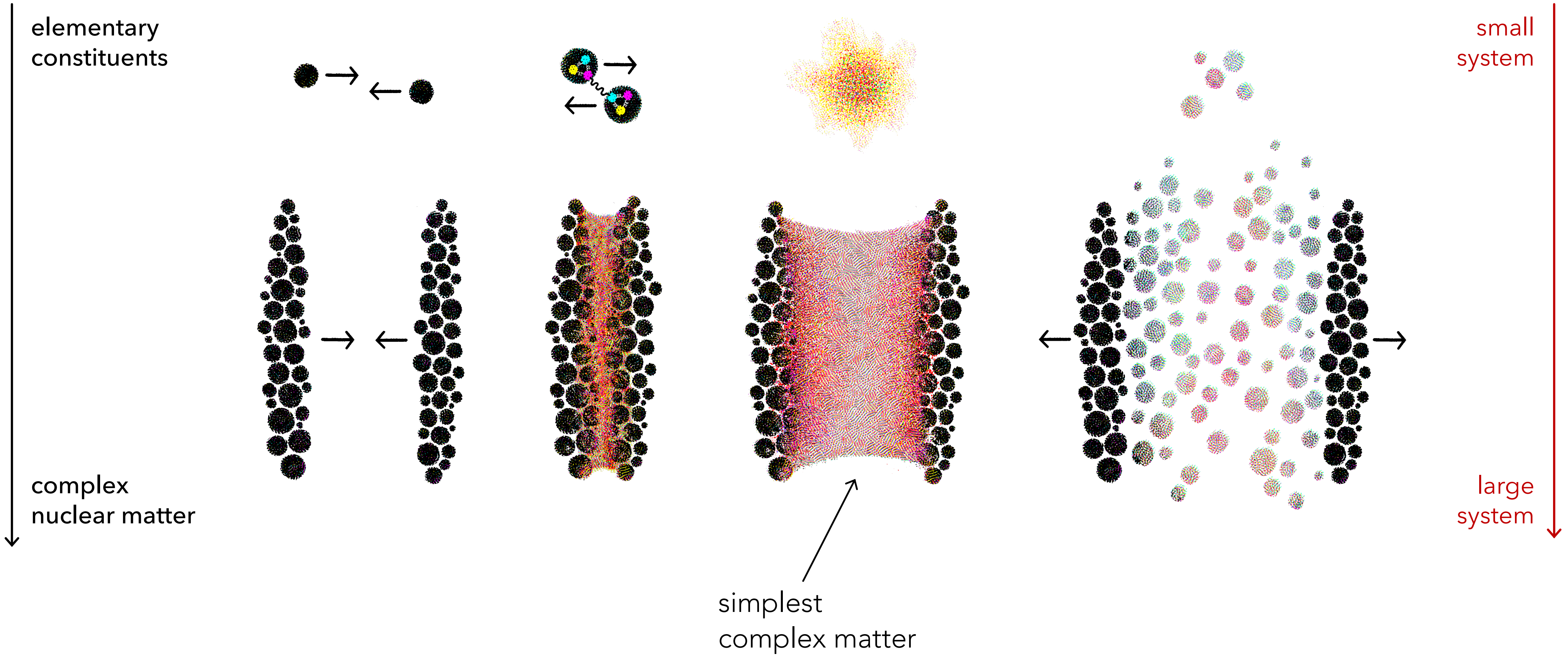
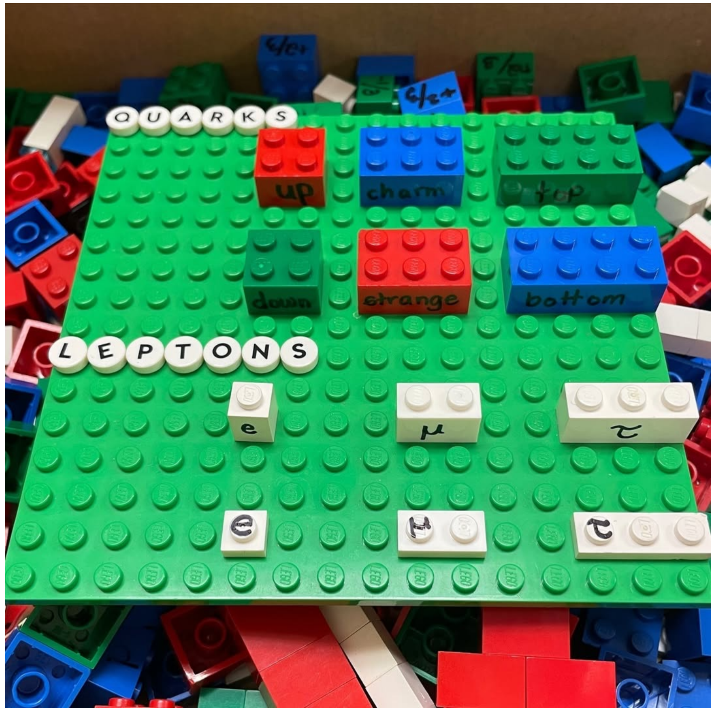
Hard probes of nuclear matter and Jet tomography

for 微扰量子场论及其应用前沿讲习班

Andrey Sadofyev
LIP, Lisbon

The origin of complex matter

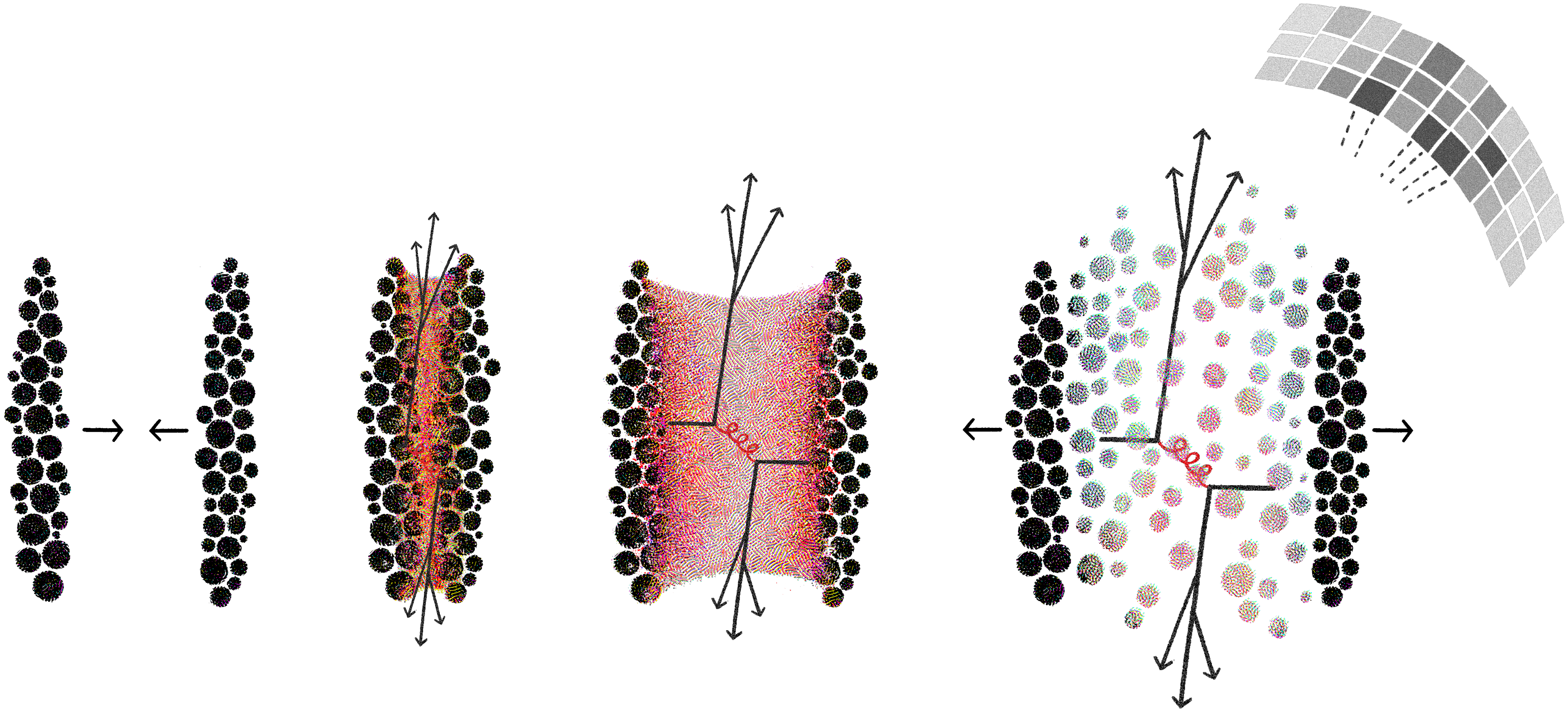



QUARKS
up
charm
top
down
strange
bottom
LEPTONS
e
μ
τ

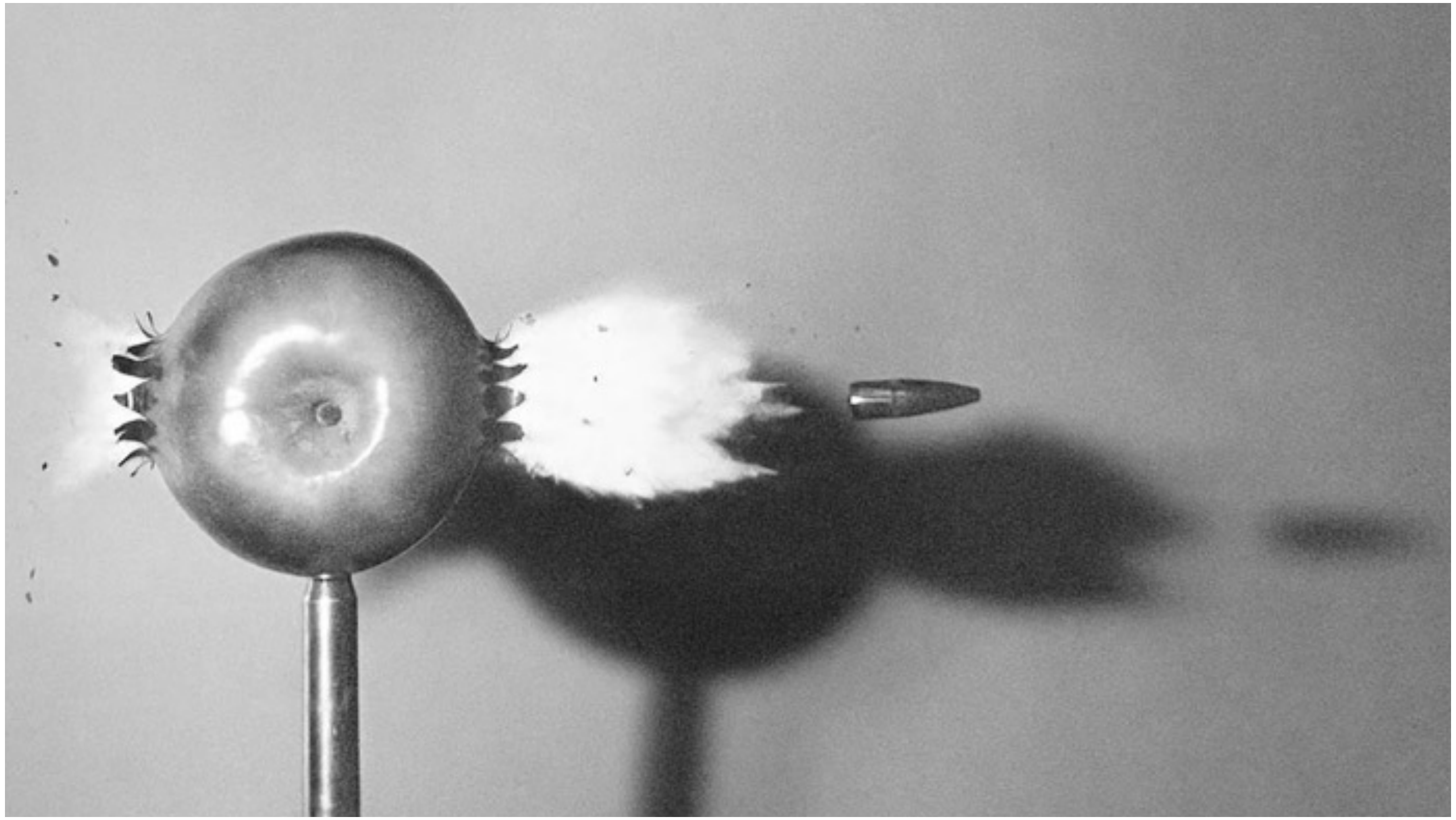
Some motivation

- Learning about QCD in Early Universe ("Big Bang" matter)
- Probing QCD phase diagram (extreme conditions)
- Probing nuclear structure (probing partons)
- Understanding matter formation (and its evolution)
- …
- Add your favorite option here

Hard probes



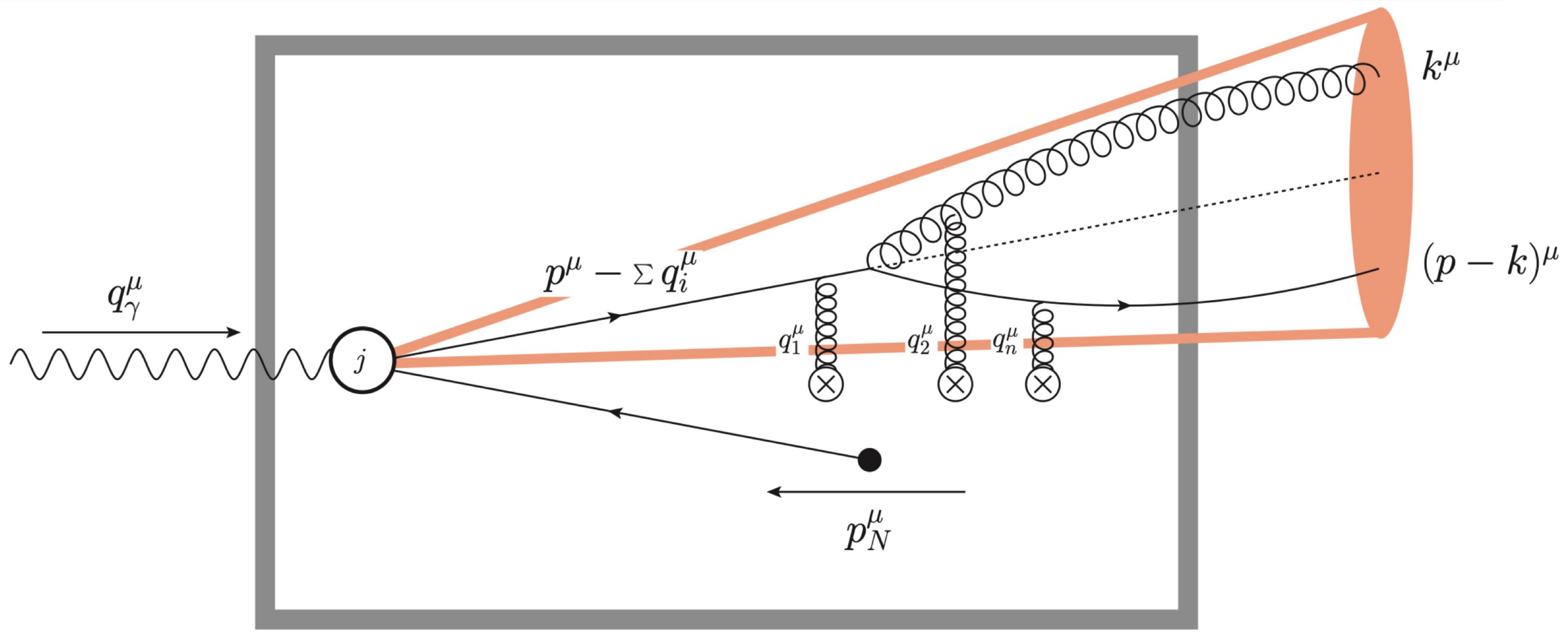
Hard probes




Final render
Ex: 75306
Se 1/1
Im 1/2
Lin: DCM
W: 4095 L: 2089
Acq tm: 09:52:07

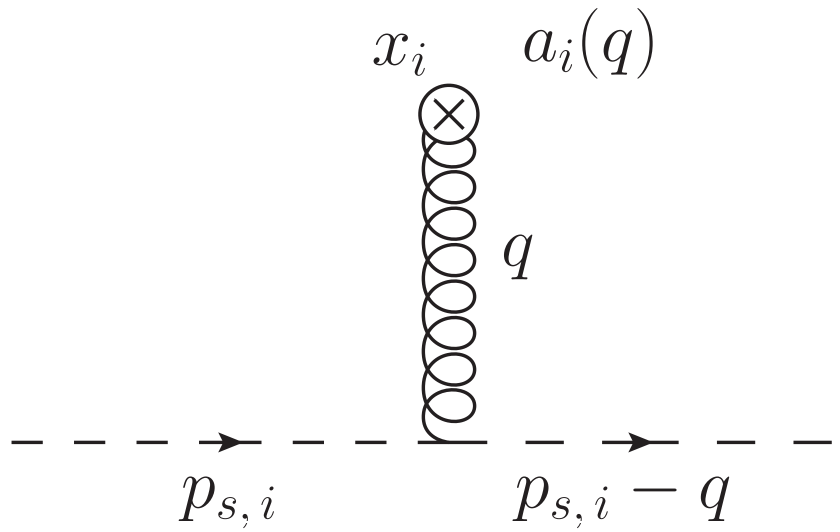
Jet quenching formalisms

R. Baier et al, NPB, 1997
B. G. Zakharov, JETP, 1997
R. Baier et al, NPB, 1998
M. Gyulassy et al, NPB, 2000
X.-F. Guo, X.-N. Wang, PRL, 2000
U. Wiedemann, NPB, 2000
M. Gyulassy et al, NPB, 2001
P. Arnold, G. Moore, L. Yaffe, JHEP, 2002
C. Salgado, U. Wiedemann, PRD, 2003



the idea goes back to Landau and Pomeranchuk (1953) and Migdal (1956)

Jet quenching formalisms

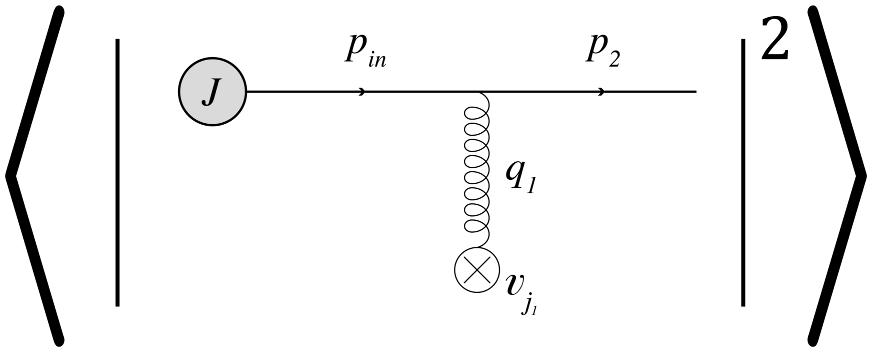


$$gA^{\mu a}_{ext}(q) = \sum_{i} e^{iq\cdot x_i}\, t^a_i\, u^\mu\, v(q)\, (2\pi)\, \delta\left(q\cdot u\right)$$

(1,0,0,0)

color sources

Jet quenching formalisms



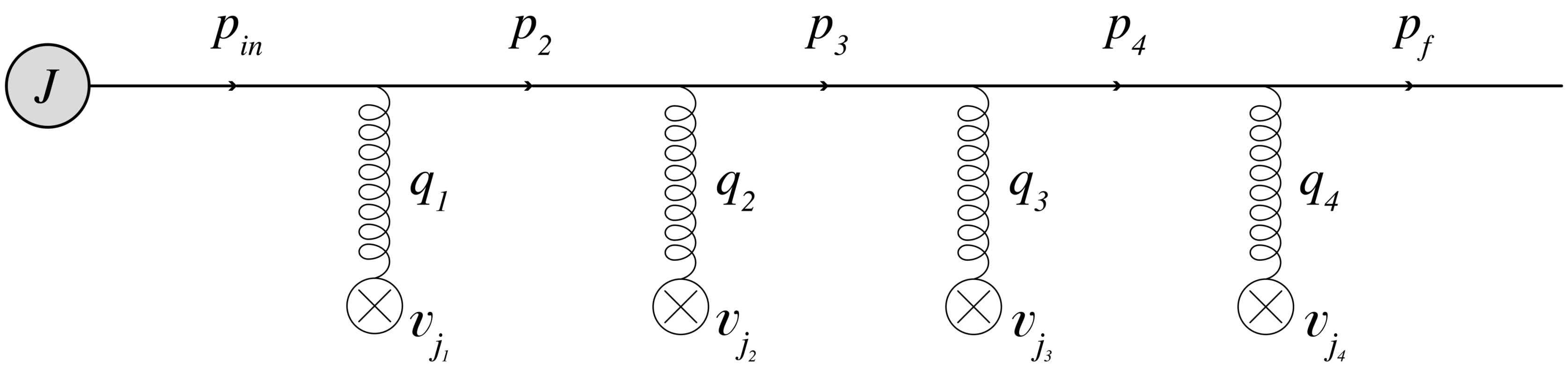
$$\langle t_i^a t_j^b \rangle = \mathcal{C}\delta_{ij}\delta^{ab}$$

color neutrality

$$\sum_i = \int d^3x\, \rho(\vec{x})$$

source averaging

Resummation (BDMPS-Z)



$$iM(p) = \int \frac{d^2\mathbf{p}_{in}}{(2\pi)^2} e^{i\frac{\mathbf{p}_f^2}{2E}L}\, G(\mathbf{p}_f, L; \mathbf{p}_{in}, 0)\, J\,(E, \mathbf{p}_{in})$$

$$\partial_L G(\mathbf{p}, L) = -i\frac{\mathbf{p}^2}{2E} G(\mathbf{p}, L) + i\int_{\mathbf{q}} t^a \hat{\rho}^a(\mathbf{q}, L)\, v_q\, G(\mathbf{p}-\mathbf{q}, L)$$

emergent Schrödinger equation

Resummation (BDMPS-Z)

$$W = \left\langle \mathcal{G}^{\dagger}(\bar{\mathbf{k}}, L+\epsilon; \bar{\mathbf{k}}_0, 0)\, \mathcal{G}(\mathbf{k}, L+\epsilon; \mathbf{k}_0, 0) \right\rangle$$

$$\downarrow$$

$$\partial_L W(\mathbf{p}, \mathbf{Y}) = -\frac{\mathbf{p}\cdot\boldsymbol{\nabla}_Y}{E} W(\mathbf{p}, \mathbf{Y}) - \int_{\mathbf{q}} \mathcal{V}(\mathbf{q}) W(\mathbf{p}-\mathbf{q}, \mathbf{Y})$$

emergent Boltzmann equation

$$\downarrow$$

$$\langle \mathbf{p}^2 \rangle = \int_{\mathbf{p},\mathbf{Y}} \mathbf{p}^2\, W(\mathbf{p}, \mathbf{Y}) = L \int_{\mathbf{p}} \mathbf{p}^2 \mathcal{V}(\mathbf{p}) = \boxed{\frac{\mathcal{C} g^4 \rho}{4\pi} L \log \frac{E}{\mu}}$$

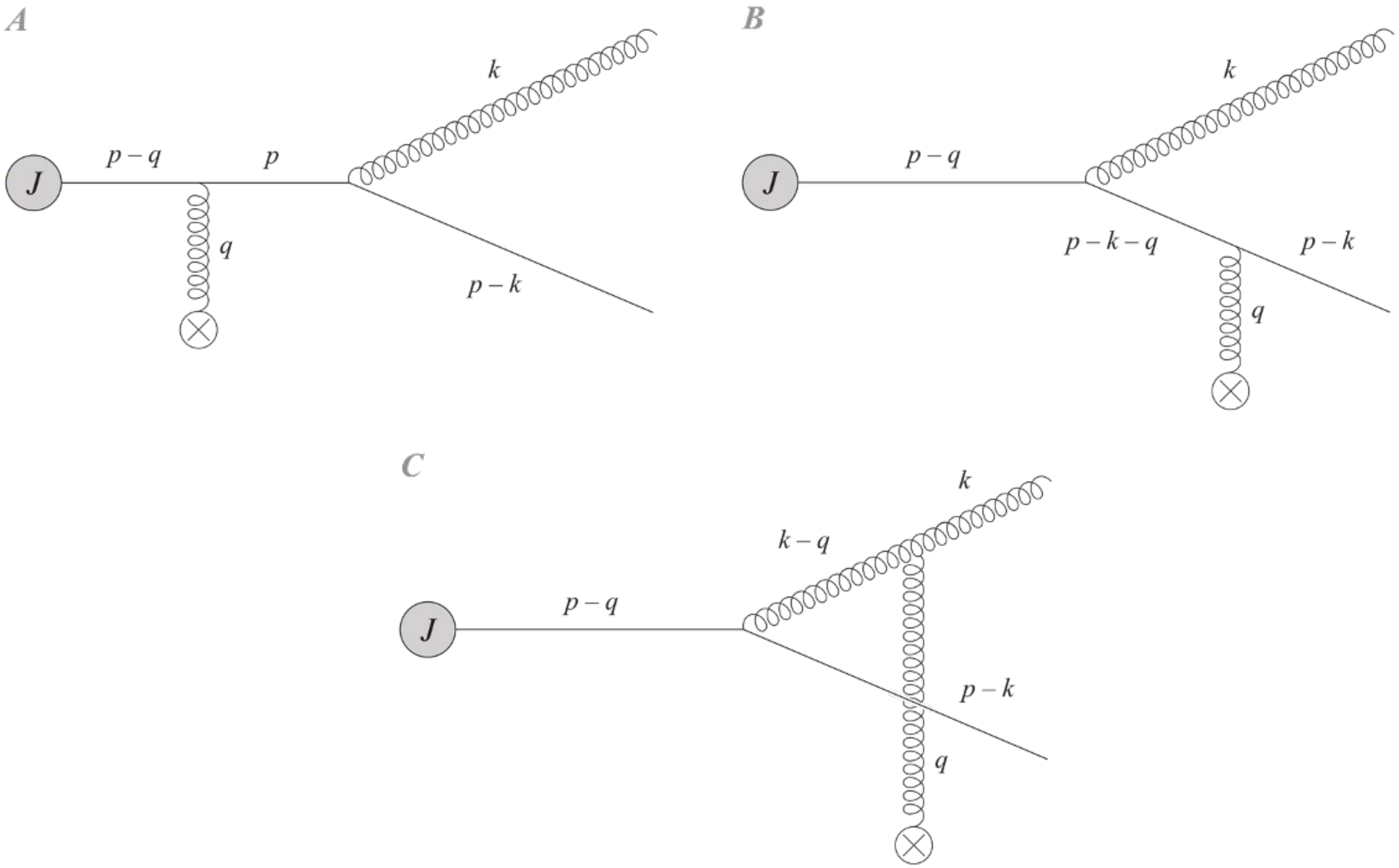
$\hat{q} L$

LABORATÓRIO DE INSTRUMENTAÇÃO
E FÍSICA EXPERIMENTAL DE PARTÍCULAS

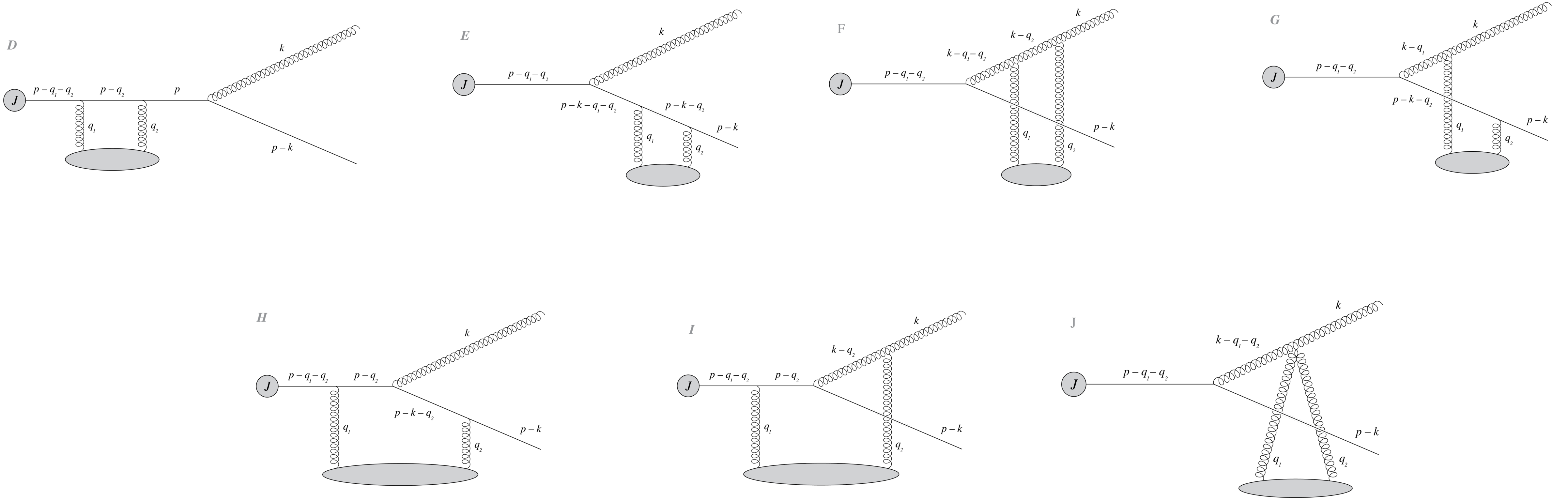
Some questions

- Where is the energy loss?
- What about the other phases of matter in HIC? For which phases that description works at all?
- Do we really need all these approximations? Do we learn anything in such an oversimplified setup?

Gluon emission

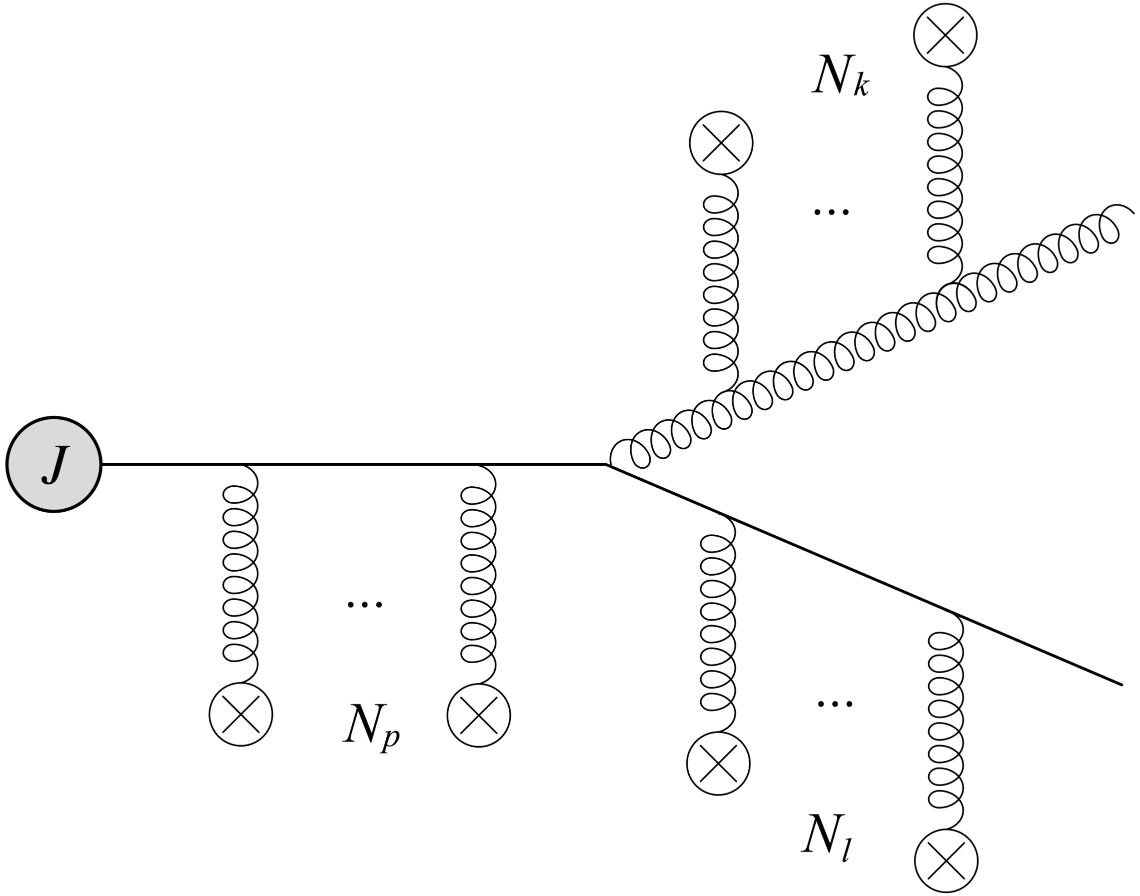


Gluon emission



J. Casalderrey-Solana, C. Salgado, lectures (Acta Phys.Polon.B), 2007

Gluon emission



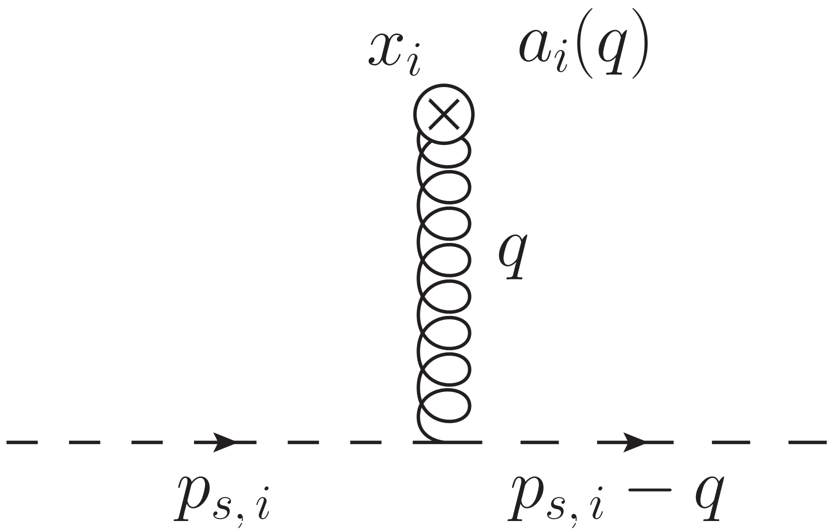
$$k \cdot \epsilon^*(k) = u \cdot \epsilon^*(k) = 0$$

$$k \simeq \left(xE, \mathbf{k}, xE - \frac{\mathbf{k}^2}{2xE} \right)$$

$$p - k \simeq \left((1-x)E, \mathbf{k}, (1-x)E - \cancel{\frac{\mathbf{k}^2}{2(1-x)E}} \right)$$

$$\frac{\mathbf{k}^2}{2xE} L \sim 1 \qquad \frac{\mathbf{k}^2}{2E} L \to 0 \qquad m_D \Delta z \gg 1$$

Jet quenching formalisms

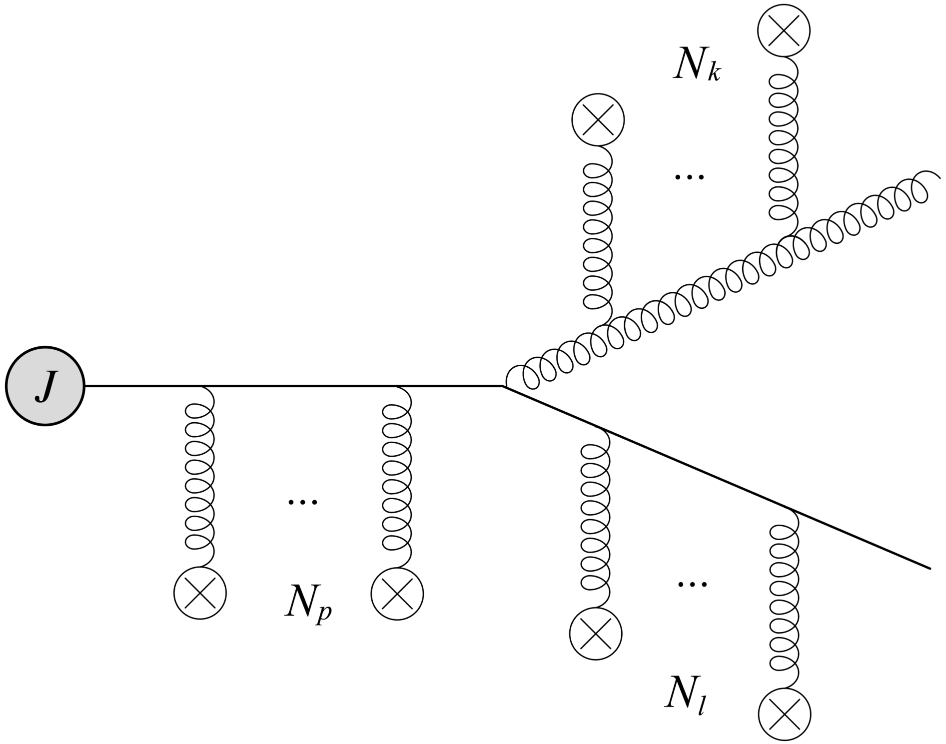


$$gA^{\mu a}_{ext}(q) = \sum_{\boxed{i}} e^{iq\cdot x_i}\, t^a_i\, \boxed{u^\mu}\, v(q)\, (2\pi)\, \delta\left(q\cdot u\right)$$

(1,0,0,-1)

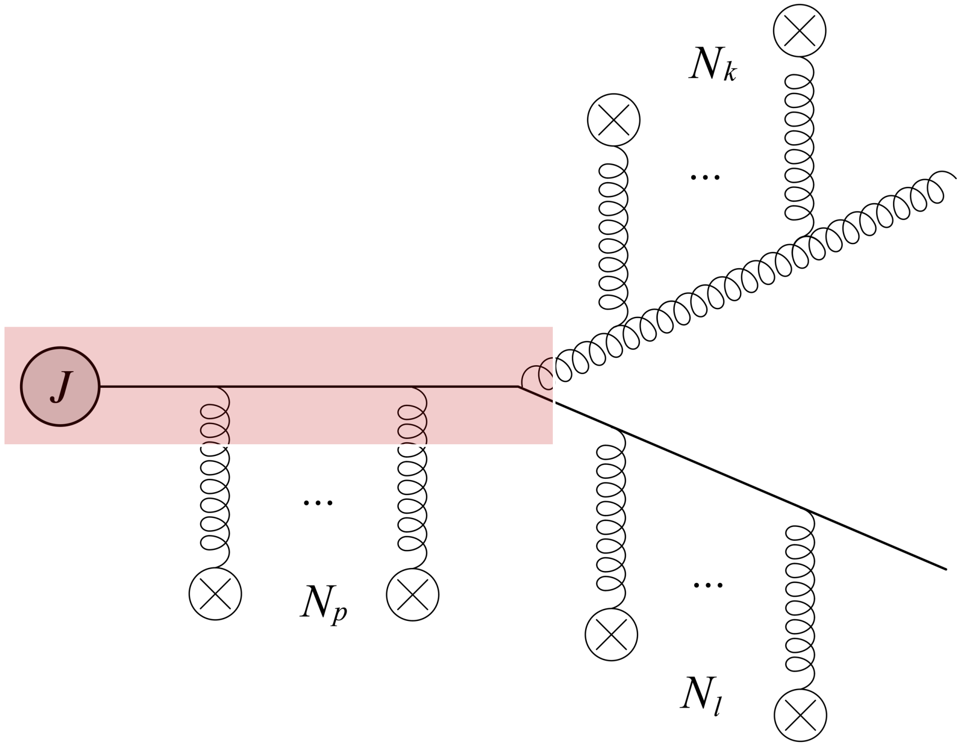
color sources

Gluon emission



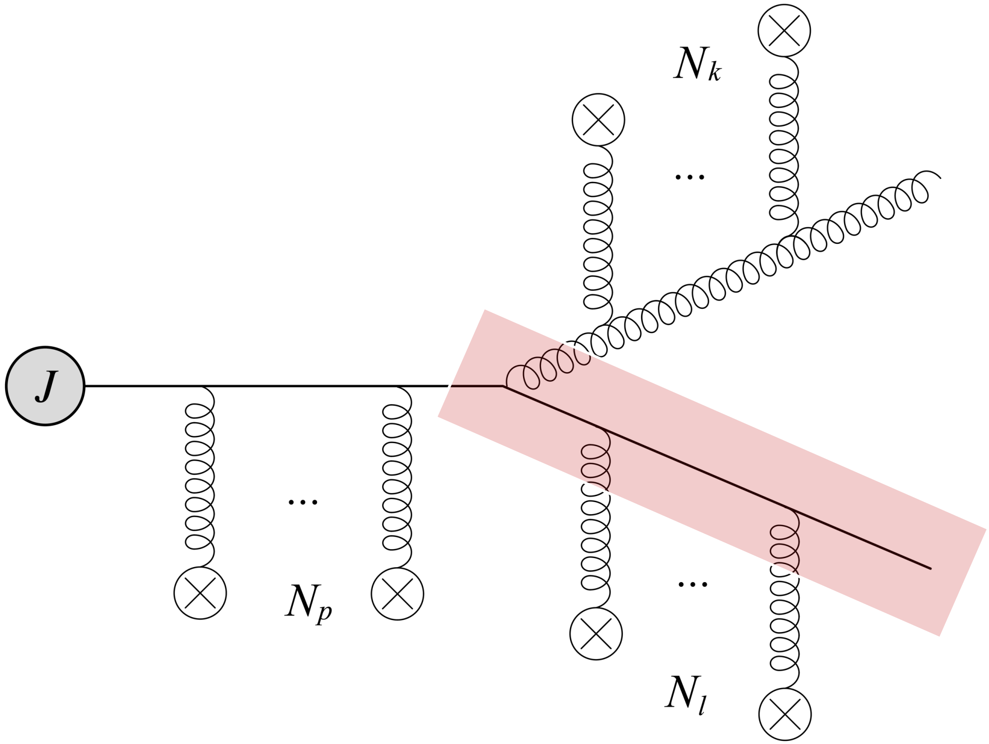
$$\prod_{n=1}^{N_p}\left[(-1)\int_{p_n} t^{a_n}_{proj} v^{a_n}(p_{n+1}-p_n)\frac{(p_{n+1}+p_n)\cdot u}{p_n^2+i\epsilon}\right]$$
$$\times\int\frac{d^4p_s}{(2\pi)^4}\,(-1)\,g t^{b_1}_{proj}\,\frac{(p_s+l_1)_{\mu_1}}{p_s^2+i\epsilon}\,(2\pi)^4\delta^{(4)}(p_s-l_1-k_1)J(p_1)$$
$$\times\prod_{r=1}^{N_k}\left[\left(-\frac{i}{g}\right)\int_{k_r}\frac{N^{\mu_r\nu_r}_{k_r}}{k_r^2+i\epsilon}\Gamma^{b_r b_{r+1} c_r}_{\nu_r\mu_{r+1}\alpha_r}(k_r,-k_{r+1})u^{\alpha_r}\,v^{c_r}(k_{r+1}-k_r)\right]$$
$$\times\prod_{m=1}^{N_l}\left[(-1)\int_{l_m} t^{d_m}_{proj} v^{d_m}(l_{m+1}-l_m)\frac{(l_{m+1}+l_m)\cdot u}{l_m^2+i\epsilon}\right]\epsilon_k^{*\mu_{N_k+1}}$$

Gluon emission



$$
\begin{aligned}
&\prod_{n=1}^{N_p}\left[i\int_{\mathbf{p}_n,x_n} t^{a_n}_{proj}\hat{\rho}^{a_n}(\mathbf{x}_n,z_n)\,v(\mathbf{p}_{n+1}-\mathbf{p}_n)\theta_{n,n-1}\,e^{-i\mathbf{x}_n\cdot(\mathbf{p}_{n+1}-\mathbf{p}_n)}e^{-iz_n(\tilde{p}_{n+1,z}-\tilde{p}_{nz})}\right]J(\tilde{p}_1)\\
&=\prod_{n=1}^{N_p}\left[i\int_{\mathbf{p}_n,x_n} t^{a_n}_{proj}\hat{\rho}^{a_n}(\mathbf{x}_n,z_n)\,v(\mathbf{p}_{n+1}-\mathbf{p}_n)\theta_{n,n-1}\,e^{-i\mathbf{x}_n\cdot(\mathbf{p}_{n+1}-\mathbf{p}_n)}\right]J(\tilde{p}_1)\\
&=\int_{\mathbf{p}_1,\mathbf{x}_1}\prod_{n=1}^{N_p}\left[i\int_{z_n} t^{a_n}_{proj}v^{a_n}\left(\mathbf{x}_1,z_n\right)\theta_{n,n-1}\right]J(\tilde{p}_1)e^{-i\mathbf{x}_1\cdot(\mathbf{p}_s-\mathbf{p}_1)}\\
&\Rightarrow\int_{\mathbf{p}_1,\mathbf{x}_1}\mathcal{P}\exp\left[i\int_0^{z_s}d\tau t^{a}_{proj}v^{a}\left(\mathbf{x}_1,\tau\right)\right]J(\tilde{p}_1)e^{-i\mathbf{x}_1\cdot(\mathbf{p}_s-\mathbf{p}_1)}\\
&=\int_{\mathbf{p}_1,\mathbf{x}_1}\tilde{\mathcal{W}}_{in}(\mathbf{x}_1;z_s,0)J(\tilde{p}_1)e^{-i\vec{x}_1\cdot(\mathbf{p}_s-\mathbf{p}_1)}\,,
\end{aligned}
$$

Gluon emission



$$
\prod_{m=1}^{N_l}\left[i\int_{\mathbf{l}_m,x_m} t^d_{proj}\hat{\rho}^d(\mathbf{x}_m,z_m)v(l_{m+1}-l_m)\theta_{l,l-1}e^{-i\mathbf{x}_m\cdot(\mathbf{l}_{m+1}-\mathbf{l}_m)}e^{-iz_m(\tilde{l}_{m+1,z}-\tilde{l}_{m,z})}\right]
$$

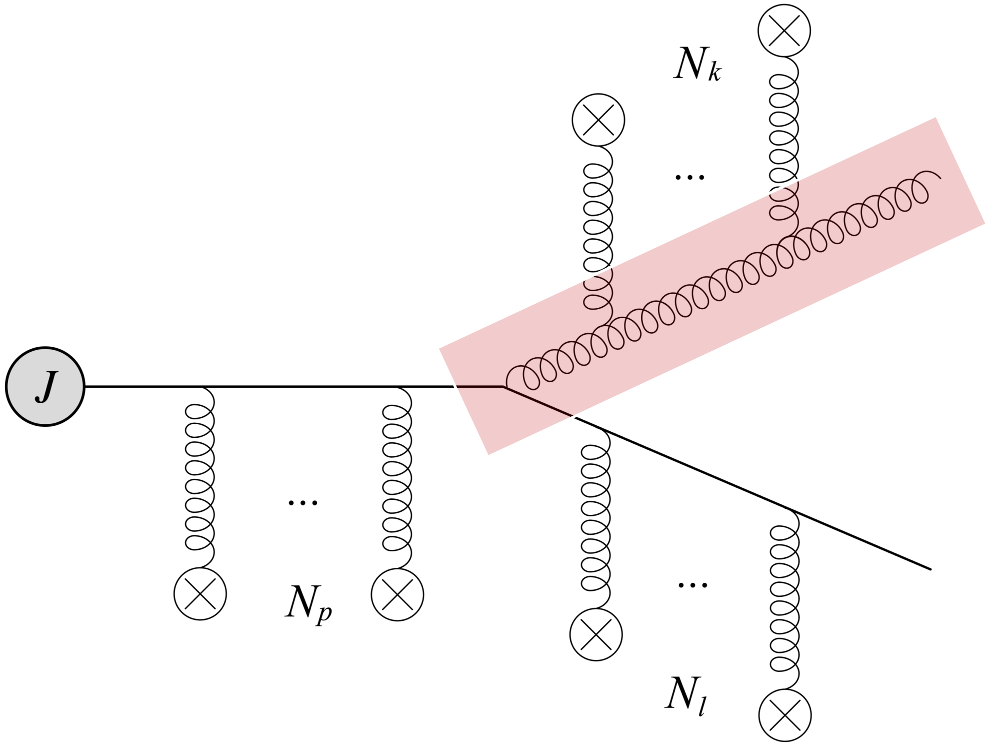
$$
=\prod_{m=1}^{N_l}\left[i\int_{\mathbf{l}_m,x_m} t^d_{proj}\hat{\rho}^d(\mathbf{x}_m,z_m)v(l_{m+1}-l_m)\theta_{l,l-1}e^{-i\mathbf{x}_m\cdot(\mathbf{l}_{m+1}-\mathbf{l}_m)}\right]
$$

$$
=\int_{\mathbf{l}_1,\mathbf{x}_1}\prod_{m=1}^{N_l}\left[i\int_{z_m} t^d_{proj}v^d\left(\mathbf{x}_1,z_m\right)\theta_{l,l-1}\right]e^{-i\mathbf{x}_1\cdot(\mathbf{l}-\mathbf{l}_1)}
$$

$$
\Rightarrow\int_{\mathbf{l}_1,\mathbf{x}_1}\mathcal{P}\exp\left[i\int_{z_s}^{\infty}d\tau\, t^d_{proj}v^d\left(\mathbf{x}_1,\tau\right)\right]e^{-i\mathbf{x}_1\cdot(\mathbf{l}-\mathbf{l}_1)}
$$

$$
=\int_{\mathbf{l}_1,\mathbf{y}_1}\tilde{\mathcal{W}}_f(\mathbf{y}_1;\infty,z_s)e^{-i\mathbf{y}_1\cdot(\mathbf{l}-\mathbf{l}_1)}
$$

Gluon emission



$$\prod_{r=1}^{N_k}\left[N_{k_r}^{\mu_r\nu_r}\Gamma^{b_r b_{r+1} c_r}_{\nu_r\mu_{r+1}\alpha_r}(k_r,-k_{r+1})u^{\alpha_r}\right]\ \epsilon^{*\mu_{N_k+1}}(k)\to N_{k_r}^{\mu_1\nu}\epsilon^*_\nu(k)$$

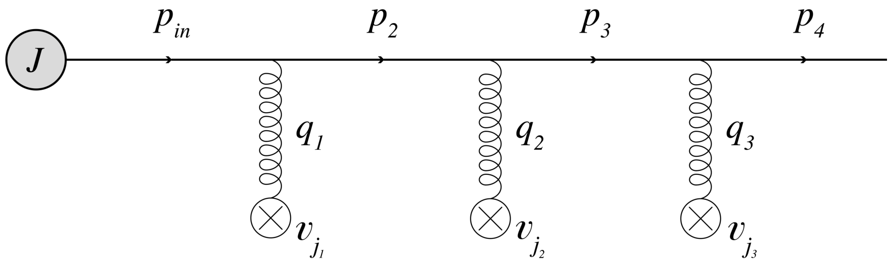
$$\downarrow$$

$$\prod_{r=1}^{N_k}\left[i\int_{\mathbf{k}_r,x_r} T^a_{proj}\hat{\rho}^a(\mathbf{x}_r,z_r)\,v(\mathbf{k}_{r+1}-\mathbf{k}_r)\theta_{r,r-1}\,e^{-i\mathbf{x}_r\cdot(\mathbf{k}_{r+1}-\mathbf{k}_r)}e^{-iz_r(\tilde{k}_{r+1,z}-\tilde{k}_{rz})}\right]$$

$$=\prod_{r=1}^{N_k}\left[i\int_{\mathbf{k}_r,x_r} T^a_{proj}v^a(\mathbf{x}_r,z_r)\,\theta_{r,r-1}\,e^{-i\mathbf{x}_r\cdot(\mathbf{k}_{r+1}-\mathbf{k}_r)}e^{iz_r\frac{\mathbf{k}^2_{r+1,z}-\mathbf{k}^2_{rz}}{2xE}}\right]$$

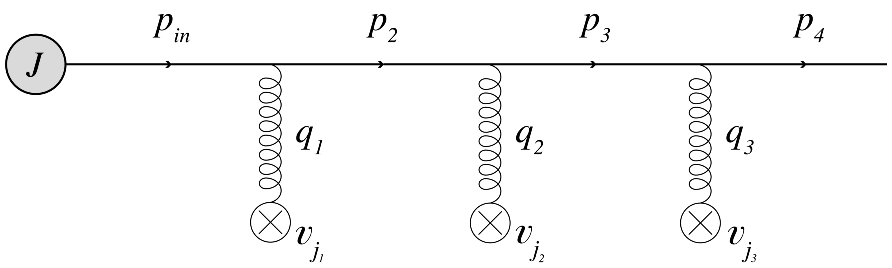
$$\Rightarrow G(\mathbf{k},z_f;\mathbf{k}_1,z_s)e^{i\frac{\mathbf{k}^2}{2xE}z_f}e^{-i\frac{\mathbf{k}_1^2}{2xE}z_s}$$

Gluon emission



$$\longrightarrow \quad \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r} \exp\left(\frac{iE}{2}\int_0^L d\tau\, \dot{\mathbf{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\mathbf{r}(\tau),\tau)\right)$$

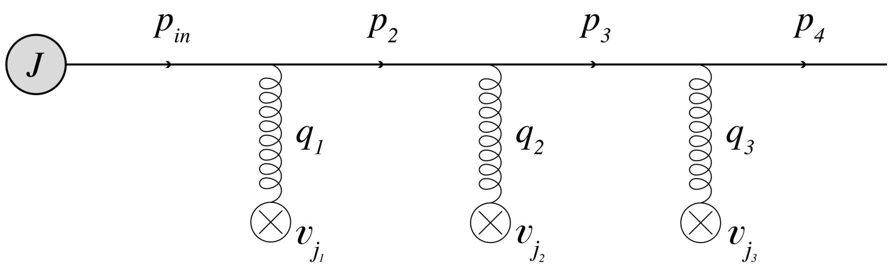
Gluon emission



$$\mathcal{W} = \mathcal{P}\exp\left(-i\int dx^\mu\, t^a_{\mathrm{proj}}\, A^a_\mu(\mathbf{r}(\tau),\tau)\right)$$

$$\int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right) \boxed{\mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\mathbf{r}(\tau),\tau)\right)}$$

Gluon emission



$$\mathcal{W} = \mathcal{P}\exp\left(-i\int dx^\mu\, t^a_{\mathrm{proj}}\, A^a_\mu(\mathbf{r}(\tau),\tau)\right)$$

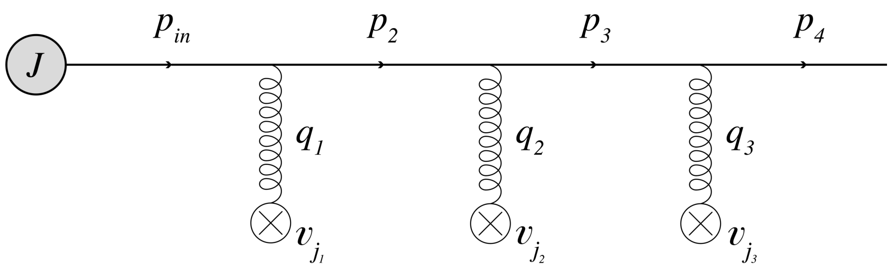
$$\longrightarrow \quad \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right)\boxed{\mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\mathbf{r}(\tau),\tau)\right)}$$

if E is large, then $E\ddot{\mathbf{r}} = F \rightarrow \dot{\mathbf{r}} = 0$

$$\downarrow$$

$$iM(p) = \int_{\mathbf{xl}} e^{-i(\mathbf{p}-\mathbf{l})\cdot\mathbf{x}}\mathcal{W}(\mathbf{x})J(E,\mathbf{l})$$

Gluon emission



$$\mathcal{W} = \mathcal{P}\exp\left(-i\int dx^\mu\, t^a_{\text{proj}}\, A^a_\mu(\mathbf{r}(\tau),\tau)\right)$$

$$\longrightarrow \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right)\boxed{\mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\mathbf{r}(\tau),\tau)\right)}$$

if E is large, then $E\ddot{\mathbf{r}} = F \to \dot{\mathbf{r}} = 0$

$\downarrow$

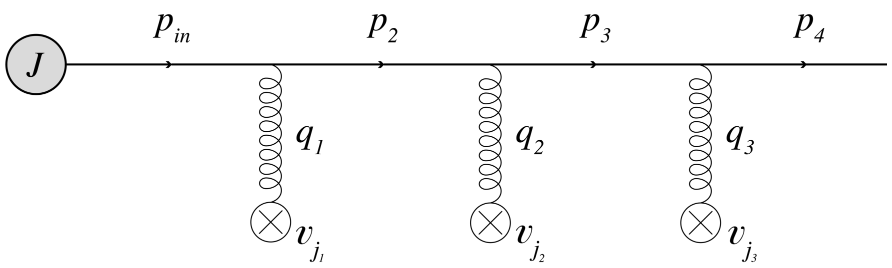
$$iM(p) = \int_{\mathbf{xl}} e^{-i(\mathbf{p}-\mathbf{l})\cdot\mathbf{x}}\mathcal{W}(\mathbf{x})J(E,\mathbf{l})$$

we don’t need the LPM phases to get $\hat{q}$

$\downarrow$

$$\partial_L W(\mathbf{p},\mathbf{Y}) = -\cancel{\frac{\mathbf{p}\cdot\boldsymbol{\nabla}_Y}{E}}W(\mathbf{p},\mathbf{Y}) - \int_{\mathbf{q}}\mathcal{V}(\mathbf{q})W(\mathbf{p}-\mathbf{q},\mathbf{Y})$$

Gluon emission



$$\mathcal{W} = \mathcal{P}\exp\left(-i\int dx^\mu\, t^a_{\text{proj}}\, A^a_\mu(\mathbf{r}(\tau),\tau)\right)$$

$$\longrightarrow \int_{\mathbf{x}_0}^{\mathbf{x}_L}\mathcal{D}\mathbf{r}\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right)\boxed{\mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\mathbf{r}(\tau),\tau)\right)}$$

if E is large, then $E\ddot{\mathbf{r}} = F \to \dot{\mathbf{r}} = 0$

$$\downarrow$$

$$iM(p) = \int_{\mathbf{xl}} e^{-i(\mathbf{p}-\mathbf{l})\cdot\mathbf{x}}\mathcal{W}(\mathbf{x})J(E,\mathbf{l})$$

we don't need the LPM phases to get $\hat{q}$

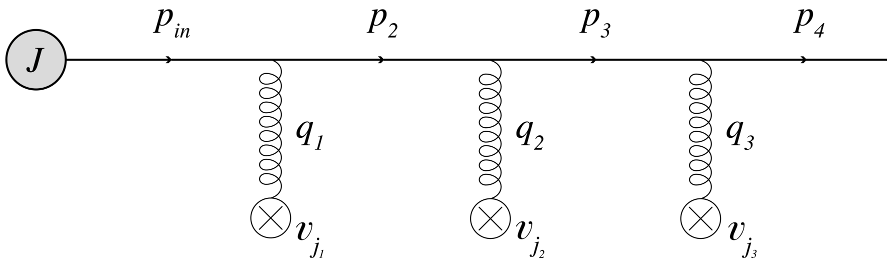
$$\downarrow$$

$$\partial_L W(\mathbf{p},\mathbf{Y}) = \cancel{-\frac{\mathbf{p}\cdot\boldsymbol{\nabla}_Y}{E}W(\mathbf{p},\mathbf{Y})} - \int_{\mathbf{q}}\mathcal{V}(\mathbf{q})W(\mathbf{p}-\mathbf{q},\mathbf{Y})$$

$$\downarrow$$

$$\langle\mathbf{p}^2\rangle = \int_{\mathbf{p},\mathbf{Y}}\mathbf{p}^2\, W(\mathbf{p},\mathbf{Y}) = L\int_{\mathbf{p}}\mathbf{p}^2\mathcal{V}(\mathbf{p}) = \frac{\mathcal{C}g^4\rho}{4\pi}L\,\log\frac{E}{\mu}$$

Gluon emission

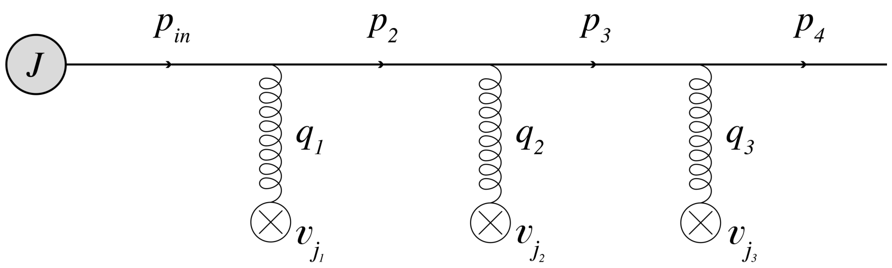


$$\mathcal{W} = \mathcal{P}\exp\left(-i\int dx^\mu\, t^a_{\mathrm{proj}}\, A^a_\mu(\mathbf{r}(\tau),\tau)\right)$$

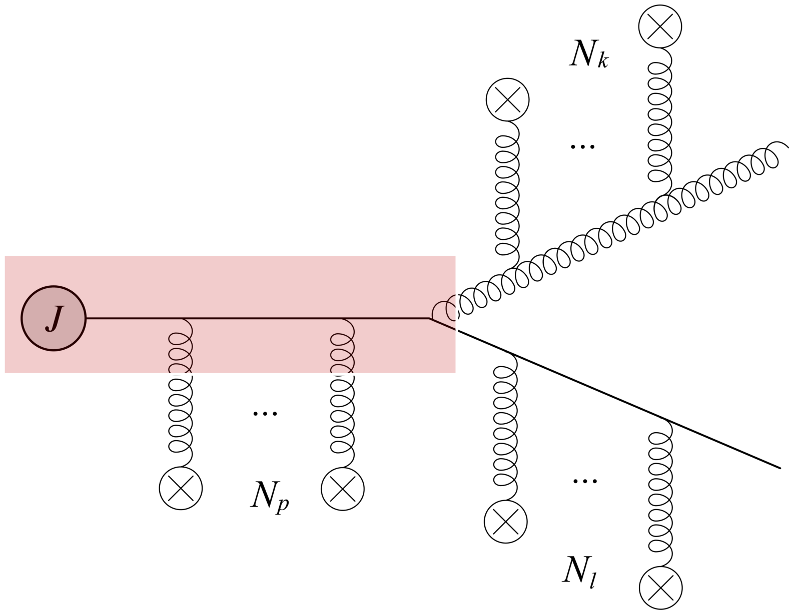
$$\longrightarrow \quad \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\,\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\mathbf{r}(\tau),\tau)\right)$$

Gluon emission

$$\mathcal{W} = \mathcal{P}\exp\left(-i\int dx^{\mu}\, t^{a}_{\mathrm{proj}}\, A^{a}_{\mu}(\mathbf{r}(\tau),\tau)\right)$$



$$\longrightarrow \quad \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right)\mathcal{P}\exp\left(-i\int_0^L d\tau\, t^{a}_{\mathrm{proj}} v^{a}(\mathbf{r}(\tau),\tau)\right)$$



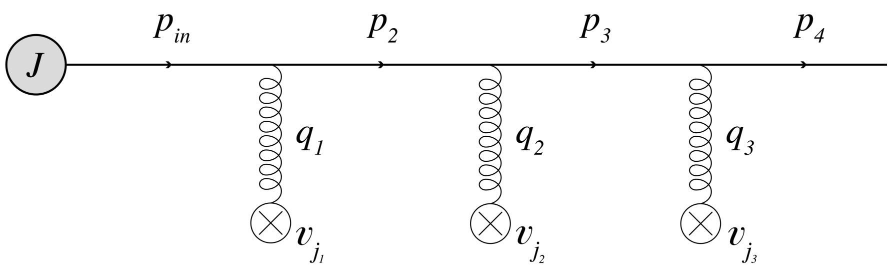
$$\longrightarrow \quad -\frac{g}{xE}\lim_{z_f\to\infty}\int_0^{\infty} dz_s \int d^2x_0\, e^{-i\mathbf{x}_0\cdot\mathbf{l}_f} J(\mathbf{x}_0)$$

$$\times\, \mathcal{W}(\mathbf{x}_0;\infty,z_s)\, t^{a}_{proj}\, \mathcal{W}(\mathbf{x}_0;z_s,0)\, e^{i\frac{\mathbf{k}_f^2}{2\omega}z_f}$$

$$\times \left[\epsilon\cdot\vec{\nabla}_{\mathbf{x}_0}\mathcal{G}^{ba}\left(\mathbf{k}_f, z_f;\mathbf{x}_0, z_s\right)\right]$$

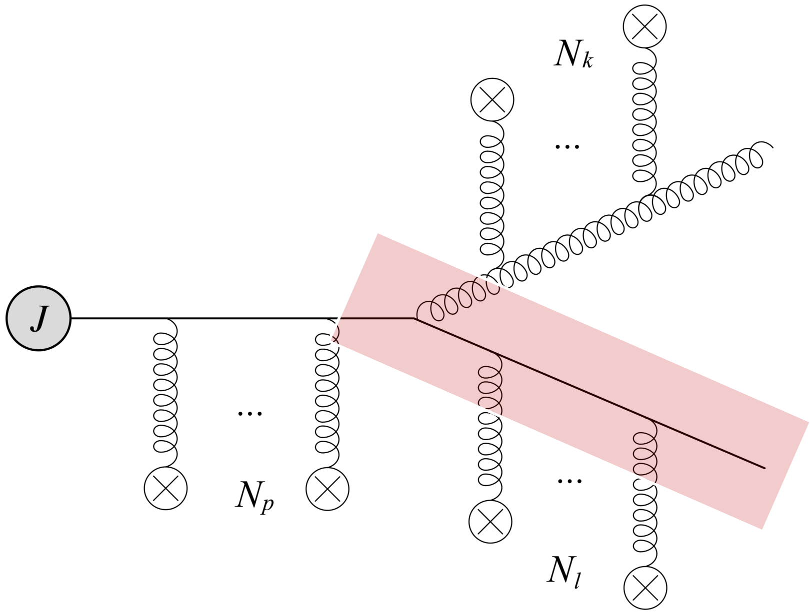
small-x limit, i.e. $\omega/E \ll 1$

Gluon emission



$$\mathcal{W} = \mathcal{P}\exp\left(-i\int dx^\mu\, t^a_{\text{proj}}\, A^a_\mu(\mathbf{r}(\tau),\tau)\right)$$

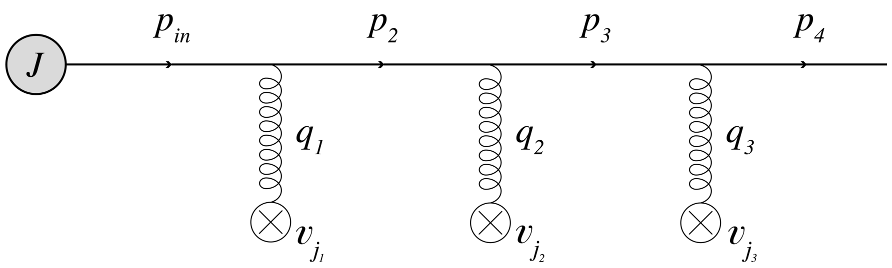
$$\longrightarrow \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\,\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right)\boxed{\mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\mathbf{r}(\tau),\tau)\right)}$$



$$\longrightarrow -\frac{g}{xE}\lim_{z_f\to\infty}\int_0^\infty dz_s\int d^2x_0\, e^{-i\mathbf{x}_0\cdot\mathbf{l}_f}J(\mathbf{x}_0)$$
$$\times\,\boxed{\mathcal{W}(\mathbf{x}_0;\infty,z_s)}\,t^a_{proj}\,\boxed{\mathcal{W}(\mathbf{x}_0;z_s,0)}\,e^{i\frac{\mathbf{k}_f^2}{2\omega}z_f}$$
$$\times\left[\epsilon\cdot\vec{\nabla}_{\mathbf{x}_0}\mathcal{G}^{ba}\left(\mathbf{k}_f,z_f;\mathbf{x}_0,z_s\right)\right]$$

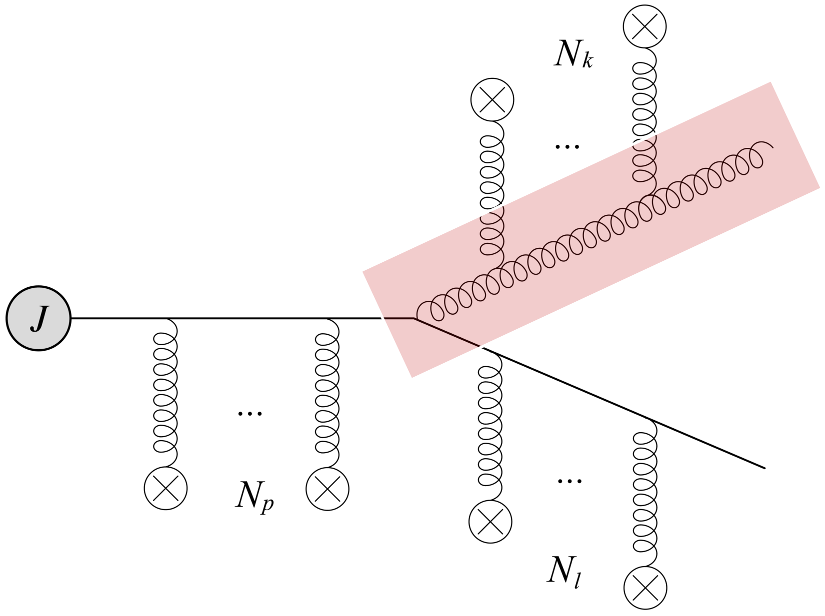
small-x limit, i.e. $\omega/_{E} \ll 1$

Gluon emission



$\longrightarrow$

$$\boxed{\int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\, \exp\left(\frac{iE}{2}\int_0^L d\tau\, \dot{\mathbf{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\mathbf{r}(\tau),\tau)\right)}$$

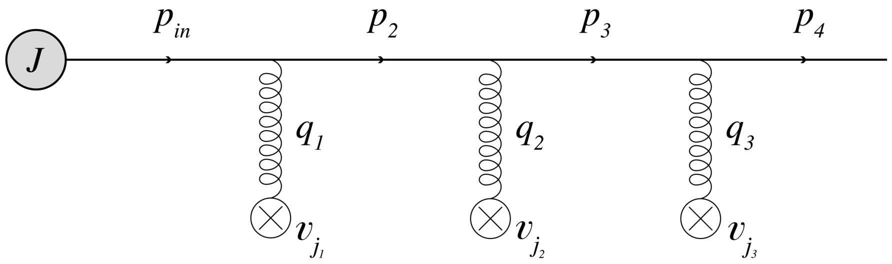


$\longrightarrow$

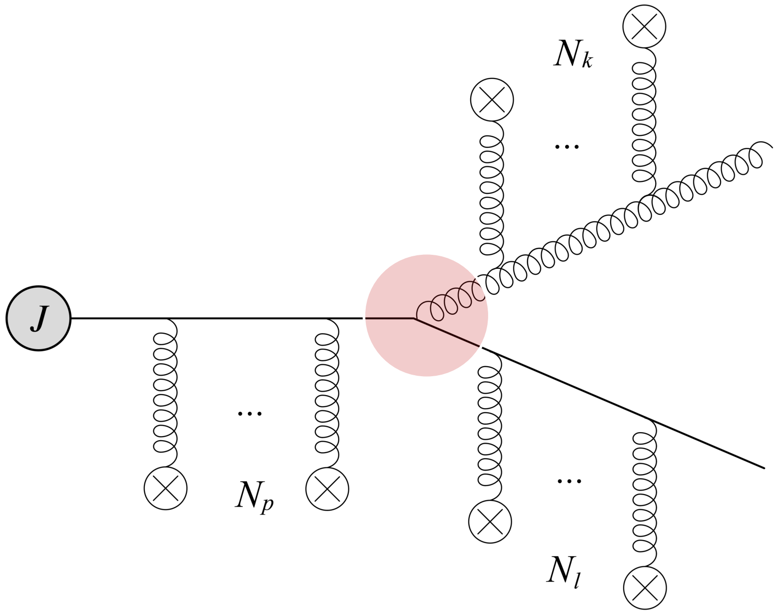
$$-\frac{g}{xE}\lim_{z_f\to\infty}\int_0^\infty dz_s \int d^2x_0\, e^{-i\mathbf{x}_0\cdot\mathbf{l}_f} J(\mathbf{x}_0)$$
$$\times\, \mathcal{W}(\mathbf{x}_0;\infty,z_s)\, t^a_{proj}\, \mathcal{W}(\mathbf{x}_0;z_s,0)\; e^{i\frac{\mathbf{k}_f^2}{2\omega}z_f}$$
$$\times \left[\epsilon\cdot\vec{\nabla}_{\mathbf{x}_0}\boxed{\mathcal{G}^{ba}\left(\mathbf{k}_f,z_f;\mathbf{x}_0,z_s\right)}\right]$$

small-x limit, i.e. $\omega/_{E} \ll 1$

Gluon emission



$$\int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\,\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right)\mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\mathbf{r}(\tau),\tau)\right)$$



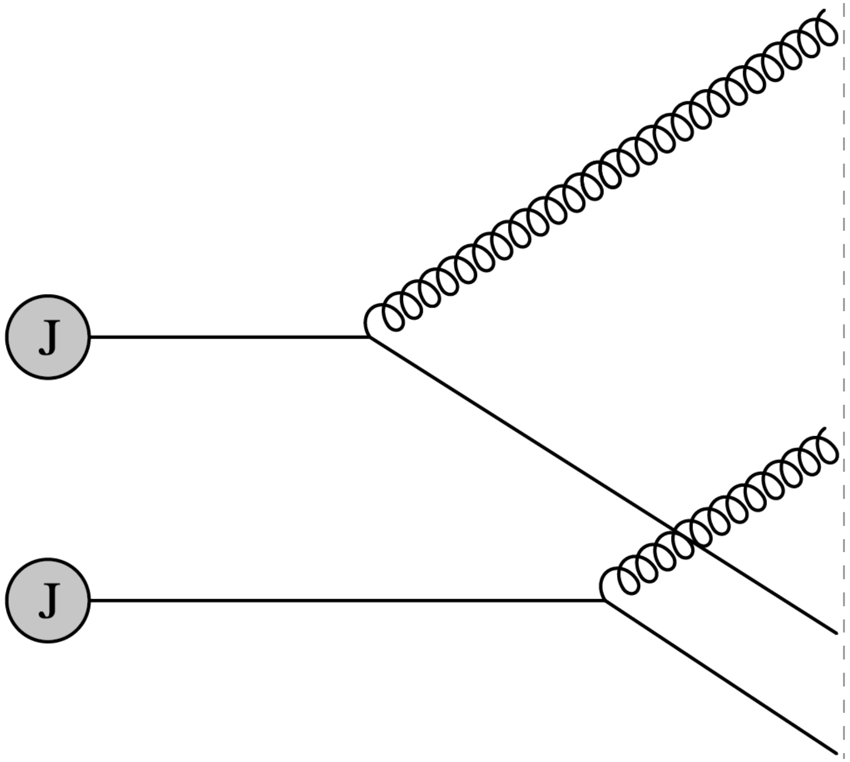
$$-\frac{g}{xE}\lim_{z_f\to\infty}\int_0^\infty dz_s \int d^2x_0\, e^{-i\mathbf{x}_0\cdot\mathbf{l}_f} J(\mathbf{x}_0)$$

$$\times\,\mathcal{W}(\mathbf{x}_0;\infty,z_s)\, t^a_{proj}\,\mathcal{W}(\mathbf{x}_0;z_s,0)\; e^{i\frac{\mathbf{k}_f^2}{2\omega}z_f}$$

$$\times\left[\epsilon\cdot\vec{\nabla}_{\mathbf{x}_0}\mathcal{G}^{ba}\left(\mathbf{k}_f,z_f;\mathbf{x}_0,z_s\right)\right]$$

small-x limit, i.e. $\omega/E \ll 1$

Gluon emission

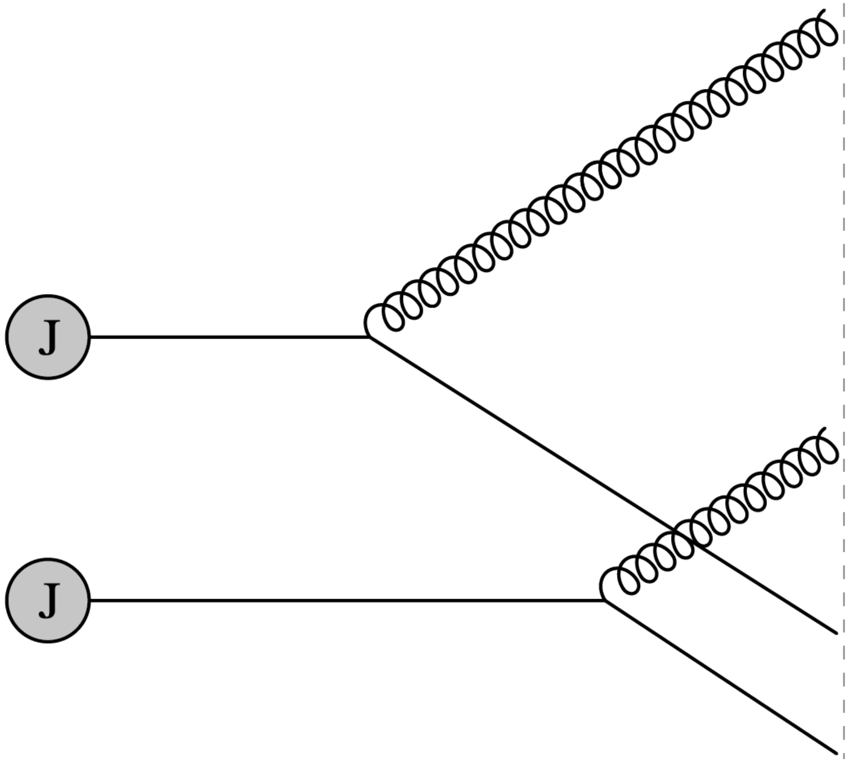


$$2(2\pi)^3\omega E\frac{d\mathcal{N}}{d\omega dEd^2\mathbf{k}} = \lim_{z_f\to\infty}\frac{\alpha_s}{N_c\,\omega^2}\int_0^\infty dz\int_0^\infty d\bar{z}\int_{\mathbf{x}_{in}}|J(\mathbf{x}_{in})|^2$$
$$\times\left\langle \mathrm{Tr}\left[\mathcal{W}(\mathbf{x}_{in};\infty,z)\,t^a_{proj}\,\mathcal{W}(\mathbf{x}_{in};z,0)\,\left[\nabla_{\alpha,\mathbf{x}_{in}}\mathcal{G}^{ba}\left(\mathbf{k}_f,z_f;\mathbf{x}_{in},z\right)\right]\right.\right.$$
$$\left.\left.\times\left(\mathcal{W}(\mathbf{x}_{in};\infty,\bar{z})\,t^{\bar{a}}_{proj}\,\mathcal{W}(\mathbf{x}_{in};\bar{z},0)\,\left[\nabla_{\alpha,\mathbf{x}_{in}}\mathcal{G}^{b\bar{a}}\left(\mathbf{k},z_f;\mathbf{x}_{in},\bar{z}\right)\right]\right)^\dagger\right]\right\rangle$$

$$\downarrow$$

$$\mathrm{Tr}\Big[\mathcal{W}(\mathbf{x}_{in};\infty,z)\,t^a_{proj}\,\mathcal{W}(\mathbf{x}_{in};z,0)\mathcal{W}^\dagger(\mathbf{x}_{in};\bar{z},0)\,t^{\bar{a}}_{proj}\,\mathcal{W}^\dagger(\mathbf{x}_{in};\infty,\bar{z})\Big]$$
$$=\mathrm{Tr}\Big[\mathcal{W}(\mathbf{x}_{in};\bar{z},z)\,t^a_{proj}\,\mathcal{W}^\dagger(\mathbf{x}_{in};\bar{z},z)\,t^{\bar{a}}_{proj}\Big]$$
$$=\frac{1}{2}\mathcal{W}_A^{\dagger a\bar{a}}(\vec{x}_{in};\bar{z},z)$$

Gluon emission

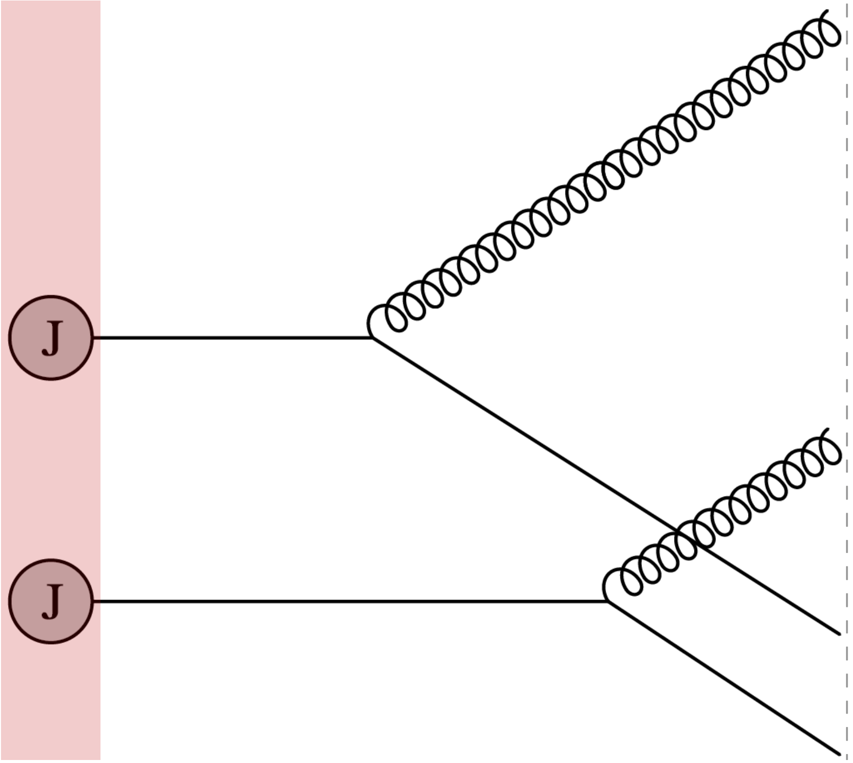


$$2(2\pi)^3\omega E\frac{d\mathcal{N}}{d\omega dEd^2\mathbf{k}} = \lim_{z_f\to\infty}\frac{\alpha_s}{N_c\,\omega^2}\int_0^\infty dz\int_0^\infty d\bar{z}\int_{\mathbf{x}_{in}}|J(\mathbf{x}_{in})|^2$$
$$\times\left\langle \mathrm{Tr}\left[\mathcal{W}(\mathbf{x}_{in};\infty,z)\,t^a_{proj}\,\mathcal{W}(\mathbf{x}_{in};z,0)\,\left[\nabla_{\alpha,\mathbf{x}_{in}}\mathcal{G}^{ba}\left(\mathbf{k}_f,z_f;\mathbf{x}_{in},z\right)\right]\right.\right.$$
$$\left.\left.\times\left(\mathcal{W}(\mathbf{x}_{in};\infty,\bar{z})\,t^{\bar{a}}_{proj}\,\mathcal{W}(\mathbf{x}_{in};\bar{z},0)\,\left[\nabla_{\alpha,\mathbf{x}_{in}}\mathcal{G}^{b\bar{a}}\left(\mathbf{k},z_f;\mathbf{x}_{in},\bar{z}\right)\right]\right)^\dagger\right]\right\rangle$$

$$\downarrow$$

$$2(2\pi)^3\omega E\frac{d\mathcal{N}}{d\omega dEd^2\mathbf{k}} = \lim_{z_f\to\infty}\frac{\alpha_s}{N_c\,\omega^2}\,\mathrm{Re}\int_0^\infty d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{x}_{in}}|J(\mathbf{x}_{in})|^2$$
$$\times\left\langle\left[\nabla_{\alpha,\mathbf{x}_{in}}\mathcal{G}^{ba}\left(\mathbf{k},z_f;\mathbf{x}_{in},z\right)\right]\mathcal{W}_A^{\dagger\,a\bar{a}}(\mathbf{x}_{in};\bar{z},z)\left[\nabla_{\alpha,\mathbf{x}_{in}}\mathcal{G}^{\dagger\,\bar{a}b}\left(\mathbf{k},z_f;\mathbf{x}_{in},\bar{z}\right)\right]\right\rangle$$

Gluon emission

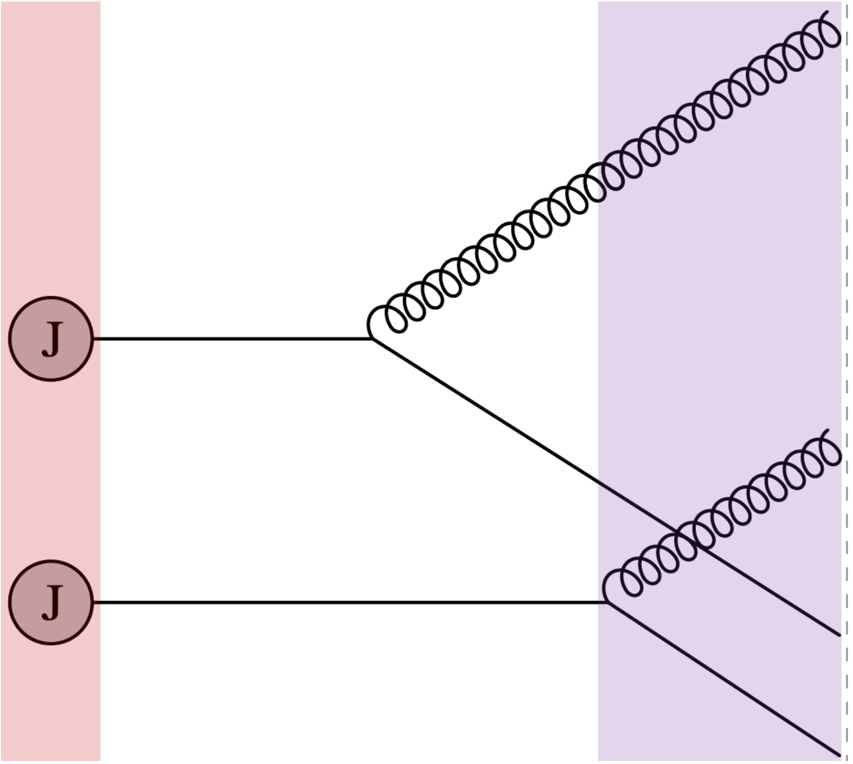


$$
(2\pi)^2\omega\frac{d\mathcal{N}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s}{N_c\,\omega^2}\mathrm{Re}\int_0^\infty d\bar z\int_0^{\bar z} dz \int_{\mathbf{x}_0,\mathbf{y}} |J(\mathbf{x}_0)|^2 \Bigg[(\nabla_{\mathbf{x}}\cdot\nabla_{\overline{\mathbf{x}}}) \times \Big\langle \mathcal{G}^{bc}\left(\mathbf{k},L;\mathbf{y},\overline{z}\right)\mathcal{G}^{\dagger\,\bar a b}\left(\mathbf{k},L;\overline{\mathbf{x}},\overline{z}\right)\Big\rangle\Big\langle \mathcal{G}^{ca}\left(\mathbf{y},\overline{z};\mathbf{x},z\right)\mathcal{W}_A^{\dagger\,a\bar a}(\mathbf{x}_0;\bar z,z)\Big\rangle\Bigg]\Bigg|_{\mathbf{x}=\overline{\mathbf{x}}=\mathbf{x}_0}
$$

broadening of the gluon

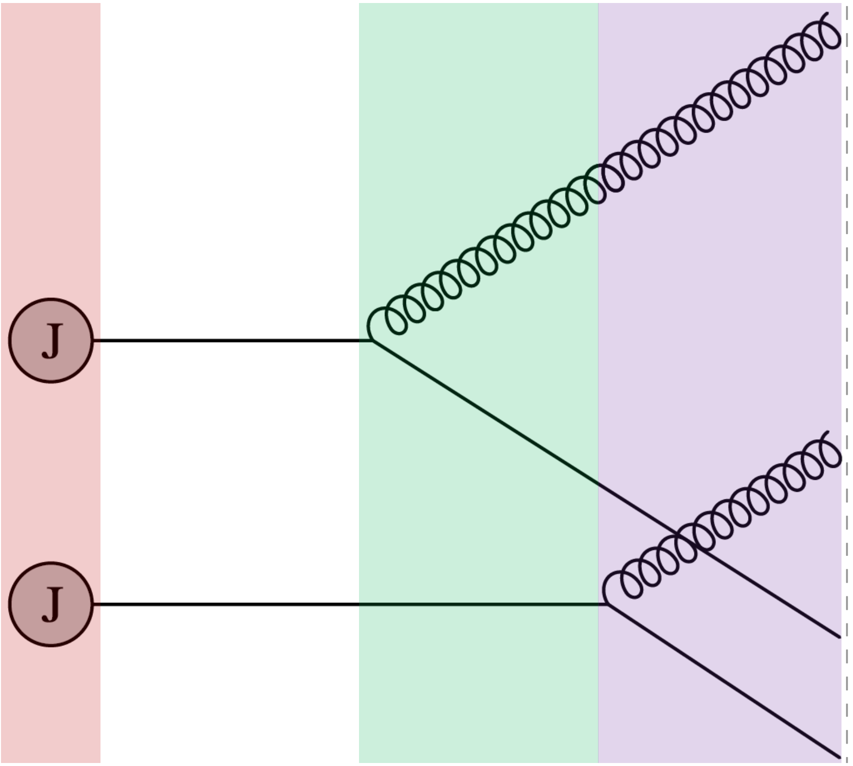
emission kernel

Gluon emission



$$(2\pi)^2\omega\frac{d\mathcal{N}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s}{N_c\,\omega^2}\mathrm{Re}\int_0^\infty d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{x}_0,\mathbf{y}} |J(\mathbf{x}_0)|^2\Bigg[(\nabla_{\mathbf{x}}\cdot\nabla_{\overline{\mathbf{x}}})$$
$$\times\underbrace{\left\langle\mathcal{G}^{bc}\left(\mathbf{k},L;\mathbf{y},\overline{z}\right)\mathcal{G}^{\dagger\,\bar{a}b}\left(\mathbf{k},L;\overline{\mathbf{x}},\overline{z}\right)\right\rangle}_{\text{broadening of the gluon}}\underbrace{\left\langle\mathcal{G}^{ca}\left(\mathbf{y},\overline{z};\mathbf{x},z\right)\mathcal{W}_A^{\dagger\,a\bar{a}}(\mathbf{x}_0;\bar{z},z)\right\rangle}_{\text{emission kernel}}\Bigg]\Bigg|_{\mathbf{x}=\overline{\mathbf{x}}=\mathbf{x}_0}$$

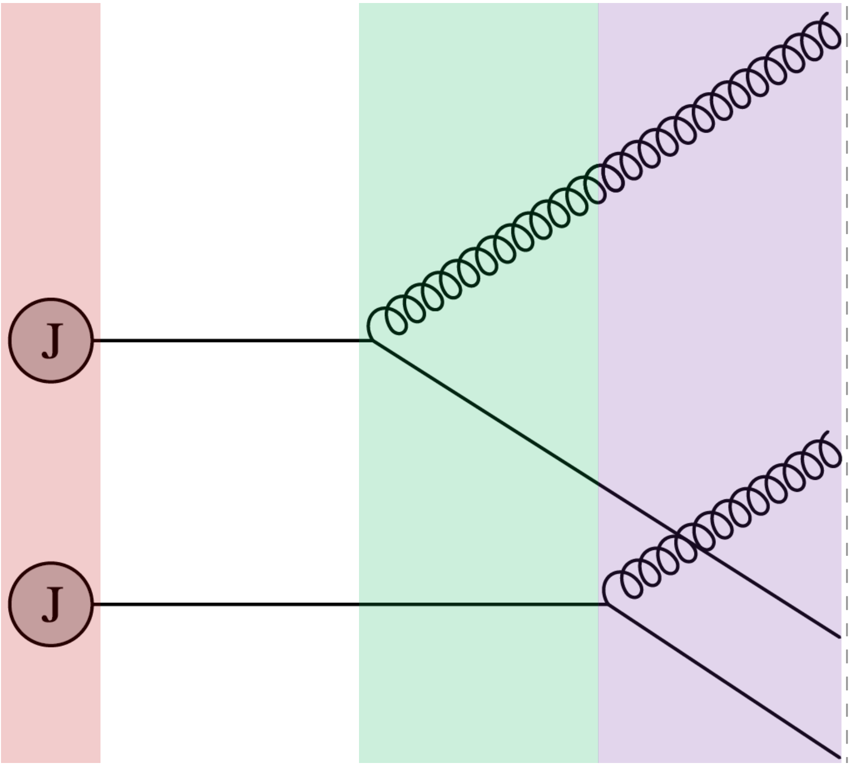
Gluon emission



$$(2\pi)^2\omega\frac{d\mathcal{N}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s}{N_c\,\omega^2}\mathrm{Re}\int_0^{\infty} d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{x}_0,\mathbf{y}} |J(\mathbf{x}_0)|^2 \Bigg[(\nabla_{\mathbf{x}}\cdot\nabla_{\overline{\mathbf{x}}})$$

$$\times \underbrace{\left\langle \mathcal{G}^{bc}\left(\mathbf{k},L;\mathbf{y},\overline{z}\right)\mathcal{G}^{\dagger\,\bar{a}b}\left(\mathbf{k},L;\overline{\mathbf{x}},\overline{z}\right)\right\rangle}_{\text{broadening of the gluon}}\underbrace{\left\langle \mathcal{G}^{ca}\left(\mathbf{y},\overline{z};\mathbf{x},z\right)\mathcal{W}_A^{\dagger\,a\bar{a}}(\mathbf{x}_0;\bar{z},z)\right\rangle}_{\text{emission kernel}}\Bigg]\Bigg|_{\mathbf{x}=\overline{\mathbf{x}}=\mathbf{x}_0}$$

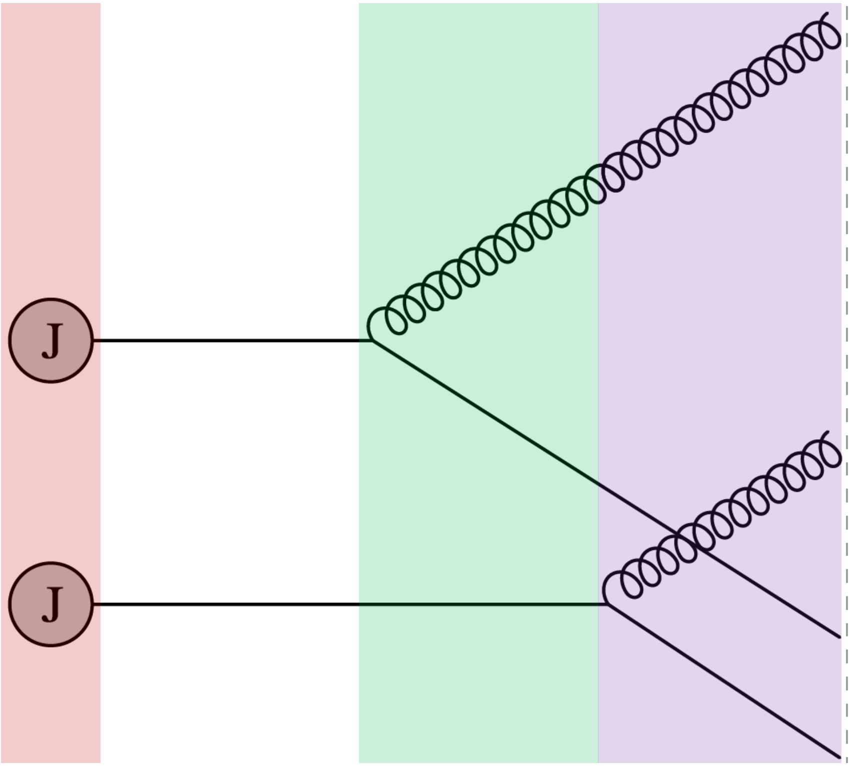
Gluon emission



$$(2\pi)^2\omega\frac{d\mathcal{N}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s}{N_c\,\omega^2}\mathrm{Re}\int_0^\infty d\bar{z}\int_0^{\bar{z}} dz \int_{\mathbf{x}_0,\mathbf{y}} |J(\mathbf{x}_0)|^2 \Big[(\nabla_{\mathbf{x}}\cdot\nabla_{\bar{\mathbf{x}}}) \times \underbrace{\Big\langle \mathcal{G}^{bc}(\mathbf{k},L;\mathbf{y},\bar{z})\,\mathcal{G}^{\dagger\,\bar{a}b}(\mathbf{k},L;\bar{\mathbf{x}},\bar{z})\Big\rangle}_{\text{broadening of the gluon}} \underbrace{\Big\langle \mathcal{G}^{ca}(\mathbf{y},\bar{z};\mathbf{x},z)\,\mathcal{W}_A^{\dagger\,a\bar{a}}(\mathbf{x}_0;\bar{z},z)\Big\rangle}_{\text{emission kernel}} \Big]\Big|_{\mathbf{x}=\bar{\mathbf{x}}=\mathbf{x}_0}$$

$$\left(\partial_L - \frac{\partial_{\mathbf{x}}^2}{2xE} + i\mathcal{V}(\mathbf{x},t)\right)\mathcal{K}(\mathbf{x},t;\mathbf{y},s) = i\delta^{(2)}(\mathbf{x}-\mathbf{y})\delta(t-s)$$

Gluon emission



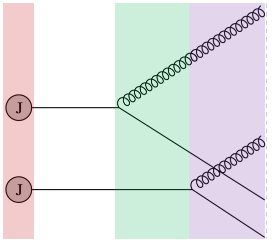
$$(2\pi)^2\omega\frac{d\mathcal{N}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s}{N_c\,\omega^2}\mathrm{Re}\int_0^\infty d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{x}_0,\mathbf{y}} |J(\mathbf{x}_0)|^2\Big[(\nabla_{\mathbf{x}}\cdot\nabla_{\bar{\mathbf{x}}})$$

$$\times\underbrace{\left\langle\mathcal{G}^{bc}\left(\mathbf{k},L;\mathbf{y},\bar{z}\right)\mathcal{G}^{\dagger\,\bar{a}b}\left(\mathbf{k},L;\bar{\mathbf{x}},\bar{z}\right)\right\rangle}_{\text{broadening of the gluon}}\underbrace{\left\langle\mathcal{G}^{ca}\left(\mathbf{y},\bar{z};\mathbf{x},z\right)\mathcal{W}_A^{\dagger\,a\bar{a}}(\mathbf{x}_0;\bar{z},z)\right\rangle}_{\text{emission kernel}}\Big]\Big|_{\mathbf{x}=\bar{\mathbf{x}}=\mathbf{x}_0}$$

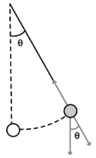
$$\left(\partial_L - \frac{\partial^2_{\mathbf{x}}}{2xE} + i\mathcal{V}(\mathbf{x},t)\right)\mathcal{K}(\mathbf{x},t;\mathbf{y},s) = i\delta^{(2)}(\mathbf{x}-\mathbf{y})\delta(t-s)$$

$$\partial_L W(\mathbf{p},\mathbf{Y}) = -\frac{\mathbf{p}\cdot\boldsymbol{\nabla}_Y}{E}W(\mathbf{p},\mathbf{Y}) - \int_{\mathbf{q}}\mathcal{V}(\mathbf{q})W(\mathbf{p}-\mathbf{q},\mathbf{Y})$$

Gluon emission



$$\mathcal{V} \propto \mu^2 x^2 \log \frac{1}{\mu^2 x^2} \longrightarrow \mathcal{V} = \frac{\hat{q}}{4} x^2$$

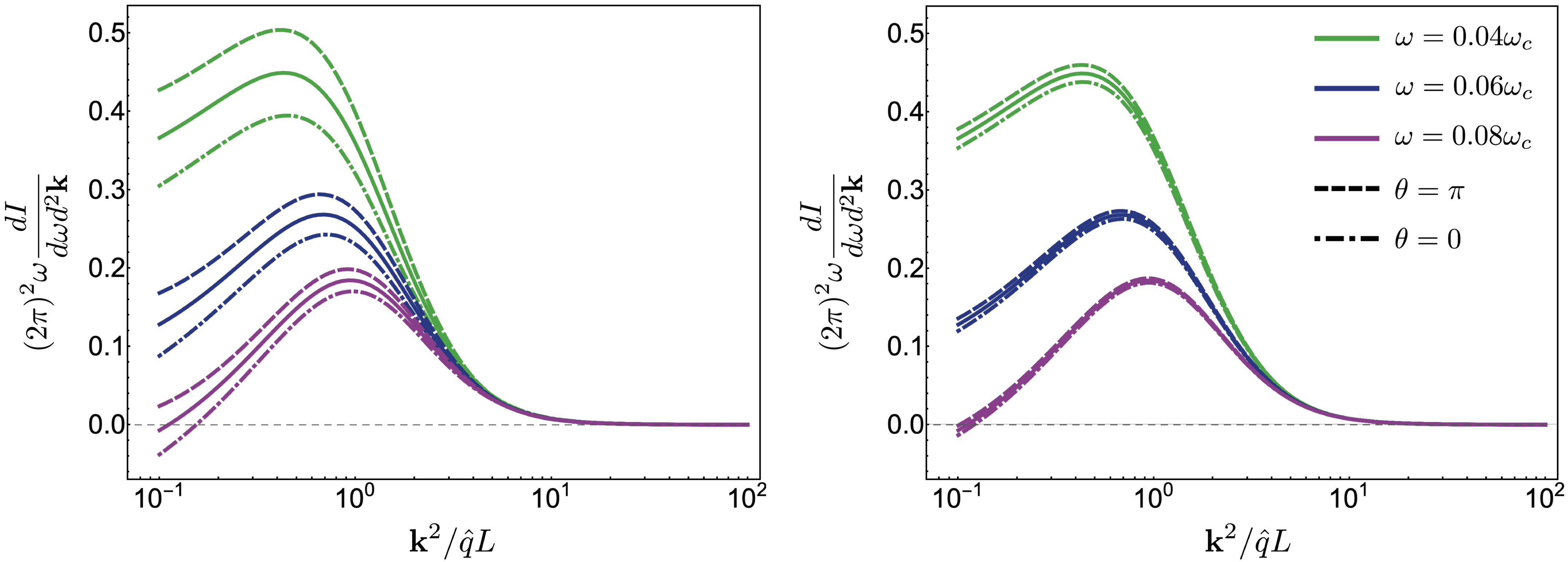


$$(2\pi)^2 \omega E \frac{dN}{d\omega dE d^2\mathbf{k}} = (2\pi)^2 \omega \frac{dI}{d\omega d^2\mathbf{k}} \, E \frac{dN_0}{dE}$$

$$\frac{1-i}{2}\sqrt{\frac{\hat{q}}{\omega}}$$

$$(2\pi)^2 \omega \frac{dI}{d\omega d^2\mathbf{k}} = 8\alpha_s C_F \,\mathrm{Re}\left[\Omega \int_0^L d\bar{z} \, \frac{e^{\frac{\mathbf{k}^2 \tan \Omega \bar{z}}{2i\omega\Omega - \hat{q}(L-\bar{z}) \tan \Omega \bar{z}}}}{\hat{q}(L-\bar{z}) \tan \Omega \bar{z} - 2i\omega\Omega} - \frac{1}{\mathbf{k}^2}\left(1 - e^{\frac{\mathbf{k}^2 \tan \Omega L}{2i\omega\Omega}}\right)\right]$$

Gluon emission



Energy loss

$$(2\pi)^2\omega\frac{d\mathcal{N}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s}{N_c\,\omega^2}\mathrm{Re}\int_0^L d\bar{z}\int_0^{\bar{z}} dz \int_{\mathbf{x}_0,\mathbf{y}} |J(\mathbf{x}_0)|^2\Big[(\nabla_{\mathbf{x}}\cdot\nabla_{\overline{\mathbf{x}}})$$

$$\times\left\langle\mathcal{G}^{bc}(\mathbf{k},L;\mathbf{y},\overline{z})\,\mathcal{G}^{\dagger\,\bar{a}b}(\mathbf{k},L;\overline{\mathbf{x}},\overline{z})\right\rangle\left\langle\mathcal{G}^{ca}(\mathbf{y},\overline{z};\mathbf{x},z)\,\mathcal{W}_A^{\dagger\,a\bar{a}}(\mathbf{x}_0;\bar{z},z)\right\rangle\Big]\Big|_{\mathbf{x}=\overline{\mathbf{x}}=\mathbf{x}_0}$$

$$\downarrow$$

$$\omega\frac{dI}{d\omega} = \frac{2\alpha_s C_F}{\omega^2}\mathrm{Re}\int_0^L d\bar{z}\int_0^{\bar{z}} dz\nabla_{\mathbf{y}}\cdot\nabla_{\mathbf{x}}\mathcal{K}_{med}(\mathbf{y},\bar{z};\mathbf{x},z)\Big|_{\mathbf{y}=\mathbf{z}=0} = \frac{2\alpha_s C_F}{\pi}\log|\cos\Omega L|$$

$$\downarrow$$

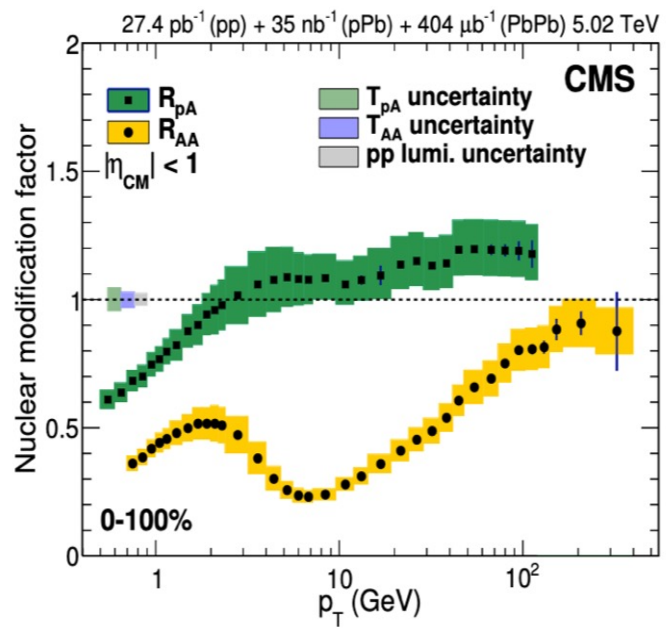
$$\Delta E = \int\omega\frac{dI}{d\omega}d\omega \sim \hat{q}L^2$$

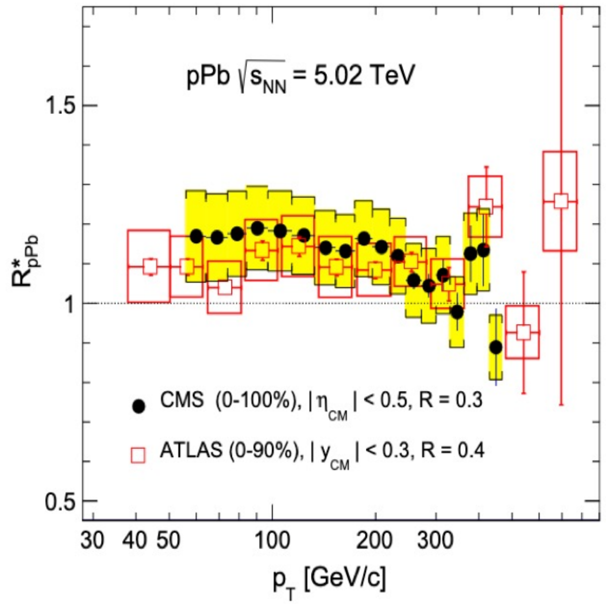
LABORATÓRIO DE INSTRUMENTAÇÃO
E FÍSICA EXPERIMENTAL DE PARTÍCULAS

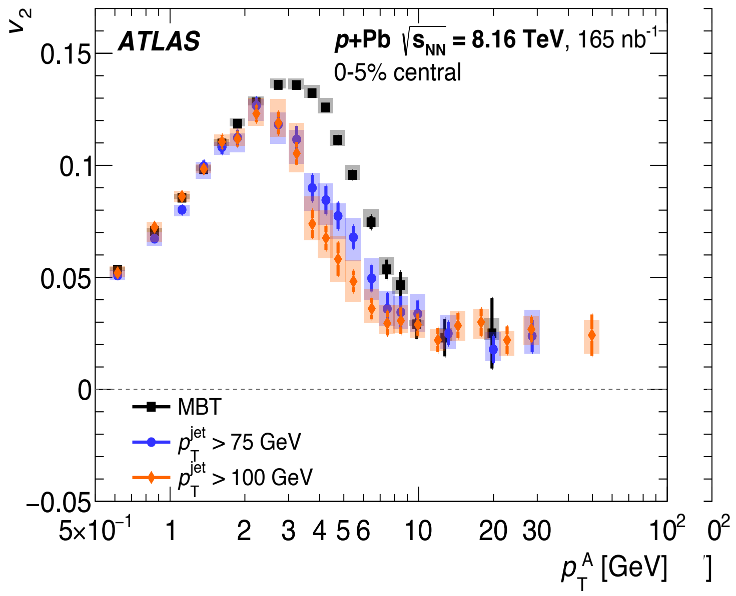
Summary

- We have derived the radiative energy loss (in its simplest form)
- Any further jet quenching calculation can be reached from this point in a similar manner
- Having the full spectrum in hand, one can study not only the energy loss but the whole jet shape modification
- In fact, this is the entry point to the state-of-the-art branch of many-body QCD applied to heavy-ion collisions, and, in general, energy loss in nuclear matter

Small systems







Notable tension in observations:

- Small systems seem to flow
- No clear signal for jet quenching