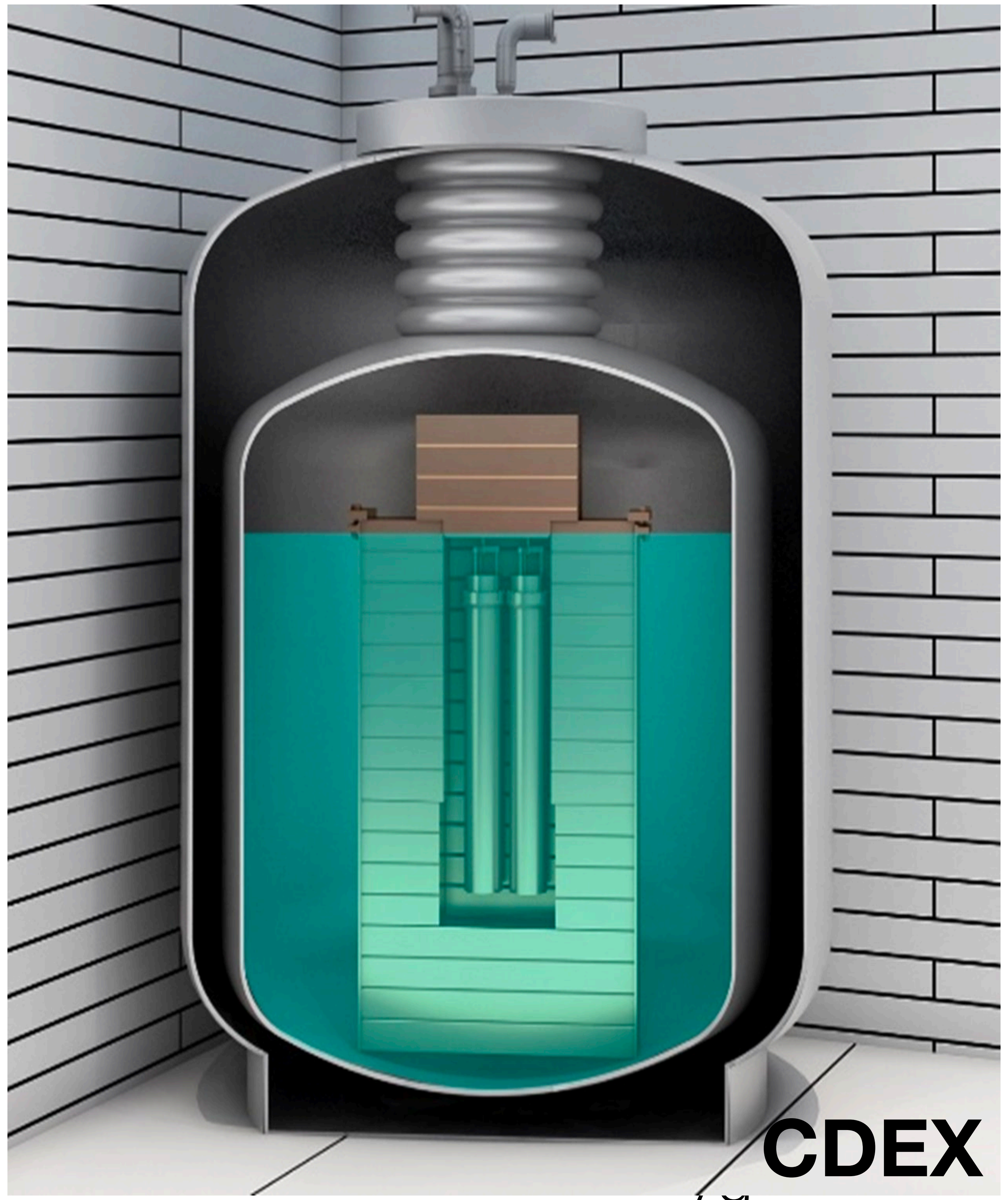

全剧终！

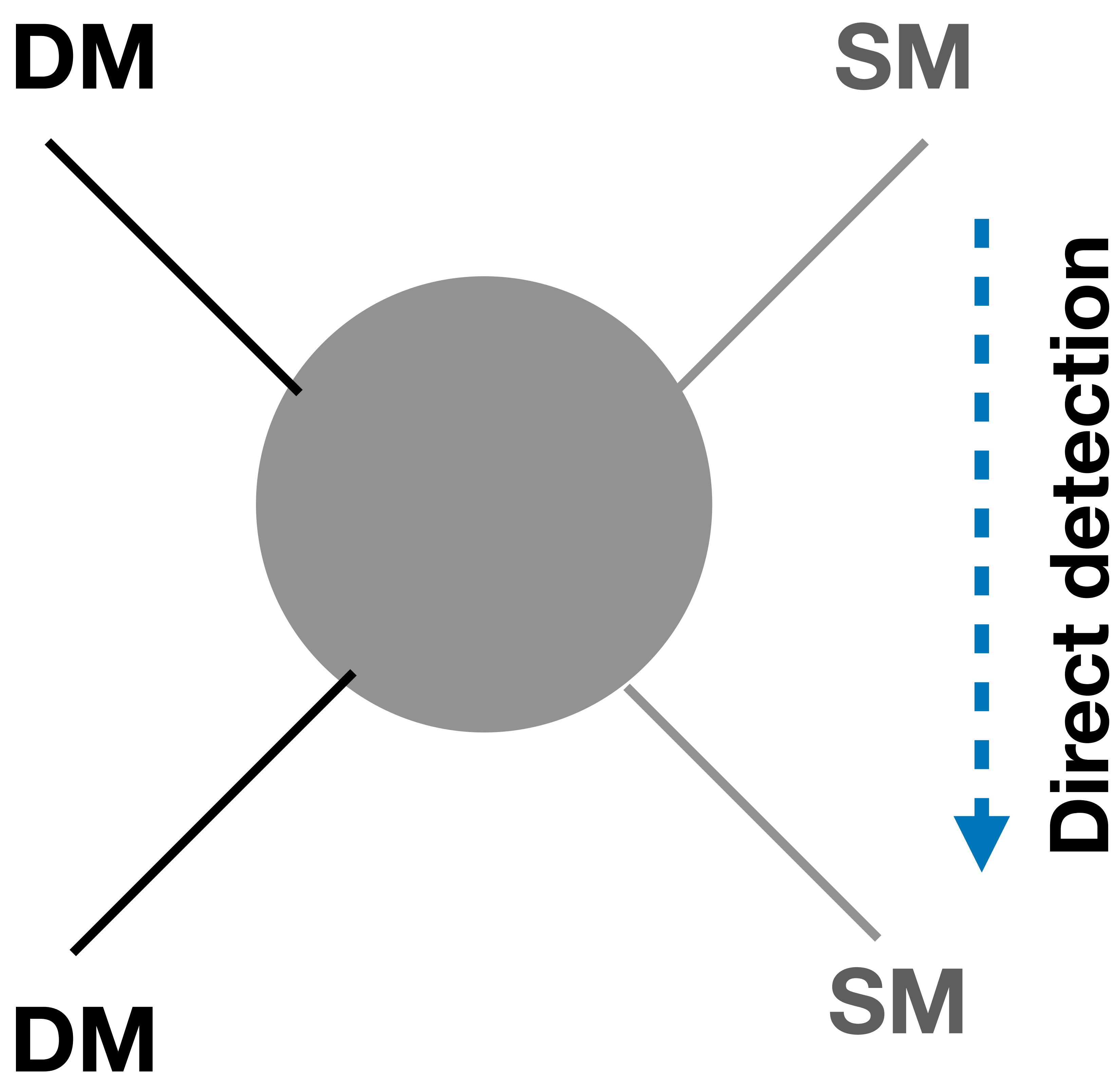
提纲

- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
    - 热退耦暗物质残余丰度计算
    - WIMP暗物质的直接探测危机
      - 6种解决方法
      - WIMP暗物质的间接探测限制
      - WIMP暗物质变种模型
    - 轻暗物质与Dark Sector

The WIMP crisis from direct detection

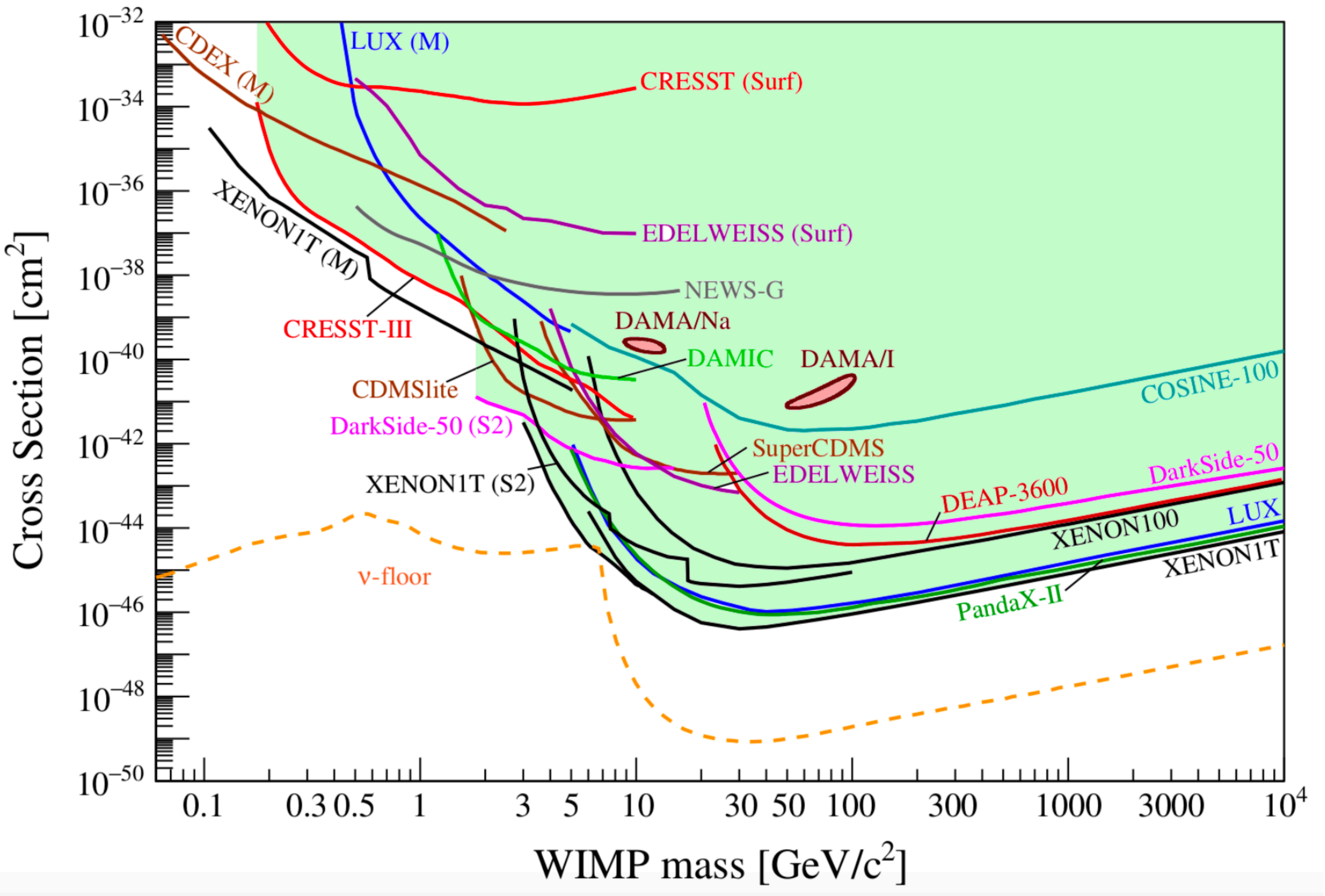
- Weakly Interacting Massive Particle
- The sizable coupling of DM to SM particles predicts sizable scattering cross-section





The WIMP crisis from direct detection

- Null result from direct detection
  - Maybe discovery in the corner?
  - Neutrino floor and beyond: directional ..
  - The rise of light dark matter ($\lesssim$ 10 GeV)
  - We focus on EW scale ($\gtrsim$ 10 GeV)



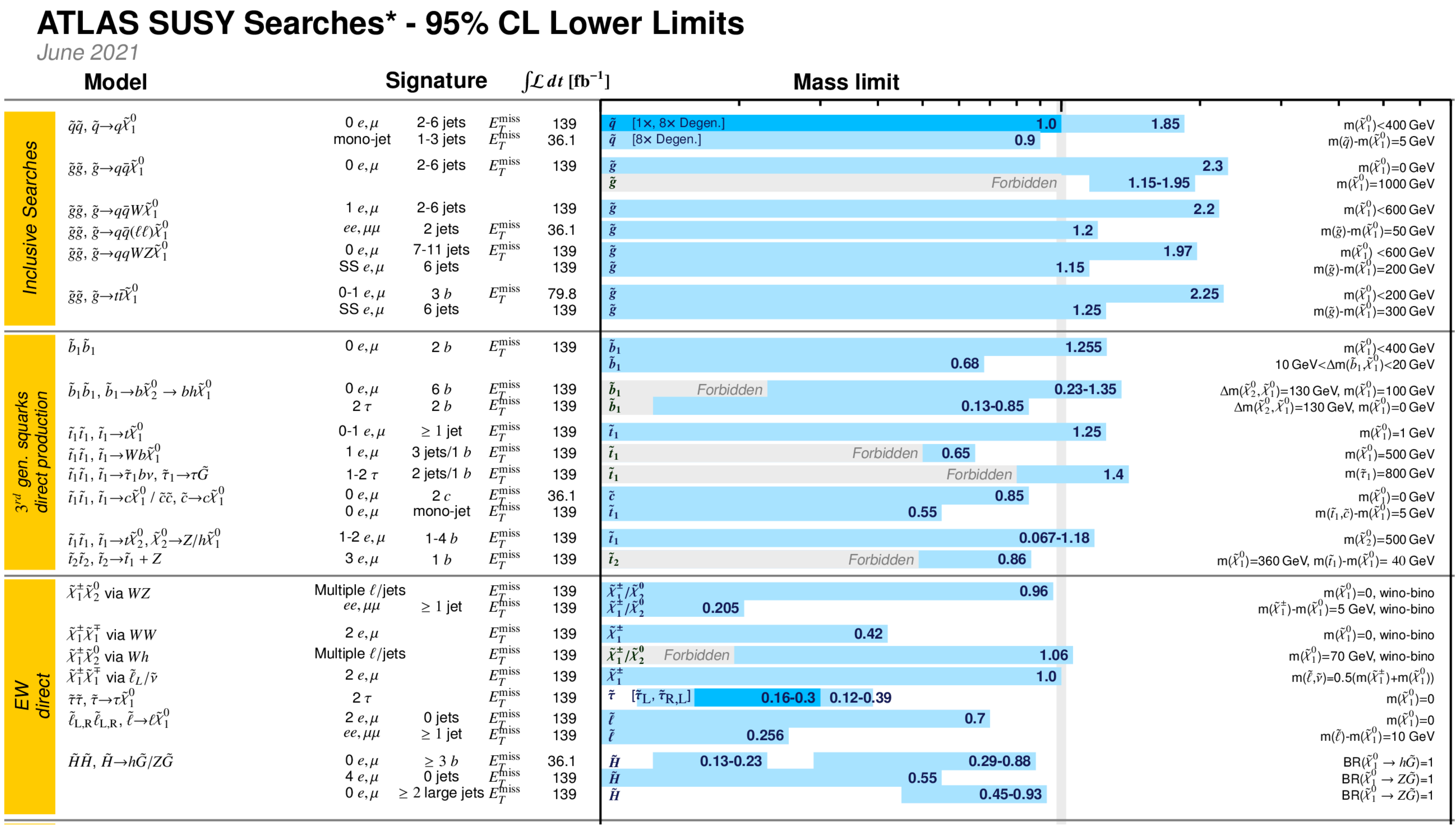
山雨欲来风满楼

APPEC Committee Report: 2104.07634

The WIMP crisis from direct detection

- Null result from direct detection
  - Maybe discovery in the corner? 转角遇到？
  - Neutrino floor and beyond: directional ..
  - The rise of light dark matter ($\lesssim$ 10 GeV)
  - We focus on EW scale ($\gtrsim$ 10 GeV)

Together with the fact that, we have not seen SUSY either.

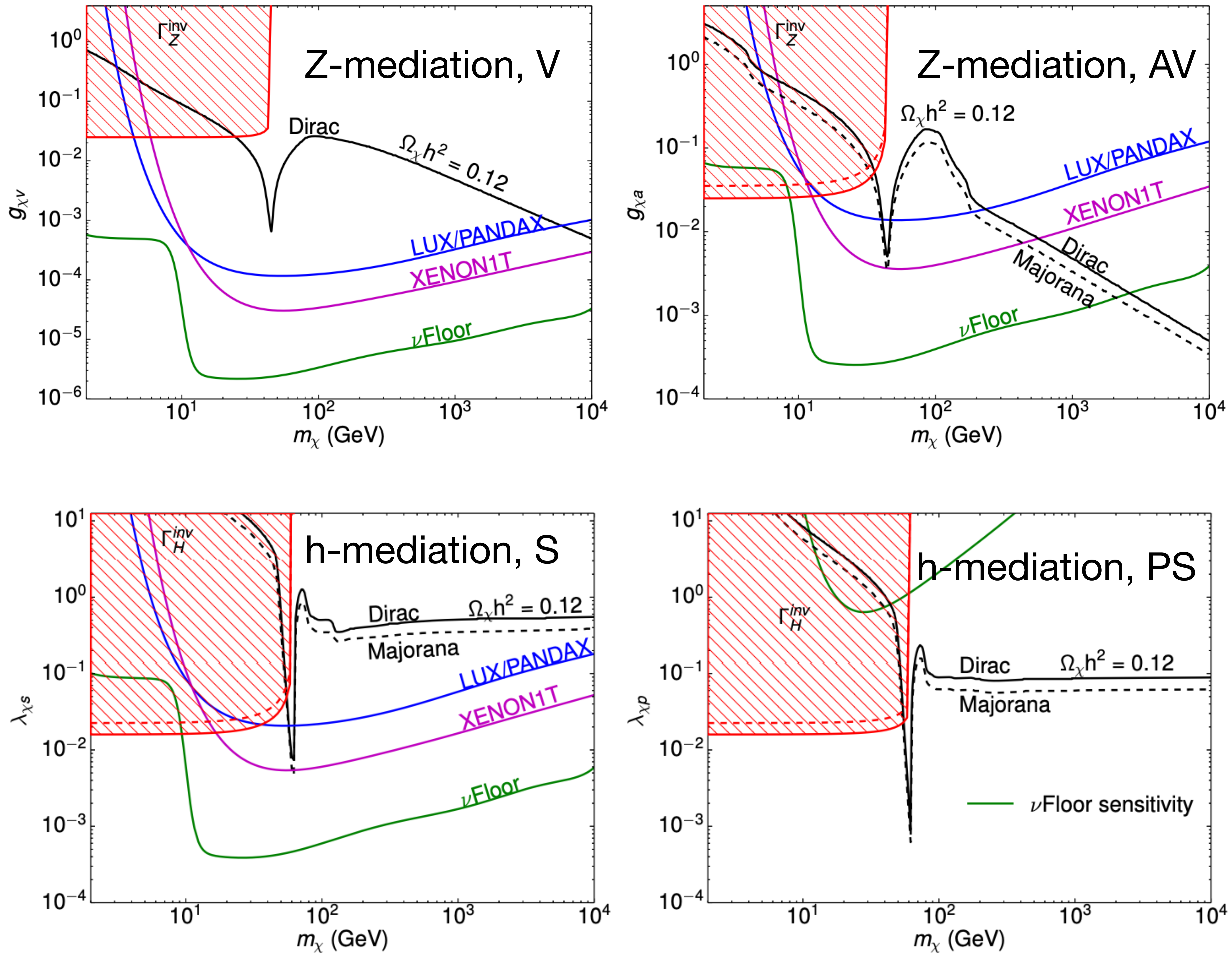


屋漏偏逢连夜雨

The WIMP crisis from direct detection

- SM Higgs and Z mediated scenario are highly constrained
- Other mediators without DD suppression is also highly constrained, e.g. A'
  - Unless in the resonant region

Toward (Finally!) Ruling Out Z and Higgs Mediated Dark Matter Models
Hooper et al, ArXiv: 1609.09079, JCAP



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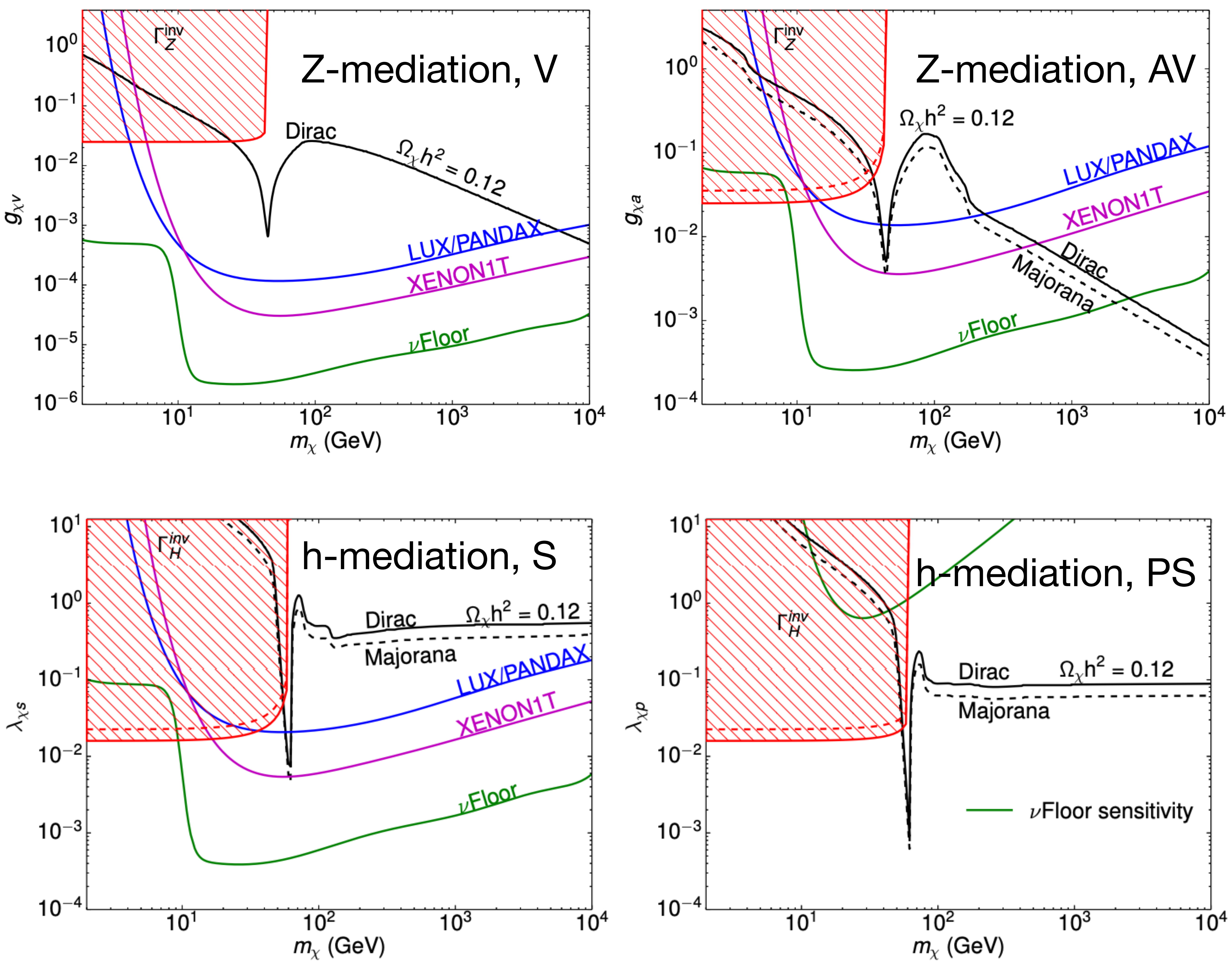
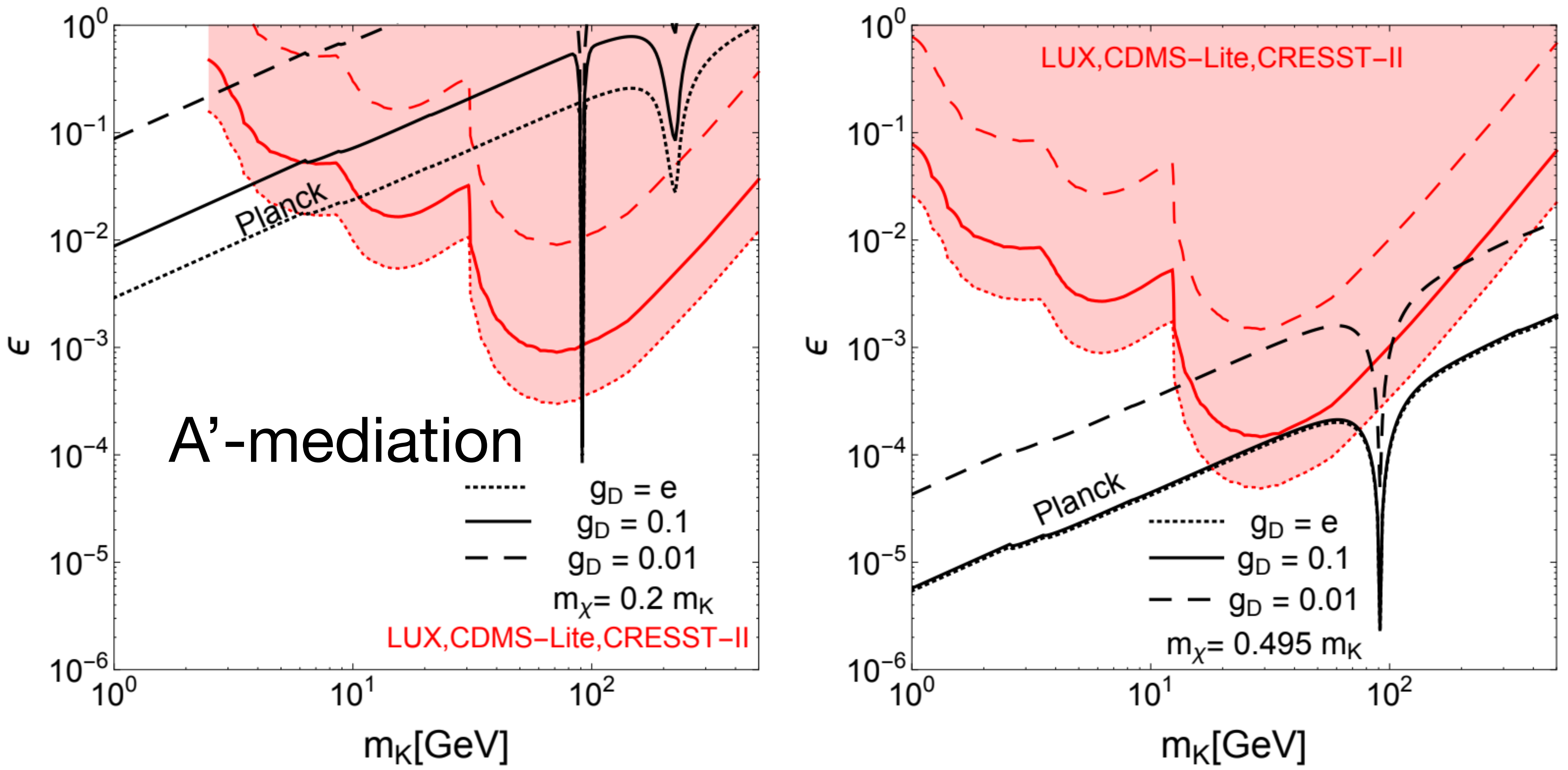


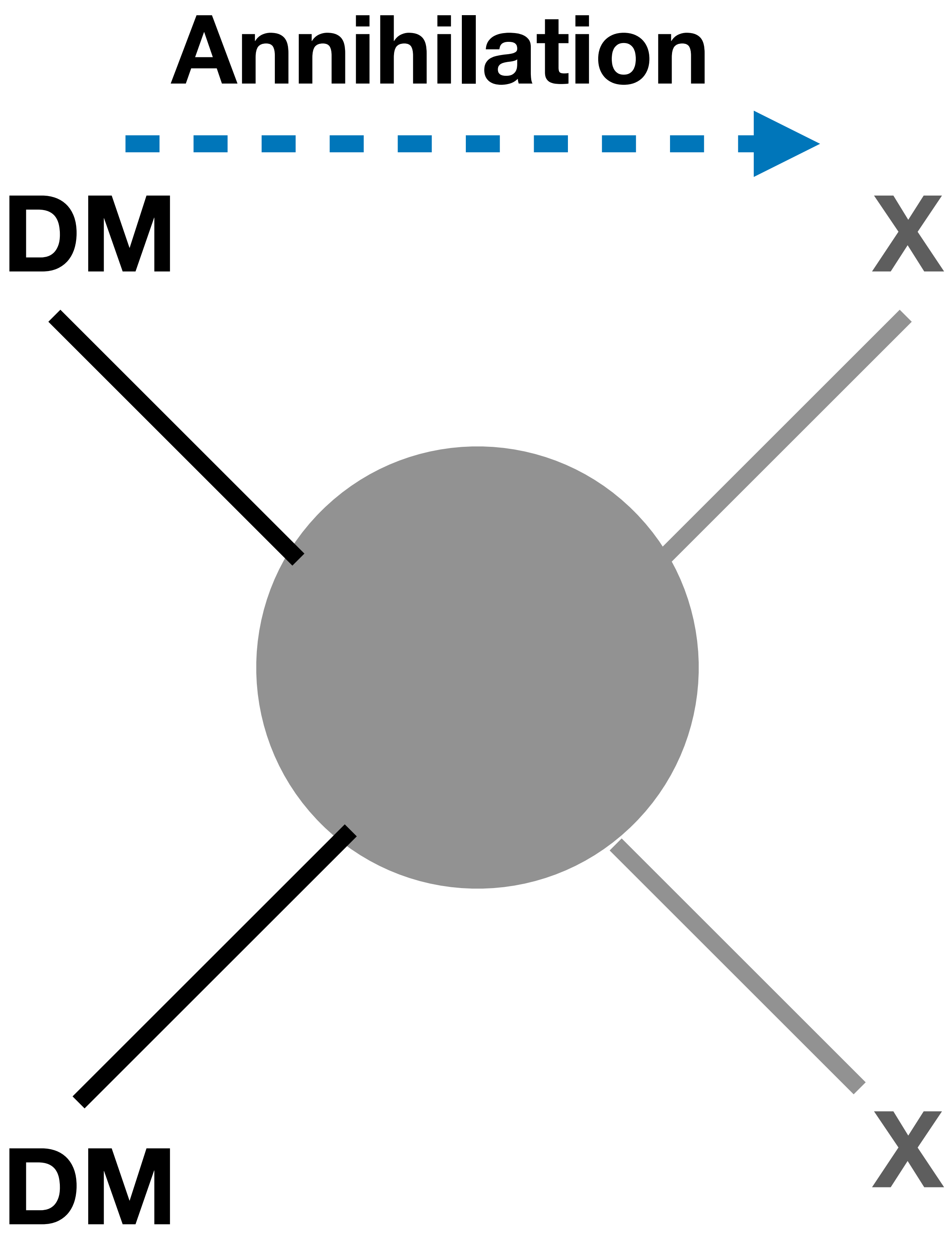
JL, X.P. Wang, F. Yu, 1704.00730, JHEP

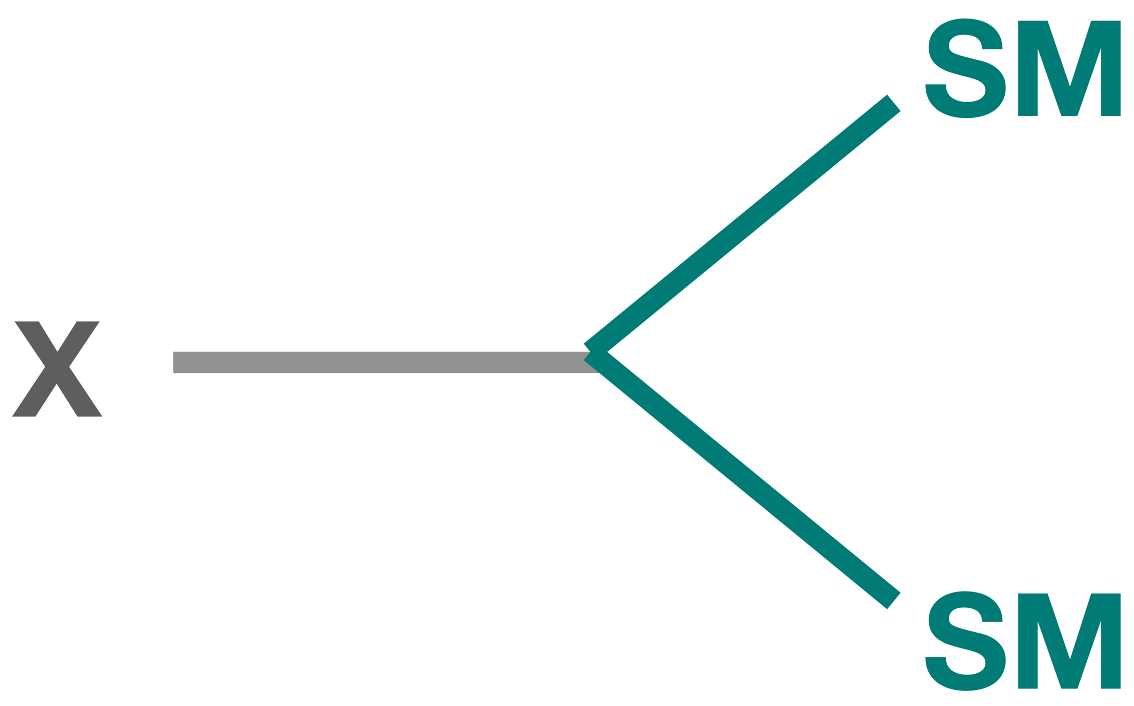


The way-out from direct detection limits

- 1. Very small coupling:

  - 1.1 Secluded dark matter (dark sector)





Dark mediator
with very small coupling to SM

The way-out from direct detection limits

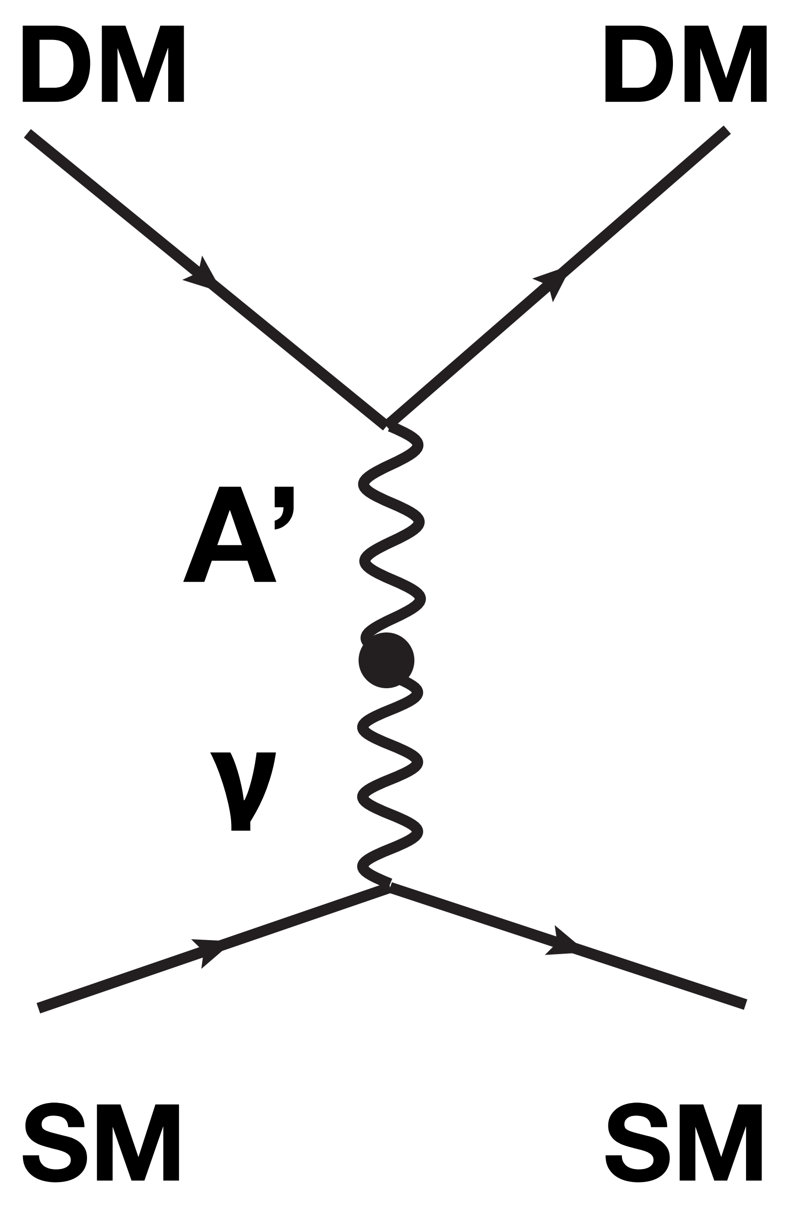
- 1.1 Secluded dark matter (dark sector)

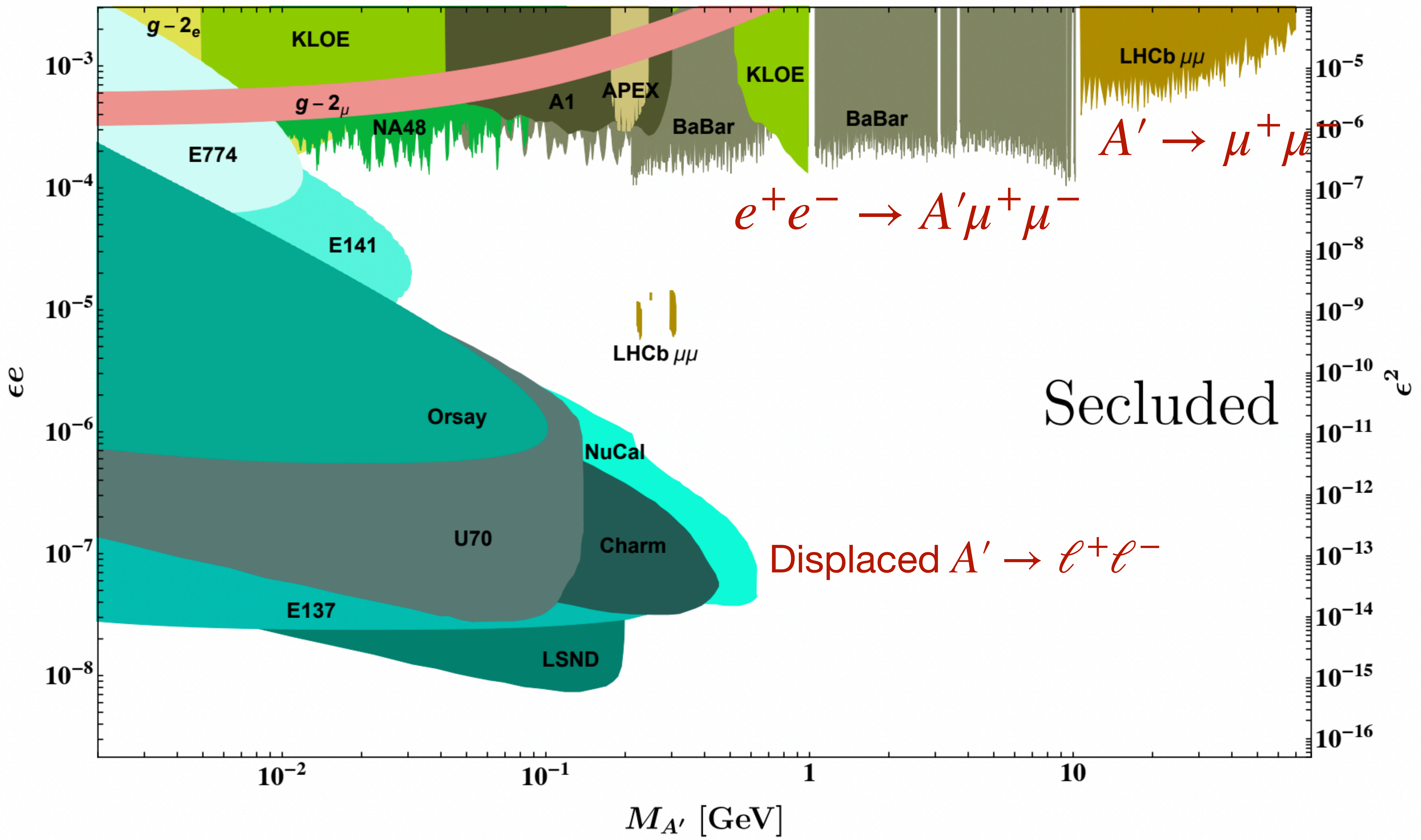
  - Looking for mediator X is easier than DM

Bauer et al: 1803.05466 (JHEP)

Dark photon A' example: visible

$$\epsilon F'_{\mu\nu} B^{\mu\nu} : \; A' \to \ell^+ \ell^-$$





The way-out from direct detection limits

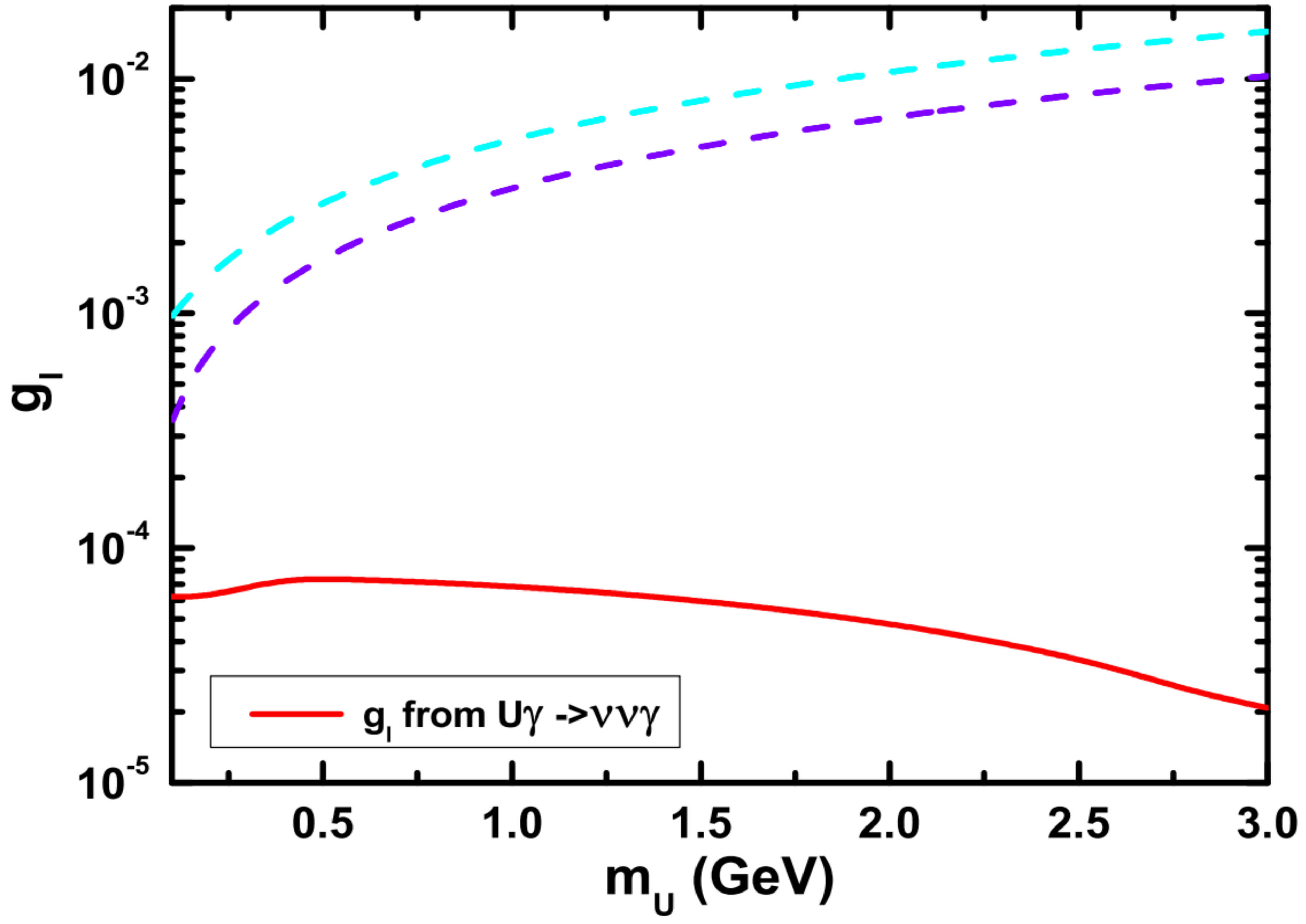
- 1.1 Secluded dark matter (dark sector)

  - Looking for mediator X is easier than DM

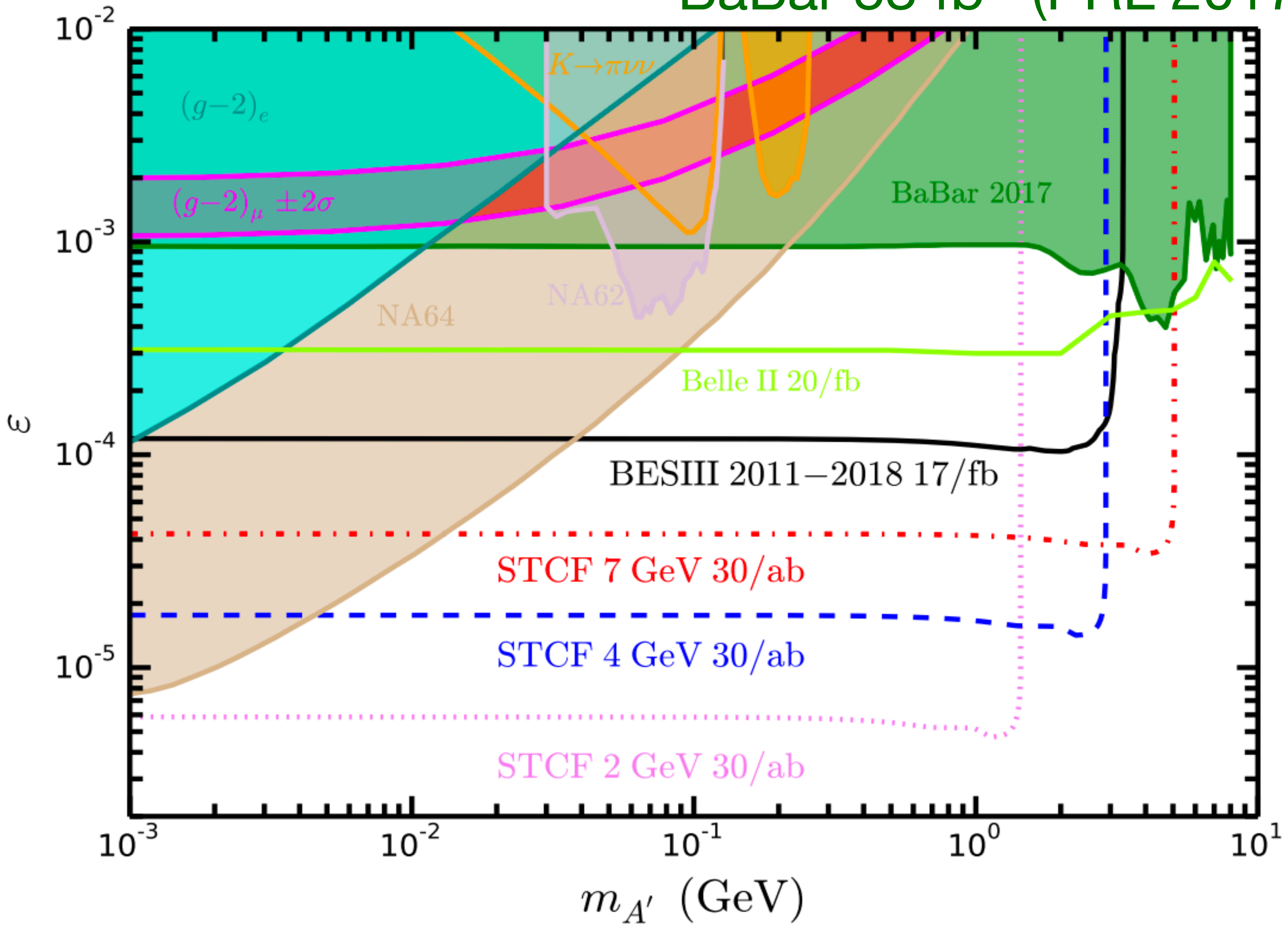
Dark photon A' example: invisible $A' \to \bar{\nu}\nu, \bar{\chi}\chi$

NA64: e beam dump, but for invisible final states

BaBar 53 fb^{-1} (PRL 2017)



PF Yin, JL, SH Zhu: 0904.4644 (PRD)



BESIII: 1907.07046 (PRD)

The way-out from direct detection limits

- 2. Suppressed scattering cross-section:
  - By velocity or momentum transfer

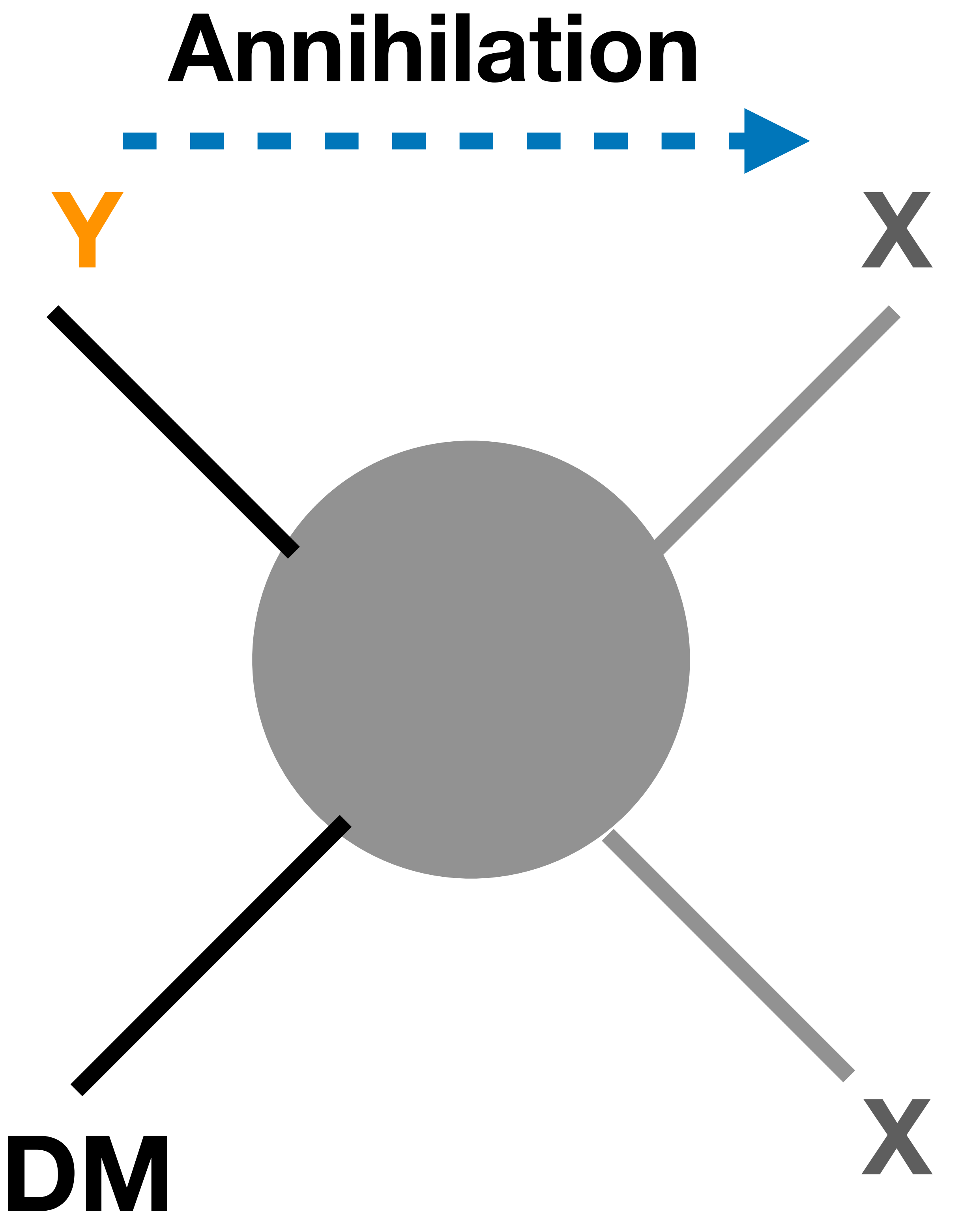
Case for Fermionic DM

Kumar & Marfatia:1305.1611 (PRD)

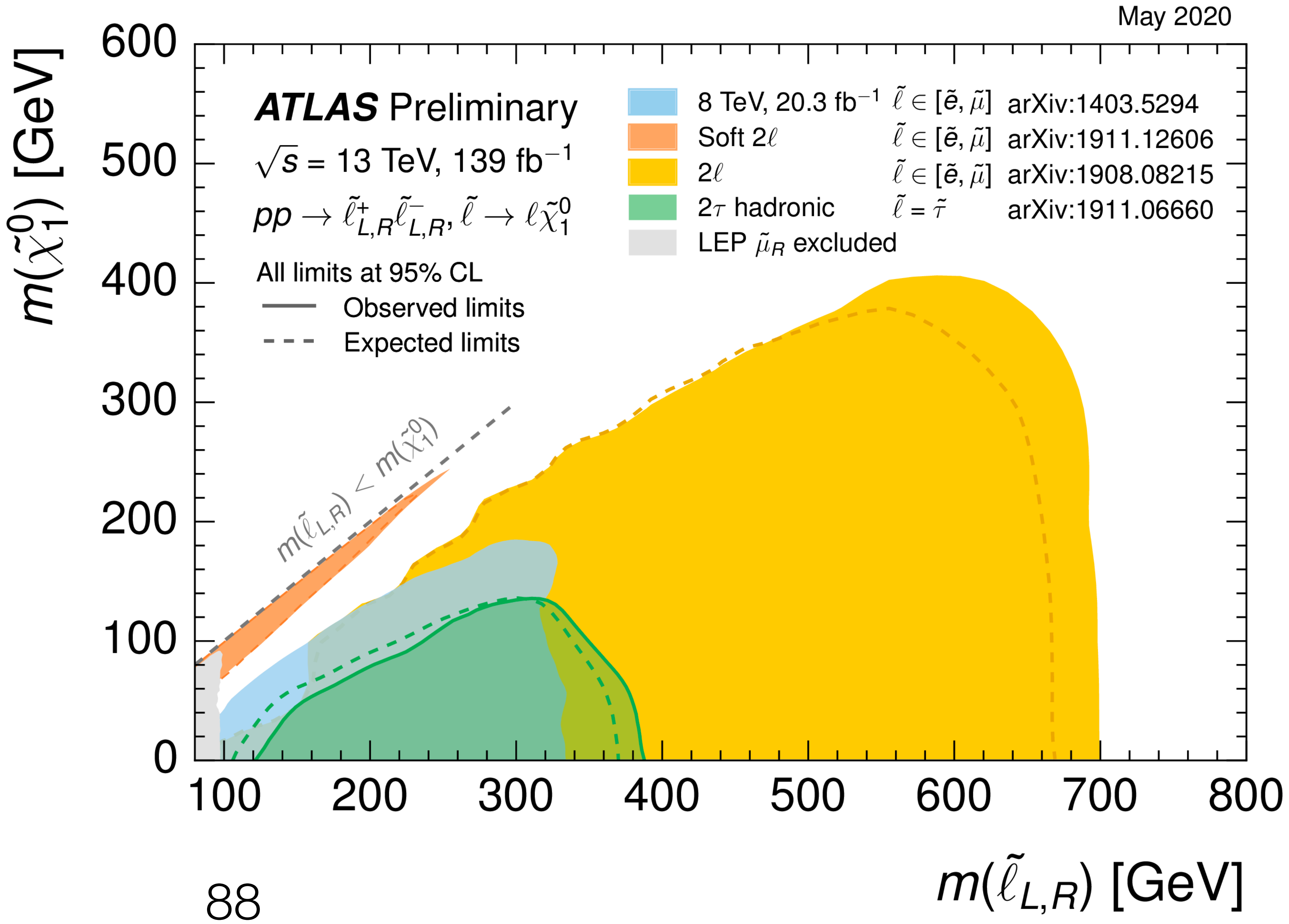
	Name	Interaction Structure	$\sigma_{\rm SI}$ suppression	$\sigma_{\rm SD}$ suppression	s-wave?
Scalar	F1	$\bar{X}X\bar{q}q$	1	$q^2 v^{\perp 2}$ (SM)	No
	F2	$\bar{X}\gamma^5 X\bar{q}q$	q^2 (DM)	$q^2 v^{\perp 2}$ (SM); q^2 (DM)	Yes
	F3	$\bar{X}X\bar{q}\gamma^5 q$	0	q^2 (SM)	No
Pseudoscalar	F4	$\bar{X}\gamma^5 X\bar{q}\gamma^5 q$	0	q^2 (SM); q^2 (DM)	Yes
Vector	F5	$\bar{X}\gamma^\mu X\bar{q}\gamma_\mu q$ (vanishes for Majorana X)	1	$q^2 v^{\perp 2}$ (SM) q^2 (SM); q^2 or $v^{\perp 2}$ (DM)	Yes
Anapole	F6	$\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu q$	$v^{\perp 2}$ (SM or DM)	q^2 (SM)	No
	F7	$\bar{X}\gamma^\mu X\bar{q}\gamma_\mu\gamma^5 q$ (vanishes for Majorana X)	$q^2 v^{\perp 2}$ (SM); q^2 (DM)	$v^{\perp 2}$ (SM) $v^{\perp 2}$ or q^2 (DM)	Yes
	F8	$\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu\gamma^5 q$	$q^2 v^{\perp 2}$ (SM)	1	$\propto m_f^2/m_X^2$
	F9	$\bar{X}\sigma^{\mu\nu}X\bar{q}\sigma_{\mu\nu}q$ (vanishes for Majorana X)	q^2 (SM); q^2 or $v^{\perp 2}$ (DM) $q^2 v^{\perp 2}$ (SM)	1	Yes
	F10	$\bar{X}\sigma^{\mu\nu}\gamma^5 X\bar{q}\sigma_{\mu\nu}q$ (vanishes for Majorana X)	q^2 (SM)	$v^{\perp 2}$ (SM) q^2 or $v^{\perp 2}$ (DM)	Yes

The way-out from direct detection limits

- 3. Coannihilation mechanism

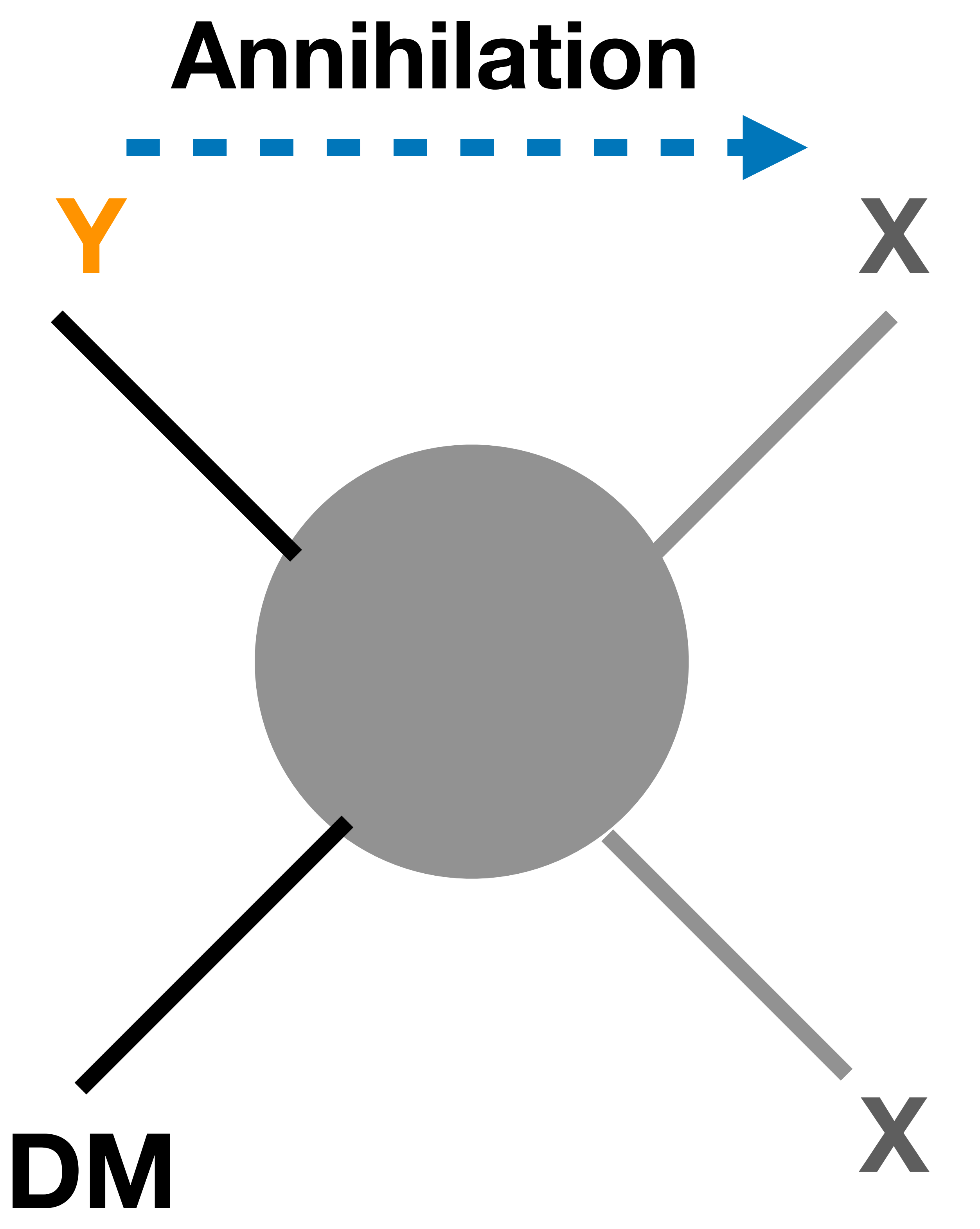


- Y has a close mass with DM
  - Y is not populated today due to decay
  - Charged Y: near degenerate spectrum of SUSY, AMSB; EW multiplet DM (2n+1, 0) ($\delta m \sim$ 166 MeV)



The way-out from direct detection limits

- 3. Coannihilation mechanism



- Y has a close mass with DM
  - Y is not populated today due to decay
  - Charged Y: near degenerate spectrum of SUSY, AMSB
  - Neutral Y: Inelastic Dark Matter

Fermionic DM with kinetic mixing A' mediator

$$\mathscr{L} = \bar{\psi} i \gamma_\mu D^\mu \psi + m \bar{\psi} \psi + \delta \overline{\psi^c} \psi / 2$$

$$\overline{\psi} \gamma_\mu \psi \simeq i(\overline{\chi}_1 \overline{\sigma}_\mu \chi_2 - \overline{\chi}_2 \overline{\sigma}_\mu \chi_1) + \frac{\delta}{2m}(\overline{\chi}_2 \overline{\sigma}_\mu \chi_2 - \overline{\chi}_1 \overline{\sigma}_\mu \chi_1).$$

$$m_{\chi_1} = m - \delta; \; m_{\chi_2} = m + \delta$$

Smith, Weiner: hep-ph/0101138 (PRD)

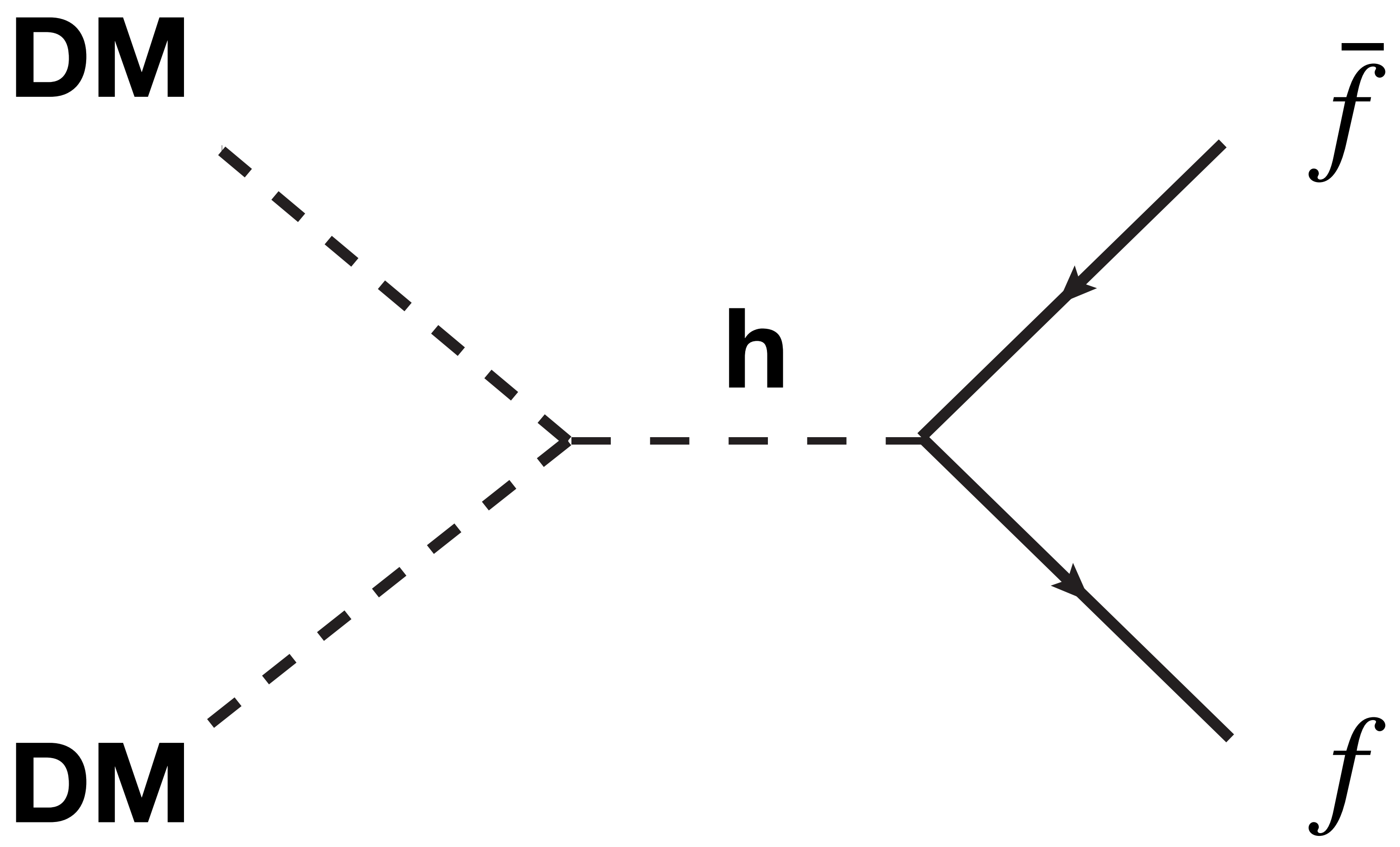
The way-out from direct detection limits

- 4. Resonant annihilation

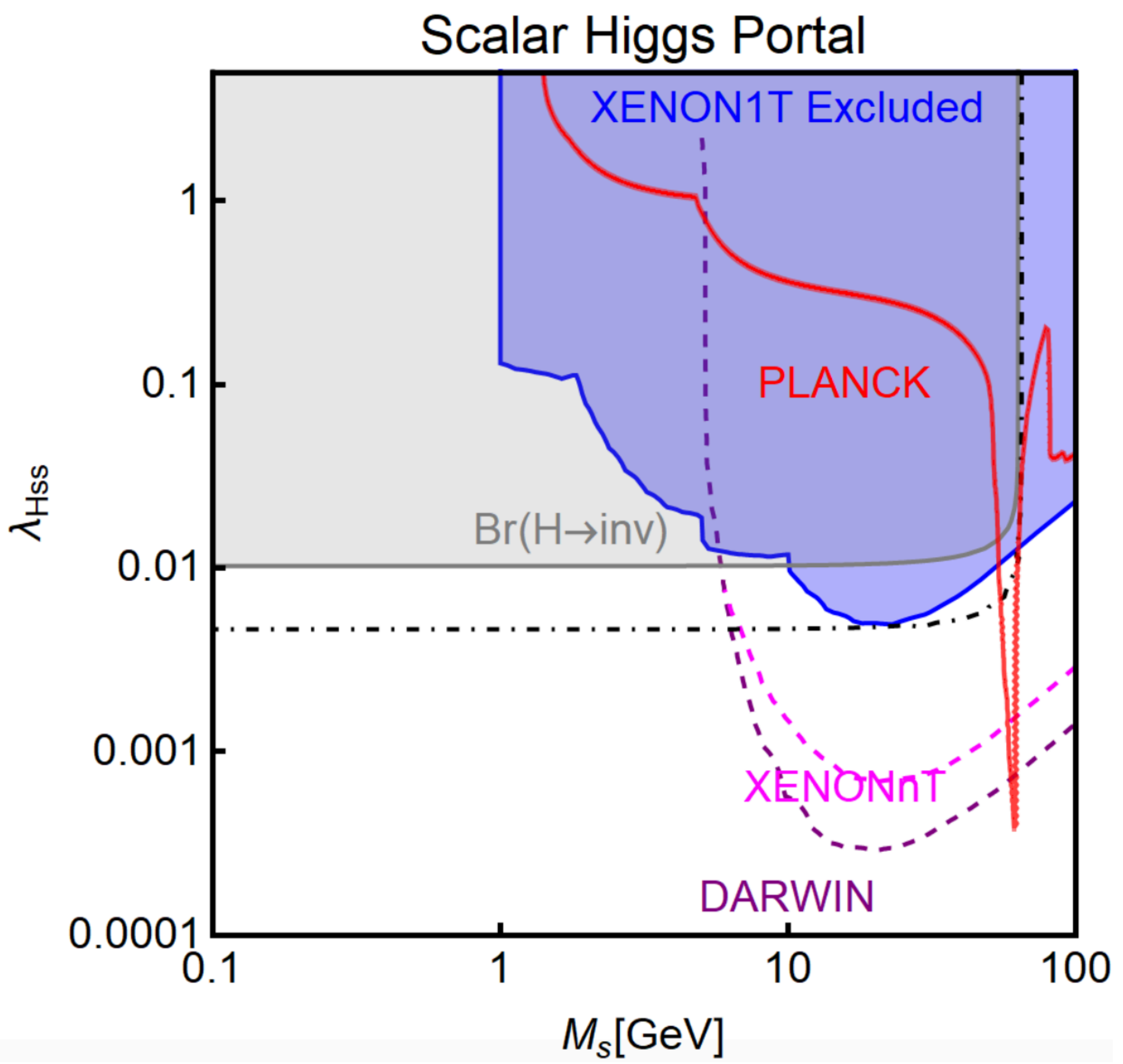
  - $2m_{\mathrm{DM}} \approx m_X$

Scalar DM (s) with a Higgs portal coupling

$$\Delta\mathcal{L}_s = -\frac{1}{2}m_s^2 s^2 - \frac{1}{4}\lambda_s s^4 - \frac{1}{4}\lambda_{Hss}\phi^\dagger\phi s^2$$



+ 2 diagrams to hh



Arcadi et al: 2101.02507

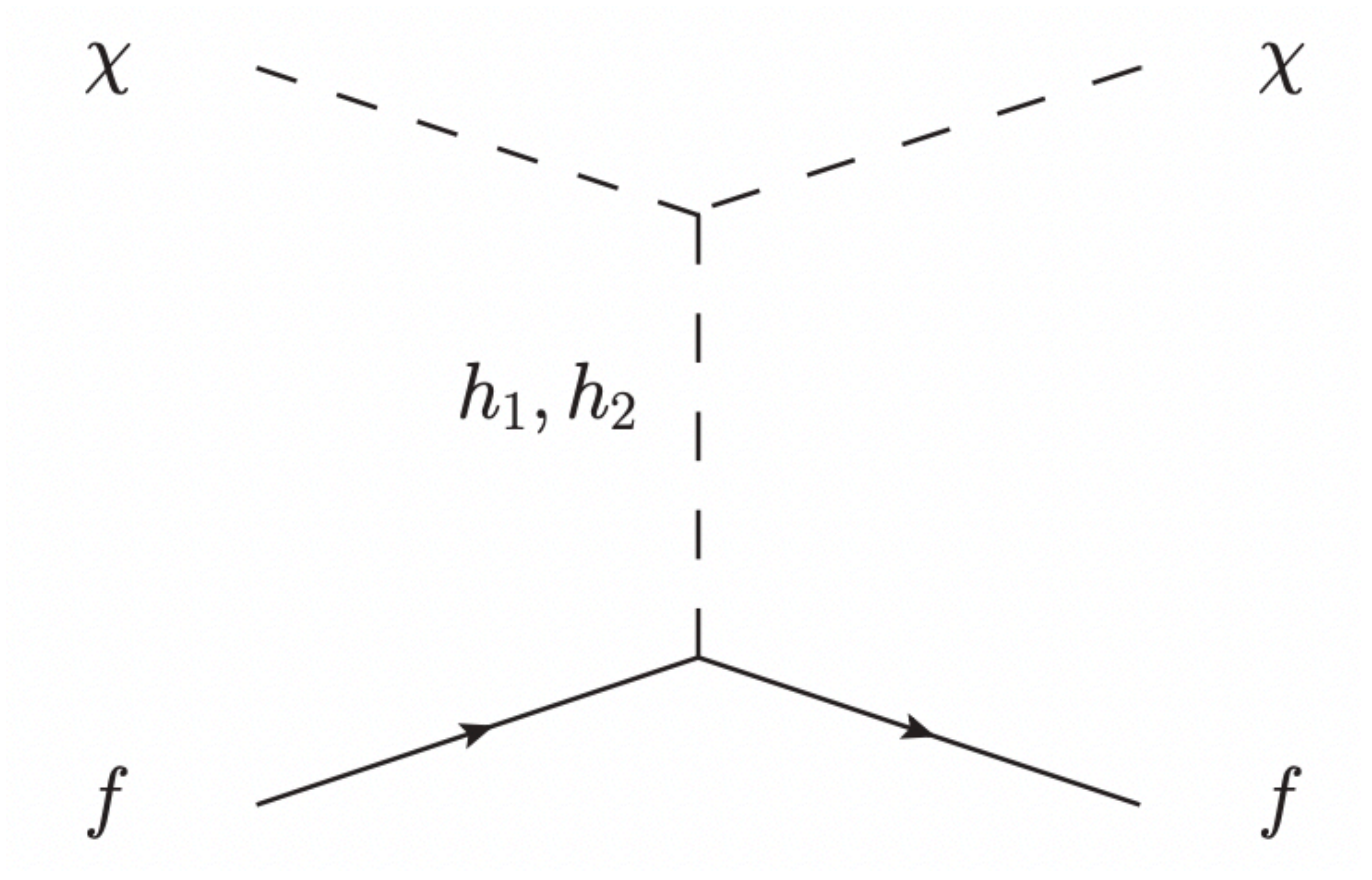
See also WL Guo, LY Wu et al 2010; B Li, YF Zhou 2015

The way-out from direct detection limits

- 5. Cancellation effect in scattering cross-section

  - SM Higgs - Dark scalar mediator cancellation

Gross, Lebedev1, Toma: 1708.02253 (PRL)



$$V_0 = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \lambda_{HS}|H|^2|S|^2 + \frac{\lambda_S}{2}|S|^4$$

$$V_{\rm soft} = -\frac{\mu_S'^2}{4} S^2 + \text{h.c.} \qquad \text{symmetry} : S \leftrightarrow S^*$$

$$S = (v_s + s + i\chi)/\sqrt{2}$$ Pseudoscalar DM

CP-even scalar mixing (s, h) $\rightarrow (h_1,\ h_2)$

$$\mathcal{L} \supset -(h_1 \cos\theta + h_2 \sin\theta) \sum_f \frac{m_f}{v} \bar{f} f \qquad \mathscr{L} \supset \frac{\chi^2}{2v_s}\left(m_{h_1}^2 \sin\theta h_1 - m_{h_2}^2 \cos\theta h_2\right)$$

$$\mathcal{A}_{dd}(t) \propto \sin\theta\cos\theta\left(\frac{m_{h_2}^2}{t - m_{h_2}^2} - \frac{m_{h_1}^2}{t - m_{h_1}^2}\right) \simeq \sin\theta\cos\theta\,\frac{t\,(m_{h_2}^2 - m_{h_1}^2)}{m_{h_1}^2\, m_{h_2}^2} \simeq 0$$

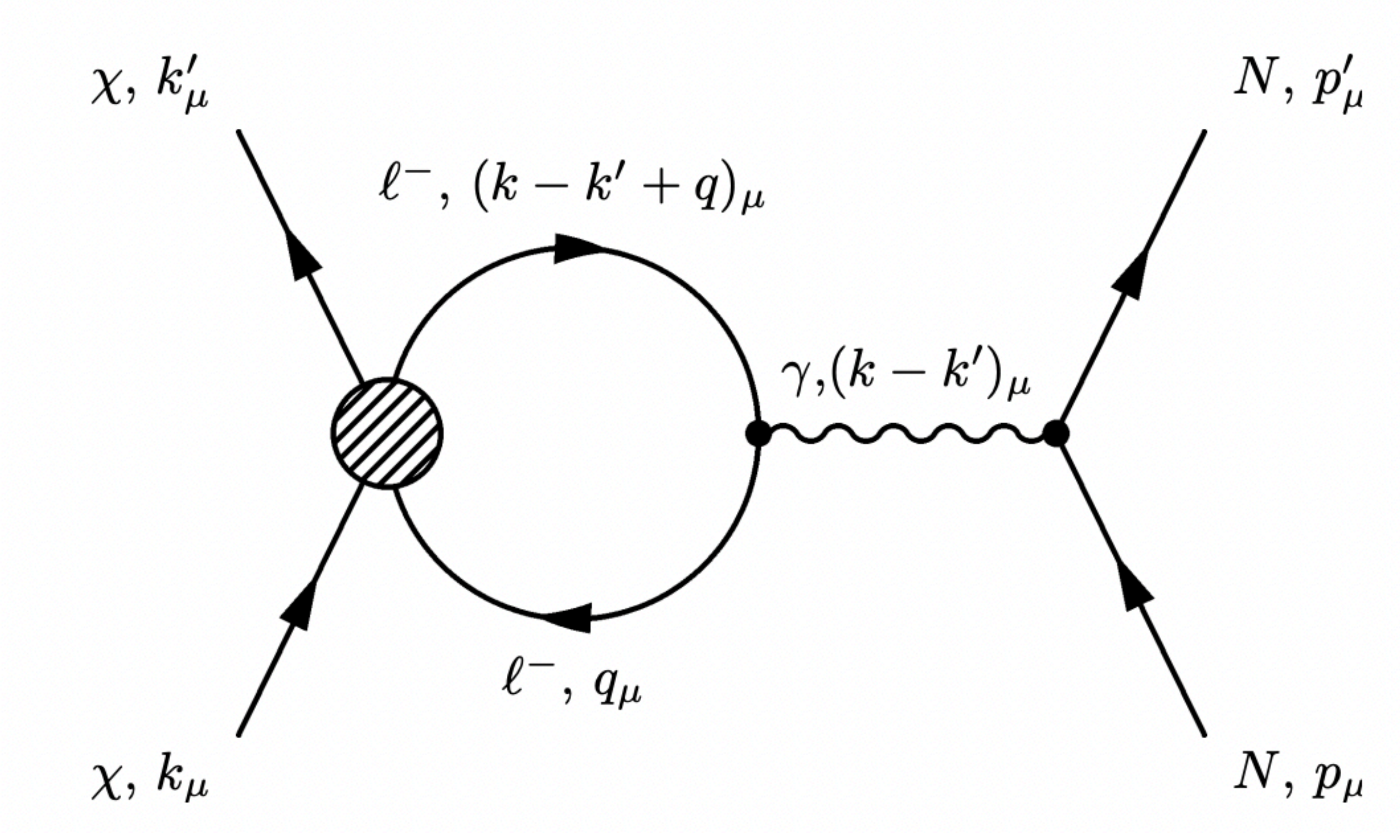
See JL, XP Wang and F Yu 1704.00730 (JHEP), for cancellation between A' - Z boson in kinetic mixing dark photon model

The amplitude is suppressed by q^2 from pseudo-goldstone nature

See an extension from Honghao Zhang et al, 2109.11499

The way-out from direct detection limits

- 6. Leptophilic models
  - Only couples to electrons, couples to nucleons at 1-loop
    - For light DM, e-DM recoils can have stringent limits (e.g. XENON1T, PANDAX, CDEX)
    - For heavy DM, neucleus-DM recoils wins over e-DM recoil



$$R^{\rm WAS} : R^{\rm WES} : R^{\rm WNS} \sim \epsilon_{\rm WAS} : \epsilon_{\rm WES}\frac{m_e}{m_N} : \left(\frac{\alpha_{\rm em} Z}{\pi}\right)^2 \sim 10^{-17} : 10^{-10} : 1$$

WAS = e kicked out

WES = e to higher energy level

WNS = nucleus recoil

The probability to find a high p electron
in the wave function is highly suppressed!

Kopp et al: 0907.3159 (PRD)

提纲

- 暗物质的物理模型
  - 可能的暗物质候选者
  - WIMP暗物质
    - 热退耦暗物质残余丰度计算
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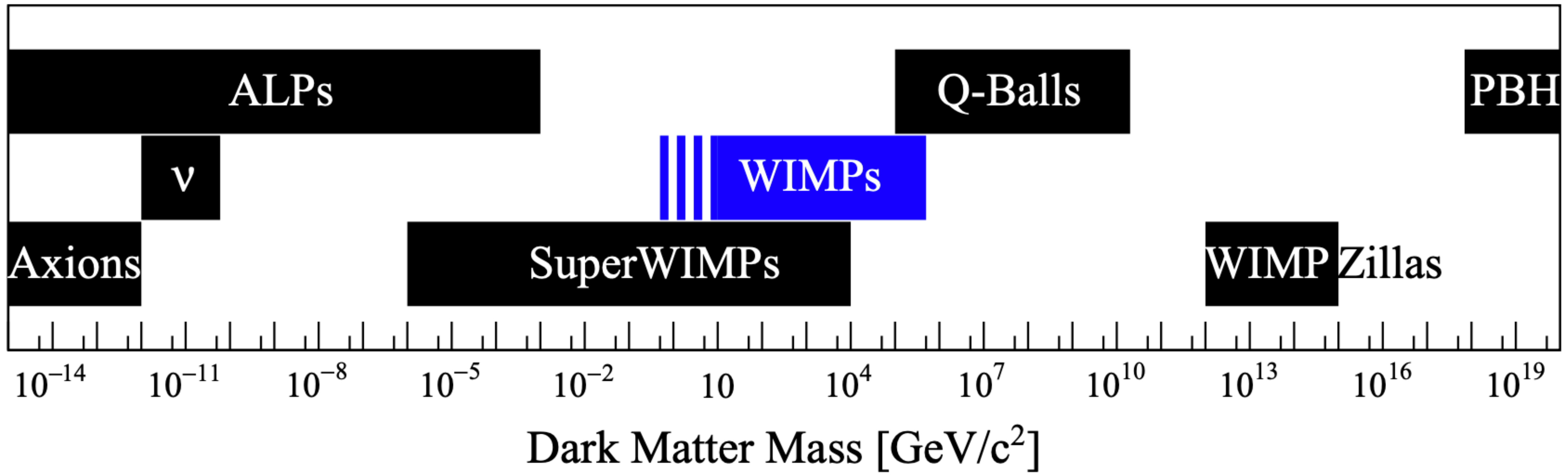
The indirect detection limits from DM annihilation



- DM starts with thermal distribution
- DM has electroweak-scale coupling
- Relic abundance is determined by freeze-out mechanism
- DM Annihilation into
  - X = Standard Model particles (direct coupling)
  - X = Dark Sector particles (secluded DM models)

The entropy of DM goes into SM sector most of the time!

(Secluded X → SM + SM)

Lower mass bound for thermal DM

- Lower bound from N_{eff} at CMB
  - Light DM freeze-out after neutrino decoupling at $T_D \approx 2.3$ MeV
  - Normally $T_{fo} \sim m_{\rm DM}/20$
  - DM entropy goes into neutrinos or e/γ, will modify T_ν/T_γ

Lower mass bound for thermal DM

- Lower bound from N_{eff} at CMB
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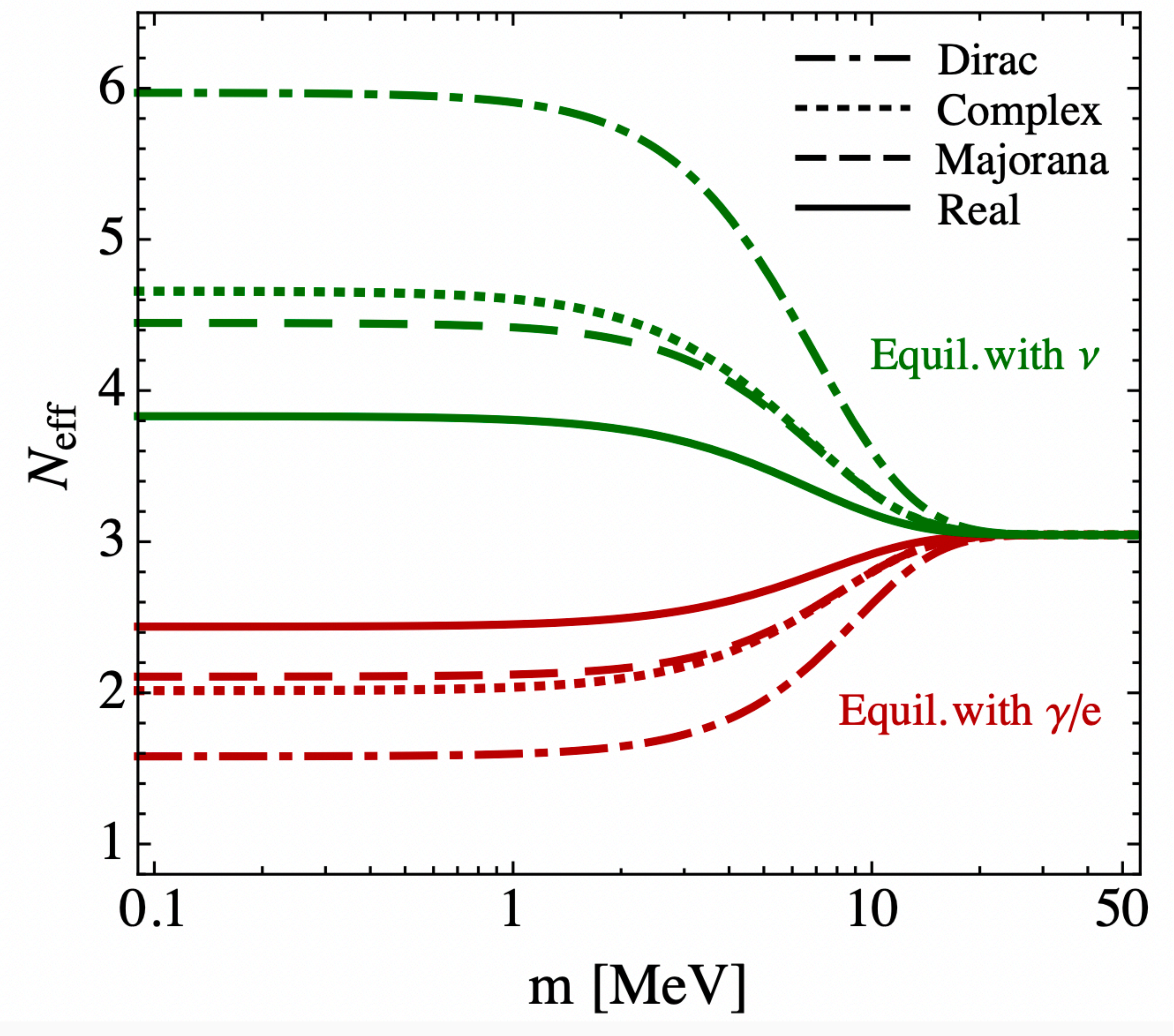
  - Normally $T_{fo} \sim m_{\rm DM}/20$
  - DM entropy goes into neutrinos or e/γ, will modify T_ν/T_γ
  - DM mass $\gtrsim 5$ MeV, depending on d.o.f.



Boehm et al: 1303.6270 (JCAP)

The annihilation cross-section expansion

- Expansion over velocity
  - S-wave
  - P-wave (L=1)
  - D-wave (L=2), due to extra chiral suppression

$$\sigma v \sim \sigma_s + \sigma_p v^2 + \sigma_d v^4 + \ldots$$

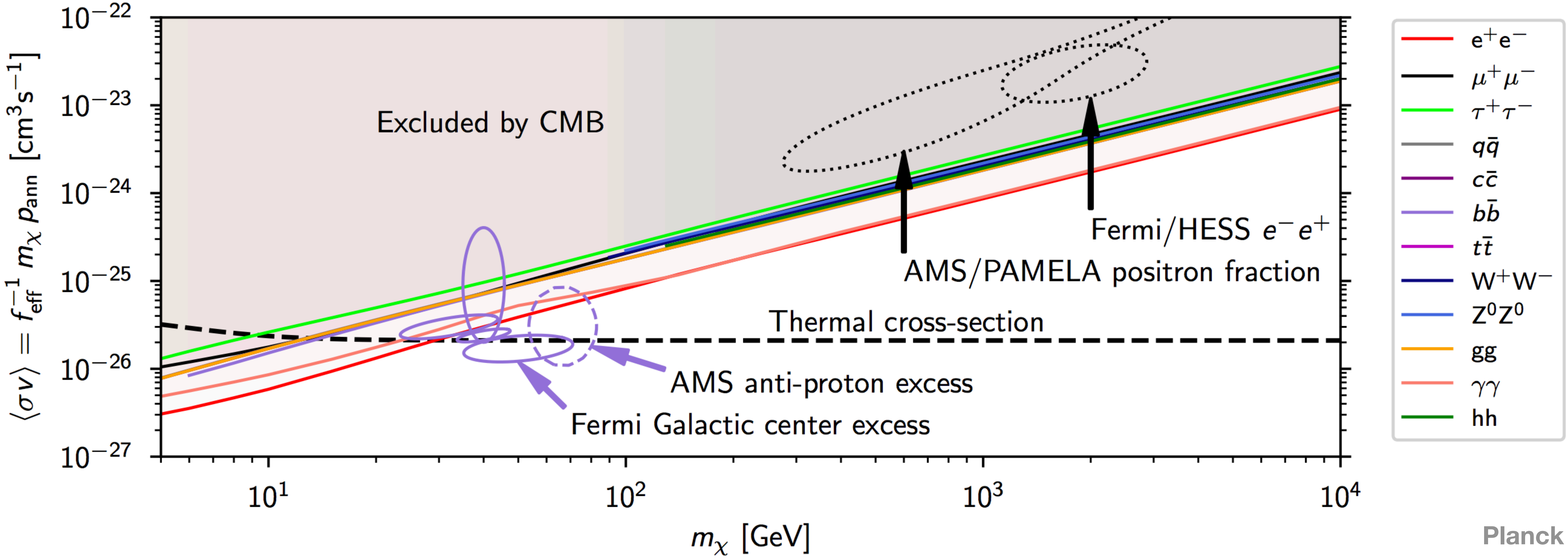
- The value of velocities at different time
  - Freeze-out: $v^2 \sim 0.25$
  - CMB: $v^2 \sim \mathrm{eV}/m_{\mathrm{DM}} \sim 10^{-5}$
  - Today: $v \sim 10^{-3} c$

Annihilation constraints from CMB

- The annihilation: $\mathrm{DM} + \mathrm{DM} \rightarrow \mathrm{SM} + \mathrm{SM}$
- The rate DM energy density converted into EM energy

$$\frac{d\rho_{\mathrm{DM}}}{dt} = m_{\mathrm{DM}} n_{\mathrm{DM}}^2 \langle \sigma v \rangle \times f_{\mathrm{eff}}$$

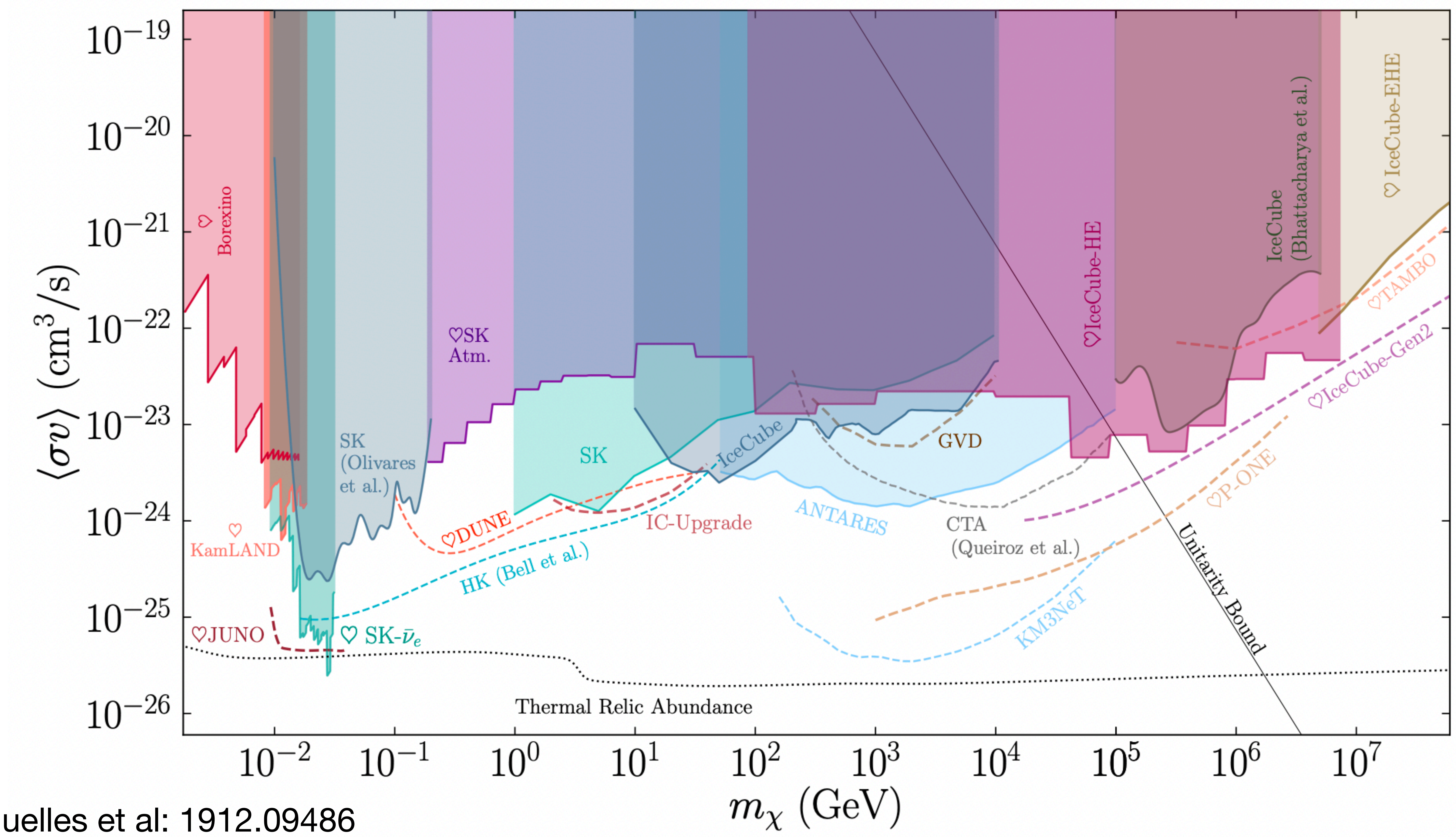
- f_{eff} : the efficiency with which the energy released in DM annihilation is absorbed by the primordial plasma



Planck 2018

How to escape CMB constraints?

- 1. Annihilation to neutrinos $(2\mathrm{DM} \rightarrow \bar{\nu}\nu)$: $f_{\mathrm{eff}} = 0$



Arguelles et al: 1912.09486

How to escape CMB constraints?

- 2. P-wave annihilation or no annihilation (asymmetric DM) but no indirect detection signal

- Expansion over velocity
  - S-wave
  - P-wave (L=1)
  - D-wave (L=2), due to extra chiral suppression
  - Linear v dependence?
    - Final state phase space suppression ($m_{\rm DM} \approx m_X$) from symmetry reason

$$\sigma v \sim \sigma_s + \sigma_p v^2 + \sigma_d v^4 + \ldots$$

- The value of velocities at different time
  - Freeze-out: $v^2 \sim 0.25$
  - CMB: $v^2 \sim \mathrm{eV}/m_{\rm DM} \sim 10^{-5}$
  - Today: $v \sim 10^{-3} c$

J Kopp, **JL**, T Slatyer, **XP Wang**, W Xue: 1609.02147 (JHEP)

How to escape CMB constraints?

- 2+. Linear v suppression
  - How about cross-section linear in v? ($\sigma v \propto v$)
    - For CMB, linear v is enough to be safe
    - For indirect detection
      - Cluster, v ~ 1000 km/s ~ 3 x 10^{-3}
      - Galaxy, v ~ 220 km/s ~ 1 x 10^{-3}
      - Dwarfs, v ~ 10 km/s ~ 3 x 10^{-5}
    - Detectable in Cluster and Galaxy, not in Dwarfs

J Kopp, **JL**, T Slatyer, **XP Wang**, W Xue: 1609.02147 (JHEP)

Linear v to escape CMB limits

- Cross-section linear in v

$$\mathrm{DM} + \mathrm{DM} \to \mathrm{X} + \mathrm{X} \qquad \langle \sigma v \rangle = \frac{1}{4m_{DM}^2} \int dPS_2 |\mathbf{M}|^2$$

- If $m_{MD} = m_X$, then the two-body phase space

$$\int dPS_2 = \frac{1}{8\pi} v$$

- For s-wave annihilation, this gives

$$\langle \sigma v \rangle \approx \frac{1}{2} \sigma_0 v$$

- In practice, not exact degenerate

$$\Delta = m_{\mathrm{DM}} - m_{\mathrm{X}}$$

$$\langle \sigma v_{\mathrm{rel}} \rangle \simeq \sigma_0 \sqrt{\frac{v_{\mathrm{rel}}^2}{4} + \frac{2\Delta}{m_{\mathrm{DM}}}}$$

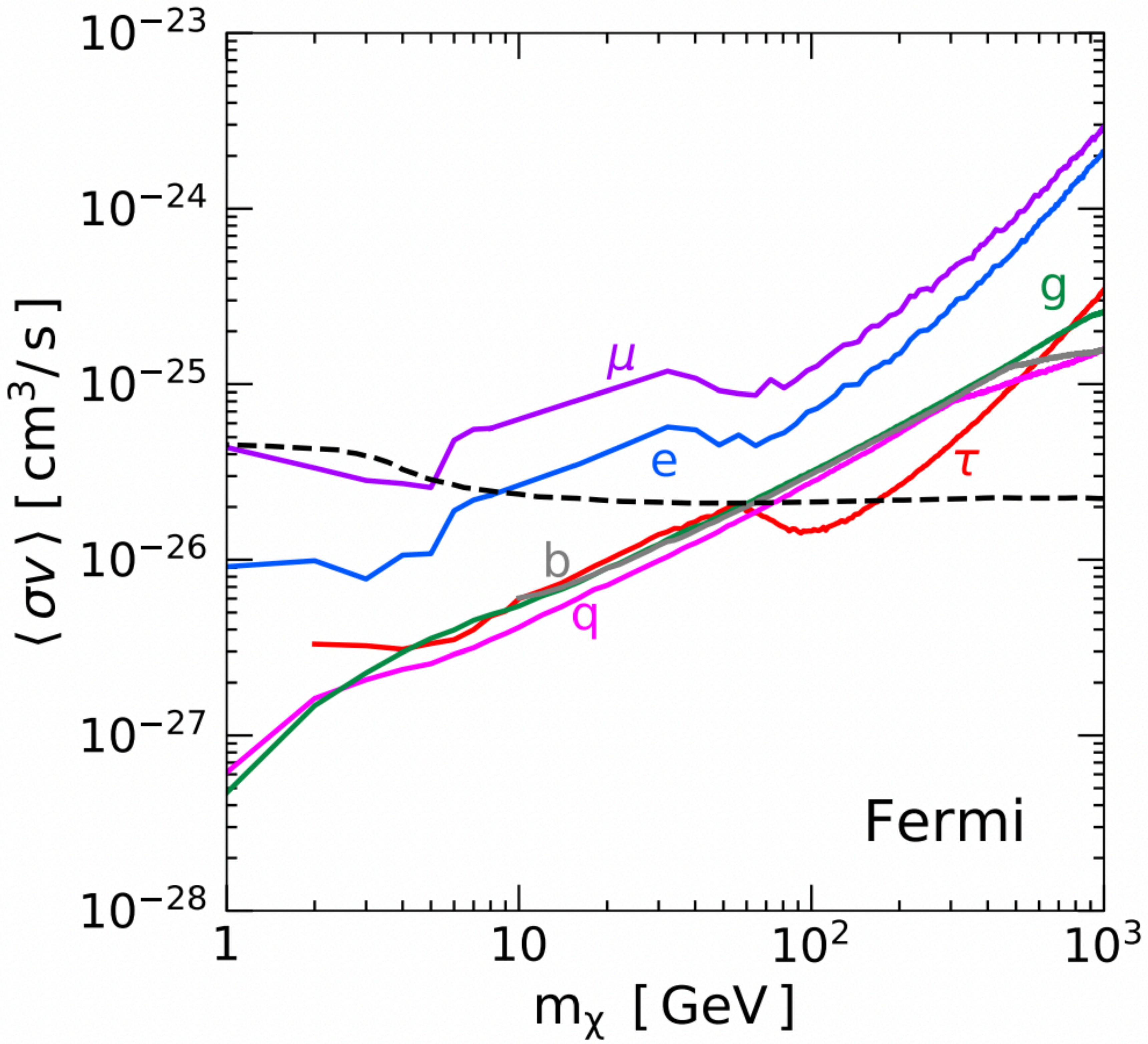
- Model building for $\Delta \ll m_{DM}$

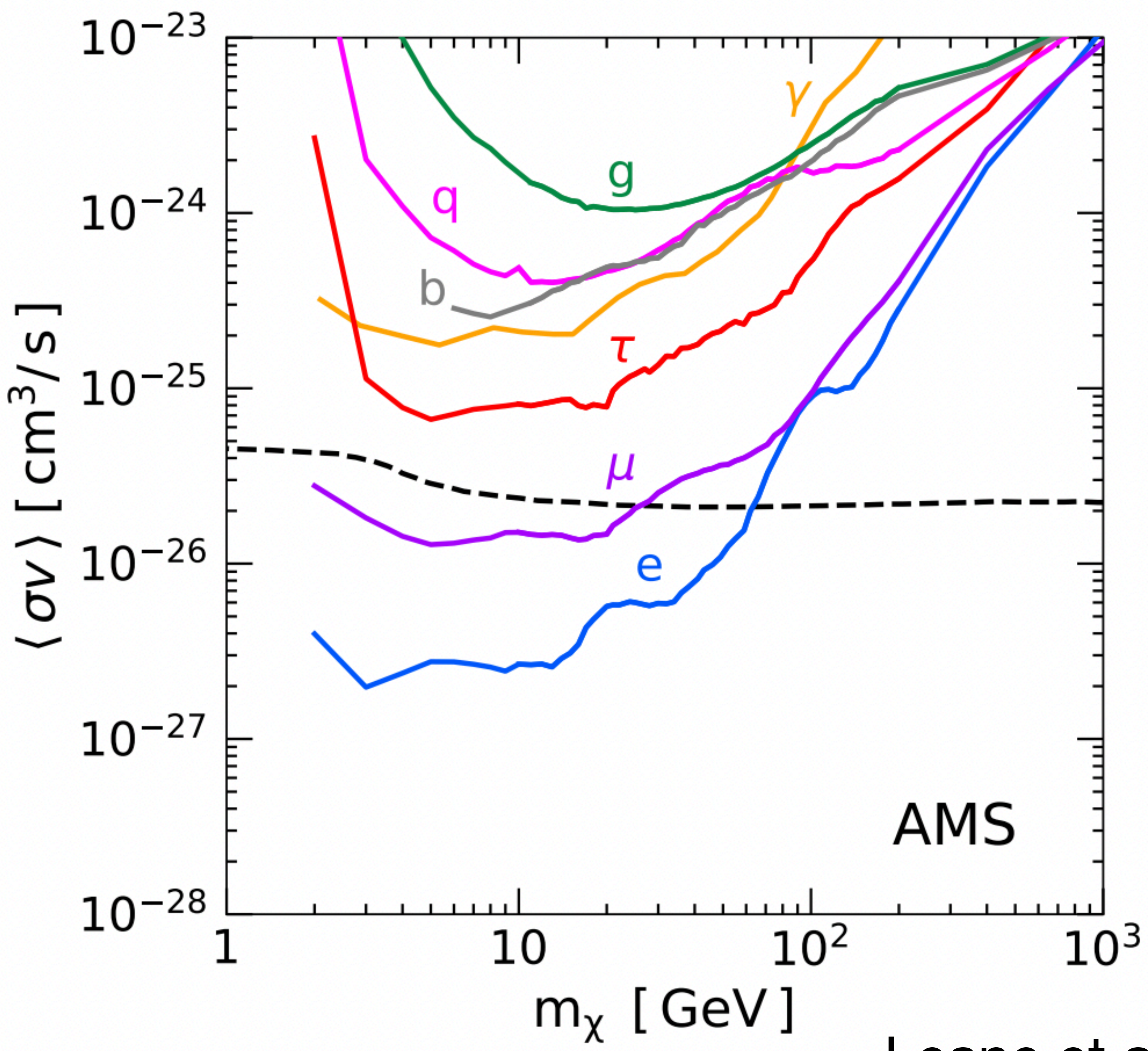
- Symmetry reason
  - Custodial symmetry: dark SU(2) vector DM $\Delta < 0$
  - Chiral symmetry: dark pion DM $\Delta > 0$
  - Supersymmetry: NMSSM setup 1901.02018

J Kopp, **JL**, T Slatyer, **XP Wang**, W Xue: 1609.02147 (JHEP)

Other indirect limits

- CMB limits only works for DM mass $\lesssim 10$ GeV
- Indirect limits from AMS-02, DAMPE, Fermi-LAT

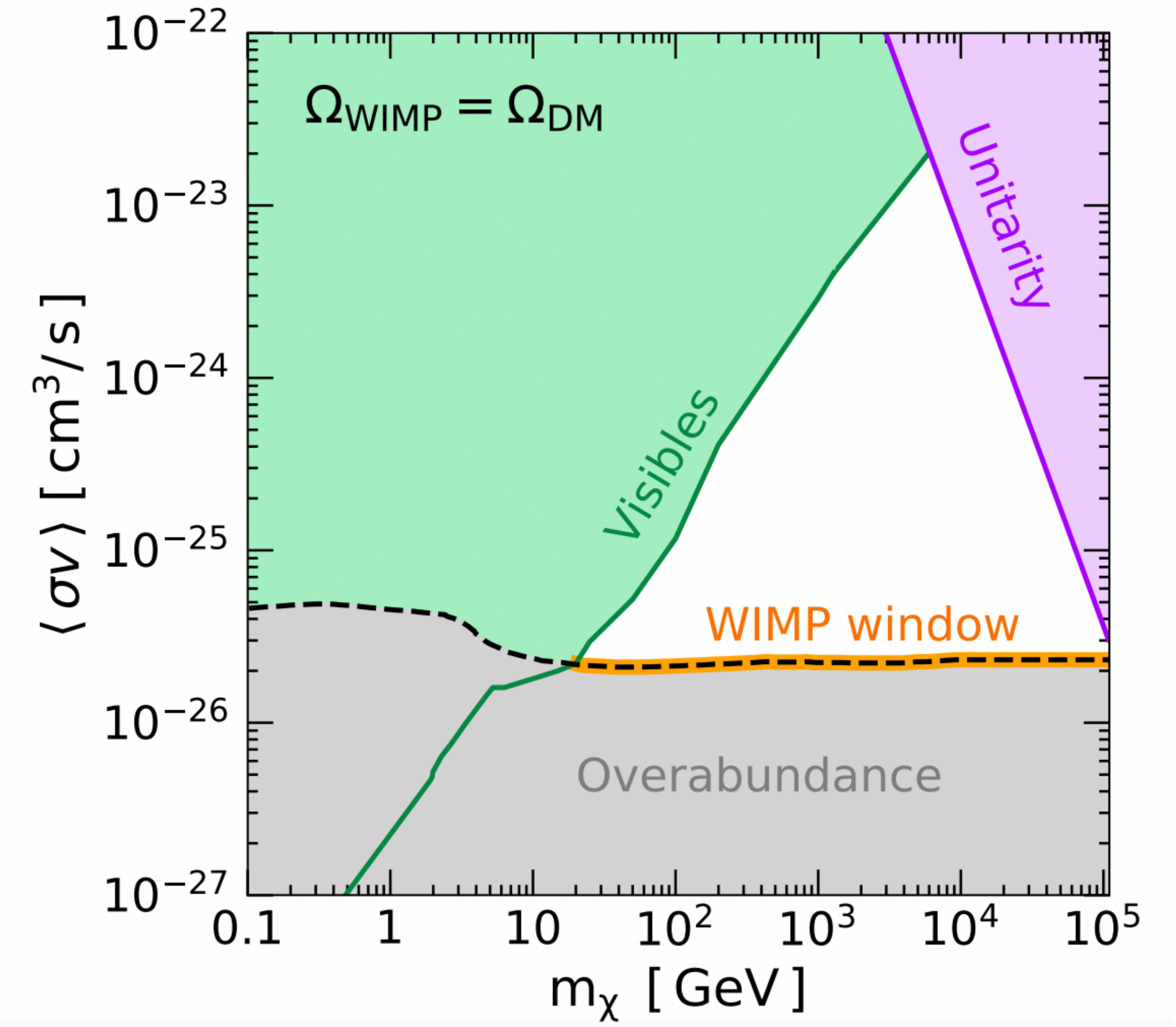




Leane et al: 1805.10305 (PRD)

The WIMP limits from indirect detection

- WIMP mass $\gtrsim 10$ GeV is still viable



GeV-Scale Thermal WIMPs: Not Even Slightly Dead

Leane et al: 1805.10305 (PRD)

总结

- 暗物质是一种不与可见物质相互作用的有质量的未知物质
  - 大量的天文证据以及ΛCDM的重要拼图
  - 暗物质存在各种候选者
    - WIMP暗物质：大质量弱相互作用的暗物质粒子
      - 暗物质的三种探测方法
      - 热退耦暗物质的残余丰度计算
    - 轻暗物质与Dark Sector
    - 超轻波动型暗物质 （以后有机会再讲）
- 暗物质问题是当前天文学和粒子物理学科的最重要问题之一

谢谢！