Some of recent progress in standard model effective field theory

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Disclaimer: quoted refs are not always the first ones.

Status of standard model in a few words

- No new particles found up to mass ~ 1 TeV > $\Lambda_{\mathrm{EW}} \approx 100\ \mathrm{GeV}$
  although some apparent tension exists between SM and expts.
  → SM phenomenologically very healthy
- Still, two practical issues remain to be addressed:

✓ $m_\nu < 1\ \mathrm{eV}$, believed to originate from phys well above Λ_{EW}

✓ If DM is of particle nature, SM cannot offer a candidate.

- There are more advanced theoretical challenges:
  flavor puzzle
  origin of electroweak symmetry breaking
  ……

Status of standard model in a few words

- Thus new phys is called for, which must involve particles of
  either mass $\gg \Lambda_{\mathrm{EW}}$
  $\longrightarrow$ not directly reachable at colliders
  or mass $\lesssim \Lambda_{\mathrm{EW}}$, but interacting feebly with SM particles
  $\longrightarrow$ not yet detected even in precision measurements
- Question:
  How to investigate new phys in such a circumstance?

Modern view of standard model

- All quantum field theories are effective field theories appropriate to a certain range of energy scales.
- SM is based on QFT.

  It should be considered the leading part of an EFT appropriate to

  $E \leq \Lambda_{\mathrm{EW}}$.
- SM is successful because it parameterizes all possible interactions permitted by gauge symmetry and renormalizability.

  It is self-contained in that it is "closed" under renormalization.

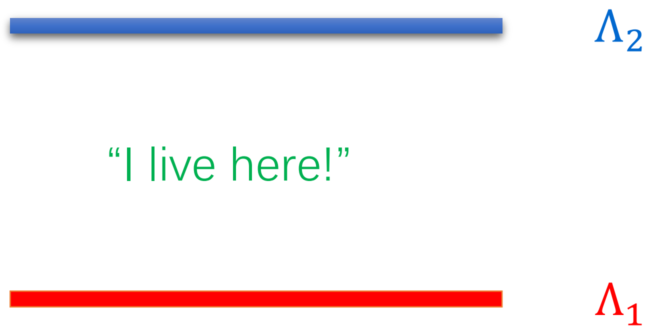
  — a very important property for

  self-consistency and predictability.

EFT： general discussion

- An EFT is an infinite tower of effective interactions organized by their relative importance.
- Given an accuracy expected for a measurement, only a finite number of effective interactions are important, which are also self-contained in a similar sense as in a renormalizable theory.

- An EFT defined in an energy range $\Lambda_1 < E < \Lambda_2$ is always a low-energy EFT relative to Λ_2.



EFT: general discussion

Three essential elements to specify an EFT:

- Dynamical degrees of freedom.

  — *what are experimentally prepared and produced?*

- Symmetries as a guiding principle for constructing interactions.

  — *most sacred are gauge symmetries and dynamically broken symmetries*

- A power counting rule assessing what would be more important.

  — *low-energy EFT: importance decreases with increasing power of*

  p/Λ_2 *in amplitude* $\leftrightarrow$ ∂/Λ_2 *in Lagrangian*

➡ ✓ to establish a basis of effective interactions/operators at each order in low-energy expansion;

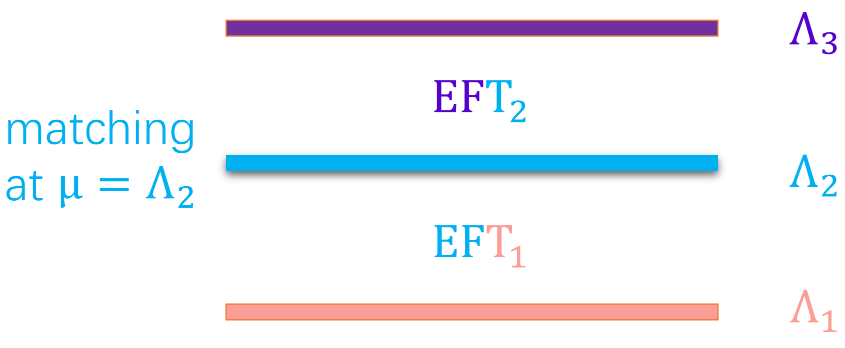
✓ to renormalize them to improve perturbation calc, i.e., RGE

EFT: general discussion

- Usually, the characteristic scale of a physical process lies well below the scale at which the mechanism for the process occurs.

  — *a sequence of EFTs is required to connect data with physical origin*

  ⟹ matching is required at the boundary of two neighboring EFTs to connect them



- Two types of matching:

✓Strong dynamics involved

— completely new dynamical DoFs appear,

e.g., chiral symmetry breaking in QCD at Λ_χ

✓Perturbative interactions only

— from $\mu > \Lambda_2$ to $\mu < \Lambda_2$, integrate out heavy fields of mass $O(\Lambda_2)$.

How EFT works:

$$K^- \to \pi^+ l^- l^-$$

$K^- \to \pi^+ l^- l^-$: general

The process occurs at $\mu \sim 10^2$ MeV.

It violates lepton number

— its mechanism is new phys at $\mu \gg \Lambda_{EW}$

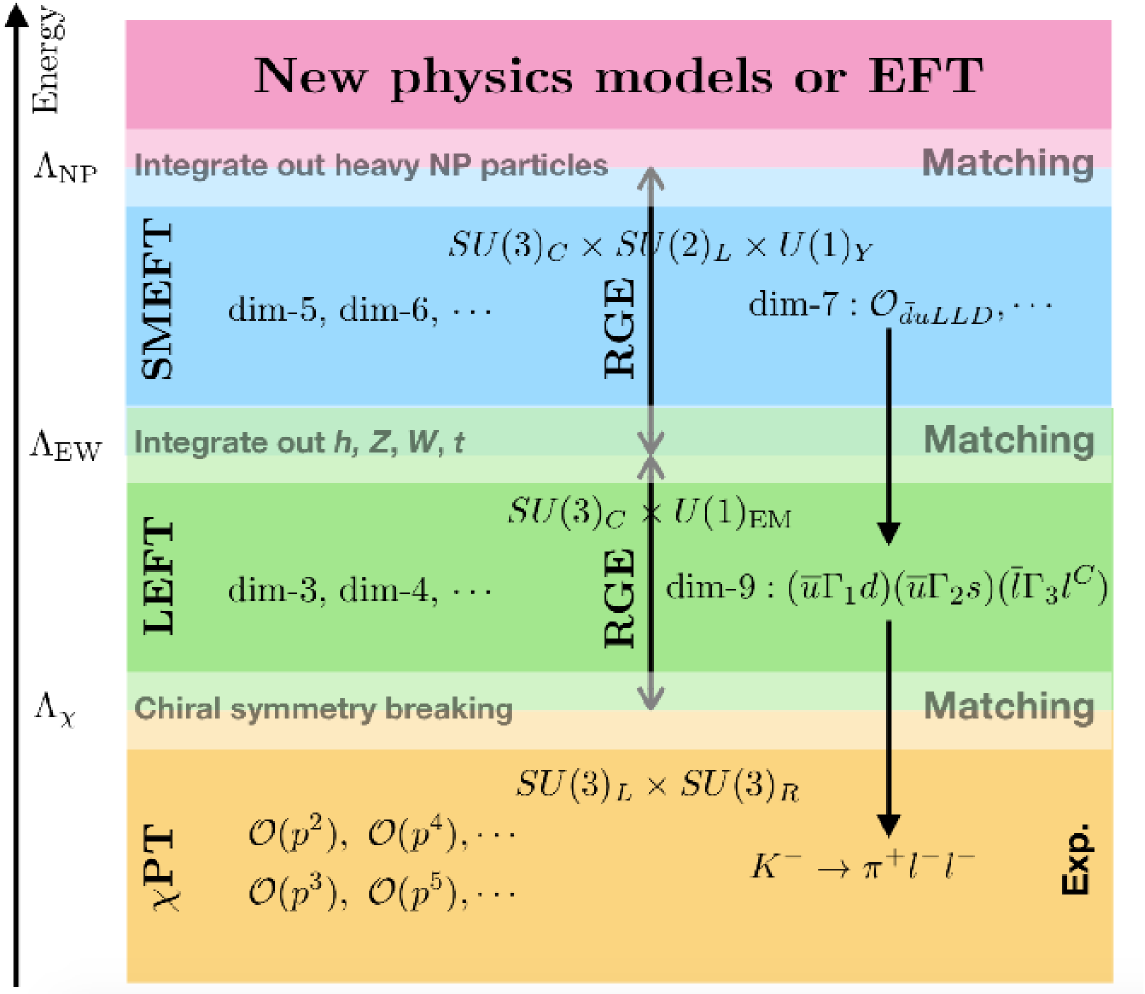
→ a sequence of EFTs required to connect them

- A sequence of EFTs: SMEFT, LEFT, χPT

✓Bases of operators and RGE in each EFT;

✓Matching between SMEFT and LEFT, and between LEFT and χPT.

- Matching between SMEFT and your desired new phys model.



$K^- \to \pi^+ l^- l^-$: final answer

Symbolically,

LNV Wilson coefficient at Λ_{NP}

$\mathscr{A}(K^- \to \pi^+ l^- l^-)$ is a sum of products:

$$\Downarrow$$

$$f_\chi R_\chi \quad \otimes \quad R_{\text{LEFT}} \quad \otimes \quad R_{\text{SMEFT}} \quad \otimes \quad C_{\text{LNV}}(\Lambda_{\text{NP}})$$

$$\Uparrow \qquad\qquad \Uparrow \qquad\qquad \Uparrow$$

matching between	matching between	matching between
χPT and LEFT	LEFT and SMEFT	SMEFT and NP

f_χ: χPT amplitude with low-energy strong constants (expt's, lattice, etc)

R_χ: RGE in χPT, $\Lambda_\chi \to m_K$

R_{LEFT}: RGE in LEFT, $\Lambda_{\text{EW}} \to \Lambda_\chi$

R_{SMEFT}: RGE in SMEFT, $\Lambda_{\text{NP}} \to \Lambda_{\text{EW}}$

$K^- \to \pi^+ l^- l^-$: general again

I outline how the result is obtained. For a complete analysis, see:

Liao-Ma-Wang, 1909.06272, 2001.07378

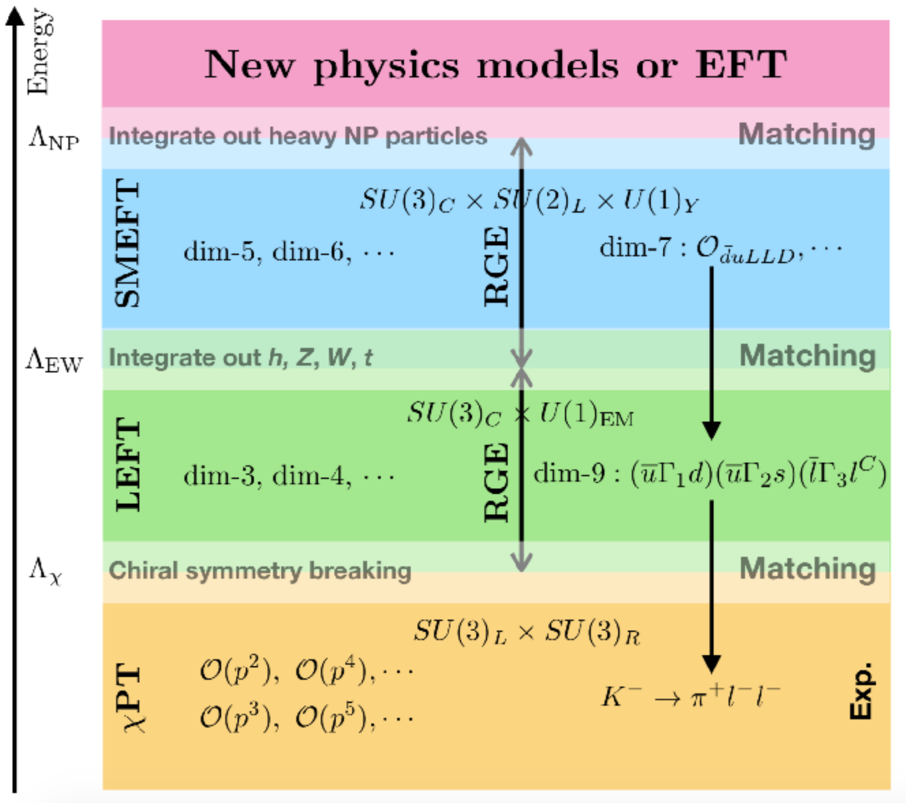
Contributions are classified into

✓ long-distance part (LD): with light ν exchanged between initial and final particles

✓ short-distance part (SD): without

I focus on less important but simpler SD part to save time.

I start from low-energy EFT (LEFT), then match it with chiral perturbation theory (χPT), and then match it with standard model EFT (SMEFT). Finally, I match SMEFT to a NP model for illustration.



$K^- \to \pi^+ l^- l^-$: LEFT

LEFT (SD) operators start with dim-9:

$K^- \to \pi^+$: $4q$; $2l$ ➡ dim=6*3/2=9

basis of dim-9 $|\Delta L| = 2$ operators ➡

$$\mathcal{L}_{\rm LEFT}^{|\Delta L|=2} = \sum_i C_i \mathcal{O}_i + \text{H.C.}$$

relevant for $K^- \to \pi^+ l^- l^-$:

$$\mathcal{O}^{LLLL,S/P}_{udus},\ \mathcal{O}^{LRLR,S/P}_{udus},\ \tilde{\mathcal{O}}^{LRLR,S/P}_{udus},\ \mathcal{O}^{LRLL,A}_{uiuj},\ \tilde{\mathcal{O}}^{LRLL,A}_{uiuj},$$
$$\mathcal{O}^{RRRR,S/P}_{udus},\ \mathcal{O}^{RLRL,S/P}_{udus},\ \tilde{\mathcal{O}}^{RLRL,S/P}_{udus},\ \mathcal{O}^{RLRR,A}_{uiuj},\ \tilde{\mathcal{O}}^{RLRR,A}_{uiuj},$$

$$\mathcal{O}^{LRRR,A}_{uiuj},\ \tilde{\mathcal{O}}^{LRRR,A}_{uiuj},\ \mathcal{O}^{LRRL,S/P}_{udus},\ \tilde{\mathcal{O}}^{LRRL,S/P}_{udus},$$
$$\mathcal{O}^{RLLL,A}_{uiuj},\ \tilde{\mathcal{O}}^{RLLL,A}_{uiuj},\ \mathcal{O}^{LRRL,S/P}_{usud},\ \tilde{\mathcal{O}}^{LRRL,S/P}_{usud},$$

Notation	Operator	Notation	Operator
$\mathcal{O}^{LLLL,S/P}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L)[\overline{u^s_L}\gamma_\mu d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\mathcal{O}^{RRRR,S/P}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R)[\overline{u^s_R}\gamma_\mu d^t_R](j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}^{LLLL,T}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L)[\overline{u^s_L}\gamma^\nu d^t_L](j_{\mu\nu}^{\alpha\beta})$	$\mathcal{O}^{RRRR,T}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R)[\overline{u^s_R}\gamma^\nu d^t_R](j_{\mu\nu}^{\alpha\beta})$
$\tilde{\mathcal{O}}^{LLLL,T}_{prst}$	$(\overline{u^p_L}\gamma^\mu d^r_L][\overline{u^s_L}\gamma^\nu d^t_L)(j_{\mu\nu}^{\alpha\beta})$	$\tilde{\mathcal{O}}^{RRRR,T}_{prst}$	$(\overline{u^p_R}\gamma^\mu d^r_R][\overline{u^s_R}\gamma^\nu d^t_R)(j_{\mu\nu}^{\alpha\beta})$
$\mathcal{O}^{LRLR,S/P}_{prst}$	$(\overline{u^p_L} d^r_R)[\overline{u^s_L} d^t_R](j^{\alpha\beta}/j_5^{\alpha\beta})$	$\mathcal{O}^{RLRL,S/P}_{prst}$	$(\overline{u^p_R} d^r_L)[\overline{u^s_R} d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$
$\tilde{\mathcal{O}}^{LRLR,S/P}_{prst}$	$(\overline{u^p_L} d^r_R][\overline{u^s_L} d^t_R)(j^{\alpha\beta}/j_5^{\alpha\beta})$	$\tilde{\mathcal{O}}^{RLRL,S/P}_{prst}$	$(\overline{u^p_R} d^r_L][\overline{u^s_R} d^t_L)(j^{\alpha\beta}/j_5^{\alpha\beta})$
$\mathcal{O}^{LRLR,T}_{prst}$	$(\overline{u^p_L} i\sigma^{\mu\nu} d^r_R)[\overline{u^s_L} d^t_R](j_{\mu\nu}^{\alpha\beta})$	$\mathcal{O}^{RLRL,T}_{prst}$	$(\overline{u^p_R} i\sigma^{\mu\nu} d^r_L)[\overline{u^s_R} d^t_L](j_{\mu\nu}^{\alpha\beta})$
$\tilde{\mathcal{O}}^{LRLR,T}_{prst}$	$(\overline{u^p_L}\sigma^{\mu\rho} d^r_R)[\overline{u^s_L}\sigma^\nu{}_\rho d^t_R](j_{\mu\nu}^{\alpha\beta})$	$\tilde{\mathcal{O}}^{RLRL,T}_{prst}$	$(\overline{u^p_R}\sigma^{\mu\rho} d^r_L)[\overline{u^s_R}\sigma^\nu{}_\rho d^t_L](j_{\mu\nu}^{\alpha\beta})$
$\mathcal{O}^{LRLL,V/A}_{prst}$	$(\overline{u^p_L} d^r_R)[\overline{u^s_L}\gamma^\mu d^t_L](j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$	$\mathcal{O}^{RLRR,V/A}_{prst}$	$(\overline{u^p_R} d^r_L)[\overline{u^s_R}\gamma^\mu d^t_R](j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$
$\tilde{\mathcal{O}}^{LRLL,V/A}_{prst}$	$(\overline{u^p_L} d^r_R][\overline{u^s_L}\gamma^\mu d^t_L)(j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$	$\tilde{\mathcal{O}}^{RLRR,V/A}_{prst}$	$(\overline{u^p_R} d^r_L][\overline{u^s_R}\gamma^\mu d^t_R)(j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$
$\mathcal{O}^{LRRR,V/A}_{prst}$	$(\overline{u^p_L} d^r_R)[\overline{u^s_R}\gamma^\mu d^t_R](j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$	$\mathcal{O}^{RLLL,V/A}_{prst}$	$(\overline{u^p_R} d^r_L)[\overline{u^s_L}\gamma^\mu d^t_L](j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$
$\tilde{\mathcal{O}}^{LRRR,V/A}_{prst}$	$(\overline{u^p_L} d^r_R][\overline{u^s_R}\gamma^\mu d^t_R)(j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$	$\tilde{\mathcal{O}}^{RLLL,V/A}_{prst}$	$(\overline{u^p_R} d^r_L][\overline{u^s_L}\gamma^\mu d^t_L)(j_\mu^{\alpha\beta}/j_{5\mu}^{\alpha\beta})$
$\mathcal{O}^{LRRL,T}_{prst}$	$(\overline{u^p_L} i\sigma^{\mu\nu} d^r_R)[\overline{u^s_R} d^t_L](j_{\mu\nu}^{\alpha\beta})$	$\mathcal{O}^{RLLR,T}_{prst}$	$(\overline{u^p_R} i\sigma^{\mu\nu} d^r_L)[\overline{u^s_L} d^t_R](j_{\mu\nu}^{\alpha\beta})$
$\tilde{\mathcal{O}}^{LRRL,T}_{prst}$	$(\overline{u^p_L} i\sigma^{\mu\nu} d^r_R][\overline{u^s_R} d^t_L)(j_{\mu\nu}^{\alpha\beta})$	$\tilde{\mathcal{O}}^{RLLR,T}_{prst}$	$(\overline{u^p_R} i\sigma^{\mu\nu} d^r_L][\overline{u^s_L} d^t_R)(j_{\mu\nu}^{\alpha\beta})$
$\mathcal{O}^{LRRL,S/P}_{prst}$	$(\overline{u^p_L} d^r_R)[\overline{u^s_R} d^t_L](j^{\alpha\beta}/j_5^{\alpha\beta})$		
$\tilde{\mathcal{O}}^{LRRL,S/P}_{prst}$	$(\overline{u^p_L} d^r_R][\overline{u^s_R} d^t_L)(j^{\alpha\beta}/j_5^{\alpha\beta})$		

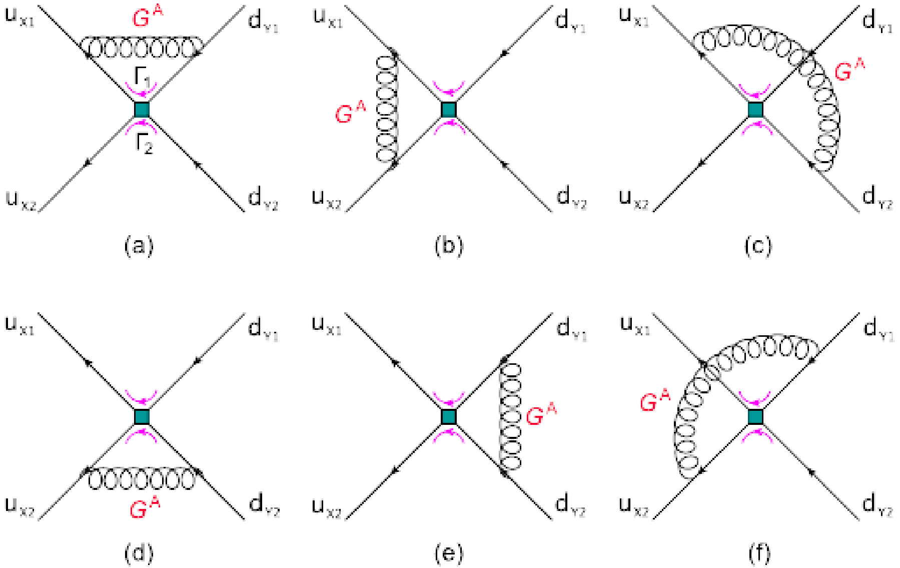
$$j^{\alpha\beta} = (\overline{l_\alpha} l^C_\beta), \quad j_5^{\alpha\beta} = (\overline{l_\alpha}\gamma_5 l^C_\beta), \quad j_{5\mu}^{\alpha\beta} = (\overline{l_\alpha}\gamma_\mu\gamma_5 l^C_\beta),$$
$$j_\mu^{\alpha\beta} = (\overline{l_\alpha}\gamma_\mu l^C_\beta), \quad j_{\mu\nu}^{\alpha\beta} = (\overline{l_\alpha}\sigma_{\mu\nu} l^C_\beta).$$

$K^- \to \pi^+ l^- l^-$: LEFT

LEFT lives in $\Lambda_\chi < \mu < \Lambda_{\rm EW}$.

It will be matched to SMEFT at $\mu = \Lambda_{\rm EW}$ and to χPT at $\mu = \Lambda_\chi$ to connect Wilson coefficients in two EFTs.

To improve perturbation theory, we sum large logarithms between $\Lambda_{\rm EW}$ and Λ_χ by renormalization group equations (RGEs):



1-loop QCD renormalization of $(\overline{u_{X_1}}\Gamma_1 d_{Y_1})(\overline{u_{X_2}}\Gamma_2 d_{Y_2})$

$$\mu\frac{d}{d\mu}\begin{pmatrix} C^{LLLL,S/P}_{prst} \\ C^{LLLL,S/P}_{ptsr}\end{pmatrix} = -\frac{\alpha_s}{2\pi}\begin{pmatrix} \frac{3}{N} & -3 \\ -3 & \frac{3}{N}\end{pmatrix}\begin{pmatrix} C^{LLLL,S/P}_{prst} \\ C^{LLLL,S/P}_{ptsr}\end{pmatrix},$$

$$\mu\frac{d}{d\mu}\begin{pmatrix} C^{LLLL,T}_{prst} \\ \tilde{C}^{LLLL,T}_{prst}\end{pmatrix} = -\frac{\alpha_s}{2\pi}\begin{pmatrix} \frac{1}{N} & -1 \\ -1 & \frac{1}{N}\end{pmatrix}\begin{pmatrix} C^{LLLL,T}_{prst} \\ \tilde{C}^{LLLL,T}_{prst}\end{pmatrix},$$

plus many more complicated RGEs

These coupled RGEs are solved to yield linear relations between $C_*^\#(\Lambda_\chi)$ and $C_*^\#(\Lambda_{\rm EW})$.

$K^- \to \pi^+ l^- l^-$: χPT

The decay takes place in the energy range $\mu < \Lambda_\chi$ appropriate for χPT.

Thus, we have to match LEFT and χPT at $\mu = \Lambda_\chi$.

Strong QCD dynamics causes dynamical chiral symmetry breaking:

$$\langle \mathrm{vac}|\bar{q}q|\mathrm{vac}\rangle \neq 0 \;\Longrightarrow\; SU_L(3)\times SU_R(3) \to SU_V(3)$$

resulting in 8 (pseudo-)Nambu-Goldstone bosons (NGBs), $\pi^{0,\pm}, K^\pm, K^0, \overline{K^0}, \eta$.

χPT is the low-energy EFT for NGBs described in terms of

$$\xi = \exp\left(\frac{i\Pi}{\sqrt{2}F_0}\right), \quad \Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

plus other non-strongly interacting light particles such as photon entering via gauge covariant derivative and field tensor, and charged leptons, neutrinos,⋯ entering as external sources.

$K^- \to \pi^+ l^- l^-$: χPT

- The matching between LEFT and χPT at $\mu = \Lambda_\chi$ is nontrivial, because dynamical degrees of freedom are completely changed by strong dynamics.
- The only guidance for matching is symmetry:
  linearly realized chiral symmetry at $\mu > \Lambda_\chi$,
  becomes nonlinearly realized at $\mu < \Lambda_\chi$.

Notation	Quark operator	chiral irrep	Hadronic operator
$\mathcal{O}^{LLLL,S/P}_{udus}$ (✓)	$(\overline{u_L}\gamma^\mu d_L)[\overline{u_L}\gamma_\mu s_L](j/j_5)$	$\mathbf{27}_L \times \mathbf{1}_R$	$\frac{5}{12} g_{27\times 1} F_0^4 (\Sigma i\partial_\mu \Sigma^\dagger)_2{}^1 (\Sigma i\partial^\mu \Sigma^\dagger)_3{}^1$
$\mathcal{O}^{RRRR,S/P}_{udus}$ (P)	$(\overline{u_R}\gamma^\mu d_R)[\overline{u_R}\gamma_\mu s_R](j/j_5)$	$\mathbf{1}_L \times \mathbf{27}_R$	$\frac{5}{12} g_{1\times 27} F_0^4 (\Sigma^\dagger i\partial_\mu \Sigma)_2{}^1 (\Sigma^\dagger i\partial^\mu \Sigma)_3{}^1$
$\mathcal{O}^{LRLR,S/P}_{udus}$ (✓)	$(\overline{u_L} d_R)[\overline{u_L} s_R](j/j_5)$	$\overline{\mathbf{6}}_L \times \mathbf{6}_R$	$-g^a_{\overline{6}\times 6} \frac{F_0^4}{4} (\Sigma^\dagger)_2{}^1 (\Sigma^\dagger)_3{}^1$
$\tilde{\mathcal{O}}^{LRLR,S/P}_{udus}$ (✓)	$(\overline{u_L} d_R][\overline{u_L} s_R)(j/j_5)$	$\overline{\mathbf{6}}_L \times \mathbf{6}_R$	$-g^b_{\overline{6}\times 6} \frac{F_0^4}{4} (\Sigma^\dagger)_2{}^1 (\Sigma^\dagger)_3{}^1$
$\mathcal{O}^{RLRL,S/P}_{udus}$ (P)	$(\overline{u_R} d_L)[\overline{u_R} s_L](j/j_5)$	$\mathbf{6}_L \times \overline{\mathbf{6}}_R$	$-g^a_{6\times \overline{6}} \frac{F_0^4}{4} (\Sigma)_2{}^1 (\Sigma)_3{}^1$
$\tilde{\mathcal{O}}^{RLRL,S/P}_{udus}$ (P)	$(\overline{u_R} d_L][\overline{u_R} s_L)(j/j_5)$	$\mathbf{6}_L \times \overline{\mathbf{6}}_R$	$-g^b_{6\times \overline{6}} \frac{F_0^4}{4} (\Sigma)_2{}^1 (\Sigma)_3{}^1$
$\mathcal{O}^{LRLL,A}_{udus}$ (✓)	$(\overline{u_L} d_R)[\overline{u_L}\gamma^\mu s_L] j_{\mu 5}$	$\overline{\mathbf{15}}_L \times \mathbf{3}_R$	$-g^a_{\overline{15}\times 3} \frac{F_0^4}{4} (\Sigma i\partial_\mu \Sigma^\dagger)_3{}^1 (\Sigma^\dagger)_2{}^1$
$\tilde{\mathcal{O}}^{LRLL,A}_{udus}$ (✓)	$(\overline{u_L} d_R][\overline{u_L}\gamma^\mu s_L) j_{\mu 5}$	$\overline{\mathbf{15}}_L \times \mathbf{3}_R$	$-g^b_{\overline{15}\times 3} \frac{F_0^4}{4} (\Sigma i\partial_\mu \Sigma^\dagger)_3{}^1 (\Sigma^\dagger)_2{}^1$
$\mathcal{O}^{LRLL,A}_{usud}$ (✓)	$(\overline{u_L} s_R)[\overline{u_L}\gamma^\mu d_L] j_{\mu 5}$	$\overline{\mathbf{15}}_L \times \mathbf{3}_R$	$-g^c_{\overline{15}\times 3} \frac{F_0^4}{4} (\Sigma i\partial_\mu \Sigma^\dagger)_2{}^1 (\Sigma^\dagger)_3{}^1$
$\tilde{\mathcal{O}}^{LRLL,A}_{usud}$ (✓)	$(\overline{u_L} s_R][\overline{u_L}\gamma^\mu d_L) j_{\mu 5}$	$\overline{\mathbf{15}}_L \times \mathbf{3}_R$	$-g^d_{\overline{15}\times 3} \frac{F_0^4}{4} (\Sigma i\partial_\mu \Sigma^\dagger)_2{}^1 (\Sigma^\dagger)_3{}^1$

⟵ Some examples (incomplete) of operator matching

$$\Sigma = \xi^2$$

F_0: decay constant in chiral limit
$g_{27\times 1}$ etc, are hadronic low-energy constants, which are obtained from data or lattice calc:

$$g_{27\times 1} = 0.38 \pm 0.08, \quad g^a_{8\times 8} = 5.5 \pm 2\,\mathrm{GeV}^2, \quad g^b_{8\times 8} = 1.55 \pm 0.65\,\mathrm{GeV}^2.$$

$K^- \to \pi^+ l^- l^-$: χPT

- Leading-order interactions in χPT:

$$\mathcal{L}_{K^-\to\pi^+l^-l^-} = \frac{1}{2}K^-\pi^-\left[c_1\left(\bar{l}l^C\right) + c_2\left(\bar{l}\gamma_5 l^C\right)\right] + \frac{1}{2}\left[c_3\partial^\mu K^-\pi^- + c_4\partial^\mu\pi^- K^-\right]\left(\bar{l}\gamma_\mu\gamma_5 l^C\right)$$
$$+\frac{1}{2}\partial^\mu K^-\partial_\mu\pi^-\left[c_5\left(\bar{l}l^C\right) + c_6\left(\bar{l}\gamma_5 l^C\right)\right],$$

where c_i are effective couplings, e.g.,

$$c_1 = g^a_{\bar{6}\times 6}F_0^2\left(C^{LRLR,S}_{udus} + C^{RLRL,S}_{udus}\right) + g^b_{\bar{6}\times 6}F_0^2\left(\tilde{C}^{LRLR,S}_{udus} + \tilde{C}^{RLRL,S}_{udus}\right)$$
$$+g^a_{8\times 8}F_0^2\left(C^{LRRL,S}_{udus} + C^{LRRL,S}_{usud}\right) + g^b_{8\times 8}F_0^2\left(\tilde{C}^{LRRL,S}_{udus} + \tilde{C}^{LRRL,S}_{usud}\right),$$
$$c_2 = g^a_{\bar{6}\times 6}F_0^2\left(C^{LRLR,P}_{udus} + C^{RLRL,P}_{udus}\right) + g^b_{\bar{6}\times 6}F_0^2\left(\tilde{C}^{LRLR,P}_{udus} + \tilde{C}^{RLRL,P}_{udus}\right)$$
$$+g^a_{8\times 8}F_0^2\left(C^{LRRL,P}_{udus} + C^{LRRL,P}_{usud}\right) + g^b_{8\times 8}F_0^2\left(\tilde{C}^{LRRL,P}_{udus} + \tilde{C}^{LRRL,P}_{usud}\right),$$

Reminder:

F_0 and $g_{..\times..}$ are hadronic low-energy constants.

$C^{LRLR,S}_{udus}$ etc are LEFT Wilson coefficients evaluated at $\mu = \Lambda_\chi$.

- Decay amplitude and width can be worked out.

$K^- \to \pi^+ l^- l^-$: SMEFT

- Since we want to relate the decay to the source of LNV at Λ_{NP}, we must match at $\mu = \Lambda_{\mathrm{EW}}$ LEFT (living in $\Lambda_\chi < \mu < \Lambda_{\mathrm{EW}}$) with SMEFT (in $\Lambda_{\mathrm{EW}} < \mu < \Lambda_{\mathrm{NP}}$), where Λ_{NP} is unknown before specifying NP.
- The SMEFT Lagrangian will be shown on next pages.
- It suffices here to say that at leading order dim-7 operators augmented with SM interactions contribute to the matching:

$$C_{udus}^{LLLL,S/P} = -2\sqrt{2}G_F V_{ud} V_{us} \left(C_{LHD1}^{ll\dagger} + 4 C_{LHW}^{ll\dagger} \right),$$

$$\tilde{C}_{udus}^{LRRL\ S/P} = -2\sqrt{2}G_F V_{us} C_{\bar{d}uLLD}^{11ll\dagger},$$

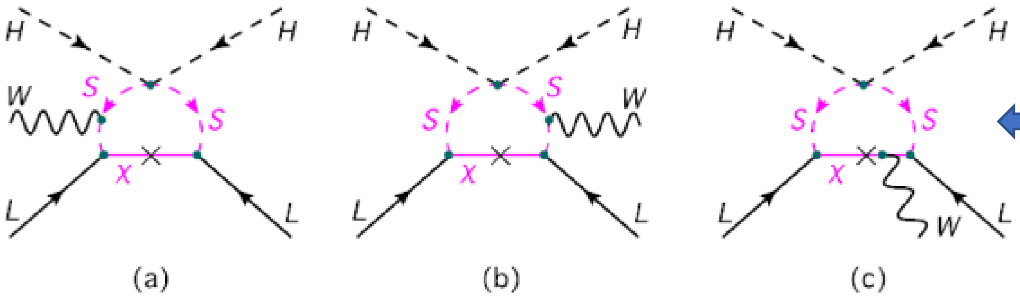
$$\tilde{C}_{usud}^{LRRL\ S/P} = -2\sqrt{2}G_F V_{ud} C_{\bar{d}uLLD}^{21ll\dagger},$$

LEFT at $\mu = \Lambda_{\mathrm{EW}}$ SMEFT

$\psi^2 H^4$		$\psi^2 H^3 D$	
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m)H^j H^n (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^i C \gamma_\mu e)H^j H^m i D^\mu H^n$
$\psi^2 H^2 D^2$		$\psi^2 H^2 X$	
$\mathcal{O}_{LHD1}$	$\epsilon_{ij}\epsilon_{mn}(L^i C D^\mu L^j)H^m (D_\mu H^n)$	$\mathcal{O}_{LHB}$	$g_1 \epsilon_{ij}\epsilon_{mn}(L^i C \sigma_{\mu\nu} L^m)H^j H^n B^{\mu\nu}$
$\mathcal{O}_{LHD2}$	$\epsilon_{im}\epsilon_{jn}(L^i C D^\mu L^j)H^m (D_\mu H^n)$	$\mathcal{O}_{LHW}$	$g_2 \epsilon_{ij}(\epsilon\tau^I)_{mn}(L^i C \sigma_{\mu\nu} L^m)H^j H^n W^{I\mu\nu}$
$\psi^4 D$		$\psi^4 H$	
$\mathcal{O}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C i D^\mu L^j)$	$\mathcal{O}_{\bar{e}LLLH}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j C L^m)H^n$
		$\mathcal{O}_{\bar{d}LQLH1}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m)H^n$
		$\mathcal{O}_{\bar{d}LQLH2}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m)H^n$
		$\mathcal{O}_{\bar{d}LueH}$	$\epsilon_{ij}(\bar{d}L^i)(uCe)H^j$
		$\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}(\bar{Q}u)(LCL^i)H^j$

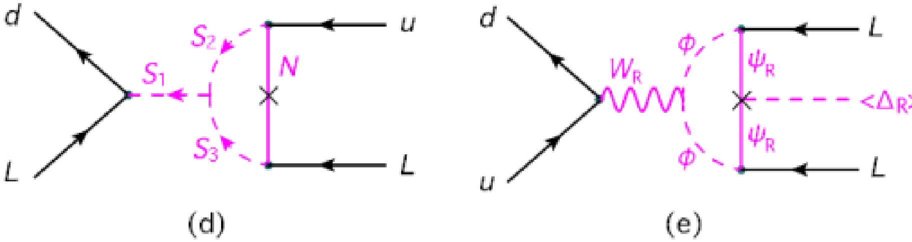
$K^- \to \pi^+ l^- l^-$: NP

- The above result for the decay based on the sequence of SMEFT-LEFT-χPT is universal, i.e., independent of NP details.
- Once NP is specified, we match it with SMEFT at $\mu = \Lambda_{\mathrm{NP}}$, so that the decay width is expressed in terms of NP parameters.
- Examples:



color-octet model for neutrino mass:
$S = (8, 2, 1/2)$, $\chi = (8, 3, 0)$
generating O_{LHW}

leptoquark model:
$S_1 = (3, 2, 1/6)$, $S_2 = (3, 1, 2/3)$,
$S_3 = (1, 2, -1/2)$, $N = (1, 1, 0)$
generating $O_{\bar{d}uLLD}$



Left-right model:
$SU_L(2) \times SU_R(2) \times U_{B-L}(1)$
$\psi_R = (N, e)$, Φ = bidoublet
Δ_R = triplet
generating $O_{\bar{d}uLLD}$

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Standard model EFT (SMEFT)

Defined between Λ_{NP} and Λ_{EW}:

- Dynamical degrees of freedom (DoFs) restricted to SM fields;
- Symmetries – $SU(3)_C \times SU(2)_L \times U(1)_Y$, no L or B conservation requirement etc;
- Power counting – expansion in p/Λ_{NP}.

SMEFT is an infinite tower of effective interactions involving higher and higher dimensional operators:

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \mathscr{L}_5 + \mathscr{L}_6 + \mathscr{L}_7 + \mathscr{L}_8 + \mathscr{L}_9 + \cdots$$

- $\mathscr{L}_{\mathrm{SM}}$: GSW 1960s
- $\mathscr{L}_5$: Weinberg 1979
- $\mathscr{L}_6$: Buchmuller-Wyler 1986; …… Grzadkowski et al 2010 – Warsaw basis
- $\mathscr{L}_7$: Lehman 2014; Liao-Ma 2016
- $\mathscr{L}_8$: Li et al, arXiv: 2005.00008; Murphy, arXiv: 2005.00059
- $\mathscr{L}_9$: Li et al, arXiv: 2007.07899; Liao-Ma, arXiv: 2007.08125

SMEFT: dim-5

- Unique Weinberg operator for Majorana m_ν, $\Delta L = 2$ Weinberg 1979

$$\varepsilon_{ij}\varepsilon_{mn}(L_p^i C L_r^m) H^j H^n$$

L : LH lepton doublet
H : Higgs doublet
i, j, m, n : SU(2) indices
p, r, s, t : flavor indices

- 1-loop RGE Babu et al 1993, Antusch et al 2001

- Responsible for "standard mass mechanism" for nuclear neutrinoless double beta ($0\nu\beta\beta$). … Cirigliano et al 2017, 2018
- No other interesting phys.

SMEFT: dim-6

- Long history on basis of operators.

  Started with Buchmüller-Wyler 1986,

  Corrected and improved by efforts by many groups,

  Culminated with Warsaw basis Grzadkowski et al 2010 –

- 63 operators $\begin{cases} 59:\ \Delta B = \Delta L = 0 \\ 4:\ \Delta B = \Delta L = 1 \end{cases}$

  without counting flavors (easy with trivial flavor relations) and Hermitian conjugate.

- 1-loop RGE by UC San Diego group in 2013, 2014 Barcelona group in 2013

- Rich phenomenology, especially for LHC phys, vast literature skipped

  Commonly quoted proton decay: $p \to e^{+}\pi^{0}$

SMEFT: dim-7

- Early partial analysis by Weinberg 1980 Weldon-Zee 1980
- 1st systematic analysis by Lehman 2014
- Final answer by Liao-Ma 2016:

  18 operators = $12\ (\Delta B = 0,\ \Delta L = 2) + 6\ (-\Delta B = \Delta L = 1)$

  Flavors not counted above; but must be done for applications –

  Nontrivial flavor relations first appear at dim 7 – involving Yukawas Liao-Ma 2019
- Consistent with independent counting by Hilbert series approach Henning et al 2015.
- 1-loop RGE Liao-Ma 2016 Liao-Ma 2019
- Phenomenology limited to L- (and B-) violating phys:

  unusual proton decay $p \to \nu\pi^+$ Liao-Ma 2016

  Liao et al, 2019,2020,2021

  various long- and short-range contri. to $0\nu\beta\beta$, $M_1^- \to M_2^+ l^- l^-$, $\tau^- \to l^+ M_1^- M_2^-$, etc

  ⋯ Cirigliano et al 2017, 2018, ⋯, Feng et al 2019

SMEFT: dim-8

- Many independent operators: Li et al, 2020; Murphy, 2020

  mostly conserve L and B, others break $\Delta B = \Delta L = 1$
- RGE done for purely bosonic operators: Chala et al, 2021; Bakshi et al, 2022
- Phenomenology partly explored, mainly with bosonic operators:

  electroweak precision data, triple gauge couplings, diboson production:

  Degrande and Li, 2023; Corbett et al, 2023

SMEFT: dim-9

- Basis of complete and independent operators established; 2 studies consistent

  Li et al, 2020; Liao-Ma, 2020

- Number of terms in $\mathscr{L}_9$: Number of operators with 3 generations:

	Number of terms in $\mathscr{L}_9$	Number of operators with 3 generations
$L = \pm 2,\ B = 0$:	384	44874
$L = 0,\quad B = \pm 2$:	10	2862
$L = \pm 3,\ B = \pm 1$:	4	486
$L = \mp 1\quad B = \pm 1$:	236	42234

  most violate both $L \pm B$ except for the last group which conserves $L + B$.

- Renormalization to be finished
- Phenomenology partly done:

  nuclear $0\nu\beta\beta$ decays, neutron-antineutron oscillation, rare nucleon decays

SMEFT: higher-dim operators less important?

- Generally yes, barring one caveat.
- L- or B-violating effects are much smaller than conserving effects

  → L or B violation should originate at a higher scale

  → Wilson coeffs. for operators of different L or B patterns cannot be compared in a model-independent manner.
- General results on L or B pattern in SMEFT:

Kobach, 2016

✓ $(\Delta B - \Delta L)/2$ and dimension d of an operator share the same odd or even nature.

✓ Imposing flavor symmetry postpones occurrence of L or B violation to a higher d:

L or B violation impossible for $d < 9$ except for $|\Delta L| = 2$;

Helset and Kobach, 2019

as a consequence, e.g., proton decay severely suppressed:

$d = 9$: 2 operators involve $3l3q$ but necessarily with c or t → tree level impossible

$d = 10$: 4-body decay with $\Delta B = -\frac{\Delta L}{3} = 1$; $d = 11$: 3-body decay with $\Delta B = \frac{\Delta L}{3} = 1$

Low-energy EFT

When $E < \Lambda_{\rm EW}$, electroweak SSB manifests itself.

Heavy particles $(h, W^{\pm}, Z^0, t)$ of mass $\sim\Lambda_{\rm EW}$ are integrated out $\rightarrow$ LEFT

Defined between $\Lambda_{\rm EW}$ and $\Lambda_{\chi} \sim 1\ {\rm GeV}$:

- Dynamical DoFs = SM fields other than above heavy ones;
- Symmetries – $SU(3)_C \times U(1)_Q$;
- Power counting – expansion in $p/\Lambda_{\rm EW}$.

Actually well applied in the past, e.g., in b phys, although not studied systematically.

Jenkins et al, 2017 ($\mathscr{L}_5$, $\mathscr{L}_6$)

Murphy, 2020 ($\mathscr{L}_8$)

$|\Delta L| = 2$ sector: Liao et al, 2019 ($\mathscr{L}_9$)

$$\mathscr{L}_{\rm LEFT} = \mathscr{L}_\nu + \mathscr{L}_{\rm QED} + \mathscr{L}_{\rm QCD} + \mathscr{L}_5 + \mathscr{L}_6 + \mathscr{L}_7 + \mathscr{L}_8 + \mathscr{L}_9 + \cdots$$

Liao et al, 2020 ($\mathscr{L}_7$)

Li et al, 2020 ($\mathscr{L}_8$, $\mathscr{L}_9$)

Attention: combined power counting in $1/\Lambda_{\rm EW}$ and $1/\Lambda_{\rm NP}$

LEFT: RGE and matching to SMEFT

To get prepared for analysis of precision measurements at low energy,
both RGE in LEFT and matching between LEFT and SMEFT are demanded.

- tree-level up to dim-6 operators in both EFTs — Jenkins et al, 1709.04486
- tree-level up to dim-7 operators in both EFTs — Liao et al, 2005.08013
- tree-level up to dim-8 operators in both EFTs: partly done, — Hamoudou et al, 2207.08856
  by either setting $H \to \mathrm{vev}$ or integrating out $h, W^{\pm}, Z$ and keeping p-indept terms
- one-loop up to dim-6 operators in both EFTs — Dekens and Stoffer, 1908.05295
  delicacy appears with evanescent operators in DR

- one-loop RGE for dim-6 operators — Jenkins et al, 1711.05270
- one-loop QCD RGE for dim-9 $|\Delta L| = 2$ operators involving $2l$ — Liao et al, 1909.06272
  for dim-9 $|\Delta L| = 2$ operators specific to $0\nu\beta\beta$ — Cirigliano et al, 1806.02780
  QCD RGE for dim-9 operators in $n\bar{n}$ oscillation: one-loop — Caswell et al, PLB122
  two-loop — Buchoff and Wagman, 1506.00647

Matching NP to SMEFT

- EFT is useful not only for bottom-up but also for top-down approach.
- Assuming NP lives at $\Lambda_{\mathrm{NP}} \gg \Lambda_{\mathrm{EW}}$ and all new particles have mass $\gg \Lambda_{\mathrm{EW}}$, its low-energy effects on SM particles can be incorporated by integrating out new particles
  — matching NP and SMEFT at $\mu = \Lambda_{\mathrm{NP}}$
- Matching in perturbation theory is a double-expansion:

✓ in inverse powers of heavy mass → higher-dim operators in SMEFT

✓ in loop expansion → Wilson coeffi., a series in couplings

- Matching at tree level:
  substituting in L_{NP} EoMs for heavy particles and expanding in inverse masses
  → tree level Wilson coeffi.

Matching NP to SMEFT at one loop

- Past years have witnessed significant progress in 1-loop matching based on:

✓ Functional approach augmented by covariant derivative expansion

✓ Loop integration by method of regions ……; Cohen-Lu-Zhang, 2011.02484

- Features:

✓ The result is directly the 1-loop contribution to $\mathcal{L}_{\rm SMEFT}$ whose operators and Wilson coeffs. are obtained simultaneously.

✓ One only has to work with NP theory without computing in SMEFT!

Examples of 1-loop functional matching

Obtain 1-loop contribution to $\mathcal{L}_{\mathrm{SMEFT}}$ by integrating out heavy

- ✓ superpartners in MSSM — Henning et al, 2014; ⋯
- ✓ singlet or triplet scalar — ⋯; Jiang et al, 2018; ⋯ ⋯; Zhang, 1610.00710
- ✓ vectorlike fermions — Huo, 1506.00840
- ✓ triplet vector boson — Brivio et al, 2108.01094
- ✓ fermions or scalars in type-I, -II, and –III neutrino seesaw models — Zhang-Zhou, 2107.12133; Du et al, 2201.04646; Li et al, 2201.05082
- ✓ dark-sector particles in scotogenic neutrino mass models — Liao-Ma, 2210.04270
- ✓ ……

Matching NP to SMEFT?

Issues:

✓ Is *the Higgs* completely responsible for electroweak SSB?

✓ Do new heavy particles gain mass completely from electroweak SSB?

They concern decoupling/nondecoupling of heavy particles and relation of *the Higgs* with would-be Nambu-Golstone bosons →

SMEFT or Higgs EFT (HEFT)?

Here I discuss briefly one example:

EFT of 2HDM   Banta et al, 2304.09884

on how to integrate out heavy particles to achieve SMEFT with better convergence.

Matching NP to SMEFT?

With 2 Higgs doublets of identical hypercharge, there is a flavor SU(2) sym mixing them.

Under flavor SU(2), scalar and Yukawa couplings rearrange themselves.

Higgs basis: only one doublet develops vev

Straight-line basis: leading order solution to classical EoM for heavy Φ_2

$\Phi_2 = k\Phi_1$ (light), with $k = v_2/v_1$

A tree-level EFT for $\Phi_1 \approx H$ is developed, which

✓ preserves $SU(3)_C \times SU(2)_L \times U(1)_Y$ (SMEFT-like),

instead of $SU(3)_C \times U(1)_Q$ (HEFT-like)

✓ expands in effective heavy mass containing $H^\dagger H$, i.e., resums vev

They found this EFT reproduces 2HDM, i.e., converges, much better than that employing Higgs basis.

Some of aspects not covered here

- In the existence of new particles of mass $< \Lambda_{\rm EW}$, SMEFT/LEFT has to be enlarged to include them as dynamical DoFs:

✓ νSMEFT, with sterile neutrinos; 1612.04527

✓ DM EFT, including axion-like particles or particles of various spin, with or without DM discrete symmetry. 2309.12166

- Higgs EFT vs SMEFT:
  Is *the Higgs boson* completely responsible for electroweak SSB? 2008.08597
  Do new particles gain mass from electroweak SSB?
- Various extensions of Hilbert series to count operators in theory with nonlinearly realized symmetry, with supersymmetry, with definite CP, etc.
- Evanescent operators in operator reduction and matching at one loop, and in RGE at two loops. 2211.09144

Some of recent development: RGE-1

- RGEs of the LEFT at two loops: d-6 BNV operators, 2505.03871
- Two-loop renormalization of quark and gluon fields in SMEFT, 2503.01954
- Anomalous Dimension of a General Effective Gauge Theory I: Bosonic Sector, 2502.14030
- Renormalization of general EFTs: Formalism and renormalization of bosonic operators, 2501.13185
- RGEs of the LEFT at two loops: d-5 effects, 2412.13251
- Two-loop running in the bosonic SMEFT using functional methods, 2410.07320
- Renormalization of the SMEFT to d-8: Fermionic interactions I, 2409.15408
- RG running of d-8 four-fermion operators in the SMEFT, 2408.15378
- Two-loop running effects in Higgs physics in SMEFT, 2408.03252

Some of recent development: RGE-2

- Leading directions in the SMEFT : Renormalization effects, 2312.09179
- LEFT below the electroweak scale: one-loop renormalization in the ’t Hooft-Veltman scheme, 2310.13051
- Positivity restrictions on the mixing of d-8 SMEFT operators, 2309.16611

Some of recent development: Matching-1

- SUSY meets SMEFT: Complete one-loop matching of general MSSM, 2506.05201
- EFT for type II seesaw model — symmetric phase v.s. broken phase —, 2504.02580
- From the EFT to the UV: the complete SMEFT one-loop dictionary, 2412.14253
- UV completion of neutral triple gauge couplings, 2408.12508
- SMEFT matching to Z' models at dimension eight, 2404.01375
- Froggatt-Nielsen Meets the SMEFT, 2402.16940
- Fermionic UV models for neutral triple gauge boson vertices, 2402.04306
- Relevance of one-loop SMEFT matching in the 2HDM, 2401.12279
- Complete UV resonances of the dimension-8 SMEFT operators, 2309.15933
- One-loop matching of the type-III seesaw model onto SMEFT, 2309.14702
- Complete tree-level dictionary between simplified BSM models and SMEFT d<=7 operators, 2307.10380
- Matching the 2HDM to the HEFT and the SMEFT: Decoupling and perturbativity, 2305.07689

Some of recent development: Matching-2

- Automation of a Matching On-Shell Calculator, 2505.21353
- A Guide to Functional Methods Beyond One-Loop Order, 2412.12270
- Efficient on-shell matching, 2411.12798
- One-loop Matching and Running via On-shell Amplitudes, 2309.10851
- Functional matching and renormalization group equations at two-loop order, 2311.13630
- EFT matching from analyticity and unitarity, 2308.00035
- A proof of concept for matchete: an automated tool for matching effective theories, 2212.04510
- Matchmakereft: automated tree-level and one-loop matching, 2112.10787

Some of recent development: Pheno

- Constraining new physics effective interactions via a global fit of ⋯ observables, 2507.06191
- Constraining four-heavy-quark operators with top-quark, ⋯ precision data, 2507.01137
- Top EFT summary plots May 2025, ATL-PHYS-PUB-2025-028 🍎
- Analytic results for electroweak precision observables at NLO in SMEFT, 2503.07724
- Constraining the SMEFT Extended with Sterile Neutrinos at FCC-ee. 2502.06972
- Global analysis of $\mu \to e$ interactions in the SMEFT, 2411.13497
- Improving the global SMEFT picture with bounds on neutrino NSI, 2411.00090
- Energy-enhanced dimension eight SMEFT effects in VBF Higgs production, 2410.21563
- $e^- + e^+ \to Z + H$ process in the SMEFT beyond leading order, 2409.11466
- Probing dimension-8 SMEFT operators through neutral meson mixing, 2409.10305
- Mapping SMEFT at high-energy colliders: from LEP and (HL-)LHC to FCC-ee, 2404.12809

Some of recent development: Other aspects

- Renormalizing Two-Fermion Operators in SMEFT via Supergeometry, 2504.18537
- Accidental symmetries, Hilbert series, and friends, 2412.05359
- Field redefinitions in classical field theory with some quantum perspectives, 2408.03369
- Understanding the SM gauge group from SMEFT, 2404.04229
- Fermion geometry and the renormalization of SMEFT, 2307.03187
- Opportunistic CP violation, 2302.07288
- Constraints on anomalous dimensions from positivity of the S matrix, 2301.09995