A low-energy EFT for 2+1 flavour QCD

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Why? What?

Chiral symmetry, lattice, EFT

Long-distance physics of QCD dominated by pions: χPT works extremely well for hadron physics. Takes inputs from lattice to fix LECs of χPT. The EFT is used for scattering amplitudes.

Approximate chiral symmetry of QCD is the reason for this success. Explored in the 1960s before discovery of QCD, and revived by Weinberg in the 1980s in the form of EFTs

Chiral symmetry controls the phase structure of QCD. Tested over years. Quantitatively used by HotQCD collaboration to extract T_c from T_{co}.

Use chiral symmetry to construct EFTs which work close to T_c. Aim: give an accurate description of low-energy thermal phenomena from lattice, then use it to help do analytical continuation of lattice results.

Construction of EFTs in matter

Use a UV cutoff Λ: only describe physics at momentum scales $k \ll \Lambda$.

Start with symmetries: in the presence of matter ($T > 0$ or $\mu > 0$) all Lorentz frames are not equivalent. Theory is Lorentz covariant. Easiest treatment: work in the rest frame of matter.

$$\text{Lorentz group} \longrightarrow \text{Rotation group} \times CPT$$

Effect: new LECs appear, thermal physics emerges automatically.

$$\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) \to \frac{1}{2}(\dot{\phi})^2 - \frac{1}{2}c_4(\nabla\phi)^2,$$

New LEC c_4. Effect: temporal propagator has pole at m, static propagator has pole at $m/\sqrt{c_4}$; difference between pole mass and screening mass.

Next choose the fields. Quark fields or meson fields? Cannot use both—double counting degrees of freedom. χPT uses mesons, NJL uses quarks. 1980s: quark EFTs $\to$ meson EFTs. **Witten, Manohar, Gervais+, SG+**

How?

Using an EFT to extend lattice studies

Quark EFT for $T > 0$ with chiral symmetry.

$$\begin{aligned}
D &= 3 \qquad & L_3 &= d_3\Lambda\overline{\psi}\psi \qquad \text{(mass term)} \\
D &= 4 \qquad & L_4 &= \overline{\psi}\partial\!\!\!/_0\psi + d_4\overline{\psi}\nabla\!\!\!\!/\,\psi \qquad \text{(kinetic terms)} \\
D &= 6 \qquad & L_6 &= \frac{1}{\Lambda^2}\sum_{i=1}^{10} d_{6,i}(\overline{\psi}\Gamma_i\psi)\,(\overline{\psi}\Gamma_i\psi) + \frac{1}{\Lambda^2}d_{6,11}\overline{\psi}\nabla^2\nabla\!\!\!\!/\,\psi \\
D &= 8 \qquad & L_8 &= \frac{1}{\Lambda^4}d_{8,0}\overline{\psi}\nabla^4\nabla\!\!\!\!/\,\psi + \frac{1}{\Lambda^4} \times 4 - \text{fermi terms with derivativ} \\
D &= 9 \qquad & L_9 &= {}'\text{t Hooft} - \text{Schaeffer terms}
\end{aligned}$$

where LECs $d_{6,\alpha}$s, $d_{8,\beta}$s, $d_{9,\gamma}$s constrained by chiral symmetry and CPT. Use all allowed terms: not a model. Adding chemical potentials straightforward.

SG+RS doi:10.1103/PhysRevD.97.036025 and SG+RS+PS in prep

The $N_f = 3$ EFT: an emergent symmetry

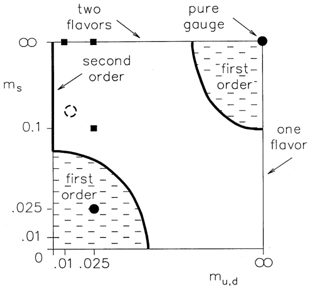
Lagrangian up to $D = 6$ has U(3)×U(3) symmetry: hence nonet of Goldstone bosons after χSB. Same emergent symmetry with $D = 8$ terms. Symmetry is broken by two $D = 9$ terms: suppressed by many powers of UV cutoff Λ.

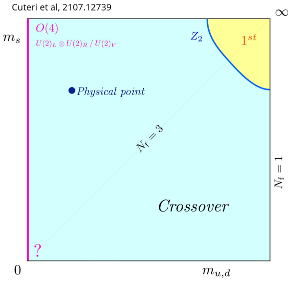
Emergent symmetry: a bug or a feature? Usually not a bug;

- baryon number conservation of standard model
- graphene and the effective Lorentz symmetry of electron gas near its Fermi surface
- emergent gauge symmetries in Hubbard models
- near degeneracies of masses and weak decays of heavy quarks in HQET

In the meson spectrum the $(m_{\eta'} - m_\eta)/(m_{\eta'} + m_\eta) \simeq 0.27$, which is large. No effect of emergent symmetry?

Emergent symmetry and the Columbia plot





Introduce a condensate, do a Hartree-Fock treatment, self consistency via gap equation, then find the effective potential. $D = 6$ and 8 terms give the Columbia plot on the right. No cubic term in the effective potential until $D = 9$. So first order region pushed to $m_\pi \simeq 1$ MeV: too small for lattice computations.

SG+RS CPOD 2024, and in prep

A lower energy theory: the meson Lagrangian

Power counting in meson Lagrangian: (1) count mass dimension if quark masses are large (2) count chiral powers otherwise. Both needed since $m_\ell \ll m_s$.

$$
\begin{array}{rcl}
\text{quadratic} & c_2^a \Lambda^2 \phi_a \phi^a & \text{mass terms} \\
 & \frac{1}{2}\dot{\phi}_a \dot{\phi}^a + \frac{1}{2} c_4^a \nabla\phi_a \nabla\phi^a & \text{kinetic terms} \\
\text{quartic} & c_{41}^{ab} (\phi_a \phi^a)(\phi_b \phi^b) & \text{couplings.}
\end{array}
$$

All terms have $D = 4$ and are LO terms in counting (1). In counting (2) only the kinetic and mass terms are LO, the meson interaction terms are part of several other NLO terms.

Only $c_2^{\eta'}$ has contribution from d_9, so this LEC can be tuned to push $m_{\eta'} > \Lambda(\mathrm{N}_f = 2+1)$ and decoupling it from the pseudo-Goldstone sector. Then ϕ^8 corresponds to the pure η state, and one recovers known spectrum.

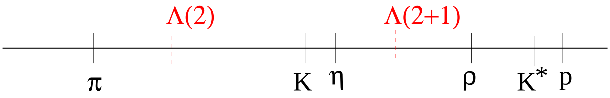
Integrating away the K and η mesons

Integrating the strange mesons at one-loop order gives an effective pion theory for $\Lambda(\mathrm{N}_f = 2) < m_K < \Lambda(\mathrm{N}_f = 2+1)$.

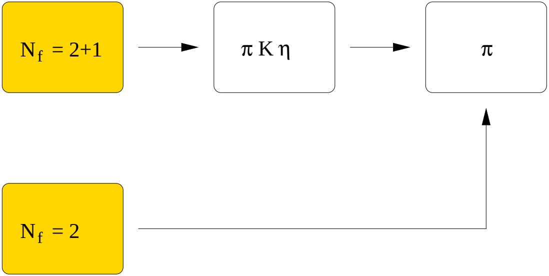
Finally one has

$$L_{pGB} = \frac{1}{2} c_2 \pi^2 + \frac{1}{2} \dot{\pi}^2 + \frac{1}{2} c_4 (\nabla \pi)^2 + c_{41} \pi^4 + \cdots$$

and the effects of the K and η mesons are hidden in these LECs. For $k \ll \Lambda(\mathrm{N}_f = 2)$ this Lagrangian works well.



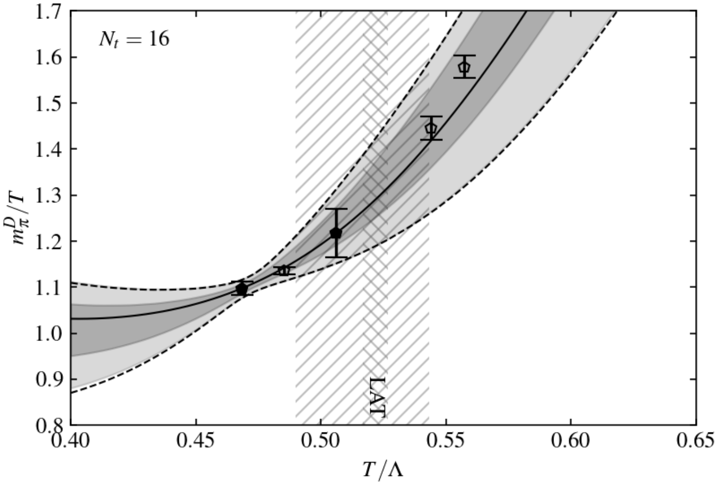
Matching to lattice and making predictions



- LECs of pion theory can be directly matched to lattice measurements of pion Debye mass m_π^D.
- LECs of pion EFT expressed in terms of LECs of quark theory.
- LECs of quark theory can be used to compute phase diagram.
- LECs of pion theory can be used to compute pressure.
- Either EFT Lagrangian can be used in Minkowski.

Hmm..

Input pion properties to fix LECs



SG+RS+PS in prep, using HotQCD doi:10.1103/PhysRevD.100.094510

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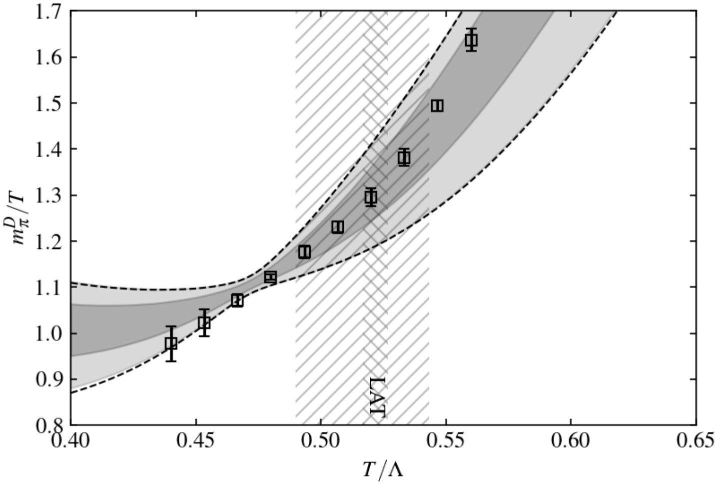

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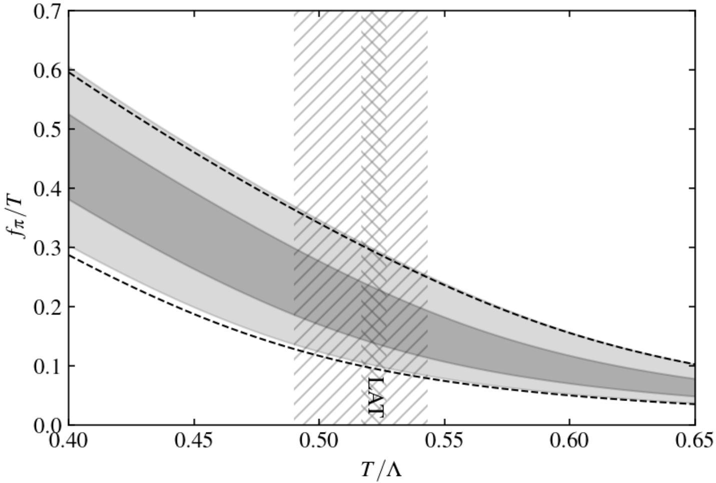
EFT predictions for thermal pions



Looking forward to measurements from HotQCD, others

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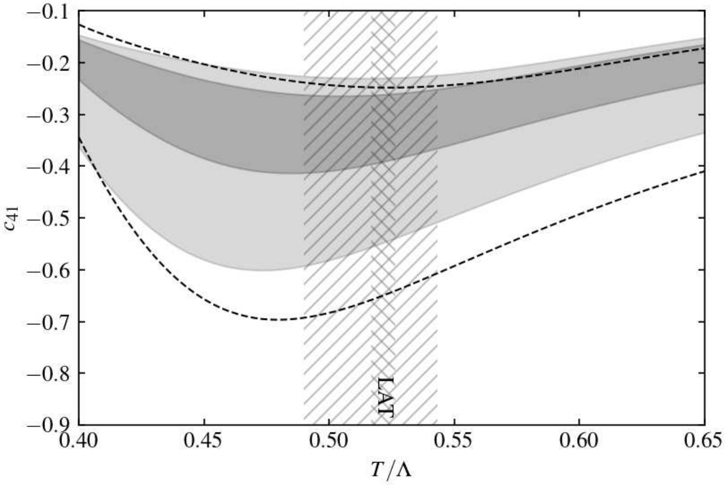

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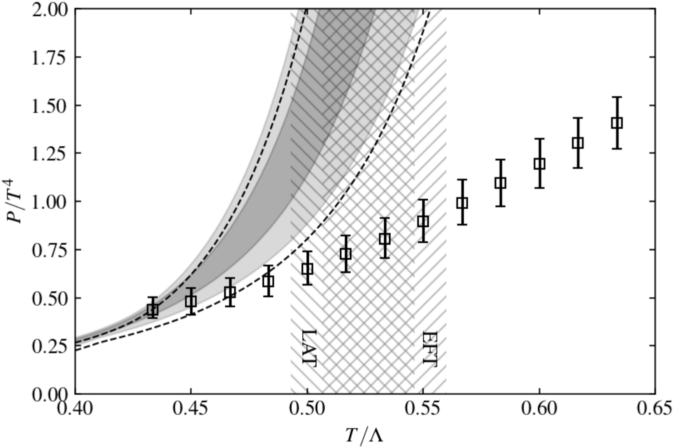
EFT predictions for the phase diagram

	T_{co} (MeV)	κ_2	κ_4
HotQCD 2017	156.5 (1.5)	0.015 (4)	-0.001 (3)
BHJW 2020	158.0 (0.6)	0.0153 (18)	0.00032 (67)
EFT	**157 (7)**	**0.0169 (20)**	**0.00014 (2)**

EFT is used to compute the phase diagram in the HF approximation. Errors in T_c for the EFT are induced from the pion Debye mass. Errors in $\kappa_{2,4}$ for the EFT are estimated from the size of strange quark loops.

Recall that the only input to the EFT was the pions screening mass at two values of T. The predictions for the phase diagram are a stringent test of the EFT.

Pressure



Uses only the LO (kinetic) terms; NLO terms in TχEFT may be needed.

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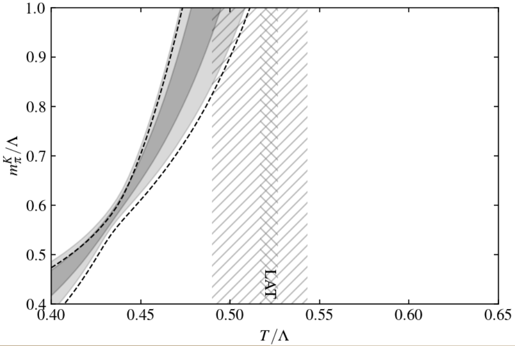
Analytic continuation: kinetic mass

In Minkowski space

$$E = \sqrt{m^2 + c_4 p^2} \simeq m + \frac{p^2}{(2m/c_4)} + \cdots$$

Pole mass (rest mass) is m, but kinetic energy needs a different mass.
Kinetic mass $m^K = m/c_4$. **SG+RS doi:10.1142/S0217751X20300215**



Increasing m_π^K imply that scattering channels which are open at low T close rapidly as T increases.

Increasing m_π^K reason why the pion decouples from the dynamics above T_c

So...

Outlook

1. Material EFTs use the same ideas as $T = 0$ EFTs but implement them differently due to two reasons: (1) change in space-time symmetries due to infinite matter (2) only reliable theory input today comes from lattice.
2. Chiral EFTs are able to take a small number of inputs from lattice computations and organize it into an accurate and complete picture of long-distance thermal physics in agreement with lattice.
3. Gains by narrowing focus: unable to say anything about physics above a pre-determined UV cutoff Λ. Changing Λ changes the LECs but does not materially change the agreement.
4. Suggests an interesting picture of pions at crossover. Pion state continuous across the crossover: pole mass and screening mass are both continuous, and show now signs of disappearing at T_{co}. However, the kinetic mass increases very rapidly, causing pions to decouple from the dynamics above T_{co}.