

对撞机物理唯象学基础与前沿

岩斌

中国科学院高能物理研究所

微扰量子场论及其前沿讲习班@山东大学

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目录

➤ 传统对撞机物理简介

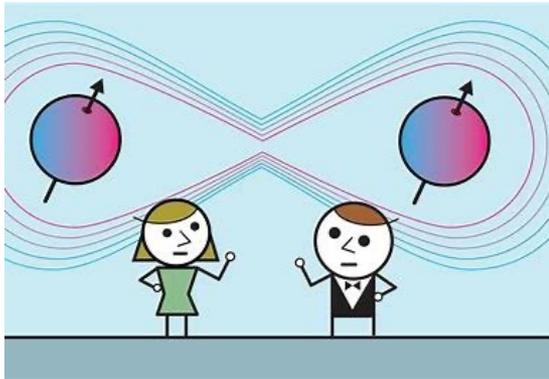
- ❑ 粒子物理基础知识回顾
- ❑ 希格斯物理简介
- ❑ 前沿进展：横向极化效应

➤ 量子纠缠和贝尔不等式破坏效应简介

- ❑ 量子纠缠和贝尔不等式破坏效应
- ❑ 对撞机上探测量子纠缠和贝尔不等式破坏
- ❑ 利用量子纠缠效应寻找超出标准模型新物理

粒子物理中的量子纠缠

- 量子纠缠和量子非局域性检验是检验量子力学基本性质的重要组成部分



- 量子纠缠是量子信息和量子计算的重要基础：如利用量子纠缠态实现量子信息的传输
- 量子纠缠在光学、固体物理、原子物理系统已经被广泛研究

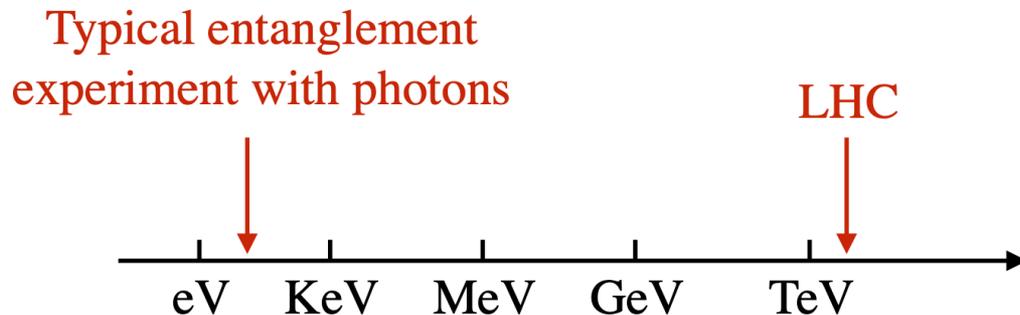


Nobel Prize in Physics awarded to Aspect, Clauser and Zeilinger for work in quantum mechanics

The Royal Swedish Academy of Sciences has decided to award the 2022 Nobel Prize in Physics to Alain Aspect, John F. Clauser and Anton Zeilinger, according to an official tweet. They have been awarded the Nobel Prize for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science.

粒子物理中的量子纠缠

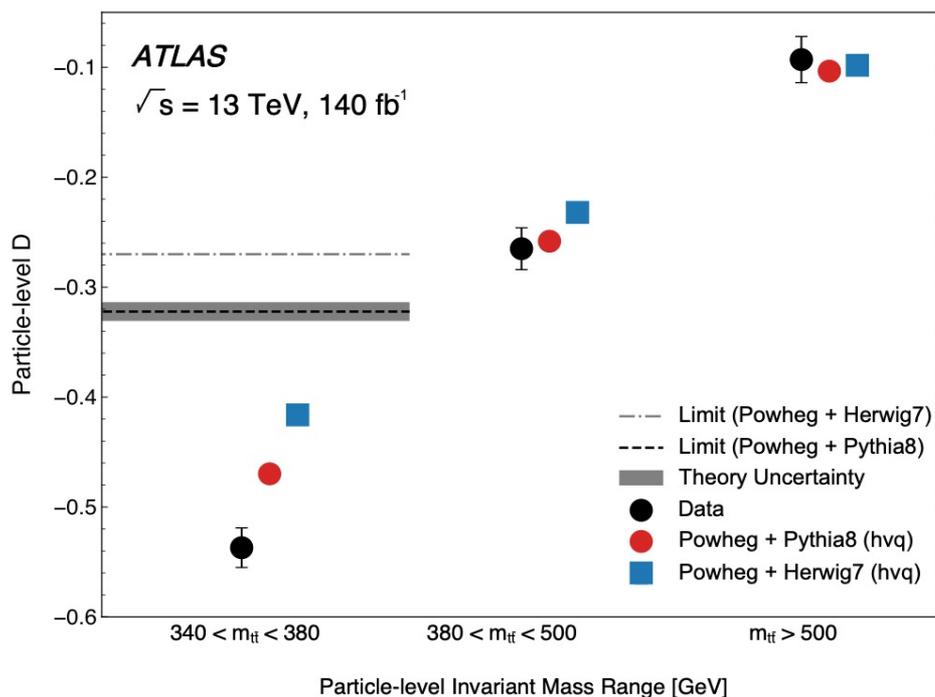
- 量子纠缠和量子非局域性检验仅在低能实验中得到了很好的检验，高能情况？



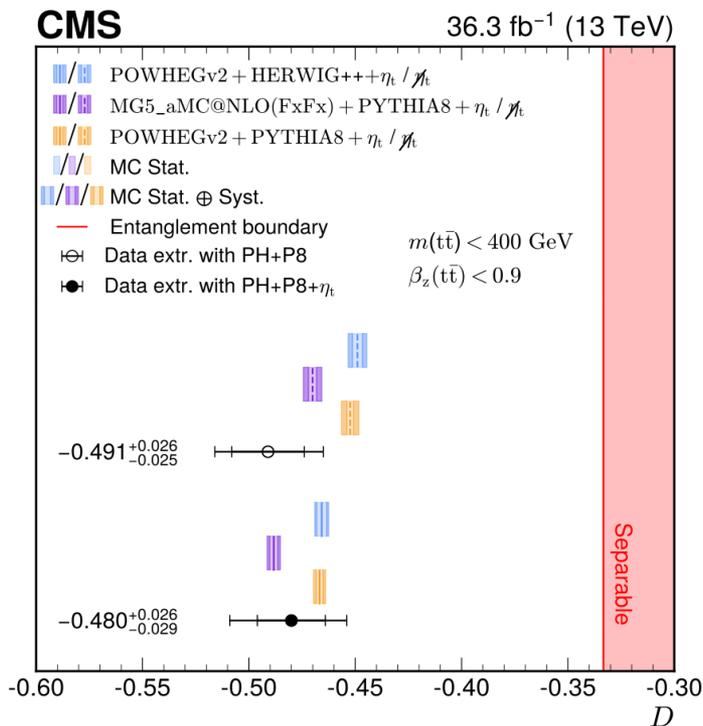
- 高能物理：物理过程可能涉及电磁、弱相互作用以及强相互作用。量子纠缠现象从未在其它相互作用中检验
- 粒子物理提供了新的检验环境和模式：粒子的自旋测量来自不稳定粒子的衰变产物分布；有质量规范玻色子存在新的自由度

粒子物理中的量子纠缠

Nature 633, 542 (2024)



Rept. Prog. Phys. 87 (2024) 117801



LHC 实验在顶夸克对产生过程中发现了量子纠缠现象

基本概念介绍

➤ 纯态：系统的量子态可以完全用一个态矢量 $|\Psi\rangle$ 描述

➤ 密度矩阵定义： $\rho = |\Psi\rangle\langle\Psi|$

基本性质：厄米性 $\rho^\dagger = \rho$

归一化 $Tr[\rho] = 1$

半正定 $\rho \geq 0$

投影算符 $\rho^2 = \rho$

➤ 混态：系统处于多个纯态的统计混合（经典概率混合），无法用单一态矢量进行描述

$$\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|, \quad \sum_i p_i = 1 \quad \rho^2 \neq \rho \quad Tr[\rho^2] < 1$$

最大混合态 $p_i = \frac{1}{d}$

自旋1/2 体系密度矩阵

➤ 单粒子体系: $\rho = \frac{1}{2} (I_2 + \vec{s} \cdot \vec{\sigma})$

自旋矢量: $\vec{s} = Tr(\vec{\sigma}\rho) = \langle \vec{\sigma} \rangle$

➤ 两粒子体系:

$$\hat{\rho} = \frac{1}{4} \left(\hat{I}_2 \otimes \hat{I}_2 + \sum_{i=1}^3 B_i^+ \hat{\sigma}_i \otimes \hat{I}_2 + \sum_{i=1}^3 B_i^- \hat{I}_2 \otimes \hat{\sigma}_i + \sum_{i,j=1}^3 C_{ij} \hat{\sigma}_i \otimes \hat{\sigma}_j \right)$$

- B_i^\pm : 单个粒子的自旋
- C_{ij} : 两粒子间的自旋关联

$$C_{ij} = Tr(\rho \sigma_i \otimes \sigma_j)$$

贝尔态的密度矩阵

➤ 自旋单态（反对称）： $|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$$\rho = |\Psi_0\rangle\langle\Psi_0| = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

➤ 自旋三重态（对称）：

$$|\Psi_1\rangle = \frac{|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle}{\sqrt{2}}, \quad C = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad |\Psi_3\rangle = -\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

$$|\Psi_2\rangle = i\frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

量子纠缠-两粒子系统

- 考虑两粒子体系的希尔伯特空间 $\mathcal{H}_A \otimes \mathcal{H}_B$ ，系统A和B可分（没有纠缠）：

$$\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

- 纯态：利用冯·诺伊曼熵（Von Neumann Entropy）判断纠缠程度

$$S(\rho) = -\text{Tr}(\rho \log \rho)$$

纠缠熵定义： $\varepsilon(\rho_{AB}) = S(\rho_A) = S(\rho_B)$

其中约化密度矩阵定义为：

$$\rho_A = \text{Tr}_B(\rho_{AB}) = \sum_n (\mathbf{1}_A \otimes_B \langle n|) \rho_{AB} (\mathbf{1}_A \otimes |n\rangle_B)$$

贝尔态的冯·诺伊曼熵

➤ 自旋单态 $|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$$\rho = |\Psi_0\rangle\langle\Psi_0| = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{系统B的基矢:}$$
$$\rho_{AB} = \begin{bmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{bmatrix}$$

约化密度矩阵:

$$\rho_A = M_{00} + M_{11} = \mathbf{1}/2$$

冯·诺伊曼熵: 贝尔态为最大纠缠态

$$S(\rho) = -\text{Tr}(\rho_A \log_2(\rho_A)) = -\sum_i \lambda_i \log_2 \lambda_i = 1$$

➤ 混态时不能用冯·诺伊曼熵去量化纠缠, 可能包含经典关联, 如下面的密度矩阵冯·诺伊曼熵与贝尔态结果相同

$$\rho_{AB}^{c.c.} = \frac{1}{2}(|0\rangle\langle 0|_A \otimes |1\rangle\langle 1|_B + |1\rangle\langle 1|_A \otimes |0\rangle\langle 0|_B)$$

量子纠缠-两粒子系统

- 判据1: 佩雷斯-霍罗德基判据, 即密度矩阵 ρ_{AB} 的部分转置矩阵是负定的, 则 ρ_{AB} 描述一个纠缠态
定义部分转置密度矩阵

$$\rho_{AB}^{T_B} = \sum_i p_i \rho_i^A \otimes (\rho_i^B)^T$$

注意: 两个半正定矩阵的直积也是半正定的

- 判据2: Concurrence (并发纠缠)

没有
纠缠

最大
纠缠

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$$0 \leq C(\rho) \leq 1$$

λ_i 矩阵 $\sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}}$ 的本征值 $\tilde{\rho} = (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2)$

W. K. Wootters, PRL 80 (1998) 2245

量子纠缠-两粒子系统

➤ 佩雷斯-霍罗德斯基判据

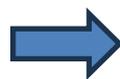
$$\rho = \frac{1}{4} \begin{bmatrix} 1 + B_3^+ + B_3^- + C_{33} & B_1^- + C_{31} - i(B_2^- + C_{32}) & B_1^+ + C_{13} - i(B_2^+ + C_{23}) & C_{11} - C_{22} - i(C_{12} + C_{21}) \\ B_1^- + C_{31} + i(B_2^- + C_{32}) & 1 + B_3^+ - B_3^- - C_{33} & C_{11} + C_{22} + i(C_{12} - C_{21}) & B_1^+ - C_{13} - i(B_2^+ - C_{23}) \\ B_1^+ + C_{13} + i(B_2^+ + C_{23}) & C_{11} + C_{22} + i(C_{21} - C_{12}) & 1 - B_3^+ + B_3^- - C_{33} & B_1^- - C_{31} - i(B_2^- - C_{32}) \\ C_{11} - C_{22} + i(C_{21} + C_{12}) & B_1^+ - C_{13} + i(B_2^+ - C_{23}) & B_1^- - C_{31} + i(B_2^- - C_{32}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix}$$

■ 对第二个子系统进行部分转置

$$\rho^{T_2} = \frac{1}{4} \begin{bmatrix} 1 + B_3^+ + B_3^- + C_{33} & B_1^- + C_{31} + i(B_2^- + C_{32}) & B_1^+ + C_{13} - i(B_2^+ + C_{23}) & C_{11} + C_{22} + i(C_{12} - C_{21}) \\ B_1^- + C_{31} - i(B_2^- + C_{32}) & 1 + B_3^+ - B_3^- - C_{33} & C_{11} - C_{22} - i(C_{12} + C_{21}) & B_1^+ - C_{13} - i(B_2^+ - C_{23}) \\ B_1^+ + C_{13} + i(B_2^+ + C_{23}) & C_{11} - C_{22} + i(C_{21} + C_{12}) & 1 - B_3^+ + B_3^- - C_{33} & B_1^- - C_{31} + i(B_2^- - C_{32}) \\ C_{11} + C_{22} + i(C_{21} - C_{12}) & B_1^+ - C_{13} + i(B_2^+ - C_{23}) & B_1^- - C_{31} - i(B_2^- - C_{32}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix}$$

■ 考虑简化情况，投影到由 $|++\rangle, |--\rangle$ 构成的子空间

$$\rho_C \equiv \begin{bmatrix} 1 + B_3^+ + B_3^- + C_{33} & C_{11} + C_{22} + i(C_{12} - C_{21}) \\ C_{11} + C_{22} + i(C_{21} - C_{12}) & 1 - B_3^+ - B_3^- + C_{33} \end{bmatrix}$$



子空间的密度矩阵出现
负本征值则原始密度矩
阵处于纠缠状态

量子纠缠-两粒子系统

- 两粒子纠缠的充分条件

$$\mathcal{P} = (B_3^+ + B_3^-)^2 + (C_{11} + C_{22})^2 + (C_{21} - C_{12})^2 - (1 + C_{33})^2 > 0$$

$$\mathcal{P} \geq \tilde{\mathcal{P}} \equiv (C_{11} + C_{22})^2 - (1 + C_{33})^2 > 0$$

$$\Delta \equiv -C_{33} + |C_{11} + C_{22}| - 1 > 0$$

$$-tr[C] > 1$$

量子纠缠-两粒子系统

- 一些简化系统，可以得到更严格的限制。考虑非极化量子态产生过程，如 $gg \rightarrow t\bar{t}$ ，对角化关联矩阵

$$\rho = \frac{1}{4} \begin{pmatrix} 1 + C_3 & 0 & 0 & C_1 - C_2 \\ 0 & 1 - C_3 & C_1 + C_2 & 0 \\ 0 & C_1 + C_2 & 1 - C_3 & 0 \\ C_1 - C_2 & 0 & 0 & 1 + C_3 \end{pmatrix} \quad \rho^{T_2} = \frac{1}{4} \begin{pmatrix} 1 + C_3 & 0 & 0 & C_1 + C_2 \\ 0 & 1 - C_3 & C_1 - C_2 & 0 \\ 0 & C_1 - C_2 & 1 - C_3 & 0 \\ C_1 + C_2 & 0 & 0 & 1 + C_3 \end{pmatrix}$$

$$\lambda_{1,2} = \frac{1}{4} (1 + C_3 \pm (C_1 - C_2))$$

$$\lambda_{3,4} = \frac{1}{4} (1 - C_3 \pm (C_1 + C_2))$$

$$\lambda_{1,2}^{T_2} = \frac{1}{4} (1 + C_3 \pm (C_1 + C_2))$$

$$\lambda_{3,4}^{T_2} = \frac{1}{4} (1 - C_3 \pm (C_1 - C_2))$$

密度矩阵要求:

$$\lambda_i \geq 0 \Rightarrow \pm C_3 + |C_1 \pm C_2| - 1 \leq 0$$

佩雷斯-霍罗德斯基判据，纠缠条件：
存在一个即可

$$\lambda_i^{T_2} < 0 \Rightarrow \pm C_3 + |C_1 \mp C_2| - 1 > 0$$

量子纠缠-两粒子系统

- Concurrence 计算

$$\rho = \frac{1}{4} \begin{pmatrix} 1 + C_3 & 0 & 0 & C_1 - C_2 \\ 0 & 1 - C_3 & C_1 + C_2 & 0 \\ 0 & C_1 + C_2 & 1 - C_3 & 0 \\ C_1 - C_2 & 0 & 0 & 1 + C_3 \end{pmatrix}$$

$$\tilde{\rho} = (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2) = \rho$$

- 计算 $\sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}}$ 本征值

$$\tilde{\lambda}_{1,2} = \frac{1}{4}(1 + C_3 \pm (C_1 - C_2))$$

$$\tilde{\lambda}_{3,4} = \frac{1}{4}(1 - C_3 \pm (C_1 + C_2))$$

- 计算 Concurrence, 结合密度矩阵的正定性条件

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) = \max(0, 2\lambda_1 - 1)$$

$$C[\rho] = \frac{1}{2} \max[-C_3 + |C_1 + C_2| - 1, 0], \quad C_3 \leq 0$$

$$C[\rho] = \frac{1}{2} \max[C_3 + |C_1 - C_2| - 1, 0], \quad C_3 \geq 0$$

量子纠缠效应的基底依赖性

- 量子纠缠效应的研究依赖于自旋密度矩阵的重构，而自旋关联的大小与参考系存在紧密关系
- 将密度矩阵按照不同基底进行展开：

$$\hat{\rho} = \rho_{\alpha\beta, \bar{\alpha}\bar{\beta}} |\alpha\bar{\alpha}\rangle \langle\beta\bar{\beta}| \quad \hat{\rho} = \rho'_{\alpha'\beta', \bar{\alpha}'\bar{\beta}'} |\alpha'\bar{\alpha}'\rangle \langle\beta'\bar{\beta}'|$$

- 基底的变换：幺正变换

$$|\alpha'\rangle \equiv |\alpha\rangle U_{\alpha\alpha'} \quad \rho'_{\beta\delta, \bar{\beta}\bar{\delta}} = \mathcal{U}_{\beta\alpha, \bar{\beta}\bar{\alpha}}^\dagger \rho_{\alpha\gamma, \bar{\alpha}\bar{\gamma}} \mathcal{U}_{\gamma\delta, \bar{\gamma}\bar{\delta}}; \quad \mathcal{U}_{\alpha\alpha', \bar{\alpha}\bar{\alpha}'} = U_{\alpha\alpha'} \otimes \bar{U}_{\bar{\alpha}, \bar{\alpha}'}$$

- Concurrence的基底依赖性

$$\sqrt{\rho'} = \sqrt{\mathcal{U}^\dagger \rho \mathcal{U}} = \sqrt{(\mathcal{U}^\dagger \sqrt{\rho} \mathcal{U})(\mathcal{U}^\dagger \sqrt{\rho} \mathcal{U})} = \mathcal{U}^\dagger \sqrt{\rho} \mathcal{U}.$$

量子纠缠效应的基底依赖性

➤ 考虑下面矩阵的变换 $\sigma_2 U^T = U^\dagger \sigma_2$

$$\begin{aligned}\tilde{\rho}' &= (\sigma_2 \otimes \sigma_2) \mathcal{U}^T \rho^* \mathcal{U}^* (\sigma_2 \otimes \sigma_2) \\ &= (\sigma_2 \otimes \sigma_2) (U^T \otimes \bar{U}^T) \rho^* (U^* \otimes \bar{U}^*) (\sigma_2 \otimes \sigma_2) \\ &= (U^\dagger \otimes \bar{U}^\dagger) (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2) (U \otimes \bar{U}) \\ &= U^\dagger (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2) \mathcal{U} \\ &= U^\dagger \tilde{\rho} \mathcal{U}\end{aligned}$$

➤ 因此基底变换下：

$$\sqrt{\rho'} \tilde{\rho}' \sqrt{\rho'} = U^\dagger (\sqrt{\rho} \tilde{\rho} \sqrt{\rho}) U, \quad \sqrt{\sqrt{\rho'} \tilde{\rho}' \sqrt{\rho'}} = U^\dagger \sqrt{\sqrt{\rho} \tilde{\rho} \sqrt{\rho}} U$$

➤ Concurrence的大小由上面的矩阵本征值决定，因此是一个**基底无关的物理量**

CHSH不等式-非定域性的直接检验

➤ 考虑希尔伯特空间 $\mathcal{H}_A \otimes \mathcal{H}_B$ 的贝尔算符

$$\hat{B}_{\text{CHSH}}(A_1, A_2, B_1, B_2) \equiv \hat{A}_1 \hat{B}_1 - \hat{A}_1 \hat{B}_2 + \hat{A}_2 \hat{B}_1 + \hat{A}_2 \hat{B}_2$$

- \hat{A}_i, \hat{B}_i 代表对两个粒子自旋在各自系统中的测量，测量结果为 ± 1
- 局域隐变量理论：决定性理论，两次自旋测量结果是固定的，不依赖彼此的测量


$$A_1 = A_2 \quad \text{or} \quad A_1 = -A_2, \quad B_1 = B_2 \quad \text{or} \quad B_1 = -B_2$$

- 局域隐变量理论： $|\langle \hat{B}_{\text{CHSH}} \rangle| \leq 2$

CHSH不等式-非定域性的直接检验

- 考虑量子力学情况：对粒子沿着方向 \vec{a} 和 \vec{b} 进行自旋测量

$$\hat{A}_i = \vec{a}_i \cdot \hat{\sigma}, \quad \hat{B}_i = \vec{b}_i \cdot \hat{\sigma} \quad \hat{A}_i^2 = 1 \text{ 和 } \hat{B}_i^2 = 1$$


$$\hat{B}_{\text{CHSH}}(A_1, A_2, B_1, B_2) \equiv \hat{A}_1 \hat{B}_1 - \hat{A}_1 \hat{B}_2 + \hat{A}_2 \hat{B}_1 + \hat{A}_2 \hat{B}_2$$

$$\hat{B}_{\text{CHSH}}^2 = 4 - [\hat{A}_1, \hat{A}_2][\hat{B}_1, \hat{B}_2]$$

- 当各自两次测量对易时，（可同时测量），测量结果满足CHSH不等式
- 不对易情况： $|\langle [\hat{A}_1, \hat{A}_2] \rangle| \leq 2 \implies |\langle \hat{B}_{\text{CHSH}} \rangle| \leq 2\sqrt{2}$
- 贝尔不等式破坏的条件为：

$$\max_{\hat{A}_1, \hat{A}_2, \hat{B}_1, \hat{B}_2} \left| \left\langle \hat{B}_{\text{CHSH}}(A_1, A_2, B_1, B_2) \right\rangle \right| > 2.$$

CHSH不等式最大破坏效应

- 考虑如下的两粒子密度矩阵和自旋测量算符

$$\hat{\rho} = \frac{1}{4} \left(\hat{I}_2 \otimes \hat{I}_2 + \sum_{i=1}^3 B_i^+ \hat{\sigma}_i \otimes \hat{I}_2 + \sum_{i=1}^3 B_i^- \hat{I}_2 \otimes \hat{\sigma}_i + \sum_{i,j=1}^3 C_{ij} \hat{\sigma}_i \otimes \hat{\sigma}_j \right)$$

$$\hat{A}_i = \vec{a}_i \cdot \hat{\vec{\sigma}}, \quad \hat{B}_i = \vec{b}_i \cdot \hat{\vec{\sigma}}$$

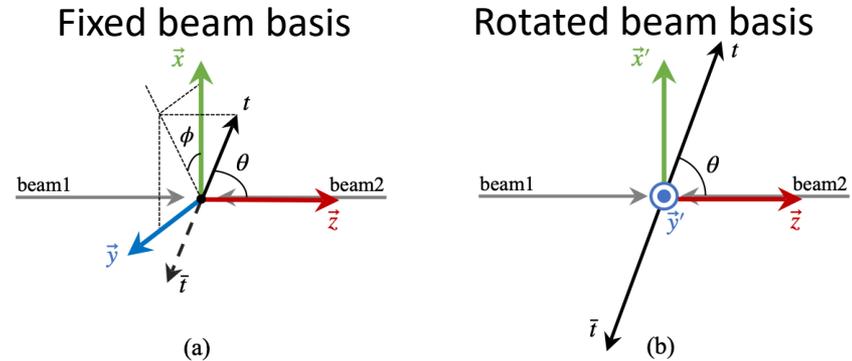
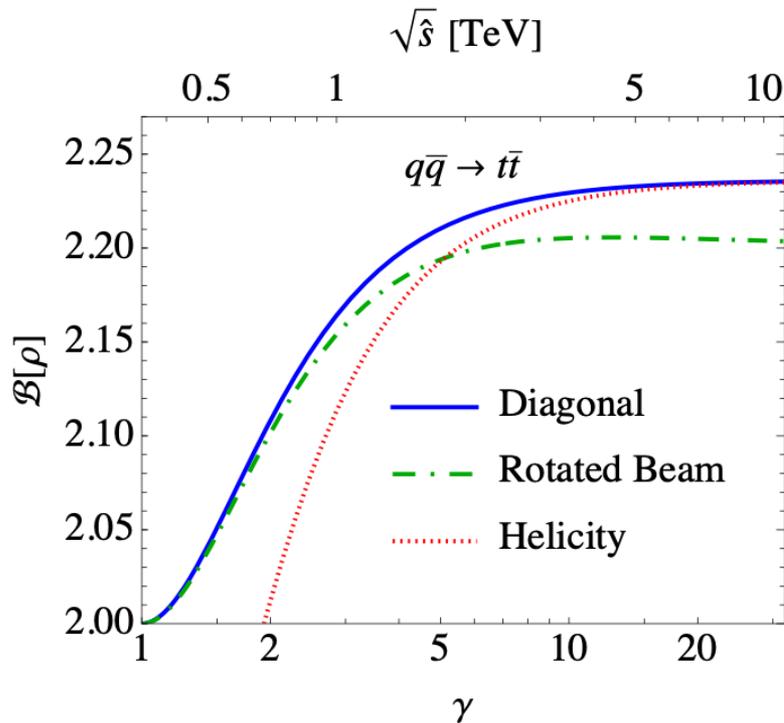
- 贝尔算符的期望值:

$$\langle \hat{\mathcal{B}}_{\text{CHSH}} \rangle = \text{Tr} \left(\hat{\rho} \hat{\mathcal{B}}_{\text{CHSH}} \right) = \vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2)$$

- 两自旋1/2系统，贝尔不等式破坏仅与两粒子的自旋关联矩阵相关，与各自的极化信息无关

贝尔变量的优化

- 贝尔变量的大小依赖于自旋密度矩阵的重构，而自旋关联的大小与参考系存在紧密关系



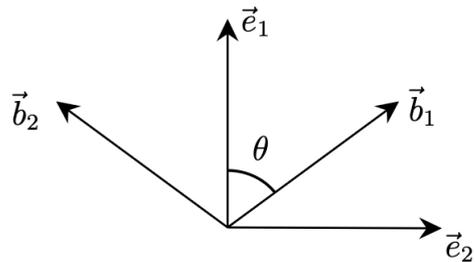
Kun Cheng, Tao Han, M. Low,
PRD 1111 (2025) 033004

贝尔变量的数值对参考系具有极强的依赖性

$$\gamma = \sqrt{\hat{s}} / (2m_t)$$

求解最大破坏效应方向

- 考虑如下正交基矢



$$\begin{cases} \vec{b}_1 + \vec{b}_2 = 2\vec{e}_1 \cos \theta \\ \vec{b}_1 - \vec{b}_2 = 2\vec{e}_2 \sin \theta \end{cases}$$

$$\begin{aligned} |\langle \hat{\mathcal{B}}_{\text{CHSH}} \rangle| &= 2 |\vec{a}_1 \cdot C \cdot \vec{e}_2 \sin \theta + \vec{a}_2 \cdot C \cdot \vec{e}_1 \cos \theta| \\ &\leq 2 \|C \cdot \vec{e}_2\| \sin \theta + \|C \cdot \vec{e}_1\| \cos \theta \quad \vec{a}_1 // C \cdot \vec{e}_2, \vec{a}_2 // C \cdot \vec{e}_1 \\ &= 2 \sqrt{\|C \cdot \vec{e}_2\|^2 + \|C \cdot \vec{e}_1\|^2} \cos(\theta - \xi) \quad \text{最大破坏效应 } \theta = \xi \end{aligned}$$

- 注意: \vec{a}_i, \vec{b}_i 是单位矢量 $\tan \xi = \frac{\|C \cdot \vec{e}_2\|}{\|C \cdot \vec{e}_1\|}$

$$\sqrt{\|C \cdot \vec{e}_2\|^2 + \|C \cdot \vec{e}_1\|^2} = \|C \cdot \vec{e}_2\| \frac{1}{\sin \xi} = \|C \cdot \vec{e}_1\| \frac{1}{\cos \xi}$$

求解最大破坏效应方向

- 利用等式: $\|C \cdot \vec{e}_1\|^2 + \|C \cdot \vec{e}_2\|^2 = \text{Tr}(C^T C) - (C^T C)_{33}$

$$|\langle \hat{\mathcal{B}}_{\text{CHSH}} \rangle| = 2|\vec{a}_1 \cdot C \cdot \vec{e}_2 \sin \theta + \vec{a}_2 \cdot C \cdot \vec{e}_1 \cos \theta|$$

$$\leq 2(\|C \cdot \vec{e}_2\| \sin \theta + \|C \cdot \vec{e}_1\| \cos \theta)$$

$$= 2\sqrt{\|C \cdot \vec{e}_2\|^2 + \|C \cdot \vec{e}_1\|^2} \cos(\theta - \xi)$$

$$\leq 2\sqrt{\mu_1^2 + \mu_2^2}$$

最大破坏
效应 $\theta = \xi$

- $\mu_{1,2}$ 为关联矩阵 $C^T C$ 最大的两个本征值。最优自旋测量方向:

$$\vec{a}_1 = \frac{C \cdot \vec{e}_2}{\|C \cdot \vec{e}_2\|}, \quad \vec{a}_2 = \frac{C \cdot \vec{e}_1}{\|C \cdot \vec{e}_1\|}, \quad \vec{b}_1 = \vec{e}_1 \cos \theta + \vec{e}_2 \sin \theta, \quad \vec{b}_2 = \vec{e}_1 \cos \theta - \vec{e}_2 \sin \theta$$

- $\vec{e}_{1,2}$ 对应两个最大本征值对应的本征向量构成的单位正交基

贝尔态的最大破坏方向

- 自旋单态

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \quad C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

- $C^T C$ 是单位矩阵，所有本征值均相同，因此方向可选择为：

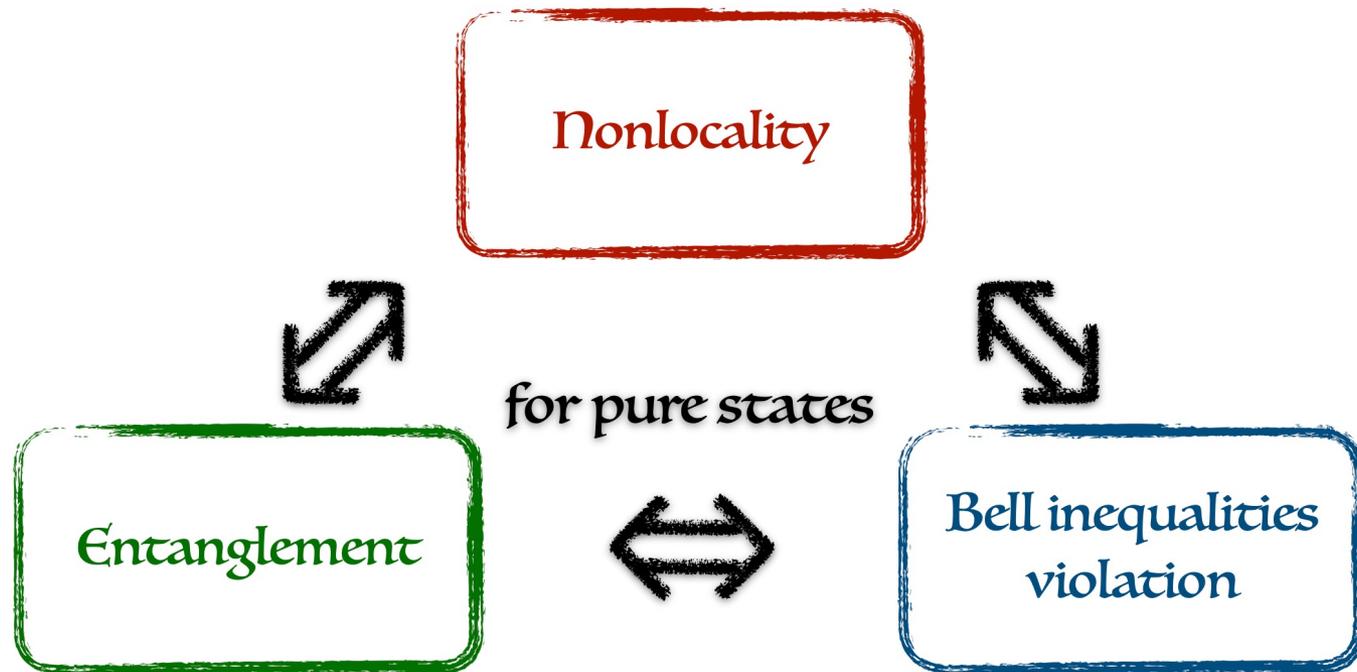
$$\vec{a}_1 = \frac{C \cdot \vec{e}_2}{\|C \cdot \vec{e}_2\|}, \quad \vec{a}_2 = \frac{C \cdot \vec{e}_1}{\|C \cdot \vec{e}_1\|}, \quad \vec{b}_1 = \vec{e}_1 \cos \theta + \vec{e}_2 \sin \theta, \quad \vec{b}_2 = \vec{e}_1 \cos \theta - \vec{e}_2 \sin \theta$$



$$\vec{a}_1 = \vec{e}_i, \quad \vec{a}_2 = \vec{e}_j, \quad \vec{b}_1 = \frac{\vec{e}_i \pm \vec{e}_j}{\sqrt{2}}, \quad \vec{b}_2 = \frac{\vec{e}_i \mp \vec{e}_j}{\sqrt{2}}, \quad (i \neq j)$$

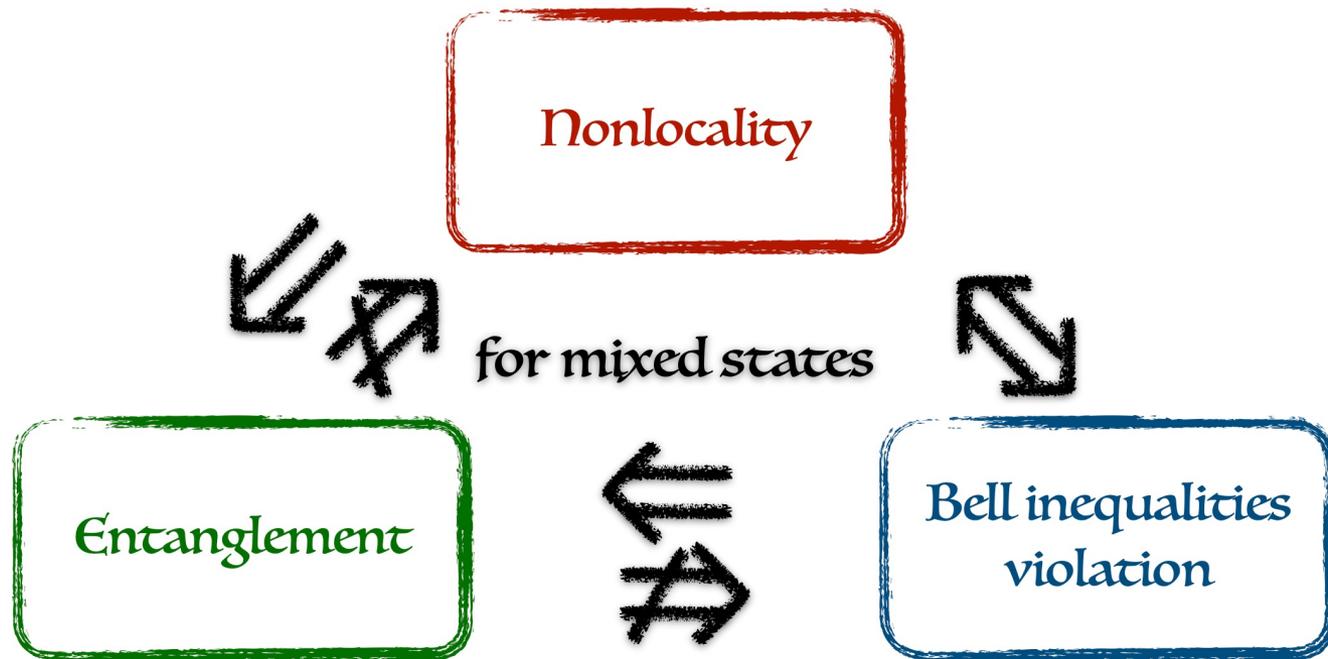
量子纠缠和贝尔不等式

- Gisin 定理：任何两个量子比特系统的**纯态纠缠**都可以在适当的测量设置下破坏贝尔不等式



量子纠缠和贝尔不等式

- 混态时量子纠缠和贝尔不等式破坏关系不成立



- 例如：Werner states

$$\rho_{\text{Werner}}(\alpha) = \alpha|\Psi_0\rangle\langle\Psi_0| + \frac{1-\alpha}{4}\hat{I}_4$$

量子纠缠和贝尔不等式

- Werner states

$$\rho_{\text{Werner}}(\alpha) = \frac{1}{4} \left(\hat{I}_2 \otimes \hat{I}_2 - \alpha \sum_i \hat{\sigma}_i \otimes \hat{\sigma}_i \right)$$

$$C_1 = C_2 = C_3 = -\alpha$$

- 正定性要求: $-\frac{1}{3} \leq \alpha \leq 1$

- 纠缠条件: $\alpha > \frac{1}{3}$

贝尔不等式的破坏是比纠缠更强的要求，所有可分态均可以用局域隐变量理论描述

- 贝尔变量最大值:

$$\left| \langle \hat{\mathcal{B}}_{\text{CHSH}} \rangle \right|_{\max} = 2\sqrt{2\alpha^2} > 2 \Rightarrow \alpha > \frac{1}{\sqrt{2}} \simeq 0.707$$

对撞机上探测量子纠缠和贝尔不等式破坏

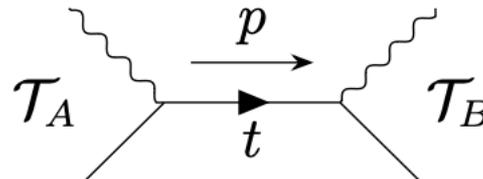
- 不稳定粒子衰变末态提取自旋关联
- 运动学特征提取自旋关联
- 非微扰函数提取自旋关联

不稳定粒子衰变末态提取自旋关联

- 对撞机环境无法直接测量粒子的自旋信息，只能通过其衰变产物进行推测
- 讨论不稳定粒子对在自旋空间的基本假设：
 - **窄宽度近似**：粒子产生和衰变过程可以因子化，因此可以定义产生和衰变过程的密度矩阵，其中产生密度矩阵对应研究系统量子信息需要的自旋关联矩阵
 - **标准模型的衰变机制**：由于母粒子的自旋信息来自于衰变末态的运动学分布，因此结果将严重依赖粒子的衰变机制

产生和衰变密度矩阵

➤ 单个顶夸克产生和衰变



$$\mathcal{M}_{A \rightarrow t \rightarrow B} = \mathcal{T}_B \frac{\not{p} + m_t}{p^2 - m_t^2 + im_t \Gamma_t} \mathcal{T}_A = \frac{1}{D_t} \mathcal{T}_B (\not{p} + m_t) \mathcal{T}_A$$

$$\int_0^\infty dp^2 \frac{1}{|p^2 - m^2 + im\Gamma|^2} = \frac{\pi}{m\Gamma} \implies \frac{1}{|p^2 - m^2 + im\Gamma|^2} \approx \frac{\pi}{m\Gamma} \delta(p^2 - m^2)$$

$$\begin{aligned} |\mathcal{M}_{A \rightarrow t \rightarrow B}|^2 &= \frac{\pi}{m_t \Gamma_t} \delta(p^2 - m_t^2) \left(\mathcal{T}_B (\not{p} + m_t) \mathcal{T}_A \right)^\dagger \left(\mathcal{T}_B (\not{p} + m_t) \mathcal{T}_A \right) \\ &= \frac{\pi}{m_t \Gamma_t} \delta(p^2 - m_t^2) \sum_{\alpha, \alpha'} \left[\mathcal{T}_B u_\alpha(p) \bar{u}_\alpha(p) \mathcal{T}_A \mathcal{T}_A^\dagger u_{\alpha'}(p) \bar{u}_{\alpha'}(p) \mathcal{T}_B^\dagger \right] \\ &= \frac{\pi}{m_t \Gamma_t} \delta(p^2 - m_t^2) \sum_{\alpha, \alpha'} \left[\mathcal{M}_{t_\alpha \rightarrow B} \mathcal{M}_{A \rightarrow t_\alpha} \mathcal{M}_{A \rightarrow t_{\alpha'}}^* \mathcal{M}_{t_{\alpha'} \rightarrow B}^* \right] \\ &= \frac{\pi}{m_t \Gamma_t} \delta(p^2 - m_t^2) \sum_{\alpha, \alpha'} R_{\alpha\alpha'} \Gamma_{\alpha\alpha'}. \end{aligned}$$

产生密度矩阵
衰变密度矩阵

通过衰变产物重构自旋信息

➤ 顶夸克衰变过程中抽取密度矩阵参数:

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_i^W} = \frac{1 + \kappa B_i \cos \theta_i^W}{2}$$

θ_i^W : W 玻色子在顶夸克静止系下运动方向与第*i*个轴的夹角

$$\langle \cos \theta_i^W \rangle \equiv \int_{-1}^1 \frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_i^W} \cos \theta_i^W d \cos \theta_i^W = \frac{\kappa_W B_i}{3}$$

➤ 顶夸克对:

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^{W^+} d \cos \theta_j^{W^-}} = \frac{1}{4} \left(1 + \kappa_W B_i^+ \cos \theta_i^{W^+} - \kappa_W B_j^- \cos \theta_j^{W^-} - \kappa_W^2 C_{ij} \cos \theta_i^{W^+} \cos \theta_j^{W^-} \right)$$

$$\langle \cos \theta_i^{W^+} \rangle = \frac{\kappa_W B_i^+}{3}, \quad \langle \cos \theta_i^{W^-} \rangle = -\frac{\kappa_W B_i^-}{3}, \quad \langle \cos \theta_i^{W^+} \cos \theta_i^{W^-} \rangle = -\frac{\kappa_W^2 C_{ij}}{9}$$

通过衰变产物重构自旋信息

- 顶夸克衰变的密度矩阵

$$\Gamma_{\alpha\beta} = M_{t\alpha \rightarrow W+b} M_{t\beta \rightarrow W+b}^* \propto \begin{pmatrix} \frac{1+\kappa_W \cos \theta_W}{2} & \frac{\kappa_W}{2} e^{i\phi_W} \sin \theta_W \\ \frac{\kappa_W}{2} e^{-i\phi_W} \sin \theta_W & \frac{1-\kappa_W \cos \theta_W}{2} \end{pmatrix}_{\alpha\beta}$$

- 参数 $\kappa_W = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} \simeq 0.39$ 表现了极化的顶夸克衰变出的W玻色子在空间分布上的方向性强弱, spin analyzing power
- 顶夸克的极化信息由产生过程决定, 因此可以通过末态衰变产物的空间分布信息提取顶夸克的极化信息 B_i
- 考虑一般性顶夸克极化状态:

$$\rho_t = \frac{1}{2} (I_2 + \sum_{i=1}^3 B_i \sigma_i) \quad \frac{1}{\Gamma} \frac{d\Gamma}{d^2\vec{\Omega}_W} = \frac{1 + \kappa_W \sum_i B_i n_i}{4\pi}$$

$$\vec{n} = (\sin \theta_W \cos \theta_W, \sin \theta_W \sin \phi_W, \cos \theta_W)$$

通过衰变产物重构自旋信息

- 不同末态粒子的 spin analysis power (Leading order)

A	ℓ^+, \bar{d}	$\nu_{\ell, u}$	W^+	b
κ_A	1	-0.3	0.39	-0.39

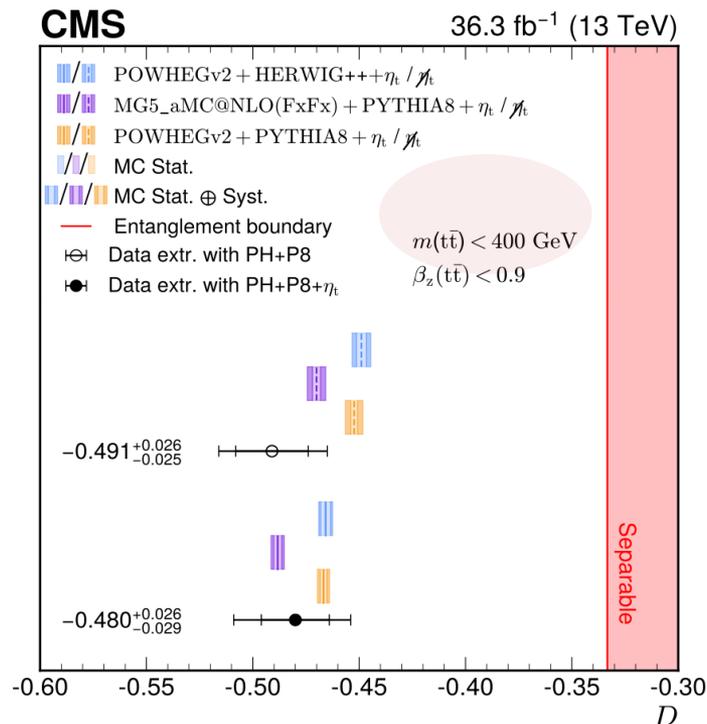
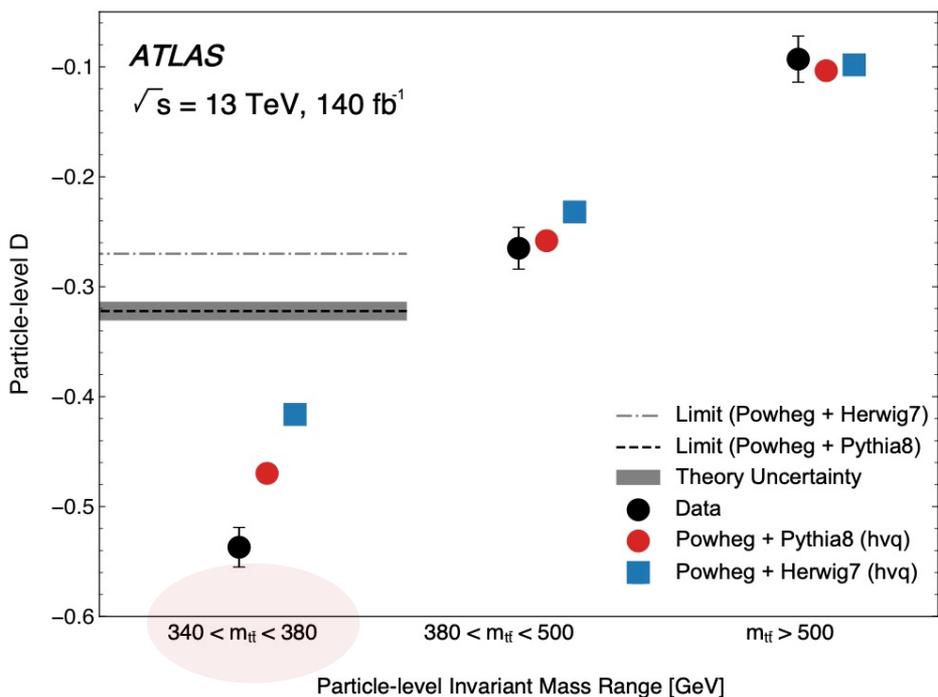
- 带电轻子的 spin analysis power 最大，最能体现出顶夸克的极化信息
- 带电轻子在顶夸克静止系的相对方位角

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D \cos\varphi) \quad D = \frac{\text{tr}[C]}{3}$$

- 纠缠的判据之一： $-\text{tr}[C] > 1$

Entangled : $D < -1/3$, separable : $D > -1/3$

实验结果



ATLAS和CMS实验组在顶夸克对阈值附近发现了明显的纠缠现象

为什么集中在阈值附近？

顶夸克对自旋纠缠效应

➤ 自旋关联矩阵 $g_L g_L / g_R g_R$ 初态 (gg-like)

$$\begin{pmatrix} \frac{\beta^2-1}{\beta^2+1} & 0 & 0 \\ 0 & \frac{\beta^2-1}{\beta^2+1} & 0 \\ 0 & 0 & -1 \end{pmatrix} \xrightarrow[\text{Boost 区间}]{\beta \rightarrow 1} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$\beta \rightarrow 0$  阈值附近

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

自旋单态, 最大纠缠

自旋态趋近于可分态, 不存在纠缠

$$\rho \propto |++\rangle \langle ++| + |--\rangle \langle --|$$

顶夸克对自旋纠缠效应

➤ 自旋关联矩阵 $g_L g_R$ 初态 (gg-unlike) / $q\bar{q}$ 初态

$$\begin{pmatrix} \frac{(2-\beta^2)s_\theta^2}{2-\beta^2s_\theta^2} & 0 & -\frac{2c_\theta s_\theta \sqrt{1-\beta^2}}{2-\beta^2s_\theta^2} \\ 0 & -\frac{\beta^2 s_\theta^2}{2-\beta^2s_\theta^2} & 0 \\ -\frac{2c_\theta s_\theta \sqrt{1-\beta^2}}{2-\beta^2s_\theta^2} & 0 & \frac{2c_\theta^2 + \beta^2 s_\theta^2}{2-\beta^2s_\theta^2} \end{pmatrix} \xrightarrow{\text{对角化}} C^{\text{diag}} = \begin{pmatrix} \frac{\beta^2 s_\theta^2}{2-\beta^2s_\theta^2} & 0 & 0 \\ 0 & -\frac{\beta^2 s_\theta^2}{2-\beta^2s_\theta^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\beta \rightarrow 0$ ↓ 对角化后
阈值附近

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

自旋态趋近于可分态, 不存在纠缠

$$\rho \propto |++\rangle \langle ++| + |--\rangle \langle --|$$

Boost 区间
中心区域 ↓ $\beta \rightarrow 1$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

自旋三重态, 最大纠缠

顶夸克对自旋纠缠效应

➤ 自旋关联矩阵

$$C_{gg \rightarrow t\bar{t}} \propto |\mathcal{M}_{gg\text{-unlike}}|^2 C_{gg\text{-unlike}} + |\mathcal{M}_{gg\text{-like}}|^2 C_{gg\text{-like}},$$

$$C^{\text{diag}} = \begin{pmatrix} \frac{\beta^2 s_\theta^2}{2 - \beta^2 s_\theta^2} & 0 & 0 \\ 0 & -\frac{\beta^2 s_\theta^2}{2 - \beta^2 s_\theta^2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \frac{\beta^2 - 1}{\beta^2 + 1} & 0 & 0 \\ 0 & \frac{\beta^2 - 1}{\beta^2 + 1} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

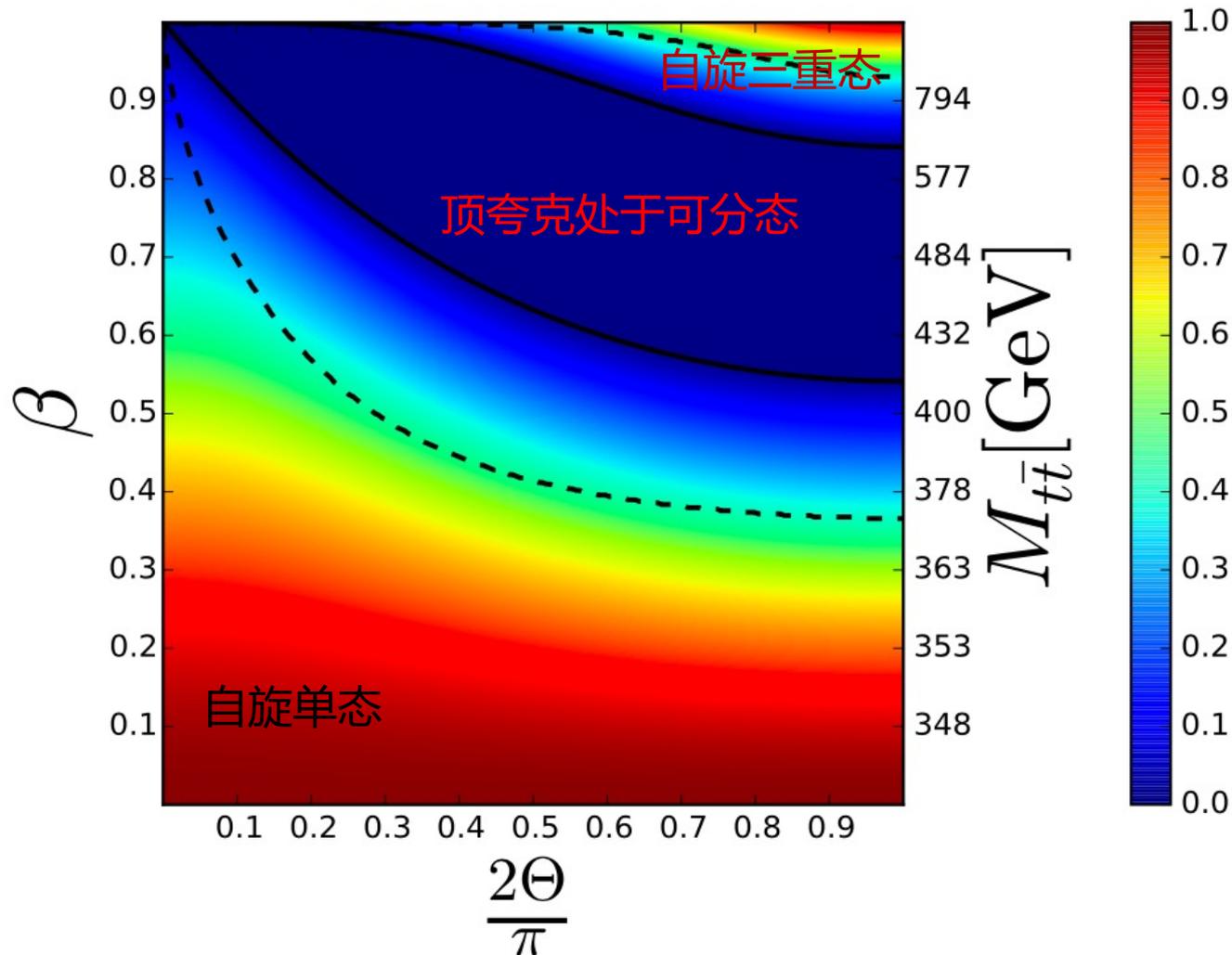


- 螺旋度相反(gg-unlike)正的自旋关联，螺旋度相同(gg-like)负的关联，两者存在严重抵消
- 阈值附近相同螺旋度(gg-like)过程主导，boost 区间则是相反螺旋度(gg-unlike)过程主导
- 阈值附近和boost区间均有利于发现纠缠现象但是中间区域由于不同过程的相互抵消导致纠缠效应很弱

顶夸克对自旋纠缠效应

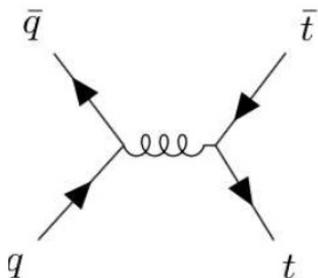
虚线: 贝尔不等式破坏边界

实线: 纠缠边界



运动学特征提取自旋关联

- 粒子自旋密度矩阵可以参数化为散射角和粒子运动的函数



$$C_{ij} = \begin{pmatrix} \frac{2c_{\Theta}^2 + \beta^2 s_{\Theta}^2}{2 - \beta^2 s_{\Theta}^2} & 0 & -\frac{2c_{\Theta} s_{\Theta} \sqrt{1 - \beta^2}}{2 - \beta^2 s_{\Theta}^2} \\ 0 & \frac{-\beta^2 s_{\Theta}^2}{2 - \beta^2 s_{\Theta}^2} & 0 \\ -\frac{2c_{\Theta} s_{\Theta} \sqrt{1 - \beta^2}}{2 - \beta^2 s_{\Theta}^2} & 0 & \frac{(2 - \beta^2) s_{\Theta}^2}{2 - \beta^2 s_{\Theta}^2} \end{pmatrix}$$

- Concurrence和贝尔变量可以根据事例的运动学信息逐个计算（近似类似贝尔三重态，贝尔变量的方向未优化）

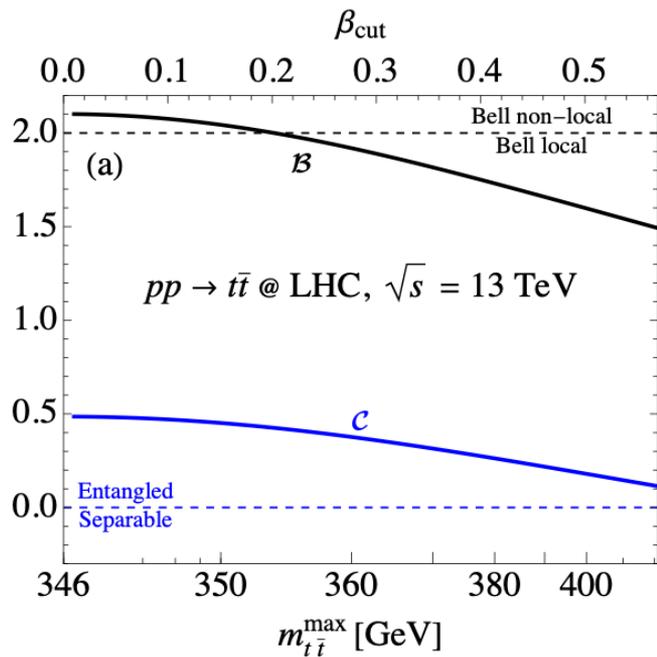
$$F_C(\Theta, \beta) = \frac{C_{11}(\Theta, \beta) + C_{33}(\Theta, \beta) - C_{22}(\Theta, \beta) - 1}{2} \quad F_B(\Theta, \beta) = \sqrt{2} |C_{33}(\Theta, \beta) - C_{22}(\Theta, \beta)|$$

$$= \frac{\beta^2 s_{\Theta}^2}{2 - \beta^2 s_{\Theta}^2} \quad = \frac{2\sqrt{2} s_{\Theta}^2}{2 - \beta^2 s_{\Theta}^2}$$

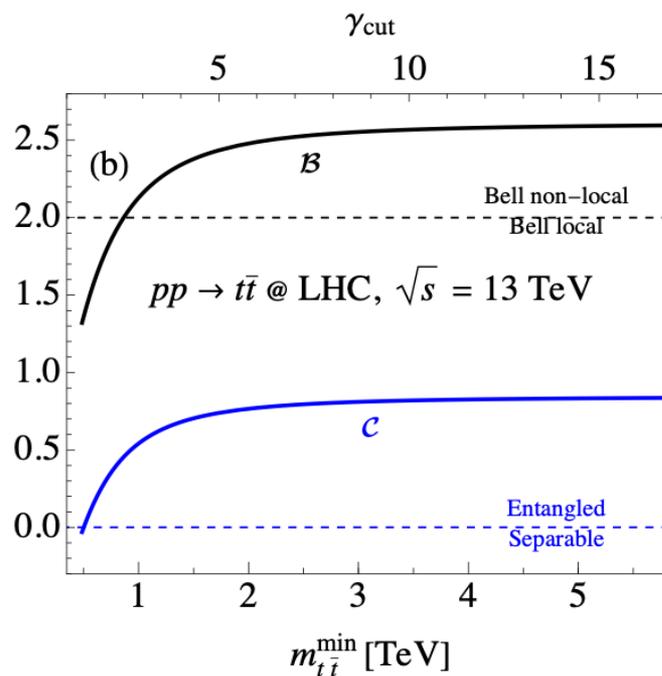
运动学特征提取自旋关联

➤ 顶夸克对完整密度矩阵:

$$\rho(\Theta, \beta) = \frac{L_{q\bar{q}}|\mathcal{M}_{q\bar{q}\rightarrow t\bar{t}}|^2\rho_{q\bar{q}\rightarrow t\bar{t}} + L_{gg}|\mathcal{M}_{gg\rightarrow t\bar{t}}|^2\rho_{gg\rightarrow t\bar{t}}}{L_{q\bar{q}}|\mathcal{M}_{q\bar{q}\rightarrow t\bar{t}}|^2 + L_{gg}|\mathcal{M}_{gg\rightarrow t\bar{t}}|^2}$$



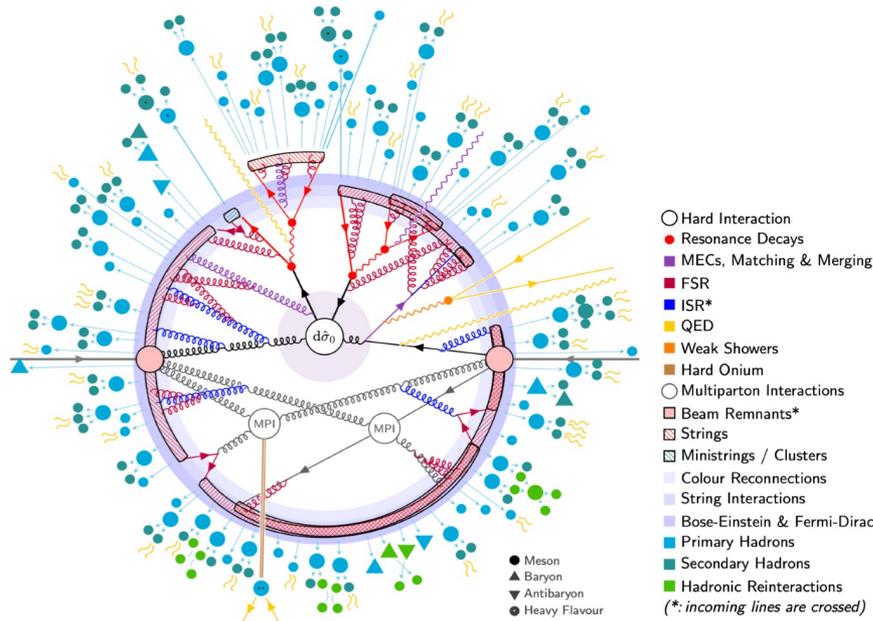
阈值附近



Boost 区间

非微扰函数提取自旋关联

□ The quark can not be a free particle due to the QCD confinement:

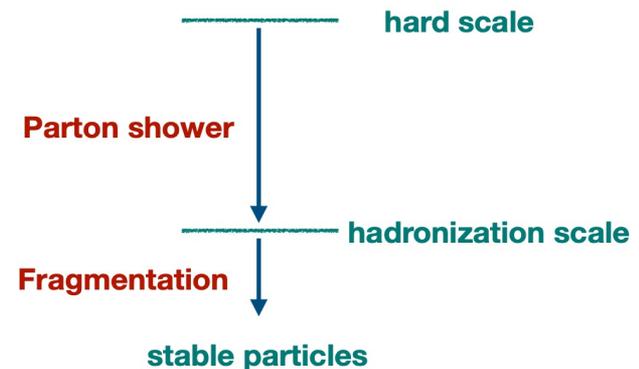


From PYTHIA 8.3

□ Hard process in high energy

□ Transition from high energy to low energy
—parton shower

□ Low energy soft regime
—fragmentation

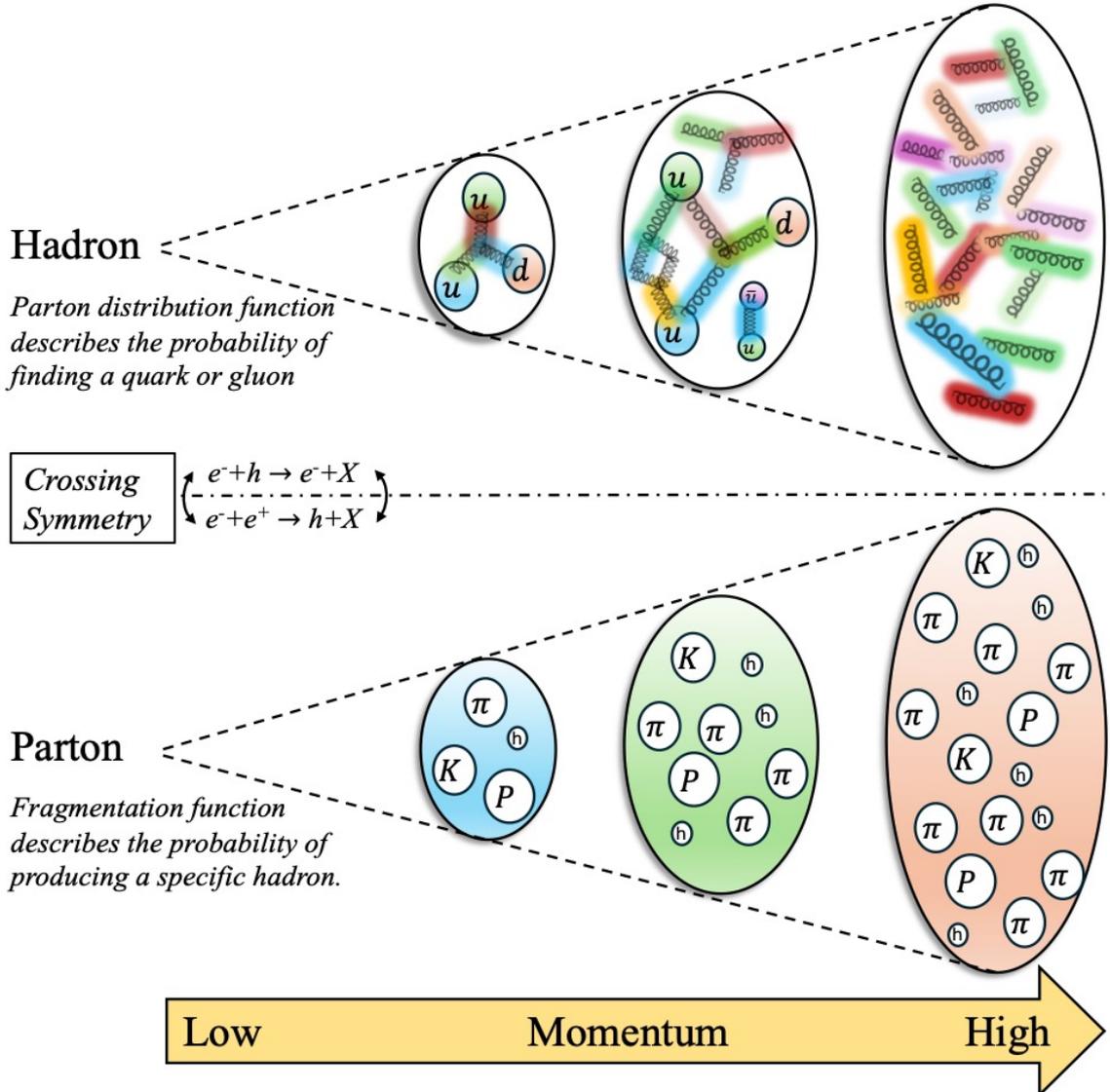


□ The light quark does not decay but instead fragments into a jet of hadrons after produced from hard scattering

□ spin information of light quarks?

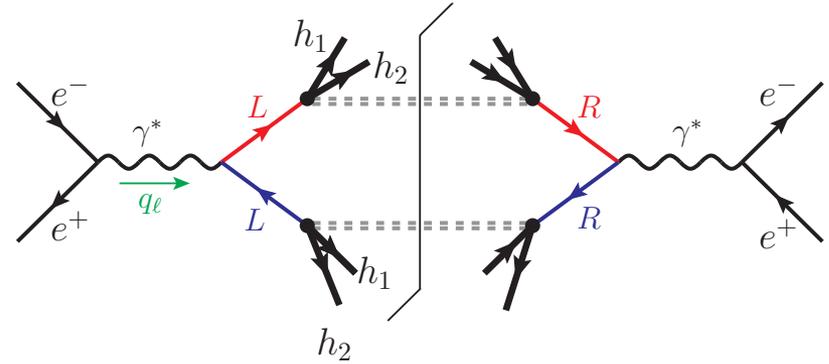
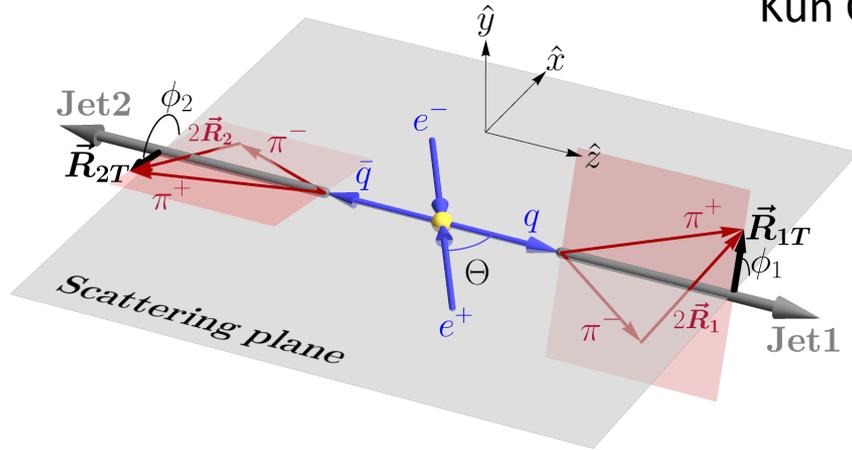
The non-perturbative functions: **the fragmentation functions**

PDF and FF



Dihadron pair production at lepton colliders

Kun Cheng and Bin Yan, PRL 135 (2025) 011902

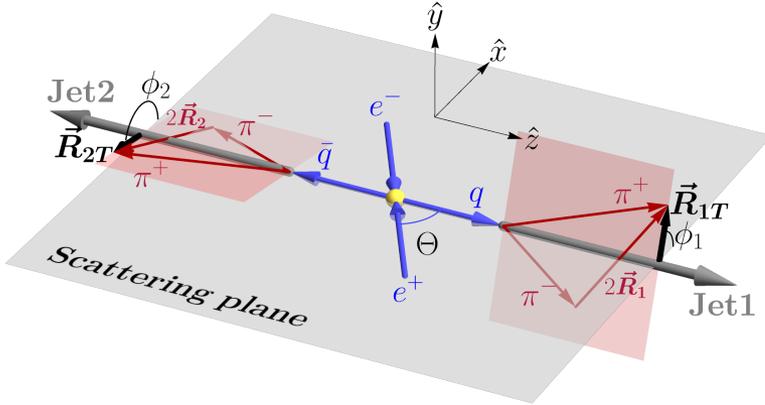


- The **transverse spin correlation** between light quarks: chiral-odd interference dihadron fragmentations (collinear factorization)
- Light quark pair are **100% correlated** in the central scattering region

$$C_{ij} = \text{diag} \left(\frac{\sin^2 \Theta}{1 + \cos^2 \Theta}, -\frac{\sin^2 \Theta}{1 + \cos^2 \Theta}, 1 \right)$$

- The **maximally entangled Bell state**: Bell inequality violation effects

Bell inequality of light quarks



J. C. Collins et al, NPB 420, 565 (1994)

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} = \sigma_{\text{hard}} \sum_{ss' \bar{s}\bar{s}'} \bar{\rho}_{ss', \bar{s}\bar{s}'} \mathcal{D}_{\pi^+\pi^-/q}^{ss'} \mathcal{D}_{\pi^+\pi^-/\bar{q}}^{\bar{s}\bar{s}'}$$

$$\frac{1}{2} \text{Tr}(\mathcal{D}_{\pi^+\pi^-/q}) = D_1^q(z_1, M_1),$$

$$\frac{1}{2} \text{Tr}(\sigma_z \mathcal{D}_{\pi^+\pi^-/q}) = 0,$$

$$\frac{1}{2} \text{Tr}(\sigma_i \mathcal{D}_{\pi^+\pi^-/q}) = -\frac{\varepsilon_T^{ij} R_{1,T}^j}{|\vec{R}_{1,T}|} H_1^{\triangleleft, q}(z_1, M_1),$$

$$\frac{d\sigma}{dz_1 dz_2 dM_1 dM_2 d\phi_1 d\phi_2} = \sigma_{\text{hard}} \left[\sum_q e_q^2 D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2) \right. \quad \text{Unpolarized diFF}$$

$$\left. + \frac{1}{2} \sum_q e_q^2 H_1^{\triangleleft, q}(z_1, M_1) H_1^{\triangleleft, \bar{q}}(z_2, M_2) \left(\mathcal{B}_- \cos(\phi_1 + \phi_2) - \mathcal{B}_+ \cos(\phi_1 - \phi_2) \right) \right] \quad \text{Transverse polarized diFF}$$

$$\mathcal{B}_{\pm} \equiv C_{xx} \pm C_{yy}$$

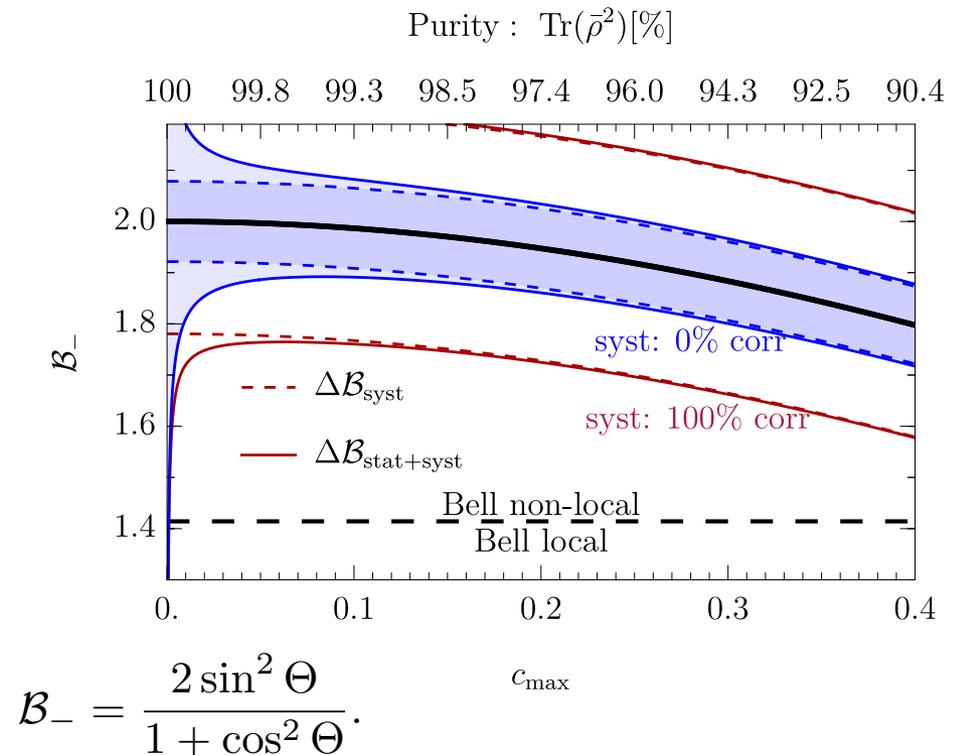
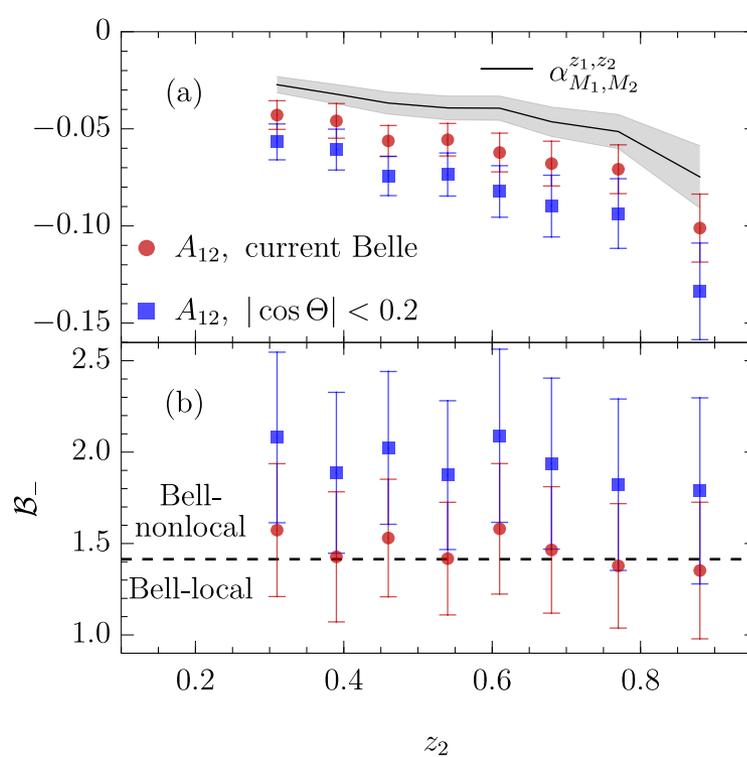
$$\mathcal{B}_+ = 0, \quad \mathcal{B}_- = \frac{2 \sin^2 \Theta}{1 + \cos^2 \Theta}.$$

$$\mathcal{B}_- = \frac{2 \langle \cos(\phi_1 + \phi_2) \rangle}{\alpha_{M_1, M_2}^{z_1, z_2}} = \frac{A_{12}}{\alpha_{M_1, M_2}^{z_1, z_2}}$$

$$\alpha_{M_1, M_2}^{z_1, z_2} = \frac{1}{2} \frac{\sum_q e_q^2 H_1^{\triangleleft, q}(z_1, M_1) H_1^{\triangleleft, \bar{q}}(z_2, M_2)}{\sum_q e_q^2 D_1^q(z_1, M_1) D_1^{\bar{q}}(z_2, M_2)}.$$

CHSH type Bell inequality $|\mathcal{B}| > \sqrt{2}$

Dihadron pair production

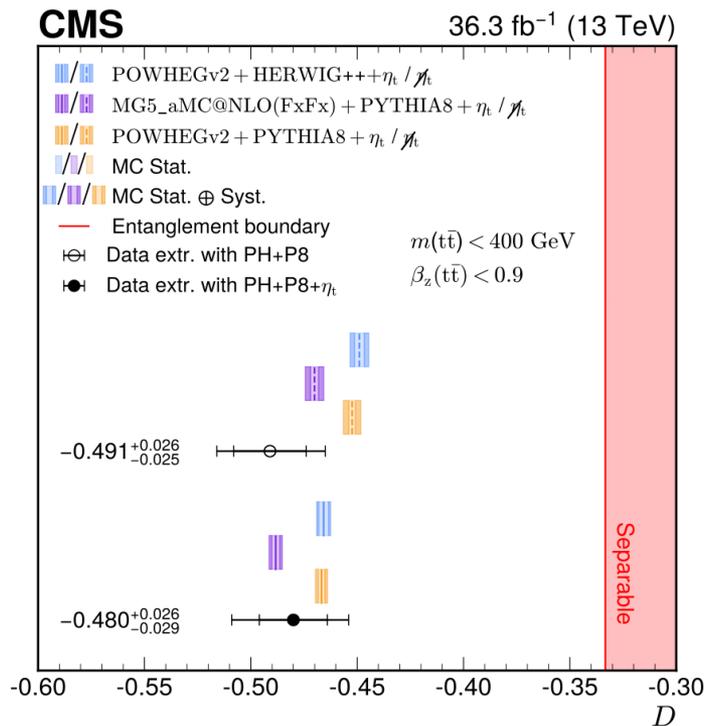
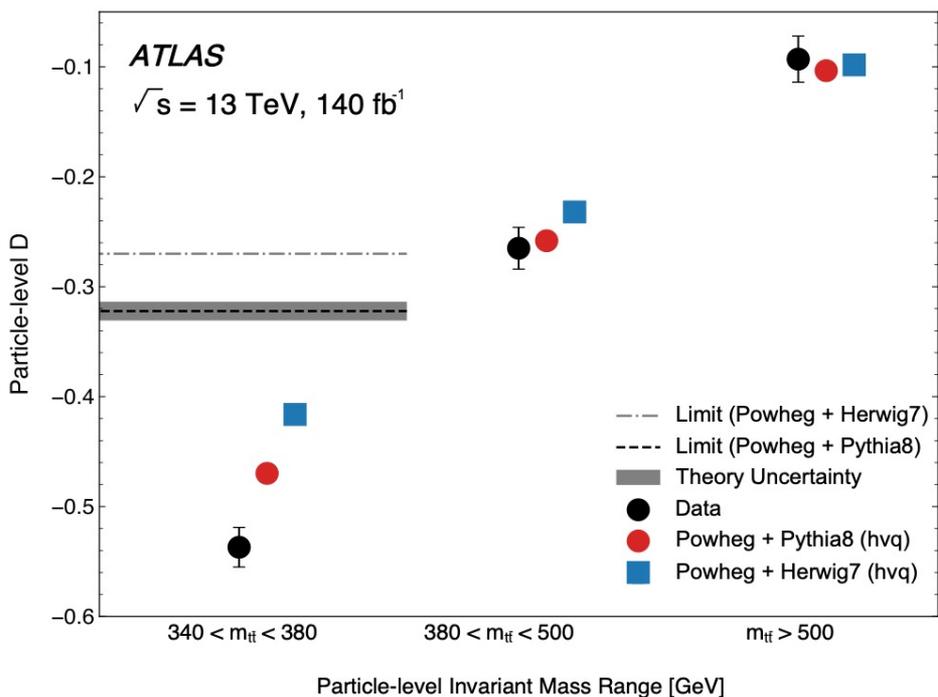


- ❖ Current data exhibiting no significant evidence of Bell inequality violation
- ❖ The optimal cuts on scattering angle will significantly improved the results
- ❖ The light quark pair would be a **highly pure spin Bell state**
- ❖ Combined results: **2.5 σ for 100% correlated systematic uncertainties and 6.7 σ for the uncorrelated case**

利用量子纠缠效应寻找超出标准模型的新物理

- 量子纠缠和贝尔不等式破坏效应的研究依赖于重构系统的自旋关联矩阵
- 新物理相互作用可能会显著改变粒子间的自旋关联效应，因此利用量子信息领域的观测量可有效检验超出标准模型的新物理效应

利用量子纠缠效应寻找超出标准模型的新物理



ATLAS测量的Concurrence与标准模型理论预言不一致CMS考虑标准模型+新的顶夸克对束缚态 pseudoscalar (自旋单态, 最大纠缠态)

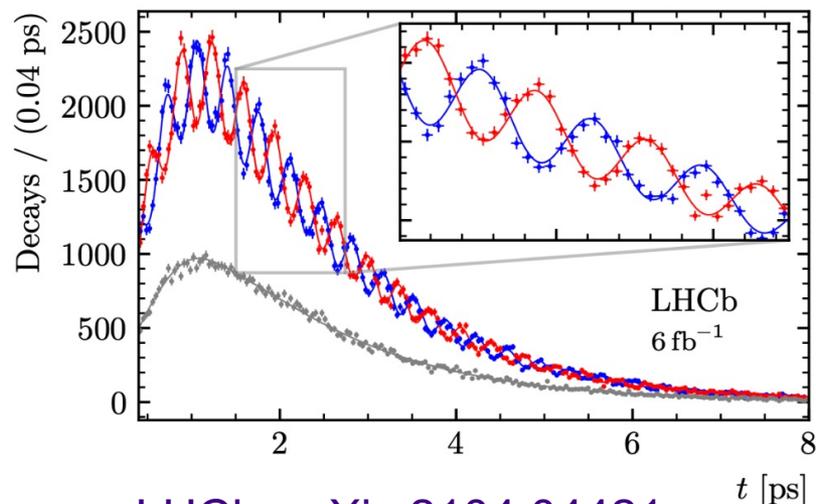
展望及前沿问题

➤ 规范玻色子的量子纠缠和贝尔不等式破坏效应

有质量规范玻色子存在额外的自由度，重构密度矩阵更具挑战性，非局域性检验考虑CGLMP不等式

➤ 味空间和同位旋空间的量子纠缠效应

— $B_s^0 \rightarrow D_s^- \pi^+$ — $\bar{B}_s^0 \rightarrow B_s^0 \rightarrow D_s^- \pi^+$ — Untagged



LHCb, arXiv:2104.04421

Kaon decay to pions:

$$|\Psi\rangle = \frac{1}{\sqrt{2 + |\gamma|^2}} [|\pi^+\pi^-\rangle + \gamma|\pi^0\pi^0\rangle + |\pi^-\pi^+\rangle]$$

M. Fabbrichesesi et al 2506.05464

展望及前沿问题

➤ 多粒子纠缠 K. Sakurai, M. Spannowsky, PRL 132 (2024) 151602

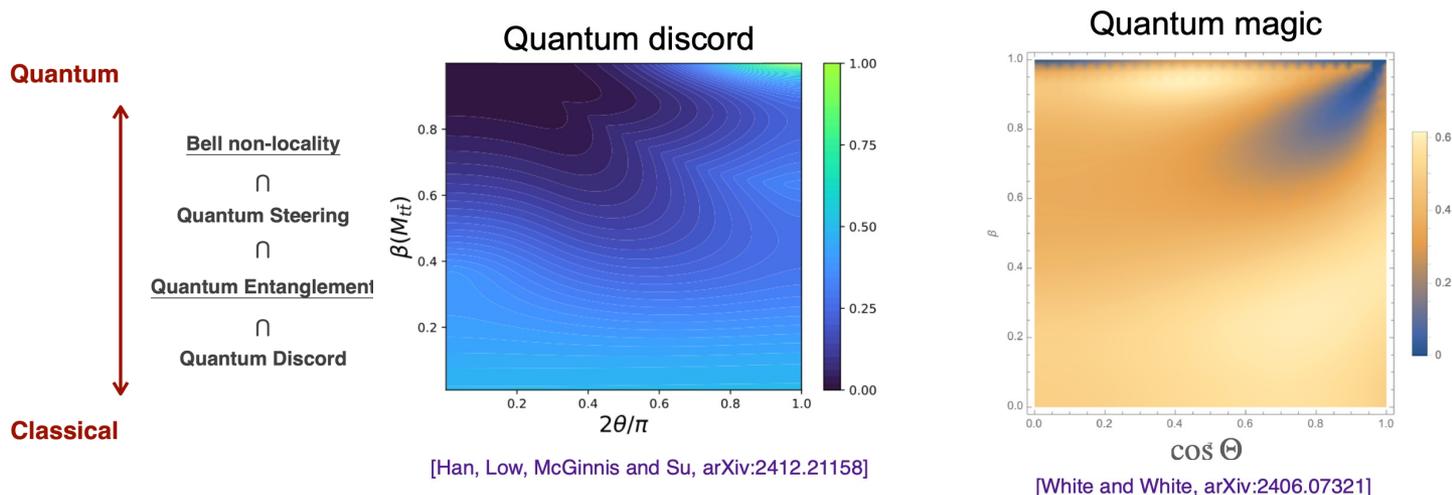
(1) 两个粒子的纠缠，将其中一个子系统求迹

$$C_{ij} = \mathcal{C}[\rho_{ij}], \quad \rho_{ij} = \text{Tr}_k(|\Psi\rangle\langle\Psi|)$$

(2) 其中一个粒子与剩下系统的纠缠

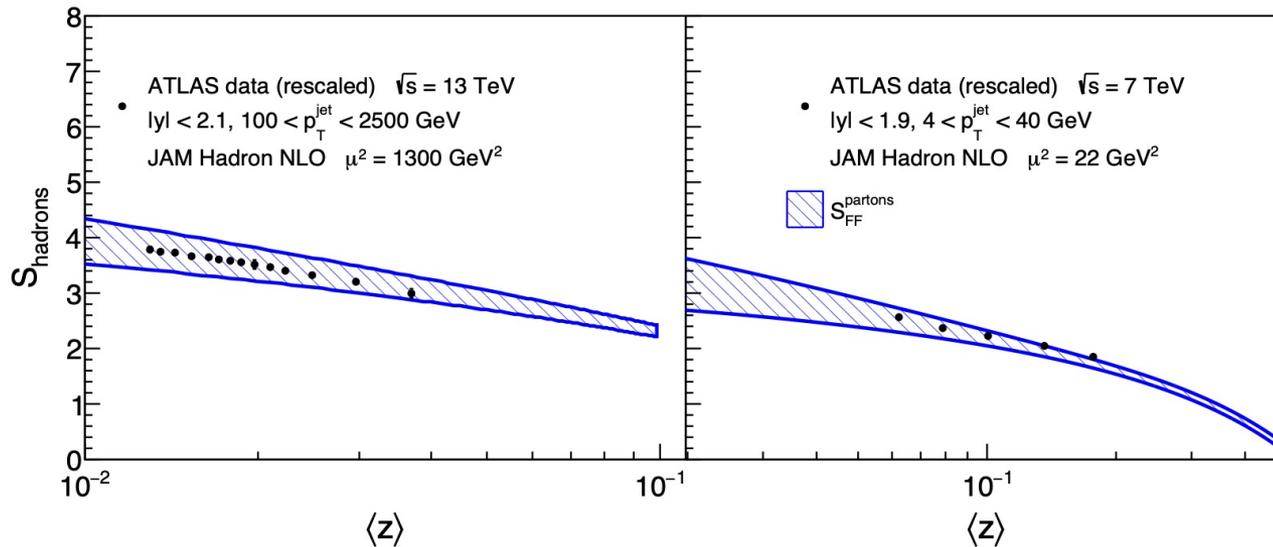
$$C_{i(kj)} = \sqrt{2(1 - \text{Tr}\rho_{kj}^2)}, \quad \rho_{kj} = \text{Tr}_i(|\Psi\rangle\langle\Psi|)$$

➤ 其它量子信息观测量在对撞机物理中的应用



扩展及前沿问题

- 量子纠缠和强子化关联 J. Datta et al, PRL 134 (2025) 111902



- 高阶辐射修正导致的退相关效应 R. Aoude et al, 2504.07030

- 量子纠缠和涌现对称性 M. Carena et al, 2505.00873

总结

- 在对撞机环境中讨论量子纠缠和贝尔不等式破坏效应对于检验量子力学基本原理至关重要
- 粒子物理和量子信息的交叉研究是高能物理领域全新的研究方向
- 目前仍然有很多悬而未决的问题亟需大家努力

例如：对于新物理寻找量子纠缠效应能否提供除自旋关联矩阵额外的信息？