Proton's inner structure and Quantum Chromodynamics

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18.07.2025. 山东大学

Content:

1. Emergence of QCD
2. Classical tests of QCD or how QCD works
3. Inner structure of proton
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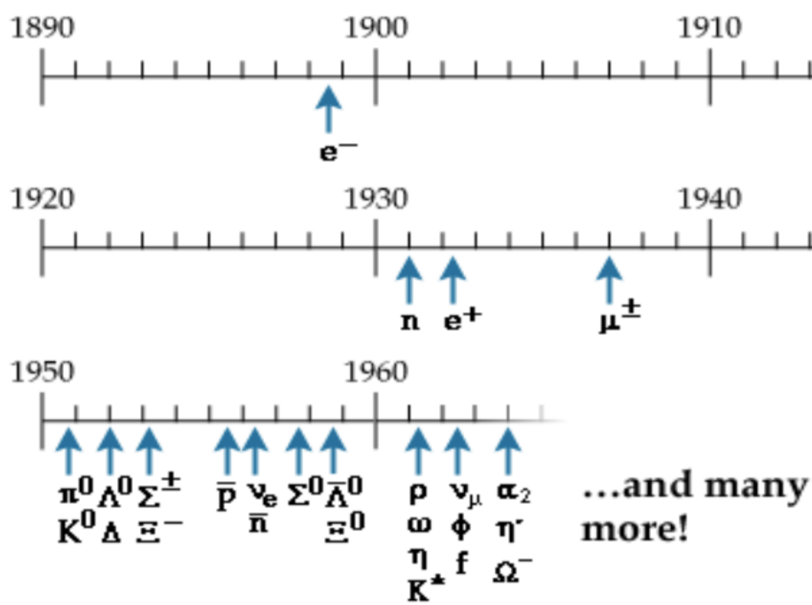
1. Emergence of QCD

Search for elementary particles:

The zoo of particles

- **Particles discovered in 1898 – 1964:**



- **Internal structure of hadrons?**

Magnetic moment: $g_{\text{proton}} \neq 2$ $g_{\text{neutron}} \neq 0$

Particle Data Group, each year, n000 pages, several hundred hadrons……

Many “elementary” hadrons have been found in experiment.

Mesons: Spin=0, 1, 2, 3, ...

Baryons: Spin=1/2, 3/2, 5/2, ...

They can not be elementary.....

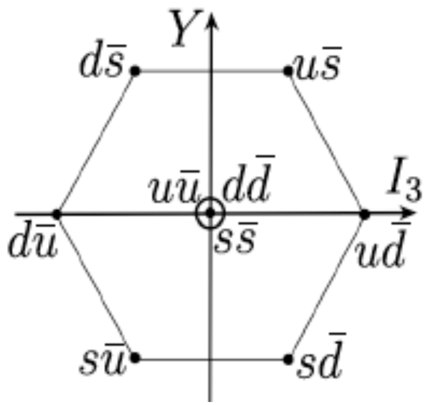
To understand these “elementary” particles, one needs in the first step a classification, like in zoology....

1964: Gellman and Zweig introduced the concept of quarks or states.
Assuming there are three states or quarks as basic vectors forming SU(3) fundamental representation $\mathbf{3}$

and the complex conjugated representation $\bar{\mathbf{3}}$

Then, any meson is a vector in the representation of the product

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \otimes \mathbf{8}$$



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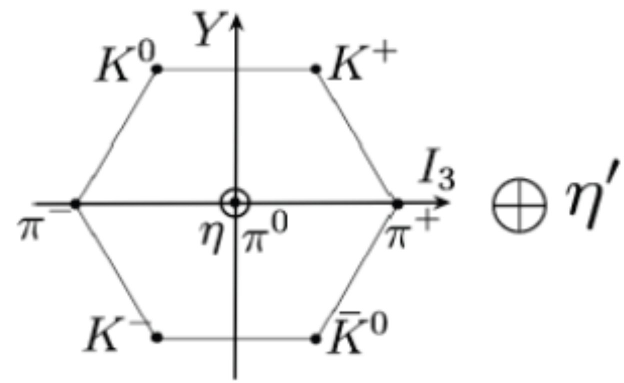
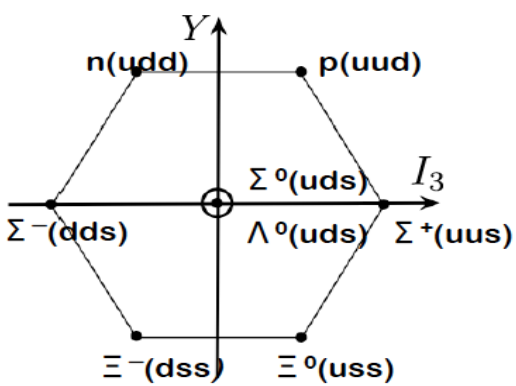
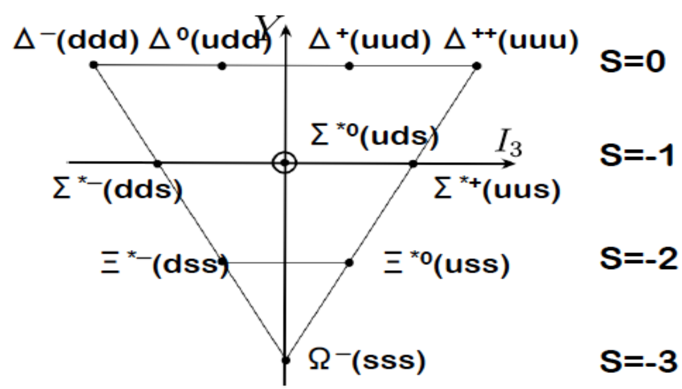


Any baryon is in the representation of the product:

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10_s} + \mathbf{8_{MS}} + \mathbf{8_{MA}} + \mathbf{1_A}$$





Eight fold way

What are quarks of Gellman & Zweig ??

If one takes “quarks” as real objects in Nature with assumed interactions between quarks, one can construct quark models to obtain many “correct- and wrong” predictions.

Remarks about quark models:

- Non-relativistic bound state
- Bound state with fixed constituents.
- Quantum mechanics.

Constituent quarks

If quarks are real objects, they must be fermions

and they must have an additional quantum numbers, called colors

Each quark has three color ➔ SU(3) global symmetry

Hadrons observed in the real world are colorless…

Gauging the SU(3) symmetry ➔ Quantum Chromodynmaic (QCD, 1970s)

$$\mathcal{L} = -\frac{1}{4} G^{\mu\nu} G_{\mu\nu} + \sum_q \left[\bar{q} i \gamma_\mu \left(\partial^\mu + i g_s G^\mu \right) q - m_q \bar{q} q \right]$$

$$q = u, d, s, c, b, t$$

8 gluon fields (gauge fields): $G^{a,\mu}(x), \quad a = 1, 2, \cdots 8.$

The quark masses:

u, d, s: light quarks $m_u \sim m_d \sim 5\mathrm{MeV}, \quad m_s \sim 120\mathrm{MeV}$

Heavy quarks: c, b and t: $m_c \sim 1.5\mathrm{GeV}, \; m_b \sim 4.8\mathrm{GeV}, \; m_t \sim 175\mathrm{GeV}$

Proton(uud) mass: $M_p = 938\mathrm{MeV}$

Coupling constant g_s

In high energy scattering involving light hadrons: E > 2 GeV one can neglect heavy quarks and masses of light quarks.
In this special case, QCD is of three massless quarks
+ massless gluons, and has only one
dimensionless parameter, the coupling constant.

In general, it is difficult to solve the theory.

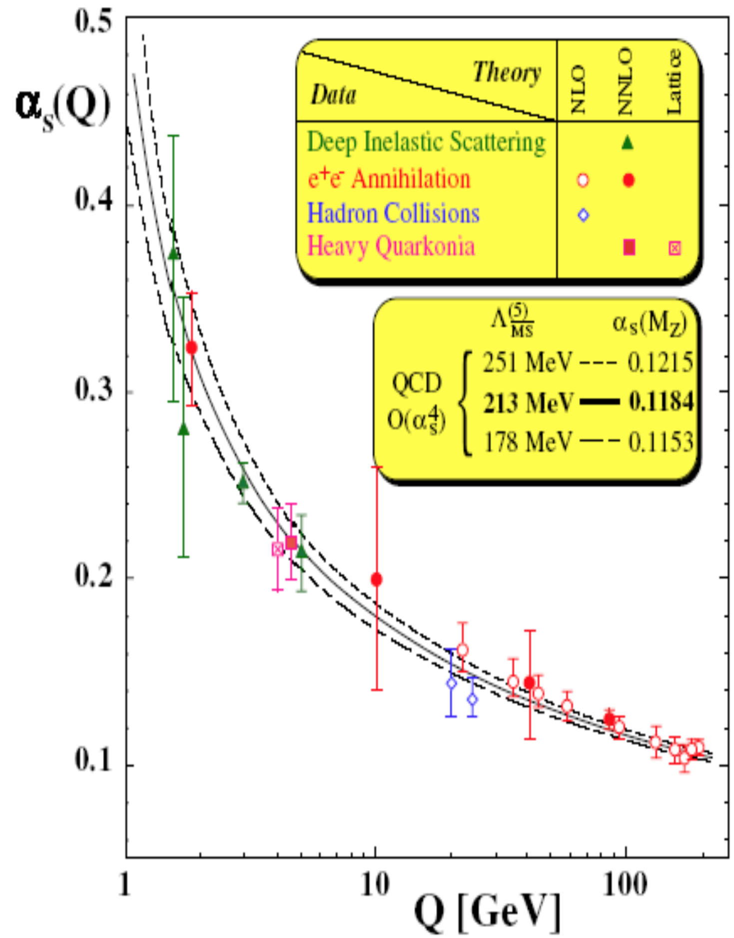
D. Gross, D. Politzer, and F. Wilczek: (1973)

The coupling becomes weaker when the distance becomes smaller.

Asymptotic freedom

$$\alpha_s(Q) = \frac{g_s^2}{4\pi}$$

Q: Energy scale $\quad Q \sim \frac{1}{r}$



An intrinsic scale

$$\Lambda_{\rm QCD} \sim 100 {\rm MeV}$$

Using perturbative theory of QCD to make theoretical predictions?

E.g., predictions of lepton + proton scattering ?

It seems there are two serious problems preventing from it:

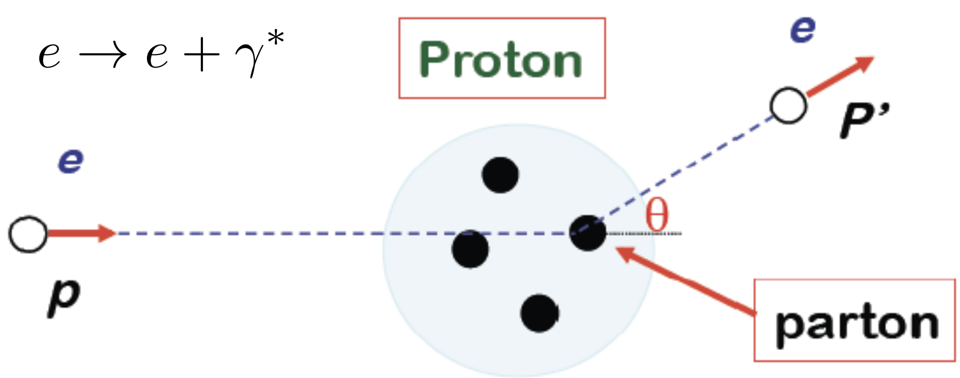
a. The lepton is point-like, but the proton is not.
We don't know the inner structure of proton……

b. Perturbative calculations of parton(quarks, gluons) scatterings always contain infrared- and collinear divergences like those in QED. But, these divergences cannot be handled like those in QED because of confinement…..

2. Classical Tests of QCD or how QCD works

DIS: electron + proton ➔ electron +X

$e \to e + \gamma^*$



Electron-proton deep inelastic scattering (DIS)

✧ Two variables: x_B, Q^2

✧ Localized probe:

$$Q^2 = -(p-p')^2 \gg 1\ \text{fm}^{-2}$$

$$\Rightarrow \quad \frac{1}{Q} \ll 1\ \text{fm}$$

$$x_B = \frac{Q^2}{2P\cdot q}$$

Consider the case:

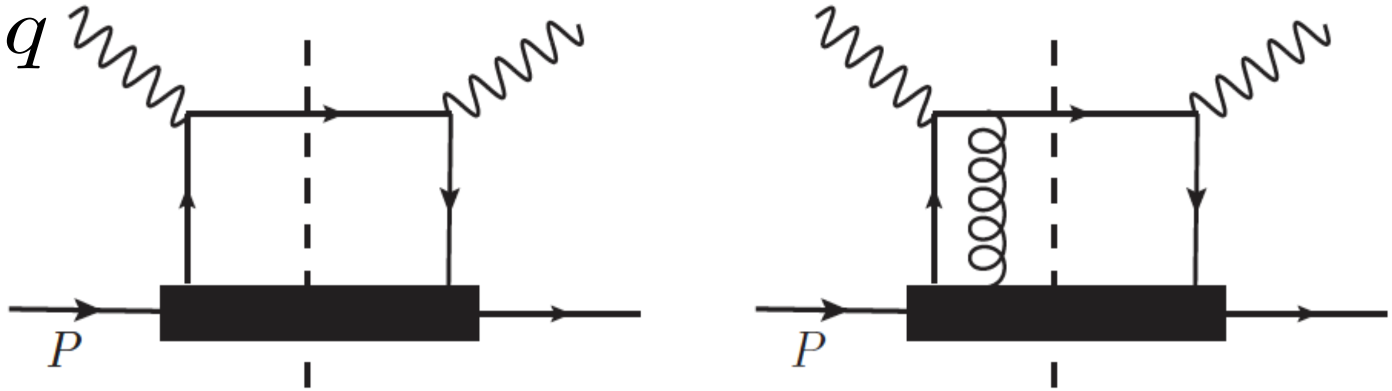
The proton has the large momentum: $P^\mu \approx (P^+, 0, 0, 0)$

(Cross-section is described with structure functions)

Light-cone coordinate system: $A^\mu = (A^+, A^-, A^1, A^2)$

$$A^+ = \frac{A^0 + A^3}{\sqrt{2}}, A^- = \frac{A^0 - A^3}{\sqrt{2}}, A^{1,2} = A^{1,2}, A_\perp^\mu = (A^1, A^2),$$

At tree-level $e \to e + \gamma^*$



+ ....

(a) (b)

Scattering: $\gamma^* + q$ $\gamma^* + q + G$ +...........

Black box: $\langle P|\bar{\psi}(x)\psi(0)|P\rangle$ $\langle P|\bar{\psi}(x)G^{a,\mu}(y)\psi(0)|P\rangle$ + ...

At leading power of 1/Q: all partons carry momentum in + direction,
i.e., the direction of the initial proton
the exchanged gluons are polarized in + direction.

Contributions from these diagrams can be summed....

The summed result for one of structure functions of DIS:

$$F_2(x, Q^2) = x f_q(x) + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2), \qquad x_B = x$$

with

$$f_q(x, \mu) = \int \frac{d\lambda}{4\pi} e^{-i\lambda x P^+} \langle P | \bar{q}(\lambda n) V^\dagger(\lambda n) \gamma^+ V(0) q(0) | P \rangle,$$

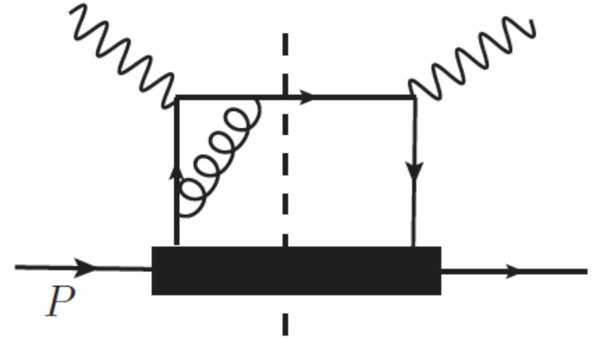
$$V(x) = P \exp\left\{ - g_s \int_0^\infty d\lambda n \cdot G(\lambda n + x) \right\}, \quad n^\mu = (0,1,0,0), \quad n \cdot G = G^+.$$

$f_q(x)$: The probability of finding in the proton a quark with the momentum fraction x. (‼ ??) Distribution
It is gauge invariant and nonperturbative................

lepton +proton ➔ $\gamma^*(q) + q(xP) \to X$

Beyond tree-level:

E.g. , the contribution at one-loop from

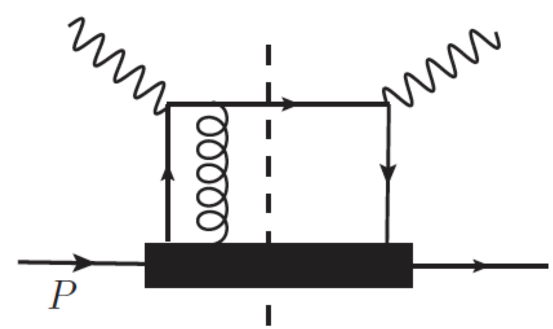


It has the divergences: a. Infrared divergence
b. Collinear divergence associated with the final quark
c. Collinear divergence associated with the initial quark

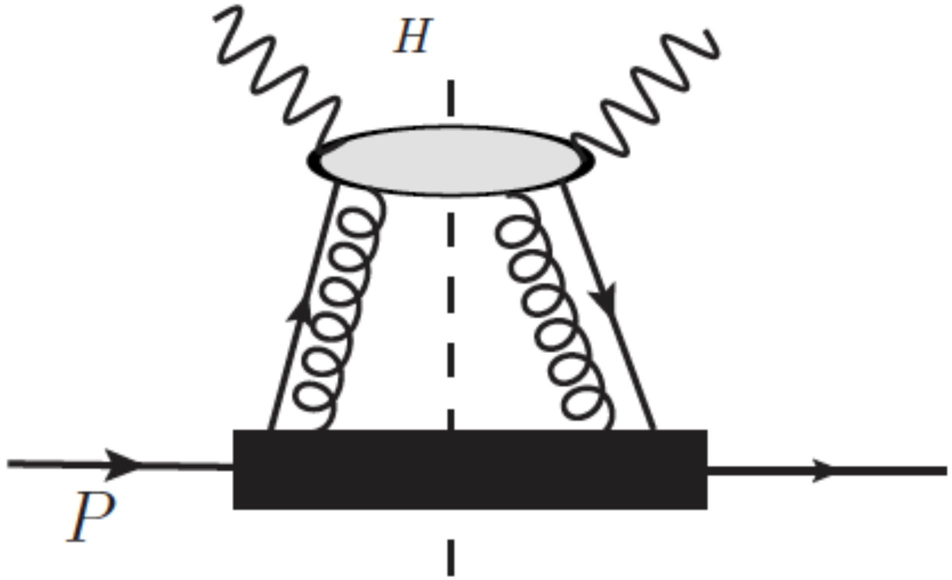
The first two divergences are cancelled after summing all one-loop contribution because the final states are summed in DIS. KLN theorem.

The third divergence is still there after summing all contribution. But it is already contained in the tree-level result, i.e., Fig. b.

Subtraction is needed. After the subtraction the perturbative result is finite.



One can show: At any order, the leading region of any diagram is given by the reduced diagram



Finally, one can prove the factorization theorem for DIS:

$$F_2(x,Q^2) = x \sum_{a=q,\bar{q},G} \int_x^1 \frac{d\xi}{\xi} C_q\left(\frac{x}{\xi}, Q^2, \mu^2\right) f_a(\xi,\mu^2) + \cdots$$

$$= x \sum_a C_a \otimes f_a + \mathcal{O}(1/Q^2),$$

$f_a(x,\mu^2)$: Parton distribution functions defined with twist-2 operators, they can not be calculated with perturbative theory, but are determined by properties or structure of proton.

$C_a(x,Q^2,\mu^2)$: Perturbative coefficient functions, they are free from any infrared- and collinear divergence, i.e., finite.

Factorization means: Effects of short-distance are factorized.
The partons here carry the momentum xP, their transverse momentum is neglected.
(Collinear factorization)

What is predicted from QCD ???

if QCD is right, one can extract pdf's—information about the structure of the hadron from measured structure functions !!

The true prediction is the Q-dependence, which can be compared with experiment.

Dimensionless and QFT:

$$C_a(x,Q^2,\mu^2) = C_a(x,Q^2/\mu^2) = C_a(x,\ln\frac{Q^2}{\mu^2}), \quad \hat{C}_a(x) = C_a(x,0), \quad (\mu^2 = Q^2)$$

$$\rightarrow \quad F_2(x,Q^2) = x \sum_{a=q,\bar{q},G} \int_x^1 \frac{d\xi}{\xi} \hat{C}_q(\frac{x}{\xi}) f_a(\xi,Q^2) + \cdots$$

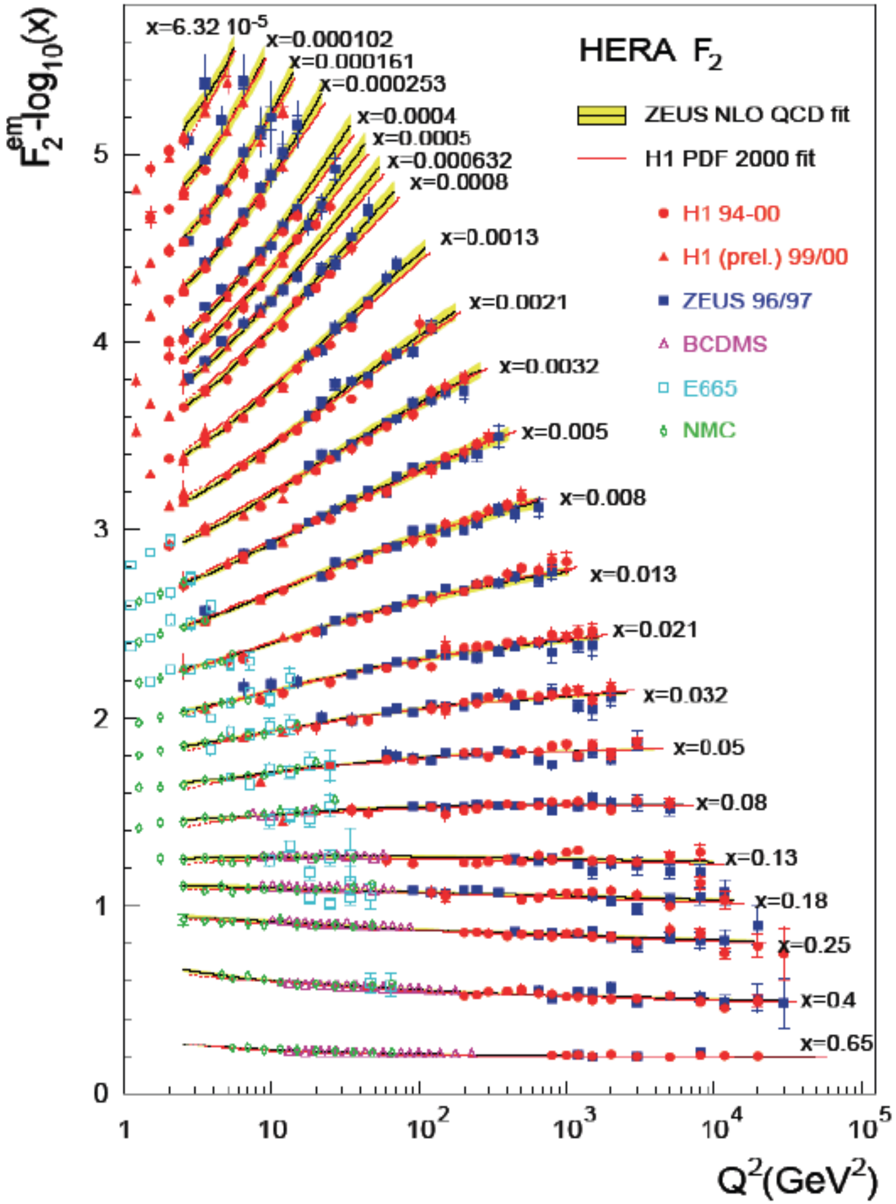
The Q-dependence is determined by that of PDF's. It is given by: (DGLAP Eq.)

$$\frac{\partial f_g(x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}\left(P_{gg}\otimes f_g(\mu) + P_{qg}\otimes f_q(\mu)\right),$$

$$\frac{\partial f_q(x,\mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}\left(P_{qq}\otimes f_q(\mu) + P_{gq}\otimes f_g(\mu)\right),$$

$P_{qq,qg,gq,gg}$: Parton splitting functions, can be calculated with perturbative theory.

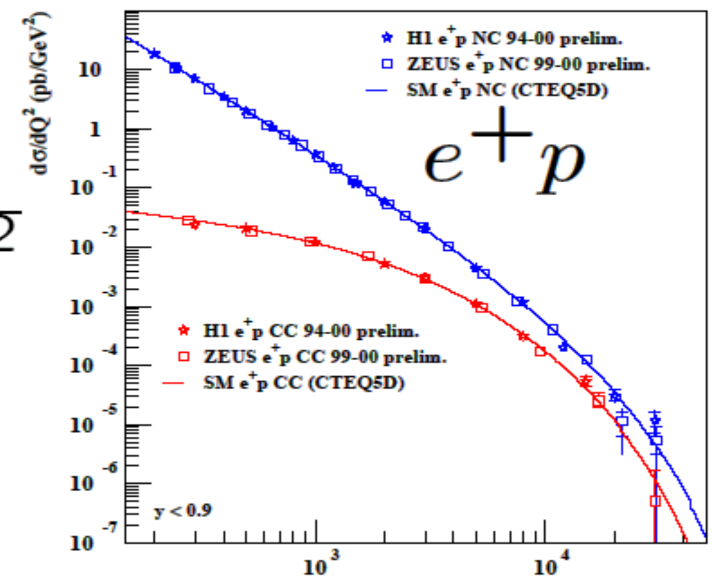
The perturbative theory predicts the Q-dependence!!



"Scaling violation"

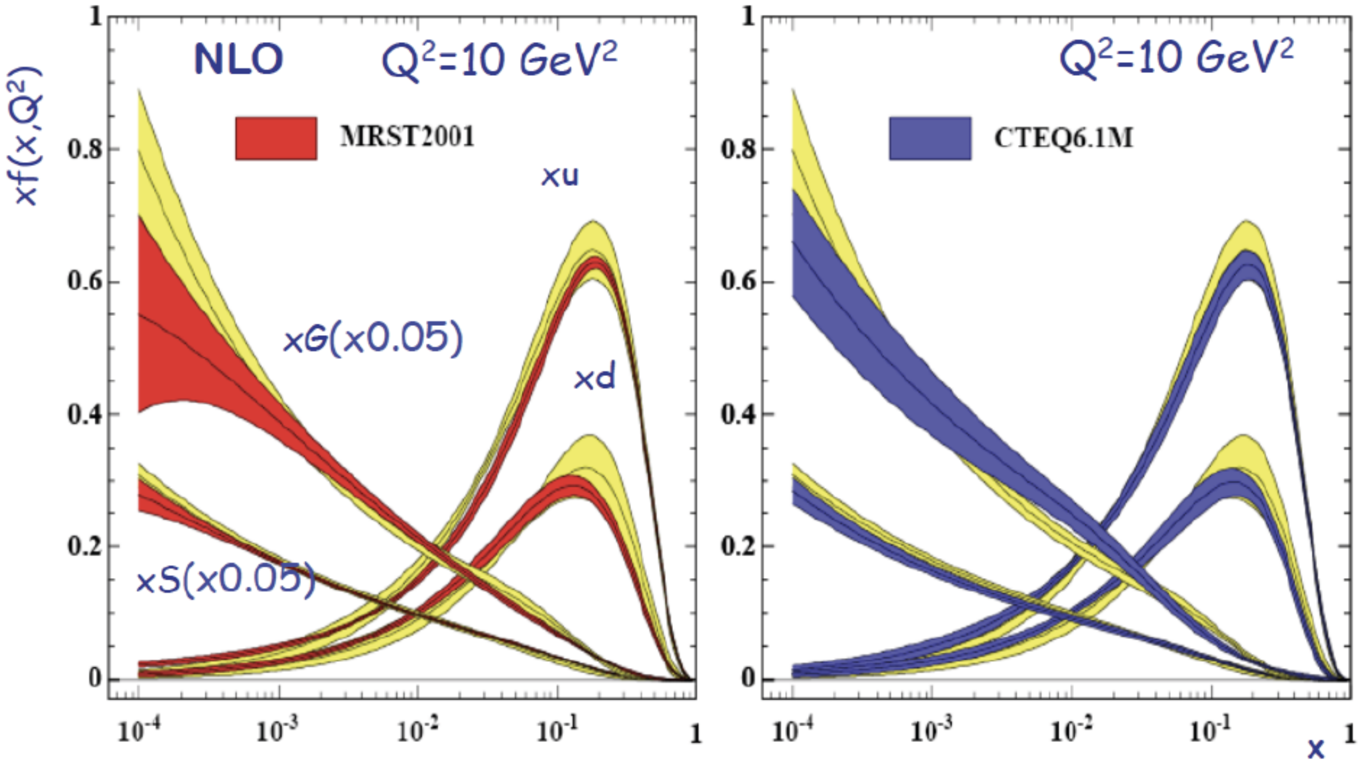
HERA experiment VS theory:
Modern days of QCD

$$\frac{d\sigma}{dQ^2}$$



➔ QCD is the correct theory

Using experimental results one can extract PDF's



Consistently fit almost all data with Q > 2GeV

One-dimensional structure, in the – direction of space-time

$$P^+, x^-$$

For polarized case of DIS, one can also establish a collinear factorization and extract from experiment the polarized pdf's.

There are interesting and unsolved problems…. One of them:

The "crisis" of proton spin:

$$\langle S_z^{\mathrm{N}} \rangle = \frac{1}{2} = J_q + J_g$$
$$= \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

- 1988: EMC: *"spin crisis"* $\Delta\Sigma = 0.12 \pm 0.17 \approx 0\,?$
- 1988–2000: SLAC, CERN, DESY: $\Delta\Sigma \approx 0.2 \ldots 0.4 > 0$

Unlike constitute quark models predicted……..

Various (collinear) factorization theorems can be established or can not be.

For processes involving large momentum transfers one may expect factorizations. Well studied cases:

$$h_A + h_B \to \ell^+ + \ell^- + X$$

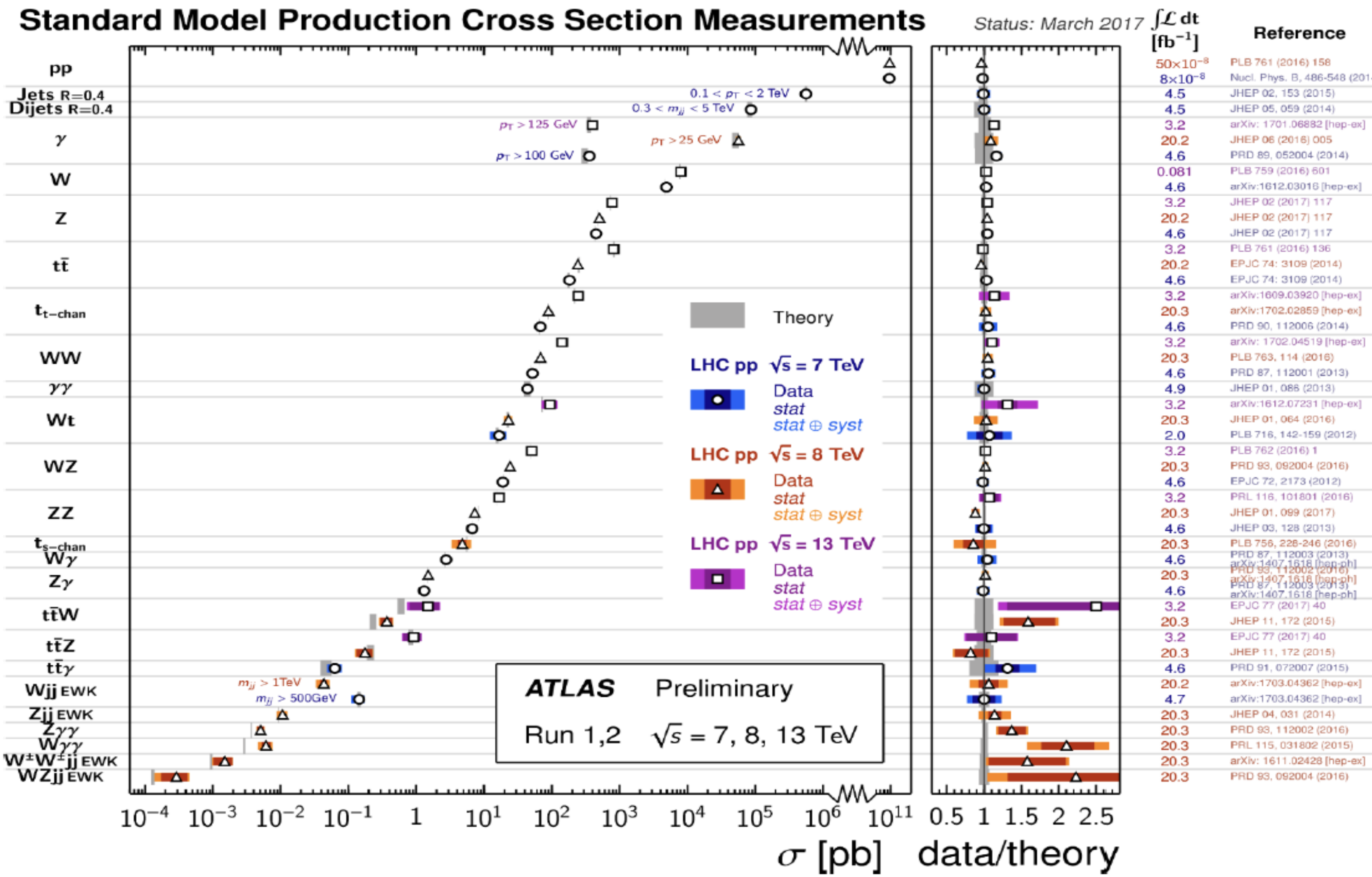
$$e^+ + e^- \to h + X$$

$$h_A + h_B \to A + X$$

$$h_A + h_B \to jet + X$$

PDF's are universal.

Unprecedented success of QCD +SM at LHC



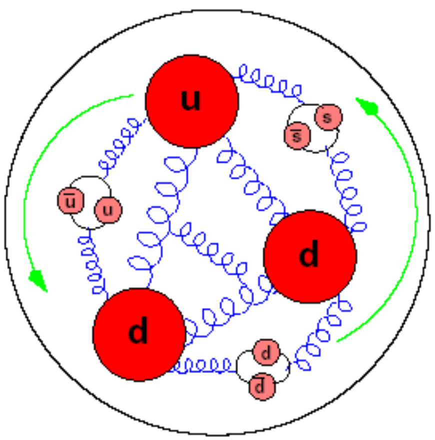
With experiments QCD as a theory of strong interaction is well tested!
Further, we learn:

Proton, and all hadrons are bound states of partons(gluons, quarks) and:

- All partons move relativistic! (quark models??)

- The number of partons is not fixed because quantum fluctuation.
(Gellman's classification is only by keeping minimum numbers of partons, by quantum numbers.)

- No free parton was found! "Quark confinement"??

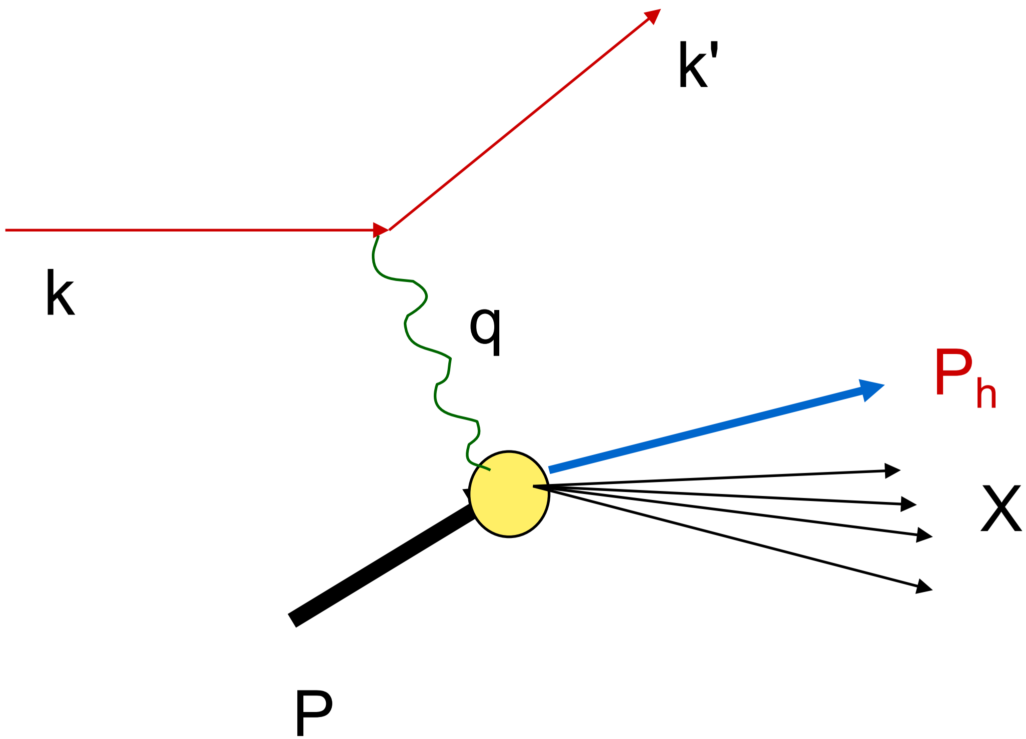


In comparison with quark models
....

The inner structure of a hadron??

3. Inner structure of proton

Semi-inclusive DIS:



- Photon momentum q is in the Bjorken limit.
- Final state hadron h can be characterized by fraction of parton momentum z and transverse momentum $P_{h\perp}$

Three cases for measured $P_{h\perp}$

A. $P_{h\perp} \sim Q$

$P_{h\perp}$ generated from QCD hard scattering, factorization theorem exists. (Standard collinear factorization)

B. $Q \gg P_{h\perp} \gg \Lambda_{QCD}$

Still perturbative, but resummation is needed.
It is important for many processes.

C. $P_{h\perp} \sim \Lambda_{QCD}$

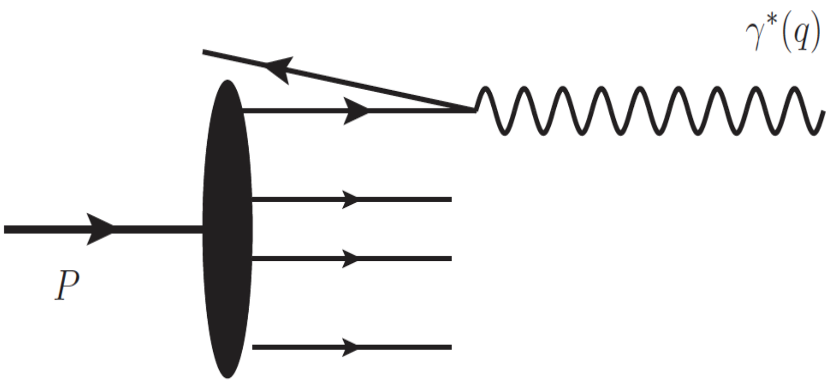
Nonperturbative! $P_{h\perp}$ is generated from partons inside of hadrons.
Transverse momenta of partons: A transparent explanation for SSA
It gives a possible way to learn
3-dimensional structure of hadrons!!!!!

A factorization theorem is needed
for the case $P_{h\perp}$ $\sim \Lambda_{QCD}$!

Head-head collision: $\gamma^* + q \to q$

In DIS, the momentum of the final quark is summed, the transverse momentum of the initial quark is neglected.....

In the case c of SIDIS, the transverse momentum of the final quark is "detected" (?), the transverse momentum of the initial quark can not be neglected, because from it the final transverse momentum comes partly.



The final quark will "become" the hadron observed in the final state.

Need:
TMD parton distribution
TMD factorization
....

The first TMD factorization was proposed and proven for electron – positron annihilation into two back-to- back jets.

J.C. Collins & D.E. Soper (1981)

The goal is to convince people that perturbative theory of QCD works!

In the case of two back-to back jets, the relative transvers momentum is small. There are in perturbative expansion large log terms like:

$$\ln^n \frac{q_\perp}{Q}$$

With "transverse momentum dependent(TMD)" factorization, such large log terms can be re-summed so that perturbative expansion is meaningful.....

The similar studies have been performed for Drell-Yan processes in the case the lepton pair has small transverse momentum.

It takes long time to prove the factorization………

J. Collins, D. Soper & G. Sterman (1982 – 1986)

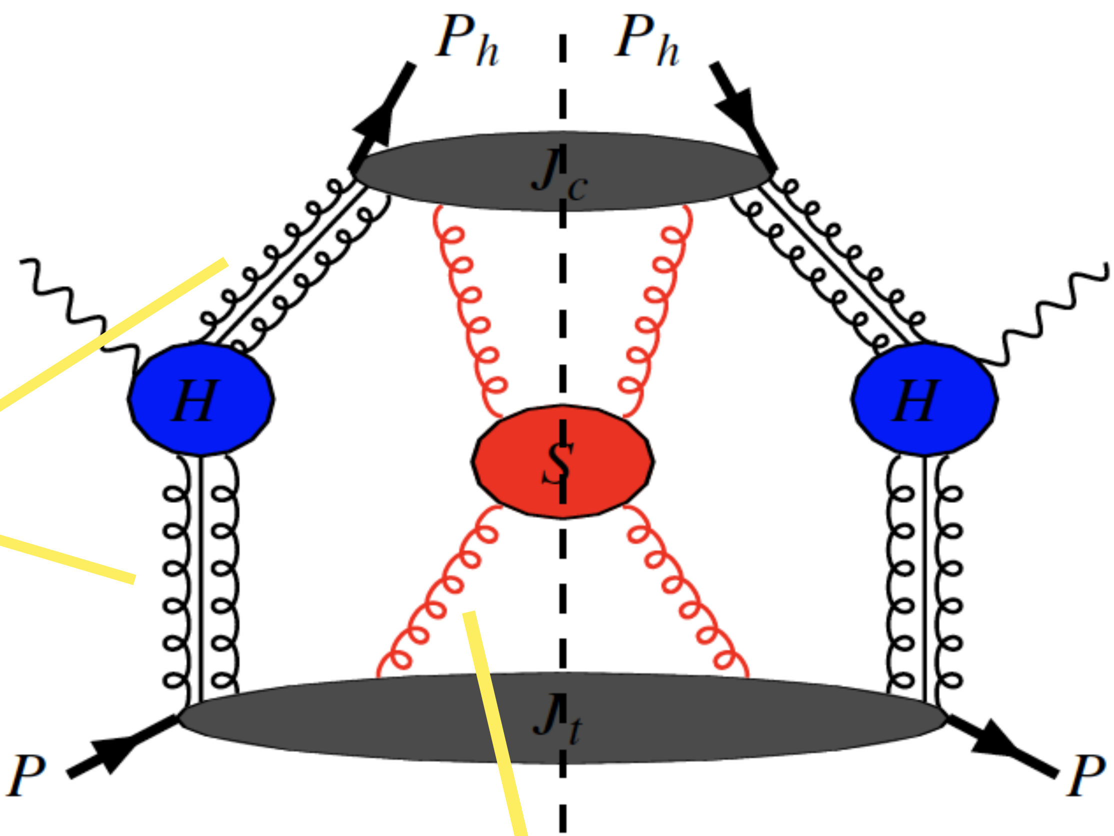
"TMD" factorization was shown in physical axis gauge(….)

The large log terms in perturbative expansion can be re-summed by using Collins-Soper equation, which now becomes well-known because LHC….

Problems of physical gauge: gauge invariance
super leading power contributions
in covariant gauges
……….

For TMD factorization of SIDIS:

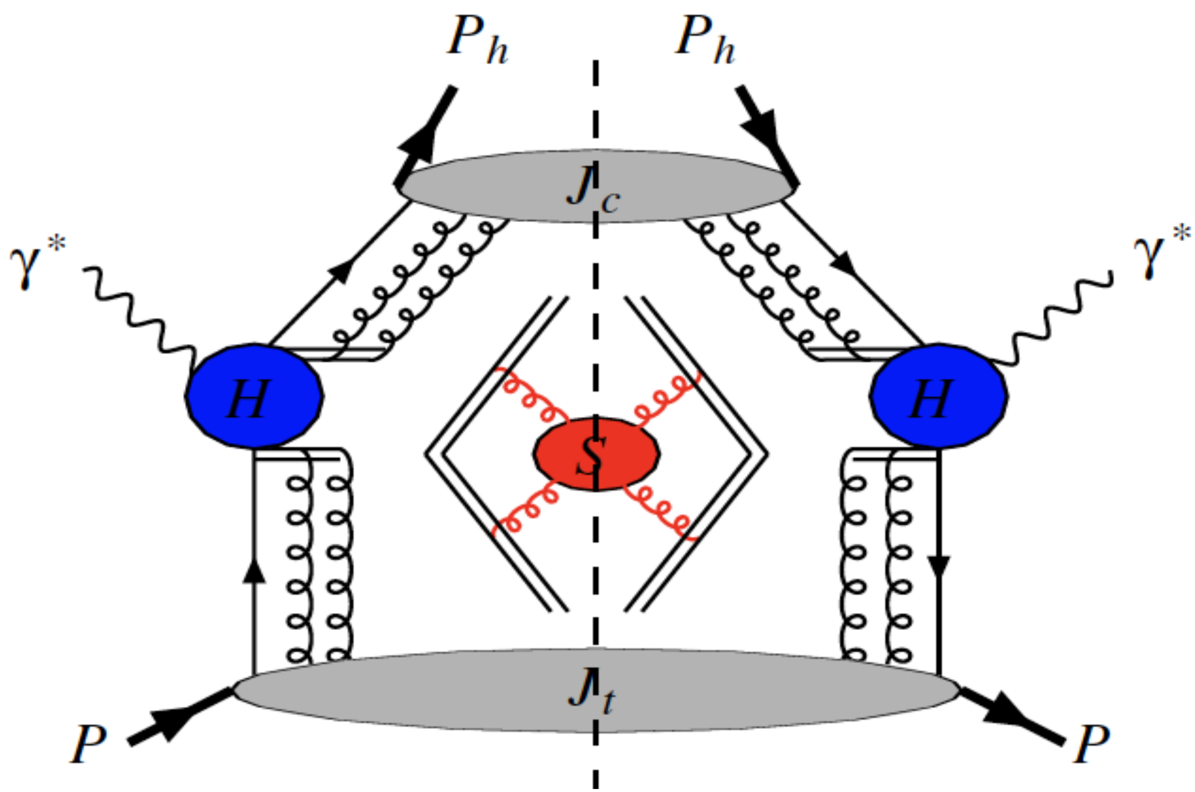
After a power-counting analysis, one finds the leading region of relevant Feynman diagrams which can be represented by the reduced diagram: (Feynman gauge)



Quark combined with collinear gluons

Many soft gluon lines…

The quark lines in the leading region stand for on-shell quark states, therefore, one can use Ward identity to factorize collinear- and soft gluons in a gauge invariant way….. One obtains:



Double lines stand for various gauge links.

A serious but interesting problem arises. The problem is:
How does one define TMD quark distributions ??

A possible definition motivated by twist-2 PDF:

$$f_{q/P}(x) = \frac{1}{2}\int \frac{dx^-}{2\pi} e^{-ixP^+x^-} \langle P|\bar{q}(x^-n)V^\dagger(x)\gamma^+ V(0)q(0)|P\rangle$$

$$= \int d^2k_\perp \left[\frac{1}{2}\int \frac{dx^- d^2x_\perp}{(2\pi)^3} e^{-ixP^+x^- - k_\perp\cdot x_\perp} \langle P|\bar{q}(x)V^\dagger(x)\gamma^+ V(0)q(0)|P\rangle \right]$$

$$= \int d^2k_\perp f_{q/P}(x, k_\perp), \quad x = x^- n + x_\perp$$

Light-cone coordinate system: $P^\mu = (P^+, 0, 0, 0), \quad n^\mu = (0, 1, 0, 0)$

Gauge Link: $V(x) = P\exp\left[-ig_s \int_0^\infty d\lambda n\cdot G(\lambda n + x)\right],$

Light-cone singularity:
From the ligh—cone gauge link one has the eikonal propagator:

$$\frac{i}{n\cdot k + i\varepsilon} = \frac{i}{k^+ + i\varepsilon} \qquad \text{divergent at } k^+ = 0$$

The divergence is cancelled in twist-2 PDF after integration over transverse momentum, but not in TMD PDF !!

How to deal with those singularities ???

One way is to introduce gauge links along the direction off the light-cone, i.e., along the direction:

$$u^\mu = (u^+, u^-, 0, 0), \quad u^+ \ll u^-$$

The TMD quark distribution is defined as:

$$q(x, k_\perp) = \frac{1}{2}\int \frac{dx^- d^2x_\perp}{(2\pi)^3} e^{-ixP^+x^- - ik_\perp\cdot x_\perp} \langle P|\bar{q}(x) V_u^\dagger(x)\gamma^+ V_u(0) q(0)|P\rangle,$$

with the gauge link $V_u(x) = P\exp\left[-ig_s\int_0^\infty d\lambda u\cdot G(\lambda u + x)\right],$

$$x^\mu = x^- n + x_\perp^\mu$$

The defined TMD quark distribution (unsubtracted) is free from light-cone singularities.

Similarly, one introduces gauge links along the direction:

$$v^\mu = (v^+, v^-, 0, 0), \quad v^+ \gg v^-$$

to define TMD quark fragmentation function, $\hat{q}(x, p_\perp)$ free from light-cone trouble

For soft gluon radiation, one introduces the soft factor

$S(l_\perp)$: Vacuum expectation value of a product of four gauge links.

The TMD factorization theorem for SIDIS: (Unpolarized case)

$$d\sigma \sim F(x,z,Q,q_\perp) \sim \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$

$$q(x,k_\perp)\hat{q}(z,p_\perp)S^{-1}(\ell_\perp)H(Q)\delta^2(z\vec{k}_\perp+\vec{p}_\perp-\vec{\ell}_\perp-\vec{P}_{h\perp})$$

TMD parton distribution, 3-dimensional

TMD parton fragmentation function

The soft-factor

The perturbative part

A compact form can be derived in b-space.

With the help of the TMD factorization, one is able to extract from experiment the 3-dim. structure of a hadron.

Ji, Ma and Yuan, 2004

The standard twist-2 PDF's do not depend on hadron's momentum.

But, TMD parton distributions do ! It is through the variable:

$$\zeta_u^2 = \frac{(2u \cdot P)^2}{u^2}, \qquad q(x, k_\perp, \zeta_u^2, \mu^2),$$

This gives the possibility to control the energy of the collision parton system.

The dependence is given by Collins-Soper equation. Hence resummation can derived in the perturbative region.

There are 8 TMD distributions in comparison with 3 twist-2 PDF's at leading power:

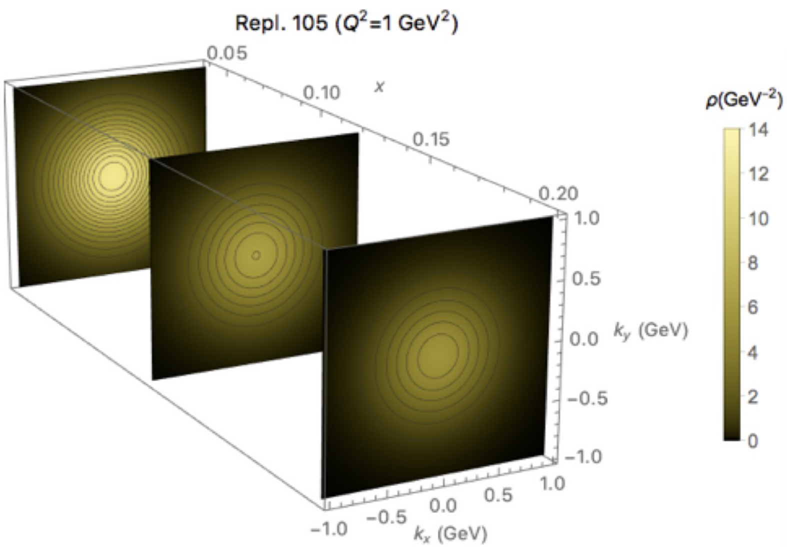
Nucleon / Quark	Unpol.	Long.	Trans.
Unpol.	$q(x, k_\perp)$		$q_T(x, k_\perp)$ (Sivers)
Long.		$\Delta q_L(x, k_\perp)$	$\Delta q_T(x, k_\perp)$
Trans.	$\delta q(x, k_\perp)$	$\delta q_L(x, k_\perp)$	$\delta q_T(x, k_\perp)$ $\delta q_T'(x, k_\perp)$

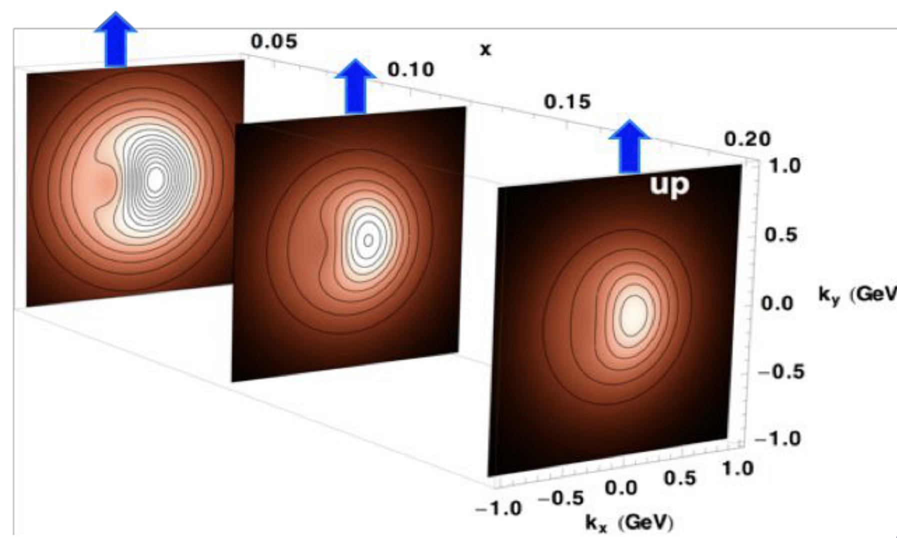
These TMD distributions provide more information that the 3 twist-2 PDF's.

Nucleon partonic structure - 3D imaging by Andrea Signori

Unpolarized proton

Transverse-polarized proton





Transverse-polarized quark distribution is distorted

Experiments:

We want to know TMD parton distributions !

Existing experiment facility: JLab, 12 GeV electron beam, fixed target

In future: Eic (US), EicC (China), Jlab-20GeV upgrade…

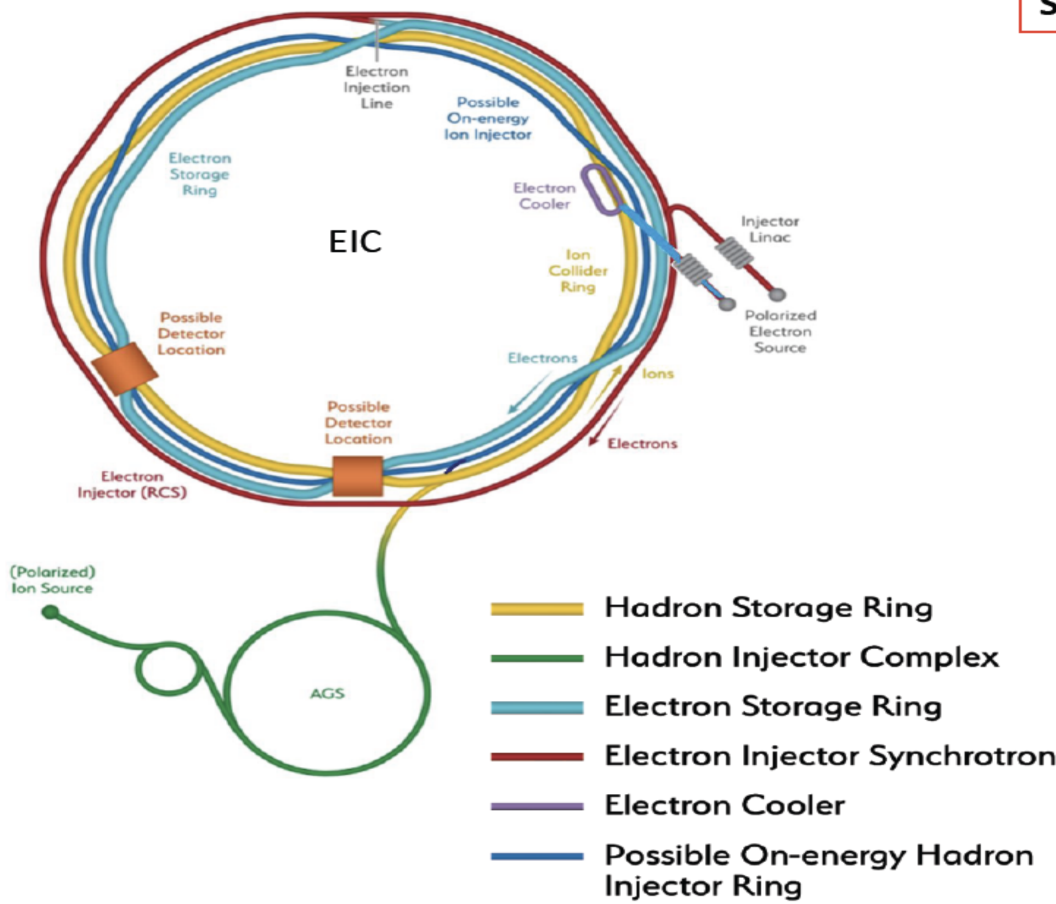
TMD physics is an important of physics programs of experiments.

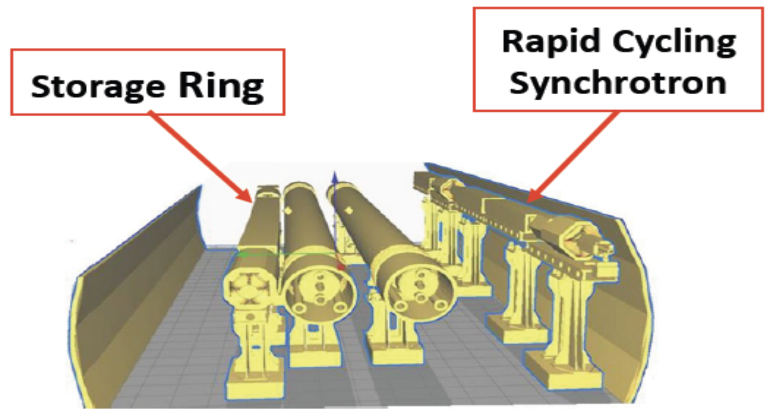
LatticeQCD: TMD parton distributions can be calculated…..

Planned facilities:

U.S. - based Electron-Ion Collider

❑ To be built at BNL:





- **Center of Mass Energies:**
  20 GeV – 141 GeV
- **Required Luminosity:**
  10^{33} - 10^{34} $cm^{-2}s^{-1}$
- **Hadron Beam Polarization:**
  80%
- **Electron Beam Polarization:**
  80%
- **Ion Species Range:**
  p to Uranium
- **Number of interaction regions:**
  up to two

Planned site: 广东惠州

Electron-Ion Collider in China (EicC)

EicC-I New construction

- polarization ion source
- Siberia snake for FRing
- e injector

SRF Linac-ring

4~5 passes

- eRing

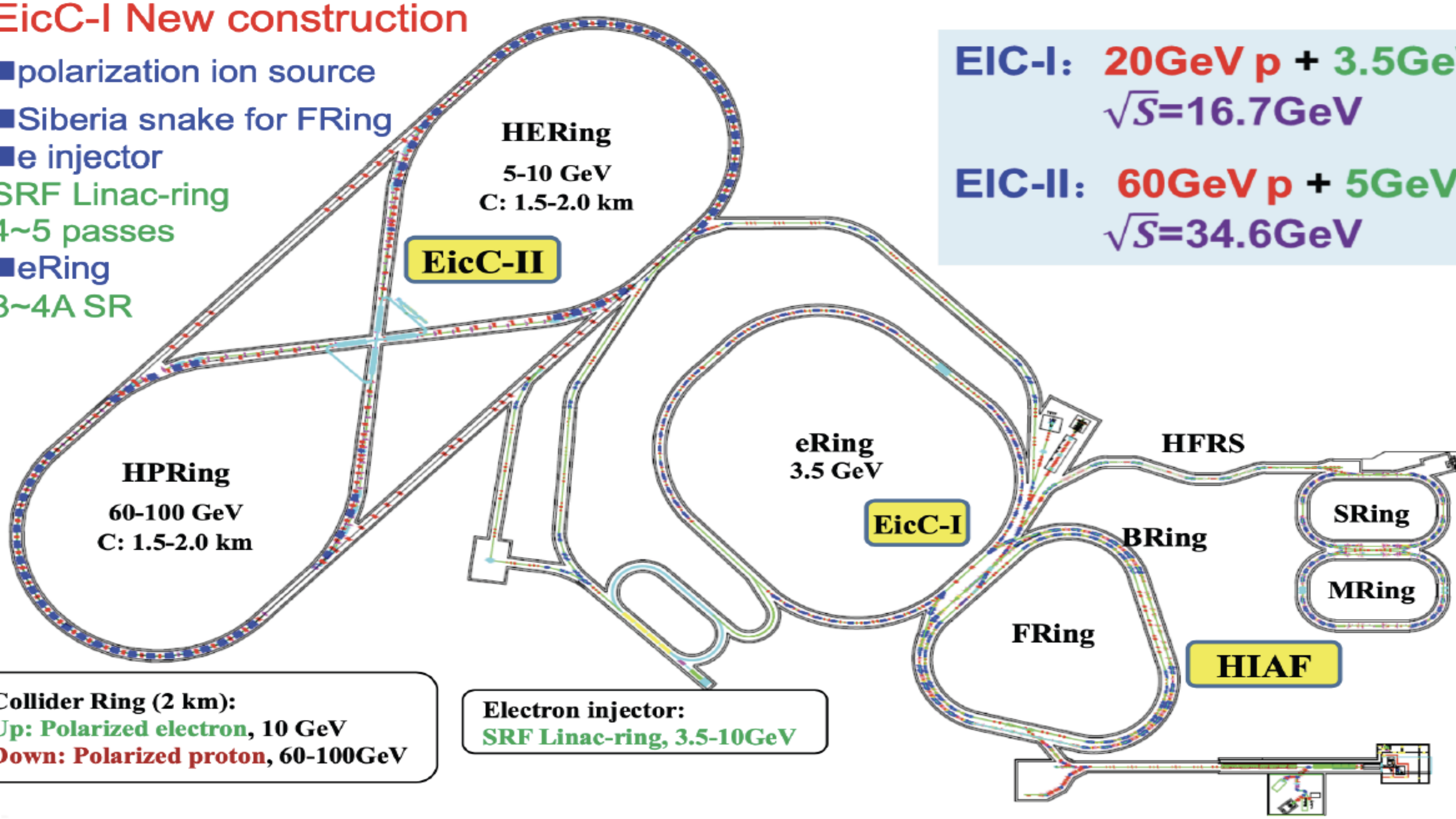
3~4A SR

EIC-I： 20GeV p + 3.5GeV e

$\sqrt{S}$=16.7GeV

EIC-II： 60GeV p + 5GeV e

$\sqrt{S}$=34.6GeV



Collider Ring (2 km):
Up: Polarized electron, 10 GeV
Down: Polarized proton, 60-100GeV

Electron injector:
SRF Linac-ring, 3.5-10GeV

4. Summary

With factorization theorems, one can use perturbative theory of QCD' to make reliable predictions

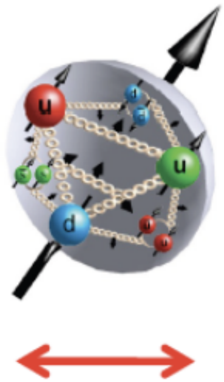
With TMD factorization theorems for a class of processes, one can "detect" the inner structure of hadrons, e.g., proton

The challenge of QCD: Confinement

- **Hadron properties in terms of dynamics of quarks and gluons:**

Hadron properties

Charge,
Mass,
Spin,
Magnetic moment,
…



QCD

Quarks
Color,
Flavor,
Charge,
Mass,
Spin,
…

+

Gluons
Color,
Spin,
…

Thank you !