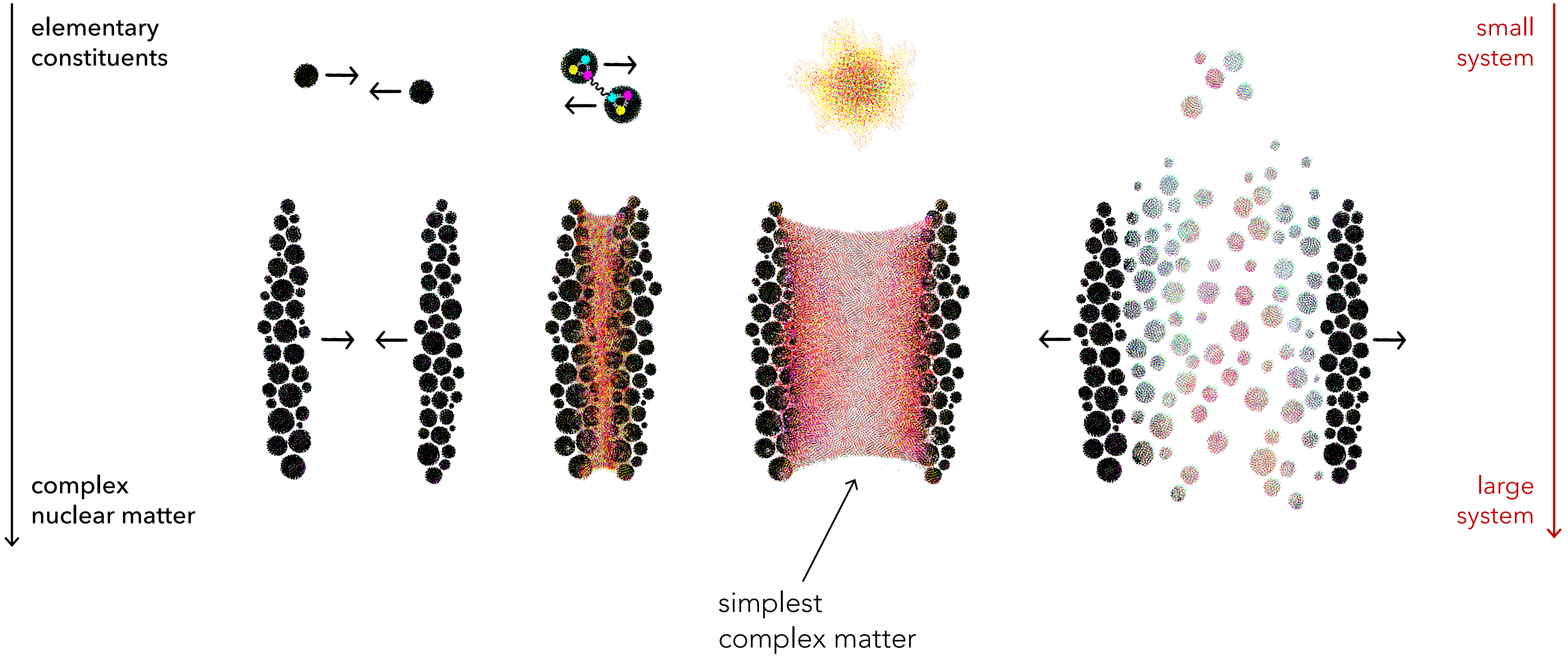
# Hard probes of nuclear matter and Jet tomography

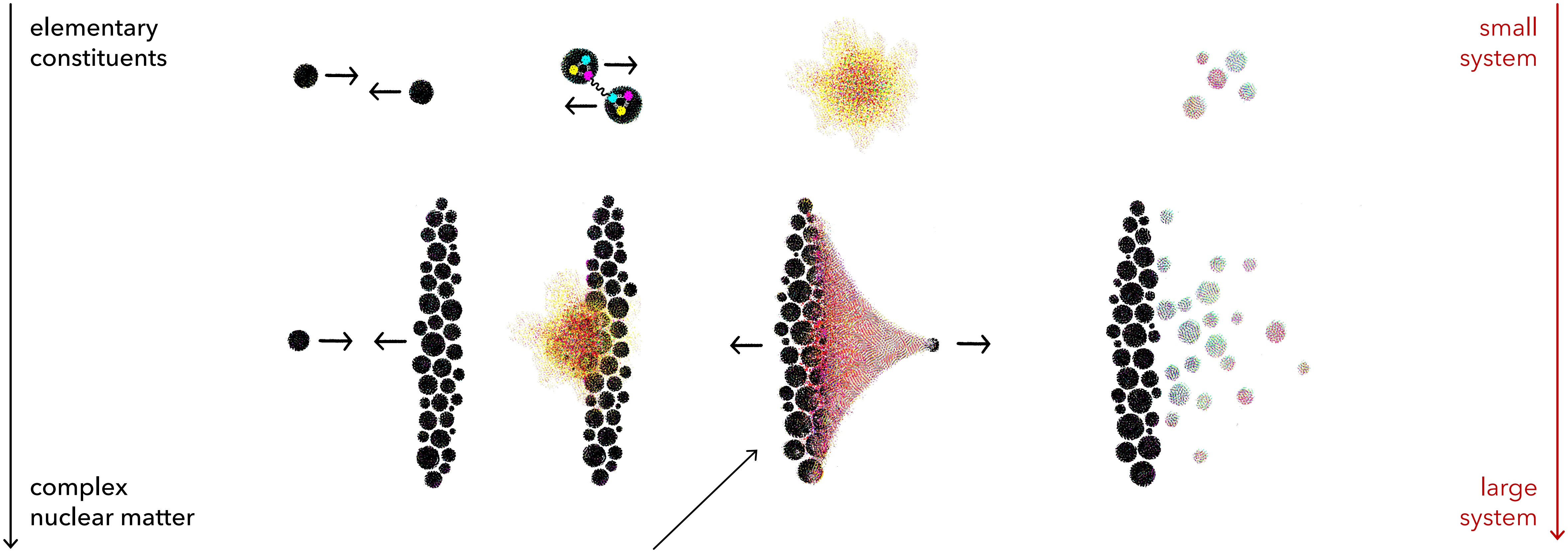
for 微扰量子场论及其应用前沿讲习班

**Andrey Sadofyev**
LIP, Lisbon

# The origin of complex matter



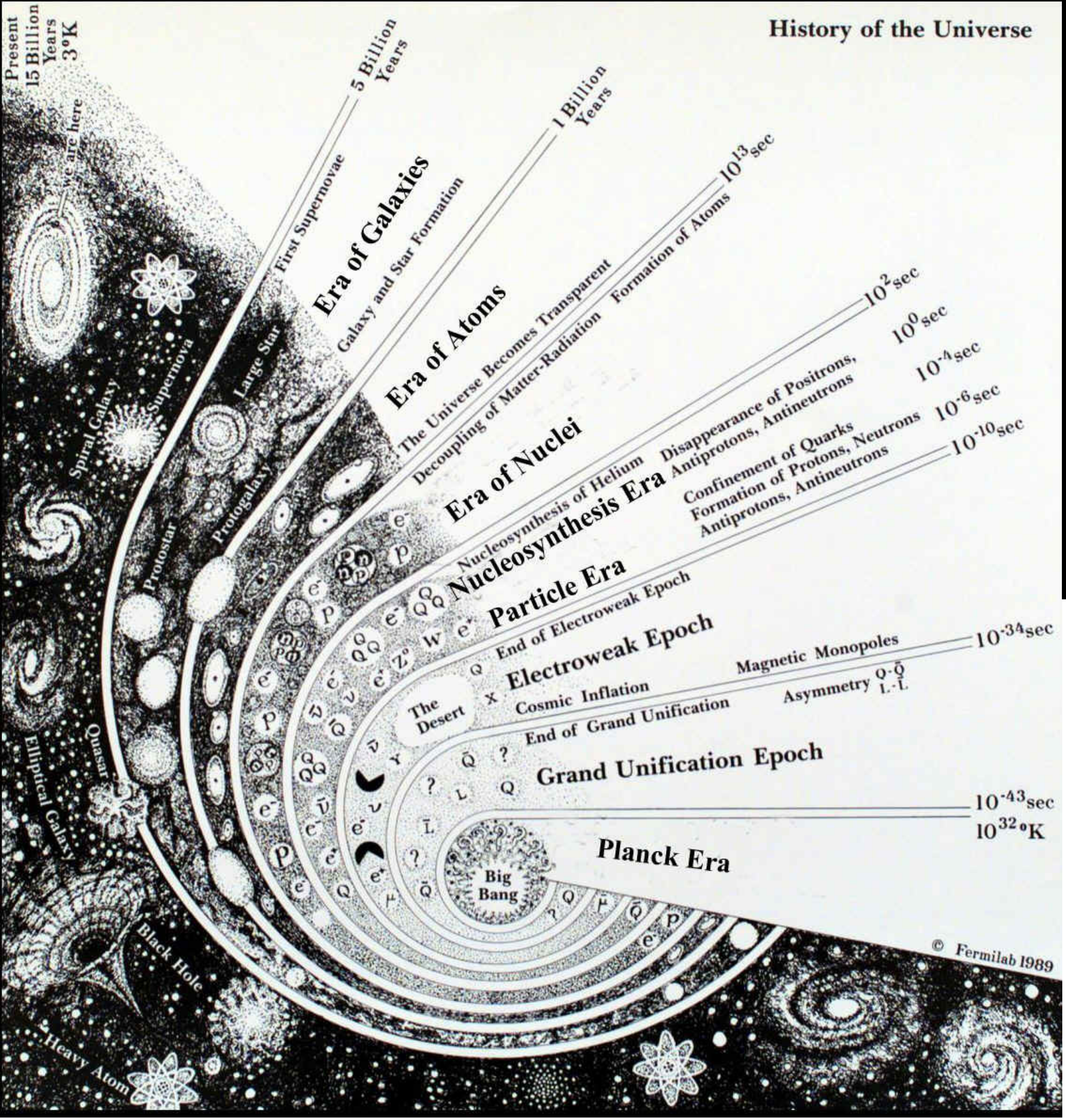
# The origin of complex matter



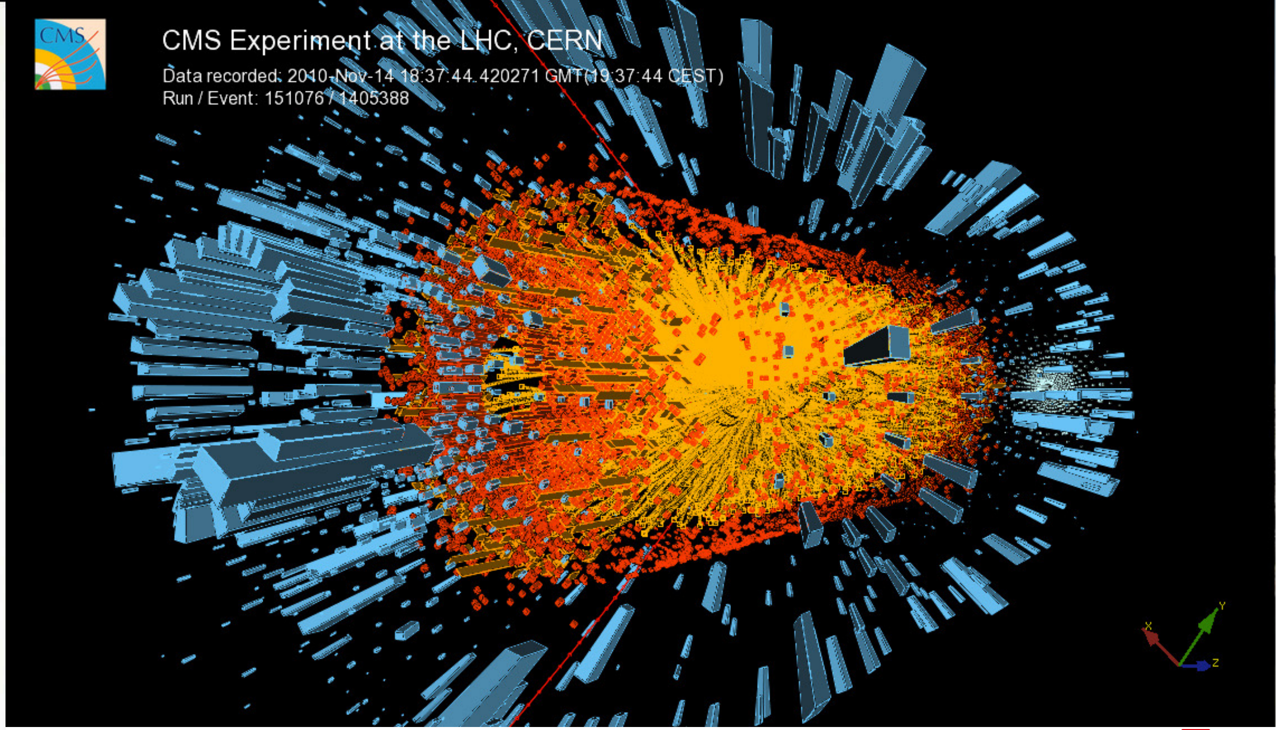

small system:
non-equilibrium matter
from elementary to complex

# Some motivation

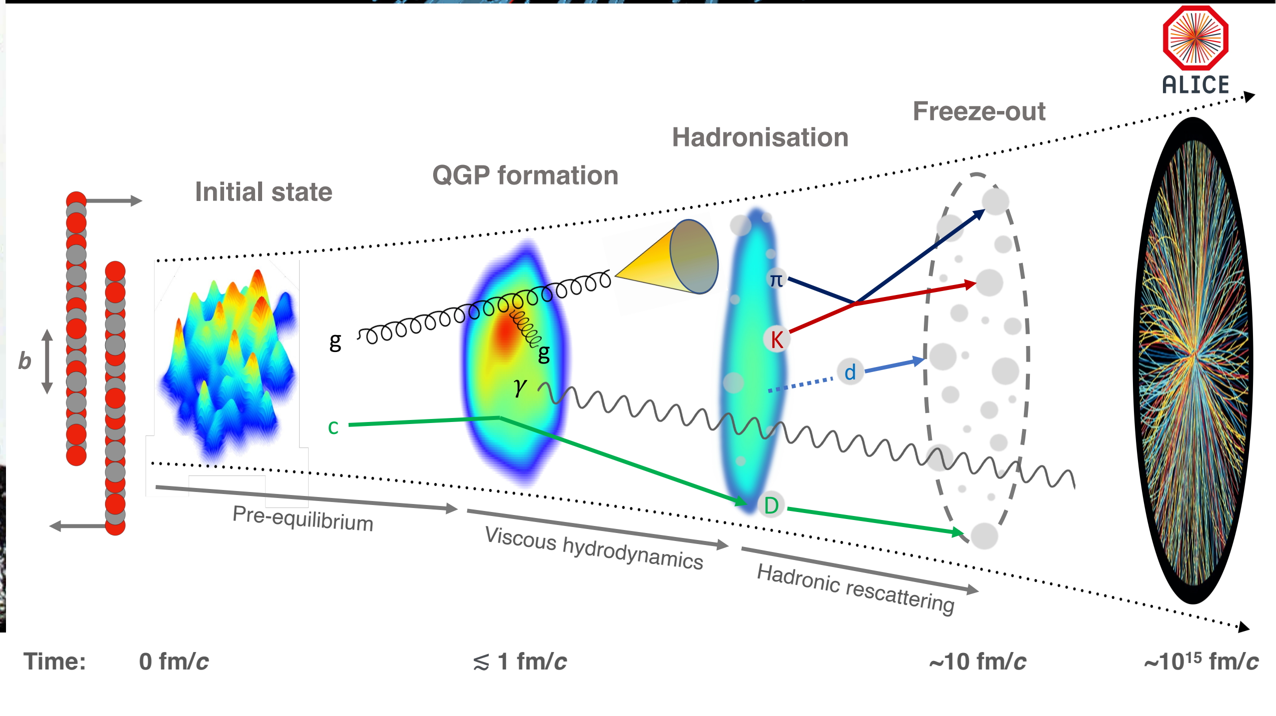
- Learning about QCD in Early Universe ("Big Bang" matter)



History of the Universe
Present
15 Billion Years
3°K
5 Billion Years
1 Billion Years
$10^{13}$sec
$10^{2}$sec
$10^{0}$sec
$10^{-4}$sec
$10^{-6}$sec
$10^{-10}$sec
$10^{-34}$sec
$10^{-43}$sec
$10^{32}$°K
We are here
First Supernovae
Era of Galaxies
Galaxy and Star Formation
Era of Atoms
The Universe Becomes Transparent
Decoupling of Matter-Radiation
Formation of Atoms
Era of Nuclei
Nucleosynthesis of Helium
Nucleosynthesis Era
Disappearance of Positrons, Antiprotons, Antineutrons
Confinement of Quarks
Formation of Protons, Neutrons
Antiprotons, Antineutrons
Particle Era
End of Electroweak Epoch
Electroweak Epoch
The Desert
Cosmic Inflation
Magnetic Monopoles
Asymmetry
End of Grand Unification
Grand Unification Epoch
Planck Era
Big Bang
Spiral Galaxy
Supernova
Large Star
Protogalaxy
Protostar
Quasar
Elliptical Galaxy
Black Hole
Heavy Atom
© Fermilab 1989

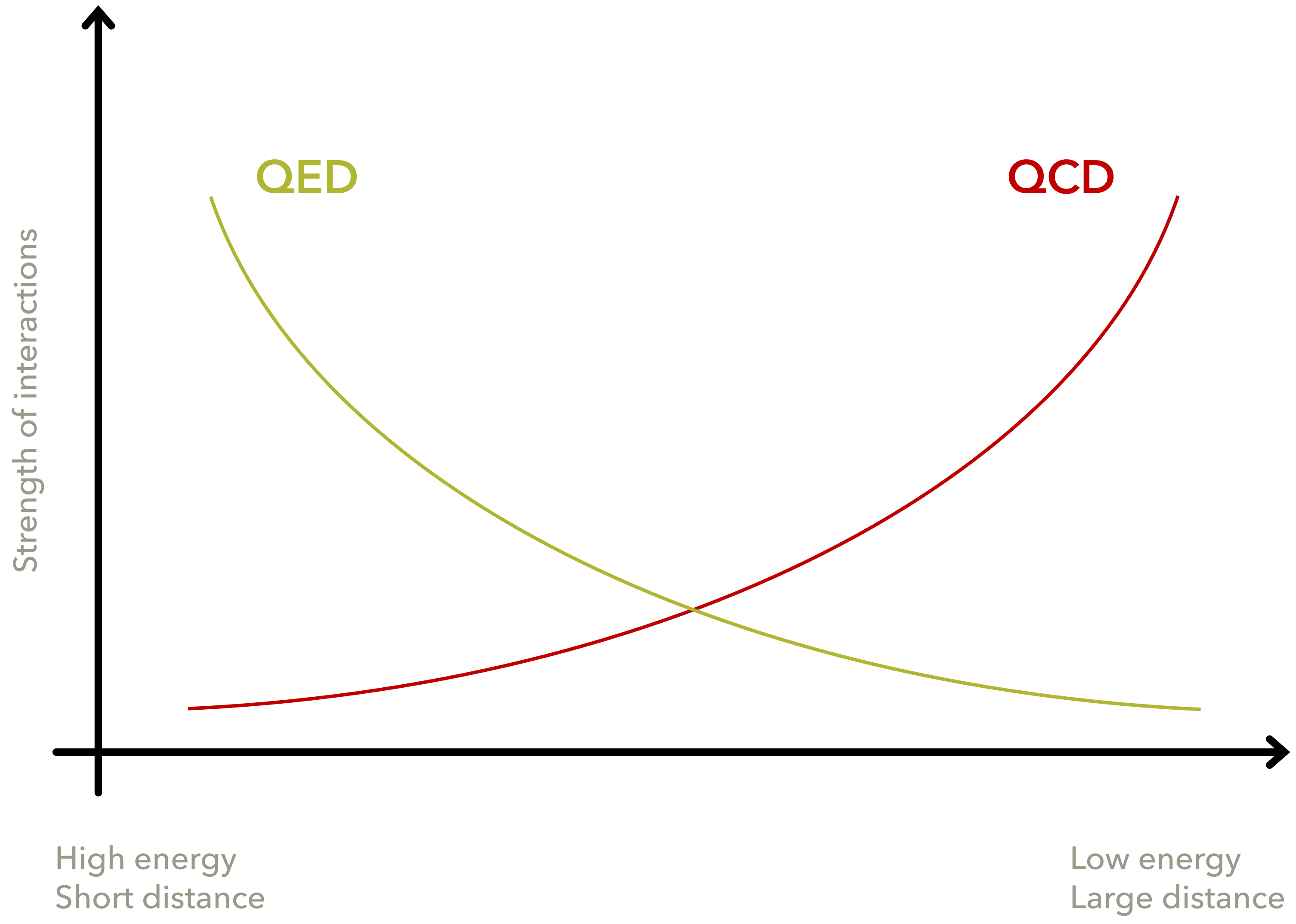
CMS Experiment at the LHC, CERN
Data recorded: 2010-Nov-14 18:37:44.420271 GMT(19:37:44 CEST)
Run / Event: 151076 / 1405388

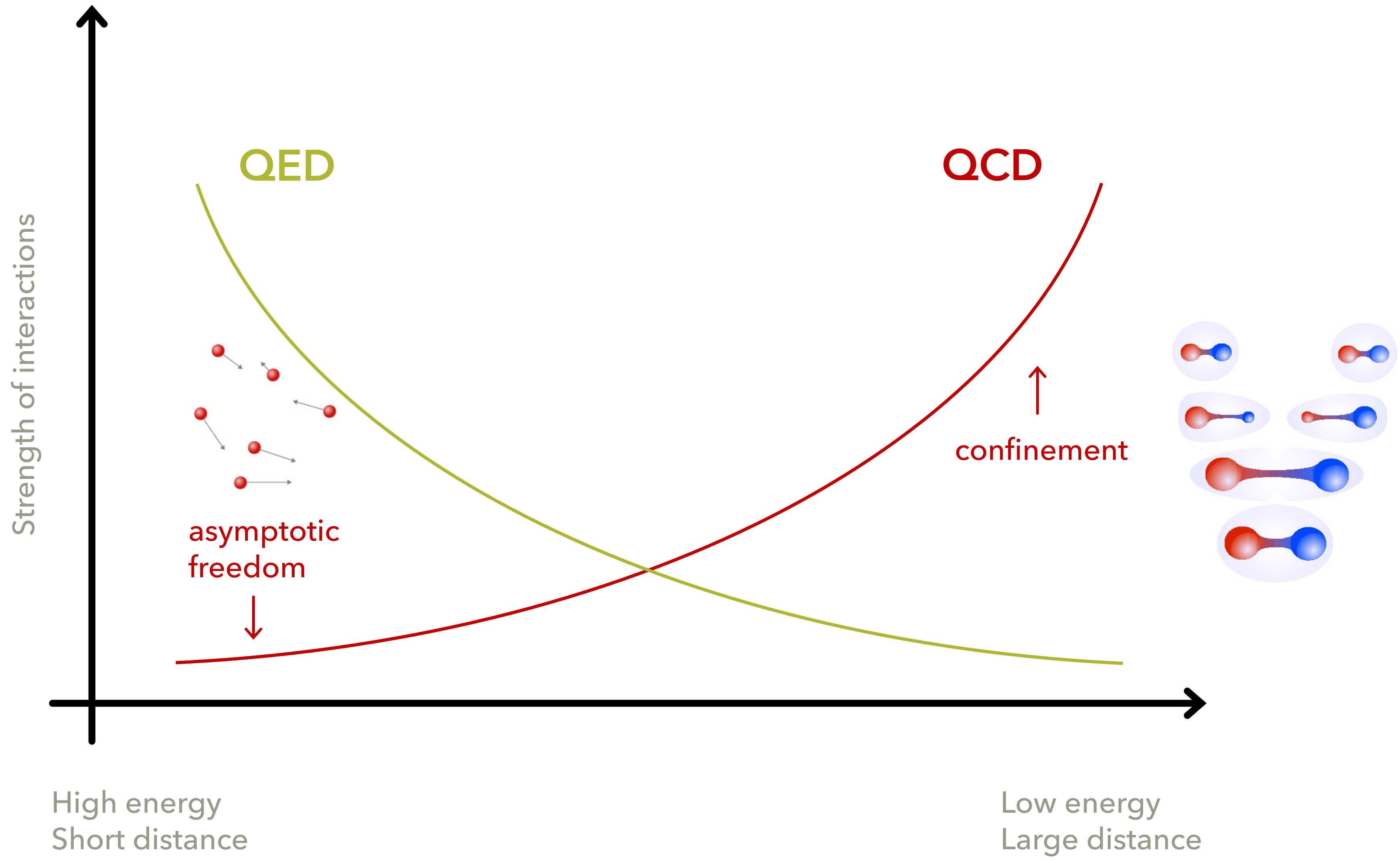



ALICE
Initial state
QGP formation
Hadronisation
Freeze-out
b
g
g
γ
c
π
K
d
D
Pre-equilibrium
Viscous hydrodynamics
Hadronic rescattering
Time:
0 fm/c
≲ 1 fm/c
~10 fm/c
~$10^{15}$ fm/c

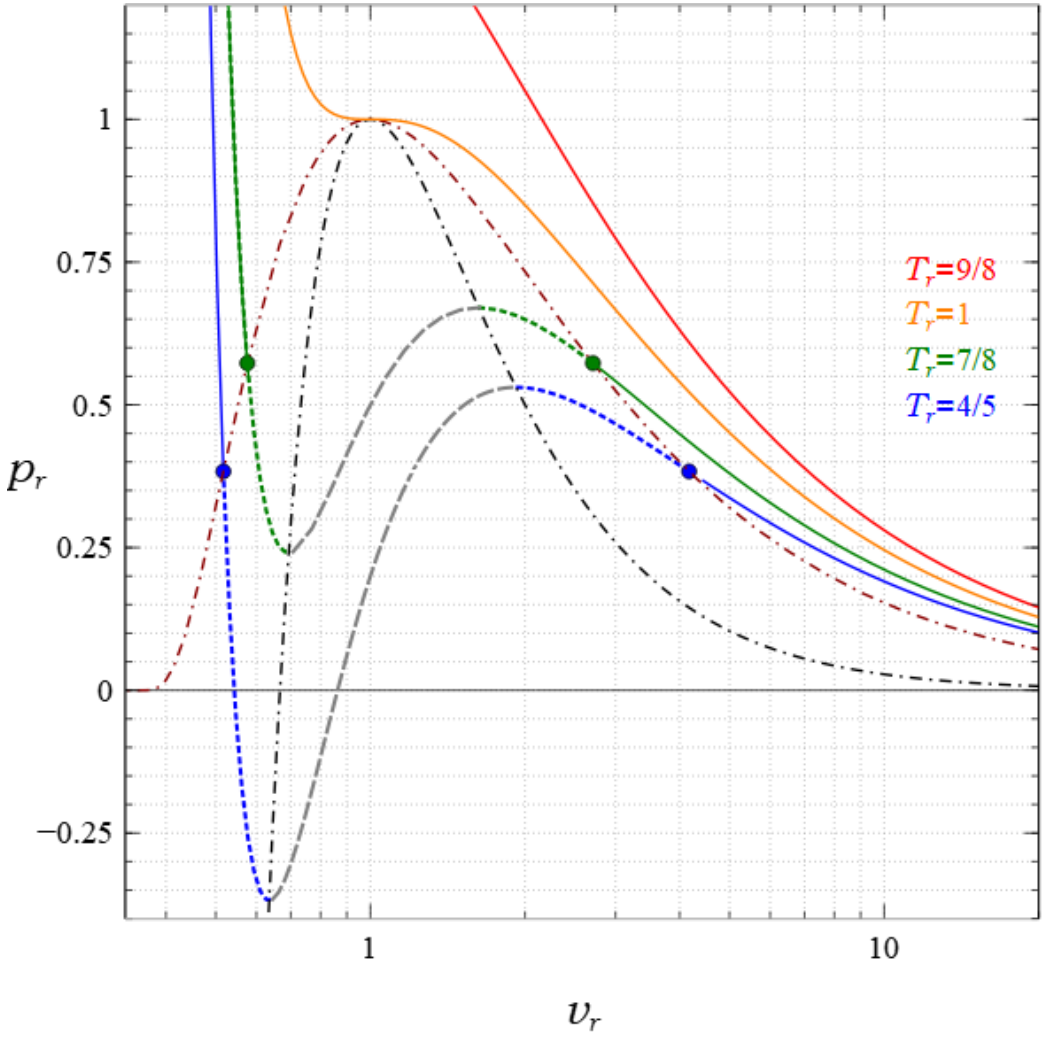
# Some motivation

- Learning about QCD in Early Universe (”Big Bang” matter)
- Probing QCD phase diagram (extreme conditions)

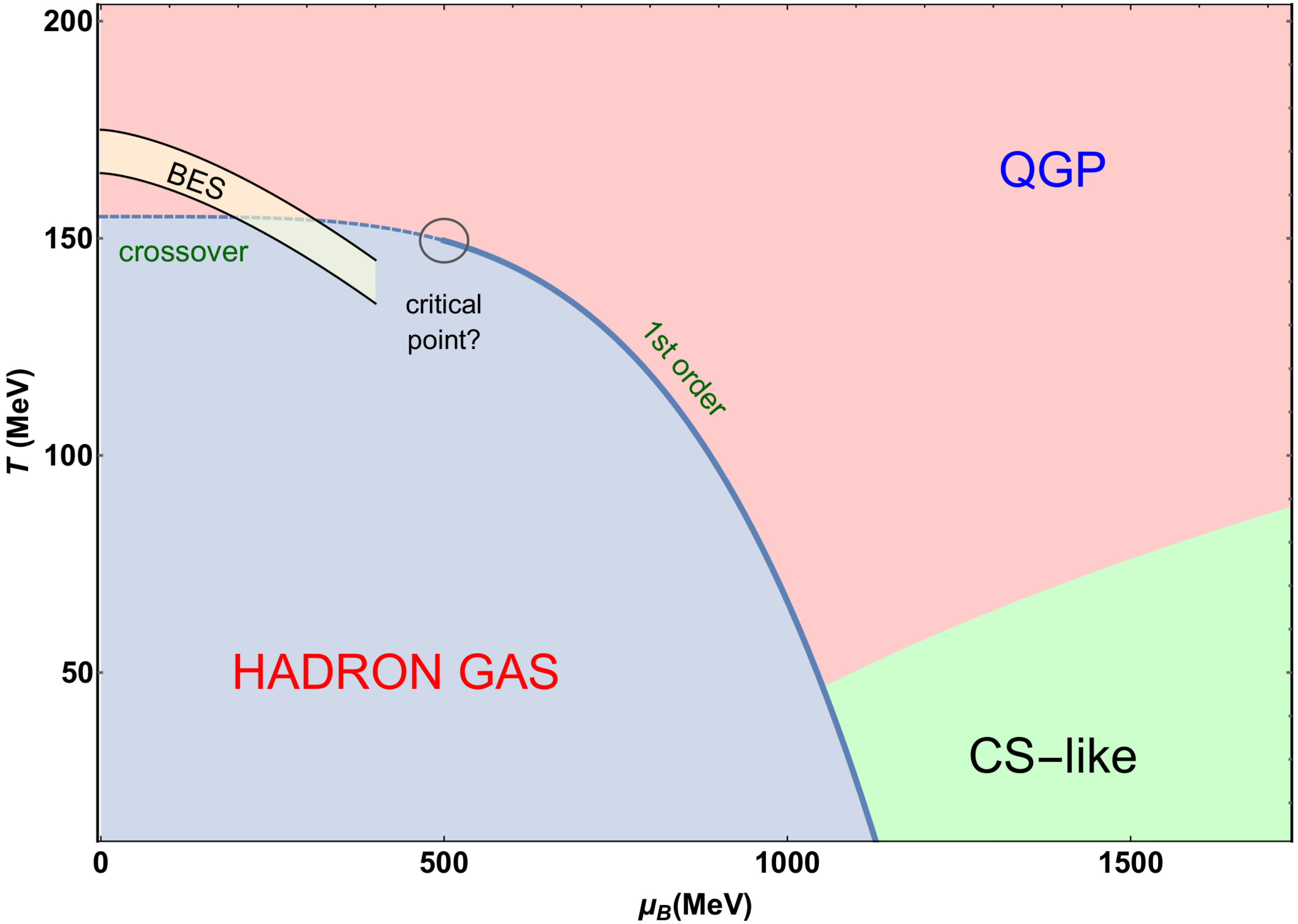

QED
QCD
Strength of interactions
High energy
Short distance
Low energy
Large distance


QED
QCD
Strength of interactions
asymptotic
freedom
confinement
High energy
Short distance
Low energy
Large distance

## van der Waals gas




## QCD matter

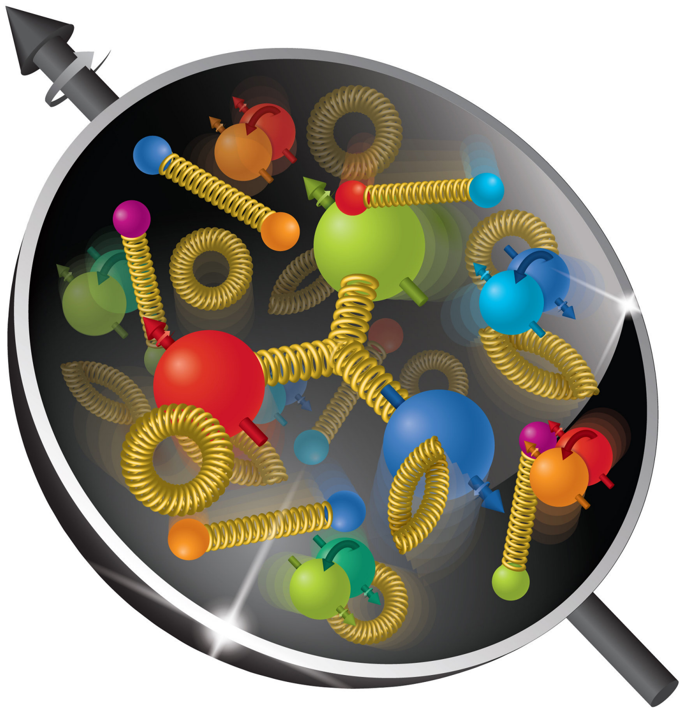


# Some motivation

- Learning about QCD in Early Universe ("Big Bang" matter)
- Probing QCD phase diagram (extreme conditions)
- Probing  nuclear structure (probing partons)

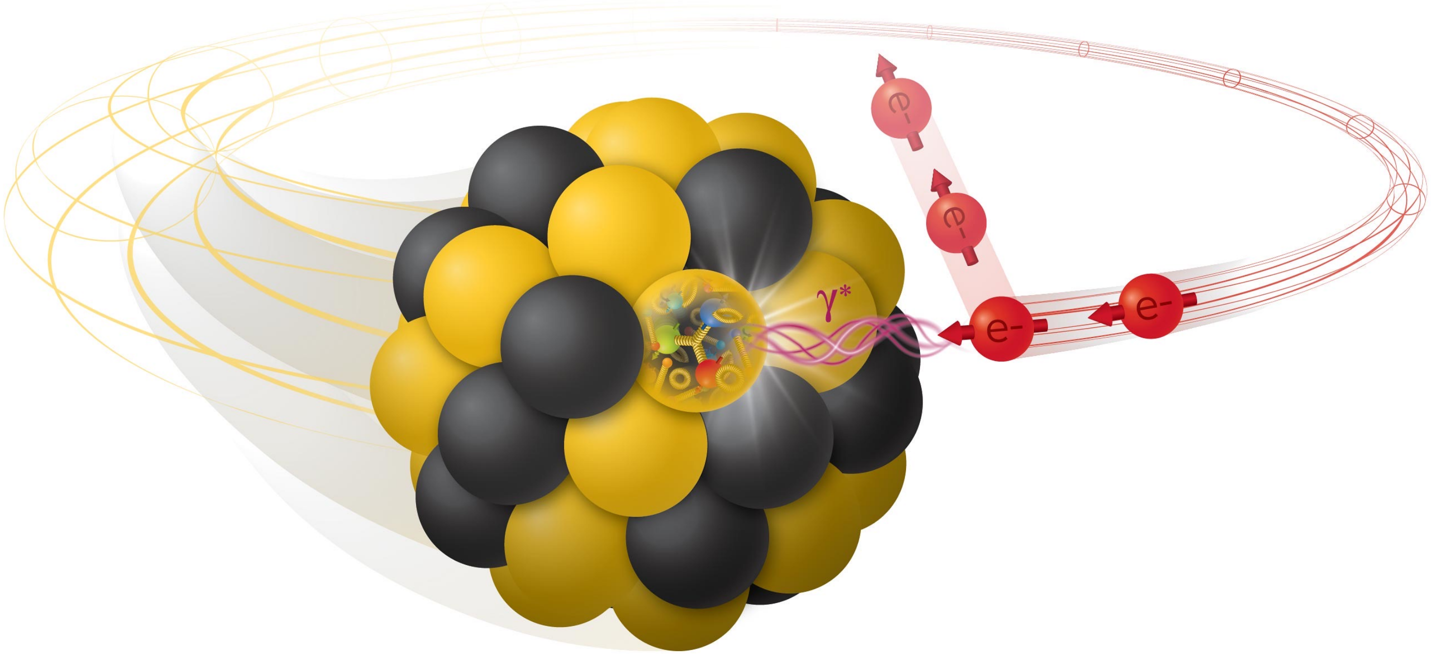
where does the spin
of proton come from?

↓



↑

where does the mass
of proton come from?

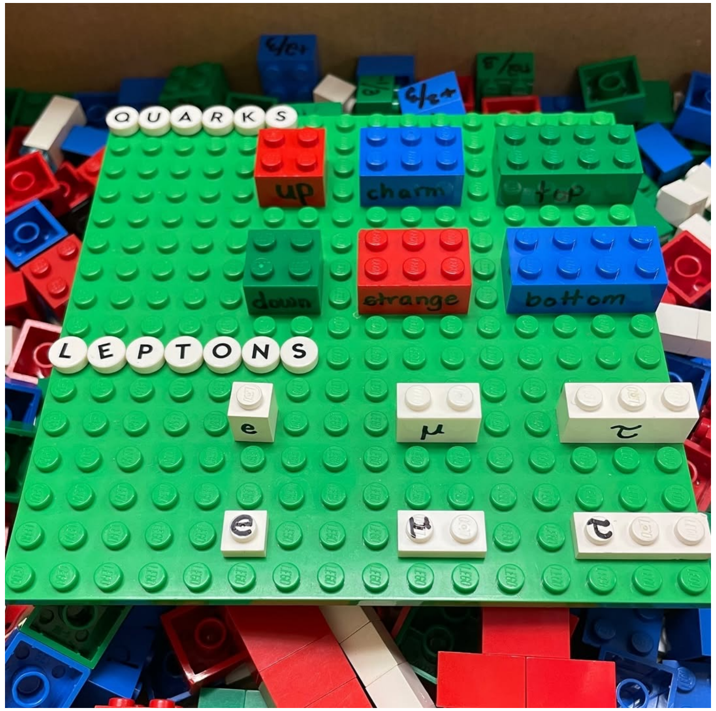


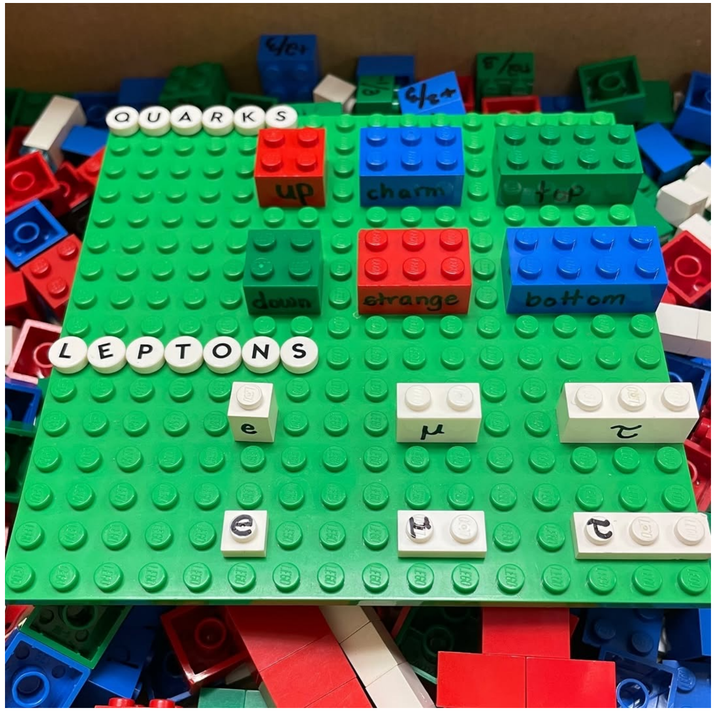

↑

Do we understand
how nuclei are formed?

# Some motivation

- Learning about QCD in Early Universe ("Big Bang" matter)
- Probing QCD phase diagram (extreme conditions)
- Probing  nuclear structure (probing partons)
- Understanding matter formation (and its evolution)


QUARKS
up
charm
top
down
strange
bottom
LEPTONS
e
μ
τ


QUARKS
up
charm
top
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strange
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LEPTONS
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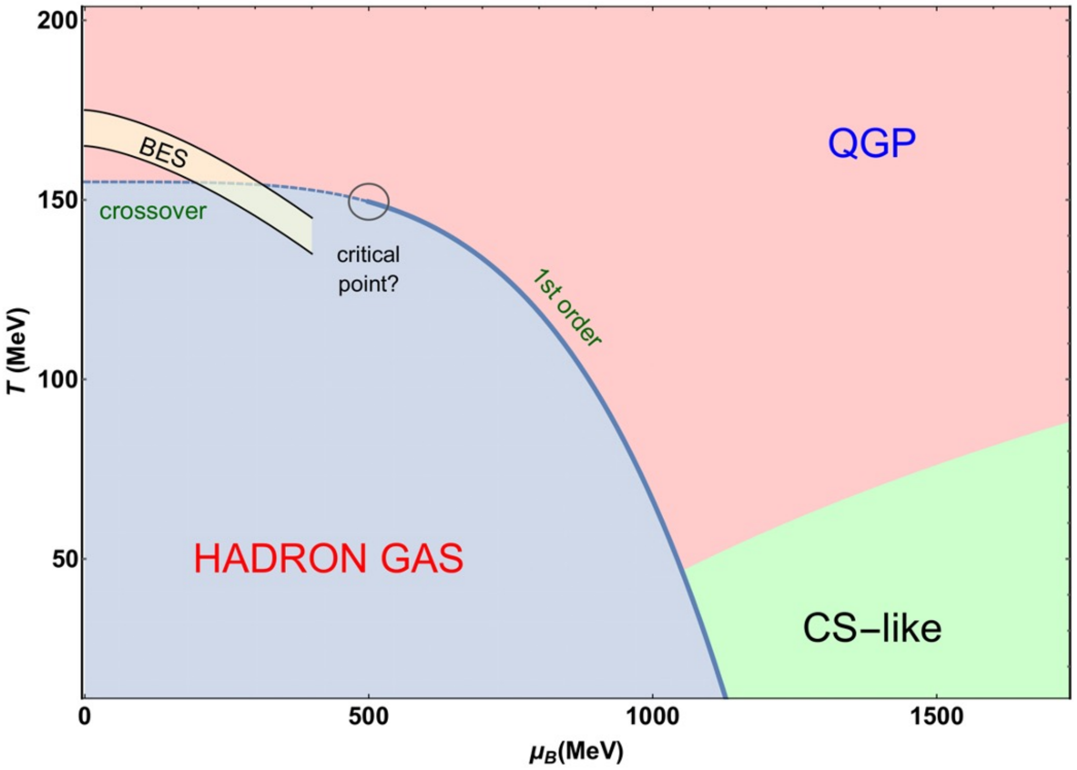
μ
τ


200
150
100
50
T (MeV)
0
500
1000
1500
μB(MeV)
BES
crossover
critical point?
1st order
QGP
HADRON GAS
CS–like

# Some motivation

- Learning about QCD in Early Universe ("Big Bang" matter)
- Probing QCD phase diagram (extreme conditions)
- Probing  nuclear structure (probing partons)
- Understanding matter formation (and its evolution)

K. Rajagopal et al., review, 2018

# Some motivation

- Learning about QCD in Early Universe ("Big Bang" matter)
- Probing QCD phase diagram (extreme conditions)
- Probing  nuclear structure (probing partons)
- Understanding matter formation (and its evolution)
- …

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- Learning about QCD in Early Universe ("Big Bang" matter)
- Probing QCD phase diagram (extreme conditions)
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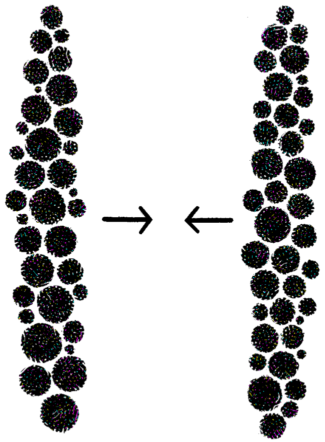
- Understanding matter formation (and its evolution)
- …
- Add your favorite option here

# Stages of QCD matter in HIC



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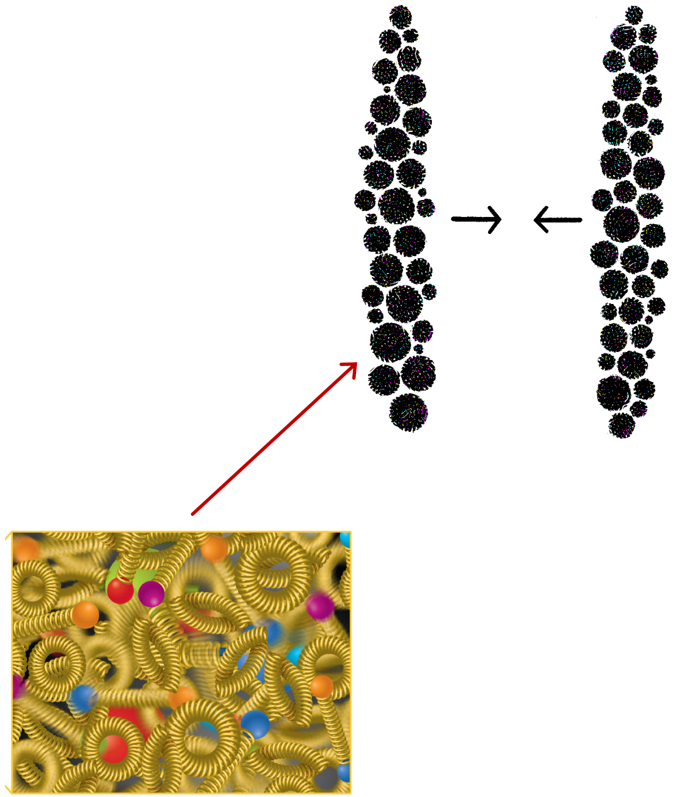
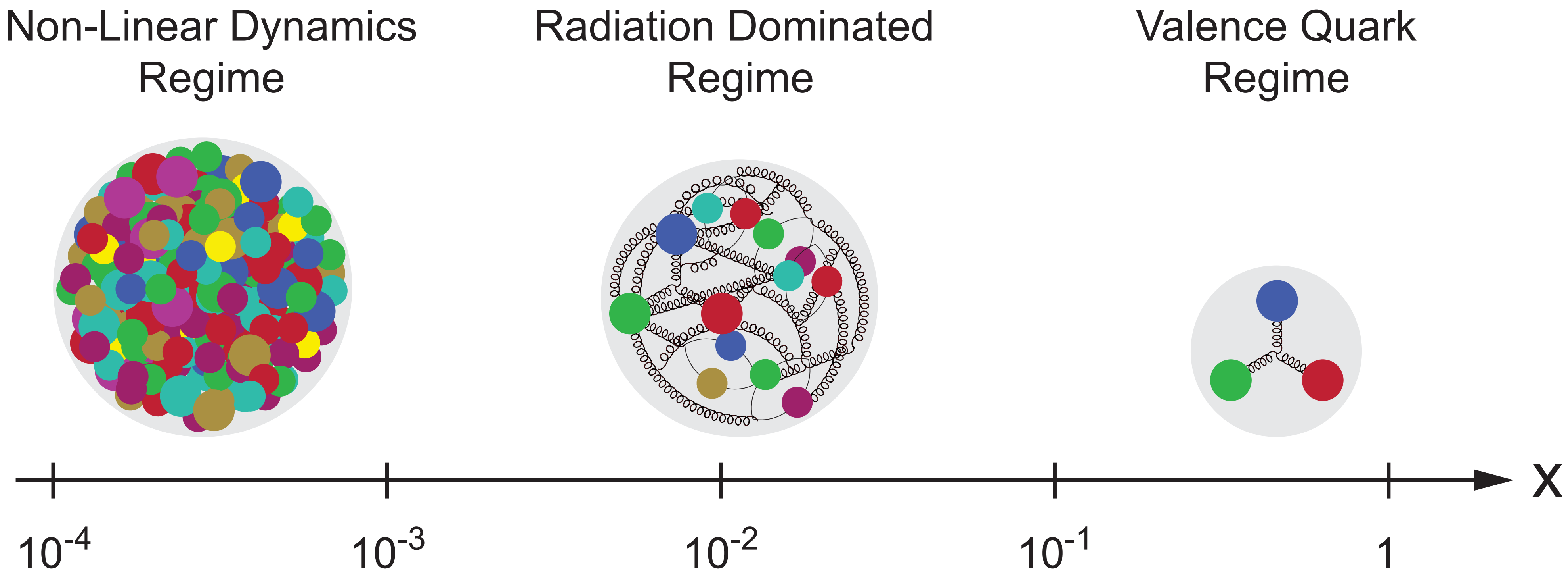
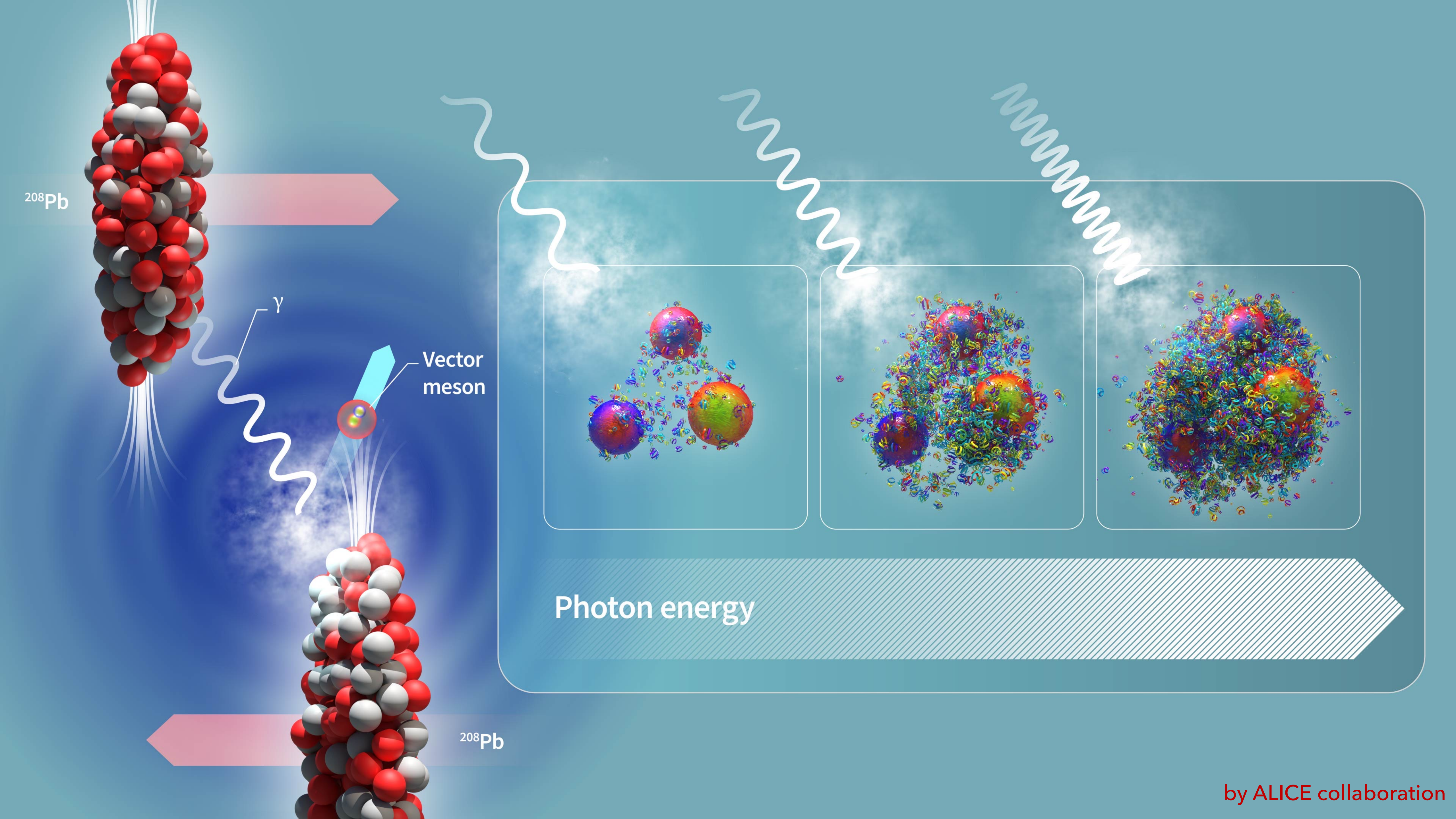


# Color glass condensate

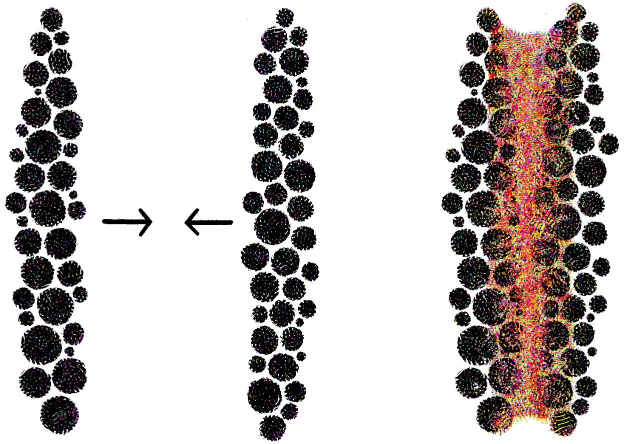



$$dP \sim \frac{\alpha_s}{x} dx$$ → large Q (resolution) small x (high density) → $$D_\mu F^{\mu\nu} = J^\nu$$



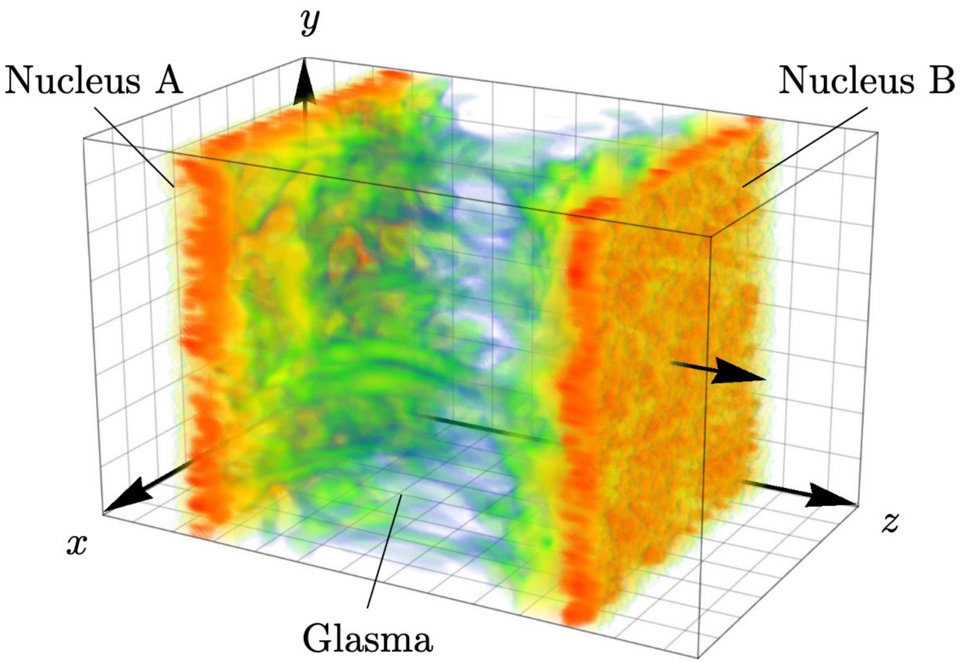
$^{208}$Pb
γ
Vector meson
$^{208}$Pb
Photon energy
by ALICE collaboration

# Stages of QCD matter in HIC

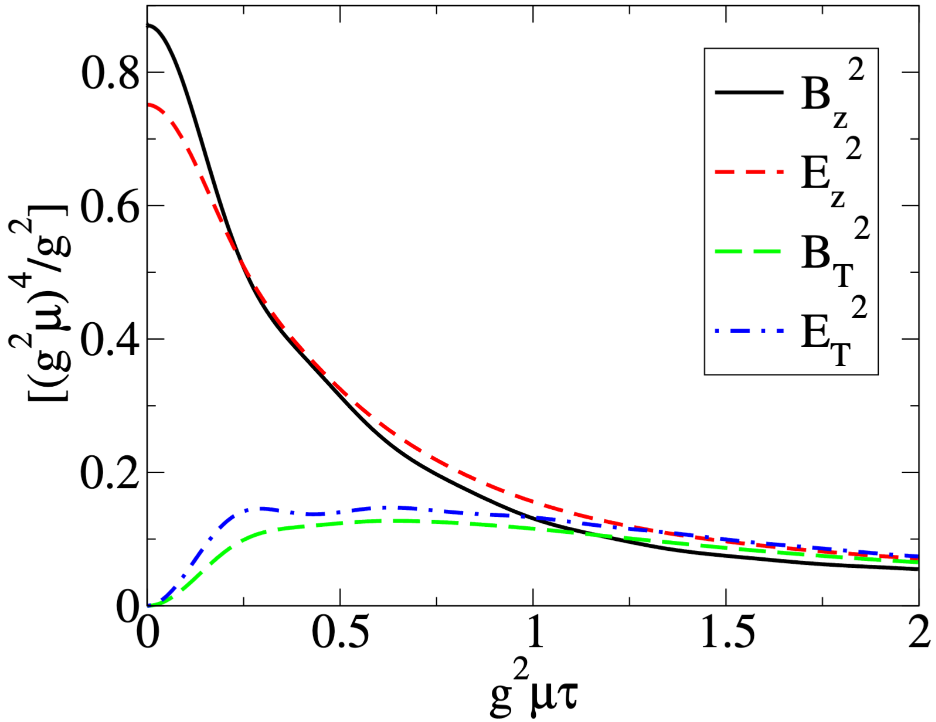


# Glasma

from a paper by Andreas Ipp et al.

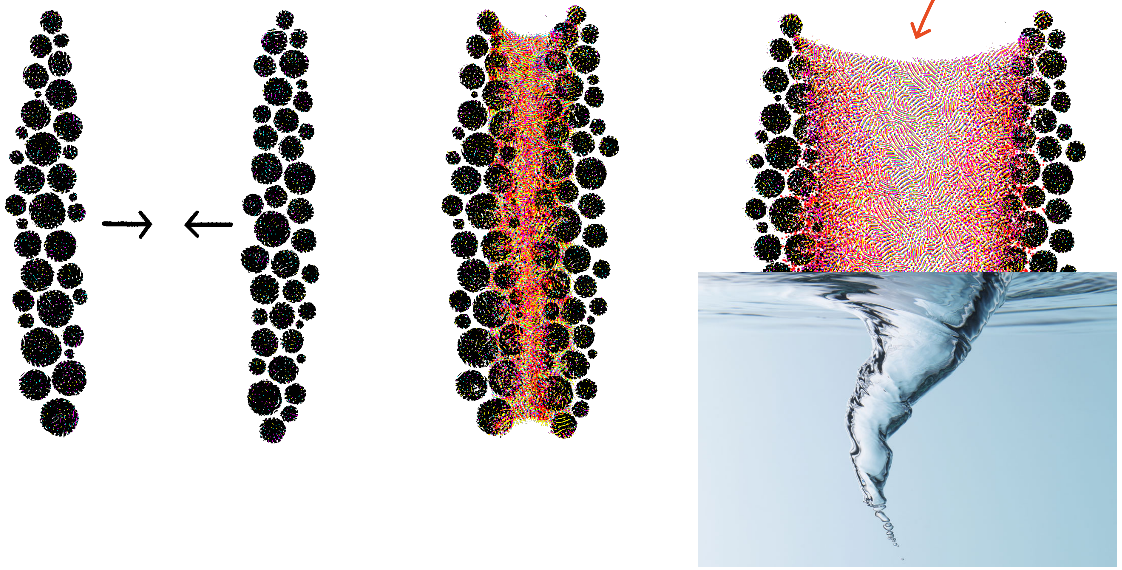



→

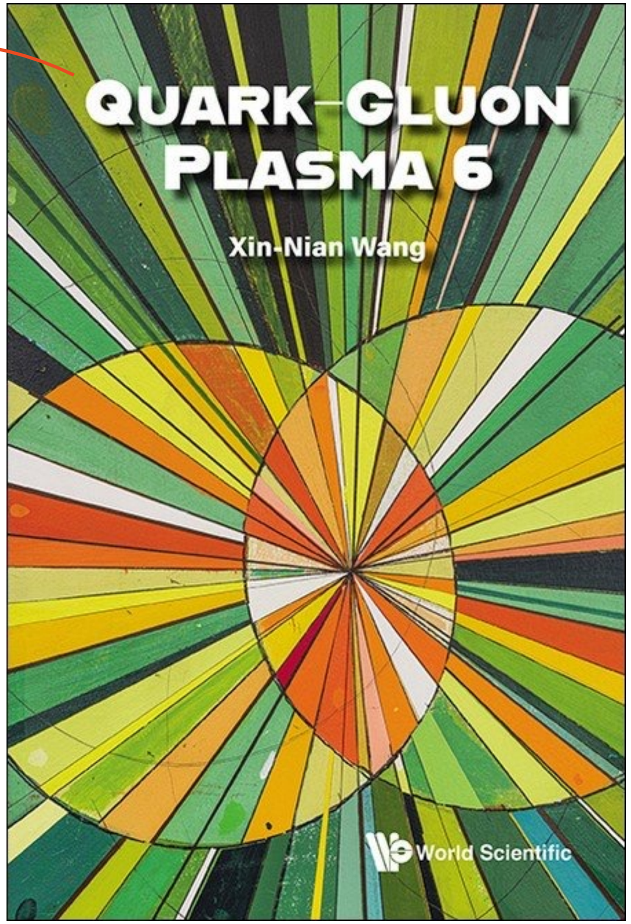


# Stages of QCD matter in HIC

# Quark-Gluon Plasma

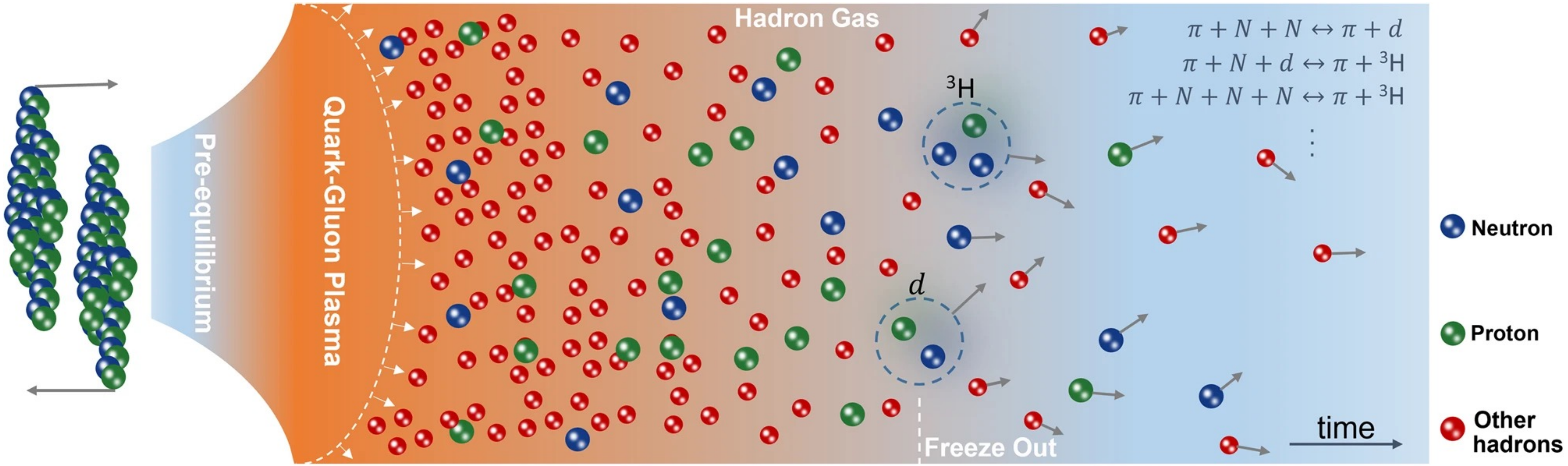


relativistic fluid



# Stages of QCD matter in HIC

# Hadron gas

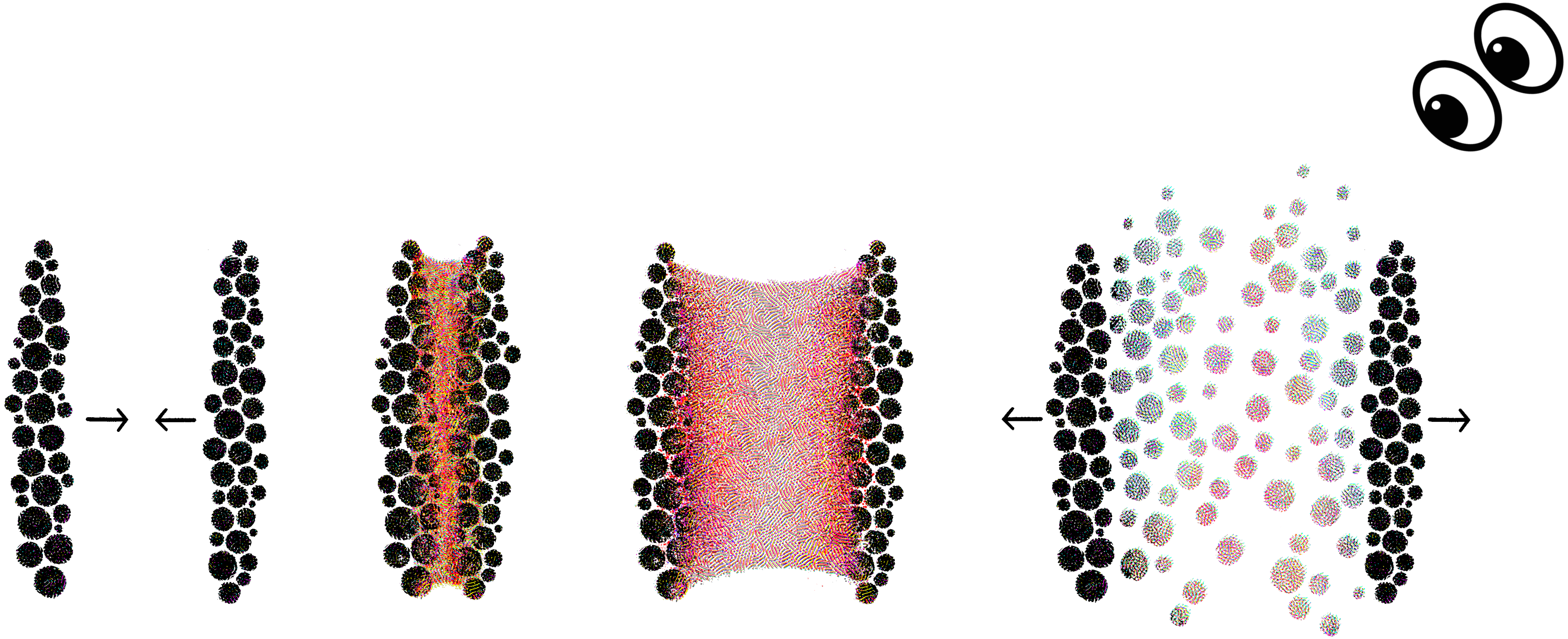



from K.-J. Sun, R. Wang, C.M. Ko, Y.-G. Ma, C. Shen, Nature Commun., 2024

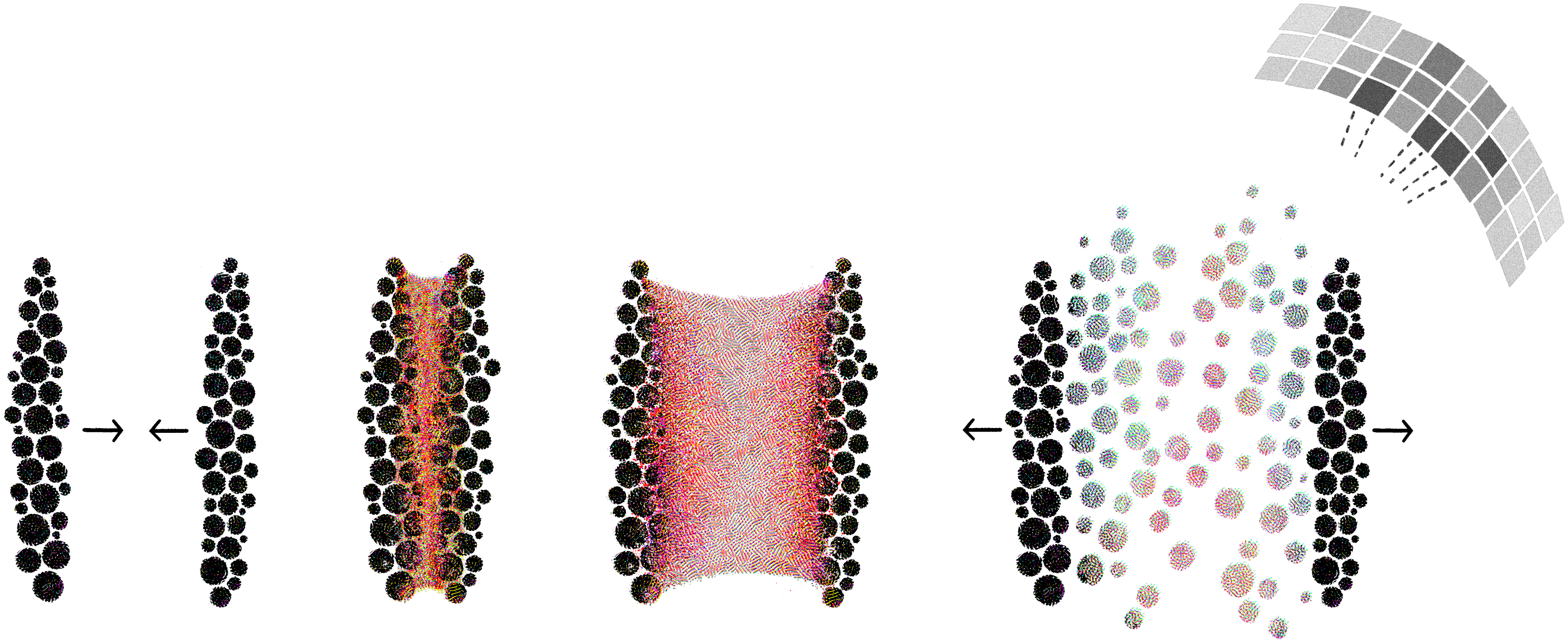
# Stages of QCD matter in HIC

- Nuclear matter produced in HIC undergoes a multiphase evolution
- We have some ideas about how it behaves during these stages, but they are mostly qualitative
- Disentangling detailed information from the experimental picture is challenging
- So, one of the central questions in the field is: How do we probe this matter? (What should we measure? What should we calculate?)

# Probes of the matter

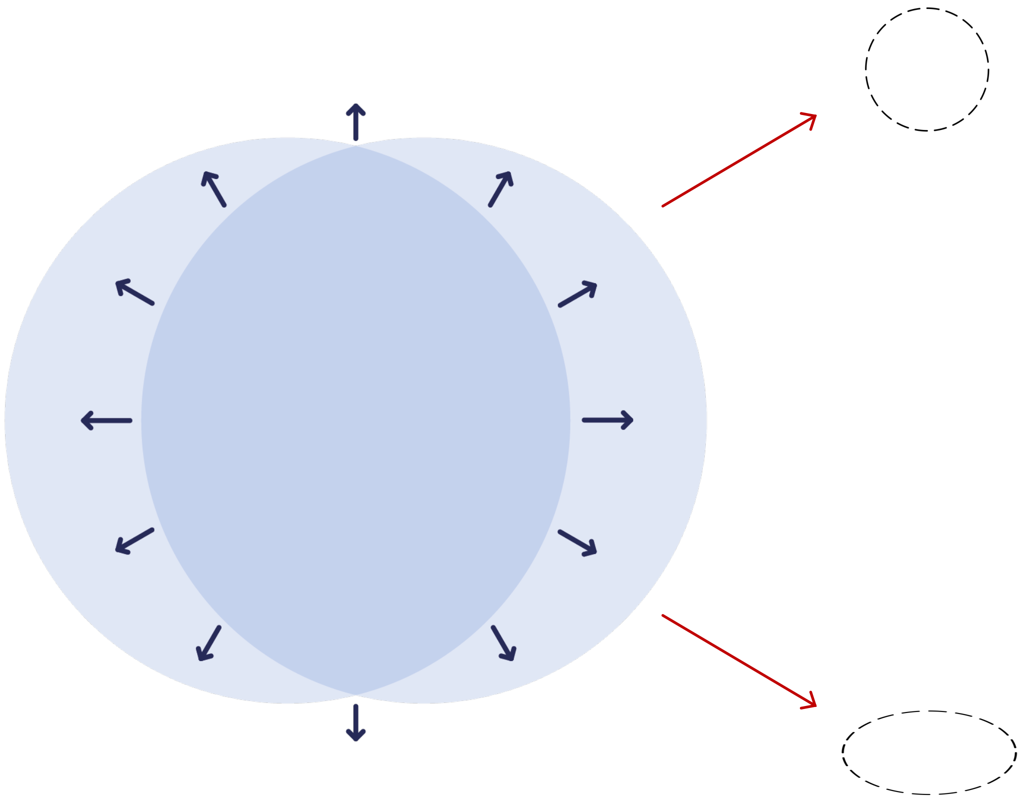


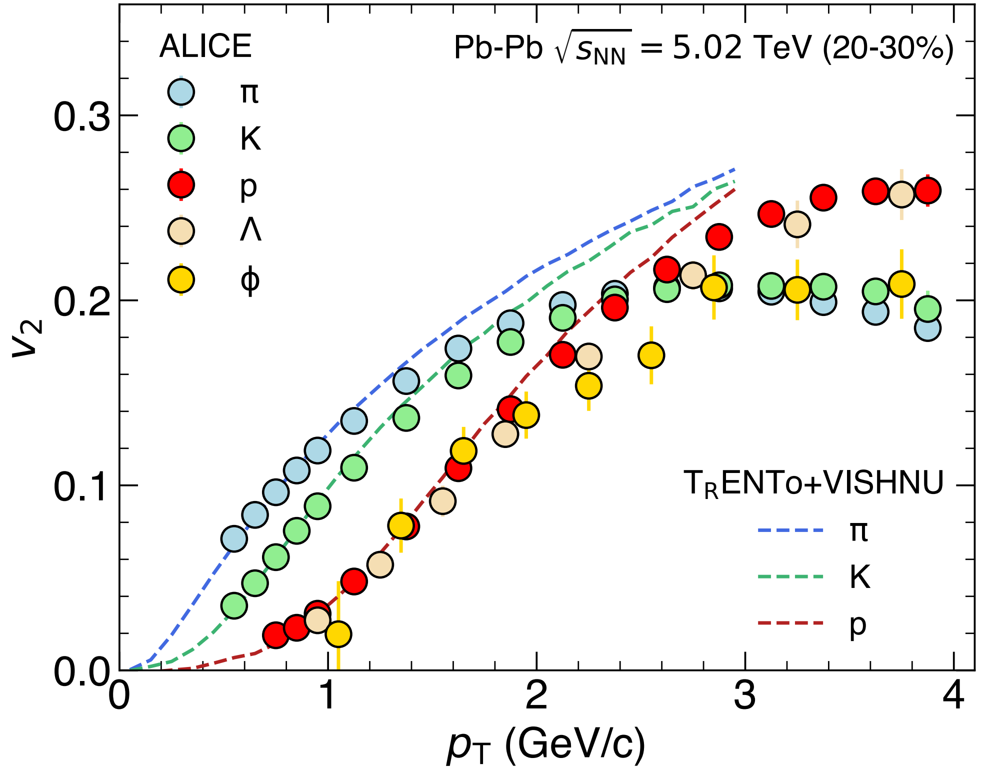
# Probes of the matter



Hot QCD White Paper, arxiv: 2023.17254

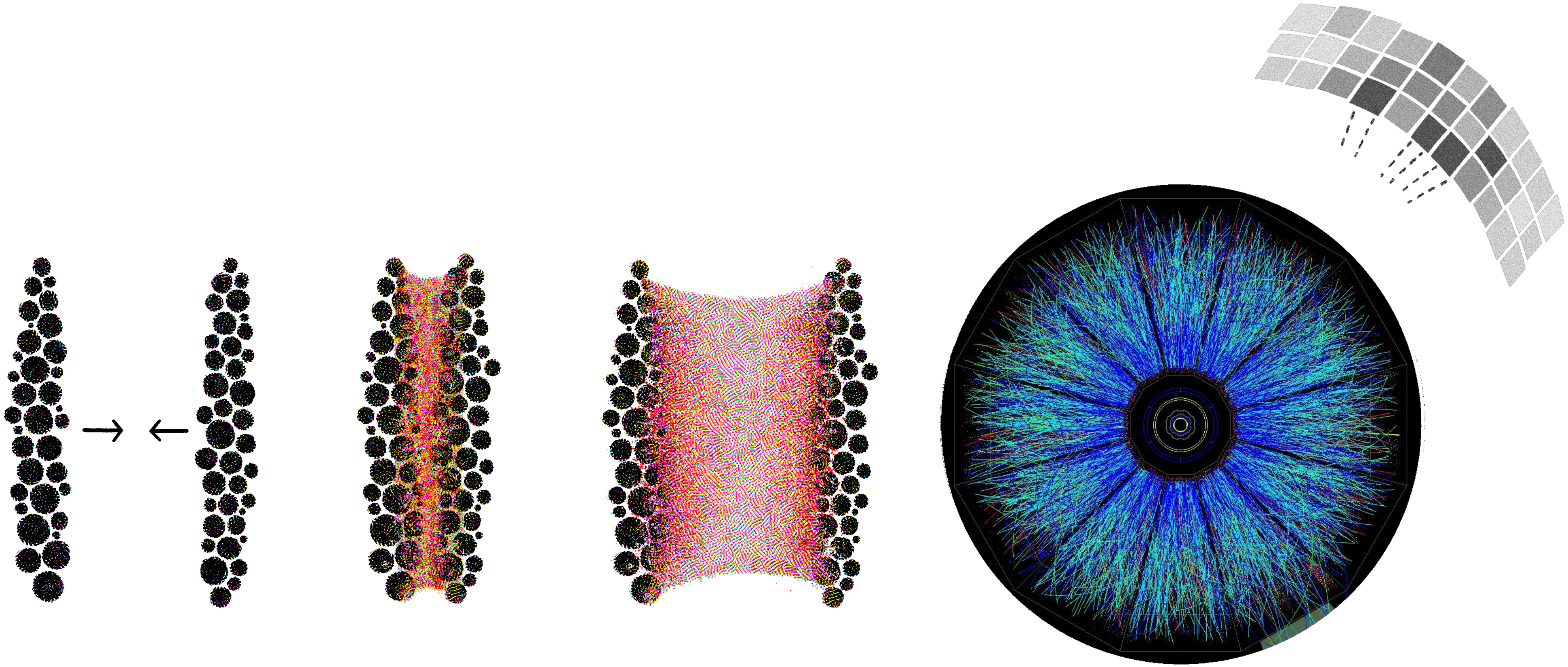
# Probes of the matter



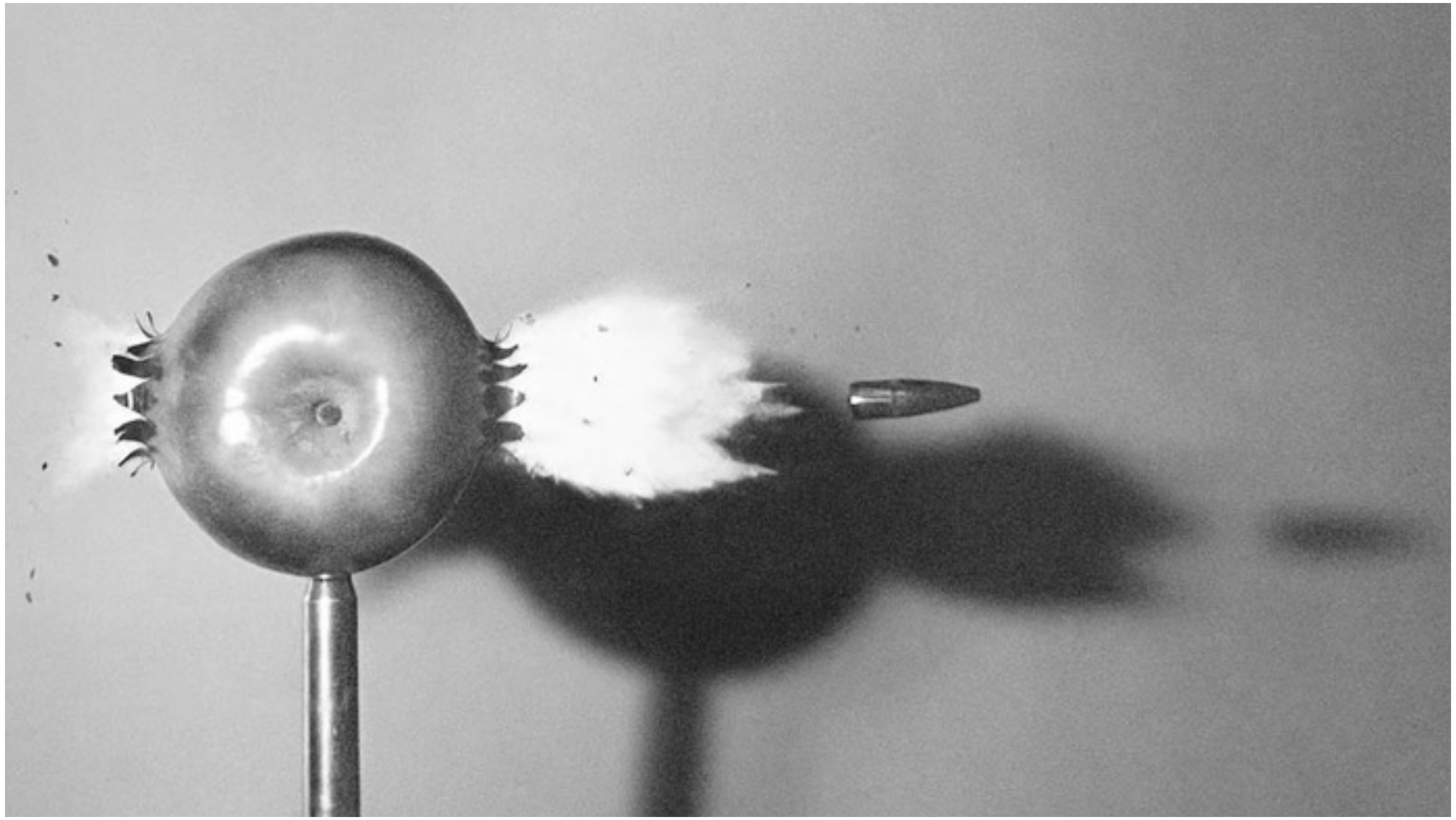



$$\frac{dN}{d\phi} \propto 1 + 2v_1 \cos\left[\phi - \Psi_{RP}\right] + 2v_2 \cos\left[2(\phi - \Psi_{RP})\right] + \ldots$$

# Probes of the matter

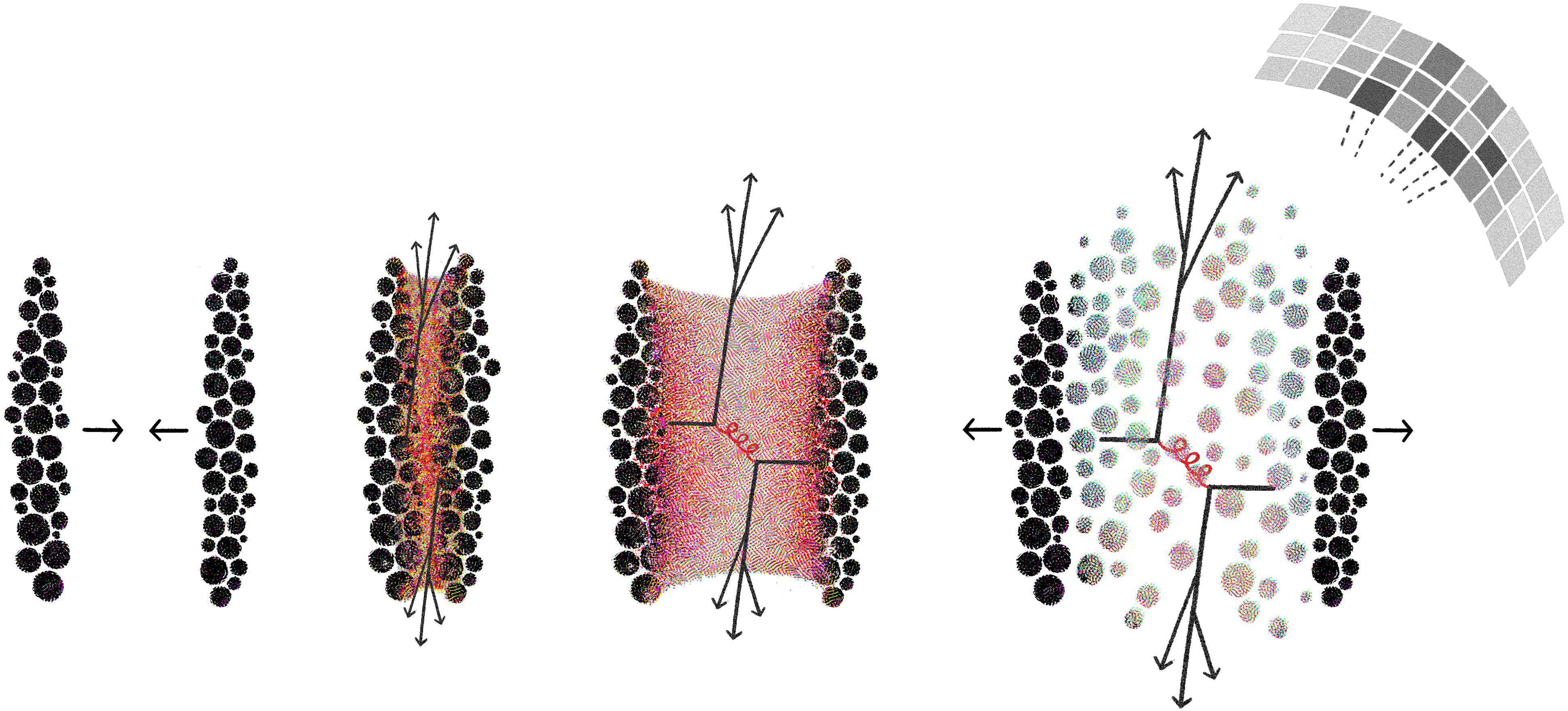


# Hard probes

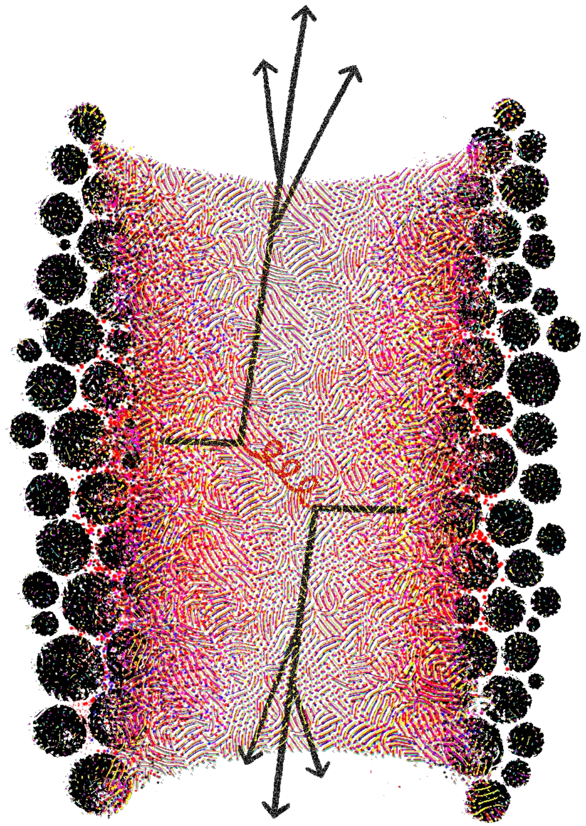


Final render
Ex: 75306
Se 1/1
Im 1/2
Lin: DCM
W: 4095 L: 2089
Acq tm: 09:52:07

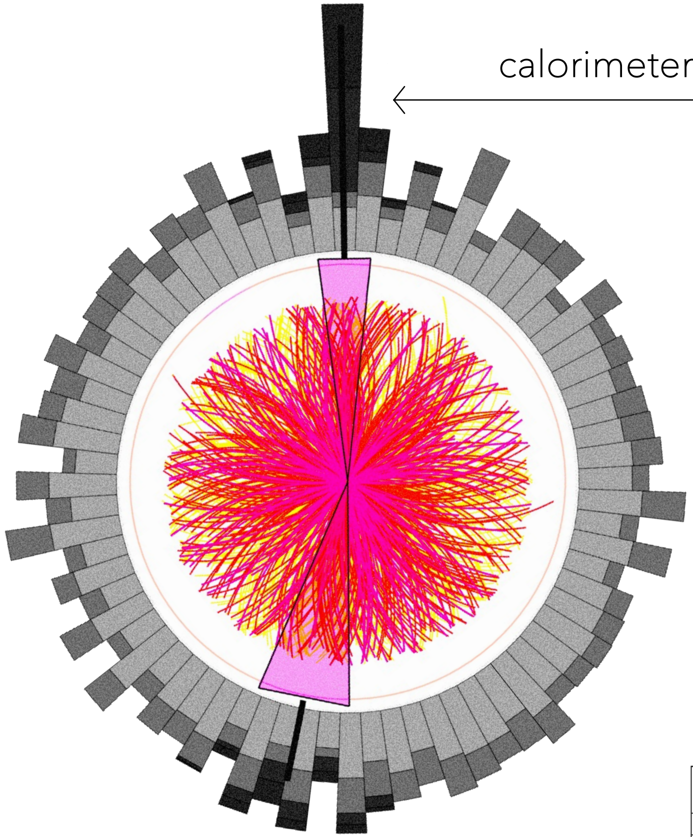
# Hard probes



# Hard probes

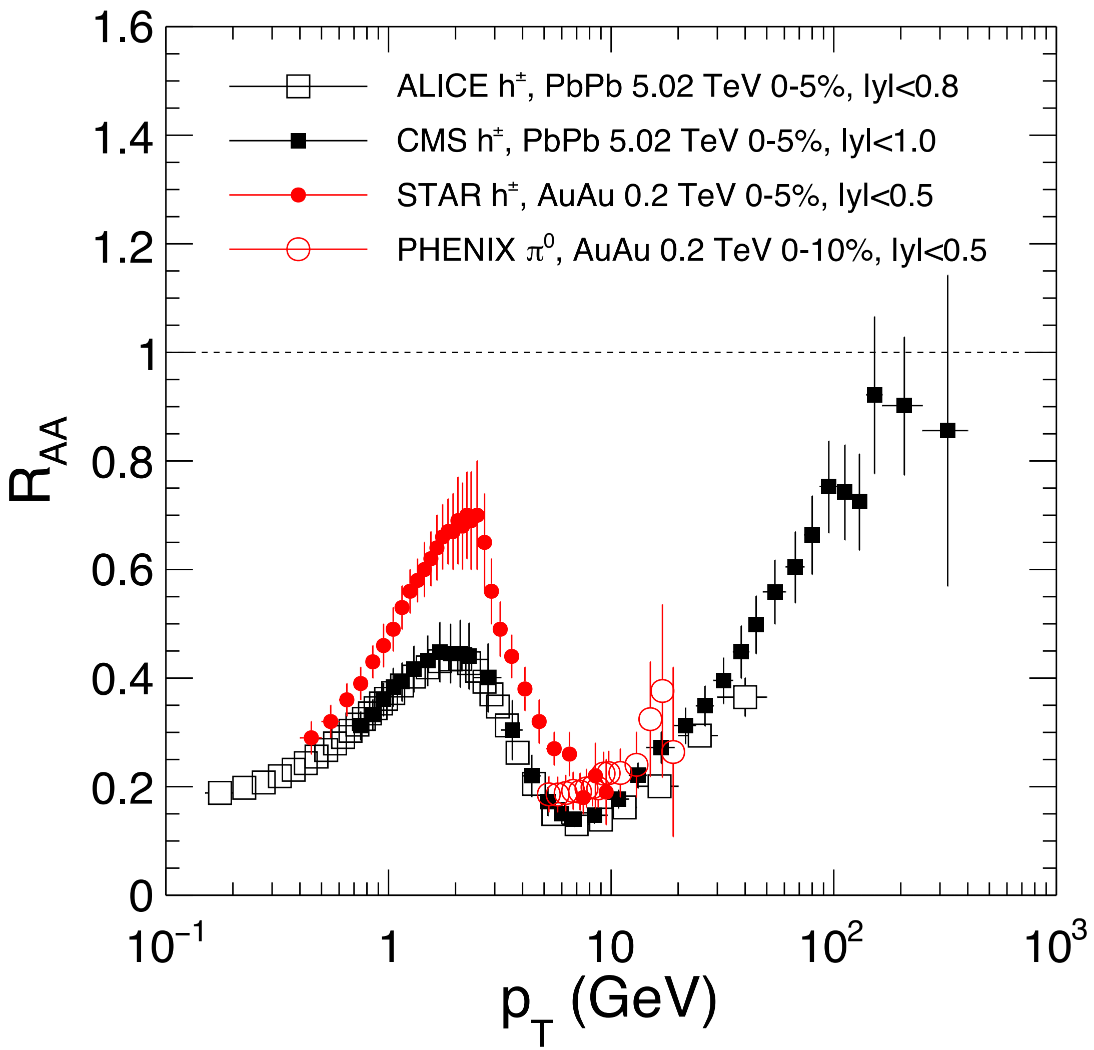


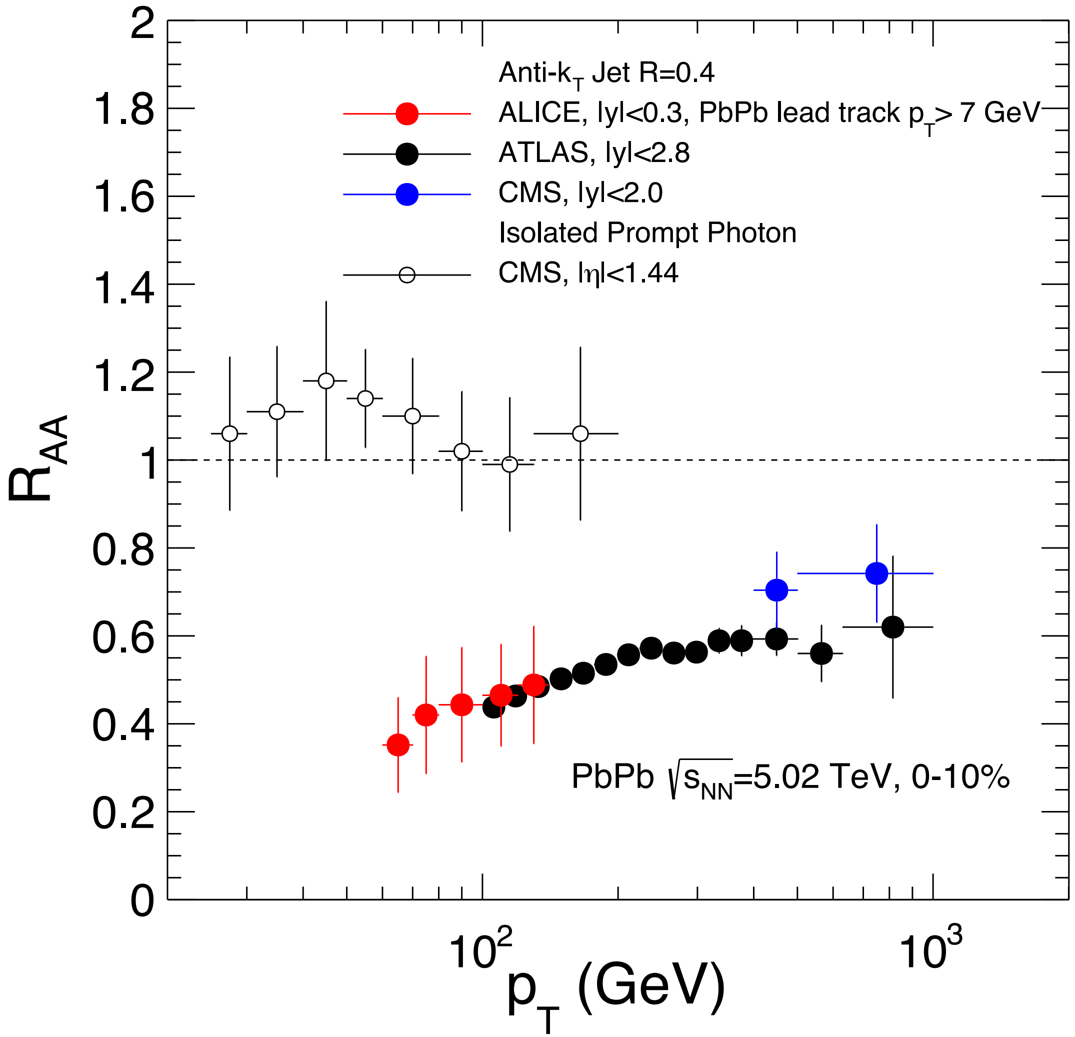
jet quenching



calorimeter tower

jets lose energy as they propagate through nuclear matter

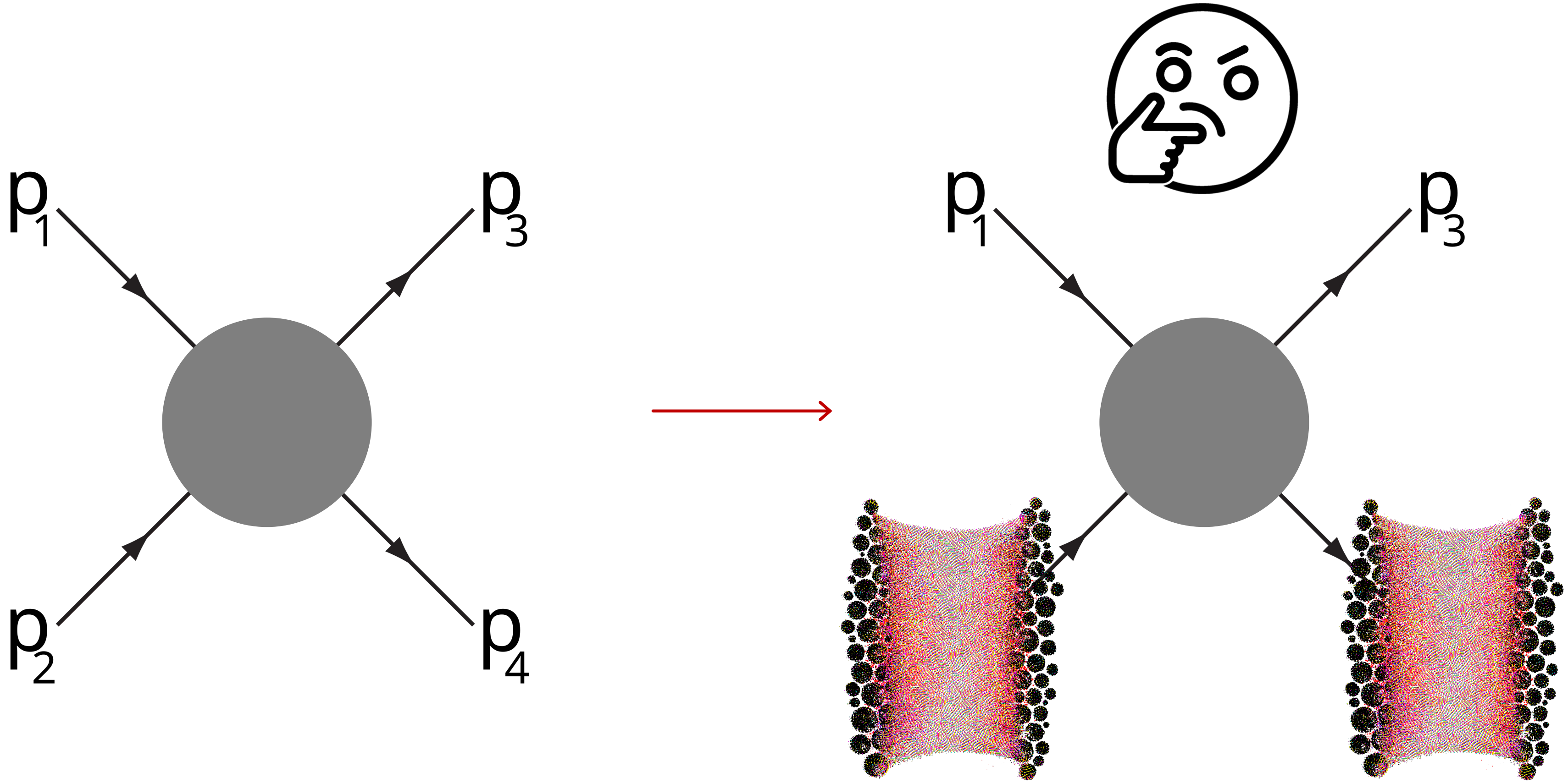

ALICE h±, PbPb 5.02 TeV 0-5%, lyl<0.8
CMS h±, PbPb 5.02 TeV 0-5%, lyl<1.0
STAR h±, AuAu 0.2 TeV 0-5%, lyl<0.5
PHENIX π0, AuAu 0.2 TeV 0-10%, lyl<0.5
RAA
pT (GeV)


Anti-kT Jet R=0.4
ALICE, lyl<0.3, PbPb lead track pT> 7 GeV
ATLAS, lyl<2.8
CMS, lyl<2.0
Isolated Prompt Photon
CMS, lηl<1.44
PbPb √sNN=5.02 TeV, 0-10%
RAA
pT (GeV)

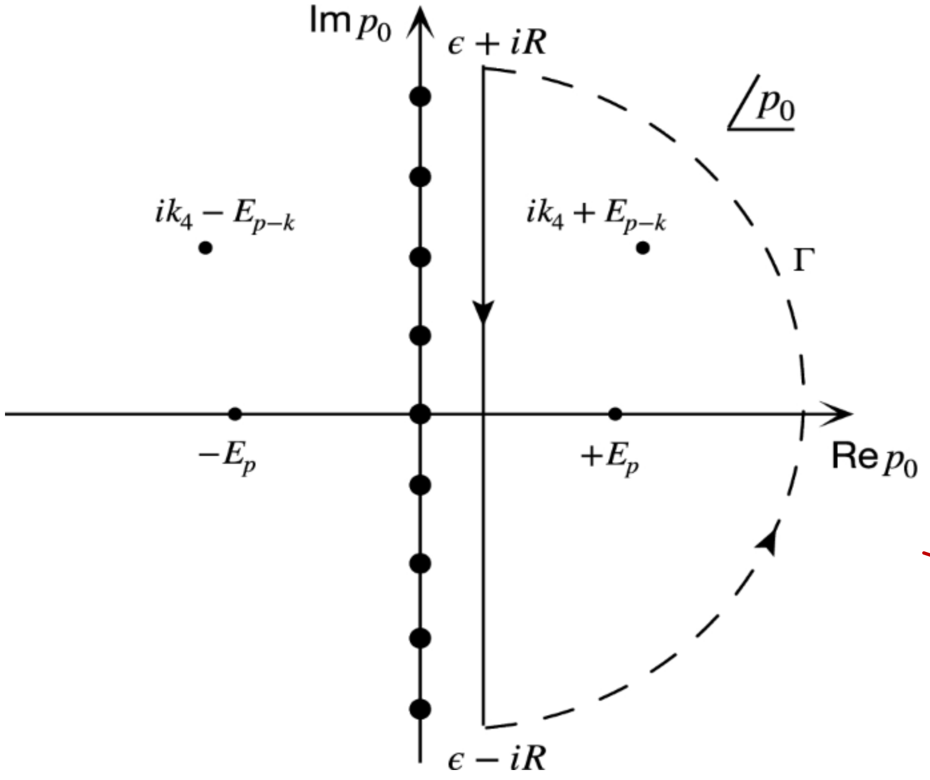
# Probes of the matter

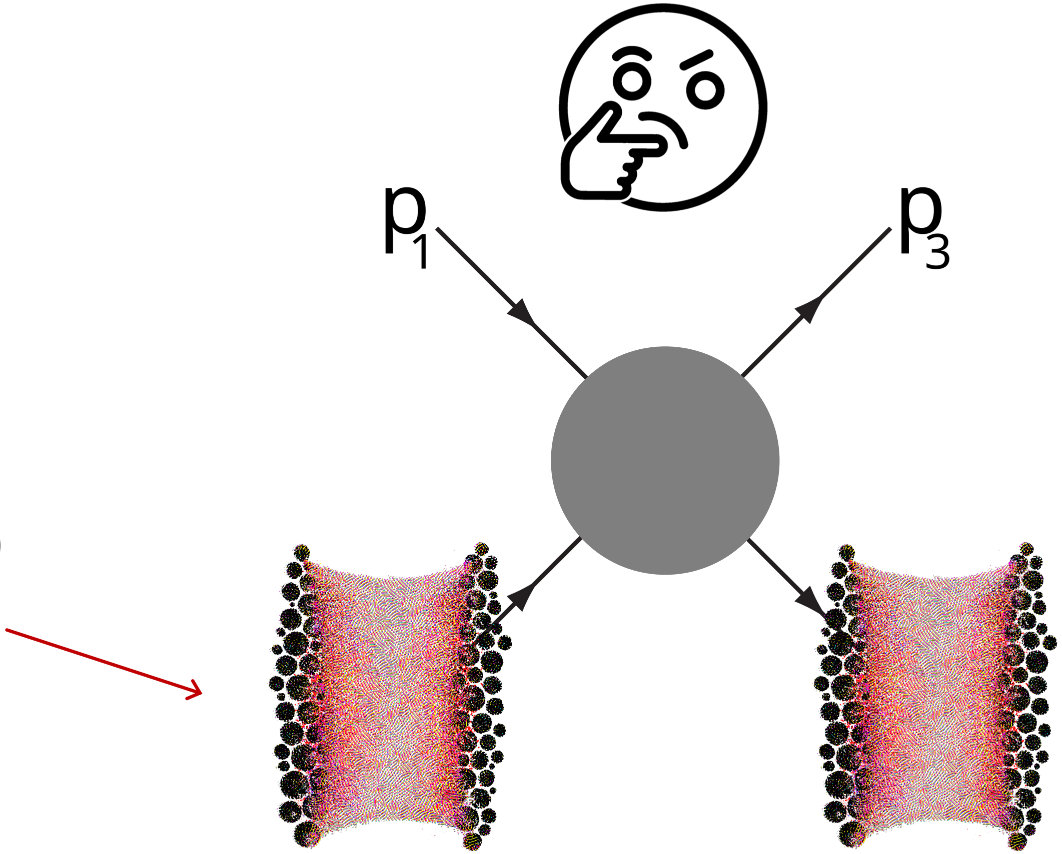
- We only observe the final-state particles that reach the detectors – and even that, in a limited way.
- We try to infer the system’s dynamics from their distributions and correlations, looking for specific signatures.
- Soft particles offer better statistics, but are harder to use for reconstructing the system's evolution in detail.
- Hard probes provide tomographic access, but they’re rare and, in some ways, even harder to interpret.

# Jet quenching formalisms



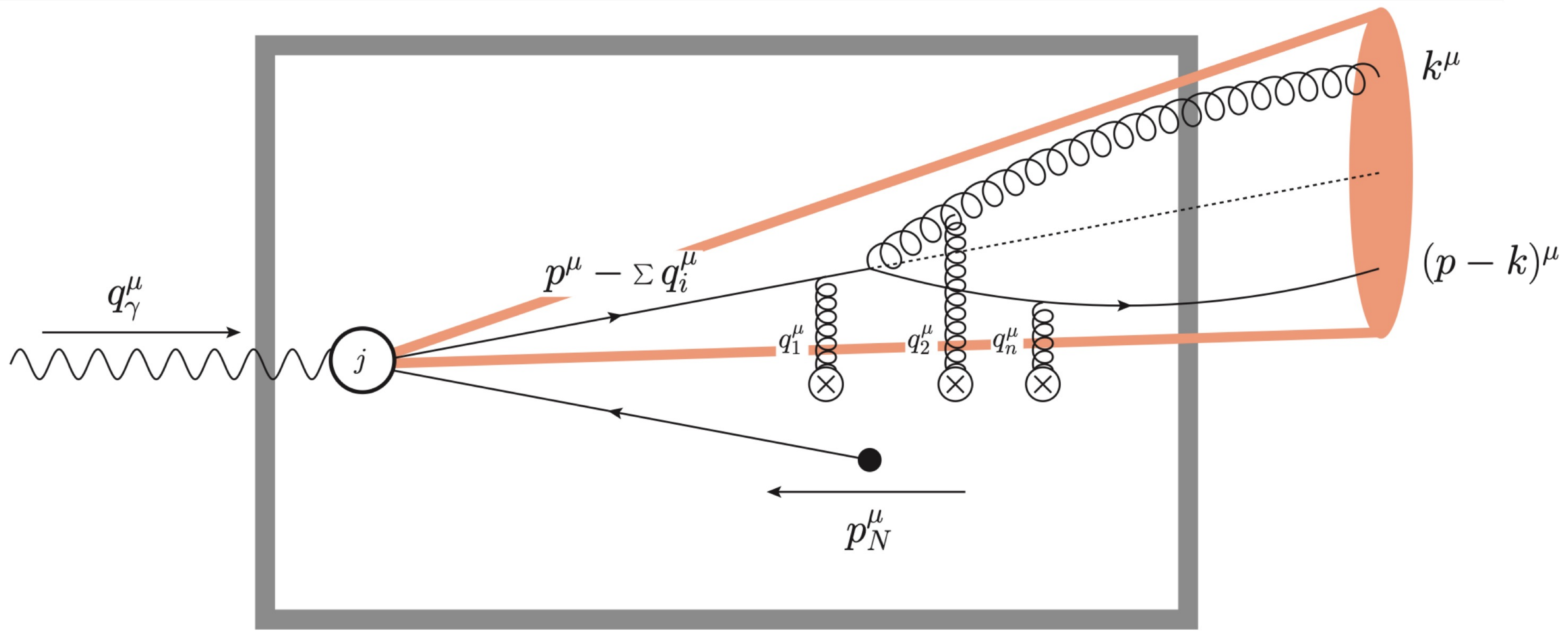
# Jet quenching formalisms



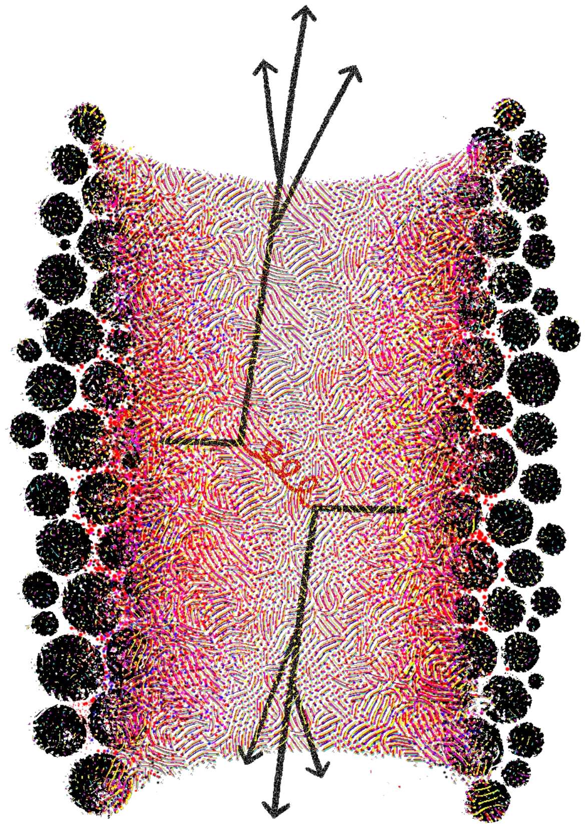
# Jet quenching formalisms

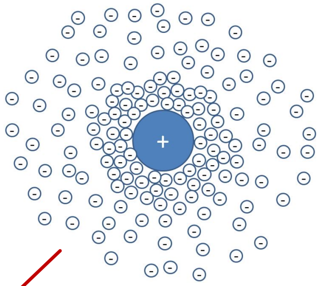
R. Baier et al, NPB, 1997
B. G. Zakharov, JETP, 1997
R. Baier et al, NPB, 1998
M. Gyulassy et al, NPB, 2000
X.-F. Guo, X.-N. Wang, PRL, 2000
U. Wiedemann, NPB, 2000
M. Gyulassy et al, NPB, 2001
P. Arnold, G. Moore, L. Yaffe, JHEP, 2002
C. Salgado, U. Wiedemann, PRD, 2003




the idea goes back to Landau and Pomeranchuk (1953) and Migdal (1956)

# Jet quenching formalisms





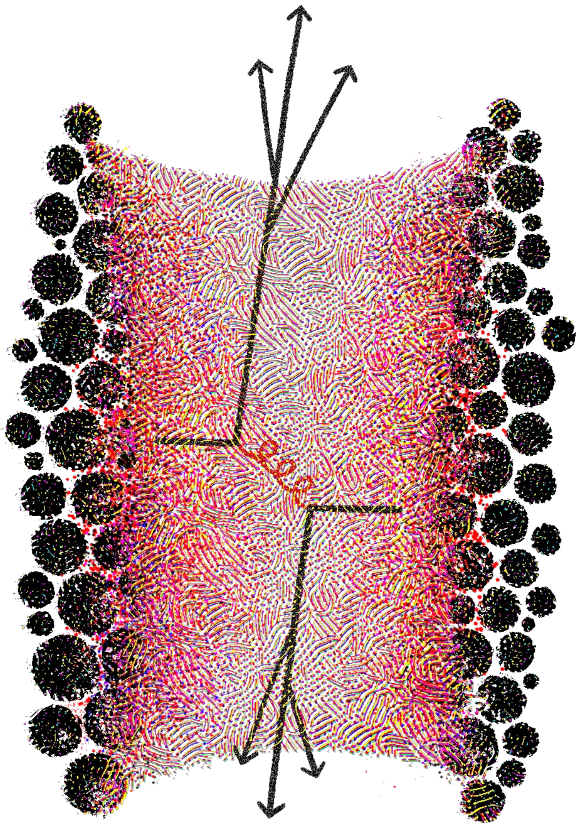
some model for
stochastic sources

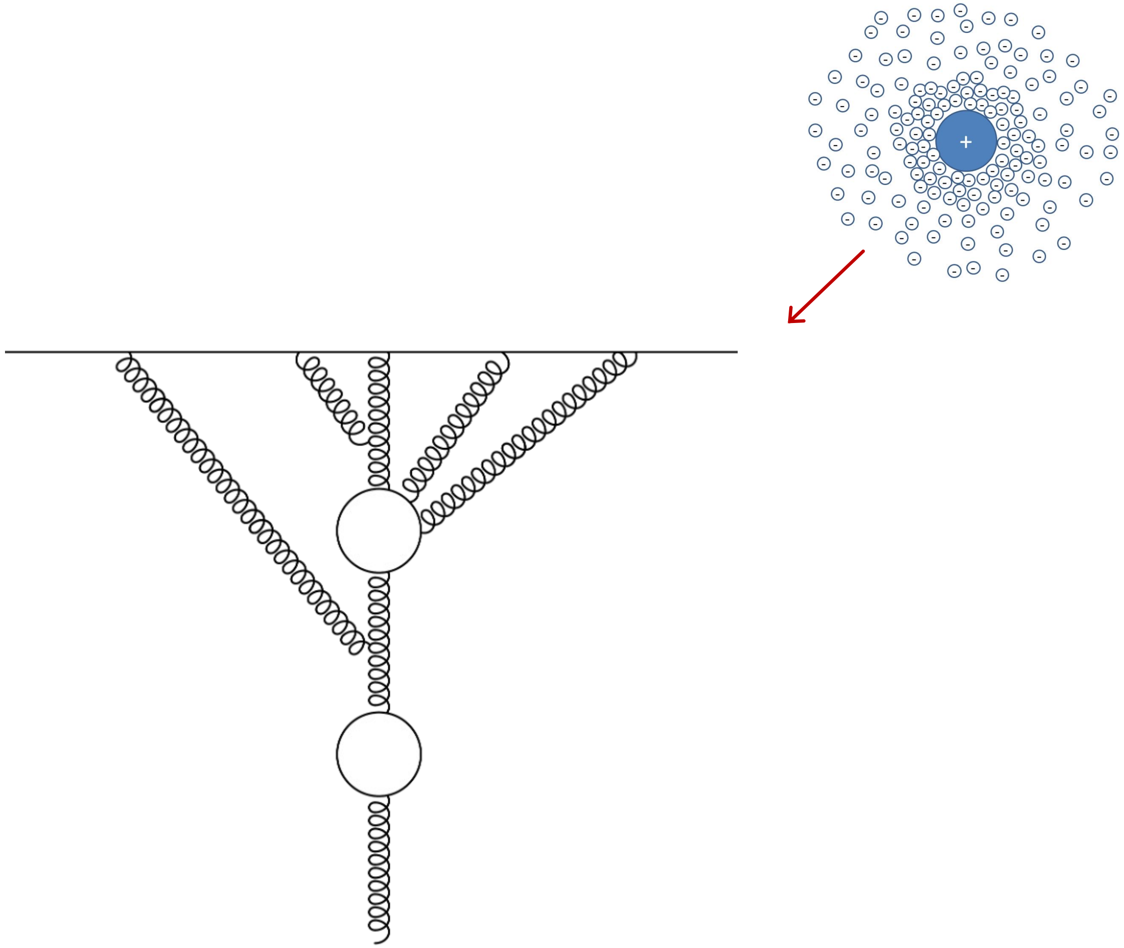
$$D_\mu F^{\mu\nu} = J^\nu + J^\nu_{ind}$$

the Debye mass

$$\phi \sim \frac{1}{r} \longrightarrow \phi \sim \frac{e^{-\mu r}}{r}$$

# Jet quenching formalisms





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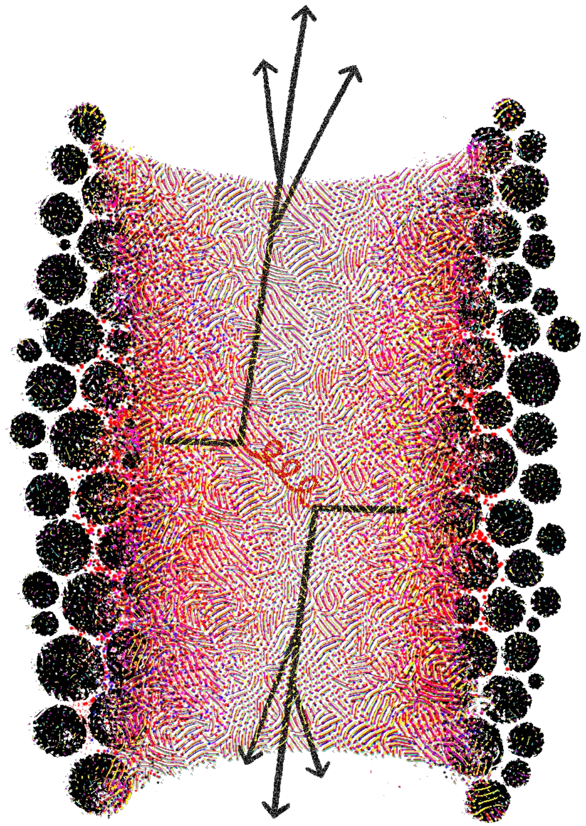

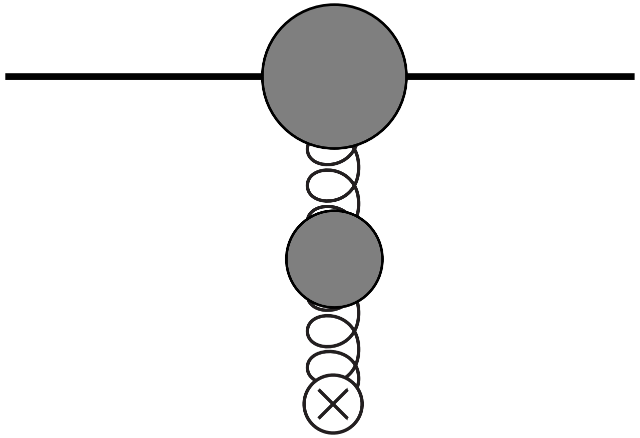

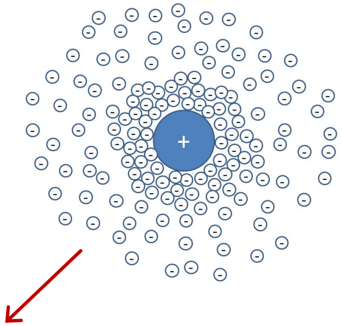

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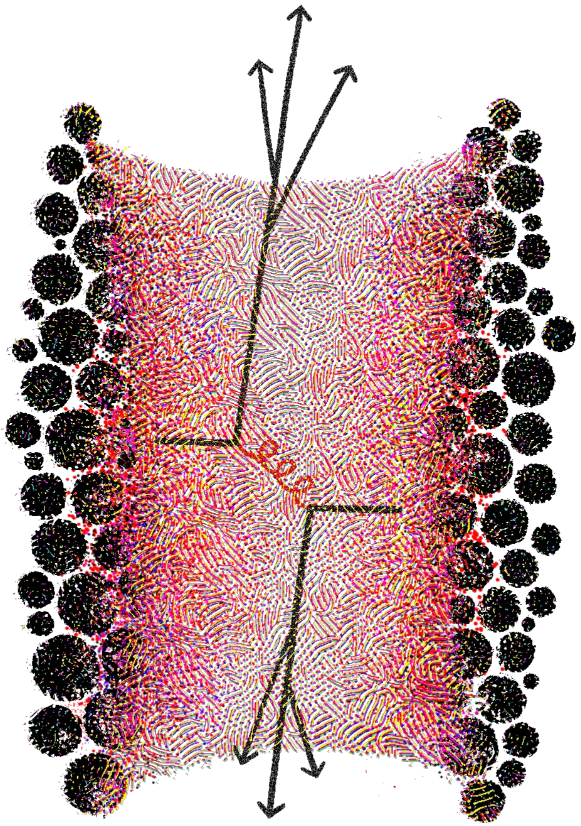
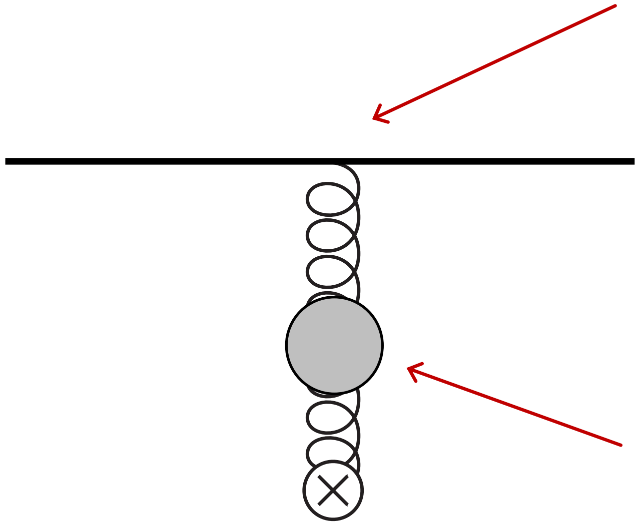
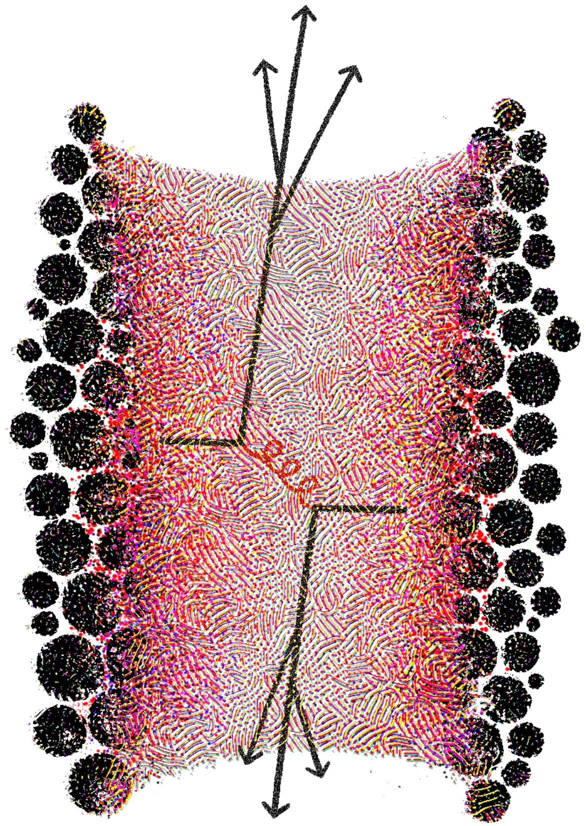
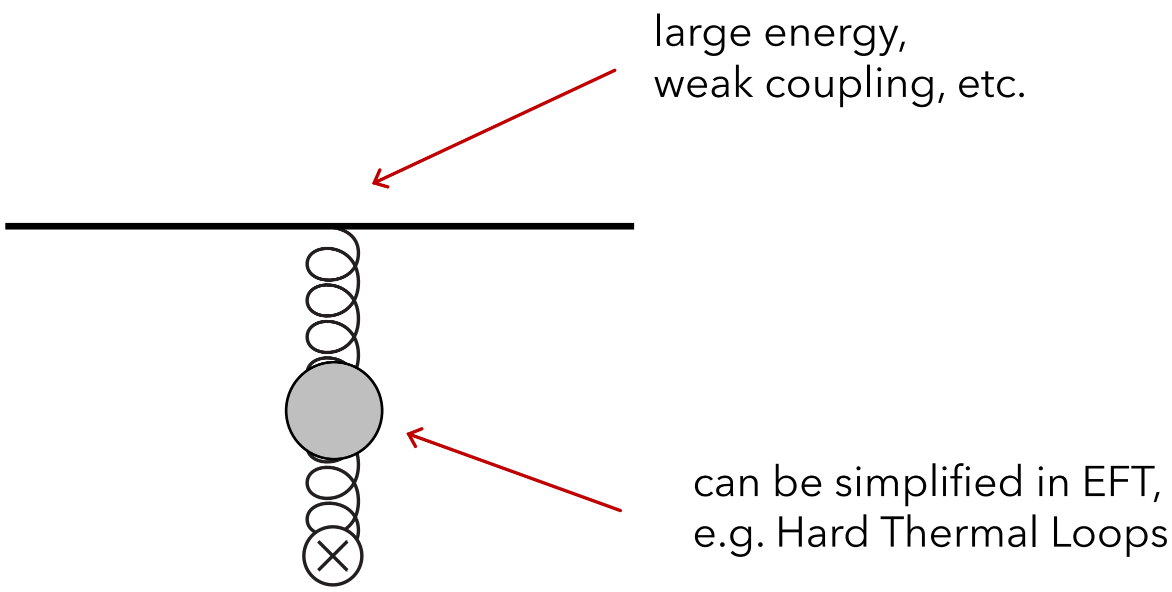





$$\Pi^{\mu\nu} = m_D^2 \left\{ -\delta_\mu^0 \delta_\nu^0 + \omega \int \frac{d\Omega}{4\pi} \frac{v_\mu v_\nu}{\omega - \mathbf{v} \cdot \mathbf{q} + i\epsilon} \right\}$$

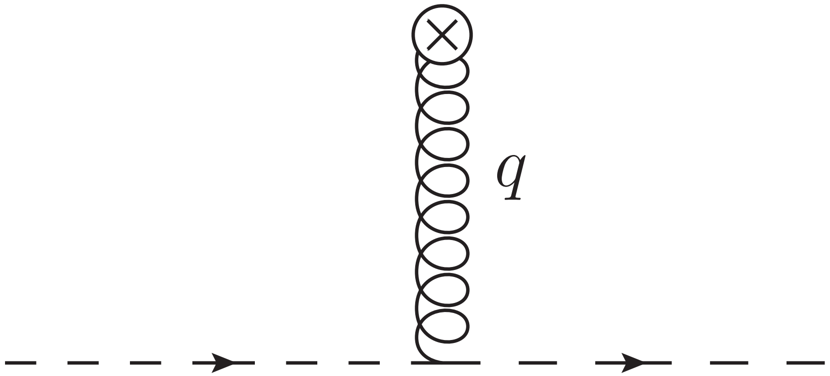
# Jet quenching formalisms





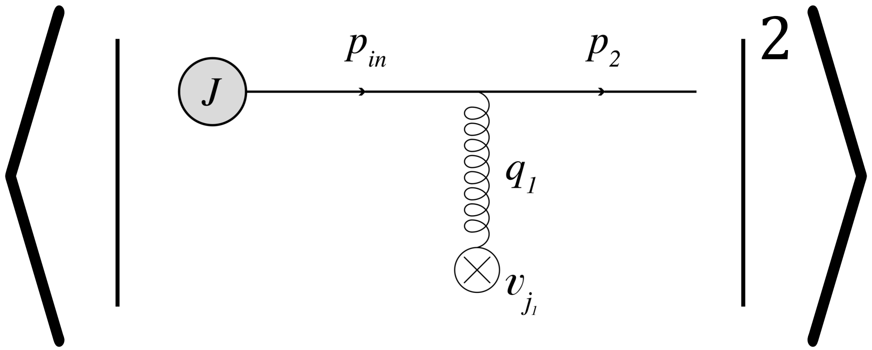

$$v = \frac{g^2}{q^2 - m_D^2}$$

# Jet quenching formalisms




$$A^{\mu a}(q) = \sum_i (ig\, t_i^a)\, e^{iq\cdot x_i}\, (2p_{s\,i} - q)_\nu \frac{-ig^{\mu\nu}}{q^2 - \mu_i^2 + i\epsilon}\, (2\pi)\, \delta\Big( (p_{s\,i} - q)^2 - M^2 \Big)\,.$$

# Jet quenching formalisms




$$\langle t_i^a t_j^b \rangle = \mathcal{C}\delta_{ij}\delta^{ab}$$

color neutrality

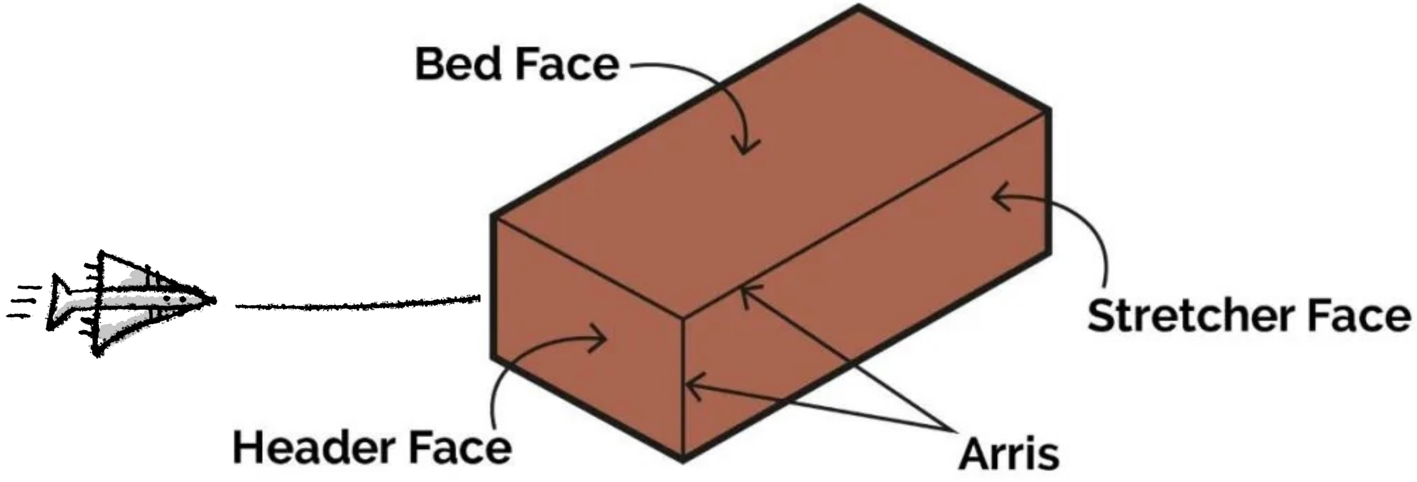
$$\sum_i = \int d^3x\, \rho(\vec{x})$$

source averaging

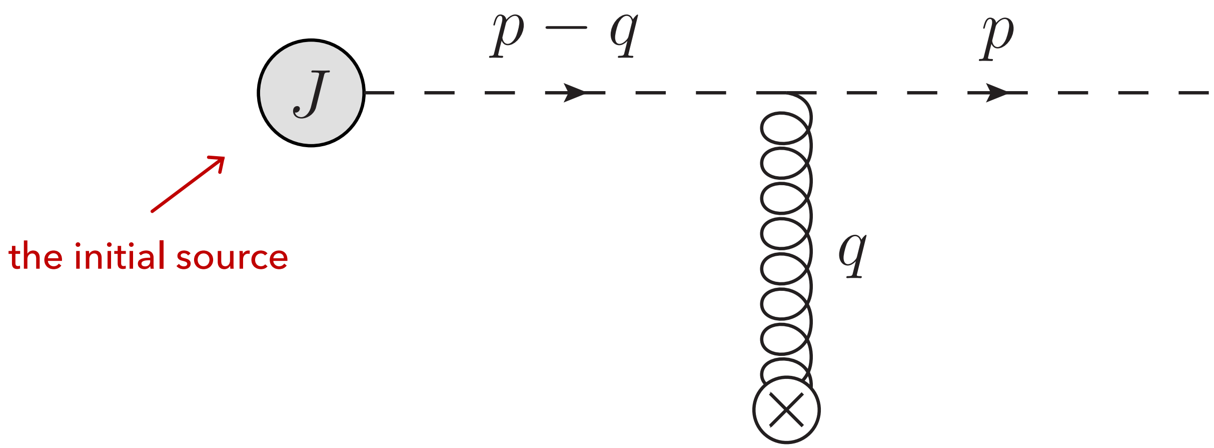
# Jet quenching formalisms

- Most theoretical jet quenching formalisms on the market are based on a variation of this picture or can be related with it.
- They differ by further approximations simplifying the algebra, e.g. finite number of interactions, more restricted kinematics, etc.
- In what follows, we will look at the simplest examples of jet quenching processes and try to see how the different formalisms are related.

# Simple example



# Simple example




the initial source

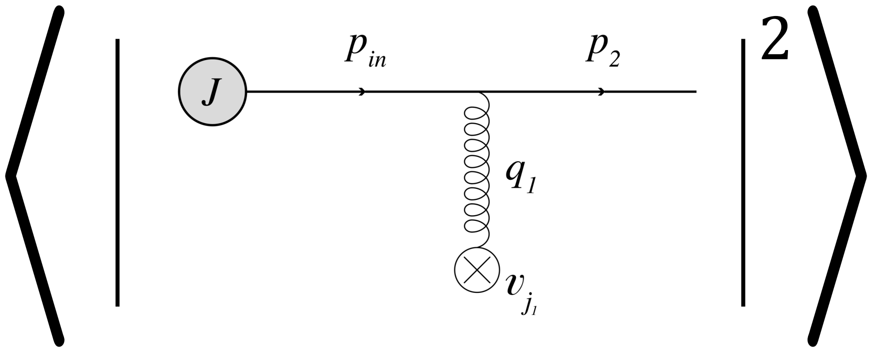
$$iM_1(p) = \int \frac{d^4q}{(2\pi)^4} \left[ig\, t^a_{\text{proj}} A^{\mu a}_{\text{ext}}(q)\ (2p-q)_\mu\right] \left[\frac{i}{(p-q)^2 + i\epsilon}\right] J(p-q)$$

$$gA^{\mu a}_{ext}(q) = \sum_i e^{iq\cdot x_i}\, t^a_i\, u^\mu\, v(q)\,(2\pi)\,\delta\,(q\cdot u)$$

color sources

(1,0,0,0)

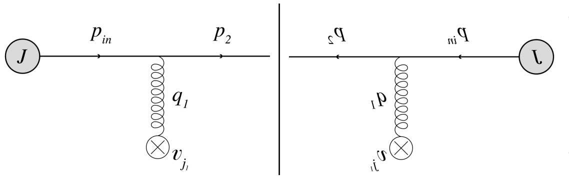
# Simple example

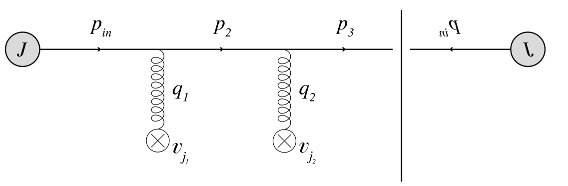



$$\langle |M_{SB}|^2\rangle \sim \int d^2q\, d^2\bar{q} \int d^3x\, \rho(z) e^{-i(q-\bar{q})_\perp \cdot x_\perp}\, v_q v_{\bar{q}}\, J(p-q) J^\dagger(p-\bar{q})$$

# Simple example

$$E\frac{d\mathcal{N}}{d^3p} \simeq f(E)\delta^{(2)}(\mathbf{p}) + \Big\langle \quad \Big\rangle + \Big\langle \quad \Big\rangle$$




$$\mathcal{V}(\mathbf{q}) = -\mathcal{C}\rho\left(|v(\mathbf{q})|^2 - \delta^{(2)}(\mathbf{q})\int_l |v(\mathbf{l})|^2\right)$$

$$E\frac{d\mathcal{N}^{(1)}}{d^3p} = \int_0^L dz \int_q \mathcal{V}(\mathbf{q})\, f(E)\delta^{(2)}(\mathbf{p}-\mathbf{q})$$

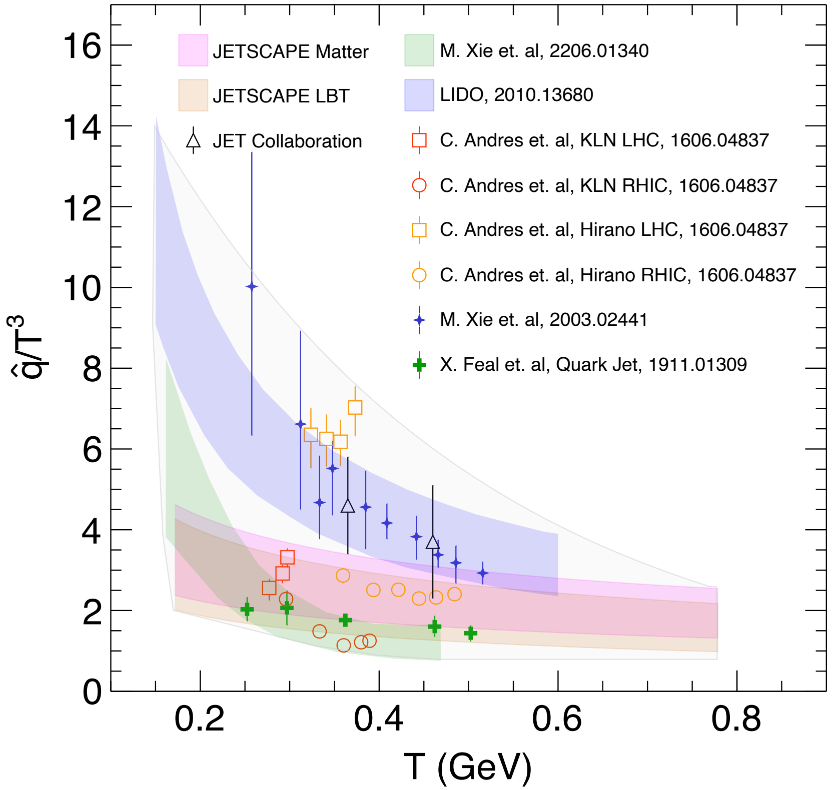
controls most of the pheno simulations $\longrightarrow$

$$\hat{q} = \frac{\partial}{\partial L}\langle p_\perp^2\rangle = \frac{\mathcal{C}g^4\rho}{4\pi\mu^2}L\,\frac{\mu^2}{L}\log\frac{E}{\mu}$$

opacity $\chi$

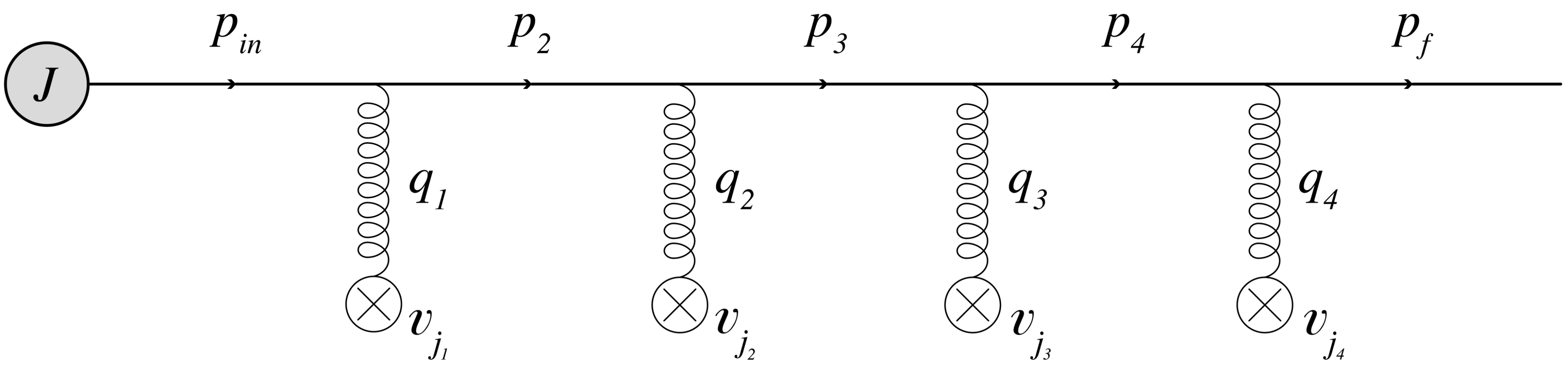
# Jet quenching parameter

- We see jets only in the final state, but they carry imprints of the medium they traversed during the system's evolution.
- Describing their interaction with the medium requires an effective framework – typically QCD scatterings off background stochastic fields.
- $\hat{q}$ is the first quantity to appear in jet quenching calculations, playing a central role in phenomenology.
- However, $\hat{q}$ is notoriously difficult to measure or reliably estimate in simulations.




Theoretical uncertainties in $\hat{q}$ extracted in different works

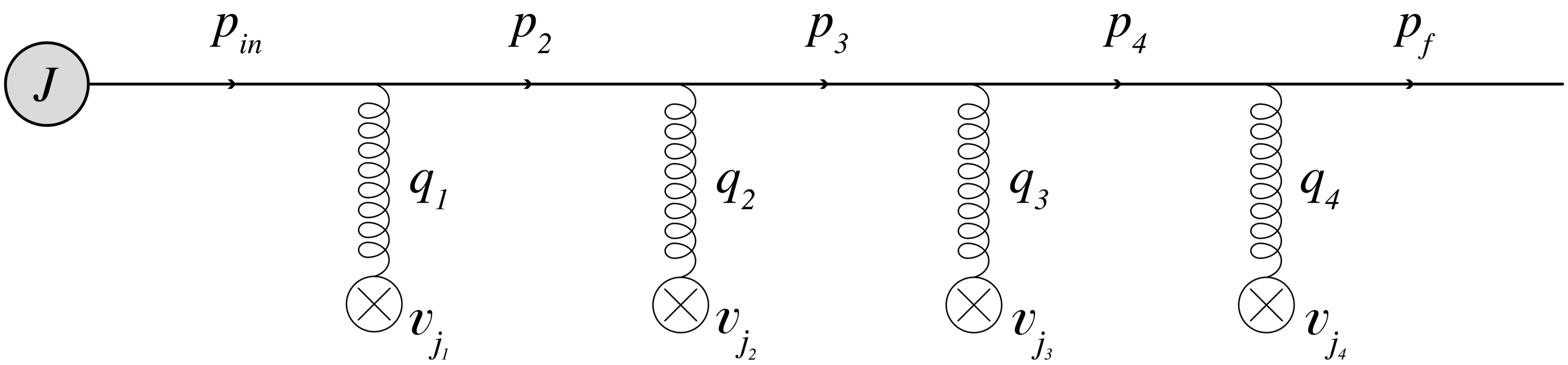
# Resummation (Baier, Dokshitzer, Mueller, Peigne, Schiff, Zakharov)




$$iM(p) = \int \frac{d^2\mathbf{p}_{in}}{(2\pi)^2} e^{i\frac{\mathbf{p}_f^2}{2E}L}\, G(\mathbf{p}_f, L; \mathbf{p}_{in}, 0)\, J\left(E, \mathbf{p}_{in}\right)$$

↑ a single particle propagator

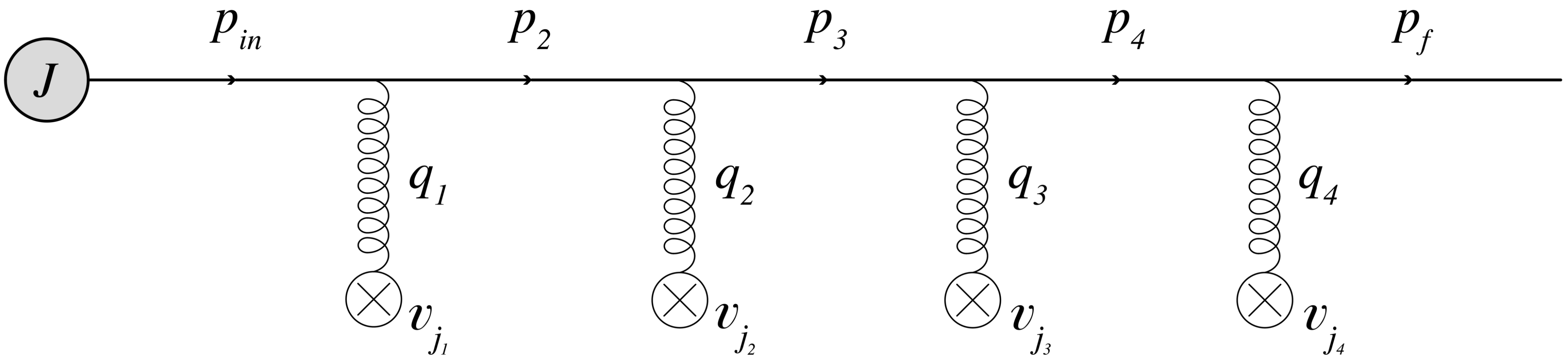
# Resummation (BDMPS-Z)




$$\partial_L G(\mathbf{p}, L) = -i\frac{\mathbf{p}^2}{2E}G(\mathbf{p}, L) + i\int_{\mathbf{q}} t^a \hat{\rho}^a(\mathbf{q}, L)\, v_q\, G(\mathbf{p}-\mathbf{q}, L)$$

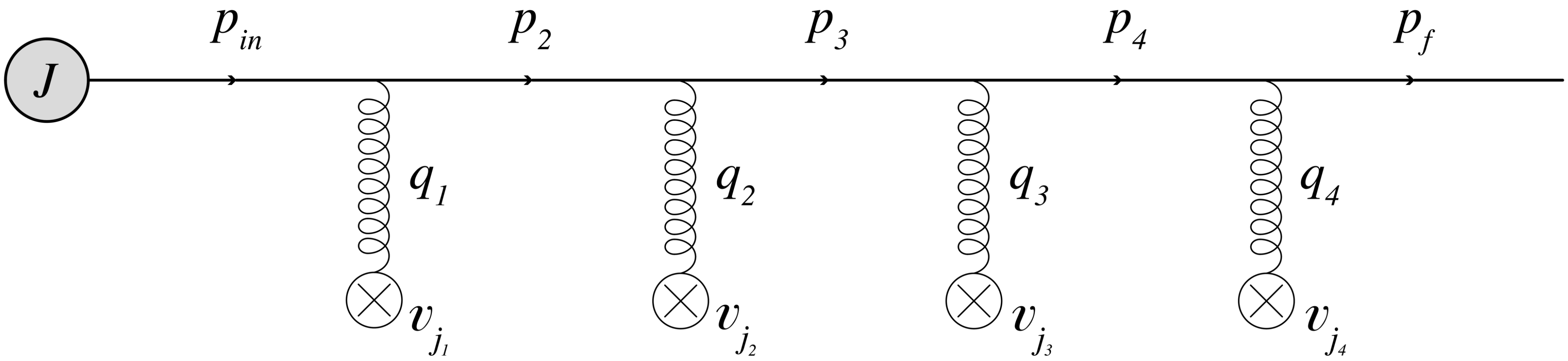
emergent Schrödinger equation

# Resummation (BDMPS-Z)




$$G(\mathbf{x}_L, L; \mathbf{x}_0, 0) = \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r} \exp\left(\frac{iE}{2}\int_0^L d\tau\, \dot{\mathbf{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\mathbf{r}(\tau),\tau)\right)$$

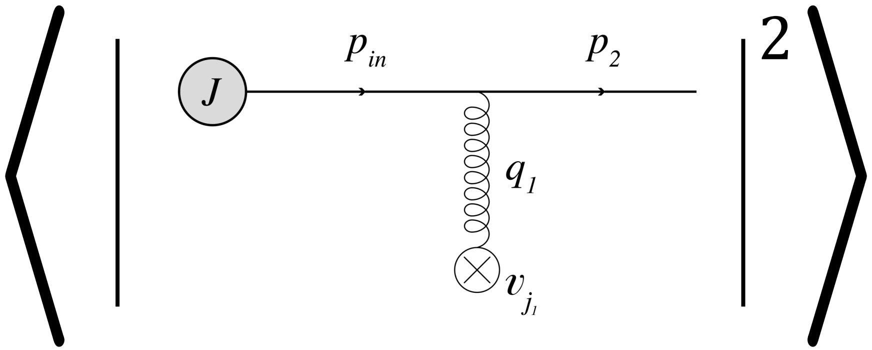
# Resumation (BDMPS-Z)




$$D = \int_0^\infty dT\, e^{-m^2 T} \int_{x(0)=x_{in}}^{x(T)=x_f} \mathcal{D}x\, \exp\left\{ -\int_0^T \left( \frac{1}{4}\dot{x}^2 + ie\dot{x}\cdot A(x(\tau)) \right) d\tau \right\}$$

very similar to the worldline formalism by Schwinger

# Resummation (BDMPS-Z)



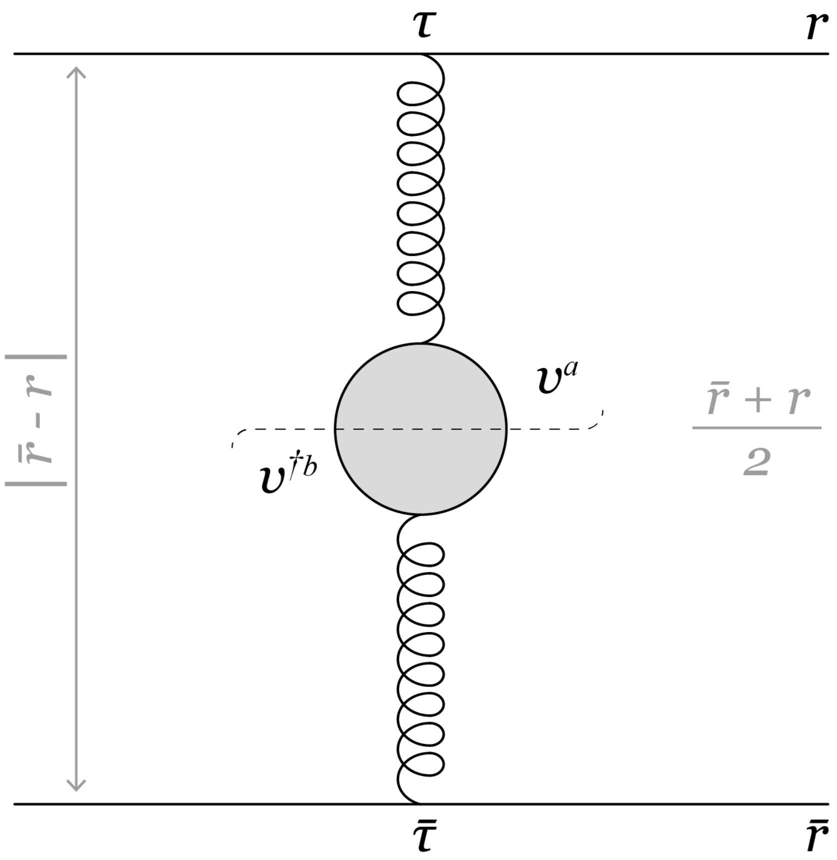

$$\langle t_i^a t_j^b \rangle = \mathcal{C}\delta_{ij}\delta^{ab}$$

color neutrality

$$\sum_i = \int d^3x \, \rho(\vec{x})$$

source averaging

# Resummation (BDMPS-Z)




$$W = \left\langle \mathcal{G}^\dagger(\bar{\mathbf{k}}, L+\epsilon; \bar{\mathbf{k}}_0, 0)\, \mathcal{G}(\mathbf{k}, L+\epsilon; \mathbf{k}_0, 0) \right\rangle$$
$$= \int_{\mathbf{l},\bar{\mathbf{l}}} \left\langle \mathcal{G}^\dagger(\bar{\mathbf{k}}, L+\epsilon; \bar{\mathbf{l}}, L)\, \mathcal{G}(\mathbf{k}, L+\epsilon; \mathbf{l}, L) \right\rangle$$
$$\times \left\langle \mathcal{G}^\dagger(\bar{\mathbf{l}}, L; \bar{\mathbf{k}}_0, 0)\, \mathcal{G}(\mathbf{l}, L; \mathbf{k}_0, 0) \right\rangle$$

↓

$$\partial_L W(\mathbf{p}, \mathbf{Y}) = -\frac{\mathbf{p}\cdot\boldsymbol{\nabla}_Y}{E} W(\mathbf{p}, \mathbf{Y}) - \int_{\mathbf{q}} \mathcal{V}(\mathbf{q}) W(\mathbf{p}-\mathbf{q}, \mathbf{Y})$$

# Resummation (BDMPS-Z)

$$\partial_L W(\mathbf{p},\mathbf{Y}) = -\frac{\mathbf{p}\cdot\boldsymbol{\nabla}_Y}{E} W(\mathbf{p},\mathbf{Y}) - \int_{\mathbf{q}} \mathcal{V}(\mathbf{q}) W(\mathbf{p}-\mathbf{q},\mathbf{Y})$$

$$\downarrow$$

$$\langle \mathbf{p}^2 \rangle = \int_{\mathbf{p},\mathbf{Y}} \mathbf{p}^2\, W(\mathbf{p},\mathbf{Y}) = L \int_{\mathbf{p}} \mathbf{p}^2 \mathcal{V}(\mathbf{p}) = \frac{\mathcal{C} g^4 \rho}{4\pi} L \,\log\frac{E}{\mu}$$

$\hat{q}L$

# Resummation (BDMPS-Z)

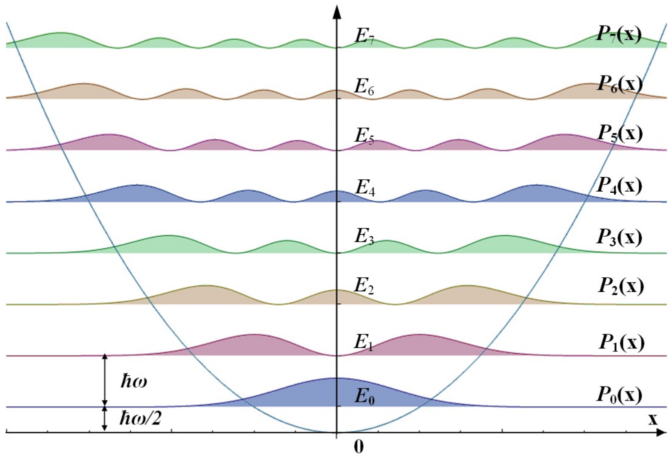
- There are several highly nontrivial analogies here:
  - an emergent Schrödinger equation for G
  - an emergent Boltzmann equation for W

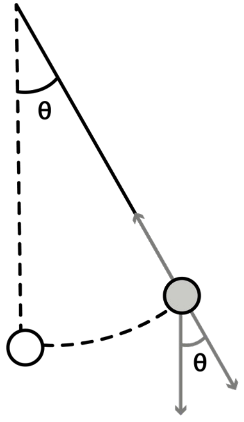
- Another widely used approximation:

$$\mathcal{V} \simeq \frac{\hat{q}_0}{4} x^2 \log \frac{1}{\mu^2 x^2} \simeq \frac{\hat{q}}{4} x^2$$

- The emergent KT precisely matches with the AMY construction (at least in this simple example).

- In this simplest case, one can easily verify that the resummed answer agrees with the perturbative expansion order by order.



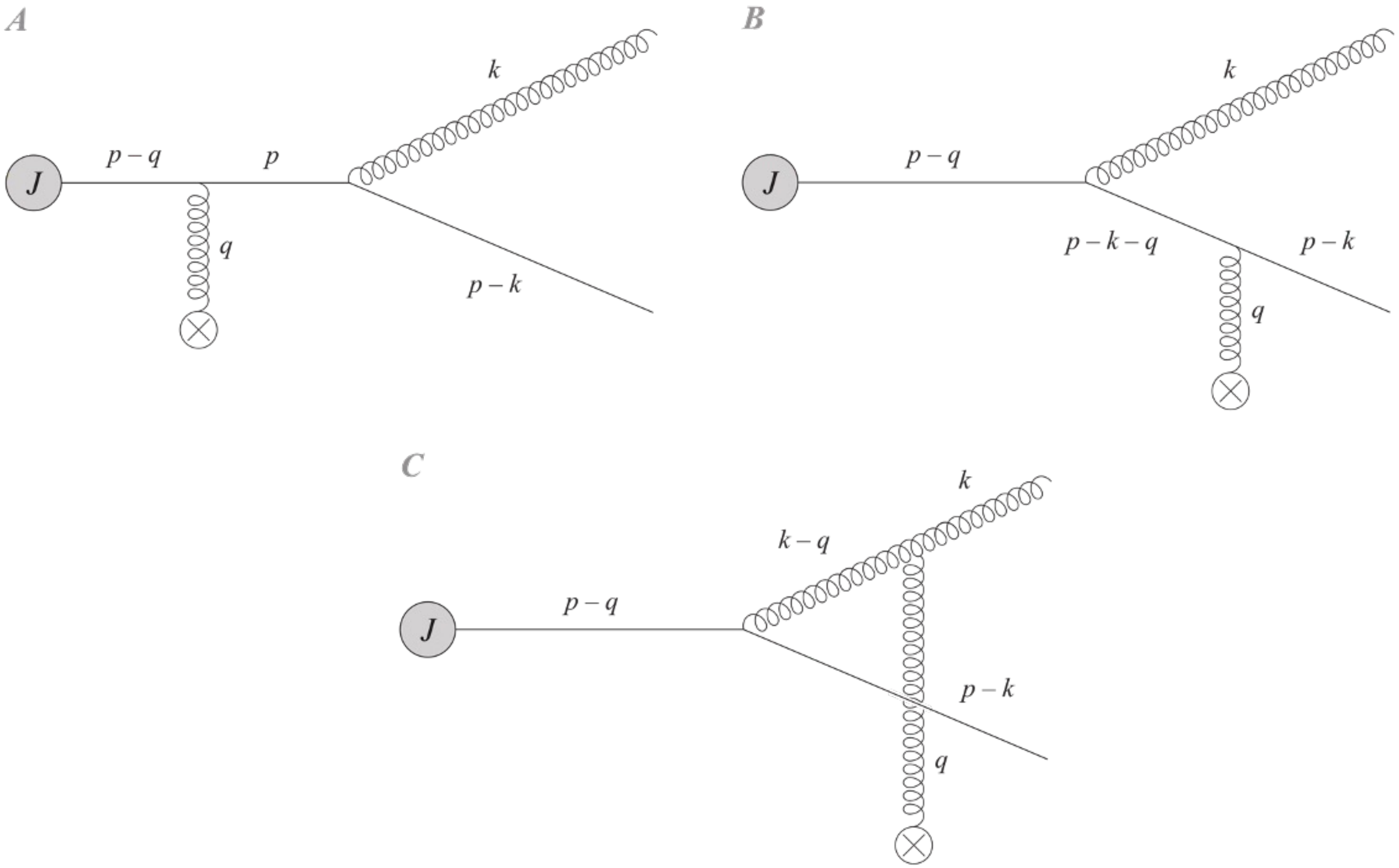




$$\left(\partial_L + \frac{\mathbf{p}\cdot\boldsymbol{\nabla}_Y}{E} - \frac{\hat{q}}{4}\partial_{\mathbf{p}}^2\right) W(\mathbf{p},\mathbf{Y}) = 0$$

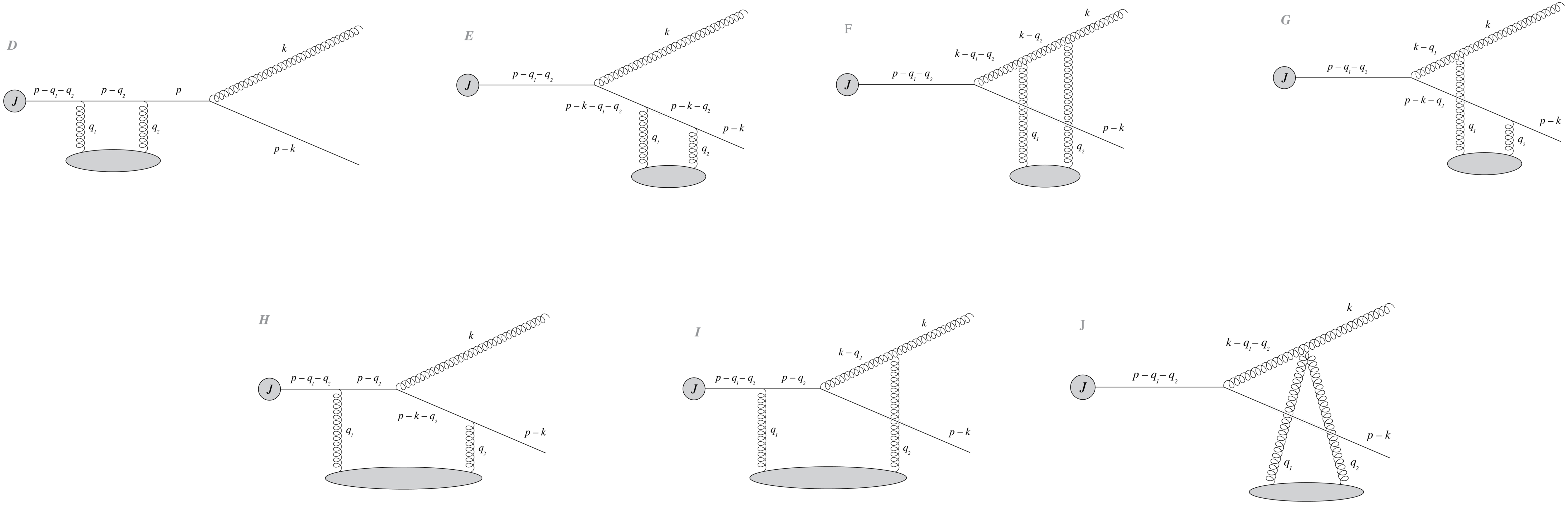
# Some questions

- Where is the energy loss?
- What about the other phases of matter in HIC? For which phases that description works at all?
- Do we really need all these approximations? Do we learn anything in such an oversimplified setup?

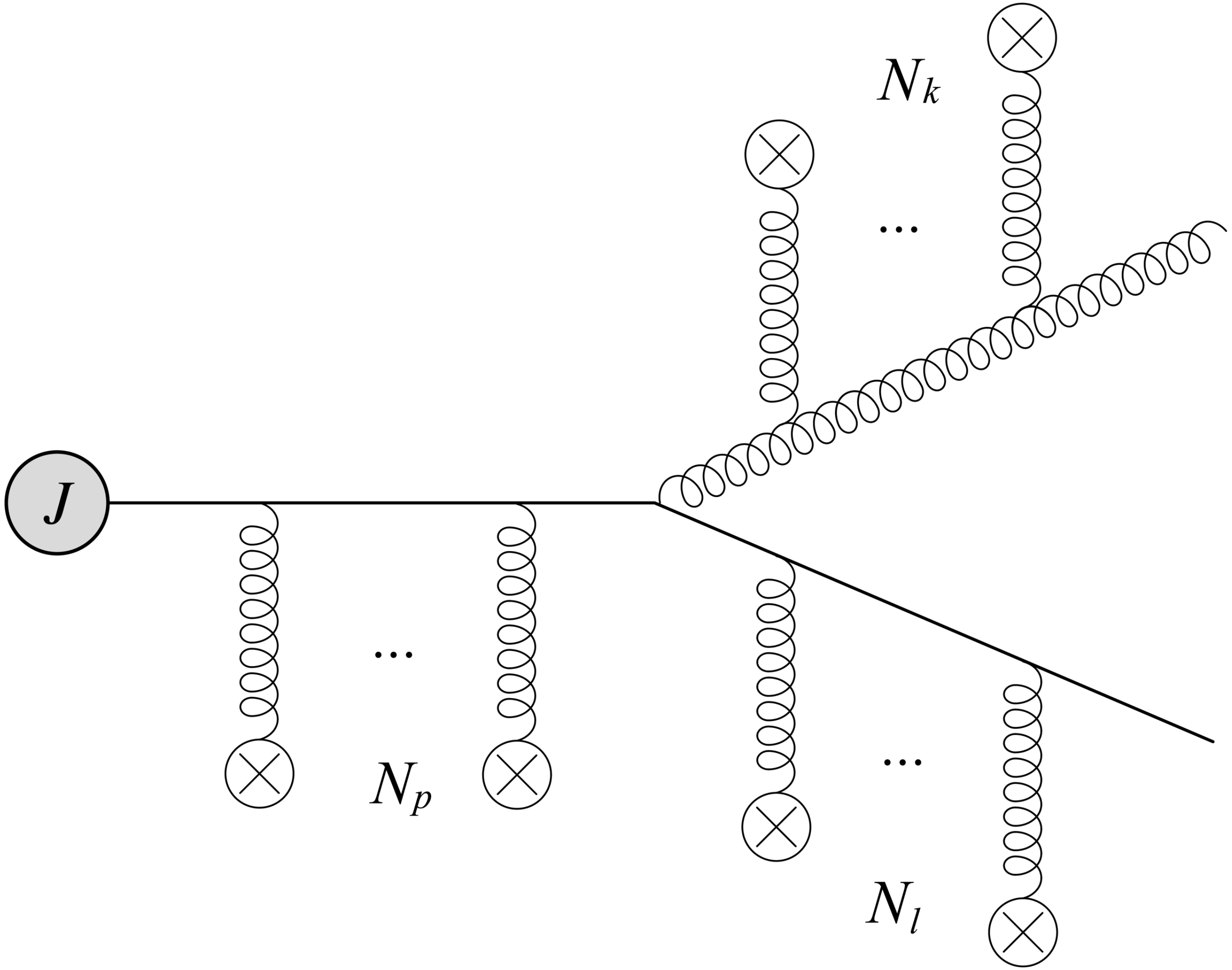
# Gluon emission



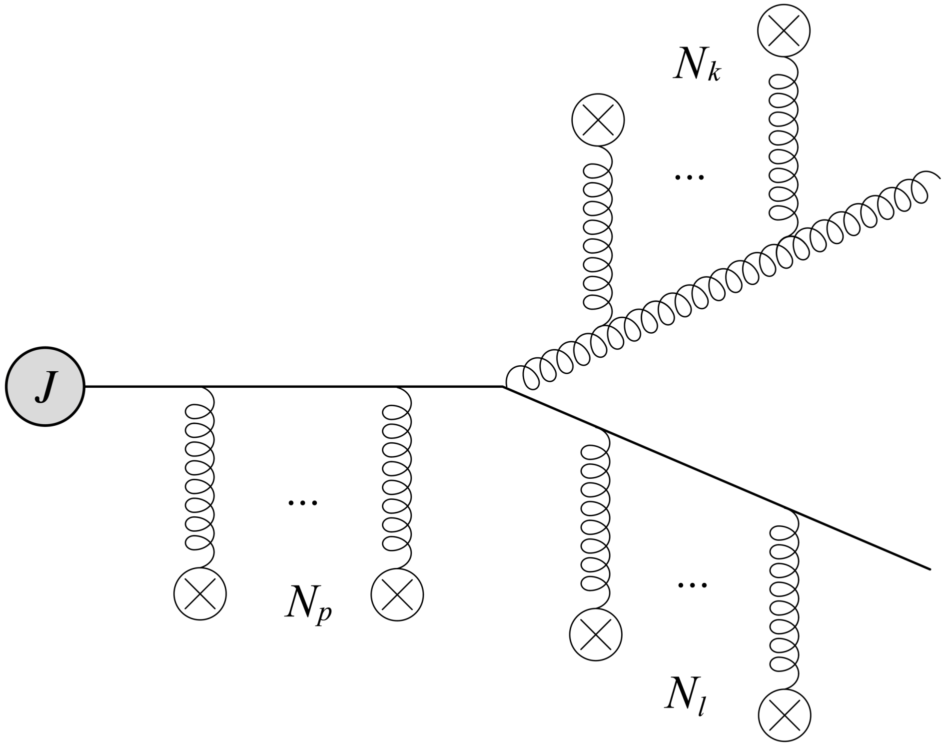
# Gluon emission



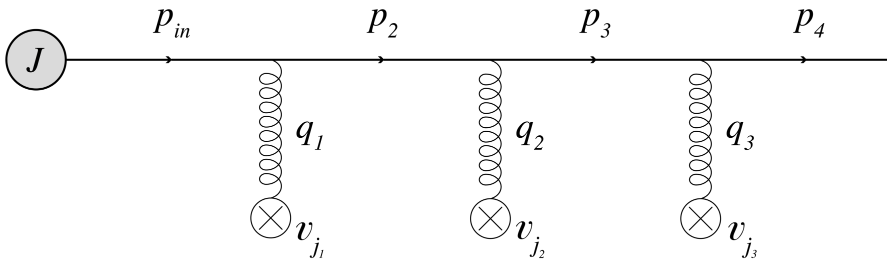
# Gluon emission



# Gluon emission

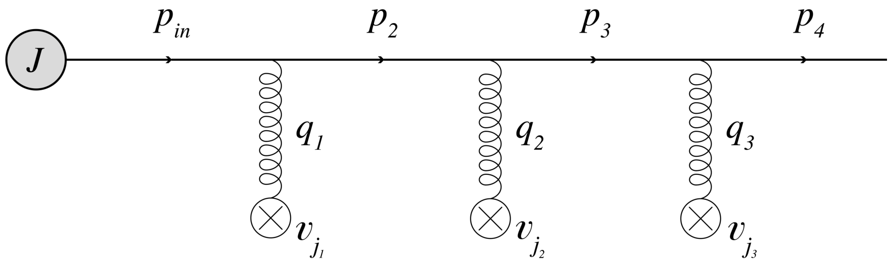


# Gluon emission




$$\longrightarrow \quad \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r} \exp\left( \frac{iE}{2} \int_0^L d\tau\, \dot{\mathbf{r}}^2 \right) \mathcal{P} \exp\left( -i \int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\mathbf{r}(\tau), \tau) \right)$$

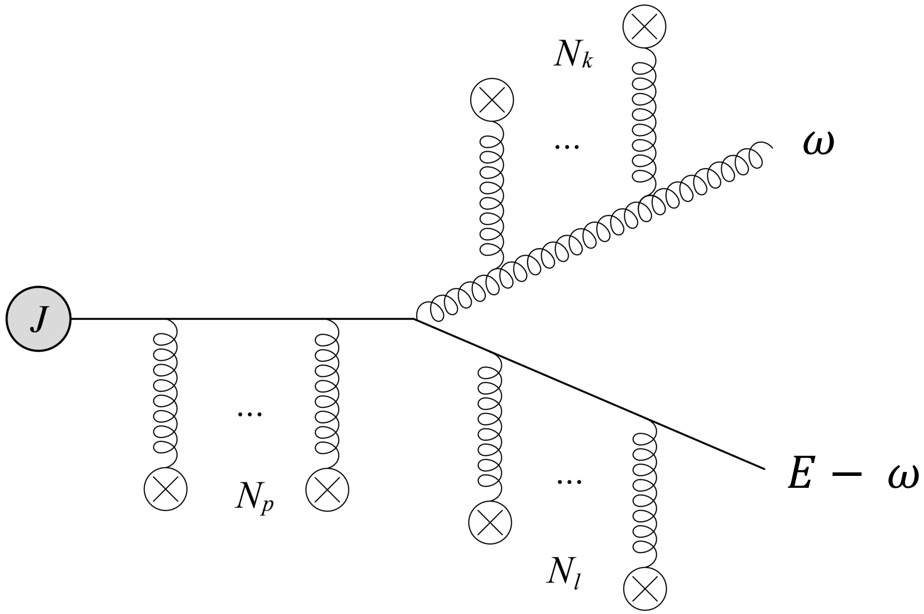
# Gluon emission




$$\mathcal{W} = \mathcal{P}\exp\left(-i\int dx^\mu\, t^a_{\rm proj}\, A^a_\mu(\mathbf{r}(\tau),\tau)\right)$$

$\longrightarrow$

$$\int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r}\,\exp\left(\frac{iE}{2}\int_0^L d\tau\,\dot{\mathbf{r}}^2\right)\mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\mathbf{r}(\tau),\tau)\right)$$




$\longrightarrow$

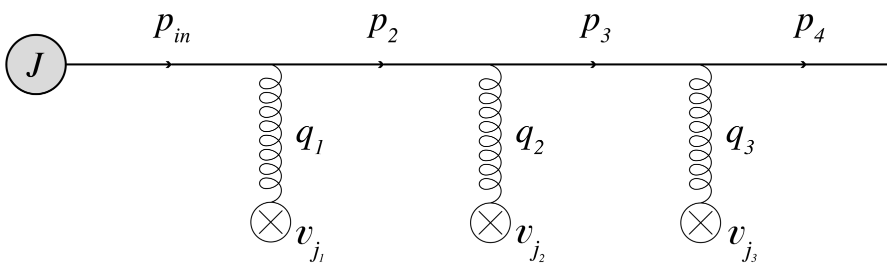
$$-\frac{g}{xE}\lim_{z_f\to\infty}\int_0^\infty dz_s \int d^2x_0\, e^{-i\mathbf{x}_0\cdot\mathbf{l}_f} J(\mathbf{x}_0)$$

$$\times\, \mathcal{W}(\mathbf{x}_0;\infty,z_s)\, t^a_{proj}\, \mathcal{W}(\mathbf{x}_0;z_s,0)\; e^{i\frac{\mathbf{k}_f^2}{2\omega}z_f}$$

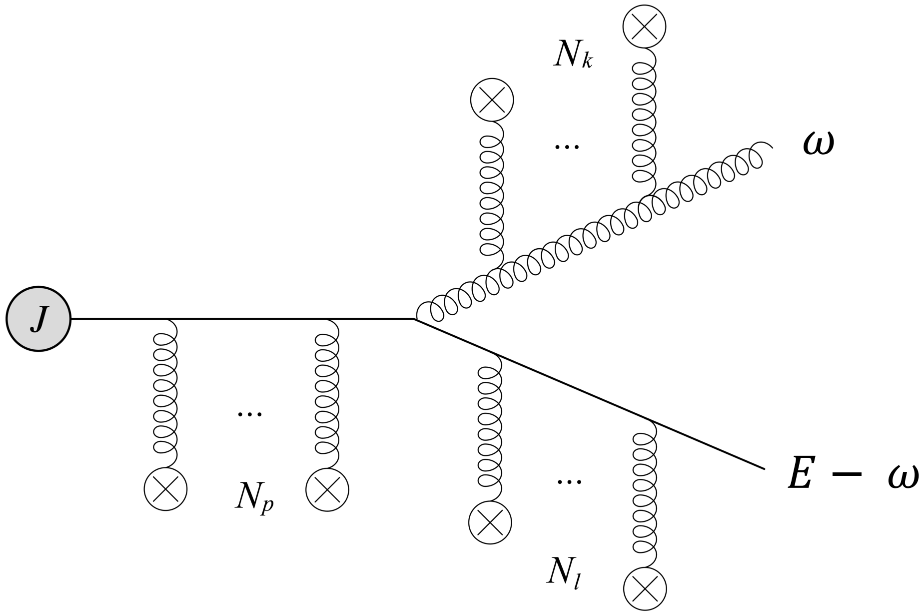
$$\times\left[\epsilon\cdot\vec{\nabla}_{\mathbf{x}_0}\mathcal{G}^{ba}\left(\mathbf{k}_f,z_f;\mathbf{x}_0,z_s\right)\right]$$

small-x limit, i.e. $\omega/E \ll 1$

# Gluon emission




$$\longrightarrow \quad \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r} \exp\left(\frac{iE}{2}\int_0^L d\tau\, \dot{\mathbf{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\mathbf{r}(\tau),\tau)\right)$$



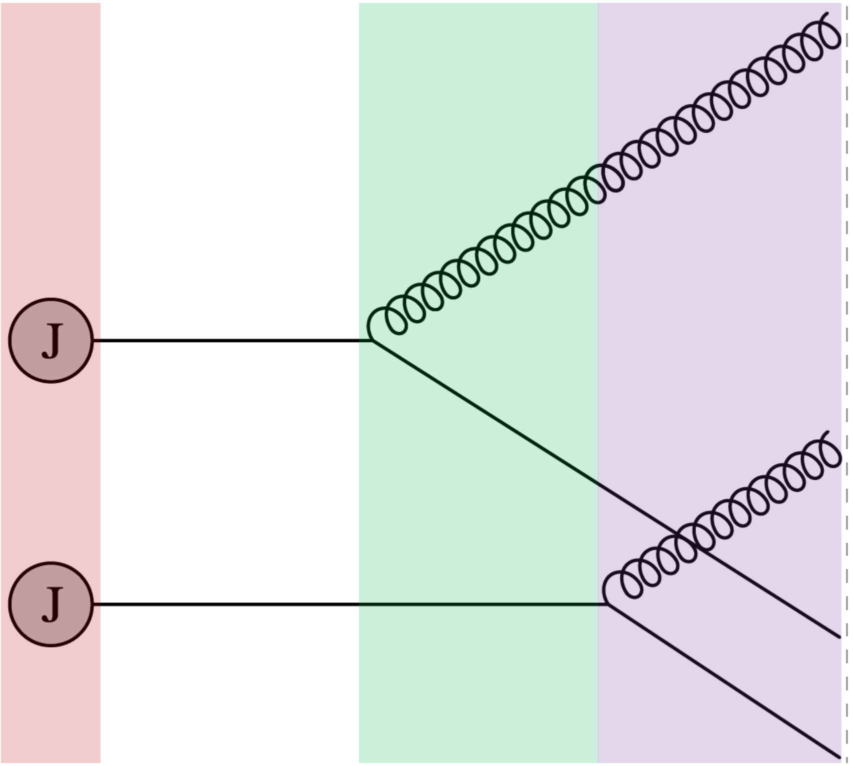

$$\longrightarrow \quad -\frac{g}{xE}\lim_{z_f\to\infty}\int_0^\infty dz_s \int d^2x_0\, e^{-i\mathbf{x}_0\cdot\mathbf{l}_f} J(\mathbf{x}_0)$$

$$\times\, \mathcal{W}(\mathbf{x}_0;\infty,z_s)\, t^a_{proj}\, \mathcal{W}(\mathbf{x}_0;z_s,0)\; e^{i\frac{\mathbf{k}_f^2}{2\omega}z_f}$$

$$\times\left[\epsilon\cdot\vec{\nabla}_{\mathbf{x}_0}\mathcal{G}^{ba}(\mathbf{k}_f,z_f;\mathbf{x}_0,z_s)\right]$$

small-x limit, i.e. $\omega/E \ll 1$

# Gluon emission




$$(2\pi)^2\omega\frac{d\mathcal{N}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s}{N_c\,\omega^2}\mathrm{Re}\int_0^L d\bar{z}\int_0^{\bar{z}} dz\int_{\mathbf{x}_0,\mathbf{y}} |J(\mathbf{x}_0)|^2 \Big[(\nabla_{\mathbf{x}}\cdot\nabla_{\bar{\mathbf{x}}}) \times \Big\langle \mathcal{G}^{bc}(\mathbf{k},L;\mathbf{y},\bar{z})\,\mathcal{G}^{\dagger\,\bar{a}b}(\mathbf{k},L;\bar{\mathbf{x}},\bar{z})\Big\rangle\Big\langle \mathcal{G}^{ca}(\mathbf{y},\bar{z};\mathbf{x},z)\,\mathcal{W}_A^{\dagger\,a\bar{a}}(\mathbf{x}_0;\bar{z},z)\Big\rangle\Big]\Big|_{\mathbf{x}=\bar{\mathbf{x}}=\mathbf{x}_0}$$

broadening of the gluon

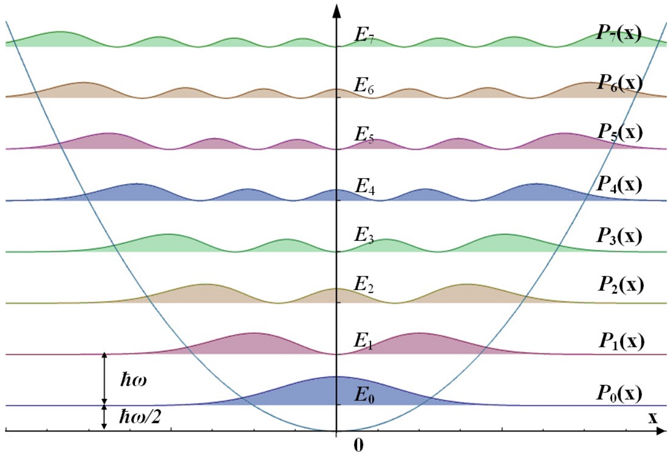
emission kernel

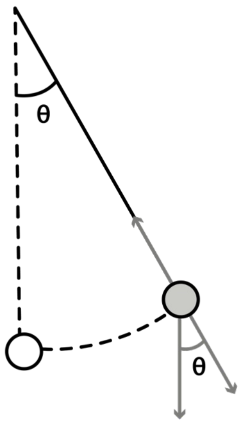
$$\mathcal{K}(\mathbf{x},t;\mathbf{y},s) = \frac{1}{N_c^2-1}\int_{\mathbf{r}(s)=\mathbf{y}}^{\mathbf{r}(t)=\mathbf{x}} \mathcal{D}\mathbf{r}\,\exp\left[i\frac{\omega}{2}\int_s^t d\tau\,\dot{\mathbf{r}}^2\right] \times \mathrm{Tr}\,\mathcal{P}\exp\left[iT^c\int_s^t d\tau\,\mathcal{V}(\mathbf{r}(\tau)-\mathbf{x}_{in})\right]$$

# Gluon emission

$$\left(\partial_t - \frac{\partial^2_{\mathbf{x}}}{2\omega} + i\mathcal{V}(\mathbf{x},t)\right)\mathcal{K}(\mathbf{x},t;\mathbf{y},s) = i\delta^{(2)}(\mathbf{x}-\mathbf{y})\delta(t-s)$$

$$\mathcal{K}(\mathbf{x},t;\mathbf{y},s) = \frac{1}{N_c^2-1}\int_{\mathbf{r}(s)=\mathbf{y}}^{\mathbf{r}(t)=\mathbf{x}} \mathcal{D}\mathbf{r}\, \exp\left[i\frac{\omega}{2}\int_s^t d\tau\, \dot{\mathbf{r}}^2\right] \mathrm{Tr}\,\mathcal{P}\exp\left[iT^c\int_s^t d\tau\,\mathcal{V}(\mathbf{r}(\tau)-\mathbf{x}_{in})\right]$$




$$\mathcal{V} \propto \mu^2 x^2 \log\frac{1}{\mu^2 x^2} + \mathcal{O}(\mu^4 x^4)$$

↓

$$\mathcal{V} = \frac{\hat{q}}{4}x^2$$

misses harder scatterings
but captures the scalings