Constraining the location of a critical point using Taylor series expansions

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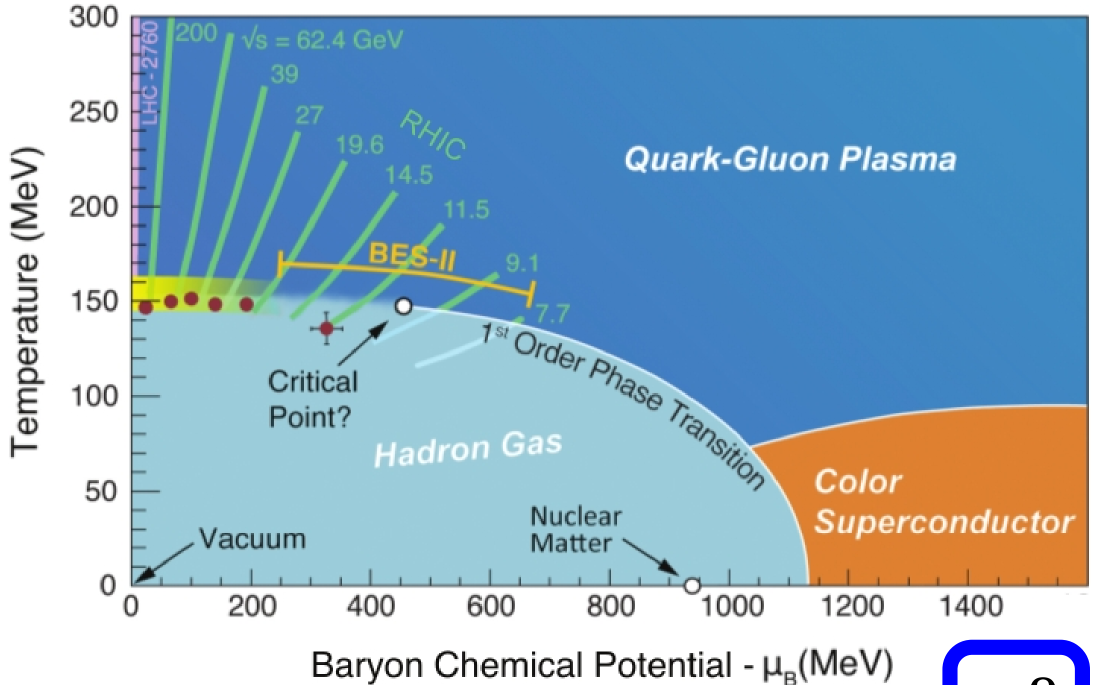
- QCD phase diagram – constraining the location of a critical endpoint
- Critical behavior in the chiral limit at vanishing chemical potential
- Universal critical behavior close to the CEP:
  Z(2) scaling functions and STAR fixed target data

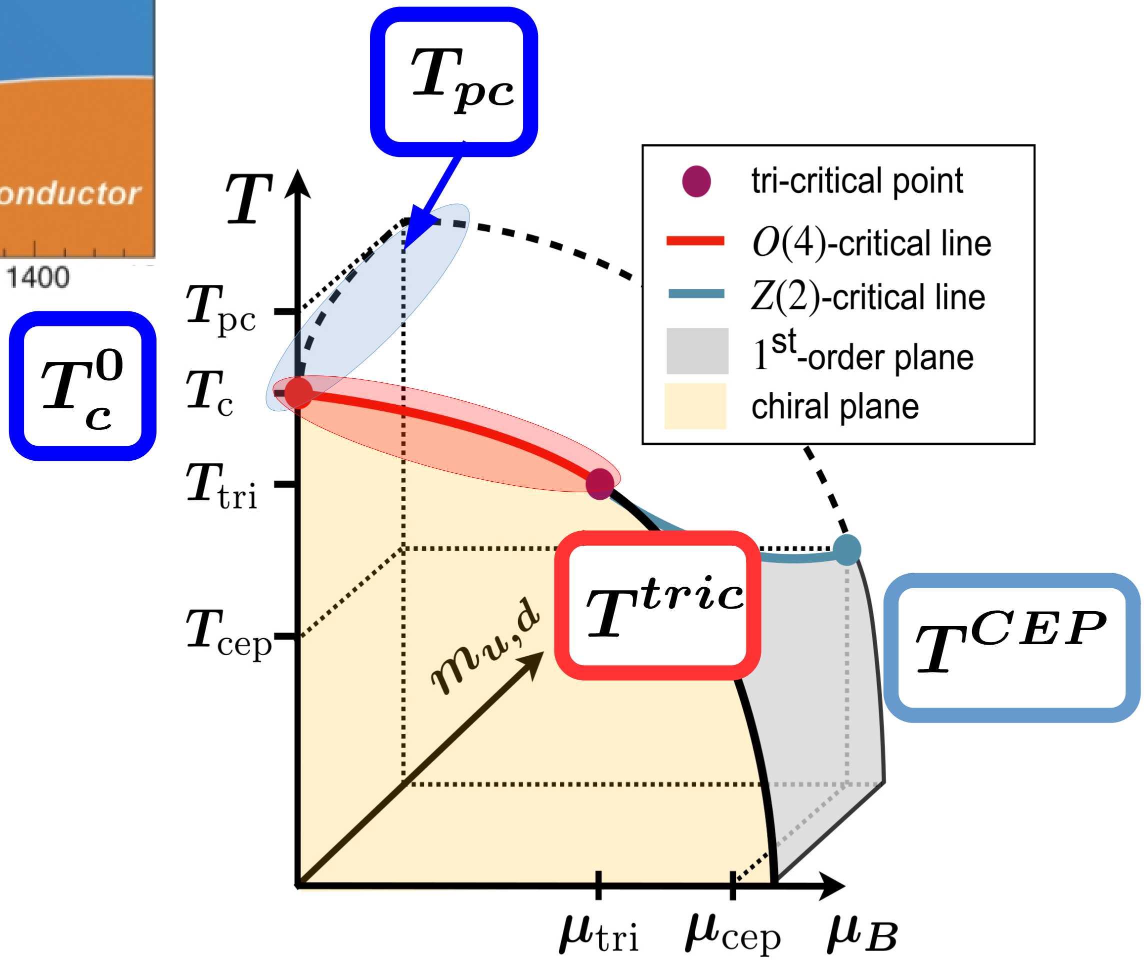
CRC-TR 211 Strong-interaction matter under extreme conditions
DFG Deutsche Forschungsgemeinschaft

Exploring the phase diagram of strong-interaction matter



constraining the location of the CEP through information on the chiral phase transition in QCD

Where is the critical point?



Critical behavior in the chiral limit of QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

$$t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_l}{T}\right)^2$$

$$h \sim \frac{m_l}{T_c}$$

Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic mixed thermal

$$\frac{\partial^2 \ln Z}{\partial h^2} \qquad \frac{\partial^2 \ln Z}{\partial h\, \partial t} \qquad \frac{\partial^2 \ln Z}{\partial t^2}$$

$$\sim \left(\frac{m_l}{T_c}\right)^{1/\delta-1} \qquad \sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta} \qquad \sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta}$$

$$\sim -0.79 \qquad \sim -0.34 \qquad \sim +0.11$$

divergence: strong moderate none

O(4) critical exponents

$$\alpha = -0.21$$
$$\beta = 0.38$$
$$\delta = 4.82$$

Chiral observables in QCD

– chiral condensate: $\langle\bar\psi\psi\rangle_l = \frac{\partial P/T}{\partial m_l/T}$, $\langle\bar\psi\psi\rangle_l = (\langle\bar\psi\psi\rangle_u + \langle\bar\psi\psi\rangle_d)/2$

– chiral order parameter: $M = \frac{2}{f_K^4}\left[m_s\langle\bar\psi\psi\rangle_l - m_l\langle\bar\psi\psi\rangle_s\right]$

$$m_l = (m_u + m_d)/2$$

– chiral susceptibility: $\chi_M = m_s\left(\frac{\partial M}{\partial m_u} + \frac{\partial M}{\partial m_d}\right)$ **magnetic**

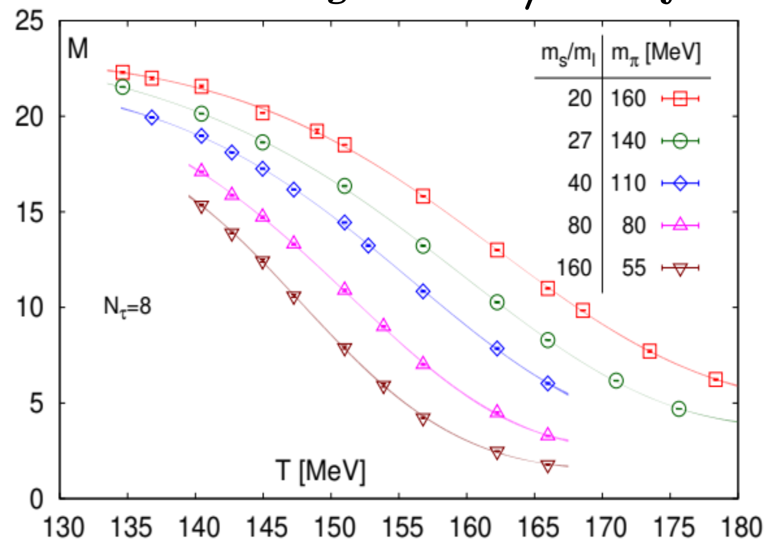
– mixed chiral susceptibility: $\chi_t = T\frac{\partial M}{\partial T}$ **mixed**

– conserved charge fluctuations: $\chi_X = T^2\frac{\partial^2 P/T^4}{\partial\mu_X^2}\Big|_{\mu_X=0}$ **thermal**

$$X = B,\ S, ...$$

The Chiral PHASE TRANSITION in (2+1)-flavor QCD

$$M \sim m_s \partial \ln Z / \partial m_l$$



m_s fixed, physical

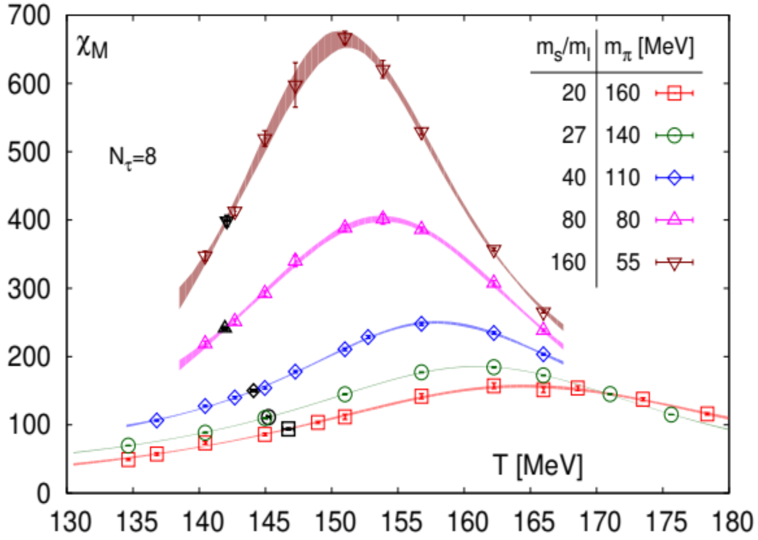
$m_l \Rightarrow 0$

"magnetic" susceptibility

$$\chi_M \sim \frac{\partial^2 \ln Z}{\partial m_l^2}$$

$$\sim (m_s/m_l)^{0.79}$$

$$(160/27)^{0.79} \sim 4$$

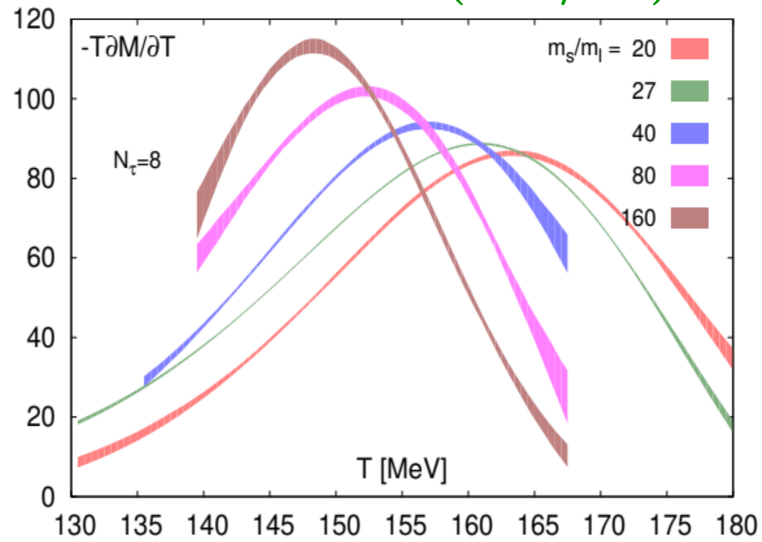


"mixed" susceptibility

$$\chi_t \sim \frac{\partial^2 \ln Z}{\partial T \partial m_l}$$

$$\sim (m_s/m_l)^{0.34}$$

$$(160/27)^{0.34} \sim 1.8$$

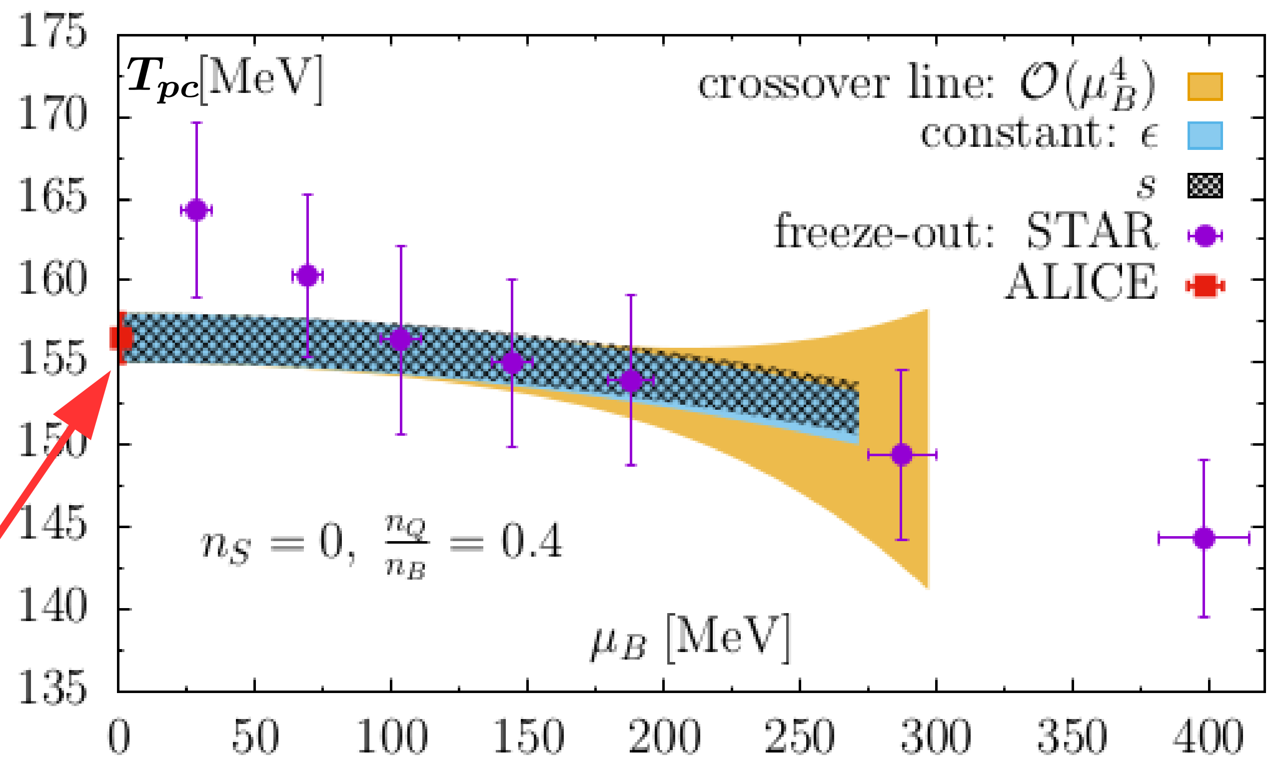


H.T. Ding et al. (HotQCD), PRL 123 (2019) 15, arXiv:1903.04801

Pseudo-critical line for physical quark mass values

$$T_{pc}(\mu_B) = T_{pc}(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T_{pc}(\mu_B)}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_{pc}(\mu_B)}\right)^4 + \dots\right)$$

phase diagram at physical values of the quark masses



STAR: arXiv:1701.07065

A. Andronic et al., Nature 561 (2018) 321

$$T_{pc} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

$\kappa_2 = 0.012(4)$
$\kappa_4 = 0.000(4)$

A. Bazavov et al. [HotQCD], Phys. Lett. B795 (2019), arXiv:1812.08235

$$T_{pc} = (158.0 \pm 0.6)\ \mathrm{MeV}$$

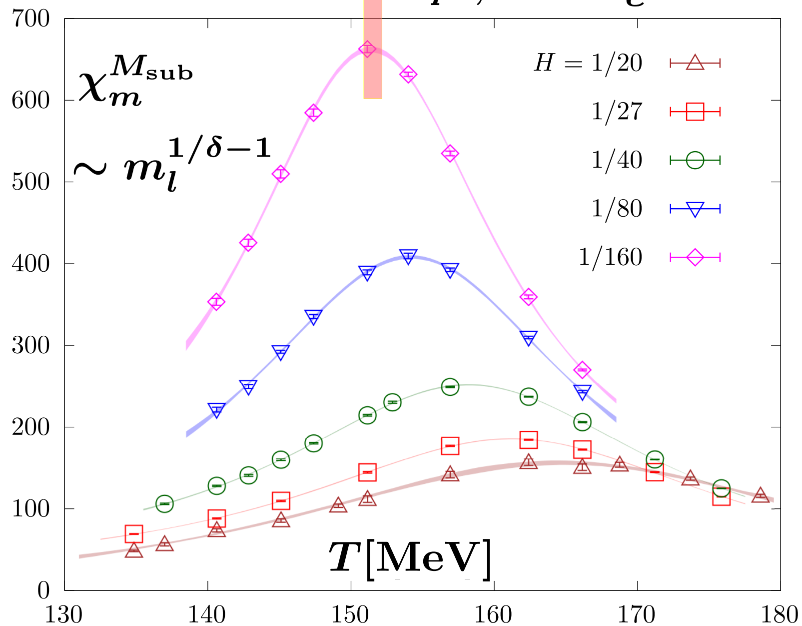
$\kappa_2 = 0.0153(18)$
$\kappa_4 = 0.00032(67)$

S. Borsanyi, et al, PRL 125 (2020) arXiv:2002.02821

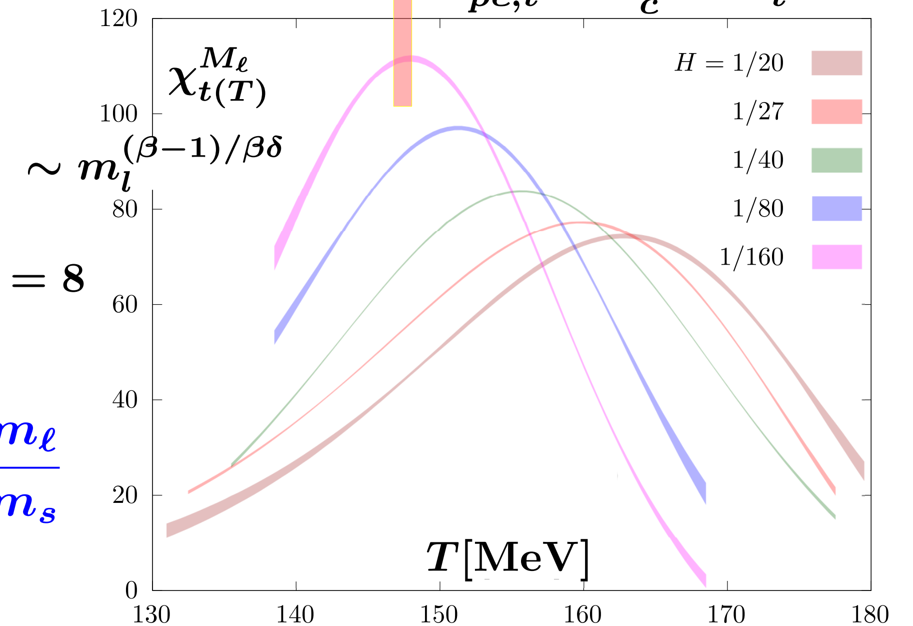
Apporaching the chiral limit

Pseudo-critical temperatures from susceptibilities

$T_{pc,m} - T_c^0 \sim z_m H^{1/\beta\delta}$



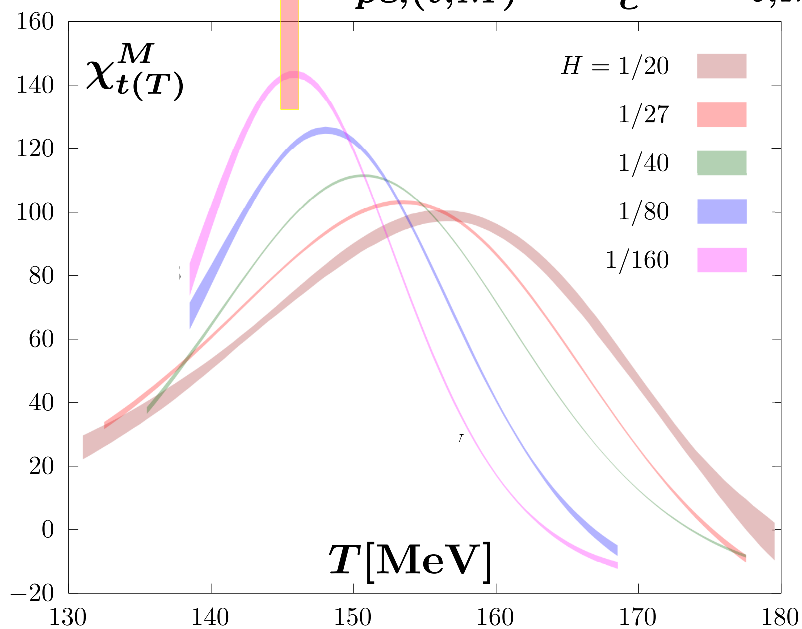
$T_{pc,t} - T_c^0 \sim z_t H^{1/\beta\delta}$



$N_\tau = 8$

$$H = \frac{m_\ell}{m_s}$$

$T_{pc,(t,M)} - T_c^0 \sim z_{t,M} H^{1/\beta\delta}$



– even in the asymptotic scaling regime T_{pc} is not a unique value

– depends on observable

$$T_{pc,x}(H) = T_c^0 \left(1 + \frac{z_x}{z} H^{1/\beta\delta}\right) \quad x = m, t, (t, M)$$

$$3d, O(2) : z_m = 1.6675(65)$$
$$z_t = 0.7991(96)$$
$$z_{t,M} = 0.629(10)$$

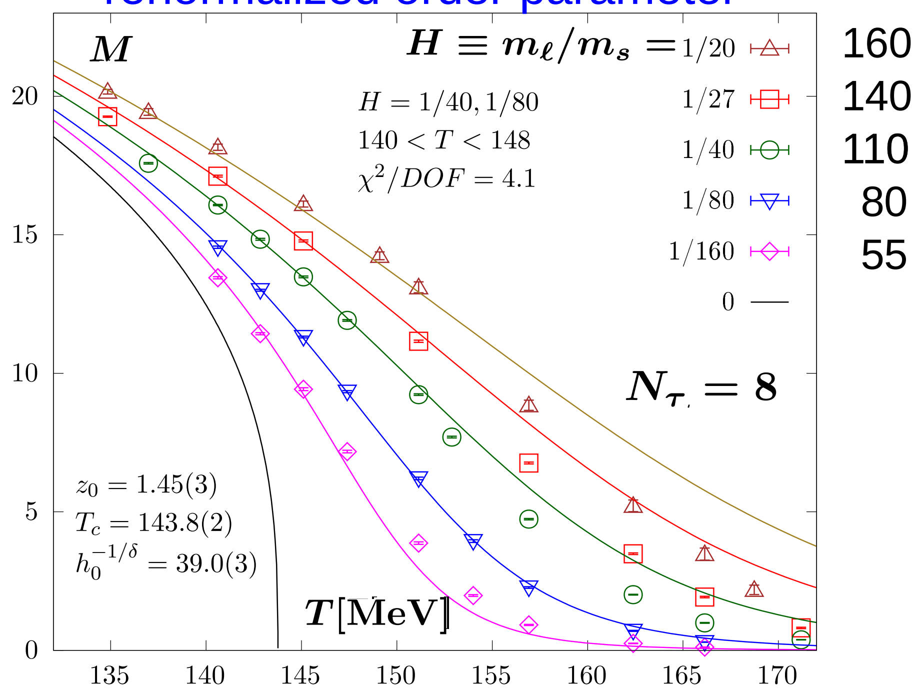
Chiral symmetry restoration

$m_{u,d} \rightarrow 0$ ⟹ $SU(2)_L \times SU(2)_R$ unbroken for $T \geq T_c$

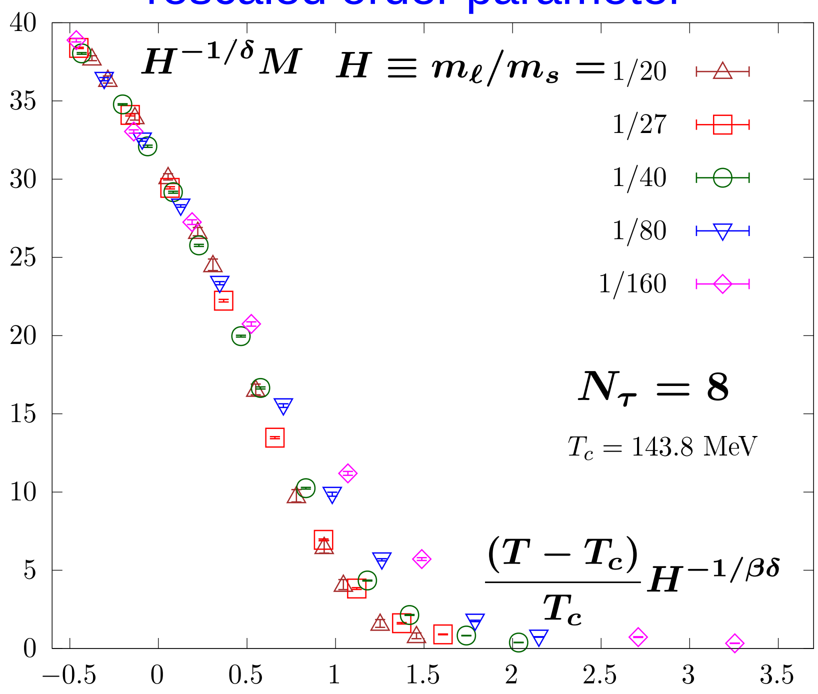
improved order parameter with clear connection to universal scaling functions:

$$M \equiv M_\ell - H\chi_\ell = h^{1/\delta}\left(f_G(z) - f_\chi(z)\right) + \tilde{f}_r(T,H)$$

renormalized order parameter



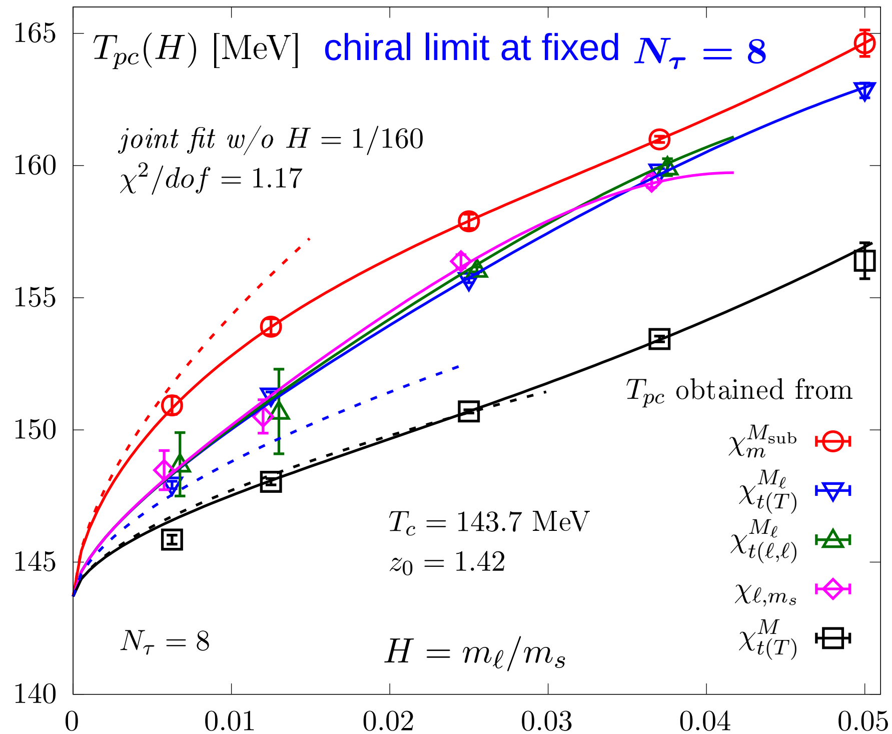
rescaled order parameter



H.T.Ding, O. Kaczmarek, FK, P. Petreczky, Mugdha Sarkar, C. Schmidt, Sipaz Sharma, arXiv:2403.09390

Pseudo-critical and critical temperatures

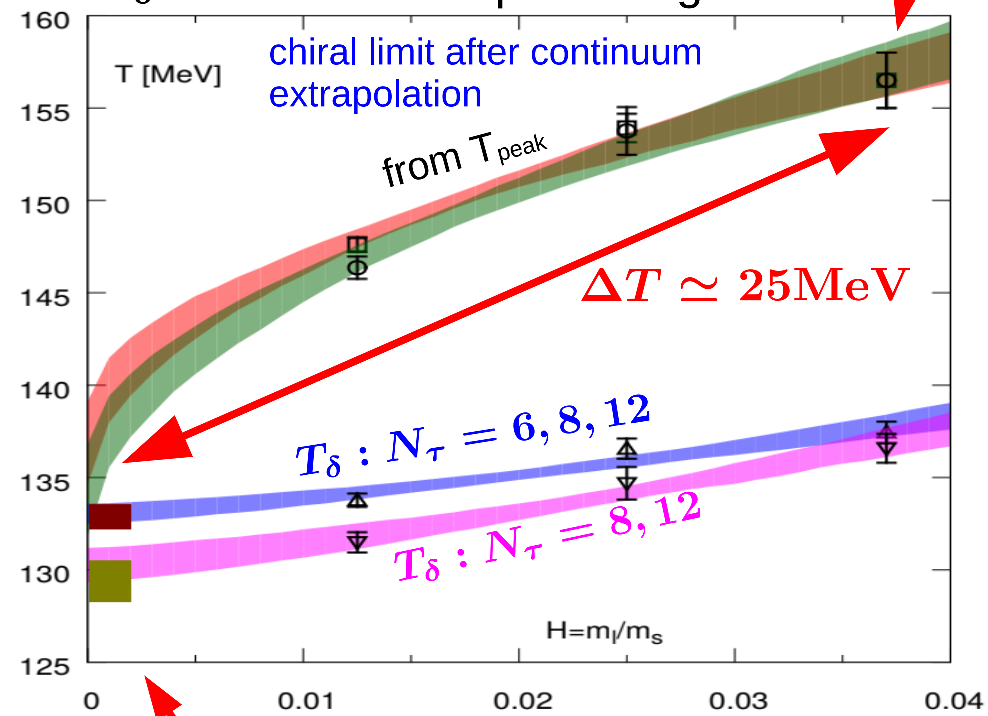
$$T_{pc}(H) = T_c + z_x T_c H^{1/\beta\delta} \quad , \quad x = \text{peak}, \delta, ...$$



H.T.Ding, O. Kaczmarek, FK, P. Petreczky, Mugdha Sarkar, C. Schmidt, Sipaz Sharma, arXiv:2403.09390

physical masses

T_δ : T at ~60% of peak height



H.T Ding et al (HotQCD), arXiv:1903.04801

Anirban Lahiri et al, arXiv:2010.15593

physical masses

$$T_{pc}^{phys} = (156.5 \pm 1.5)\ \text{MeV}$$

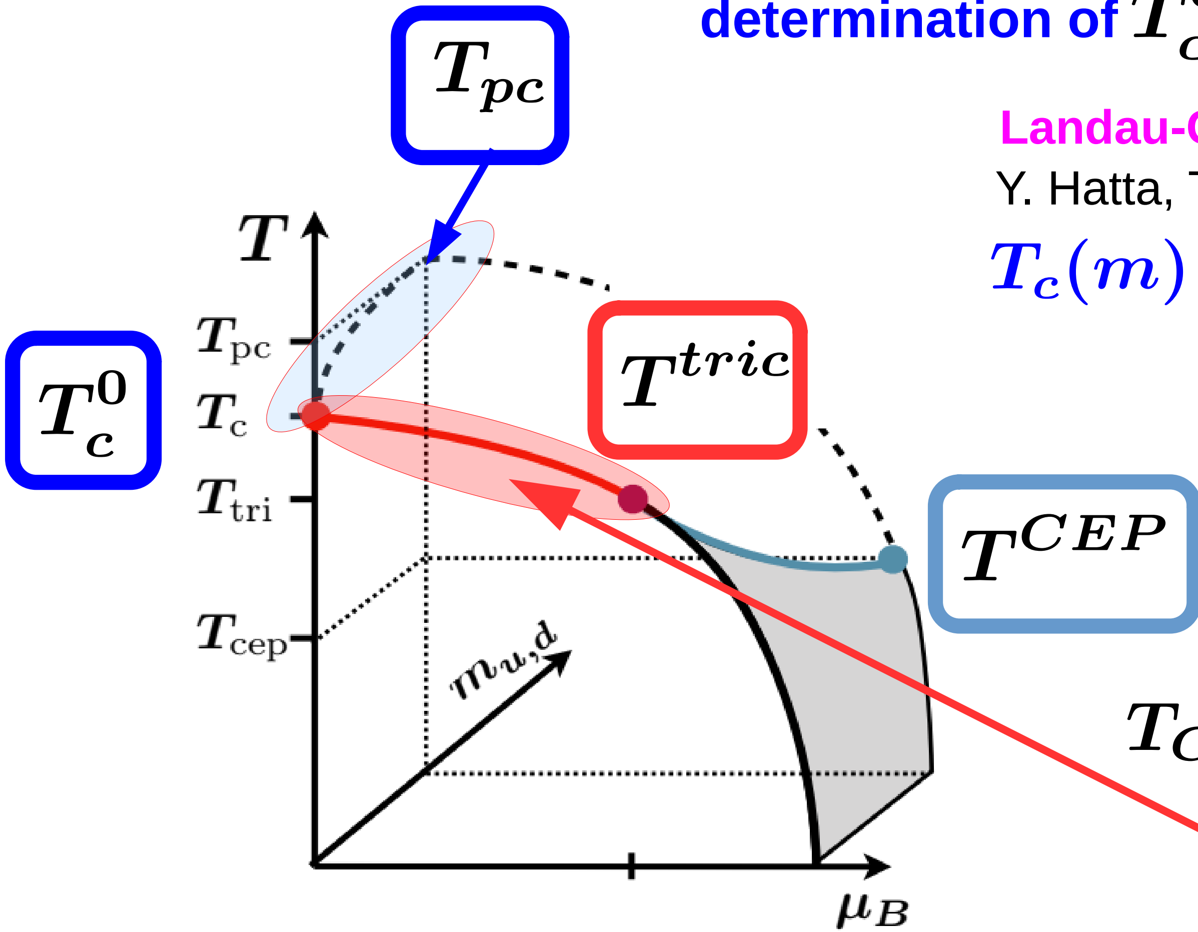
A. Bazavov et al (HotQCD), arXiv:1812.08235

chiral limit extrapolations

$$T_c^0 = 132^{+3}_{-6}\ \text{MeV}$$

also: A. Y. Kotov et al., arXiv: 2105.09842

The Chiral PHASE TRANSITION in (2+1)-flavor QCD at non-vanishing chemical potential



determination of T_c^0 puts an upper limit on T^{CEP}

Landau-Ginzburg potential:

Y. Hatta, T. Ikeda, Phys. Rev. D67 (2003) 014028

$$T_c(m) = T^{tric} - c\ m^{2/5}$$

$\rm non-universal,\ but\ c > 0$

$$T_{CEP} < T^{tric} < T_c^0 < T_{pc}^0$$

curvature in the chiral limit;
weak quark mass dependence:

H.-T. Ding et al. PRD 109 (2024) 114516

Random Matrix Model — A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.M. Verbaarschot, Phys. Rev. D58 (1998) 096007

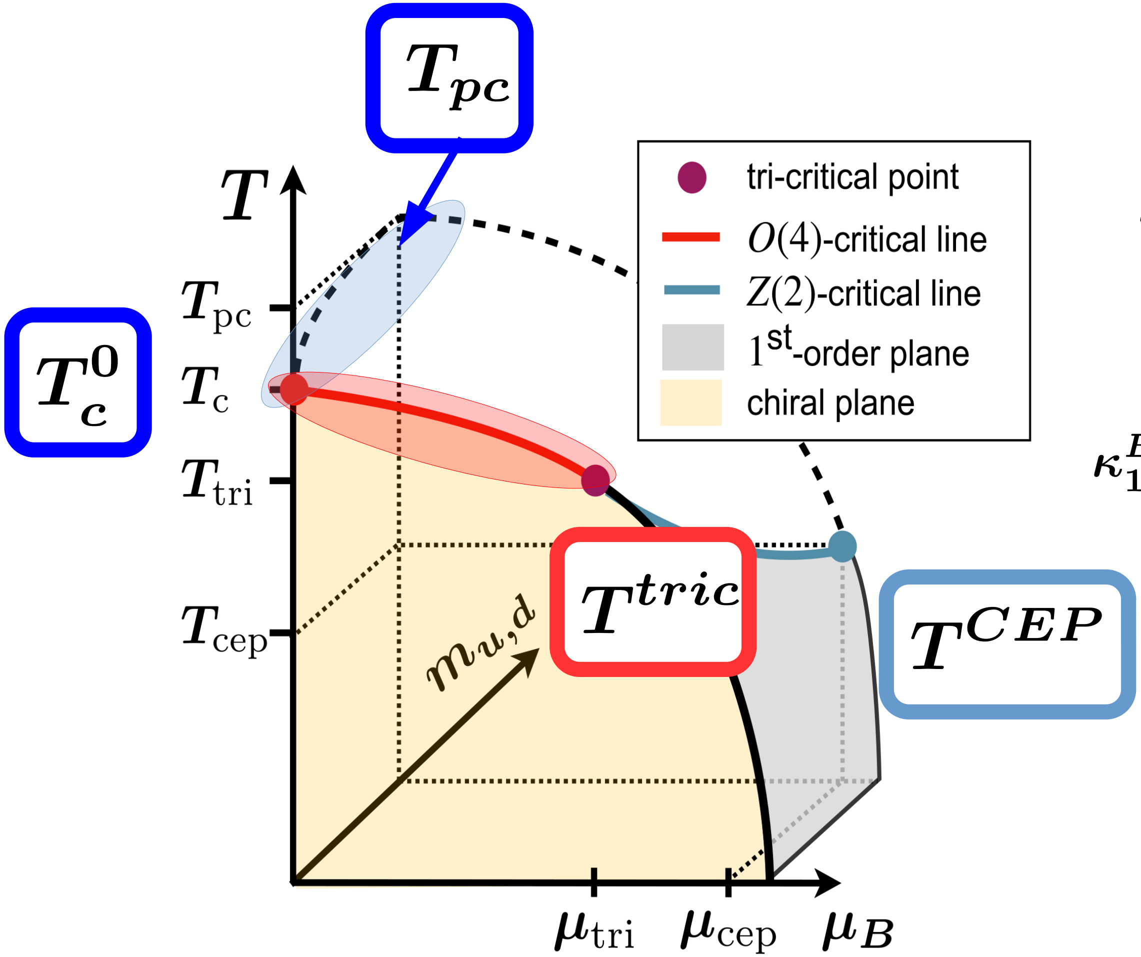
QCD motivated — M. Stephanov, Phys. Rev. D73 (2006) 094508

NJL — M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

Critical temperatures at non-zero chemical potential
– curvature of the critical line in the chiral limit –

$$T_{pc,x}(H) = T_c^0\left(1 + \frac{z_x}{z_0} H^{1/\beta\delta} - \kappa_2^B (\mu_B/T)^2 + \ldots\ldots\right) \quad , \; H = \frac{m_\ell}{m_s} \; , \; z_0 = \frac{h_0^{1/\beta\delta}}{t_0}$$

$$T_c(\mu_B, \ldots) = T_c^0\left(1 - \kappa_2^B \left(\frac{\mu_B}{T}\right)^2 - \kappa_{11}^{BS} \frac{\mu_B}{T}\frac{\mu_S}{T} - \kappa_2^S \left(\frac{\mu_S}{T}\right)^2 + \ldots\right)$$



curvature coefficients

$$\kappa_2^f(H) = \frac{1}{2T_c}\left(\frac{\partial^2 M_\ell/\partial\hat{\mu}_f^2}{\partial M_\ell/\partial T}\right)_{(T_c,\vec{\mu}=0)}$$

$$f = B, S$$

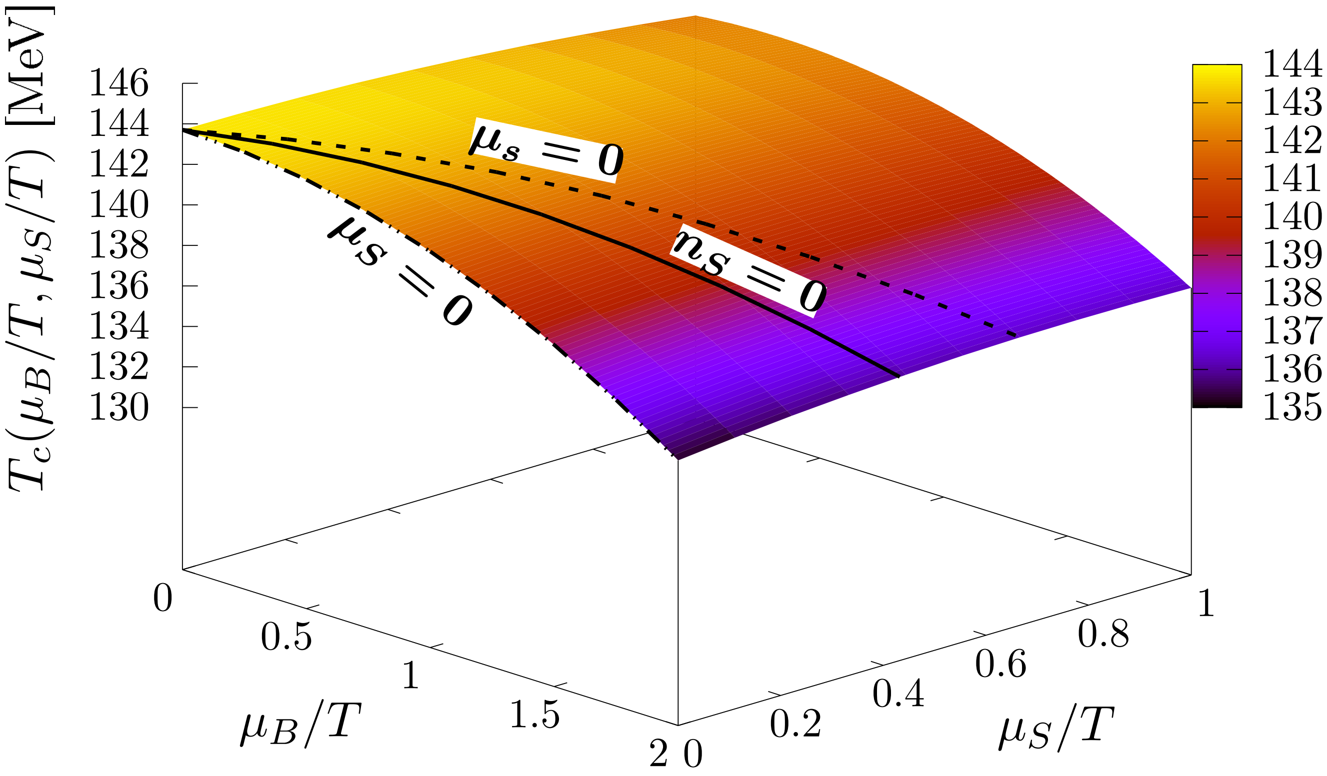
$$\kappa_{11}^{BS}(H) = \frac{1}{2T_c}\left(\frac{\partial^2 M_\ell/\partial\hat{\mu}_B\partial\hat{\mu}_S}{\partial M_\ell/\partial T}\right)_{(T_c,\vec{\mu}=0)}$$

H.T.Ding, O. Kaczmarek, FK, P. Petreczky, Mugdha Sarkar, C. Schmidt, Sipaz Sharma, arXiv:2403.09390

The critical surface in the $\mu_B - \mu_S$ plane

chiral limit extrapolated

$$\kappa_2^{\mu_S=0} > \kappa_2^{n_S=0} > \kappa_2^{\mu_s=0}$$



physical mass results

$$\kappa_2^B = 0.015(4)$$
$$\kappa_2^{n_S=0} = 0.012(2)$$

HotQCD: arXiv:1812.08235

$$\mu_S = 0: \quad \kappa_2^{\mu_S=0} = \kappa_2^B = 0.015(1)$$

$$n_S = 0: \quad \kappa_2^{n_S=0} = \kappa_2^B \left(1 + s_1^2 \frac{\kappa_2^S}{\kappa_2^B} + 2s_1 \frac{\kappa_{11}^{BS}}{\kappa_2^B}\right) = 0.895(31)\kappa_2^B$$

$$\mu_s = 0: \quad \kappa_2^{\mu_s=0} = \kappa_2^B \left(1 + \frac{1}{9}\frac{\kappa_2^S}{\kappa_2^B} + \frac{2}{3}\frac{\kappa_{11}^{BS}}{\kappa_2^B}\right) = 0.972(19)\kappa_2^{n_S=0}$$

The Chiral PHASE TRANSITION in (2+1)-flavor QCD at non-vanishing chemical potential

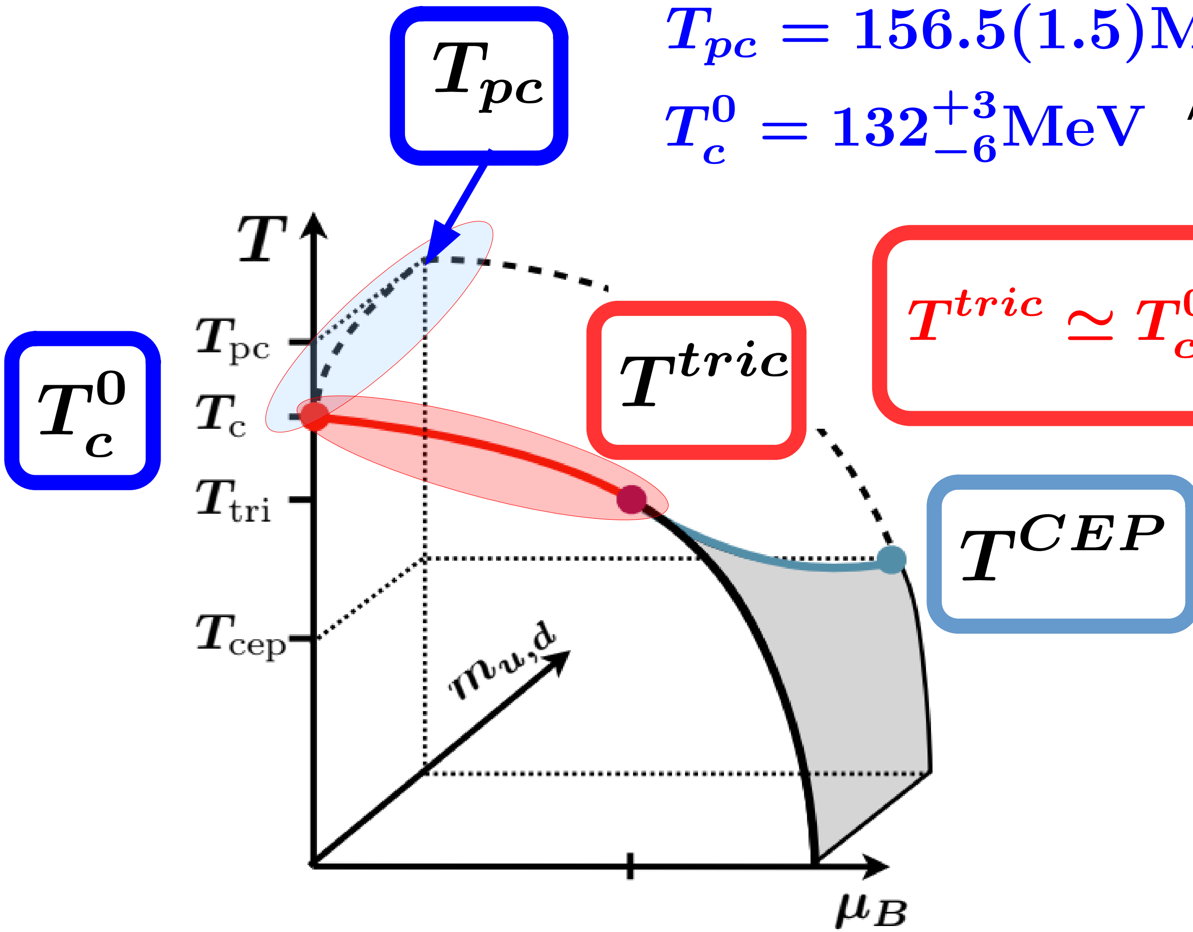
determination of T_c^0 puts an upper limit on T^{CEP}

$$T_{pc} = 156.5(1.5)\mathrm{MeV}$$

$$T_c^0 = 132^{+3}_{-6}\mathrm{MeV}$$

A. Bazavov et al (HotQCD), arXiv:1812.08235

H.T. Ding et al (HotQCD), arXiv:1903.04801



$$T^{tric} \simeq T_c^0 - 1.9(6)\ \mathrm{MeV} \left(\frac{\mu_B}{T_c}\right)^2_{tric}$$

Taylor series suggests

$$(\mu_B/T)_{tric} \geq 2$$

$$T^{CEP} < T^{tric} < T_c^0$$

$$\boxed{T^{CEP} < 125\ \mathrm{MeV}}$$

the temperature range below T_c^0 is most important for getting information on a possible CEP

upper limit on T^{CEP} puts constraint on HIC searches for the CEP

– pseudo-critical temperatures at physical quark mass values

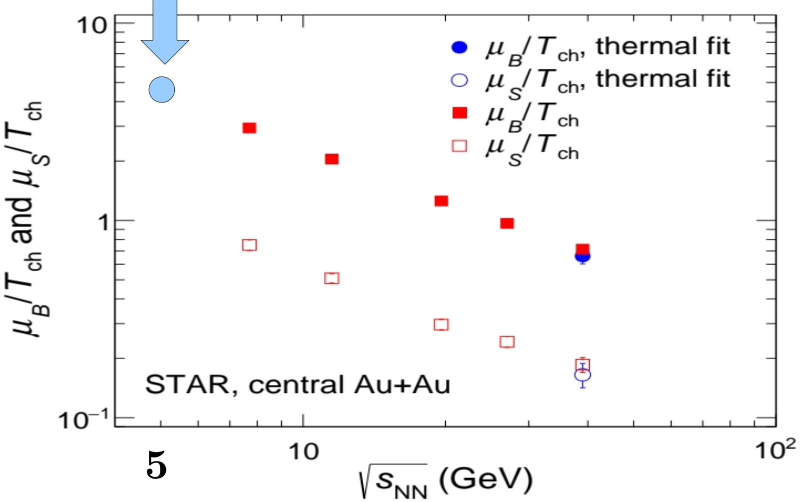
$$T_{pc}(\mu_B) = 156.5(1.5)\text{MeV} \left(1 - 0.012(4)\left(\frac{\mu_B}{T}\right)^2 - / + \ldots\right)$$

to reach $T < 132\text{MeV}$ $T < 125\text{MeV}$ $T < 110\text{MeV}$

need $\mu_B/T \simeq 3.6$ $\mu_B/T \simeq 4$ $\mu_B/T \simeq 5$

$\mu_B \geq 476\text{MeV}$ $\mu_B \geq 500\text{MeV}$ $\mu_B \gtrsim 550\text{MeV}$

my extrap.



Search for the Critical End Point requires beam energies

$$\sqrt{s_{NN}} \leq 5\ \text{GeV}$$

STAR freeze-out parameter, arXiv:1906.03732
also: HADES collaboration, arXiv:1512.07070

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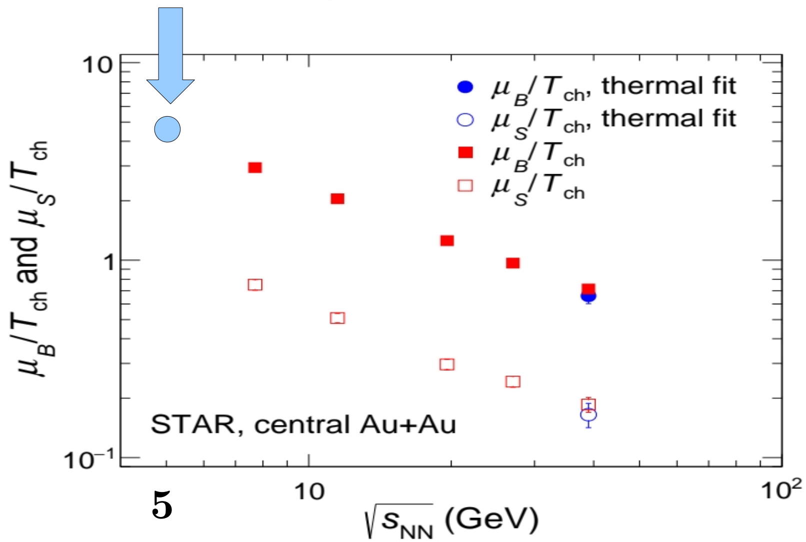

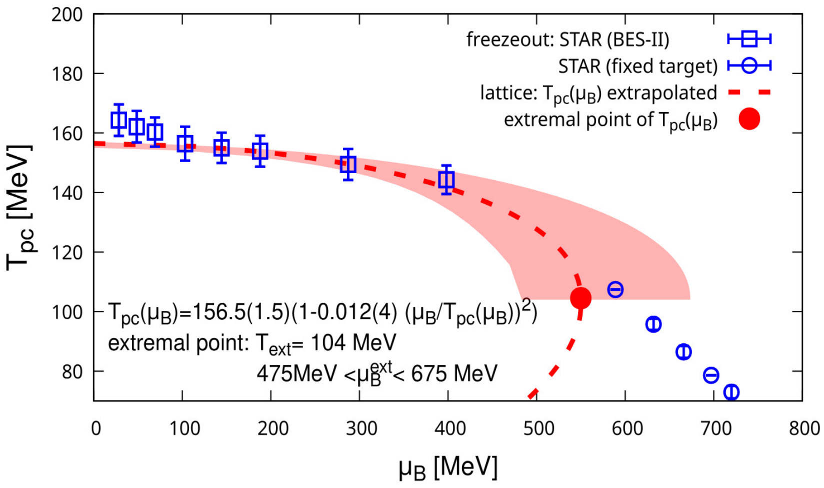

Constraint on allowed region for location of the CEP

Ding,..Mugdha Sarkar,... et al., arXiv:2403.09390

– based on determination of $T_c(\mu_B)$ and apparent convergence of Taylor series for $\mu_B/T \leq 2$

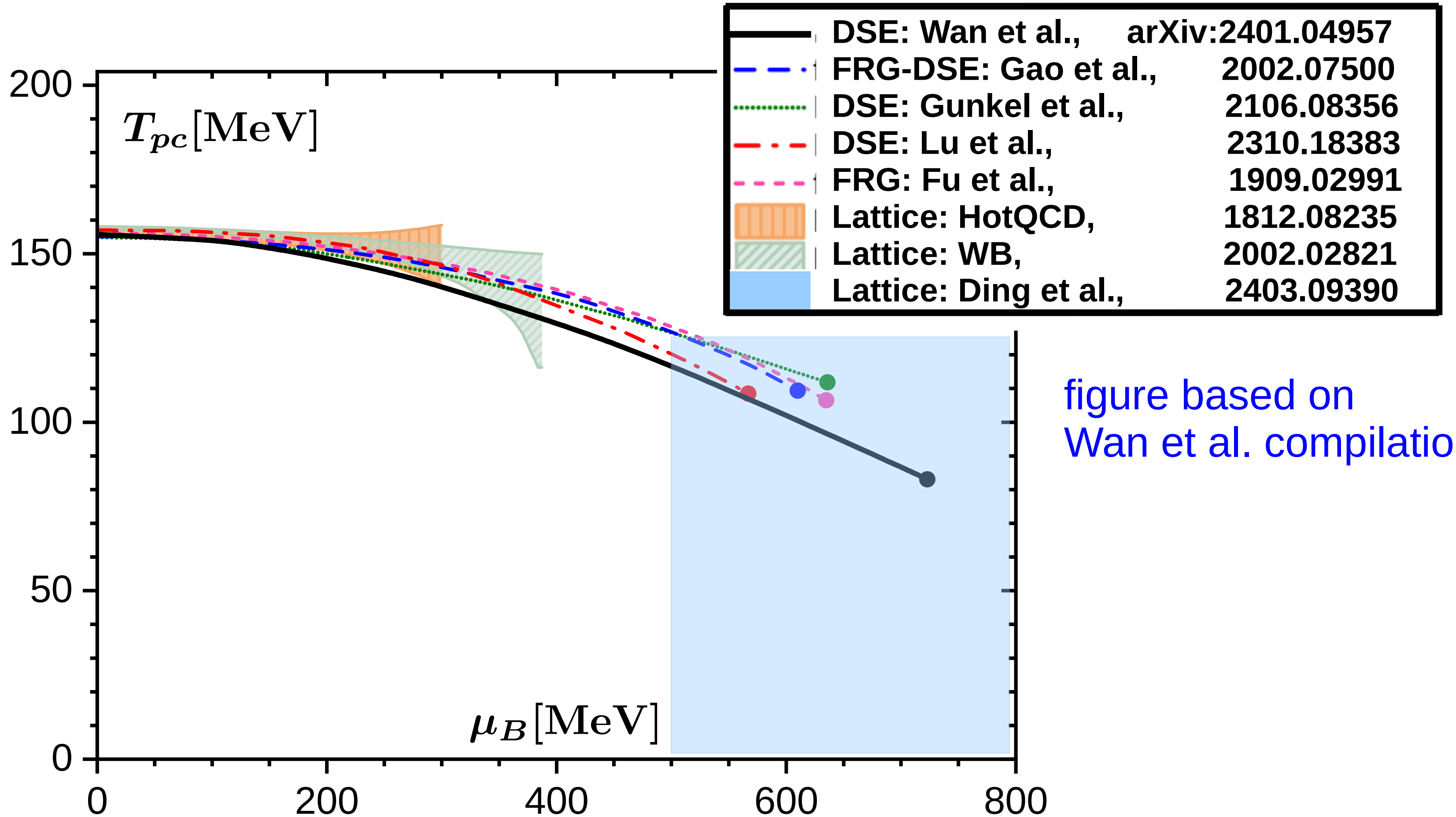

figure based on
Wan et al. compilation

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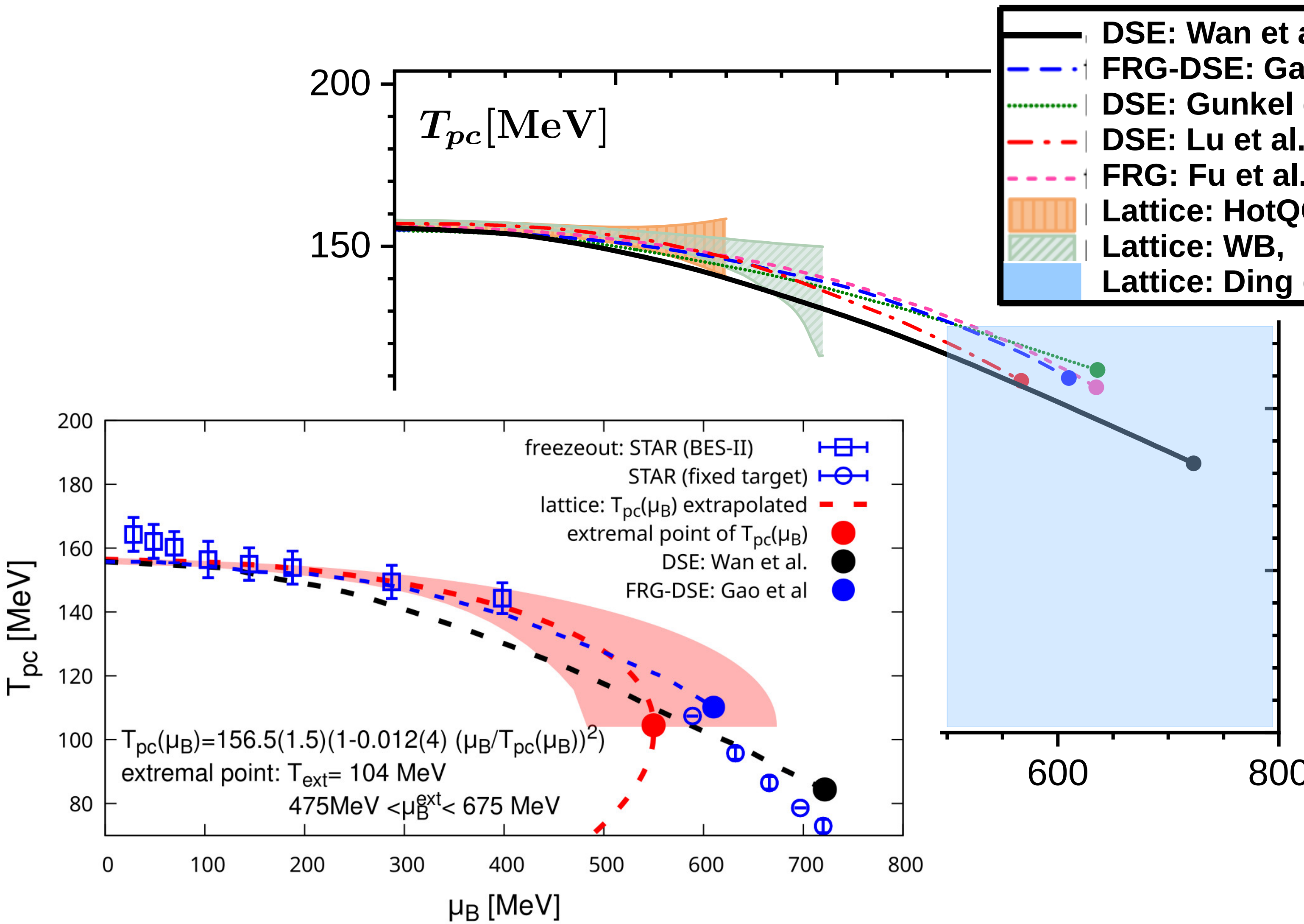


figure based on
Wan et al. compilation

Taylor series for finite-density QCD
Radius of Convergence and Pade approximants

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\boxed{\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k}$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

constrained Taylor series, demanding strangeness neutrality: $n_S = 0$

and: $n_Q / n_B = 0.4$

$$\begin{aligned} \hat{\mu}_Q(T, \mu_B) &= q_1(T)\, \hat{\mu}_B + q_3(T)\, \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5) \\ \hat{\mu}_S(T, \mu_B) &= s_1(T)\, \hat{\mu}_B + s_3(T)\, \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5) \end{aligned}$$

$$\boxed{\frac{P}{T^4} = \sum_{k=0}^{\infty} \frac{1}{k!} \tilde{\chi}_k^B(T) \left(\frac{\mu_B}{T}\right)^k}$$

Comparing Taylor series and Pade resummation

Taylor series

$$\frac{\Delta p}{T^4} \equiv \frac{p(T,\mu_B)}{T^4} - \frac{p(T,0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T) \left(\frac{\mu_B}{T}\right)^{2k}$$

$$= \frac{P_2^2}{P_4} \left(\bar{x}^2 + \bar{x}^4 + c_{6,2}\bar{x}^6 + c_{8,2}\bar{x}^8 + ...\right)$$

use $P_2 > 0,\ P_4 > 0$

$$\bar{x} \equiv \sqrt{P_4/P_2}\ (\mu_B/T)$$

$$c_{6,2} = \frac{P_6 P_2}{P_4^2} \quad , \quad c_{8,2} = \frac{P_8 P_2^2}{P_4^3}$$

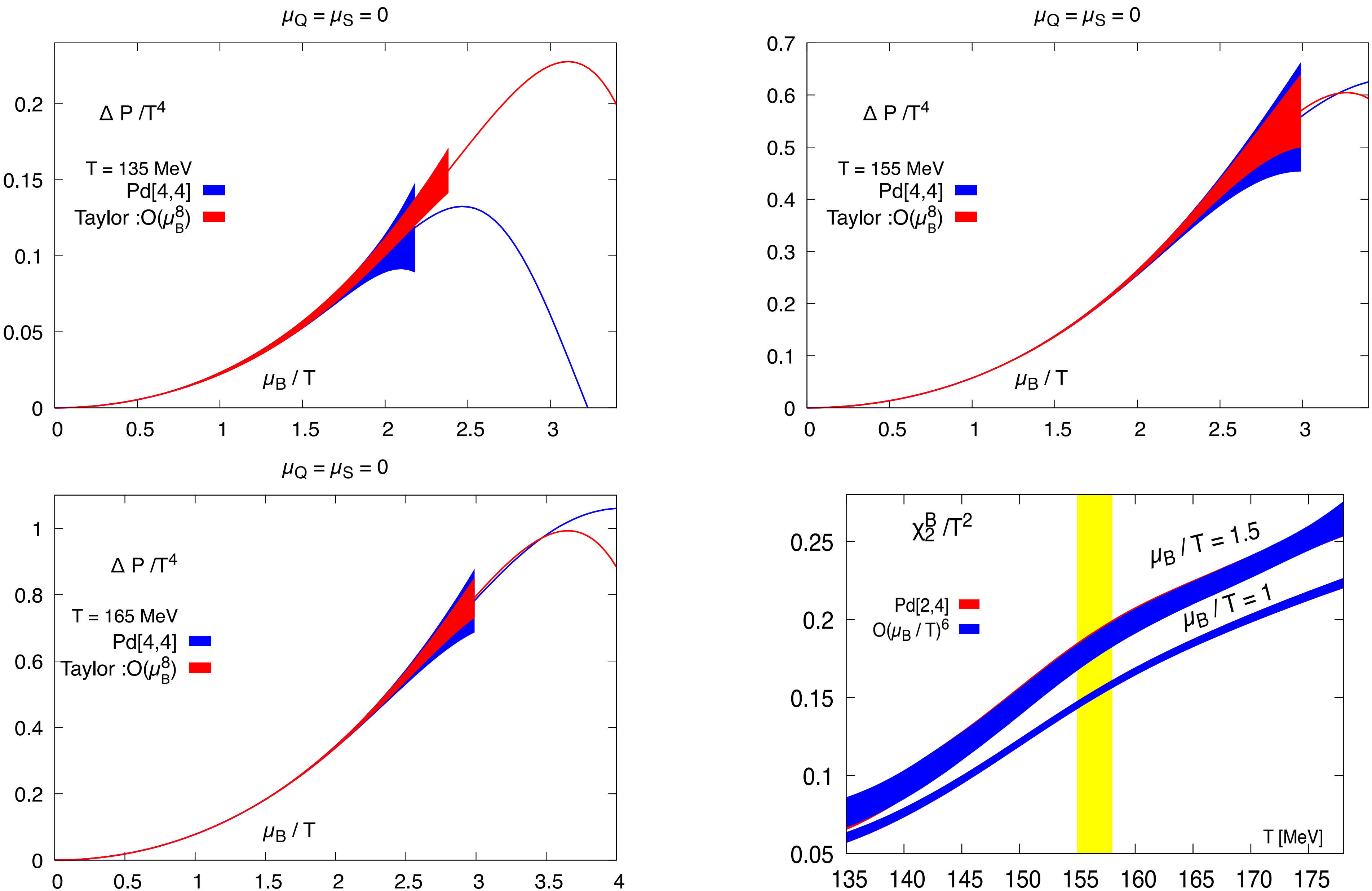
Radius of convergence (Mercer-Roberts estimator):

$$r_{c,n} = \left| \frac{P_{n-2}P_{n+2} - P_n^2}{P_{n+4}P_n - P_{n+2}^2} \right|^{1/4} , \qquad r_{c,4} = \sqrt{\frac{12\tilde{\chi}_2^B}{\tilde{\chi}_4^B}} \left| \frac{1 - c_{6,2}}{c_{6,2}^2 - c_{8,2}} \right|^{1/4} ,$$

Pade approximation

$$P_{[4,4]} = \frac{(1 - c_{6,2})\bar{x}^2 + (1 - 2c_{6,2} + c_{8,2})\,\bar{x}^4}{(1 - c_{6,2}) + (c_{8,2} - c_{6,2})\bar{x}^2 + (c_{6,2}^2 - c_{8,2})\bar{x}^4}$$

Taylor series and Pade approximants



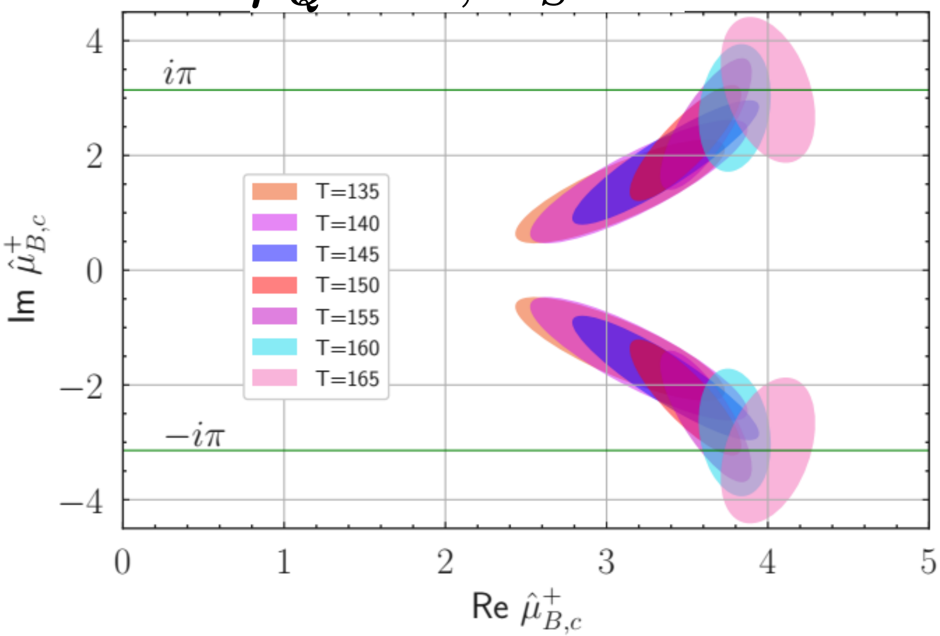
D. Bollweg et al (HotQCD), Phys.Rev.D 105 (2022) 074511

Poles of [n,n] Pade approximants in QCD

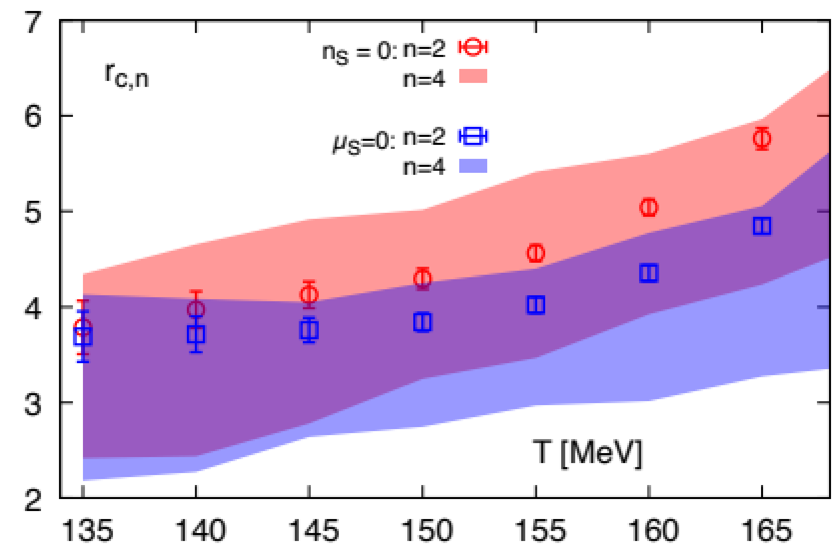
$$\hat{\mu}^{\pm}_{B,c} = \pm r_{c,4} e^{\pm i\Theta_{c,4}}\ ,\ \ r_{c,4} = \sqrt{\frac{12\tilde{\chi}_2^B}{\tilde{\chi}_4^B}} \left| \frac{1 - c_{6,2}}{c_{6,2}^2 - c_{8,2}} \right|^{1/4}\ ,\ \ c_{2k,2} = \frac{2\tilde{\chi}_2^B}{(2k)!\tilde{\chi}_{2k}^B} \left(\frac{12\tilde{\chi}_2^B}{\tilde{\chi}_4^B} \right)^{k-1}$$

complex poles move to real axis as temperature decreases

$$\mu_Q = 0\ ,\ n_S = 0$$



distance of complex pole closest to the origin is identical to the **Mercer-Roberts estimator** for the radius of convergence



HotQCD, arXiv:2202.09184

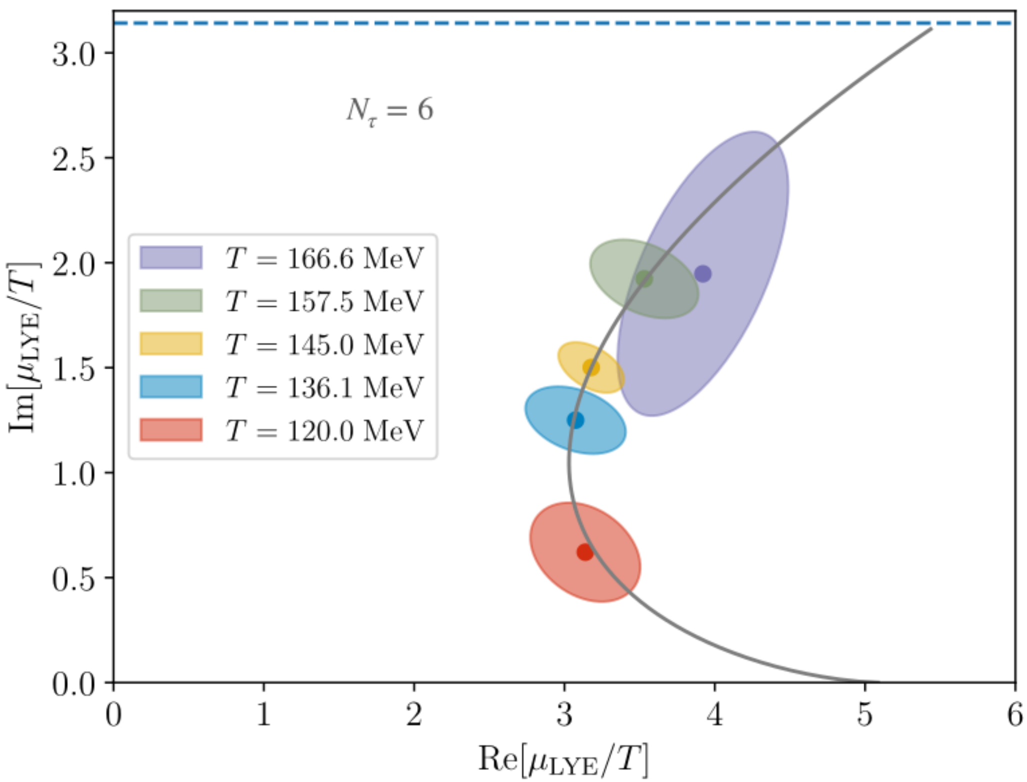
within current errors poles on the real axis (critical point) are possible only for $T \leq 135\mathrm{MeV}\ ,\ \mu_B/T > 2.5$

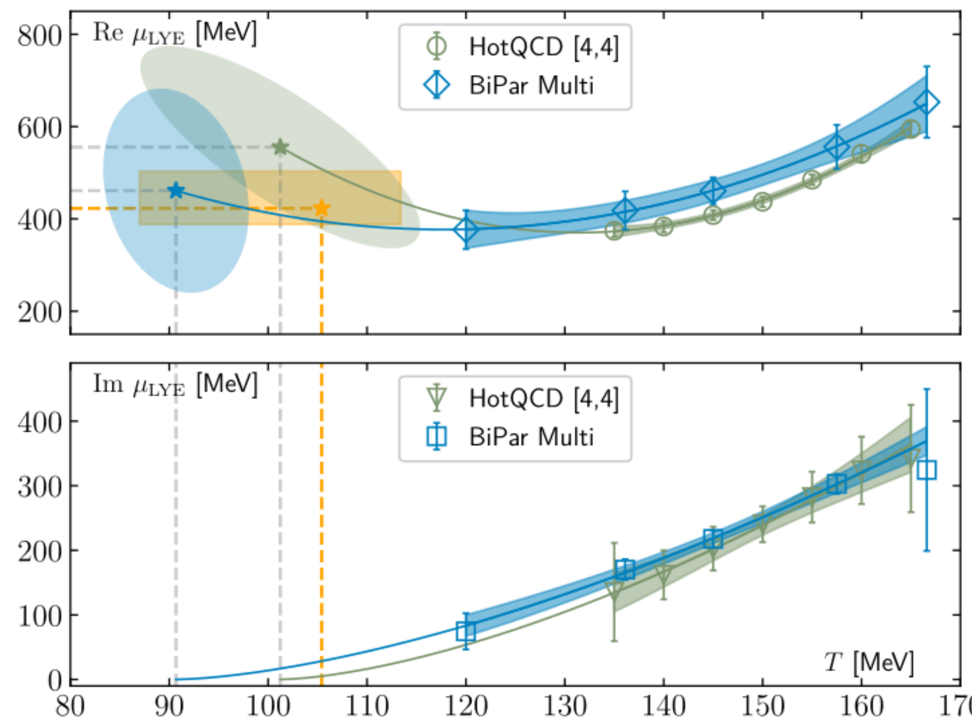
higher statistics will sharpen the constraint

Lee-Yang edge singularities from imaginary chemical potential calculations

slide: C. Schmidt, ECT*, Trento, Italy, Sep 8-12, 2025

- Perform one fit for $N_\tau = 8$ (LT=4) and $\mathcal{O}(10^5)$ fits for $N_\tau = 6$ (LT=6)





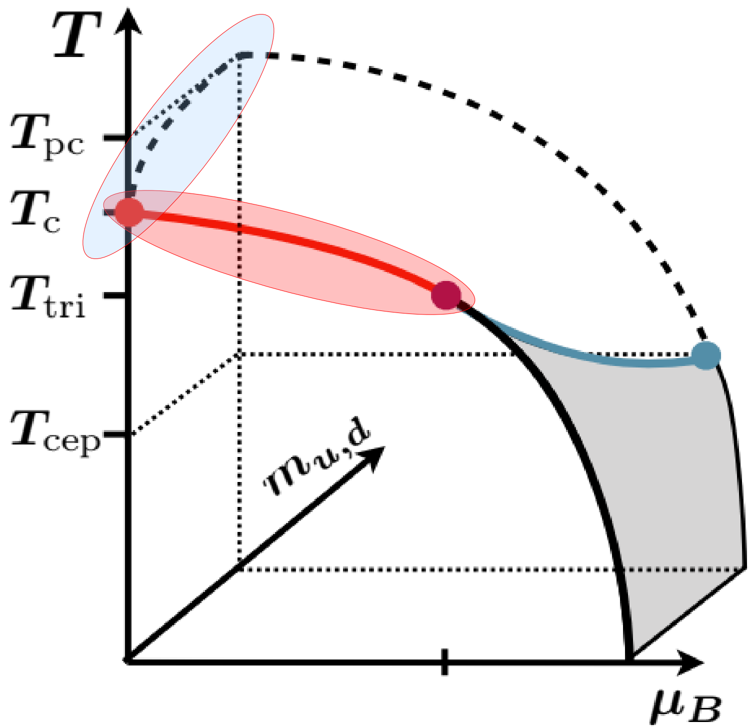
$$\mathrm{Re}[\mu_{\mathrm{LYE}}] = \mu_B^{\mathrm{CEP}} + c_1 \Delta T + c_2 \Delta T^2$$
$$\mathrm{Im}[\mu_{\mathrm{LYE}}] = c_3 \Delta T^{\beta\delta},$$

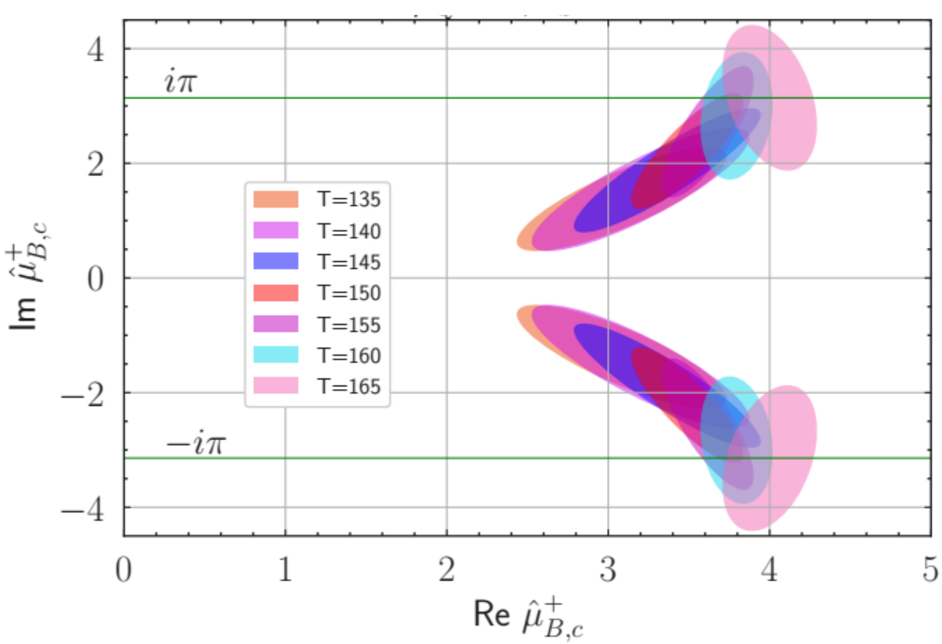
- Ellipses show 1σ confidence region, using the Pearson correlation coefficient
- $N_\tau = 6$ singularities show here are chosen on the basis of the χ^2 of the scaling fit ("best fit")
- Orange box shows the AIC weighted result for $N_\tau = 6$, based on $\mathcal{O}(10^5)$ scaling fits

D.A. Clarke (Bielefeld-Parma), arXiv:2405.10196

$$(T^{cep}, \mu_B^{cep}) = (105^{+8}_{-18}\mathrm{MeV}, 422^{+80}_{-35}\mathrm{MeV})$$

Conclusions - I





What we learned so far about the CEP in QCD from lattice QCD calculations:

I) the critical temperature is below $T_c = 132^{+3}_{-6}$ MeV

II) the corresponding critical chemical potential is likely to be above 500 MeV

→ Taylor expansions need to be resummed in order to reach higher μ_B/T without relying on extrapolations

– radius of convergence obtained from Taylor series agrees well with pole closest to the origin found from Pade approximants

– no CEP for $\mu_B/T \leq 2.5$

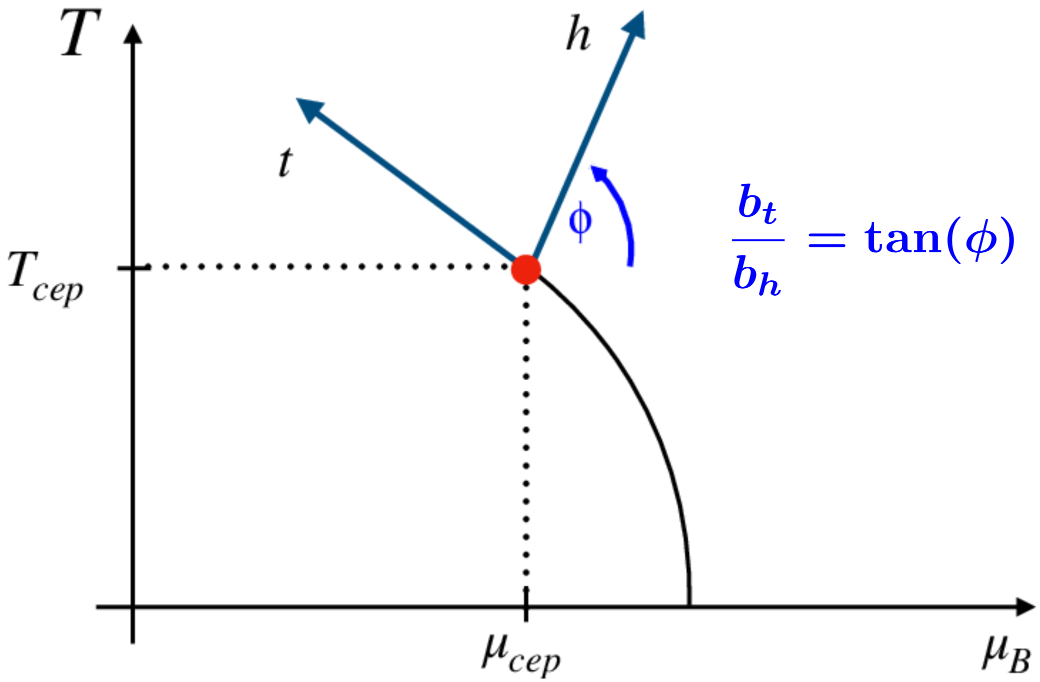
– CEP not in the BES-II range (in collider mode)

– EoS (pressure & number density) well controlled for

$$\mu_B/T \leq 2.0 \;\forall T > 135 \text{ MeV}$$

(larger range for higher T)

Universal critical behavior of QCD in the vicinity of the CEP



$$\frac{b_t}{b_h} = \tan(\phi)$$

$$t \sim \frac{T - T_{cep}}{T_{cep}} + b_t \frac{\mu^2 - \mu_{cep}^2}{\mu_{cep}^2}$$

$$h \sim \frac{T - T_{cep}}{T_{cep}} + b_h \frac{\mu^2 - \mu_{cep}^2}{\mu_{cep}^2}$$

Note: symmetry restored region, z>0, corresponds to t>0

conserved charge fluctuations: $\chi_n^B = - \left. \frac{\partial^n f/T^4}{\partial \hat{\mu}_B^n} \right| \quad , \quad \hat{\mu}_B \equiv \frac{\mu_B}{T}$

singular contribution in Z(2) universality class: $\chi_n^B = - \frac{\partial^n f_{sing}/T^4}{\partial \hat{\mu}_B^n}$

$$f_{sing} = h^{2-\alpha} f_f(z) \quad , \quad z = z_0 \frac{t(T,\mu)}{|h(T,\mu)|^{1/\beta\delta}}$$

Critical behavior of QCD in the vicinity of the CEP

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

$$t \sim (T-T_c)/T_c + b_t(\mu^2-\mu_c^2)/\mu_c^2$$

$$h \sim (T-T_c)/T_c + b_h(\mu^2-\mu_c^2)/\mu_c^2$$

Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic — mixed — thermal

$$\frac{\partial^2 \ln Z}{\partial h^2} \qquad \frac{\partial^2 \ln Z}{\partial h\, \partial t} \qquad \frac{\partial^2 \ln Z}{\partial t^2}$$

$$\sim \left(\frac{m_l}{T_c}\right)^{1/\delta-1} \qquad \sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta} \qquad \sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta}$$

$$\sim -0.79 \qquad \sim -0.43 \qquad \sim -0.07$$

divergence: strong — moderate — very moderate

Z(2) critical exponents

$$\alpha = +0.108$$
$$\beta = \quad 0.326$$
$$\delta = \quad 4.805$$

Critical behavior of QCD in the vicinity of the CEP

– universal critical behavior in cumulants in the vicinity of CEP is quite different from that in the chiral limit

– already second order cumulants diverge, dominant singular behavior of cumulants is "magnetization like" at CEP whereas it is "energy like" in the chiral limit

– HOWEVER: sub-leading corrections to the magnetization-like divergence are energy-like. Their calculation requires knowledge of the universal free energy scaling function (C. Nonaka, M Asakawa, Phys.Rev.C71(2005)044904), e.g.

$$\chi_1(T,\mu) = A b_h \, h^{1/\delta} \left(f_G(z) + \frac{b_t}{b_h} f_f'(z) \; h^{1-1/\beta\delta} \right)$$

although parametrically suppressed, it can give a sizable modification of the scaling function, in particular when

$\phi \simeq \pi/2$ cumulants almost energy-like i.e. divergence in $\chi_2(T,\mu_B)$ will show up only very close to CEP

J. Goswami, FK, S. Mitra, M. Sarkar, Sipaz Sharma, C.Schmidt, work in progress

Critical behavior of QCD in the vicinity of the CEP

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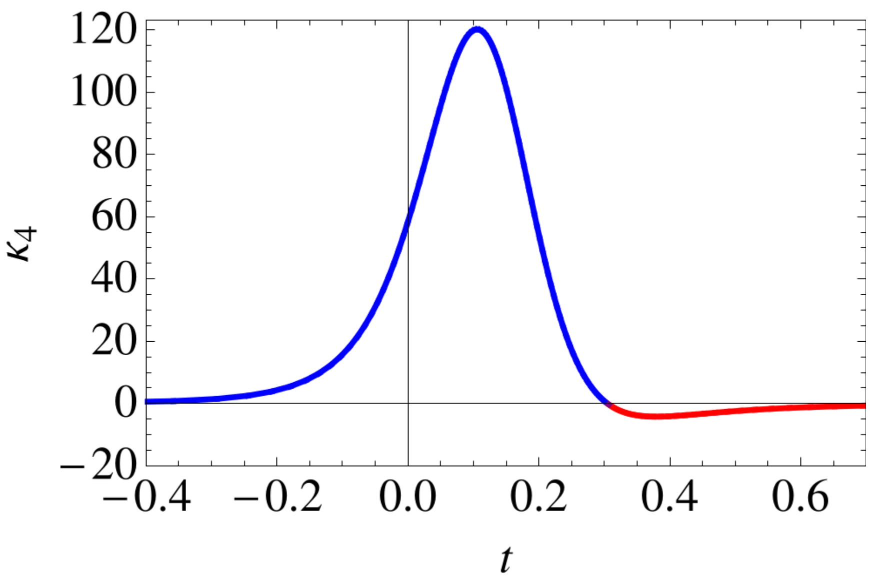
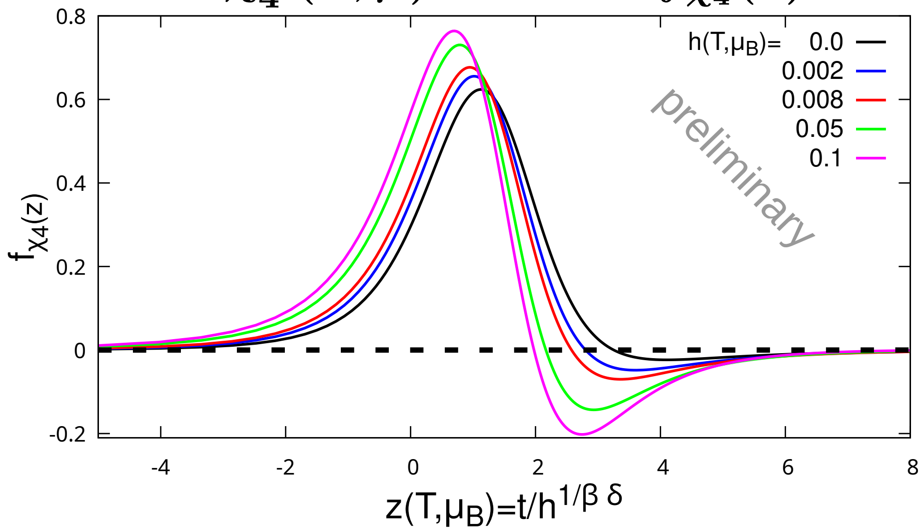


M.A. Stephanov, PRL, 107, (2011) 052301

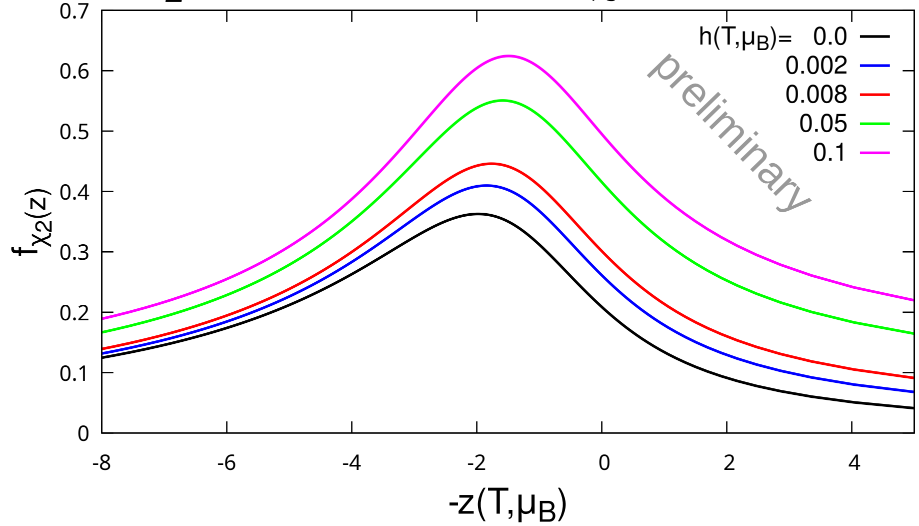
taking into account sub-leading corrections $\chi_4^B(T,\mu) = h^{1/\delta-3} f_{\chi_4}(z) + \text{reg.}$

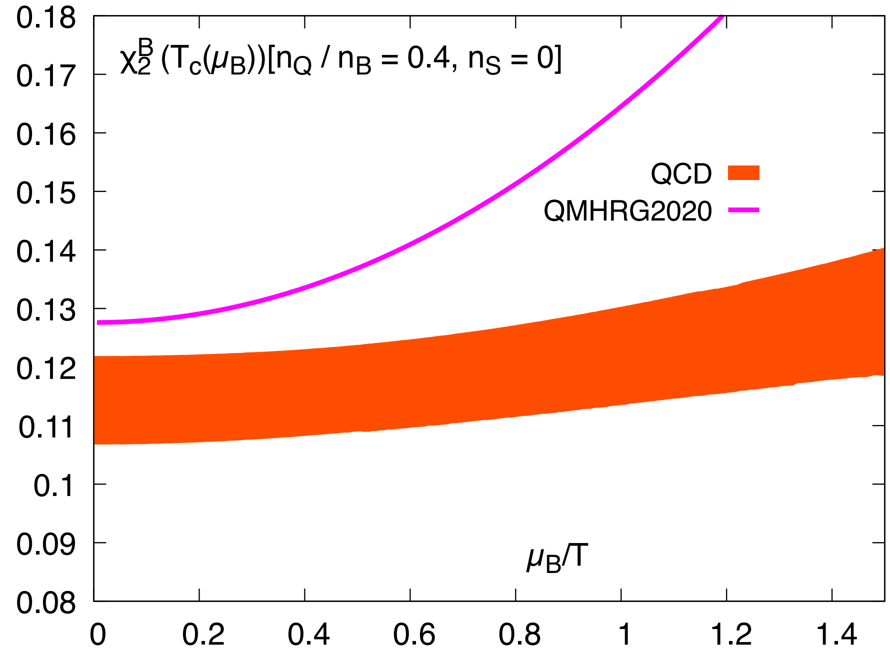


J. Goswami, FK, S. Mitra, M. Sarkar, Sipaz Sharma, C.Schmidt, work in progress

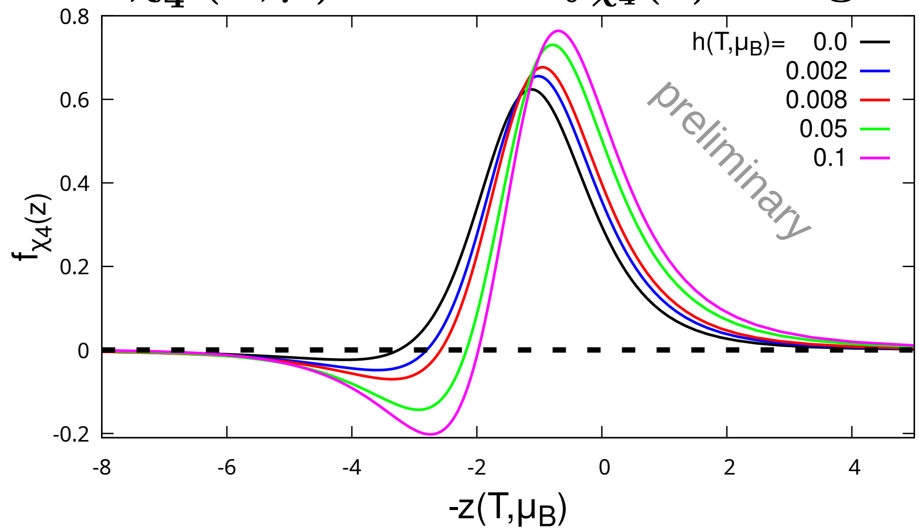
Z(2) scaling functions in the vicinity of the CEP

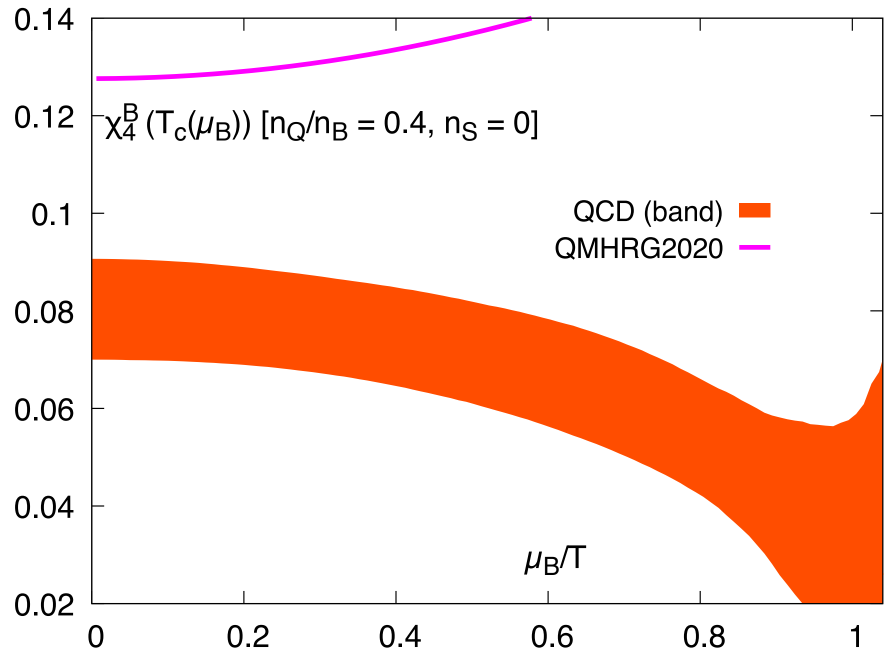
$$\chi_2^B(T,\mu) = h^{1/\delta-1} f_{\chi_2}(z) + \text{reg.}$$





$$\chi_4^B(T,\mu) = h^{1/\delta-3} f_{\chi_4}(z) + \text{reg.}$$

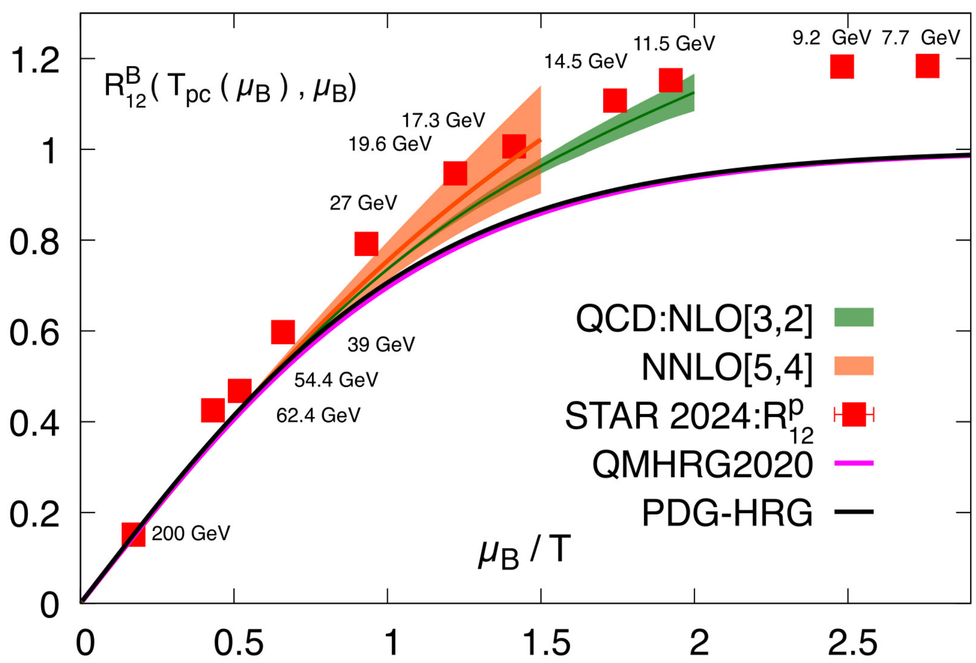




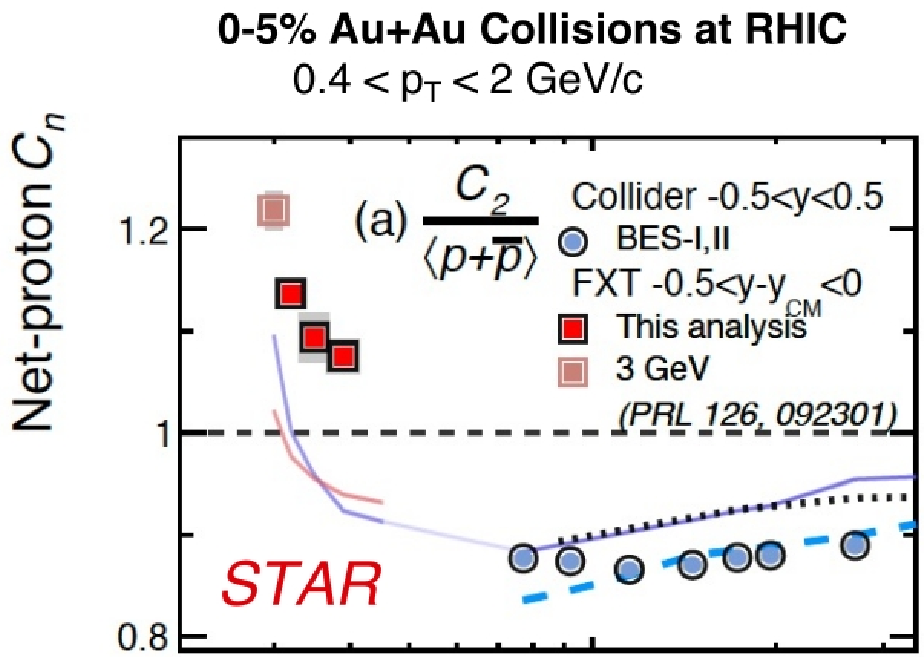
J. Goswami, FK, S. Mitra, M. Sarkar, Sipaz Sharma, C.Schmidt, work in progress

Inverse net-baryon number fluctuations

QCD on the pseudo-critical line vs. STAR data at freeze-out



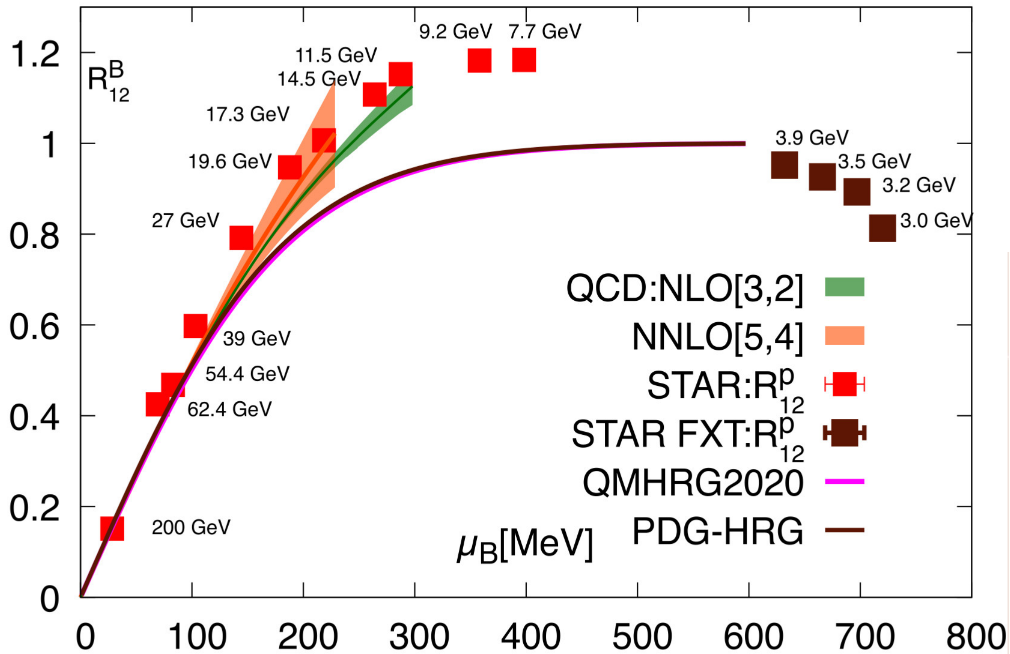
D. Bollweg et al., arXiv:2407.09335
Phys. Rev. D 110 (2024) 5



Z. Sweger (STAR Coll.),
Quark Matter 2025

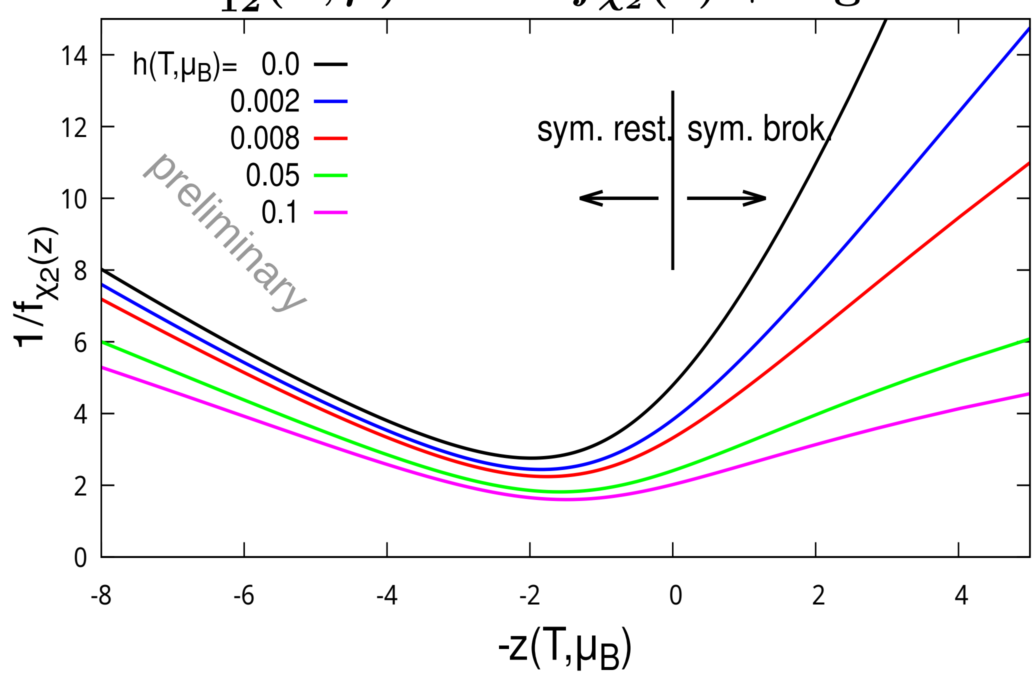
Note: Already at 7.7GeV the total to net proton ratio is close to unity,
STAR, PRL. 135 (2025) 142301

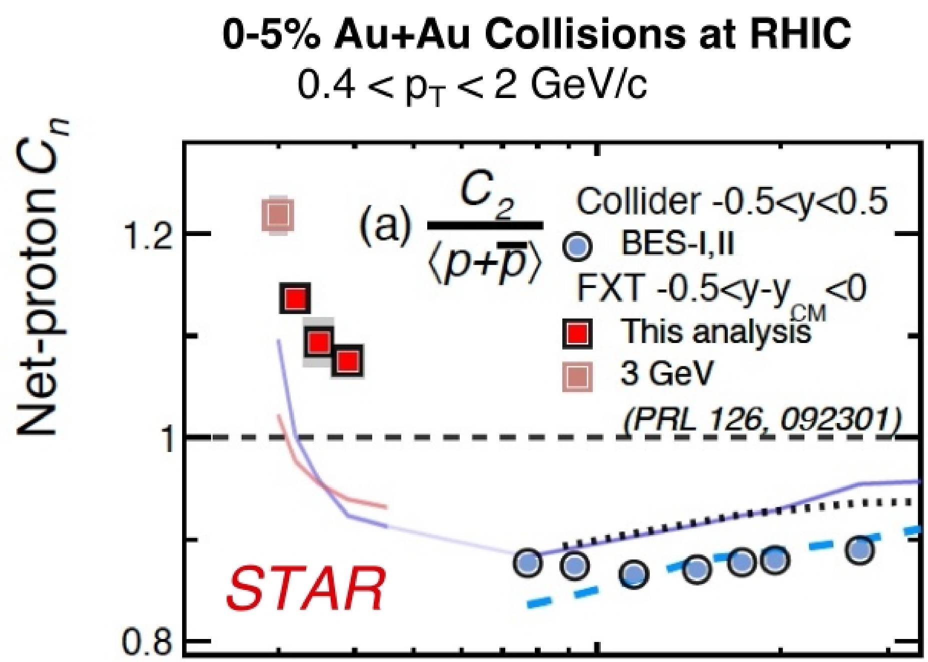
Inverse net-baryon number fluctuation ratio



QCD on the pseudo-critical line vs. STAR data at freeze-out

$$R_{12}^B(T,\mu)\ h^{1-1/\delta} f_{\chi_2}(z) + \text{reg.}$$

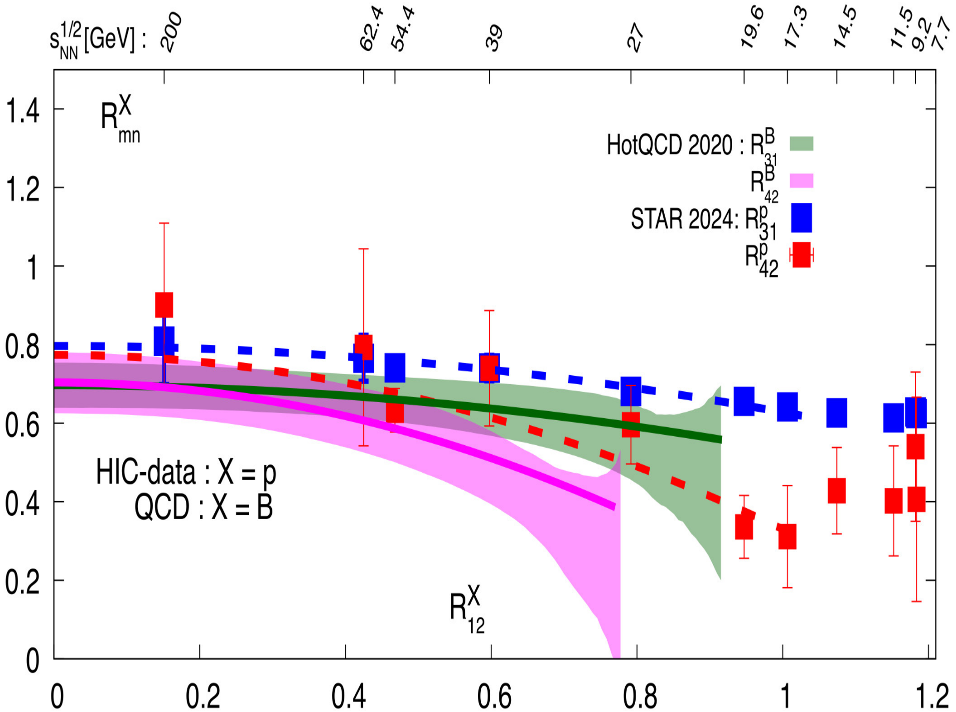


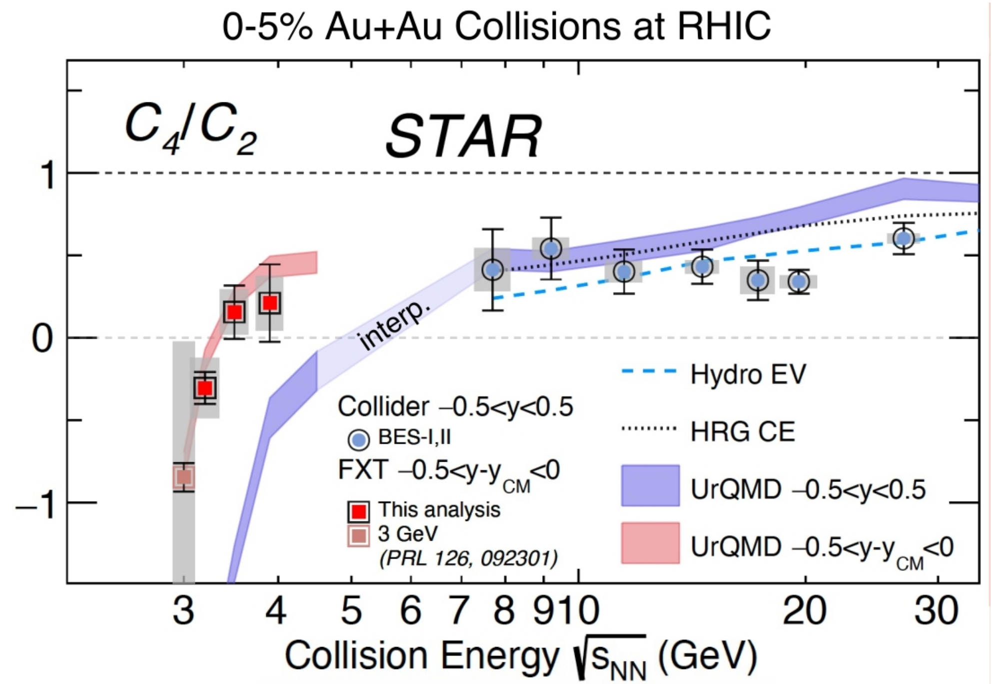


Z. Sweger (STAR Coll.), Quark Matter 2025

Note: Already at 7.7GeV the total to net proton number ratio is close to unity, STAR, PRL. 135 (2025) 142301

Kurtosis ratio: QCD on the pseudo-critical line vs. STAR data at freeze-out





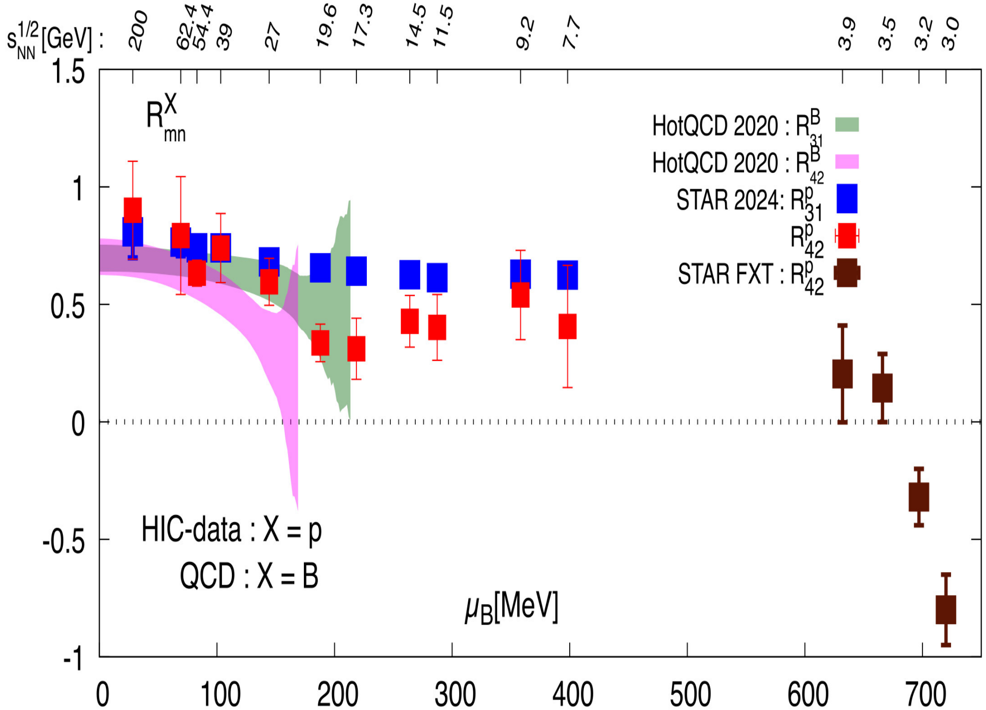
HotQCD 2017, 2020:

$$R^B_{31} = 0.70(x) - 0.169(x)\left(R^B_{12}\right)^2$$

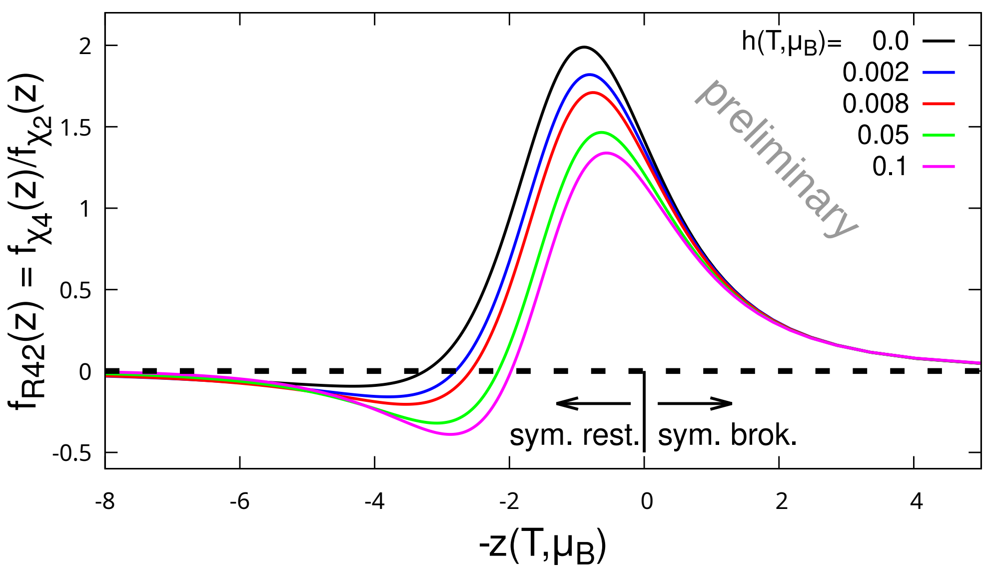
$$R^B_{42} = 0.70(x) - 0.56(x)\left(R^B_{12}\right)^2$$

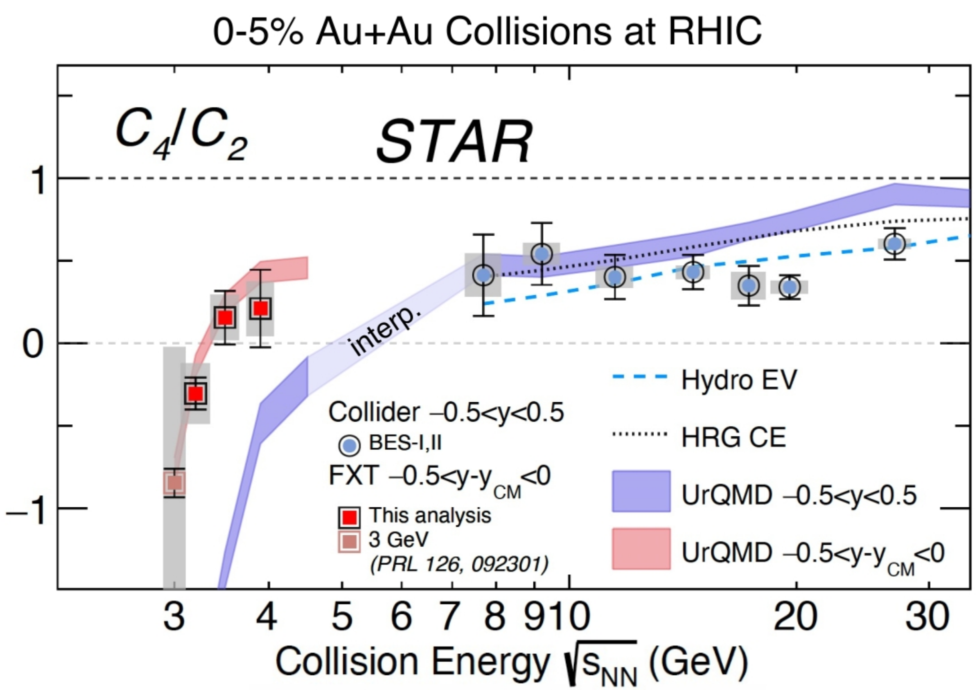
Z. Sweger (STAR Coll.), Quark Matter 2025

Kurtosis ratio: QCD on the pseudo-critical line vs. STAR data at freeze-out



$$R^B_{42}(T,\mu) = h^{-2} f_{R42}(z) + \text{reg.}$$





Z. Sweger (STAR Coll.),
Quark Matter 2025

Higher order net baryon-number cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101 (2020) 074502
PRD 96 (2017) 074510

constrained Taylor series, demanding strangeness neutrality:

$n_S = 0$ and $n_Q/n_B = 0.4$

$$R^B_{nm} = \frac{\chi^B_n(T,\mu_B)}{\chi^B_m(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B}{\sum_{l=1}^{l_{max}} \tilde{\chi}^{B,l}_m(T)\hat{\mu}^l_B}$$

STAR data on proton-number ratio $R^p_{12}(\sqrt{s_{NN}})$ presented at CPOD 2024 by A. Pandav

mean over variance

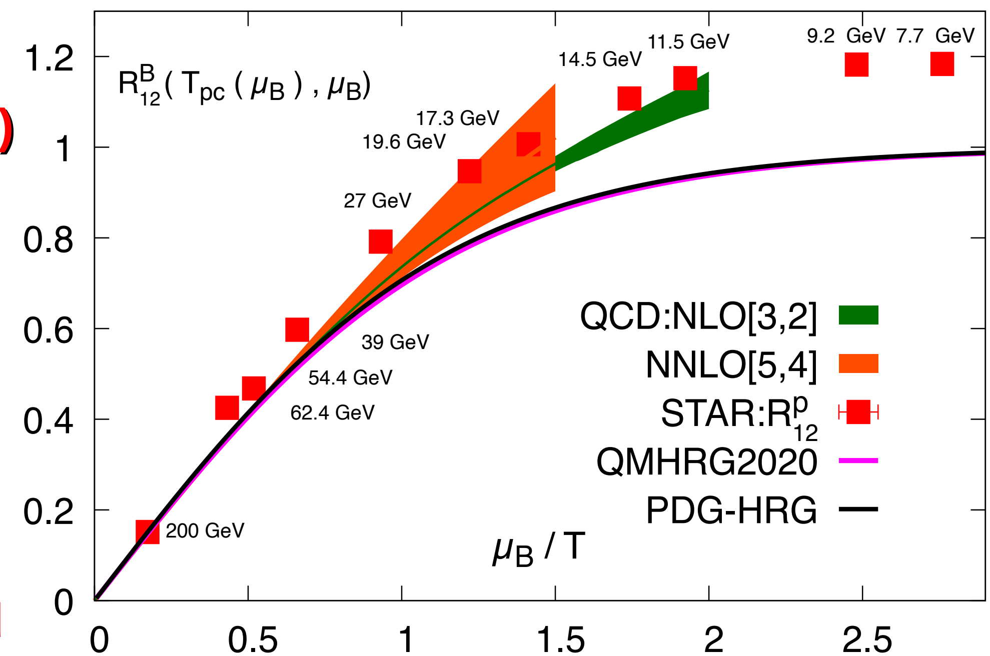
$$R^X_{12} = (M/\sigma^2)_X = \frac{\chi^X_1}{\chi^X_2}$$

X=B (QCD)
P (STAR)

- used STAR results on hadron yields to convert $\sqrt{s_{NN}} \Leftrightarrow \mu_B$

  L. Adamczyk (STAR), PRC 96 (2017) 044904

- **thermal conditions determined from STAR proton cumulants at freeze-out are consistent with those determined in QCD from baryon cumulants on the pseudo-critical line**



D. Bollweg et al., arXiv:2407:09335
Phys. Rev. D 110 (2024) 5