重庆大学
CHONGQING UNIVERSITY

What's the Principle of Maximum Conformality and Its Recent Progresses

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In collaboration with
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10 PhDs have been trained

OUTLINE

- **The QCD scale-setting problem —— very important**
  - The first round PK: /FAC/PMS/BLM/
  - The second round PK: /seBLM/PMC/

推动前行
深化课题

- **The Principle of Maximum Conformality**
  - Examples – /Resummation/Differential/UHO/
- **Summary and Outlook**

学术发展需要PK
一台大戏，多重启示
争议推动前行

夸克模型诞生记：老谋深算盖尔曼

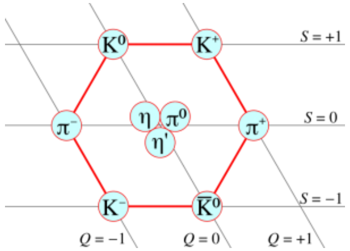
—— 邢志忠

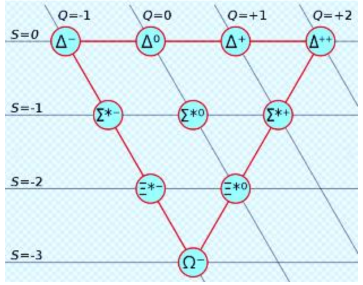
1962年共同参会，预言Ω- ？

1953, 盖尔曼/尼曼 提出奇异量子数

盖尔曼提出介子八重态、重子十重态：1961.01预印本-1961.03接收-1962.02发表-1969次
尼曼（导师萨拉姆）类似工作，1960年底完成，未发预印本，1961.08发表-878次

PK1



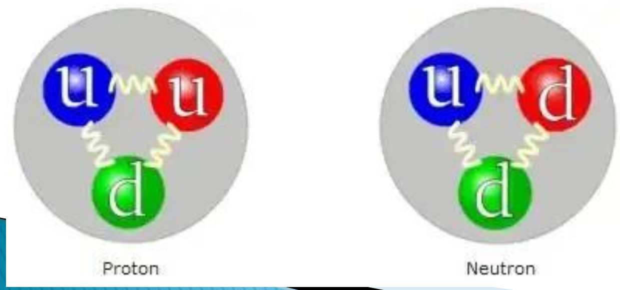


彼德曼提出类似分数电荷粒子：1963年底投稿，最早提出
但1965.03发表于《核物理(法语)》-无影响力-21次

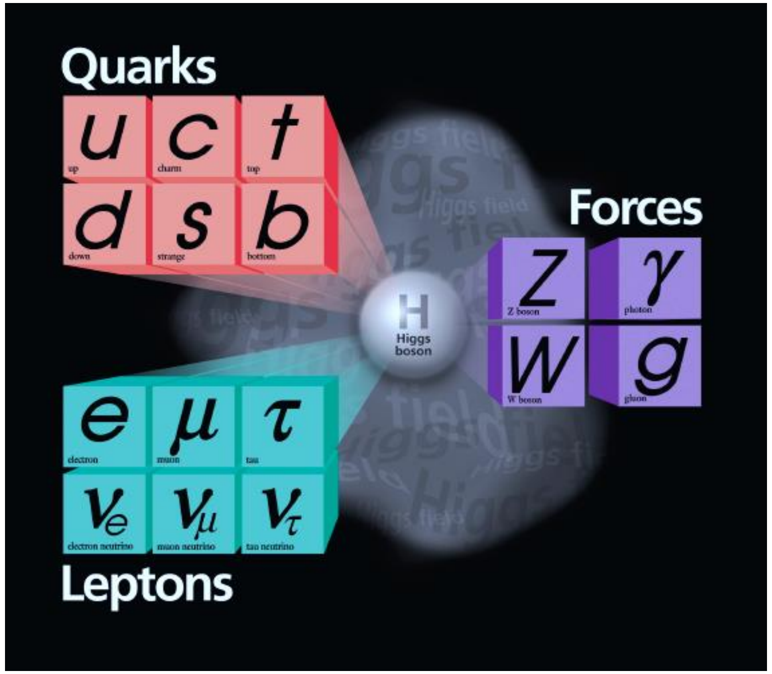
盖尔曼提出夸克模型(2页)-《物理快报》-4536次：
1964.01投稿，02发表；致谢提及1963.03已有想法-时间提前

PK2

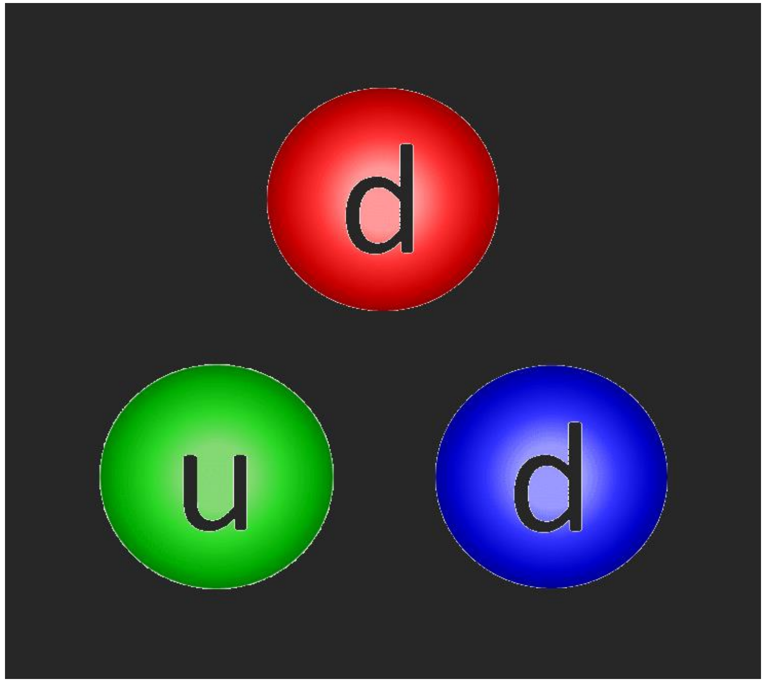
茨威格提出类似模型：1964.01(26页)和.02(80页)预印本
(CERN主任范霍夫特阻止投稿美国，放弃投稿 ?!)-900次



持之以恒，抢占先机
终成正果



标准模型



强相互作用 QCD

重要问题之一

如何获得可靠且精确的微扰QCD理论预言值

前提：相信微扰论

获得有限阶高精度微扰论预言的三要素

一、**发展高效微扰论高阶计算方法** ＝＝ **基础**

目标：完成更高阶微扰计算，获足够高阶信息；完成特定相空间处理处理

包括振幅计算、Feynman积分化简、多体相空间、自动化程序、重求和、碎裂机制等

二、**找到正确的重整化能标设定方案** ＝＝ **核心**

目标：最大程度地获得有限阶下的精确预言

与重整化能标及重整化方案无关的能标设定方案极其重要

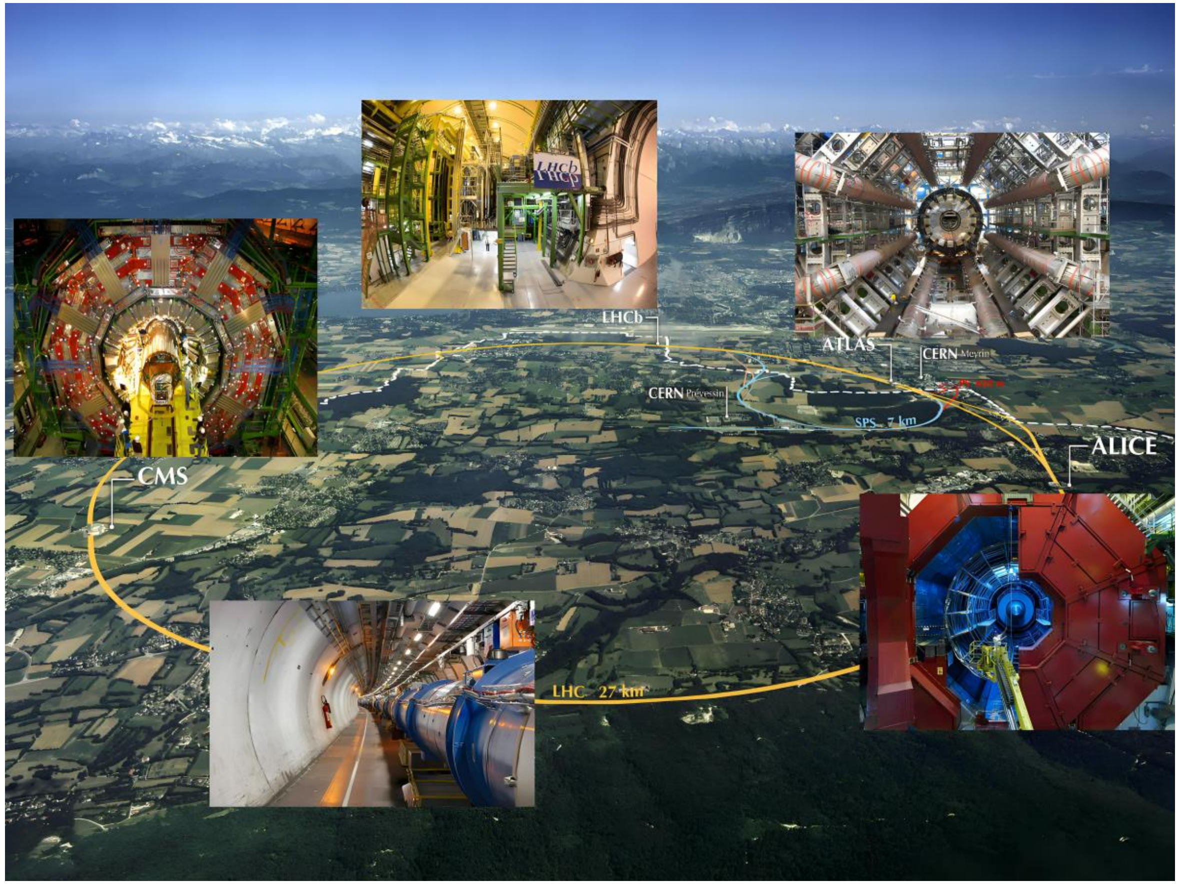
三、**找到估算未知高阶项贡献可靠方法** ＝＝ **延拓**

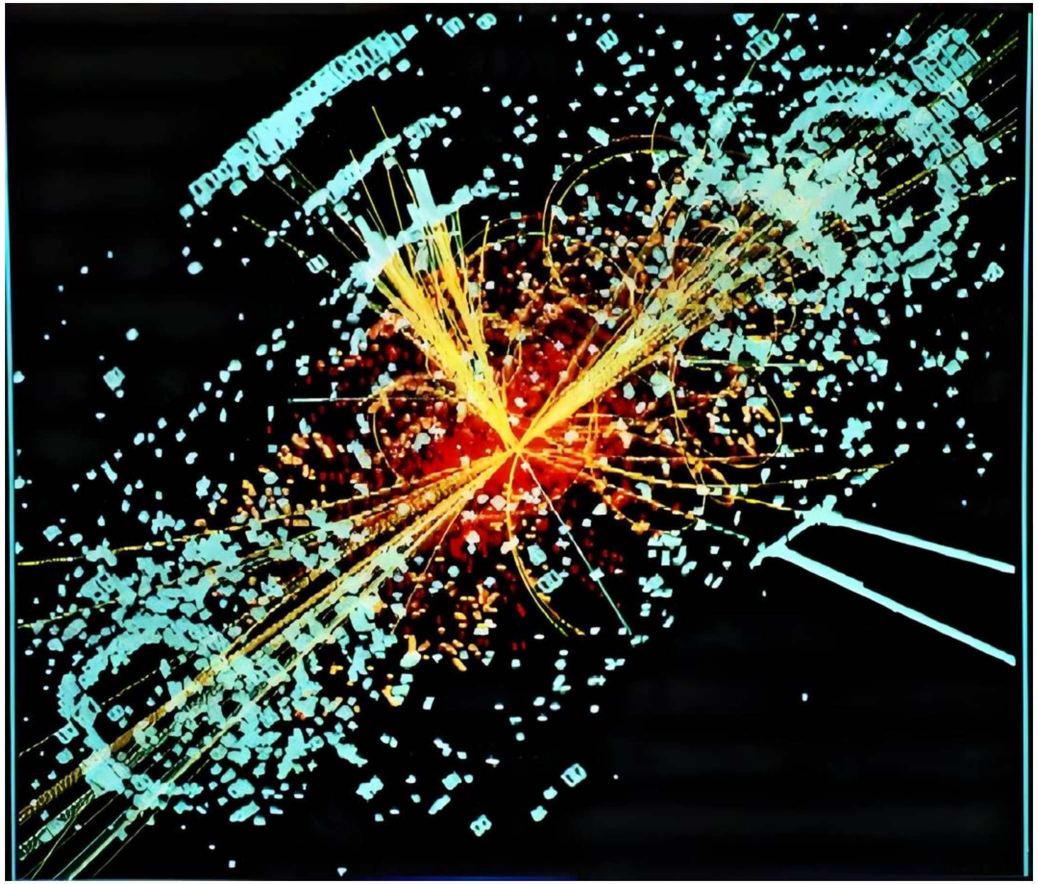
目标：基于已知微扰序列，估算未知阶贡献，帮助判断有无新物理

已知序列若具备更好微扰收敛性，有助于压低未知高阶项所导致的理论不确定性

注：与方案选择无关，不妨碍存在收敛性最好的方案

➢ The QCD scale-setting problem


LHCb
ATLAS
CERN Meyrin
CERN Prévessin
SPS 7 km
ALICE
CMS
LHC 27 km

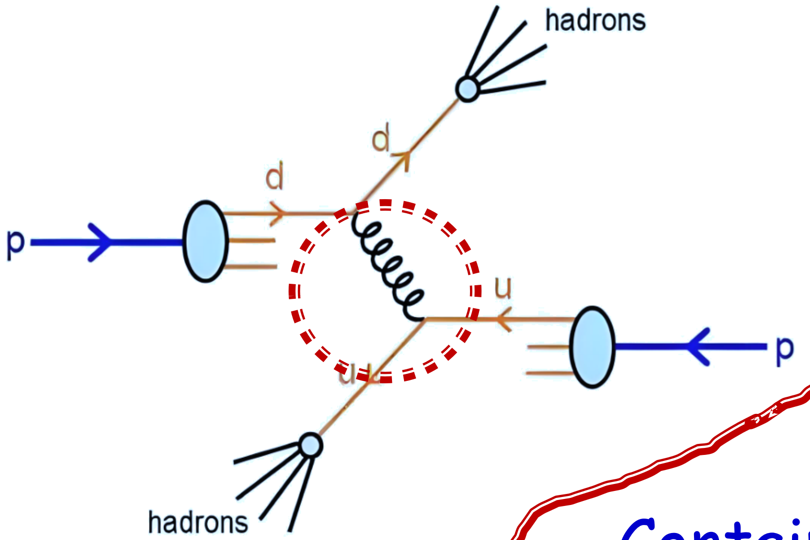


Factorization Picture
for the Fixed-order pQCD predictions at hadronic colliders

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

Parton Distribution Functions

Fragmentation Functions



Perturbative partonic cross section (virtual & real radiation)

$$\hat{\sigma} = \sigma_0[1 + \alpha_s + \alpha_s^2 + \cdots]$$

Contains α_s^n of tree level process

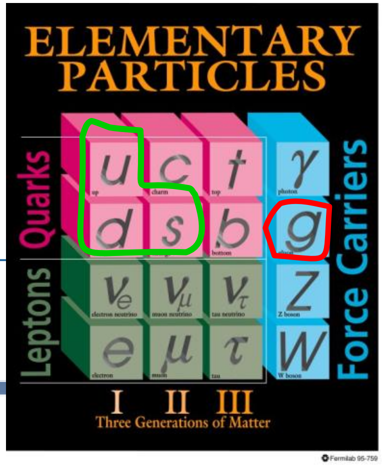
NLO

NNLO

QCD渐近自由

保证高能区微扰可算

重点关注：微扰序列本身的性质



量子色动力学—强相互作用的基本理论

量子色动力学重要性质

研究方向一

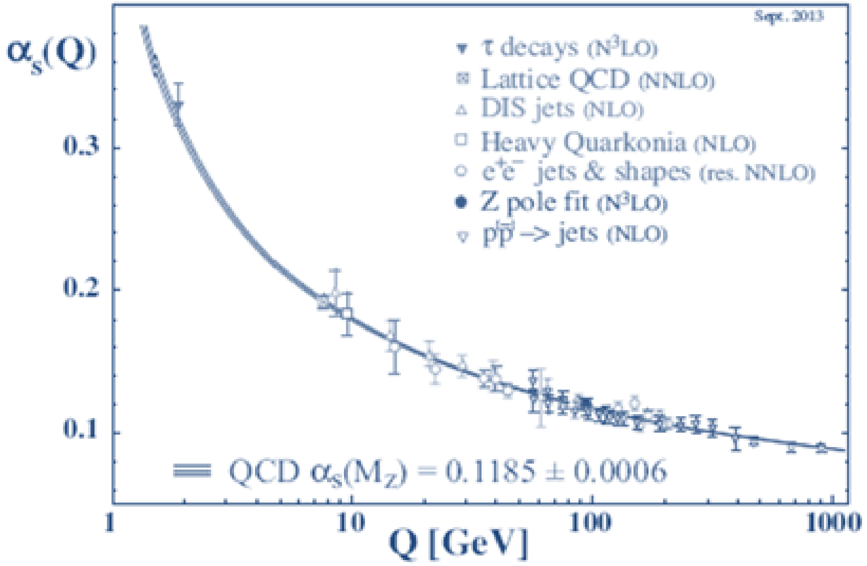
■ 夸克禁闭（不存在自由夸克）

$$\Lambda_{QCD} \sim 1/R_{hadr} \sim 0.2\text{GeV}$$

研究方向二

■ 渐近自由理论（微扰可算）

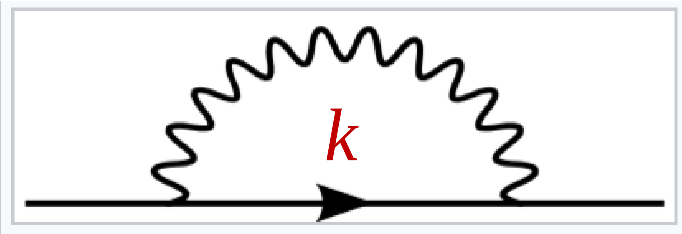
$$\alpha_s\left(Q \gg \Lambda_{QCD}\right) < 1$$

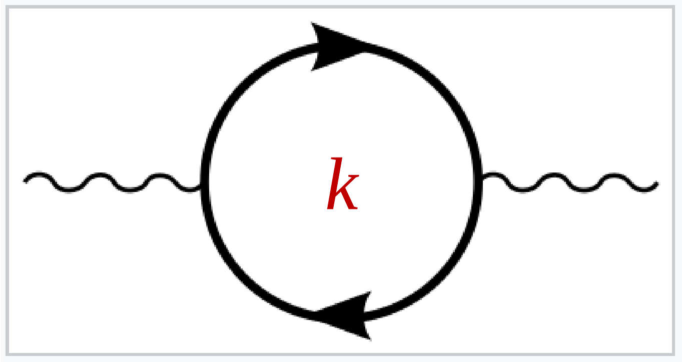


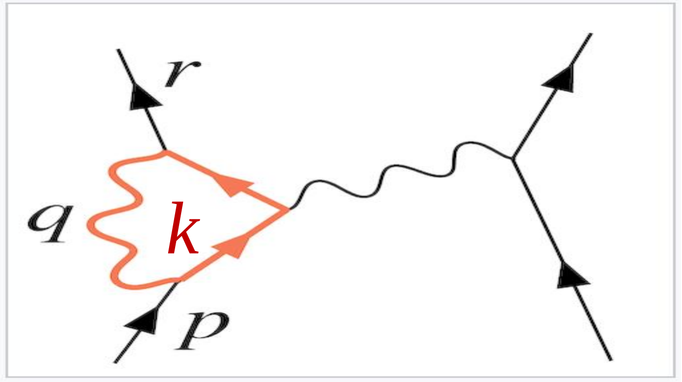
微扰高阶圈动量积分引起**发散**

$\int d^4 k$

圈动量 $\rightarrow \infty$ $\Rightarrow$ 紫外发散（UV-发散）







如何得到有限结果？解决方案通常分两步

保证微扰可靠

正规化 —分离发散积分

重整化 —划分发散（重整化方案）

—消除发散与实验对应（引入实验测量值作为基准点）

原始拉氏量中的参数（质量、场量、耦合常数等）并不对应于实验测得的物理常数，它们是并不包含虚粒子圈效应的裸量

为与现实关联，拉氏量需要用可测量及可重整的参量来表示。例如，电子电荷（对应电磁精细结构常数），可以基于某个特定能标点来定义

我们观点，还需有第三步

需设定正确重整化能标

回望历史轨迹
寻求新的思路

The Glorious Days of Physics - Renormalization of Gauge Theories

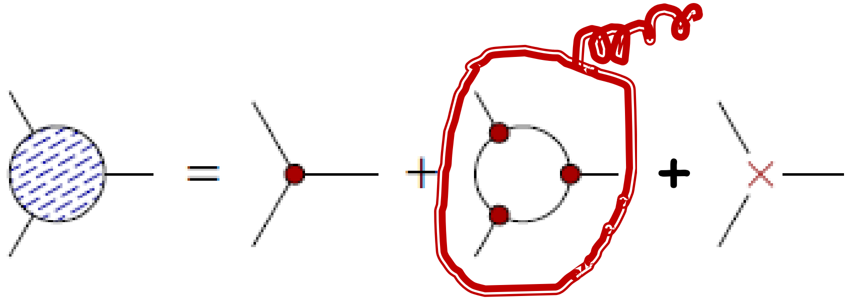
-- Gerard ’t Hooft

hep-th/9812203

- 1953年 Peterman 与 Stueckelberg - 提出**重整化群**及**重整化群不变性**的思想

耦合常数随能标跑动
振幅与减除方案无关

注：此图仅为
简单示意图



$$\Gamma = g^{\mathrm{ren}} + (g)^3 \int (\cdots) + \Delta g$$

完整振幅 = 低阶顶点g^{ren} + 单圈修正 + 抵消项Δg（吸收表观发散项）

g^{ren} 与 Δg 之间的划分是任意的， 完整振幅（无穷阶）不应该依赖于这种划分。**完整振幅与减除方式的无关性 - - 被称为重整化群不变性**

“This is a one-dimensional subgroup of the renormalization group, and it is all that is still in use today”

Quantum Electrodynamics at Small Distances

M. Gell-Mann and F. E. Low
Phys. Rev. **95**, 1300 – Published 1 September 1954

- 1954年，M.Gell-Mann 与 F.E.Low，提出QED耦合常数α的重整化群变换
  —— 引入重整化群方程（β—函数）

接受耦合常数随能标跑动观点
但指出随能标跑动至少是二阶效应

$$\mu d\alpha/d\mu = O(\alpha^2) > 0$$

$$\alpha(Q) \to \alpha = 1/137.036\cdots \quad \text{as } Q \to 0$$

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$$

精细结构常数

重求和所有真空极化图
贡献可获能标无关结果

GM-L方案 - 结论
QED不存在明显重整化能标设定问题
“1/137-本身数值小”

$\mu d\alpha/d\mu = O(\alpha^2) > 0$

争论

- 约1954年，Landau认为，QED中β-函数的右边是增函数，μ大时有突增过程，从而导致奇点的出现——朗道奇点
  —— 结论：重整化群方程方法在数学上不完善
- Gell-Mann等认为β-函数会出现零点，耦合常数会终止于该点（实验点）——正好对应裸耦合常数——可通过非微扰方式确定。

注：对QED来说，上述讨论只具科学意义，the Landau scale ~10^{286} eV

一个模型
一种方法
不可能解决所有问题
关键是谁更普适

谁更深刻

当时认为量子场的相互作用在短距离下的强度均与QED类似，为无限大；形成量子场论有缺陷的观念

- **1954年，C.N.Yang和R.Mills提出定域规范不变性，构造简单拉氏量**

$$\mathcal{L}^{\mathrm{YM}} = -\tfrac{1}{4}G_{\mu\nu}G_{\mu\nu} - \bar{\psi}(\gamma D + m)\psi$$

引入无质量矢量粒子
—不对应现实粒子

有红外散问题

构造新型

可重整模型

- **1961年，S. Glashow添上质量项解释弱相互作用**

$$\mathcal{L} = \mathcal{L}^{\mathrm{YM}} - \tfrac{1}{2}M^2A_\mu^2$$

手动放入质量
—不自然，破坏规范不变性

构造可重整模型

整体连续对称性的自发破缺必定导致无质量的标量粒子

- 1961年- Goldstone连续对称性自发破缺会产生无质量Goldstone粒子
- 1964年，P.Higgs证明定域规范理论Goldstone粒子会被有质量粒子所取代；F.Englert和R.Brout证明矢量粒子将获得质量
- 1964年，A.Salam采用YM+Higgs机制构造弱相互作用模型
- 1967年，Weinberg将电磁与弱相互作用合而为一

同时，需要解决的问题 - 证明模型的可重整性

- 1961年，基于S. Glashow模型，Veltman证明其单圈可重整性；但在继续证明所有圈的可重整性时遇到紫外困难
- 1967年，L.D.Faddeev和V.N.Popov规范场路径积分量子化加入规范约束条件，保持局域规范不变性同时消除发散
- 1970年，tHooft加入Veltman课题组，尝试分两步解决问题

I）如何重整无质量纯 Yang-Mills 体系中的振幅？＝＝微扰论框架
II）如何将这一切与质量项匹配起来？ ＝＝利用Higgs 理论

证明可重整

- 1971年，tHooft引入第五维实现无质量Yang-Mills场的单圈重整化；为维数重整化方法的前身。
- 1971-1972年，Taylor和Slavnov提出Slavnov-Taylor等式证明振幅的高阶可重整性。
- 1972年，tHooft和Veltman提出维数正规化/重整化-重整化多圈图有用工具：4-ε维处理，消除对数发散、而线性及平方发散将通过分部积分消除。振幅在复ε平面的极点通过引入规范不变的抵消项消除。

维数正规化理论可正确地描述观测到的相互作用
下一步，讨论重整化群效应

- 1970年，Callan和Symanzik独立提出方程讨论重整化群效应——指出猜想
——“所有可重整化理论的β函数与QED一致均大于零”
无法解释Bjorken于1969年发现的标度律－该标度律表明耦合常数在能标大时很小。

- 1973年，Politzer和Gross和Wilczek提出QCD理论的渐近自由性质，解释了标度律。
注：1969-赫里普洛维奇发现SU(2)规范场论渐近自由；1972-tHooft也注意到但未发表

简单小结

微扰QCD理论 – 按α_s展开估算高能物理量

$$\rho = r_0\, \alpha_s^{p}(\mu_R) + r_1\, \alpha_s^{p+1}(\mu_R) + r_2\, \alpha_s^{p+2}(\mu_R) + \ldots$$

只有计算完所有阶或足够高阶才对应物理量真实值

$k \to \infty$，导致紫外发散

重整化理论消发散

ρ_n

正规化、重整化、能标设定 → 往往忽略

准确预言具同等重要性

→ 高精度微扰预言？

微扰论预言应与人为引入的参数无关

（1953年 Peterman 与 Stueckelberg ） 重整化群不变性

物理观测量 $\frac{\partial \rho}{\partial \mu_R} \equiv 0; \frac{\partial \rho}{\partial R} \equiv 0$

等价

Standard RGI

有限阶原始序列 ρ_n

Equivalence to: $\frac{\partial \rho_n}{\partial \mu_R} \neq 0; \frac{\partial \rho_n}{\partial R} \neq 0;$ n − perturbative order, R-scheme

The initial fixed-order QCD series is non-conformal, its prediction must be scheme-and-scale dependent due to mismatching of α_s with its coefficients for an arbitrary choice of scale

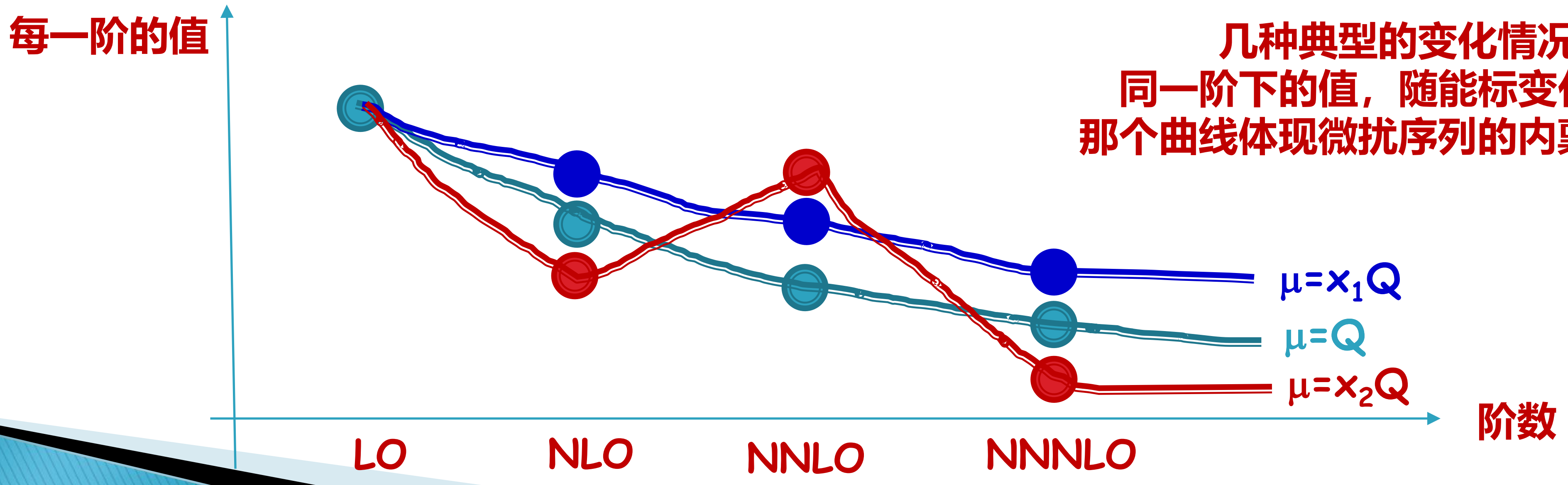
Then how to estimate the magnitude of initial series ?

"猜"—— Conventional scale-setting approach

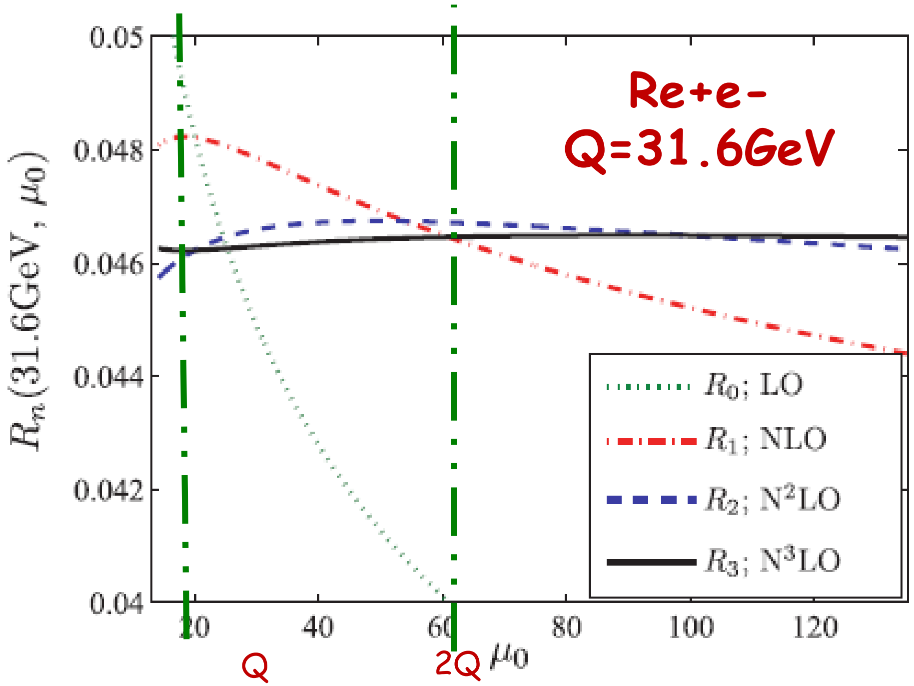
=>**"Choose"** the scale Q to be **typical momentum transfer**, or to eliminate large/dangerous logs or to get more convergent series or to agree with data

=>**"Keep it fixed"** throughout the calculation

=>**"Vary"** in a certain range, e.g. [Q/2, 2Q], [Q/3, 3Q], to discuss its uncertainty or to test its sensitivity to not-yet computed high-order contributions



一个典型的实例



In lower orders, we do not know which scale range is enough for estimating the uncertainty

We have to finish enough high-order calculations, which are complex and time-consuming

It is only an order estimation, depressing predictive power of pQCD theory

各阶能标不确定性的抵消作用，高阶时可获净（小）能标误差

收敛性强时，一切OK
不强时，低阶准确性重要性突显

只要保证强耦合常数小于1，一切初始能标选择皆有可能

【实际上，不存在所谓的中心值】

Question

Is it possible to achieve scheme-and-scale invariant pQCD prediction by using an improved series from the initial fixed-order series ?

百家争鸣，百花齐放

学习知识要善于**思考，思考，再思考……**

- ✓ 它是否符合客观规律？
- ✓ 它能否在实践中被验证？
- ✓ 它对解决问题是否有启发？

对微扰论本身的质疑从未停止

？微扰展开是否一直有效？

`tHooft, Can We Make Sense Out of QCD ?

We understand how to renormalize the theory to any finite order in the perturbation expansion, but it is expected that this expansion will diverge badly, for any value of the coupling constant.

我们的出发点是基于“**相信微扰论**”

微扰【QCD理论】提出不久，人们就已在考虑是否可解决能标着问题

早期思想：基于某种原理或某种理念来确定<u>最优重整化能标</u>

上世纪80年代，第一波热潮，诞生BLM，PMS，FAC等典型方案

1980's attempts to remove the ambiguity

【解释实验】

FAC
Effective charge

Resum all
higher-order terms

【源于类比】

BLM
Resum all
nf-term

【逆向思考】
什么样的序列符合RGI

PMS
Minimize unknown higher-order contributions

Simply require series to satisfy the RG invariance

Three typical and different ideas

目标：寻找微扰序列的最优能标和最优方案
Goal: using initial series to find optimal scheme and scale, forming new improved series

First round: BLM/PMS/FAC

Typical PK works

Sense and Nonsense in the Renormalization Scheme Dependence Problem

P. Stevenson (CERN) (Feb, 1982)

Published in: *Nucl.Phys.B* 203 (1982) 472-492

Scale Scheme Ambiguities in the Brodsky-lepage-mackenzie Procedure

W. Celmaster (Northeastern U.), Paul M. Stevenson (CERN) (Dec, 1982)

Published in: *Phys.Lett.B* 125 (1983) 493-496

INTERPRETATION OF THE BRODSKY-LEPAGE-MACKENZIE CRITERIUM

G. Grunberg (Ecole Polytechnique)
Published in: *Phys.Lett.B* 135 (1984) 455-456
Published: 1984
DOI: 10.1016/0370-2693(84)90314-9
View in: ADS Abstract Service

On Some Ambiguities in the Method of Effective Charges

G. Grunberg (Ecole Polytechnique) (Feb, 1989)

Published in: *Phys.Rev.D* 40 (1989) 680

Jet production rates at LEP and the scale of alpha-s

G. Kramer (Hamburg U.), B. Lampe (CERN) (Jul, 1990)

Published in: *Z.Phys.A* 339 (1991) 189-193

指出BLM更好

各显神通，即有合纵也有连横，异常激烈

Response to Brodsky and Lu's letter: 'On the selfconsistency of scale setting me

Paul M. Stevenson (Rice U.) (Dec 4, 1992)

e-Print: hep-ph/9211327 [hep-ph]

Method of effective charges and BLM criterion

G. Grunberg (Ecole Polytechnique) (Feb, 1992)

Published in: *Phys.Rev.D* 46 (1992) 2228-2239 • Contribution t

On Some possible extensions of the Brodsky-Lepage-MacKenzie approach beyond the next-to-leading order

G. Grunberg (Ecole Polytechnique), A.L. Kataev (Michigan U.) (May, 1991)

Published in: *Phys.Lett.B* 279 (1992) 352-358

Stevenson's Optimized Perturbation Theory Applied to Factorization and Mass Scheme Dependence*

H. DAVID POLITZER

California Institute of Technology, Pasadena, California 91125

July 1981

A serious shortcoming of PMS procedure, common to all applications, is discussed

OPTIMIZED PERTURBATION THEORY APPLIED TO FACTORIZATION SCHEME DEPENDENCE

Paul M. Stevenson (Rice U.), H.David Politzer (Caltech) (May 19, 1986)

Published in: *Nucl.Phys.B* 277 (1986) 758-763

Politzer承认推导错误，同时也指出借助近似永远无法获严格解决，但却提供最好方法

On the selfconsistency of scale setting methods

Stanley J. Brodsky (SLAC), Hung Jung Lu (Maryland U.) (Nov, 1992)

e-Print: hep-ph/9211308 [hep-ph]

Commensurate scale relations in quantum chromodynamics

Stanley J. Brodsky (SLAC), Hung Jung Lu (Maryland U.) (Apr, 1994)

Published in: *Phys.Rev.D* 51 (1995) 3652-3668 • e-Print: hep-ph/9405218 [hep-ph]

各种论文中，也有我比你更懂你的自信

Commensurate relation among different α_s

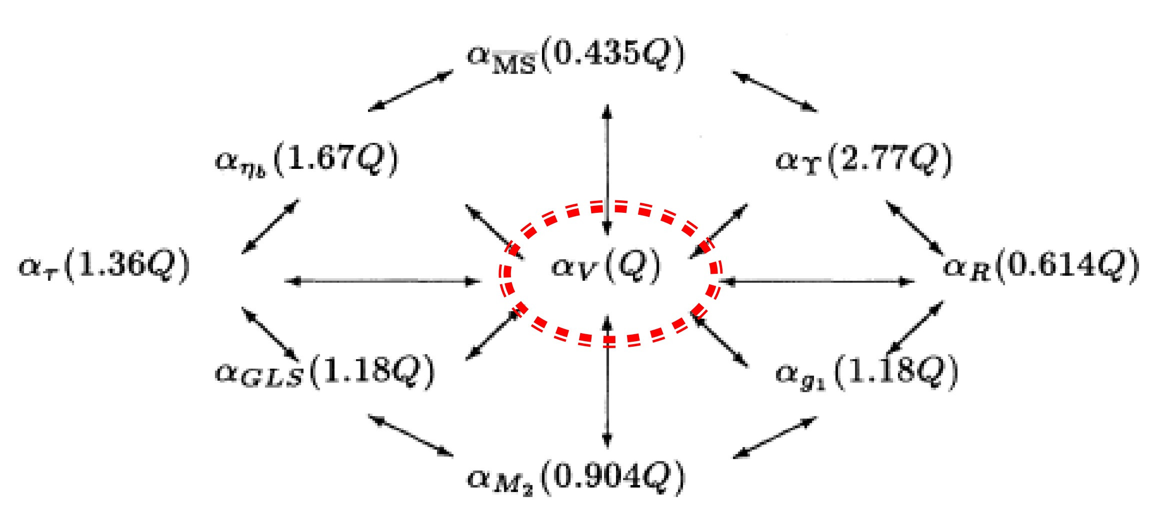
S.J. Brodsky and H.J. Lu, Phys.Rev. D51, 3652(1995);

在我看来，第一轮PK结束于1995
（已是近十五年之后）
回应对不能解决重整化方案不变性的质疑

$$R_{e^+e^-}(Q) \equiv R^0_{e^+e^-}(Q)\left[1+\frac{\alpha_s^R(Q)}{\pi}\right]$$

One-loop CSR ensures scheme invariance

TABLE I. Leading order commensurate scale relations.



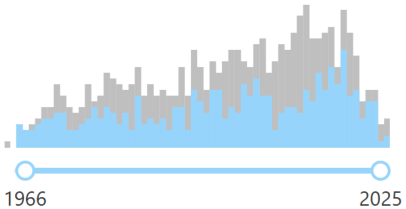
不同方案下的有效强耦合常数值不一样，但结合各自的展开系数，最终获得与方案无关的结果
---这是应对Steverson质疑的回答---

特殊情况可以将<u>所有系数也吸收进有效强耦合常数定义</u>

S.J. Brodsky

工作迄今59年

Date of paper



Number of authors

- Single author 29
- 10 authors or less 387

403 results | cite all | claim — Citation Summary | Most Recent

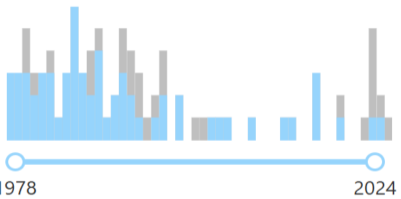
Citation Summary

Exclude self-citations

	Citeable	Published
Papers	**403**	**403**
Citations	**59,300**	**59,300**
h-index	**125**	**125**
Citations/paper (avg)	**147.1**	**147.1**

P.M. Stevenson

Date of paper



Number of authors

- Single author 28
- 10 authors or less 57

62 results | cite all | claim — Citation Summary | Most Recent

Citation Summary

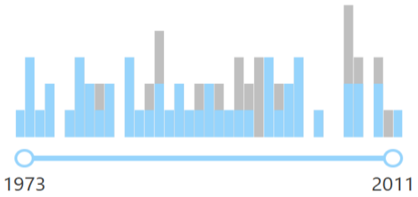
Exclude self-citations

	Citeable	Published
Papers	**62**	**62**
Citations	**4,879**	**4,879**
h-index	**31**	**31**
Citations/paper (avg)	**78.7**	**78.7**

95之后，工作很少了

G. Grunberg

Date of paper



Number of authors

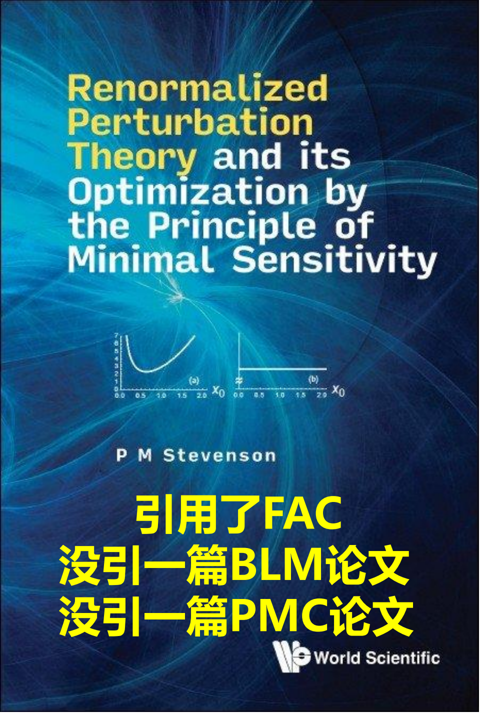
- Single author 30
- 10 authors or less 51

51 results | cite all | claim — Citation Summary | Most Recent

Citation Summary

Exclude self-citations

	Citeable	Published
Papers	**51**	**51**
Citations	**2,898**	**2,898**
h-index	**23**	**23**
Citations/paper (avg)	**56.8**	**56.8**



引用了FAC
没引一篇BLM论文
没引一篇PMC论文

Maximum

三连反击
波澜再起

Phys. Lett. B 847 (2023) 138288

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

Letter

‘Maximal conformality’ does not work

P.M. Stevenson

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ARTICLE INFO

Editor: B. Grinstein

ABSTRACT

The so-called “principle of maximal conformality” is ineffective and does nothing to resolve the renormalization-scheme-dependence problem. Some essential facts about that problem are summarized. It is stressed that RG invariance is a symmetry and that any viable method for resolving the scheme-dependence problem should be formulatable in terms of the invariants of that symmetry.

Suggesting two extreme pQCD examples to show PMC does not work

Brodsky *et al*’s defence does not work

P. M. Stevenson

T.W. Bonner Laboratory, Department of Physics and Astronomy, Rice University, Houston, TX 77251, USA

QCD Perturbation Theory: It’s not what you were taught

P. M. Stevenson

T.W. Bonner Laboratory, Department of Physics and Astronomy, Rice University, Houston, TX 77251, USA

Stevenson's papers emphasize the importance of scheme-invariance

We do agree

Comment on P. M. Stevenson, "'Maximal conformality' does not work", Phys. Lett. B 847 (2023) 138288

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Improved as an article

But Stevenson's two extreme examples inversely show the success of PMC

have since passed. Moreover, his successive comments and assumed "PMC samples" given in Ref. [18] are unfortunately full of typos and wrong deductions, indicating he does not understand the PMC at all. To show

symmetry

MDPI

Article

The Principle of Maximum Conformality Correctly Resolves the Renormalization-Scheme-Dependence Problem

Jiang Yan [1], Stanley J. Brodsky [2], Leonardo Di Giustino [3,4], Philip G. Ratcliffe [3,4], Shengquan Wang [5] and Xinggang Wu [1,*]

质疑的底气在那里？

超越微扰论，“求稳”的强词夺理

P.M. Stevenson 对RGI解读

Nucl.Phys.B 203 (1982) 472-492

Strong coupling has dual nature

【自由参数】和【展开参数】

【free parameter】【expansion coefficient】

作为**自由参数**，可以如下变换保证有限序列的值不变:

$$\alpha_s\text{-expansion} \to \alpha_s' = f(\alpha_s)\text{-expansion}$$

$$\rho_n(\alpha_s) \to \rho_n'(\alpha_s') = \rho(f^{-1}(\alpha_s'))$$

引入拓展重整化群方程(Extended RGE) 实现不同方案下耦合常数变化，从而得到与方案无关的结果

3. - FREE PARAMETERS AND EXPANSION PARAMETERS

Traditionally, the coupling constant has played a dual rôle as both a free parameter and an expansion parameter. It is very important to distinguish between these two functions.

Even in the exact theory, the predictions do not have the form of definite, arithmetical numbers. However, we can conveniently express the predictions algebraically, as known functions of an unknown, or "free", parameter. How we do this is purely a matter of convention. If we have a set of predictions $R_n(g)$ in terms of some free parameter g, then we can always re-express them in terms of some other free parameter $\lambda = f(g)$. The predictions $R_n'(\lambda)$, where the functions R_n' are given by $R_n \circ f^{-1}$, will look very different, but they express exactly the same physics. The convention adopted truly does not matter. (And it is quite irrelevant whether the free parameter has itself any physical significance.) A free parameter serves only as a book-keeping device which permits a consistent comparison of different physical quantities. Naturally, it must be fixed universally, and not varied from one process to another.

Possible to achieve scheme-invariant predictions under fixed orders

... e different. When we appro- ... eries expansion, the results ... Thus, unlike the free parameter, the expansion parameter is a choice which matters: it deserves careful consideration, and should not be made arbitrarily. (Measurability or physical significance is, in itself, no indication that a quantity will make a good expansion parameter.) Moreover, it is entirely possible that a sensible choice of expansion parameter in one case may not be sensible in another. This is a vital consideration; particularly so when there is a large set of possible expansion parameters - the renormalized coupling constants of different RS's - which are all *a priori* on an equal footing.

强耦合常数是【自由参数】观点是Peterman和Stueckelberg提出

PHYSICS REPORTS (Review Section of Physics Letters) 53, No. 3 (1979) 157–248. North-Holland Publishing Company

RENORMALIZATION GROUP AND THE DEEP STRUCTURE OF THE PROTON

A. PETERMAN

CERN, Geneva, Switzerland

重整化群方法的核心思想完全基于这样一种观察：诸如耦合常数、质量等重整化常数属于任意数学参数，**可通过随意改变重整化方案来对其进行调节**。

The spirit of the renormalization group approach lies entirely in the observation that in a specific theory the renormalized constants such as the couplings, the masses, are arbitrary mathematical parameters which can be varied by changing arbitrarily the renormalization prescription. For example, given a scale of mass μ, prescriptions can be chosen by doing subtractions of the relevant amplitudes at

Applicability of naïve perturbation expansion to non-Abelian gauge theories of strong interactions in the domain of high energy momentum transfer is one result from the renormalization group way of looking at these problems and will probably remain its only use. Non-trivial fixed points are unlikely to occur.

“在高能动量传递情形，将朴素微扰展开应用于QCD理论，是通过重整化群方法研究此类问题得到的一个结论，而**这很可能是该方法的唯一应用场景**。”

注：作为微扰展开参数，则原始微扰序列肯定与方案相关（符合RGI），因此无法判断那个能标和方案是正确的；但基于最优能标和最优方案下的新微扰序列可以 —— 因此，可认为PMS是一种有效方案

若坚持微扰论展开的思想，PMS的“求稳”必须基于如下假设才能成立

Key assumptions of PMS

Because $\{\beta_{i>=2}\}$-functions are different for different schemes

==>> **Assum**: one can use $\{\beta_{i>=2}\}$-functions inversely to label different scheme

==>>**Assume**: the pQCD approximant can be scheme invariant under scheme transformations

The expansion basis and coefficients can be transformed simultaneously to ensure the scheme invariance

$$a_s^{\mathcal{R}} \to a_s^{\mathcal{S}} \quad \text{and} \quad C_i^{\mathcal{R}} \to C_i^{\mathcal{S}}$$

$$\beta^{\mathcal{S}}(a_s^{\mathcal{S}}) = \left(\partial a_s^{\mathcal{S}} / \partial a_s^{\mathcal{R}}\right) \beta^{\mathcal{R}}(a_s^{\mathcal{R}}).$$

$$\alpha_s\text{-expansion} \to \alpha_s' = f(\alpha_s)\text{-expansion}$$

$$\rho_n(\alpha_s) \to \rho_n'(\alpha_s') = \rho(f^{-1}(\alpha_s'))$$

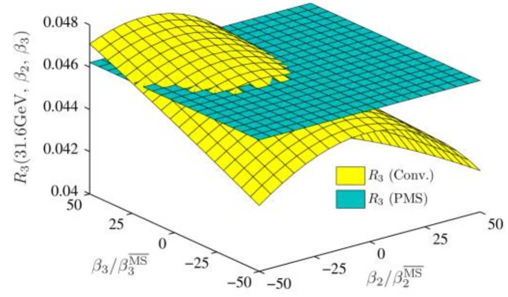
==>> **假设**所有未考虑的高阶项贡献为零 （注：导致有效性极度依赖内在收敛性）

$$\partial\varrho_n/\partial(\mathrm{RS}) = \mathcal{O}(a_s^{p+n+1}) \sim 0$$

假设之下
确实可以
——对应

PMS: the scheme-dependence and scale-dependence are treated by using the extended RGE

At n-th order, **2n+1 parameters** $\tilde{a}_s, \tilde{\tau}, \tilde{\beta}_2, \ldots, \tilde{\beta}_n, \tilde{\mathcal{C}}_1, \ldots, \tilde{\mathcal{C}}_n$



n PMS Equations $\frac{\partial \varrho_n}{\partial \tau} = 0 \quad \frac{\partial \varrho_n}{\partial \beta_m} = 0, (m \geq 2)$

One Basic RGE (β-function) $\beta(a_s) = \mu^2 \frac{\partial}{\partial \mu^2}\left(\frac{\alpha_s}{4\pi}\right) = -\sum_{i=0} \beta_i \left(\frac{\alpha_s}{4\pi}\right)^{i+2}$

n RG invariant coefficients

$$\rho_1 = \frac{1}{4} p\beta_0\tau - \mathcal{C}_1$$

$$\rho_2 = \mathcal{C}_2 - \frac{(1+p)\mathcal{C}_1^2}{2p} - \frac{\beta_1\mathcal{C}_1}{4\beta_0} + \frac{p\beta_2}{16\beta_0}$$

$$\rho_3 = 2\mathcal{C}_3 + \frac{\mathcal{C}_1^2\beta_1}{4p\beta_0} - \frac{\mathcal{C}_1\beta_2}{8\beta_0} + \frac{p\beta_3}{64\beta_0} + \frac{2(1+p)(2+p)\mathcal{C}_1^3}{3p^2} - \frac{2(2+p)\mathcal{C}_1\mathcal{C}_2}{p}$$

2n+1方程求解2n+1未知数可采用数值求解

I）确定有效耦合常数 -- 无Λ_{QCD}不确定性
II）理论预言与初始能标选择以及初始方案选择无关
Phys.Rev.D97,036024(2018)

积分常数为不变量 — 等价于 — 重整化能标和方案无关常数

$R_{e^+e^-}$ four-loop level

	n_f=3	n_f=4	n_f=5
C_1	1.6401	1.5249	1.4097
C_2	-10.284	-11.6857	-12.8047
C_3	-106.896	-92.9124	-80.0075
C_1^{PMS}	-0.458	-0.1105	0.0479
C_2^{PMS}	-1.1361	0.2103	1.3075
C_3^{PMS}	[illegible]2.2132	24.9881	16.4108

TABLE I. Coefficients for the perturbative expansion of $R_3(Q)$ before and after the PMS scale setting, where we have set Q = 1.2 GeV for n_f=3, Q = 3 GeV for n_f=4, and Q = 31.6 GeV for n_f=5.

Factorial divergence suppressed !

R_τ (four loop)

	C_1'	C_2'	C_3'
Conv.	5.2023	26.3659	127.079
PMS	0.3906	1.2380	-6.1747

TABLE V. Coefficients for the perturbative expansion of r_3^τ and after the PMS scale setting. $\mu_0 = M_\tau$.

$\Gamma(H \to b\bar{b})$ (four loop)

	C_1''	C_2''	C_3''
Conv.	29.145	41.765	-825.598
PMS	0.34376	21.2286	-142.849

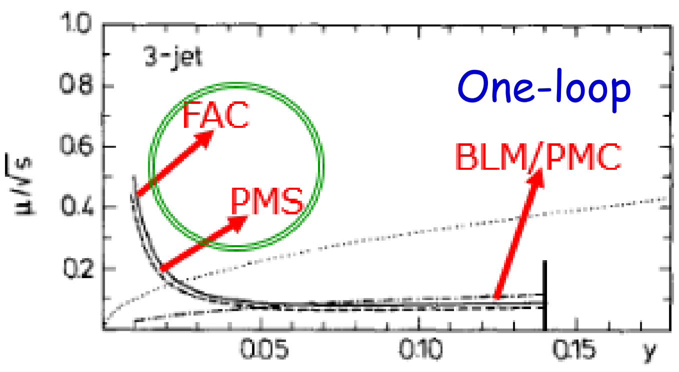
TABLE IX. Coefficients for the perturbative expansion of $\tilde{R}_3$ before and after the PMS scale setting. $\mu_0 = M_H$.

PMS prediction（limitations）

Lower-order predictions are generally incredible

weak perturbative nature

PMS cannot predict contributions from unknown higher-order terms



G. Kramer and B. Lampe, Z. Phys. C 39, 101 (1988).

$R_{e^+e^-}$ four-loop level

	LO	NLO	N^{2}LO	N^{3}LO	*total*
Conv.	0.04499	0.00285	-0.00117	-0.00033	0.04635
PMS	0.04608	0.00010	0.00013	0.00007	0.04638

Accidental series convergence

R_τ (four loop)

	LO	NLO	N^{2}LO	N^{3}LO	*total*
Conv.	0.10320	0.05541	0.02898	0.01441	0.20200
PMS	0.19935	0.01552	0.00981	-0.00975	0.21493

$\varrho^{LO}_{3,PMS} \gg \varrho^{NLO}_{3,PMS} \sim \varrho^{N^2LO}_{3,PMS} \sim \varrho^{N^3LO}_{3,PMS}$

	LO	NLO	N^{2}LO	N^{3}LO	*total*
Conv.	0.20371	0.03767	0.00194	-0.00138	0.24194
PMS	0.23967	0.00061	0.00161	-0.00046	0.24144

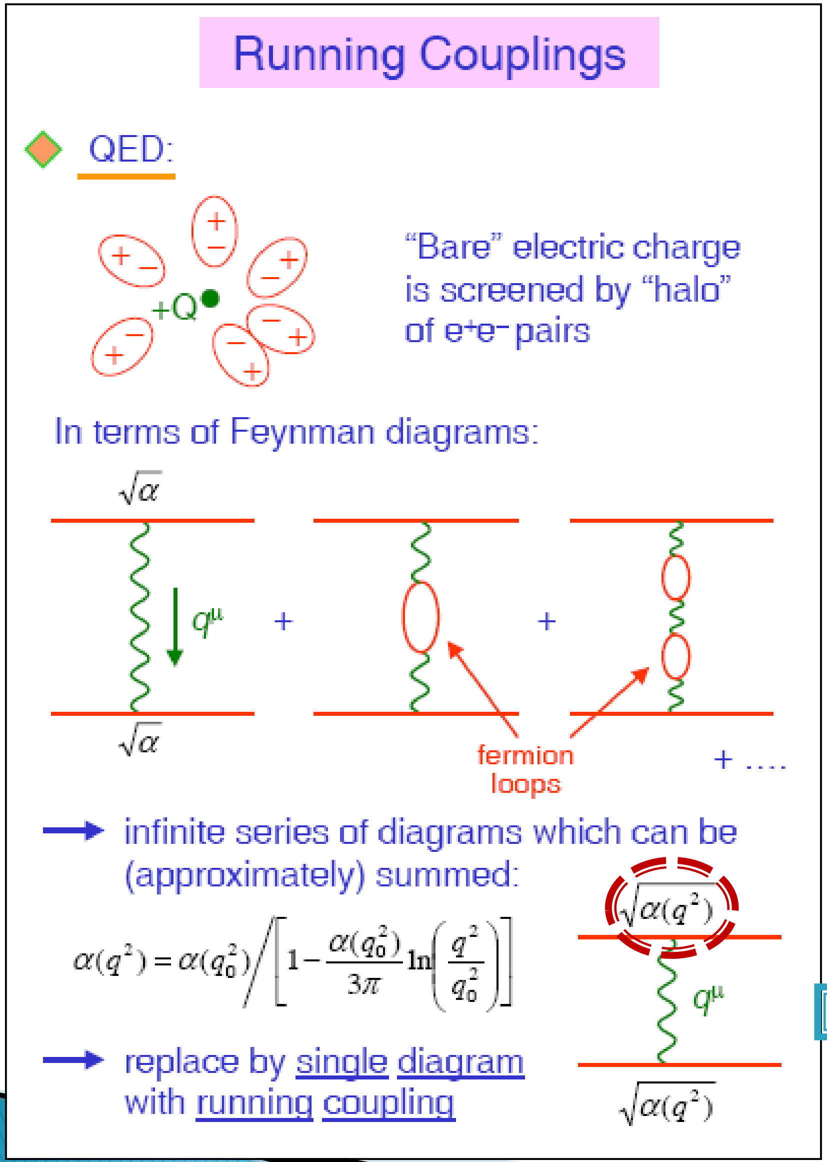
$\varrho^{NLO}_{3,PMS} < \varrho^{N^2LO}_{3,PMS}$

$\Gamma(H \to b\bar{b})$ (four loop)

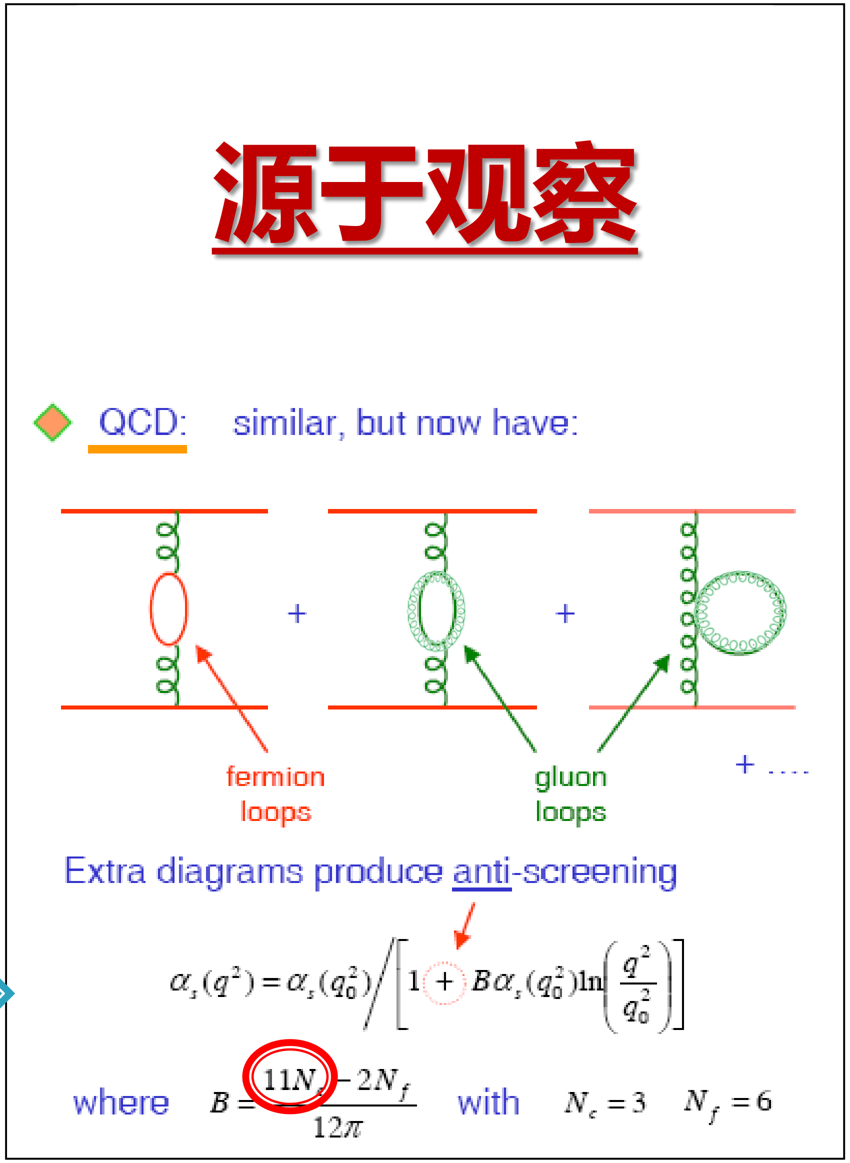
summary

The useful is Extended RGE but not PMS

The idea of BLM



Q0 – Initial scale
Resummed to achieve a scale-invariant prediction -- it is infact the chain-loop approximation



GM-L：重求和泡泡图可获得准确的α值

BLM：重求和泡泡图可获得准确的α_s值

Who wins the first round PK ？ It's surely the BLM
--- almost all of its citations are for scale-setting

ELSEVIER
Physics Letters B
Volume 95, Issue 1, 8 September 1980, Pages 70-74

Renormalization group improved perturbative QCD

G. Grunberg [1]

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA

Received 5 June 1980, Available online 16 October 2002.

FAC-approach ~600

Optimized perturbation theory

P. M. Stevenson
Phys. Rev. D **23**, 2916 – Published 15 June 1981

PMS-approach ＞ 1200

Article | References | Citing Articles (855) | PDF | Export Citation

ABSTRACT

Conventional perturbation theory gives different results in different renormalization schemes, a problem which is especially serious in quantum chromodynamics (QCD). I propose a theoretical resolution of this ambiguity which uses the full renormalization-group invariance of the theory. The idea is that, in any kind of approximation scheme which does not respect the known invariances of the exact result, the "optimum" approximant is the one that is "most invariant," i.e., least sensitive to variations in the unphysical parameters. I discuss this principle in several examples, including the Halliday-Suranyi expansion for the anharmonic oscillator. Turning to massless field theories, I identify the unphysical variables which label a particular renormalization scheme as the renormalization point μ, and the β-function coefficients. I describe how perturbative approximations depend on these unphysical variables,

引用次数高，
并不一定方法就好。
东边不亮，西边亮。

PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

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Paul B. Mackenzie
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(Received 23 November 1982)

BLM-approach ＞ 1300

I) 猜是“无奈”的选择

II) 寻找“最优能标”是一种进步，但非解决问题，只是深度隐藏问题

从重整化理论来说，重整化能标只是人为引进参数

因此，我们可以选择任意重整化能标来做数值估算

如果最优能标才能对应正确微扰论预言，那么重整化能标的自由去那里了？

III) 谁才是真正的解决方案？

BLM相对胜出；但胜出之后，路又在何方？

路在何方之一：BLM之问

Why is BLM so successful ?
What underlies it ?
How to extend it to all orders ?

介绍迄今最系统最深入的延拓方案

- The principle of Maximum Conformality （PMC）

1981年，Brodsky-Lepage-Mackenzie提出BLM机制
1992年，Gruberg-Kataev提出（错误）延拓方案认为BLM机制存在问题
1995年，Brodsky-Lu提出自洽能标对应关系CSR，拓展BLM机制到两圈
1997年-2011年，Brodsky提出采用β-函数替换BLM中nf-项想法，但不知如何做
2011年，Brodsky-Giustino明确β-函数想法，命名PMC（单圈层次重述BLM，2012年发表）

From BLM to PMC

萌芽阶段

2011年，Brodsky-Wu提出PMC－BLM对应原理，实现PMC高圈（突破进展，PMC首篇发表论文，PRD）
2012年，Brodsky-Wu将PMC用于同时解释顶夸克对总产生截面及正反不对称性分析，取得成功（PRL）
2013年，Wu-Brodsky-Mojaza完成PPNP邀请综述（国际首篇重整化能标系统综述）
2014年，Brodsky-Mojaza-Wu（BMW）给出对应原理解释，完善PMC公式体系
PMC获赞意义“接近重整化理论”（PRL）
2015年，Bi-Wu等证明简并关系普适性、PMC两种方案的微扰等价性（PLB）
Ma-Wu等基于重整化群不变性完成PMC与PMS深入对比（PRD）
Wu-Ma-Wang-Brodsky等完成RPP邀请综述、完成国内FOP综述
2016年，Shen-Wu-Brodsky等提出PMC单能标实现方案，证明不同方案的微扰等价性（PRD）
2017年，Shen-Wu-Brodsky等将自洽能标对应关系CSR推到任意高阶（PLB）
Deur-Shen-Wang-Wu-Brodsky等初步考虑将PMC应用于低能区的可能性（PLB）
2018年，Shen-Wu-Brodsky等证明PMC对于任意重整化方案均有方案无关性（PRD）
Du-Wu等提出基于PMC共形序列以及Pade估算未知高阶项方法（PRD）
2019年，Wu-Shen-Brodsky等完成PPNP邀请综述
2020年，Leonardo-Brodsky-Wang-Wu等提出PMC∞方案（PRD）
2023年，Huang-Yan-Wu-Brodsky等详细对比PMC的各种方案（NPB）
Shen-Wu等提出PMC共形序列以及Bayes估算未知高阶项方法（EPJC）
2024年，Leonardo-Brodsky-Ratcliffe-Wu-Wang等完成PPNP邀请综述
Yan-Wu等提出算符方法自动化PMC（JHEP）
2025年，Wu-Yan-Wu等提出线性回归方法估算未知高阶项贡献（arXiv）
Yan- Leonardo-Brodsky-Ratcliffe-Wang-Wu回应Stevenson质疑（Symmetry)

发展阶段

RMP-2026 ?

Progress in Particle and Nuclear Physics 72 (2013) 44–98

Contents lists available at SciVerse ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

The renormalization scale-setting problem in QCD

Xing-Gang Wu[a,*], Stanley J. Brodsky[b], Matin Mojaza[b,c]

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[b] SLAC National Accelerator Laboratory, Stanford University, CA 94039, USA
[c] CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230, Denmark

ARTICLE INFO

Keywords:
Renormalization group
Renormalization scale
BLM/PMC
QCD

ABSTRACT

A key problem in making precis
malization scale of the running
an arbitrary range and an arbit
fact, this *ad hoc* procedure give
tion scheme, and it is in confli
Predictions for physical results
theoretical conventions. We re
the renormalization scale amb

2013

IOP Publishing Reports on Progress in Physics

Rep. Prog. Phys. 78 (2015) 126201 (15pp) doi:10.1088/0034-4885/78/12/126201

Review

Renormalization group invariance and optimal QCD renormalization scale-setting: a key issues review

Xing-Gang Wu[1], Yang Ma[1], Sheng-Quan Wang[1], Hai-Bing Fu[1], Hong-Hao Ma[1], Stanley J Brodsky[2] and Matin Mojaza[3]

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[3] CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denma Denmark

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Published 29 October 2015

Invited by Dr Sharon D'Souza

2015

Progress in Particle and Nuclear Physics 108 (2019) 103706

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

The QCD renormalization group equation and the elimination of fixed-order scheme-and-scale ambiguities using the principle of maximum conformality

Xing-Gang Wu[a,*], Jian-Ming Shen[a,b], Bo-Lun Du[a], Xu-Do
Sheng-Quan Wang[c], Stanley J. Brodsky[c,*]

[a] Department of Physics, Chongqing University, Chongqing 401331, PR China
[b] School of Physics and Electronics, Hunan University, Changsha 410082, PR China
[c] SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94039, USA

ARTICLE INFO

Article history:
Available online 27 May 2019

Keywords:
Perturbative QCD calculations
Renormalization
Principle of maximum conformality

ABSTRACT

The conventional scale setting appr
tions is based on a guessed renorm
the large log-terms of the pQCD se
uncertainty. This *ad hoc* assignmen
of the QCD running coupling at ea
the choices of both the renormaliz
leads to conventional renormaliza

2019

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Review

High precision tests of QCD without scale or scheme ambiguities

The 40th anniversary of the Brodsky–Lepage–Mackenzie method

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2024

Solving QCD scale-setting problem
PMC —— step-by-step forward

Fix α_s-value, the new series naturally satisfy RGI

The idea of PMC is simple

It is β-terms that are most important, but not nf-terms

So it is important to finish correct transformation of nf-terms to β- terms

At the NLO-level, there is no ambiguity to make such transformation

重点关注
微扰论起源与核心

α_s

Basis—renormalization group equation (RGE)

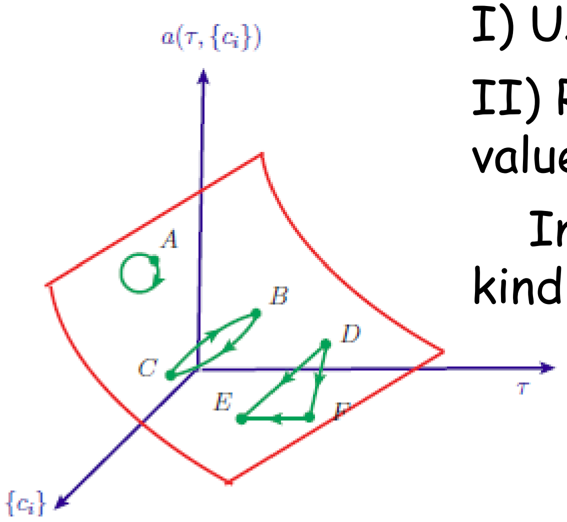
Phys.Rev.D86,054018 (2012)

$\frac{\partial \rho_n}{\partial \mu_R} \neq 0; \frac{\partial \rho_n}{\partial R} \neq 0;$ We cannot get exact constraints from those inequalities

The RGE itself is used to determine the running behavior of α_s, thus can also be adopted for fixing scale ambiguity.

I) Using RGE to determine the beta-terms at each order.

II) Resumming all the same type beta-terms, determining the exact value for exach perturbative order.



In this sense, PMC is like a way of resummation, which resums a kind of large log-terms to form a steady pQCD prediction.

But different to a pure resummation to improve the reliability, PMC also tends to solve the scale-setting problem.

PMC satisfies RGE-properties: symmetry, reflexity, transitivity

PMC tries to solve scheme-and-scale ambiguities simultaneously

The importance of RGE has already been realized by Peterman etal, but not are throughly used

> Can We Make Sense Out of Quantum Chromodynamics ?
>
> Gerard 't Hooft, 1977

It has already been realized that, ``the pQCD theory is invariant to the choice of subtraction point (μ) under a simultaneous change in alphas and μ provided that alphas satisfies RGE ! "

$$\rho_n(\mu) = \sum_{i=0}^{n} c_i(\mu)\alpha_s^i(\mu)$$

牛人未尽的思考
RGE有两层用途

Conventional treatment of the initial series, alphas值和微扰序列展开系数值的确定是**割裂**开的. At present, the RGE has only been used to fix the correct alphas value at the chosen scale μ, but **the RGE-involved β-terms of the series itself have not been adopted to determine the exact value of alphas for the considered process** !

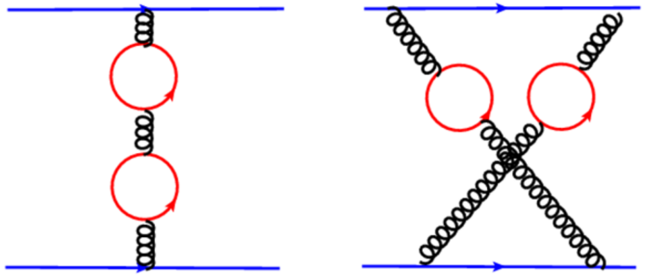
How RGE works for pQCD series

Basically, it gives the value of the concerned parameter at given scale
Secondly, it shows how to set the correct ``effect" scale of the concerned value

PMC multi scale-setting procedure

Initial pQCD series --- First step

$$\rho(Q)=\sum_{i=1}^{n}\left(\sum_{j=0}^{i-1} c_{i,j} n_f^j\right) a_s^{p+i-1}(\mu_r)+\mathcal{O}(a_s^{n+1}),$$



Transformation nf-series into RGE-involved β-series ---Second step

$$\begin{aligned}\rho(Q) &= r_{1,0}a_s(\mu_r)+(r_{2,0}+\beta_0 r_{2,1})a_s^2(\mu_r)\\ &+(r_{3,0}+\beta_1 r_{2,1}+2\beta_0 r_{3,1}+\beta_0^2 r_{3,2})a_s^3(\mu_r)\\ &+(r_{4,0}+\beta_2 r_{2,1}+2\beta_1 r_{3,1}+\frac{5}{2}\beta_1\beta_0 r_{3,2}\\ &+3\beta_0 r_{4,1}+3\beta_0^2 r_{4,2}+\beta_0^3 r_{4,3})a_s^4(\mu_r)+\mathcal{O}(a_s^5)\end{aligned}$$

Comment: different to the wrong extenson of BLM: seBLM

There are also β–terms that are pertained to Msbar running mass and etc., which should be treated separately

PMC single scale-setting procedure

$$\rho(Q)=\sum_{n\geq1} r_{n,0}\alpha(\mu)^{n+p-1}+\sum_{n\geq1}\left[(n+p-1)\alpha(\mu)^{n+p-2}\beta\right]\sum_{j\geq1}(-1)^j\Delta_n^{(j-1)}r_{n+j,j}$$

Basing on RGI:

$$r_{i,j}=\sum_{k=0}^{j} C_j^k \tilde{r}_{i-k,j-k}L^k$$

$$\tilde{r}_{i,j} = r_{i,j}|_{\mu=Q}$$

$$L = \ln\mu^2/Q^2$$

$$\rho(Q)=\sum_{n\geq1}\tilde{r}_{n,0}\alpha(\mu)^{n+p-1}+\sum_{n\geq1}\left[(n+p-1)\alpha(\mu)^{n+p-2}\beta\right]\sum_{j\geq1}(-1)^j\Delta_n^{(j-1)}\tilde{r}_{n+j,j}$$

$$+\sum_{k\geq1}L^k\sum_{n\geq1}\left[(n+p-1)\alpha(\mu)^{n+p-2}\beta\right]\sum_{j\geq k}(-1)^j C_j^k\Delta_n^{(j-1)}\tilde{r}_{n+j-k,j-k},$$

RGE-involved

Eliminate all β-terms

$$\rho(Q)=\sum_{n\geq1}\tilde{r}_{n,0}\alpha(Q_\star)^{n+p-1}$$

$$\ln\frac{Q_*^2}{Q^2}=\sum_{k=0}^{p-2}T_k a_s^k(Q_*)$$

The overall PMC scale Q* is also in perturbative form

Demonstration of scheme independence (Using the C-scheme coupling to characterize any scheme)

$$\frac{1}{a_\mu} + \frac{\beta_1}{\beta_0} \ln a_\mu = \beta_0 \left(\ln \frac{\mu^2}{\Lambda^2} - \int_0^{a_\mu} \frac{da}{\tilde{\beta}(a)} \right).$$

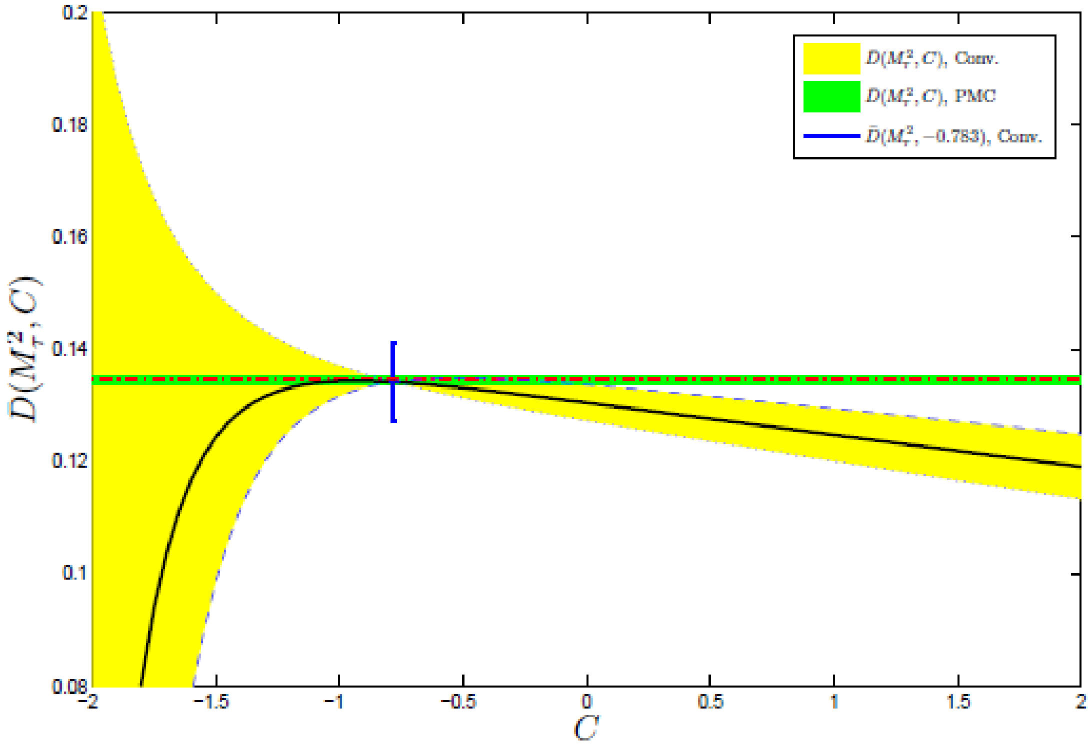
a new coupling $\hat{a}_\mu = \hat{\alpha}_s(\mu)/\pi$ via the following way

$$\frac{1}{\hat{a}_\mu} + \frac{\beta_1}{\beta_0} \ln \hat{a}_\mu = \beta_0 \left(\ln \frac{\mu^2}{\Lambda^2} + C \right),$$

New coupling constant, all scheme-dependence are introduced into C-parameter; its scale and scheme running satisfies the same RGE, which is scheme-independent

$$\hat{\beta}(\hat{a}_\mu) = \mu^2 \frac{\partial \hat{a}_\mu}{\partial \mu^2} = -\frac{\beta_0 \hat{a}_\mu^2}{1 - \frac{\beta_1}{\beta_0} \hat{a}_\mu} = -\beta_0 \hat{a}_\mu^2 \sum_{i=0}^{\infty} (\beta_1/\beta_0)^i \hat{a}_\mu^i$$

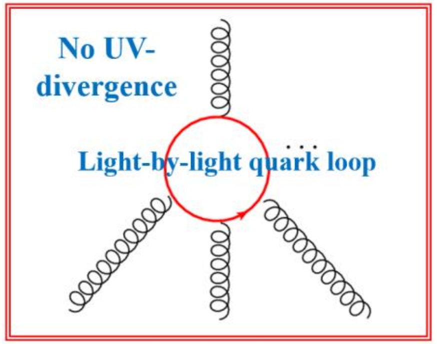
$$\frac{\partial \hat{a}_\mu}{\partial C} = \hat{\beta}(\hat{a}_\mu)$$



PMC是最好的方案吗？！

Second round —— PMC or seBLM or other ?

Some criticisms on PMC



Generalization of the Brodsky-Lepage-Mackenzie optimization within the $\{\beta\}$-expansion and the principle of maximal conformality

A. L. Kataev and S. V. Mikhailov

Phys. Rev. D **91**, 014007 – Published 6 January 2015

Still in PK——究竟需不需要区分nf-项

Ambiguities of the principle of maximum conformality procedure for hadron collider processes

Herschel A. Chawdhry* and Alexander Mitov
Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom

事实上，2013年我们就已经发现提到的问题，并于2017年提出单能标解决方案。但他们没引用，而是自行提出"Improved PMC approach"

Letter

Maximum

'Maximal conformality' does not work

P.M. Stevenson
T.W. Bonner Laboratory, Department of Physics and Astronomy, Rice University, Houston, TX 77251, USA

该文错误：错误认为PMC只是处理nf-项，没有意识到需正确使用RGE；we need to use the correct RGE under the specific scheme to do β-resummation

极度悲观：PMC都不行，没有可行的了。Any approach cannot remove the theoretical uncertainties in physical predictions, since these uncertainties ultimately arise from missing higher orders in α_s.

$$C_{NS}(a_s)D^{NS}(a_s) = 1 + \sum_{n\geq 1}\left(\frac{\beta(a_s)}{a_s}\right)^n P_n(a_s)$$

The PK is still going on for PMC and seBLM

On the CBK relation in QCD : 15 years later

A. L. Kataev

$$D^{ns}(a_s) = 1 + \sum_{n\geq 0}\left(\frac{\beta(a_s)}{a_s}\right)^n P_n(a_s)$$

2025年7月-最新的报告

2012
1011.5248

The generalized scheme-independent Crewther relation in QCD

Jian-Ming Shen [a], Xing-Gang Wu [a,*], Yang Ma [b], Stanley J. Brodsky [c]

我们于2017PLB，基于简并关系证明CBK正确，但问题的关键是如何正确利用β-项来得到正确的PMC共形序列
这一点Kataev坚持错误不改，其实质是认为他们更早提出PMC！

但PMC与seBLM是截然不同两种方法

我们的态度

坚持到底

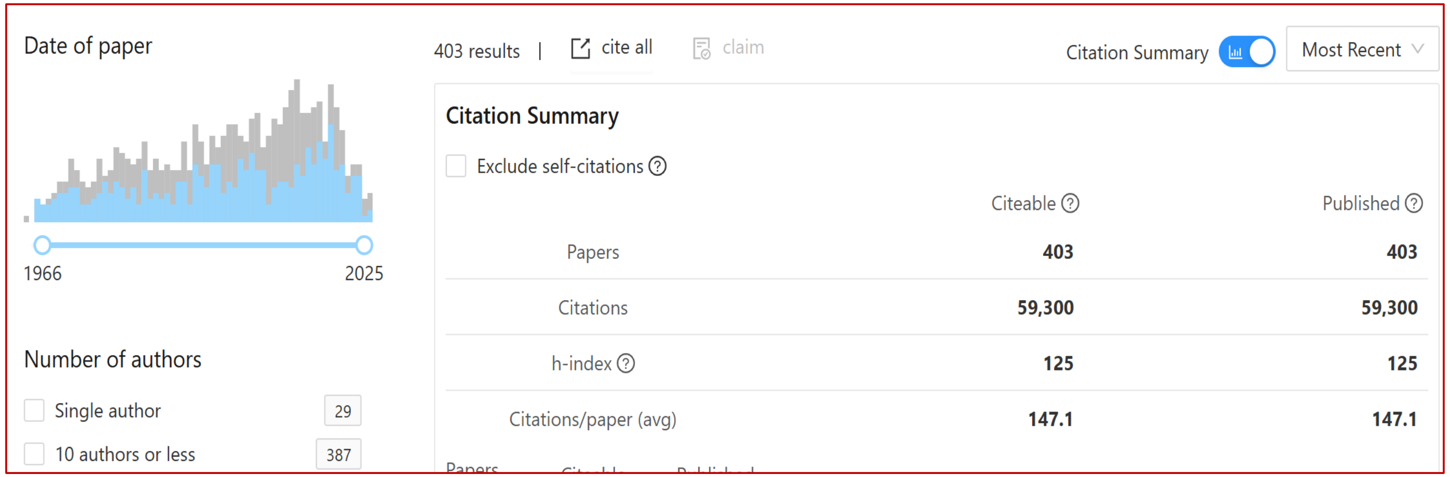
！让时间来证明一切！

《行路誓·证道》

大鹏振翼起沧溟，裂云破雾贯长虹。
一旦决志踏前路，纵令沧海扬尘，志未穷。
人间万议如星落，各执圭璋论浊清。
且放苍天为砚海，日月磨墨待书成。
岁月如刀剖伪真，古今谁见错漏隐？
试看东流不息水，千年自会辨浊泾。
笑谈纷纭何需辩，寸心若铁照汗青。
但守孤勇行到底，直待光阴判重轻。

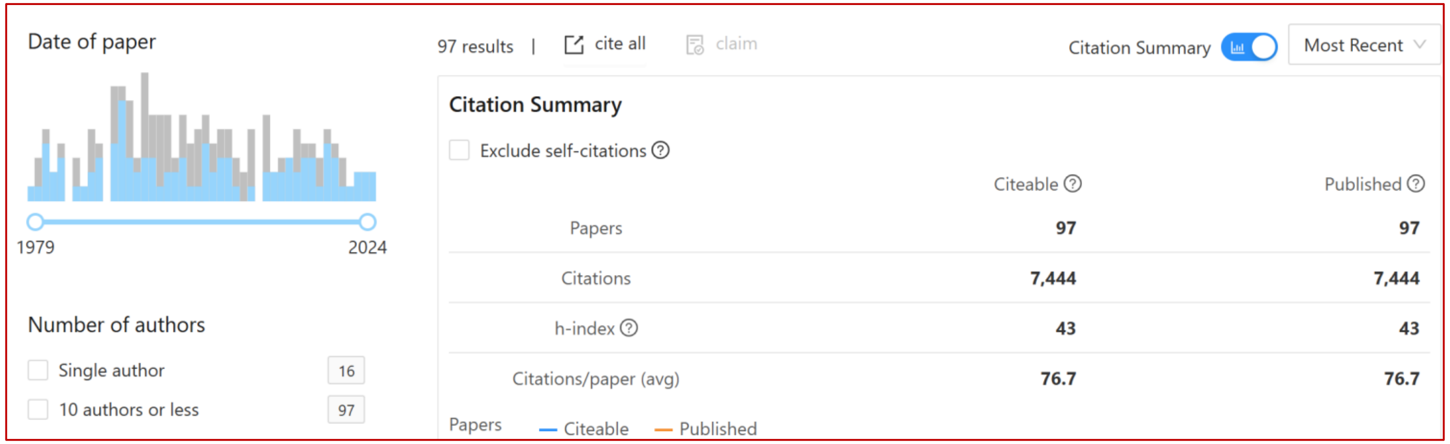
S.J. Brodsky

工作迄今59年



A.L. Kataev

工作迄今45年



X.G. Wu

工作迄今23年

"坐一望二"，微扰论依然在路上

路在何方之二：如何估算微扰论误差

- How to reduce and estimate residual scale dependence
- How to estimate magnitude of unknown terms

There are two kind of residual scale depenence

FAC和PMS均无法估算未知高阶项贡献

新的问题

An important step forward

Conventional scheme-and-scale ambiguity ===> residual scale dependence due to UHO-terms

After fixing the scale ambiguity,

what we still have to do for perturbative theory ?

It is not the end of the story, but a new beginning

路总是趟出来的

The road always comes out

Feynman, 1961, ``How to estimate higher order terms in the perturbation series without having to laboriously calculate Feynman diagrams"

In nearly every case we are reduced to computing exactly the coefficient of some specific term. We have no way to get a general idea of the result to be expected. To make my view clearer, consider, for example, the anomalous electron moment, $[\frac{1}{2}(g-2) = \alpha/2\pi - 0.328\alpha^2/\pi^2]$. We have no physical picture by which we can easily see that the correction is roughly $\alpha/2\pi$, in fact, we do not even know why the sign is positive (other than by computing it). In another field we would not be content with the calculation of the second-order term to three significant figures without enough understanding to get a rational estimate of the order of magnitude of the third. We have been computing terms like a blind man exploring a new room, but soon we must develop some concept of this room as a whole, and to have some general idea of what is contained in it. As a specific challenge, is there any method of computing the anomalous moment of the electron which, on first rough approximation, gives a fair approximation of the α term and a crude one to α^2; and when improved, increases the accuracy of the α^2 term, yielding a rough estimate to α^3 and beyond?"

Everything all shrinks into one thing

How to reliably estimate UHO contributions and with high precision

$$\rho_n^{[N/M]}(Q) = a^p \times \frac{b_0 + b_1 a + \cdots + b_N a^N}{1 + c_1 a + \cdots + c_M a^M}$$
$$= \sum_{i=1}^{n} C_i a^{p+i-1} + C_{n+1}\, a^{p+n} + \ldots$$

Several ways to estimate higher-order contributions

- Conventional: Varying scale — Rough order estimation and cannot estimate conformal contribution
- Convervative: The one-order higher shall always be smaller than the given order
- Resummation: Find a proper generating function, such as fractional function – Pade approximation
- Probability analysis： Bayesian analysis, Linear regression through the origin

Give the probability of the higher-order magnitude

P(A∩B) = P(A)*P(B|A)=P(B)*P(A|B)

Comparing with initial series, the PMC series has advantages:

Better convergence; More accurate without scheme-and-scale dependence; The coefficients have no RGE-relations; …

Thus it has good potential to do the estimation; especially it can achieve more precise prediction with less given orders.

PMC
Nearly 80 papers

PMC-Basis

- Suggestion of PMC-I, PMC-II, PMC-∞, PMCs approaches
- Demonstration of equivalence of various approaches in sense of perturbative series
- Demonstration of the degeneracy relations are general properties of QCD due to RGI
- Demonstration scheme-invariance using single parameter C-scheme coupling
- A detailed comparison of PMC, PMS and FAC using RGI
- A detailed comparison of PMC and seBLM
- Ways (Pade, Bayes, LRTO) of estimating contributions of uncalculated higher-order terms
- Characteristic Operator approach of PMCs

PMC-Applications

- Top pair total cross-section and asymmetry at hadronic colliders
- Ree-related works: Ree, Adler-function, Crewther-Relation, Gross-Llewellyn Smith sum rules
- Pomeron, introducing residual scale dependence
- QCD corrections to jets
- Heavy quarkonium production/decays, Bc decays
- B physics, Higgs decays, electro-weak
- Scheme transformations-scheme-invariant but have different convergent behavior, MOM-scheme
- Threshold effects via proper resummations
- Alphas in low-energy regions and its connections to high-energy behavior
- Differential cross-sections – showing the consistency of PMC at total and differential levels

- Recent example: threshold effect of top-pair prediction at e+e- colliders

Letter

Reanalysis of the top-quark pair production via the e^+e^- annihilation near the threshold region up to N^{3}LO QCD corrections

Jiang Yan [a], Xing-Gang Wu [a,*], Zhi-Fei Wu [a], Jing-Hao Shan [a], Hua Zhou [b]

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Chinese Physics C Vol. 48, No. 4 (2024) 043104

Revisiting the top-quark pair production at future e^+e^- colliders*

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Jian-Ming Shen (申建明)

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[2]School of Physics and Electronics, Hunan Provincial K
Hunan University
[3]Department of Physics, Chongqing Key Laboratory for Strong

Chinese Physics C Vol. 48, No. 5 (2024) 053113

Precise determination of the top-quark on-shell mass M_t via its scale-invariant perturbative relation to the top-quark $\overline{\rm MS}$ mass $\overline{m}_t(\overline{m}_t)$*

Xu-Dong Huang (黄旭东)[1†] Xing-Gang Wu
Zhi-Fei Wu (

[1]Institute of High Energy Phys
[2]Department of Physics, Chongqing Key Laboratory
[3]Department of Physics

Chinese Physics C Vol. 45, No. 11 (2021) 113102

Reanalysis of the top-quark pair hadroproduction and a precise determination of the top-quark pole mass at the LHC*

Sheng-Quan Wang(王声权)[1†] Xing-Gang Wu(吴兴刚)[2‡] Jian-Ming Shen(申建明)[3§] Stanley J. Brodsky[4‡]

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[2]Department of Physics, Chongqing University, Chongqing 401331, China
[3]School of Physics and Electronics, Hunan University, Changsha 410082, China
[4]SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

CPC (2021)
CPC (2024)
CPC (2024)
PLB (2024)

NLO QCD correction

J. Jersak, E. Laermann, and P. M. Zerwas, Phys. Rev. D 25, 1218 (1982)

N^{2}LO QCD correction

J. Gao and H. X. Zhu, Phys. Rev. Lett. 113, 262001 (2014)

L. Chen, O. Dekkers, D. Heisler, W. Bernreuther, and Z. G. Si, J. High Energy Phys. 12 (2016) 098

N^{3}LO QCD correction

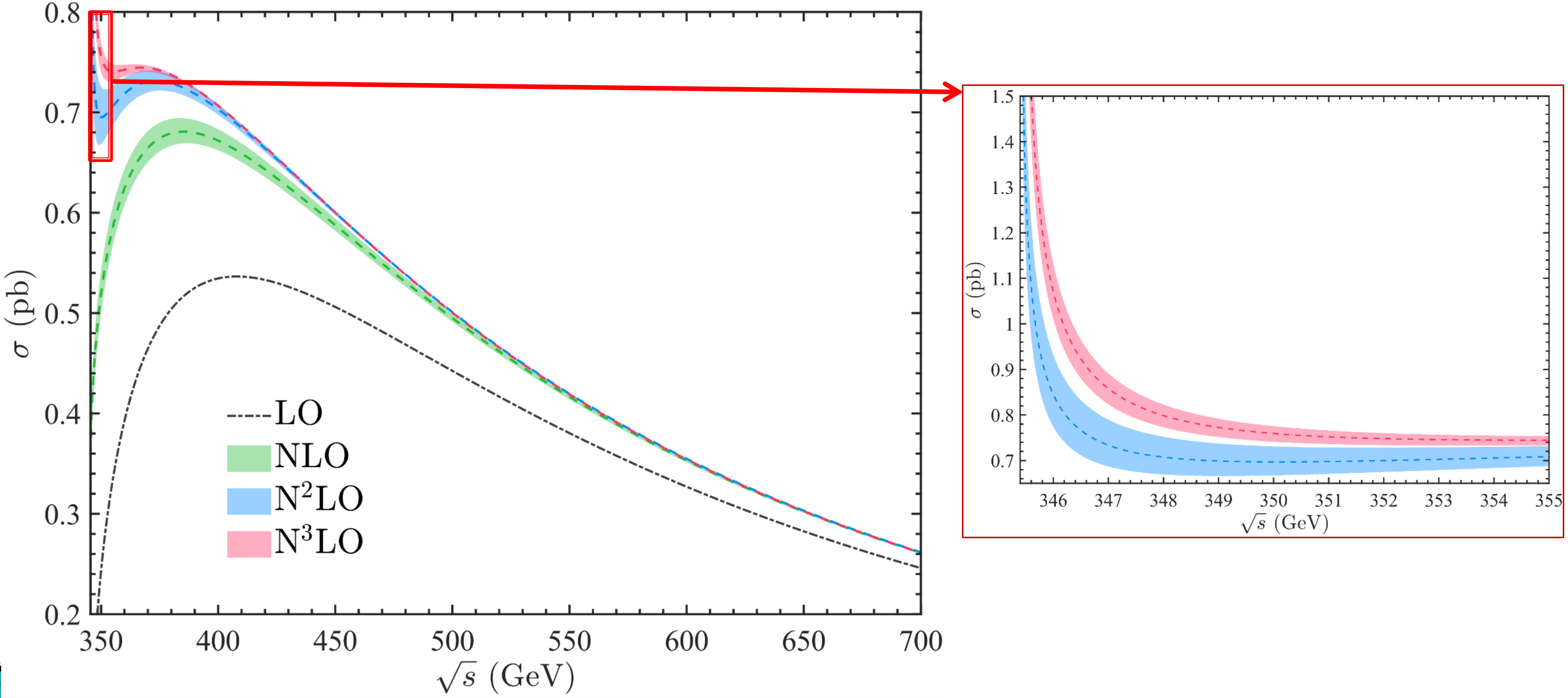
M. Fael, F. Lange, K. Schönwald, and M. Steinhauser, Phys. Rev. Lett. 128, 172003 (2022)

M. Fael, F. Lange, K. Schönwald, and M. Steinhauser, Phys. Rev. D 106, 034029 (2022)

L. Chen, X. Chen, X. Guan and Y. Q. Ma, Phys. Rev. Lett. 132, 10 (2024)

$$e^+e^- \to \gamma^* \to t\bar{t}$$

$\sqrt{s} \approx 2m_t$



Up to N[3]LO, total cross section of $e^+e^- \to \gamma^* \to t\bar{t}$ can be written as

$$\sigma = \sigma_0\left(1 + r_1\alpha_s + r_2\alpha_s^2 + r_3\alpha_s^3 + \cdots\right)$$

$$\sigma_0 = N_C \frac{4\pi\alpha^2}{3s}\frac{v(3-v^2)}{2}e_t^2$$

$$r_1 = \frac{1}{v}r_{1,v} + r_{1,+}$$

$$r_2 = \frac{1}{v^2}r_{2,v^2} + \frac{1}{v}r_{2,v} + r_{2,+}$$

$$r_3 = \frac{1}{v^2}r_{3,v^2} + \frac{1}{v}r_{3,v} + r_{3,+}$$

only numerical results !

$\boldsymbol{v = \sqrt{1 - \frac{4m_t^2}{s}}}$ **the velocity of produced quarks**

we schematically factorize total cross section as the product of Coulomb and non-Coulomb parts

PSLQ algorithm

$$\sigma = \sigma_0 \times \mathcal{R}_{\rm NC} \times \mathcal{R}_{\rm C}$$

Coulomb part

reconstructing analytic form !

Non-Coulomb part

H. R. P. Ferguson, and D. H. Bailey, RNR Technical Report RNR-91-032 (1992)
H. R. P. Ferguson, D. H. Bailey, and S. Arno, Math. Comp. 68, 351 (1999)

The QCD coupling $\alpha_s^{\rm V}(\mathbf{q}^2)$ has been introduced for describing the interaction of the non-relativistic heavy quark-antiquark pair, which is defined as the effective charge in the following Coulomb-like potential:

$$V(\mathbf{q}^2) = -4\pi C_F \frac{\alpha_s^{\rm V}(\mathbf{q}^2)}{\mathbf{q}^2},$$

where $\alpha_s^{\rm V}(\mathbf{q}^2)$ absorbs all the higher-order QCD corrections, which is related to the $\overline{\rm MS}$-scheme coupling via the following way

$$\alpha_s^{\rm V}(\mathbf{q}^2) = \alpha_s(\mu^2) + \left(a_1 - \beta_0 \ln\frac{\mathbf{q}^2}{\mu^2}\right)\alpha_s^2(\mu^2) + \left(a_2 - (2a_1\beta_0 + \beta_1)\ln\frac{\mathbf{q}^2}{\mu^2} + \beta_0^2 \ln^2\frac{\mathbf{q}^2}{\mu^2}\right)\alpha_s^3(\mu^2) + \cdots$$

$$a_1 = \frac{1}{4\pi}\left(\frac{31}{9}C_A - \frac{20}{9}T_F n_l\right)$$

$$a_2 = \frac{1}{(4\pi)^2}\left[\left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^2}{4} + \frac{22}{3}\zeta_3\right)C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\zeta_3\right)C_A T_F n_l - \left(\frac{55}{3} - 16\zeta_3\right)C_F T_F n_l + \left(\frac{20}{9}T_F n_l\right)^2\right]$$

T. Appelquist, M. Dine and I. J. Muzinich, Phys. Lett. B 69, 231 (1977)
W. Fischler, Nucl. Phys. B 129, 157 (1977)

M. Peter, Phys. Rev. Lett. 78, 602 (1997)
Y. Schroder, Phys. Lett. B 447, 321 (1999)

$$\sigma = \sigma_0 \times \mathcal{R}_{\rm NC} \times \mathcal{R}_{\rm C}$$

exactly non-conformal term

$$\mathcal{R}_{\rm C} = 1 + C_F \frac{\pi}{2v} \alpha_s^{\rm V}(sv^2) + C_F^2 \frac{\pi^2}{12v^2} \alpha_s^{\rm V,2}(sv^2) + C_F\left(\frac{\pi^3}{3v}\beta_0^2 - C_F \frac{2\zeta_3}{v^2}\beta_0\right)\alpha_s^{\rm V,3}(sv^2) + \cdots$$

PMC

$$\mathcal{R}_{\rm C}\Big|_{\rm PMC} = 1 + C_F \frac{\pi}{2v} \alpha_s^{\rm V}(Q^2_{*,\rm C}) + C_F^2 \frac{\pi^2}{12v^2} \alpha_s^{\rm V,2}(Q^2_{*,\rm C}) + 0 \times \alpha_s^{\rm V,3}(Q^2_{*,\rm C}) + \cdots$$

resum

$$\frac{X}{1-\exp(-X)}$$

Sommerfeld-Gamow-Sakharov factor

$$X = \pi\, C_F \frac{\alpha_s^{\rm V}(Q^2_{*,\rm C})}{v}$$

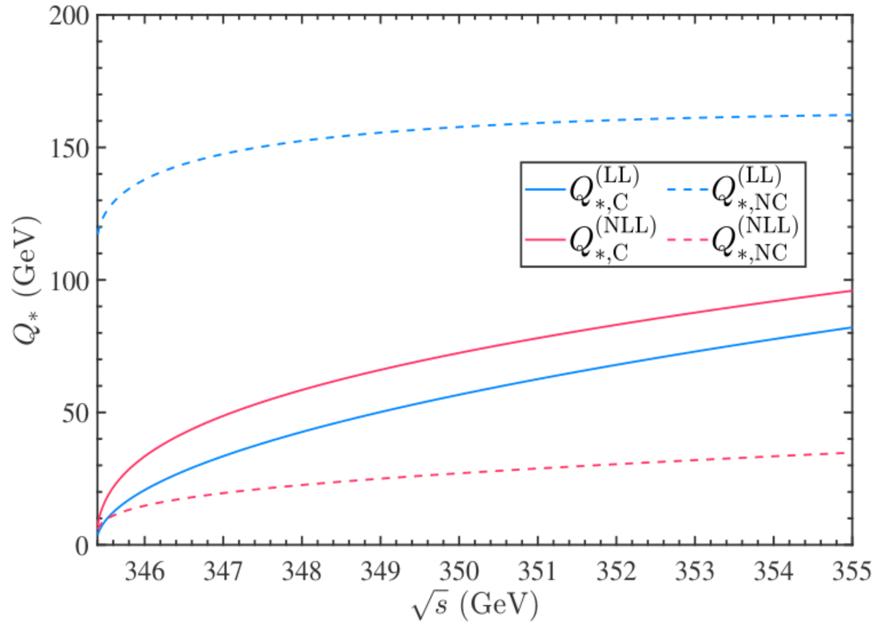
Obtained by solving NR Schrödinger equation

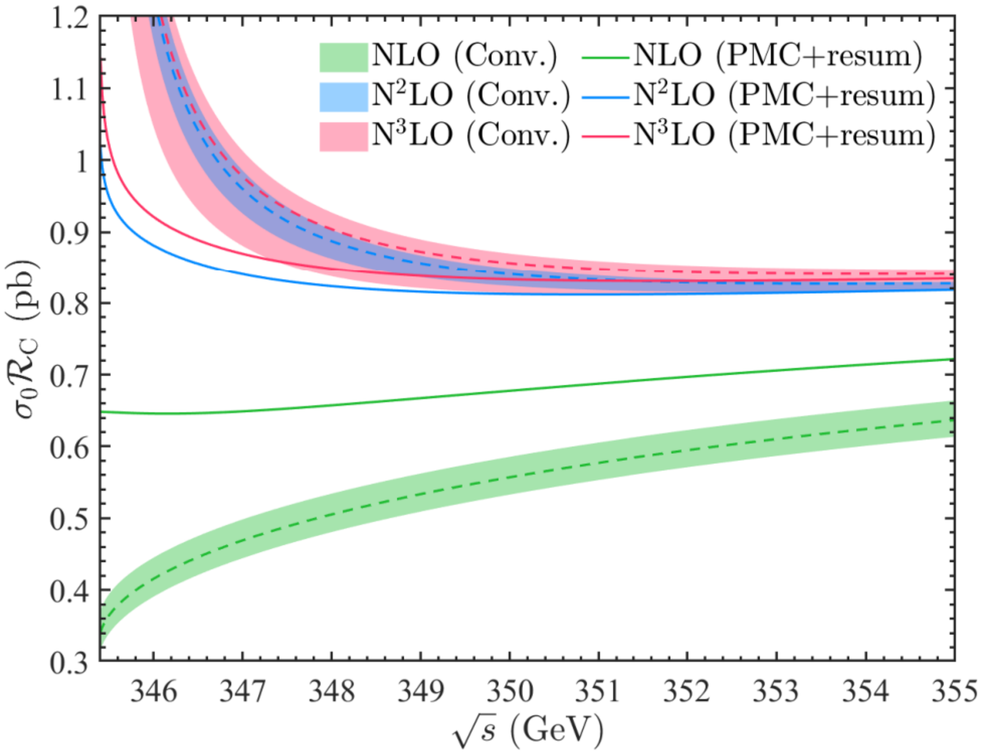
A. D. Sakharov, Zh. Eksp. Teor. Fiz. 18, 631 (1948)

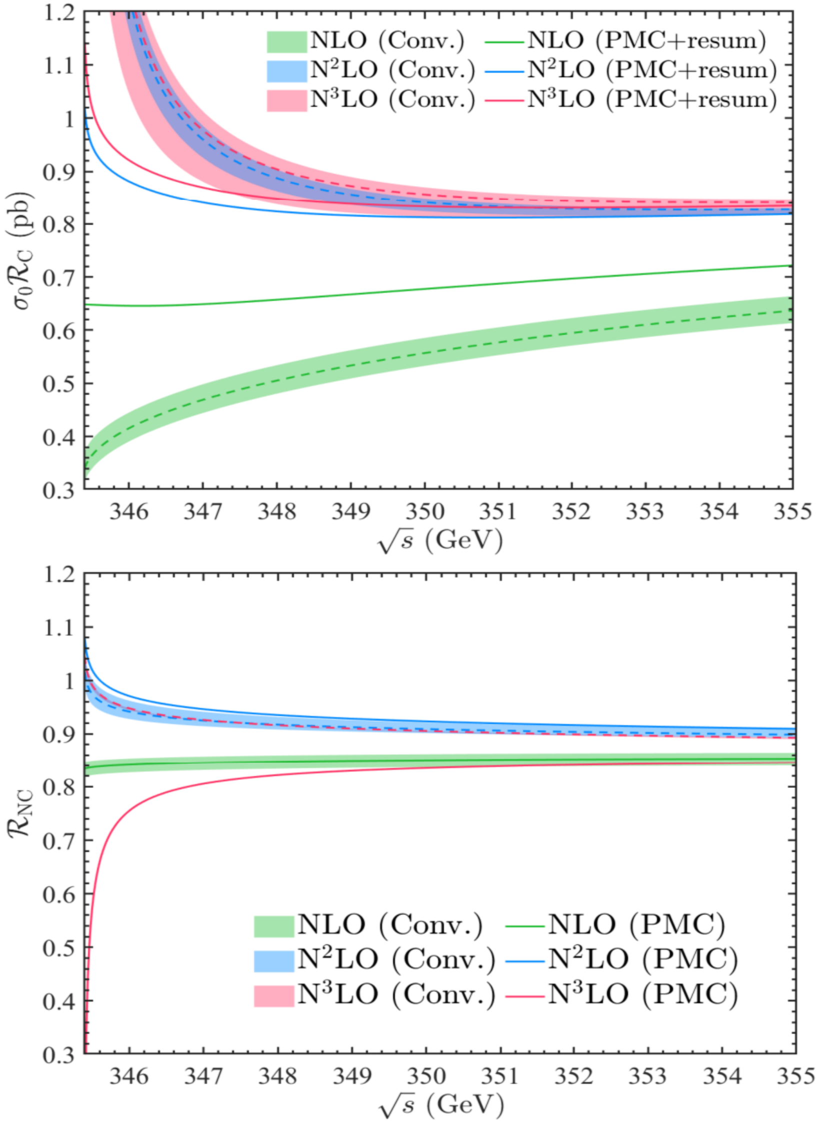
$$\frac{X}{1-\exp(-X)} = 1 + \frac{X}{2} + \frac{X^2}{12} - \frac{X^4}{720} + \cdots$$

The X^3-coefficient is exactly zero!

$$\lim_{v\to 0^+} v \frac{\pi C_F \alpha_s^V / v}{1-\exp(-\pi C_F \alpha_s^V / v)} = \pi C_F \alpha_s^V \text{ is a finite value}$$







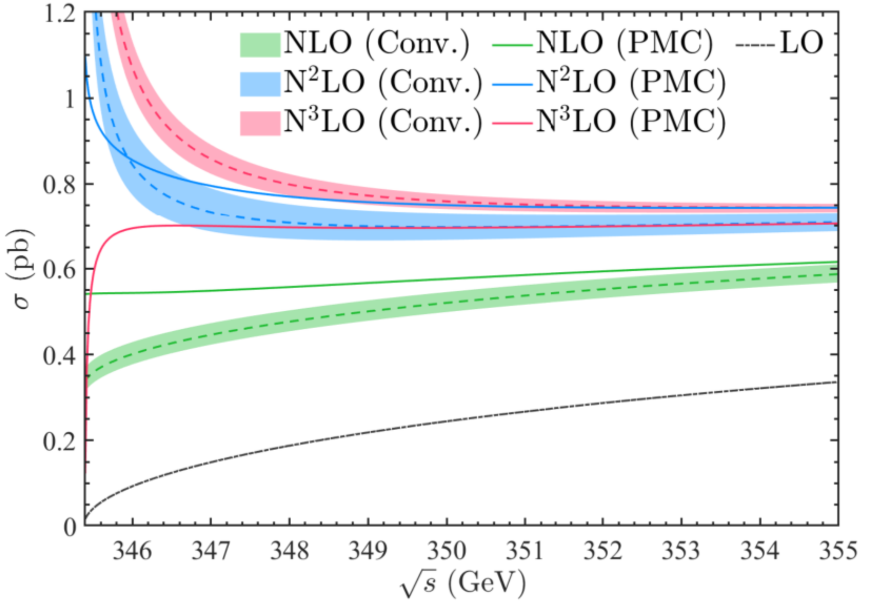
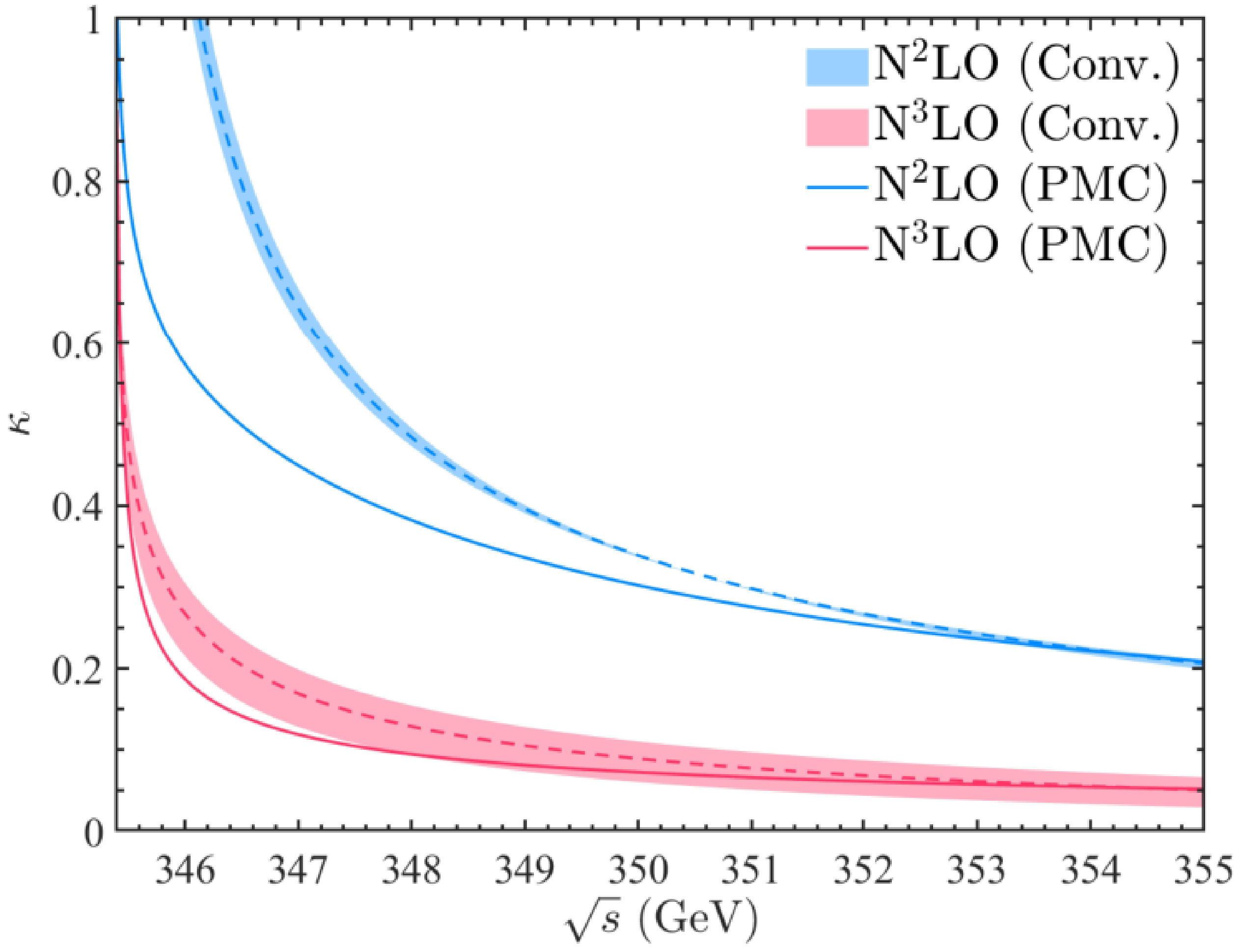
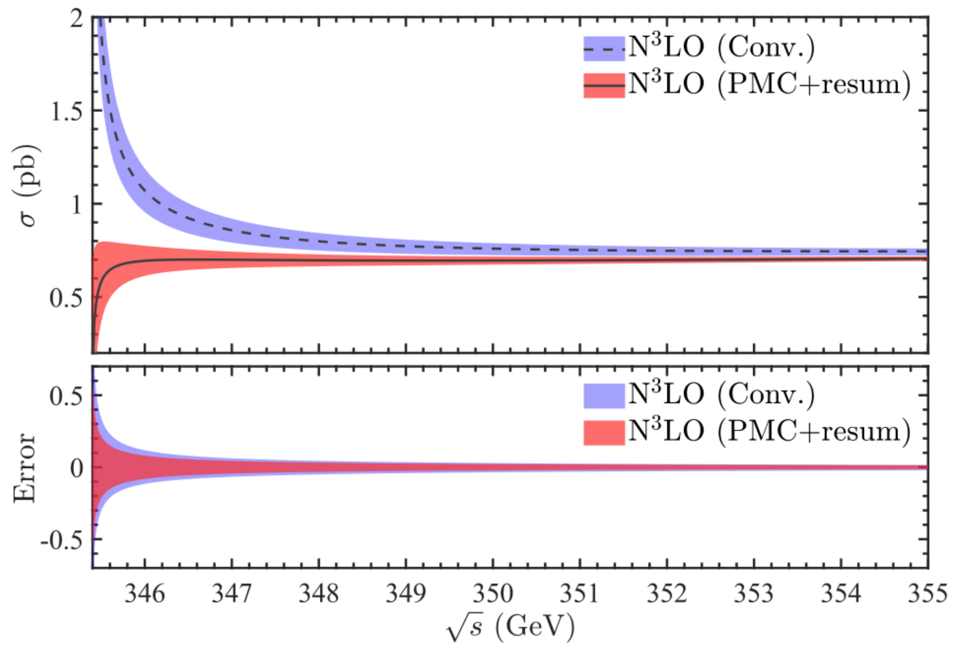


FIG. 3. (Color online) Total cross section $\sigma_{t\bar{t}}$ with different QCD corrections under conventional (dashed line) and PMCs (solid line) scale-setting approaches, respectively.

$$\kappa(\mathrm{N^iLO}) = \left|\frac{\sigma(\mathrm{N^iLO}) - \sigma(\mathrm{N^{i-1}LO})}{\sigma(\mathrm{N^{i-1}LO})}\right|$$

After applying the PMC, the scale dependence is removed and the pQCD convergence near the threshold region is improved





The PMC provids more reliable foundation for constraining predictions of UHO contributions

By applying PMC, uncertainties caused by the UHO-terms become smaller. These results confirm the importance of the PMC scale-setting approach.

Summary and Outlook

It is maximum conformality, but not strict conformality !

Up to infinite order, the predictions are scheme and scale independent, there is no scale ambiguity

At fixed-order, guessing/using typical momentum flow as the scale, one cannot get precise values, becoming an important systematic error

PMC is not simply chosen "special/effective/optimal/ scale", but basing on RGE, satisfying standard RGI, and using a general way to get an improved series that is independent of any choices of scheme and scale, resulting to a precise perturbative prediction at any fixed order

More convergent and precise series provides much better ground for estimating not-yet-computed higher-oder contributions

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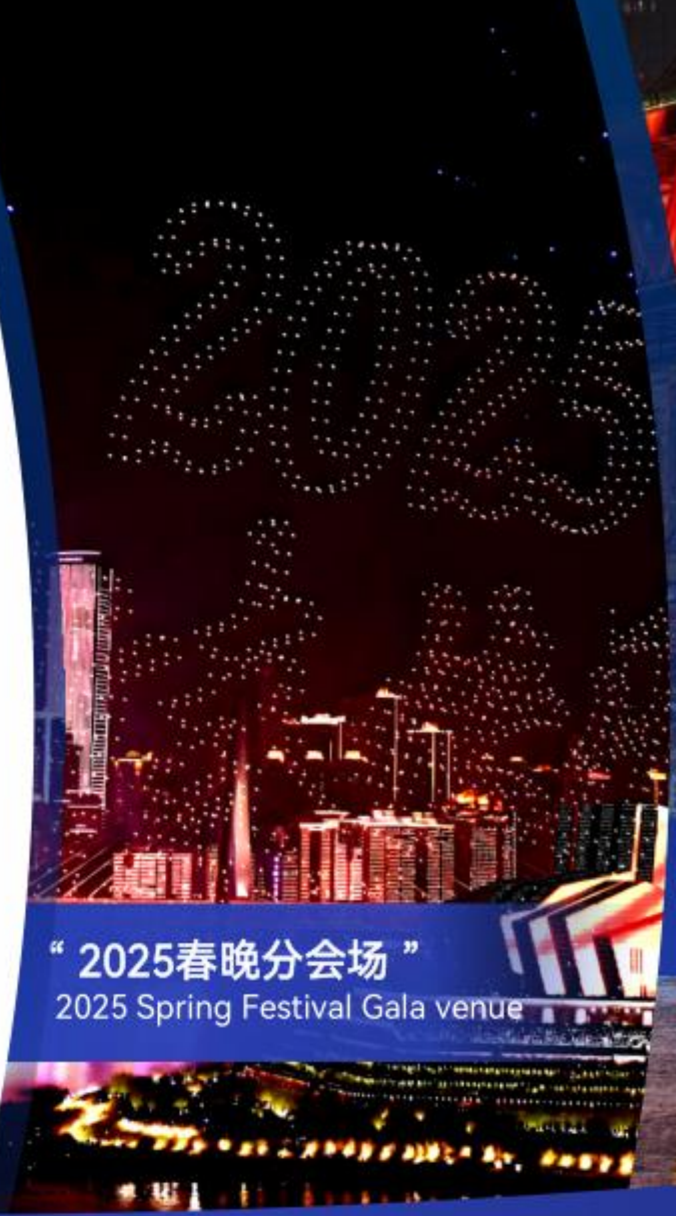
商圈/ Key Commercial Areas

朝天门
Chaotianmen

解放碑商圈
Jiefangbei

江北嘴商圈
Jiangbeizui

光环购物公园
The Ring Shopping Park



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体育服务体系
Sports service system

以数字重庆建设为牵引全面增强超大城市发展、服务、治理能力，**打造安全韧性、智慧宜居人民城市**

With the Digital Chongqing initiative as the driving force, the government will comprehensively strengthen the development, service, and governance capabilities of megacities, aiming to create a safe, resilient, smart, and livable city .


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