Nuclear Science Computing Center at CCNU

Search for the QCD critical endpoint at a low temperature of 108 MeV from lattice QCD

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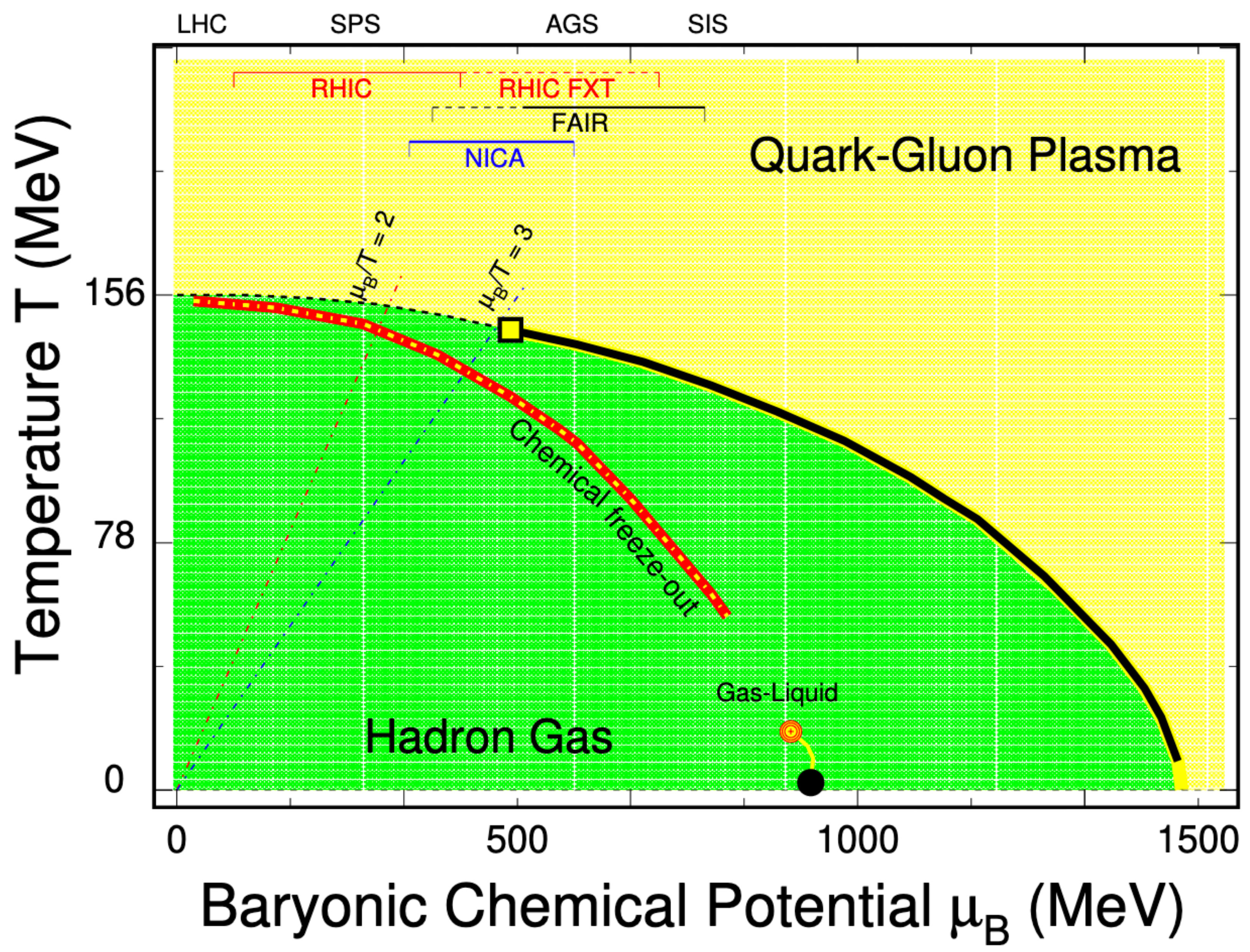
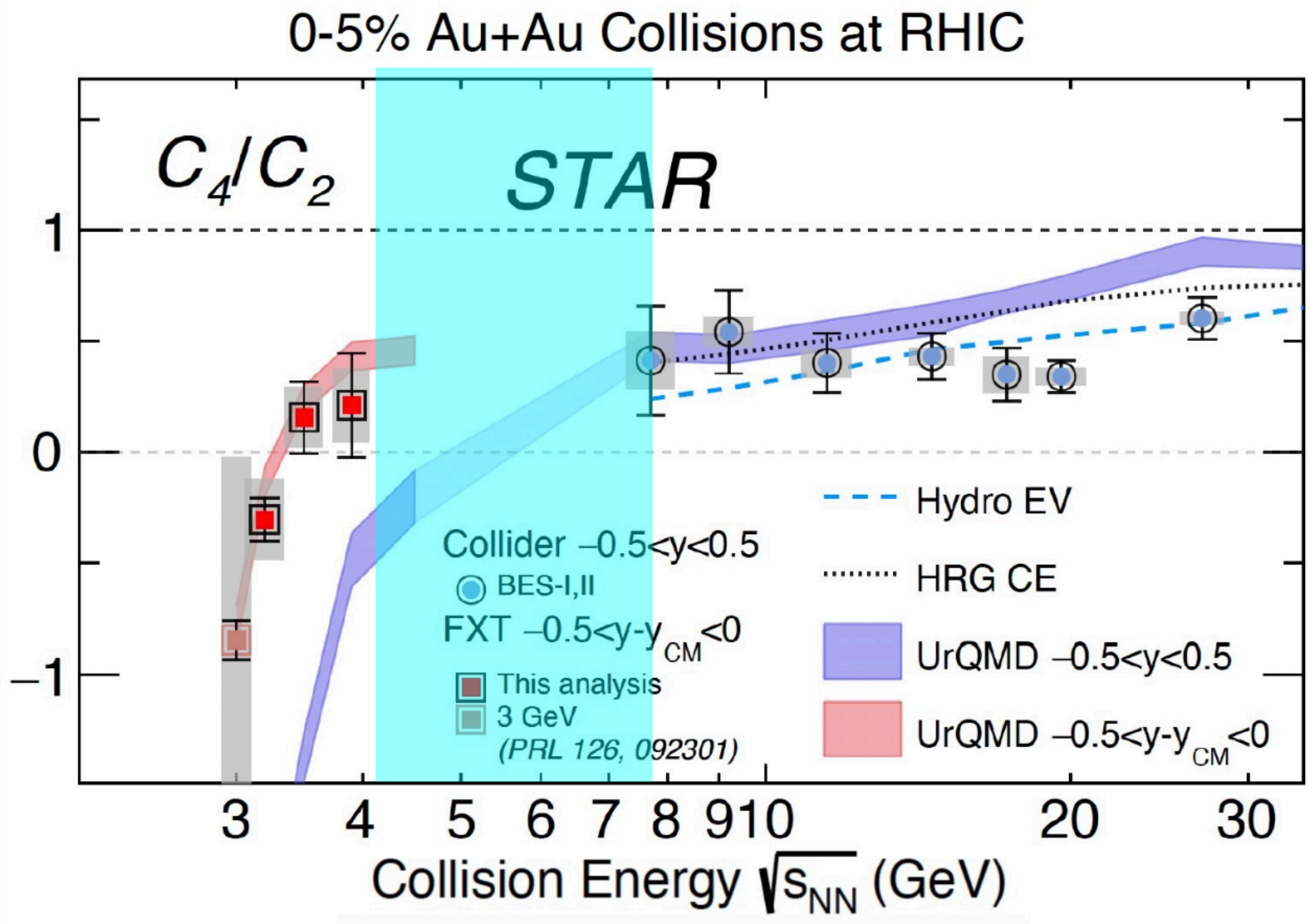
D. Bollweg, D. Clarke, HTD, J.-B. Gu, S.-T. Li, S. Mukherjee, P. Petreczky, C. Schmidt, H.-T. Shu, **Ke-Fan Ye**, work in progress

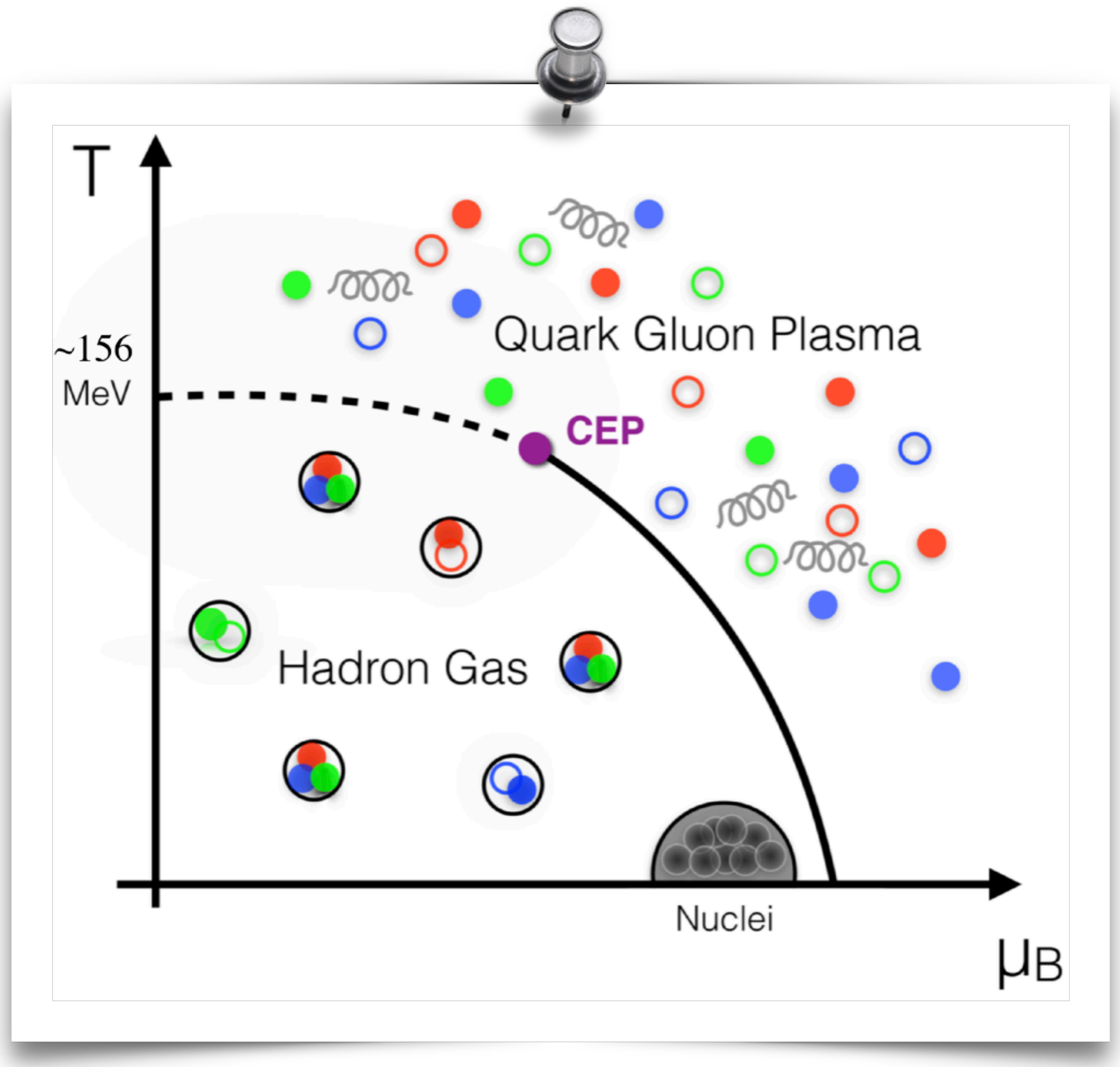
The 2nd international workshop on Physics at High Baryon Density (PHD2025)

Oct. 17, 2025 @ CCNU, Wuhan

Search for criticalities







BES

STAR 7.7-27 GeV results from arXiv: 2504.00817,PRL 25'
New results at 3.2,3.5 and 3.9 GeV from Zachary Sweger(QM2025)

HTD, F. Karsch, S. Mukherjee,
arXiv:1504.05274

Sign Problem at μ_B=/=0

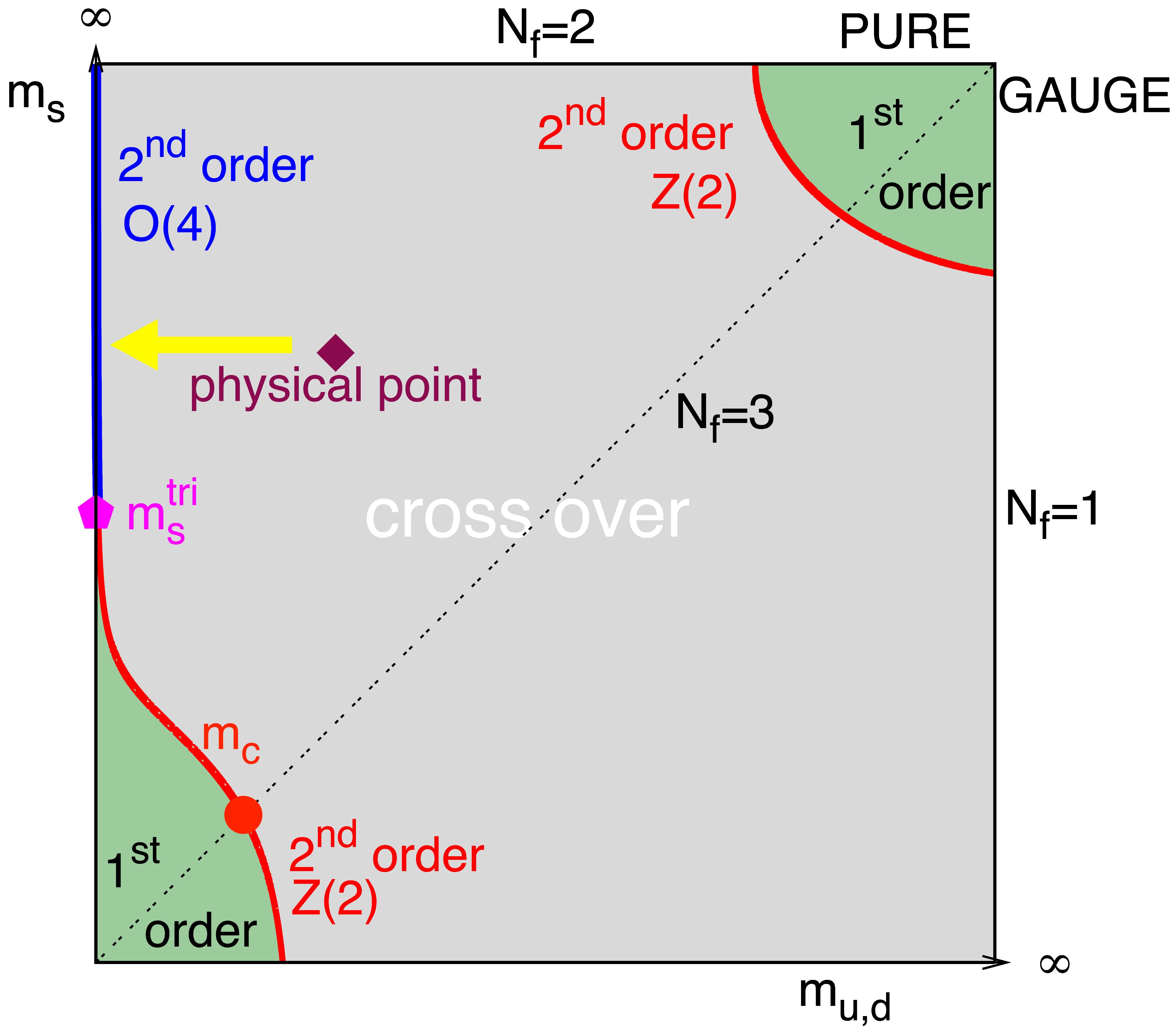
Taylor Expansion

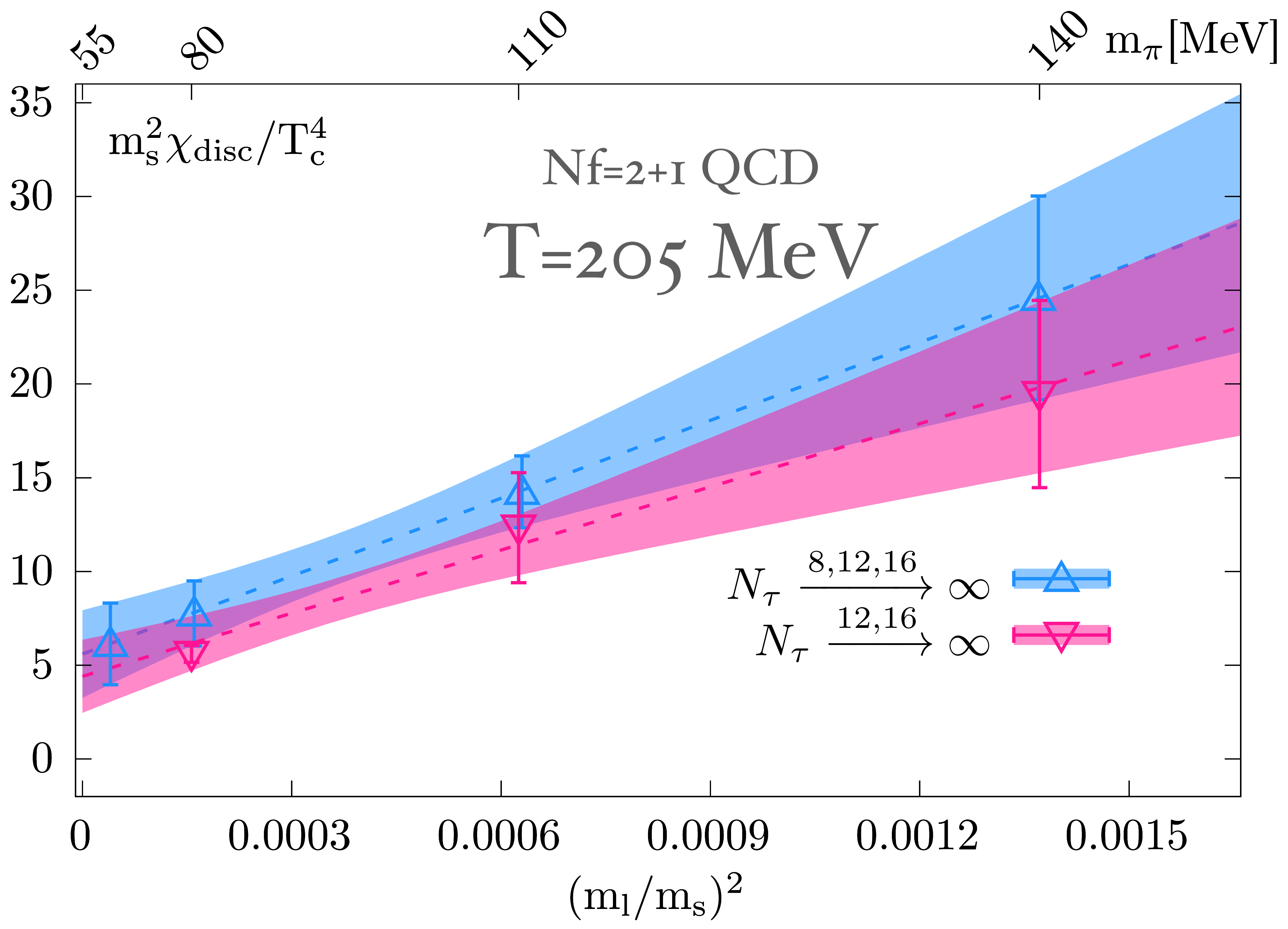
Imaginary μ_B

if the CEP exits, most likely, it maybe found in

$$\sqrt{s_{NN}} \in (3.9,\ 7.7)\ \text{GeV}$$

QCD criticality at μB=0: relevance to CEP





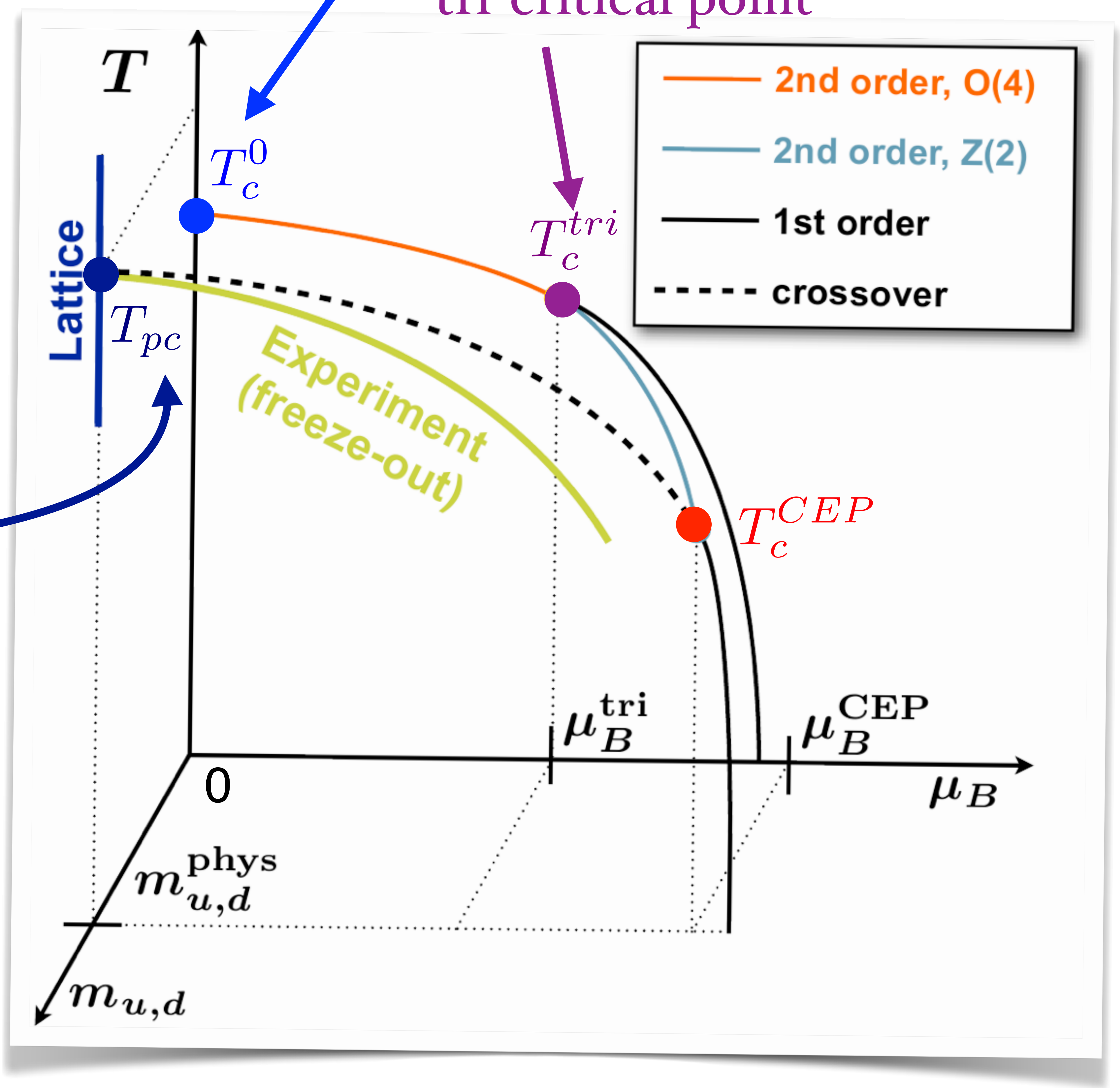
HTD et al., PRL 126 (2021) 082001, and similar findings from Alexander et al., arXiv: 2404.12298
see Nf=2 QCD results from JLQCD arXiv:2501.12675

Relevant: 2nd order chiral phase transition belonging to O(4) universality class
as axial U(1) symmetry is manifested even at T =205 MeV ($\chi_{disc} \neq 0$)

QCD phase diagram in 3D: quark mass, μB, T

Mπ→0, O(4)
Chiral phase transition T at μB=0

tri-critical point



Chiral crossover transition T at μB=0 and Mπ=140 MeV

- $T_c^0(\mu_B) = (1 - \kappa_2^B \hat{\mu}_B^2) T_c^0(0)$ decreases as μB increases rom LQCD

  $N_\tau = 8$ lattice: $\kappa_2^B = 0.015(1)$

  HTD et al., PRD 109 (2024) 114516

  P. Hegde & HTD, PoS LATTICE2015 (2016) 141

  O. Kaczmarek et al., PRD83 (2011) 014504

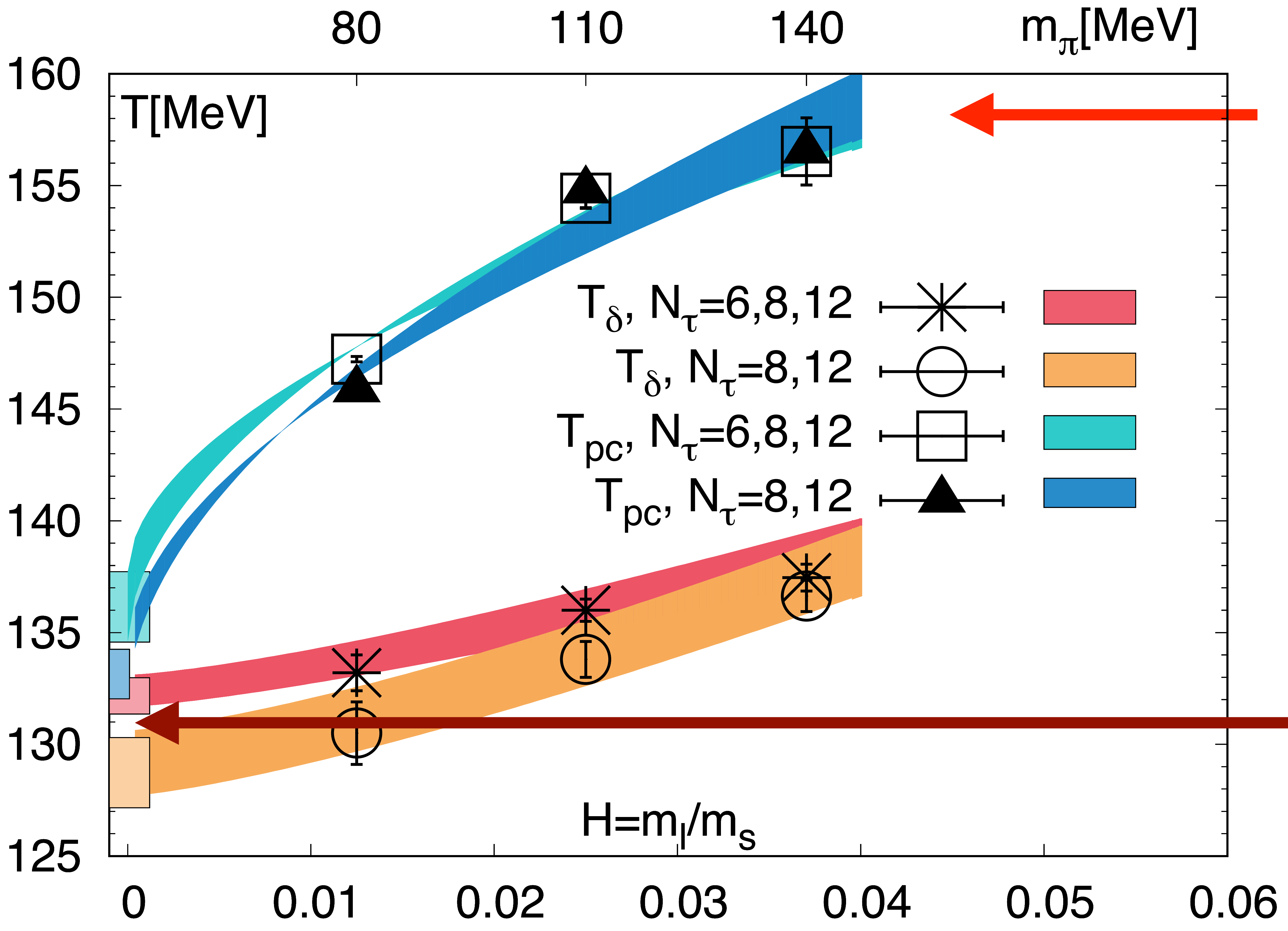
- Tri-critical mean-field:

$$T_c^{tri} - T_c^{CEP}(m_q) \propto m_q^{2/5}$$

M. A. Halasz et al, PRD 58 (1998) 096007
M. Buballa, S. Carignano, PLB791(2019)361
Y. Hatta & T. Ikeda, PRD67 (2003) 014028

Indications:

$$T_{pc}^{phys} > T_c^0 > T_c^{tri} > T_c^{CEP}$$



HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], PRL 123 (2019) 062002
HTD, arXiv:2002.11957

Chiral crossover transition temperature

$$T_{pc}^{\rm phys} = 155 - 158.6 \text{ MeV}$$

HotQCD, PLB 19'
WB, PRL 20'

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

Chiral phase transition T

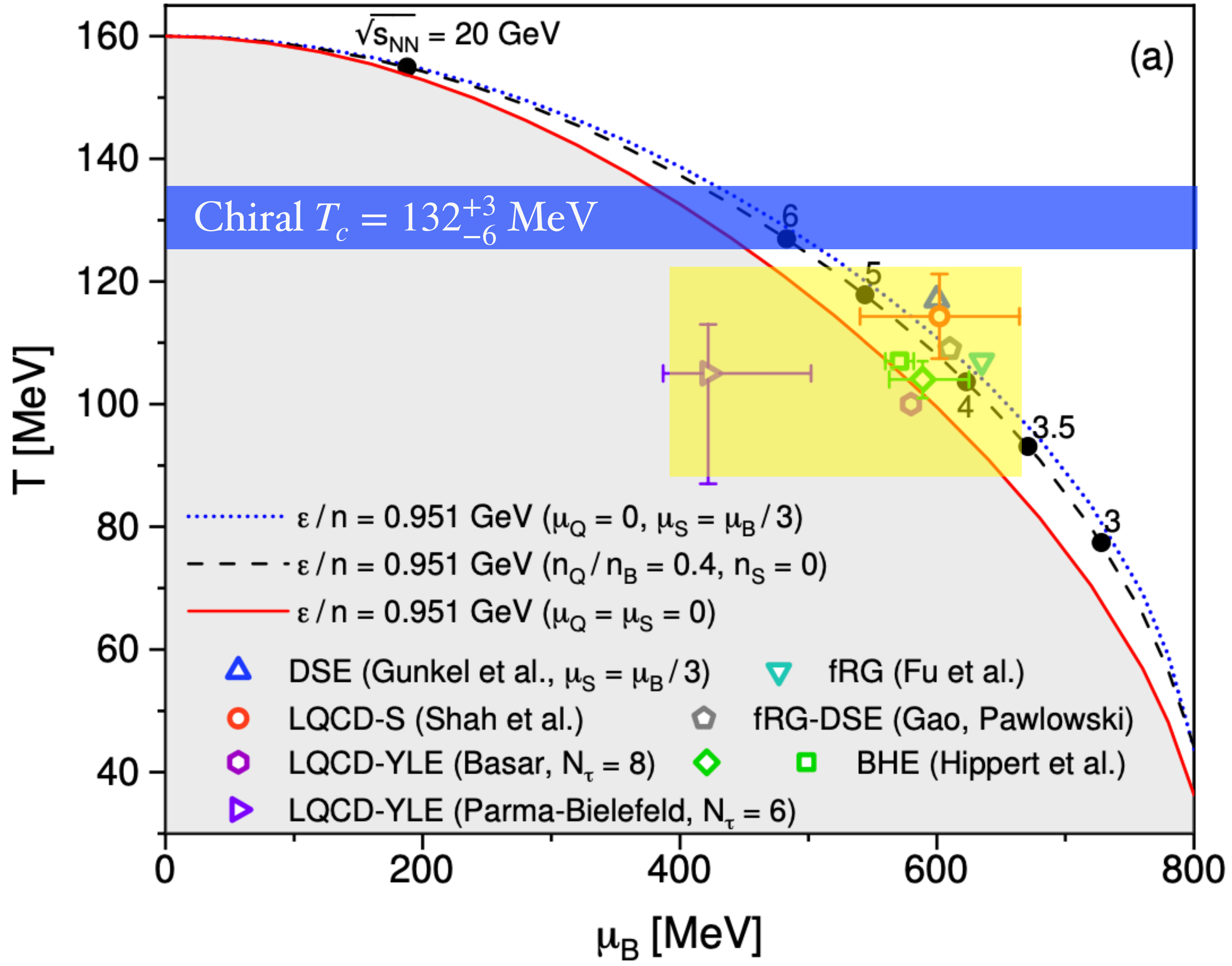
$$T_c^0 = 132^{+3}_{-6} \text{ MeV}$$

See also in QCD-inspired model calculations:
e.g. J. Berges, D. U. Jungnickel and C. Wetterich, Phys. Rev.D59, 034010 (1999)
J. Braun, B. Klein, H.-J. Pirner and A. H. Rezaeian,Phys. Rev. D73, 074010 (2006)
And consistent results using Wilson fermions from Kotov et al , Phys.Lett. B 823(2021) 136749

Indication of $T_{CEP} \lesssim 135$ MeV

See more in Frithjof Karsch's talk yesterday

Recent theoretical predictions/estimates of CEP



Lysenko et al., arXiv:2408.06473, *Phys.Rev.C* 111 (2025) 5, 054903

- Recent predictions/estimates:
  - $T_c^{CEP} \approx 83 - 120$ MeV
    $\mu_{B,c}^{CEP} \approx 387 - 664$ MeV
  - or CEP does not exist
- The region hope to find to CEP in EXP

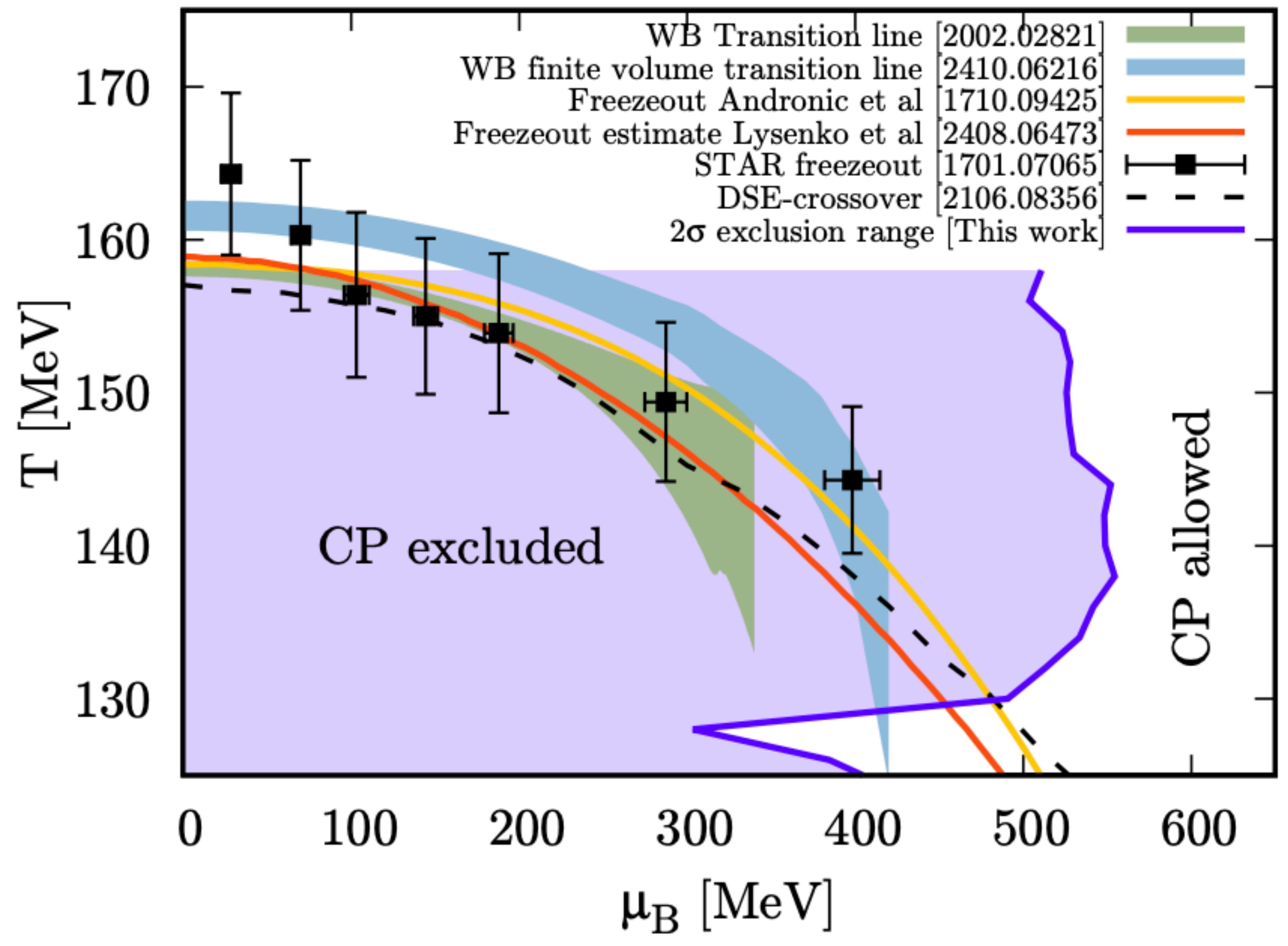
$$\sqrt{s_{NN}} \approx 3.9 - 7 \text{ GeV}$$

$$\mu_{B,c}^{CEP} \approx 420 - 632 \text{ MeV}$$

See Jan Pawlowski's talk on FRG yesterday

Recent results on the CEP search in Lattice QCD

Contour of constant entropy



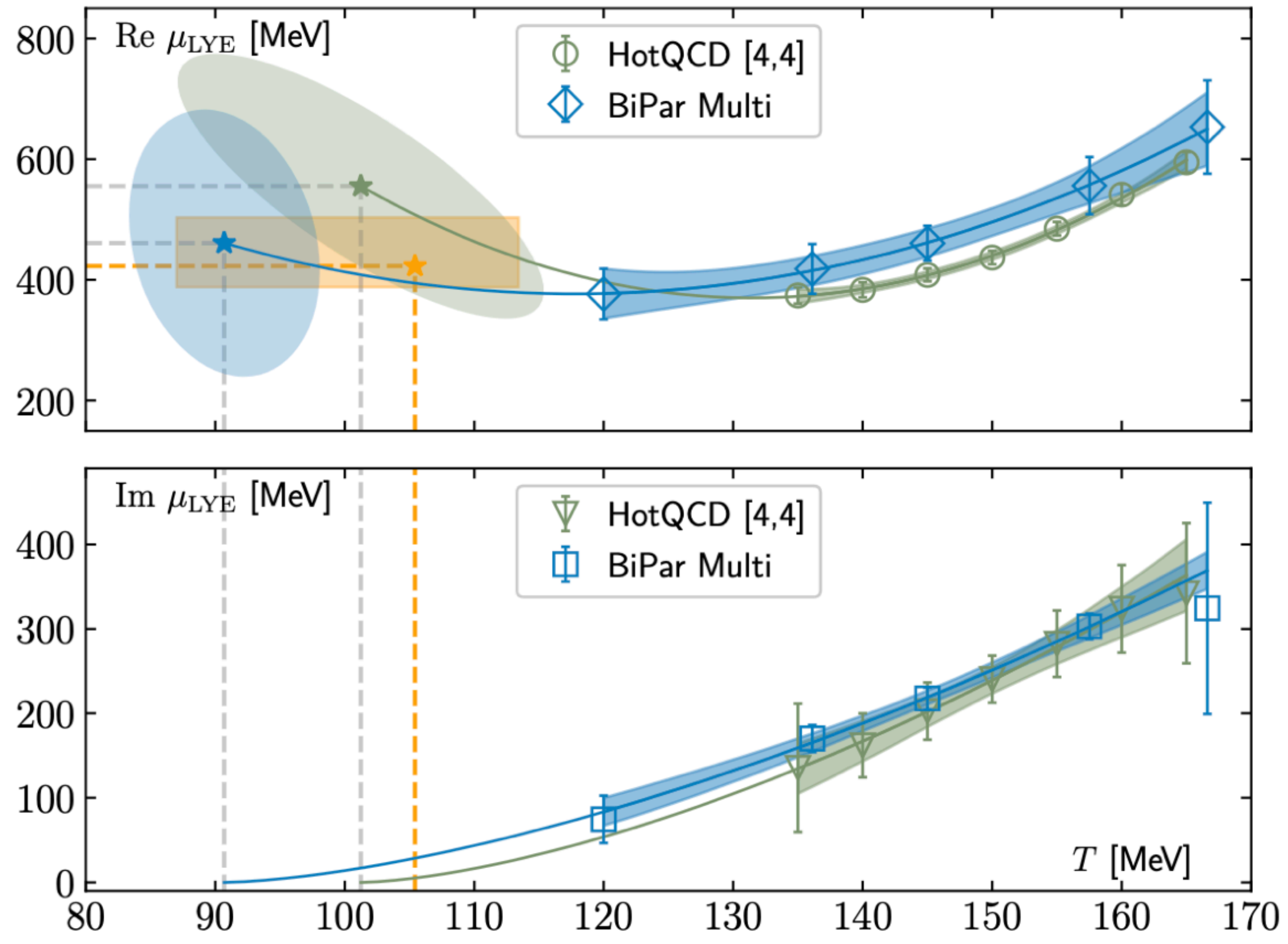
No CEP at $\mu_B < 450$ MeV at the 2σ level

Borsányi et al., arXiv:2502.10267

$$T^{\rm CEP} = 114.3 \pm 6.9 \text{ MeV}$$
$$\mu_B^{\rm CEP} = 602.1 \pm 62.1 \text{ MeV}$$

Shah et al., arXiv:2410.16206

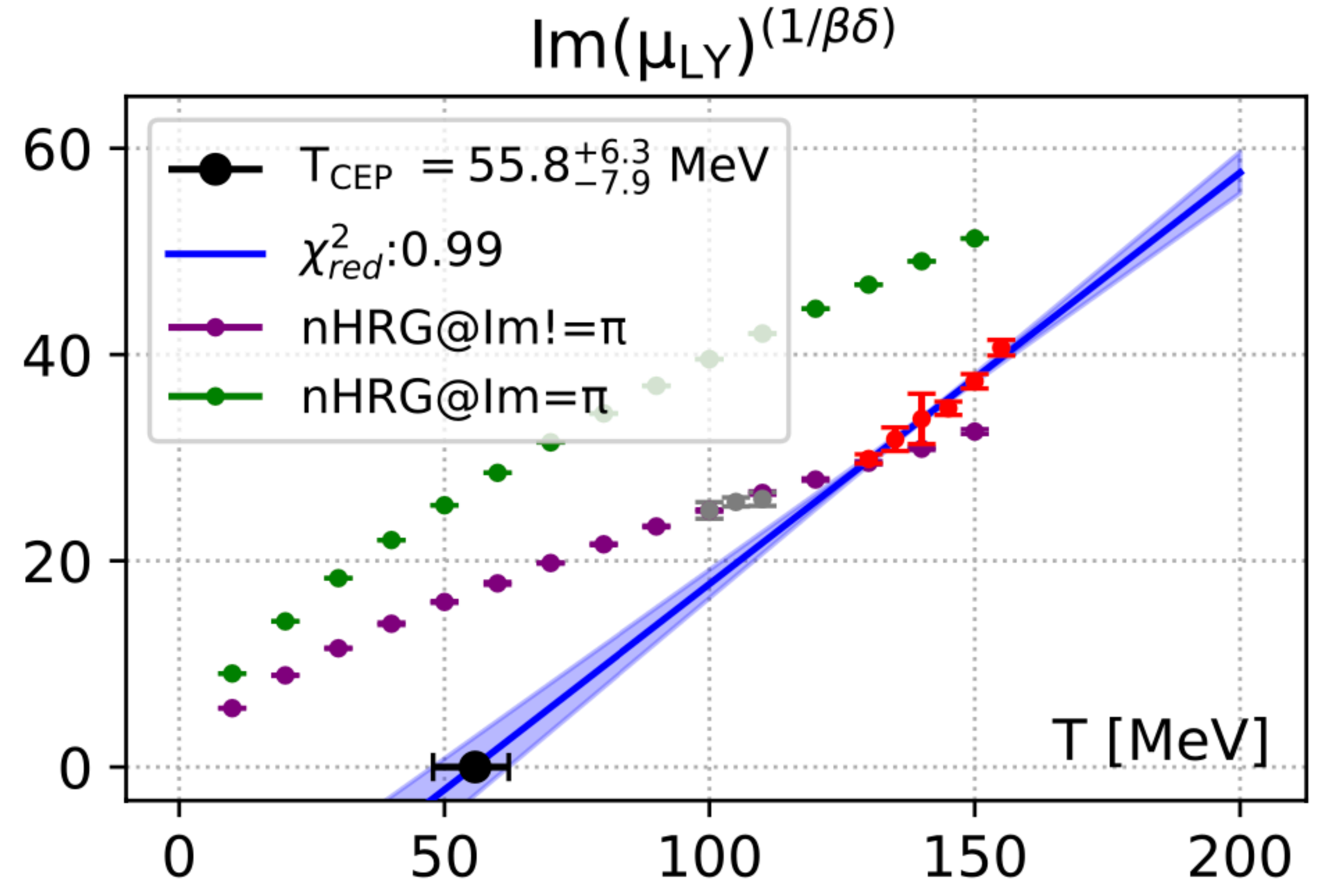
Lee-Yang edge singularity



$$T^{\rm CEP} = 105^{+8}_{-18} \text{ MeV}$$
$$\mu_B^{\rm CEP} = 422^{+80}_{-35} \text{ MeV}$$

Bielefeld-Parma, arXiv:2405.10196

Lee-Yang edge singularity



$T^{\rm CEP} < 103$ MeV
with 84% probability or
CEP does not exit

Adam et al., arXiv:2507.13254

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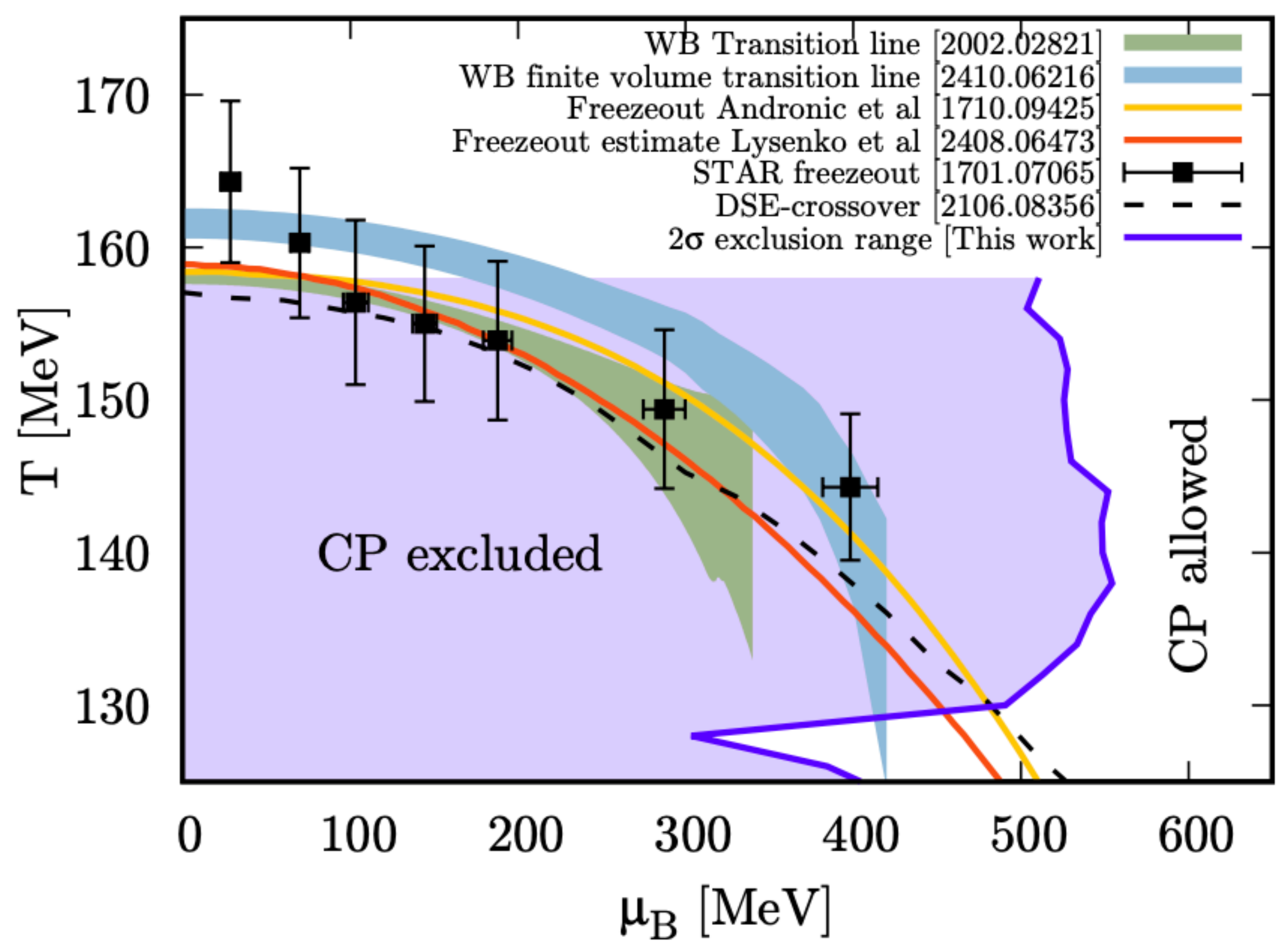

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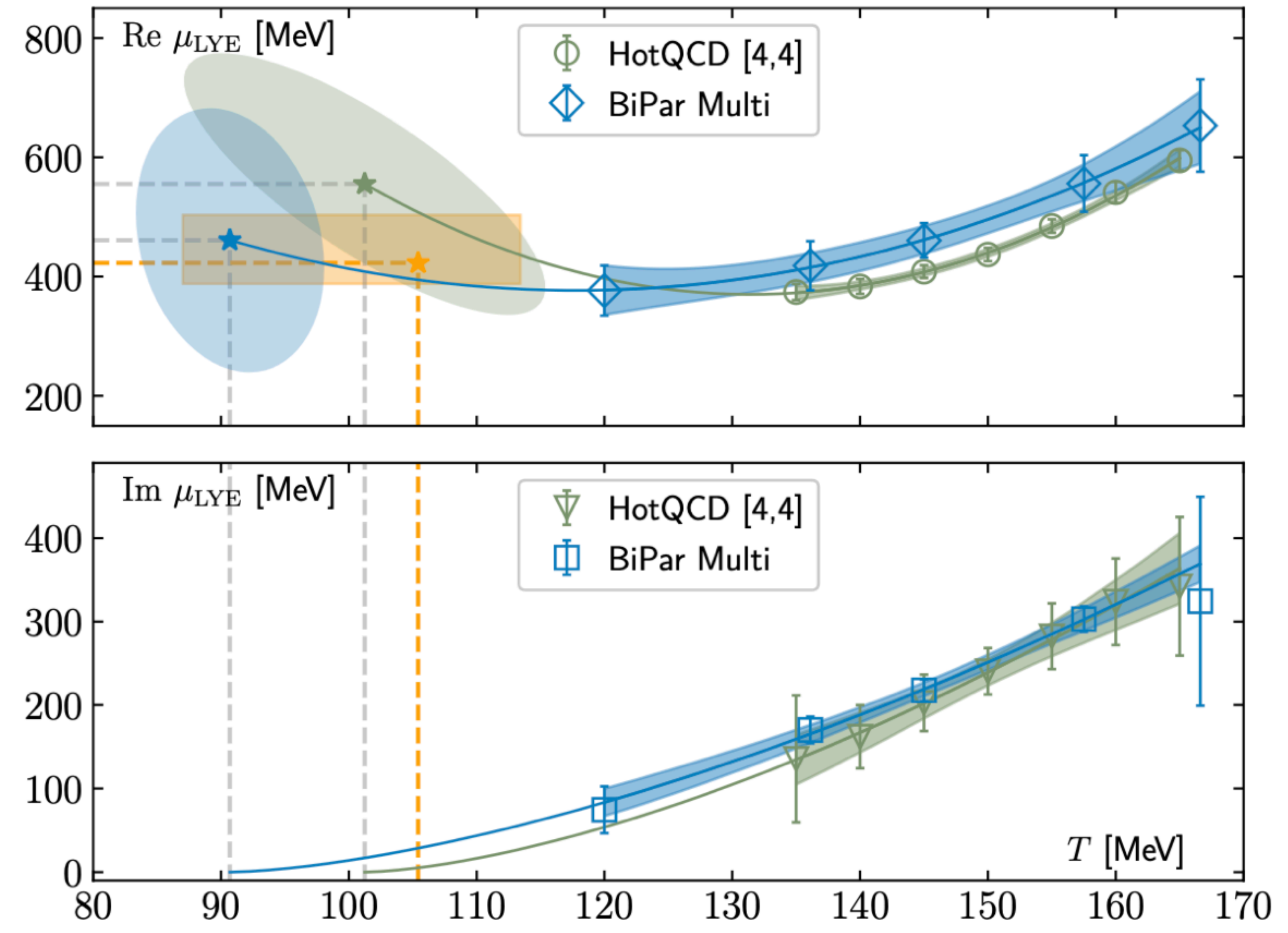

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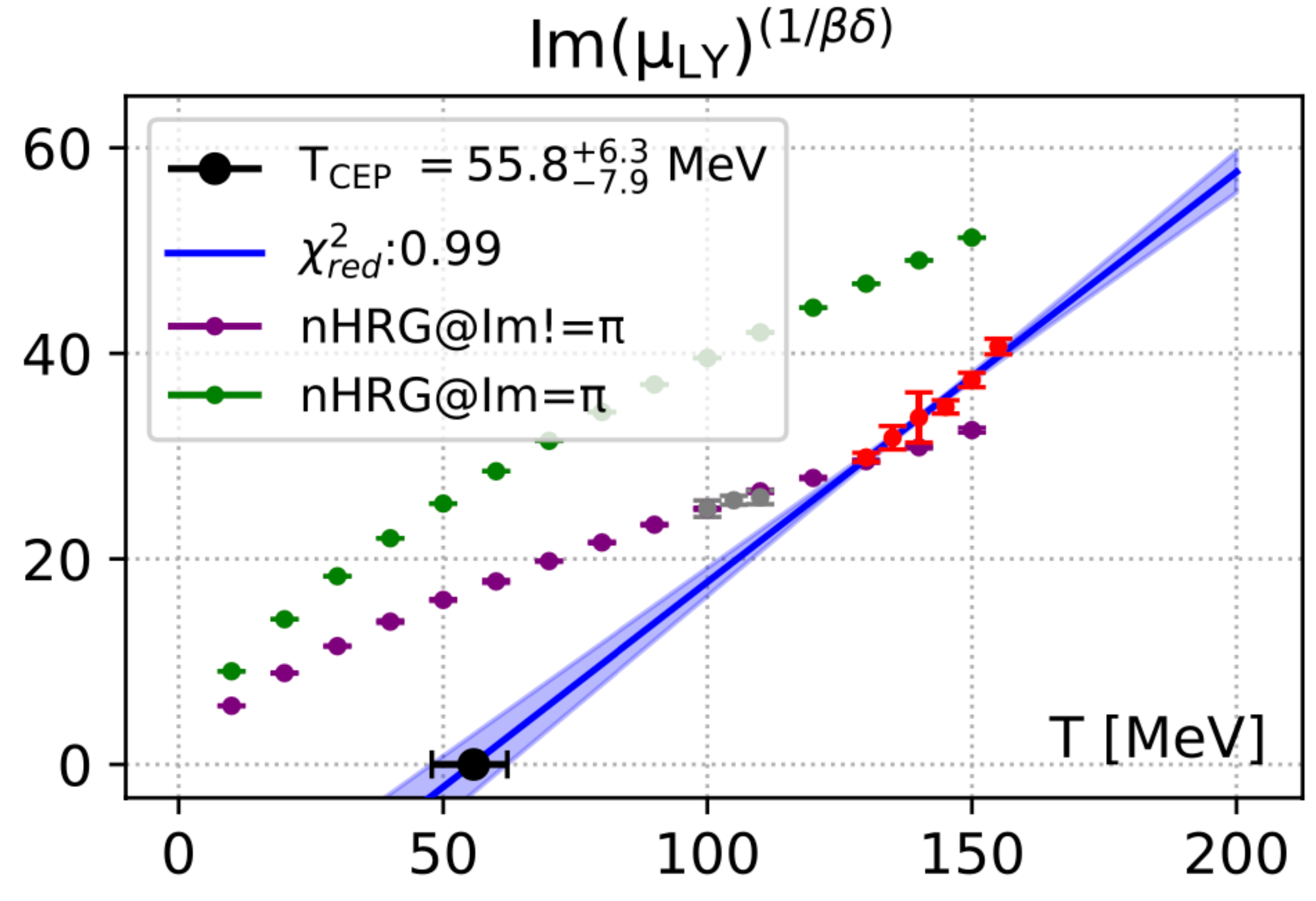


$T^{\rm CEP} < 103$ MeV
with 84% probability or
CEP does not exit

Adam et al., arXiv:2507.13254

All these analyses are done based on extrapolations from lattice QCD results at $T \gtrsim 120 - 130$ MeV

Recent theoretical predictions/estimates of CEP

- ✦ $T^{\mathrm{CEP}} = 114.3 \pm 6.9$ MeV
- ✦ $T^{\mathrm{CEP}} = 105^{+8}_{-18}$ MeV
- ✦ $T^{\mathrm{CEP}} < 103$ MeV w. 84% prob. or No CEP

Current LQCD simulations at $T \geq 120$ MeV

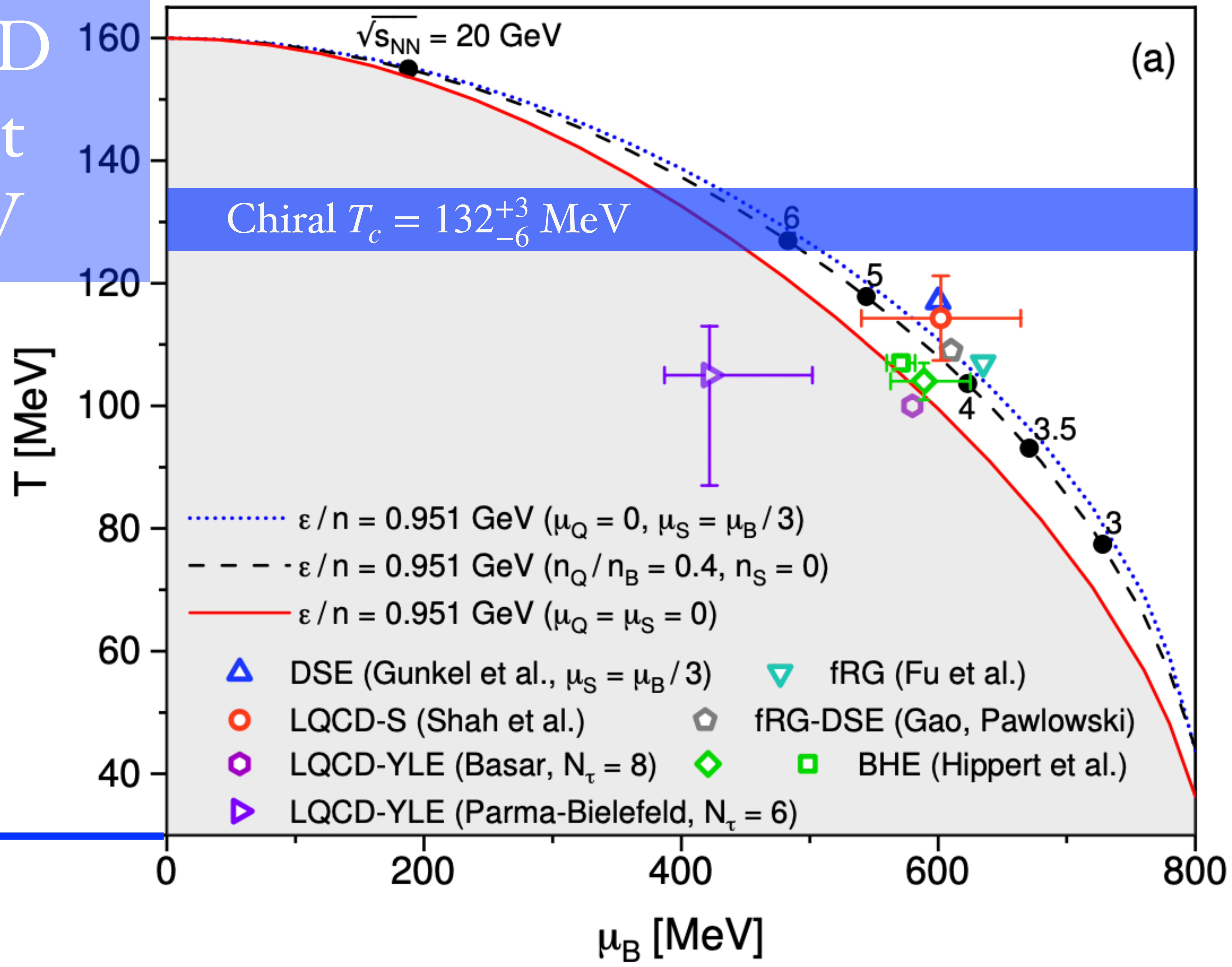
∗ fRG: $T_c^{\mathrm{CEP}} \approx 105 - 115$ MeV

- Recall recent estimates/predictions:

$$T_c^{\mathrm{CEP}} \approx 83 - 120 \text{ MeV}$$
$$\mu_{B,c}^{\mathrm{CEP}} \approx 387 - 664 \text{ MeV}$$

- How about perform a direct LQCD simulations in the currently predicted T_c^{CEP} region? E.g. 108 MeV

$Im(\mu_B)$



Lysenko et al., arXiv:2408.06473, *Phys.Rev.C* 111 (2025) 5, 054903

Lattice setup at T≈108 MeV

- N_f=2+1 QCD on 20^3x10 lattices with at T=107.7 MeV using HISQ fermions at the physical point with $\mu_u = \mu_d = \mu_s = \mu_B/3$
- 24 values of $i\mu_B \in [0, i\pi]$ each for χ^B_{1-4}
- 400k configurations for zero chemical potential, 200k for others
- Additional 32^3x10 lattices generated to check the volume dependence

NuclearScience Computing Center at CCNU + SUMMIT

Multi-point Padé fits

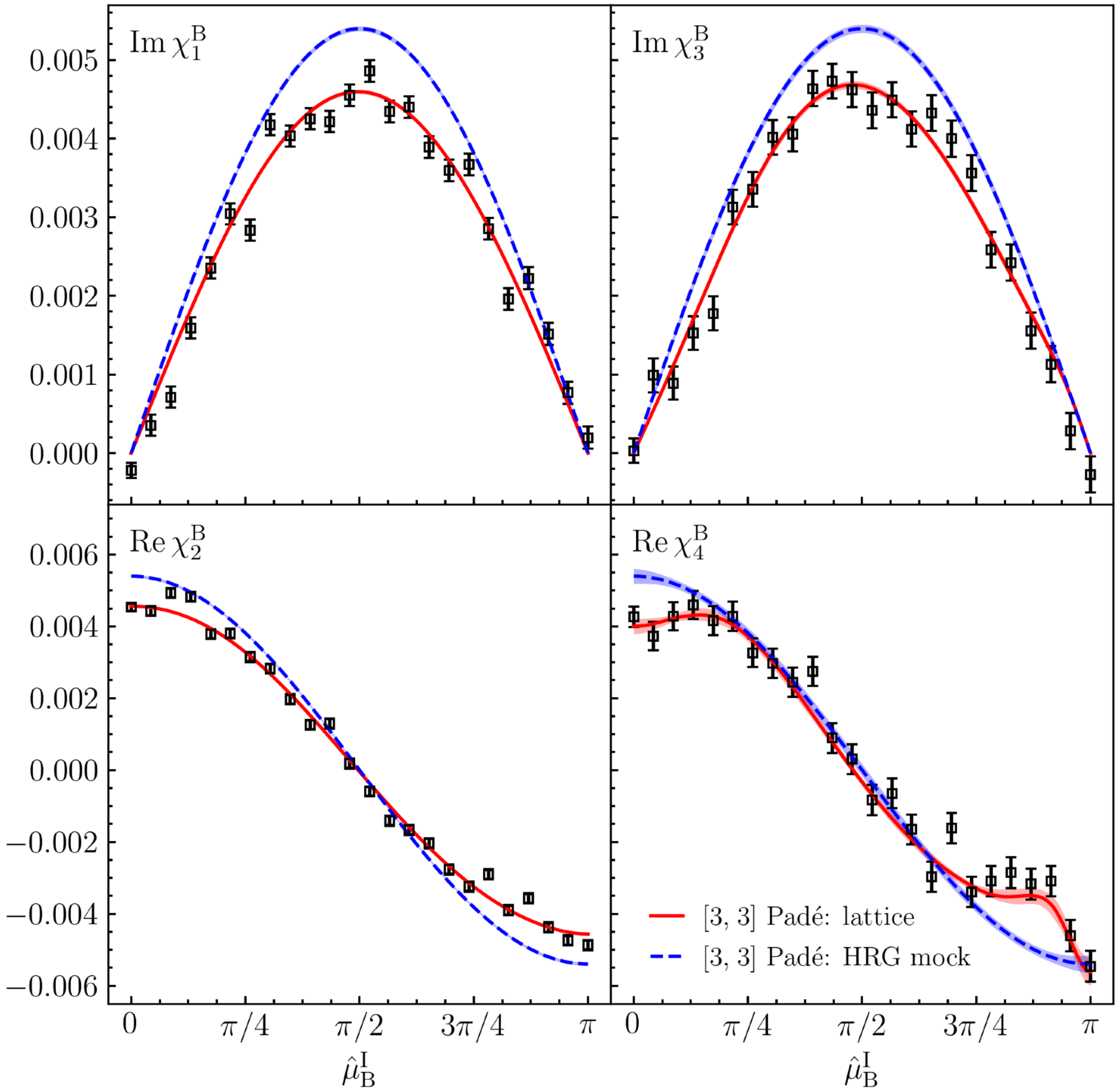
$$P(x) \equiv R_k^m(x) = \frac{\sum_{i=0}^{m} a_i x^i}{1 + \sum_{j=1}^{k} b_j x^j}, x = \cosh(\mu_B) - 1$$

- Generate N_{boot} bootstrap samples by resampling $N_{conf,i}$ configurations allowing replacement for each one of 24 discrete values of $\mu_{B,i}$, and compute mean value of χ^B_{1-4} at $\mu_{B,i}$ within each bootstrap sample

- Joint fit to χ^B_n *with* $\partial^n P(x)/\partial \mu_B^n$ *with n=1-4 in each bootstrap sample with covariance matrix obtained from the original data sets*

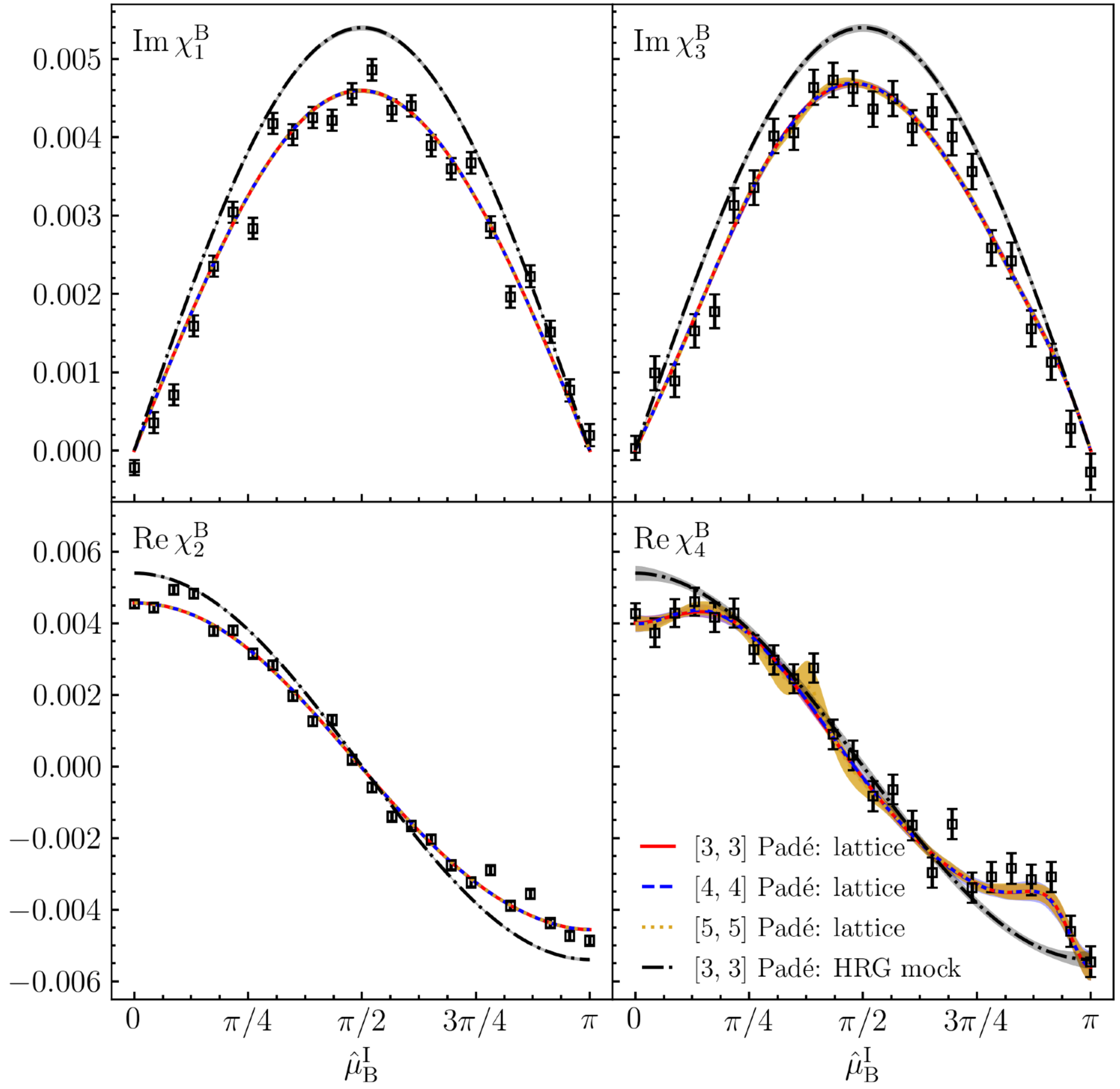
- Final uncertainties from 68% confidence interval over bootstrap fits

[3,3]



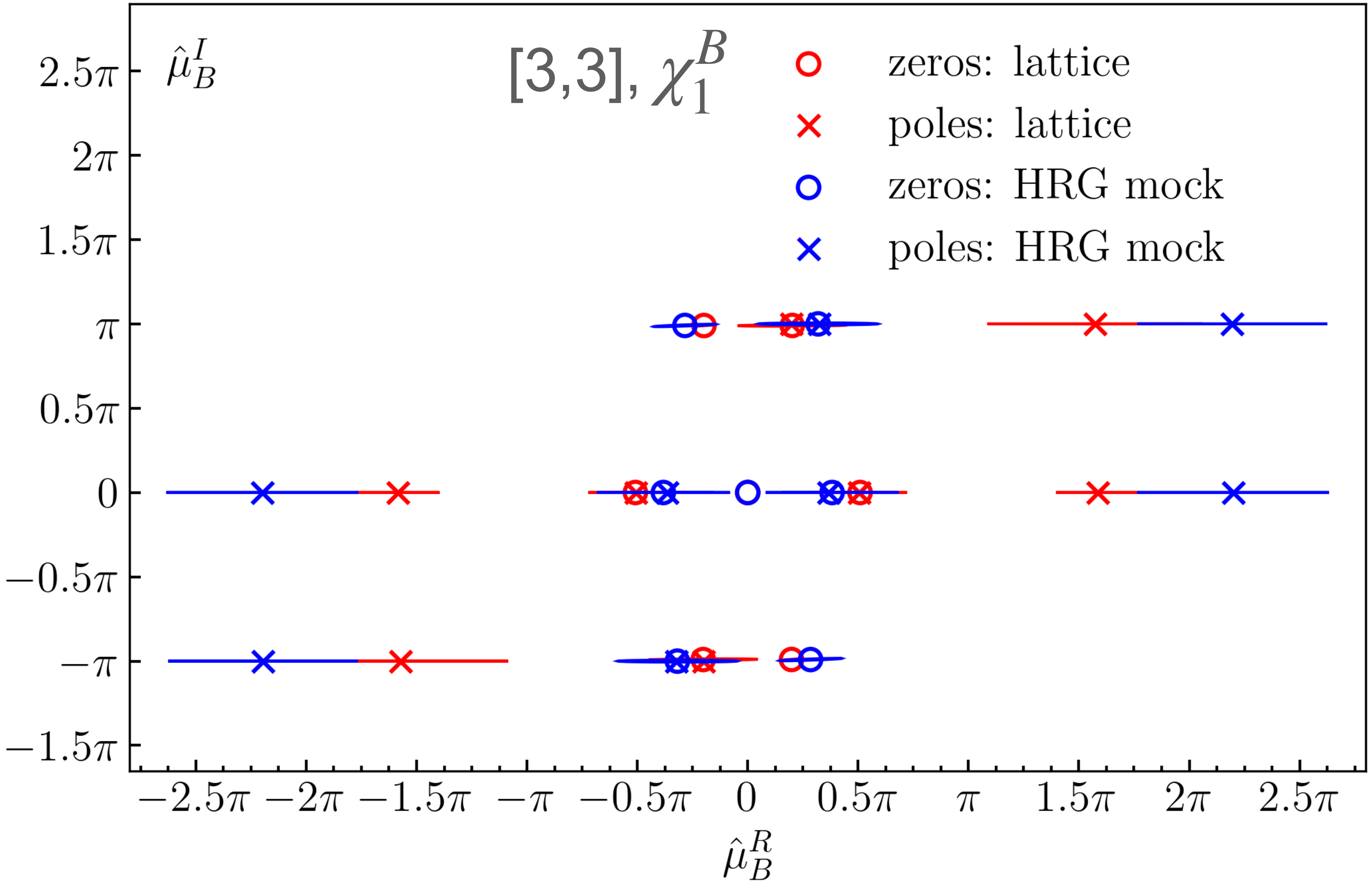
Multi-point Padé fits

$$P(x) \equiv R_n^m(x) = \frac{\sum_{i=0}^{m} a_i x^i}{1 + \sum_{j=1}^{n} b_j x^j}, x = \cosh(\mu_B) - 1$$



		Padé [3,3]		Padé [4,4]		Padé [5,5]	
		All 24 points	Random 12 points	All 24 points	Random 12 points	All 24 points	Random 12 points
χ^2/d.o.f.	lattice	4.00(55)	3.39(75)	4.06(57)	4.45(90)	4.03(57)	3.99(85)
	HRG mock	0.97(21)	0.95(30)	0.97(21)	0.94(31)	0.96(21)	0.93(31)
AICc	lattice	373(50)	156(31)	375(50)	198(36)	370(49)	178(32)
	HRG mock	101(19)	54(13)	103(18)	57(12)	105(18)	61(12)

Zero/poles extracted from Padé [3,3] analyses



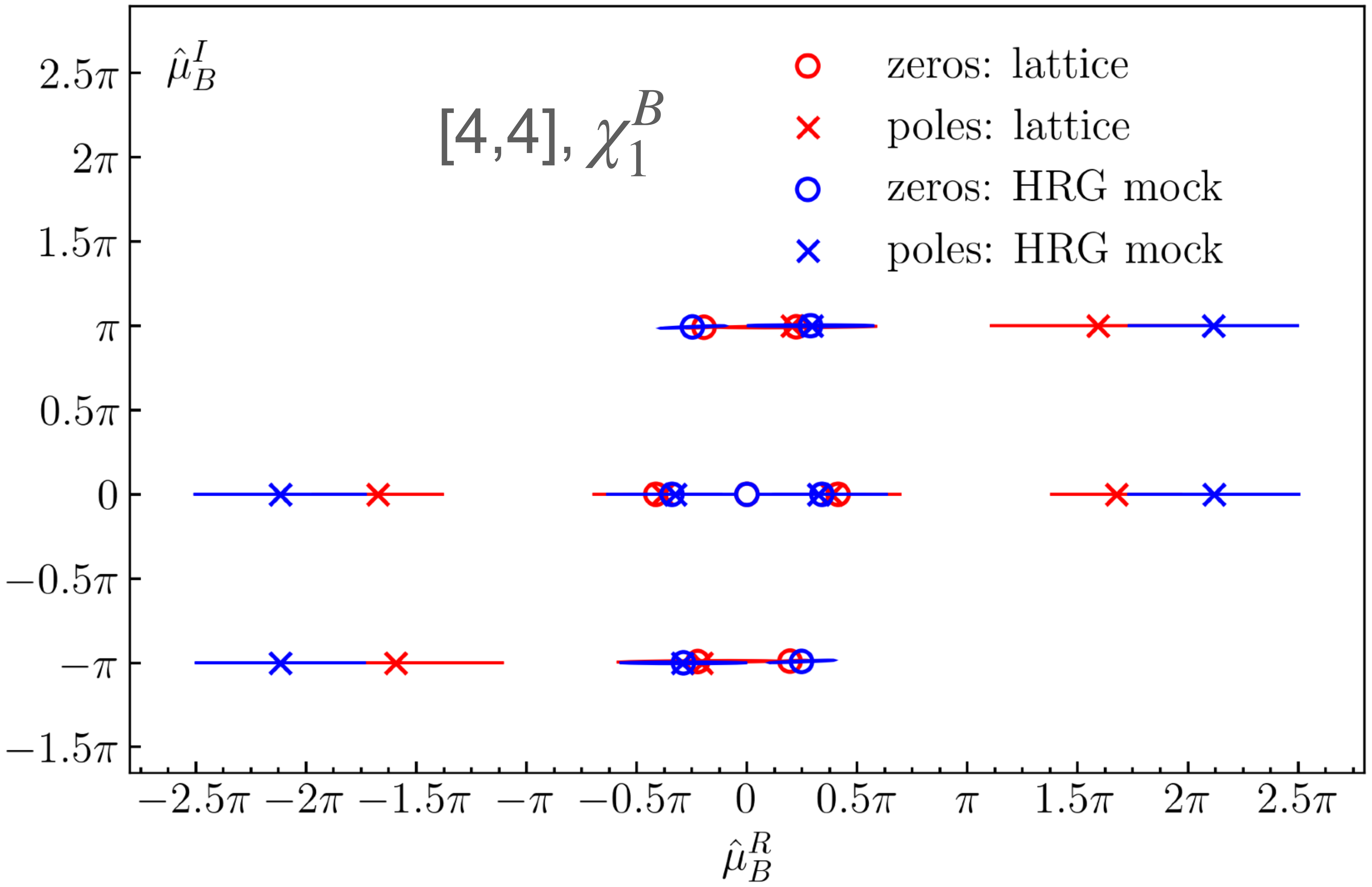
$$P(x) = \frac{\sum_{i=0}^{m} a_i x^i}{1 + \sum_{j=1}^{n} b_j x^j}$$

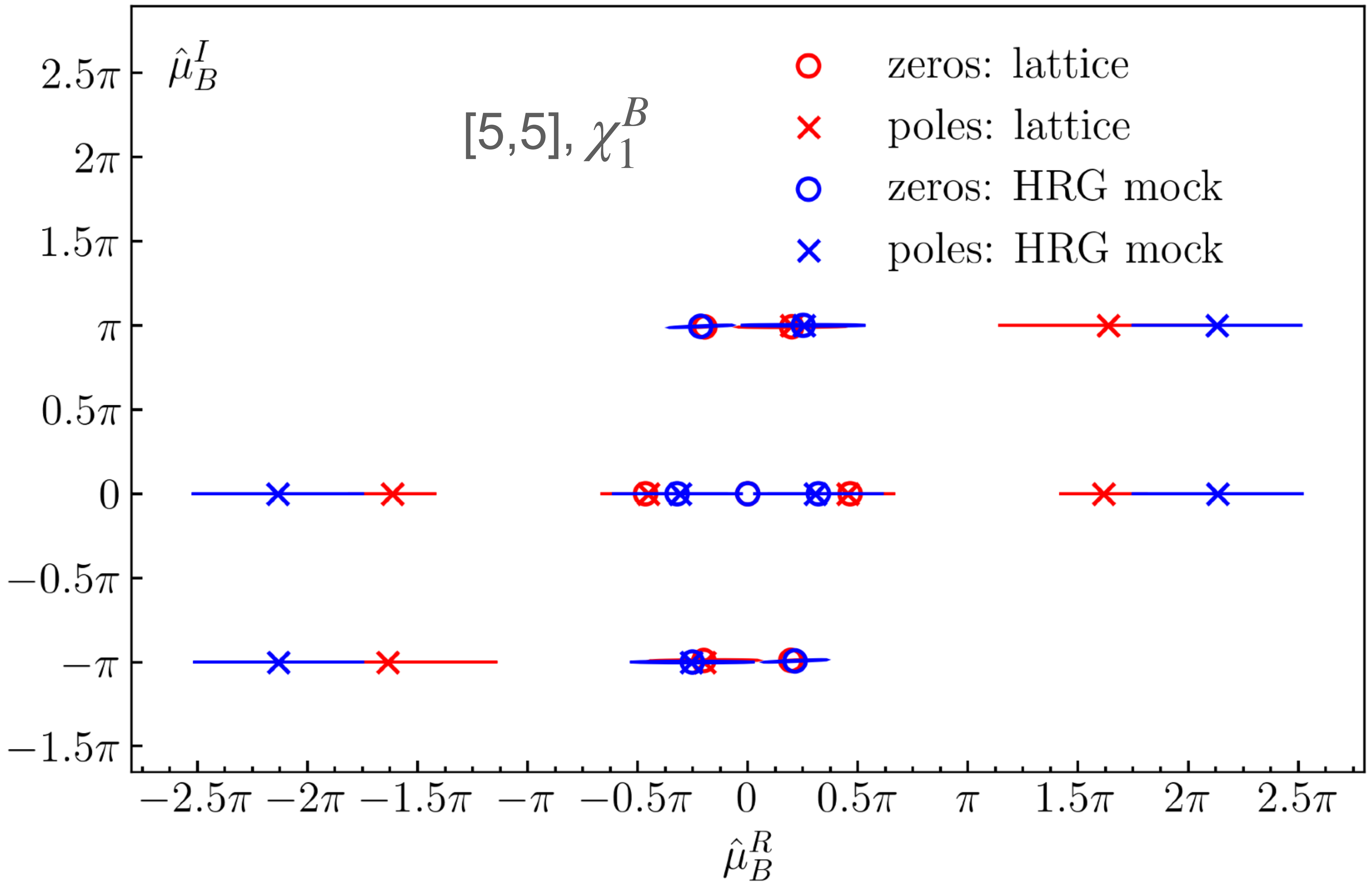
$$\chi_1^B = \partial P(x)/\partial \mu_B$$

Zeros: roots of numerator
Poles: roots of denominator

- No uncanceled poles
- Similar pattern between Lattice and HRG mock

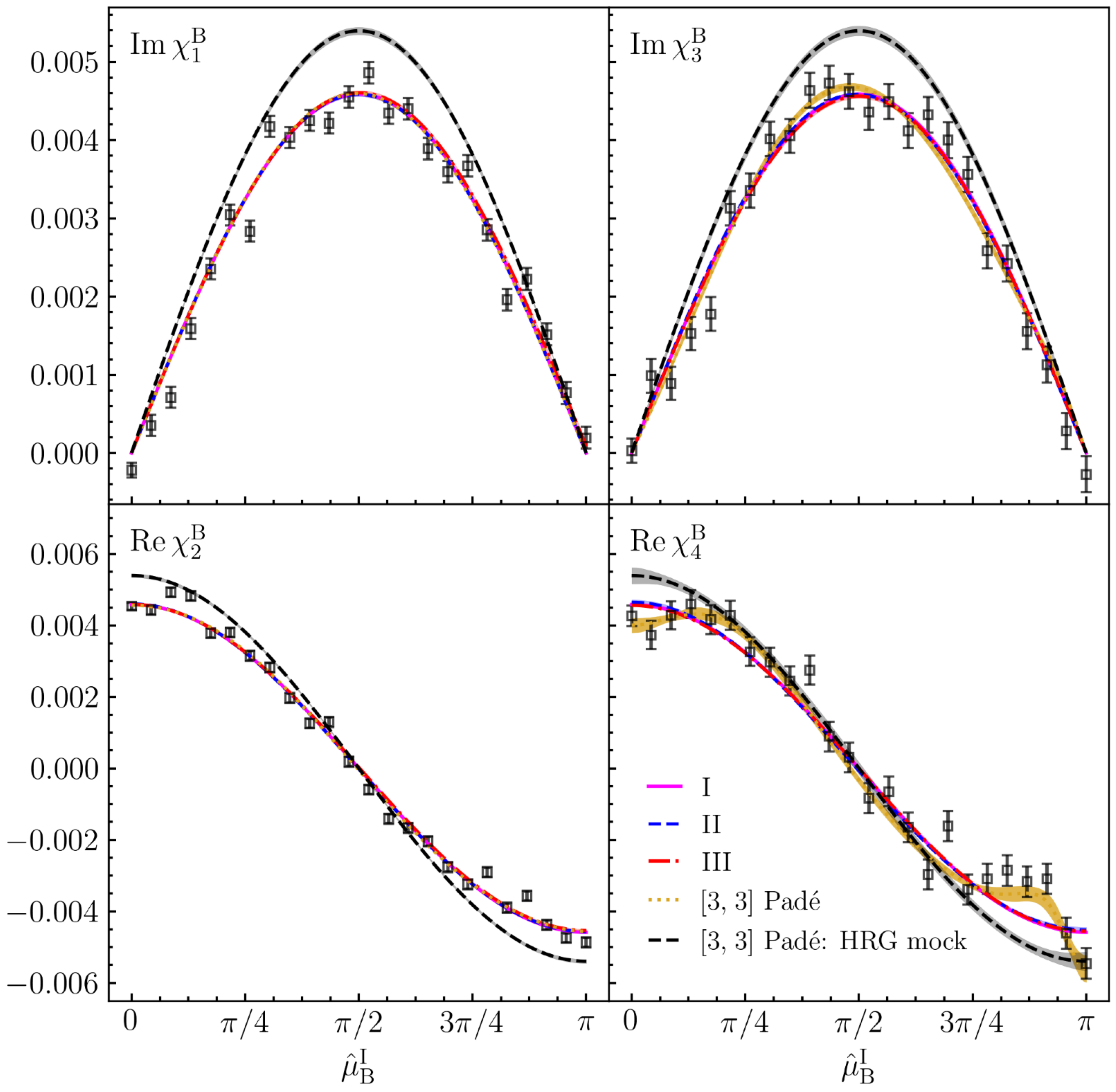
Zero/poles extracted from higher order Padé analyses





Similar observation as in the case of Padé [3,3]

Fits with physical-motivated non-critical ansatz



Non-critical ansatz for pressure:

Ansatz I: $f(\mu_B) = A\cosh(\mu_B)$

Ansatz II: $f(\mu_B) = A\cosh(\mu_B) + B\cosh(2\mu_B)$

	Lattice QCD Data			
	All 24 points		Random 12 points	
	Ansatz I	**Ansatz II**	**Ansatz I**	**Ansatz II**
Parameters				
A $(\times 10^{-3})$	4.584(35)	4.582(34)	4.528(48)	4.662(50)
B $(\times 10^{-5})$	–	0.38(64)	–	0.001(898)
Statistics				
χ^2/d.o.f.	4.20(55)	4.13(55)	4.45(82)	4.23(81)
AICc	401(52)	401(52)	211(38)	199(37)

Singularities in the real plane?

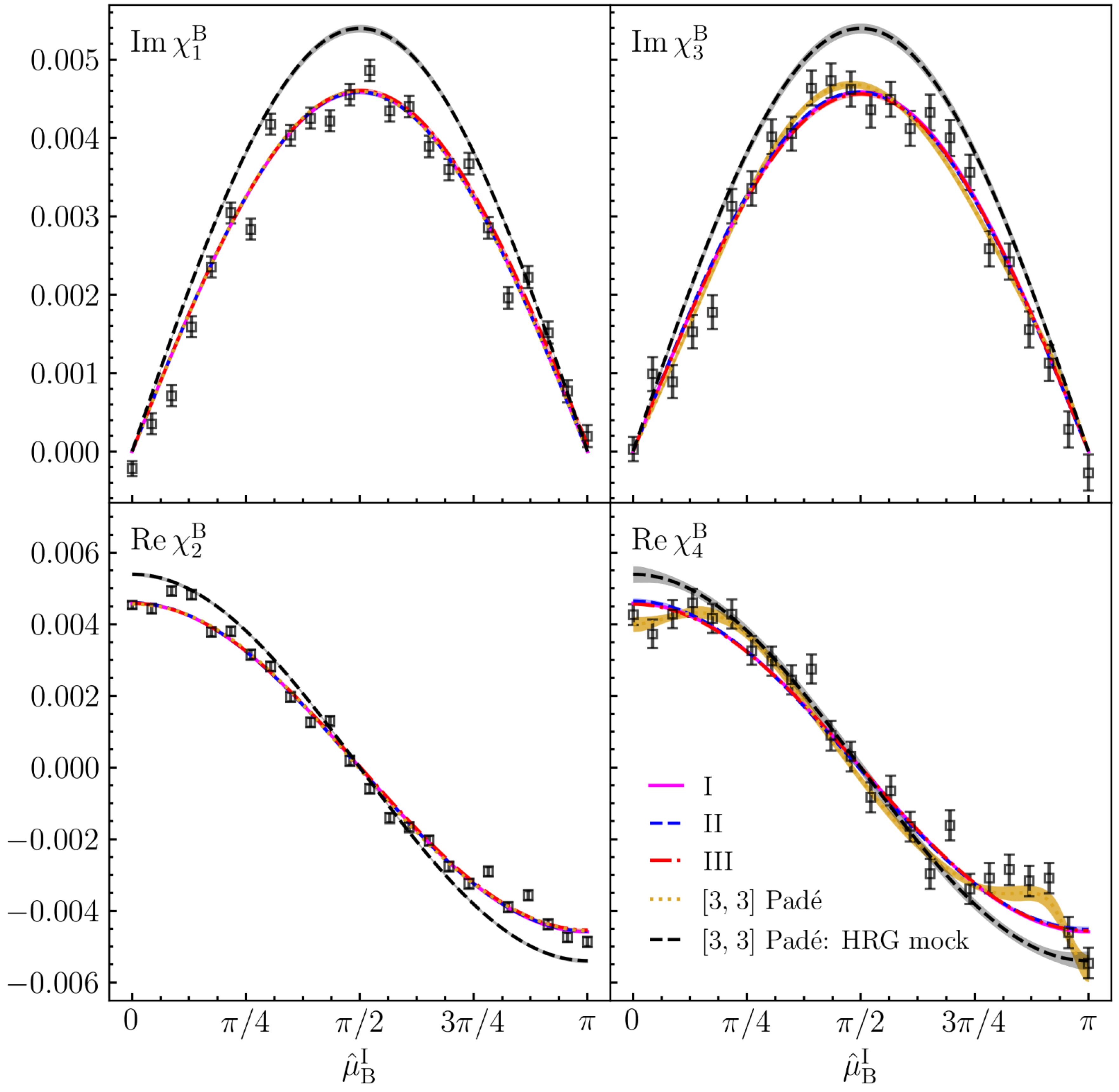
Critical ansatz III: $f(\mu_B) = A\cosh(\mu_B) + C(|\mu_B - \mu_{B,c}|^\alpha + |\mu_B + \mu_{B,c}|^\alpha)$

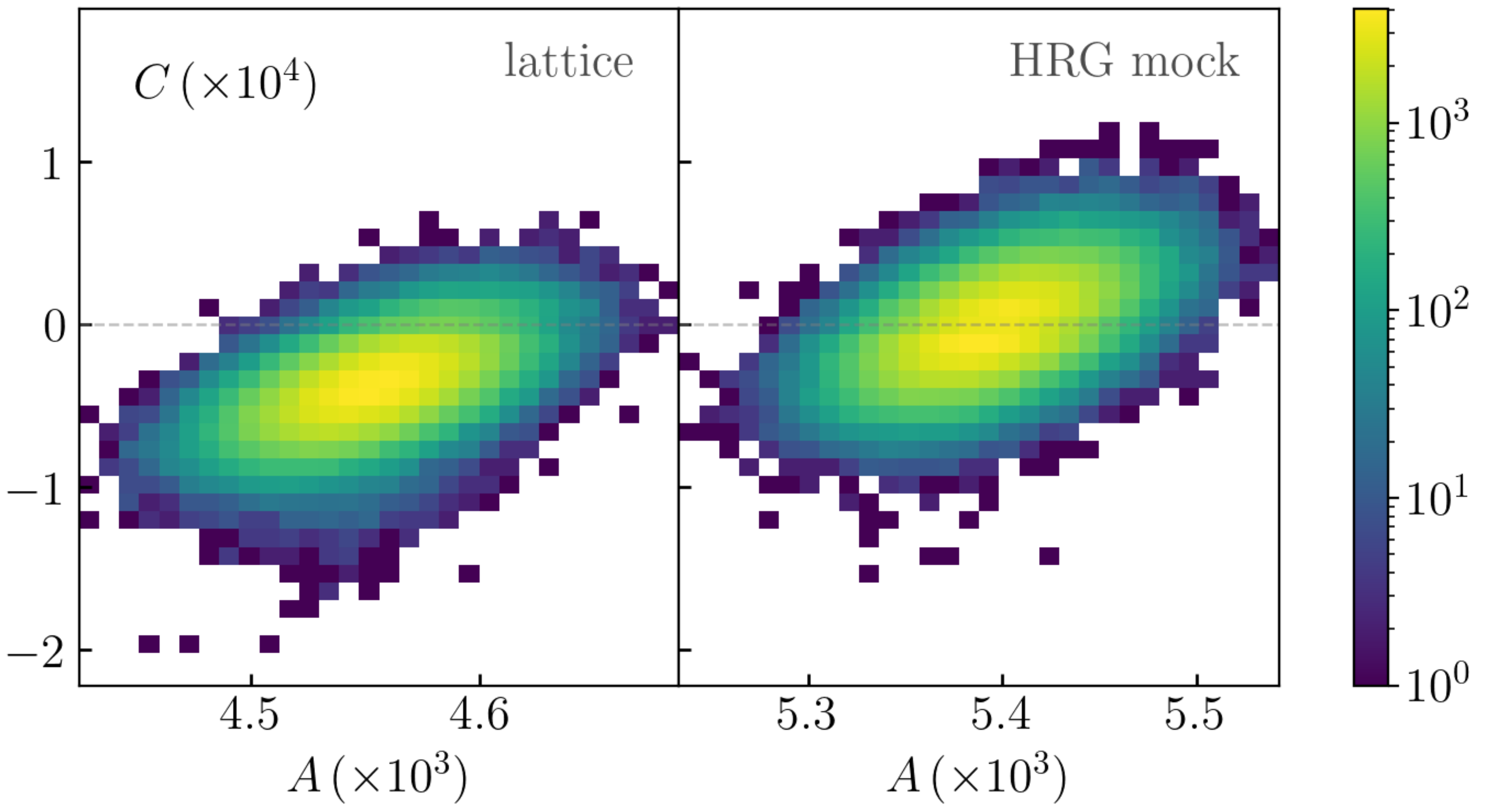
Impose lower bound of $\mu_{B,c}$: 350, 360,380, 400 MeV

$\alpha = 1 + \delta_{\mathrm{ISNG}}$ = 1.208

Hasenbusch , PRB 82(2010)174433

Karch, Schmidt & Singh, PRD109(2024)014508





Diagnostic measure of $\mu_{B,c}$ in a certain region $\mathscr{R}$

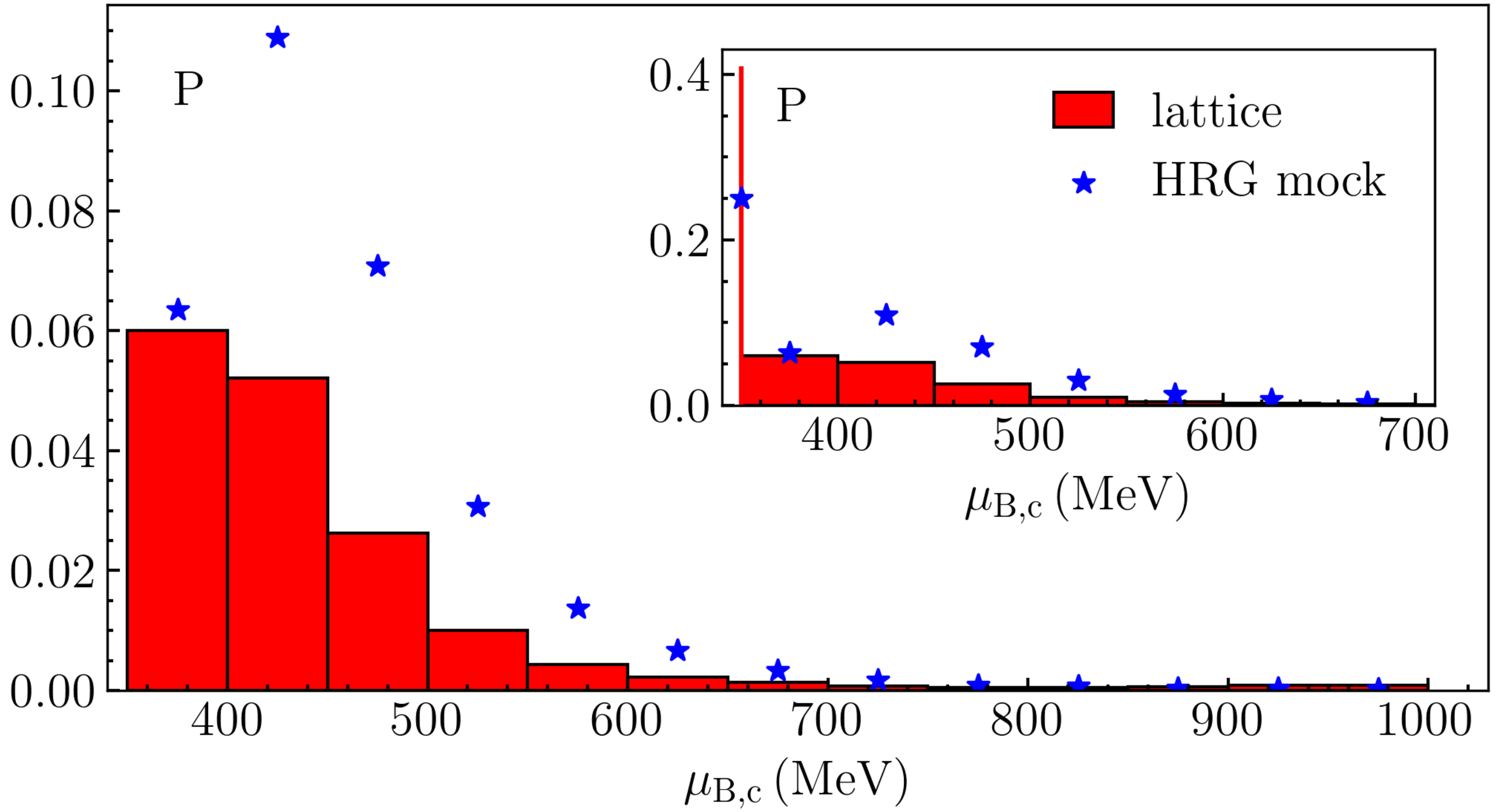
Define a Diagnostic measure P for $\mu_{B,c}$ in a region $\mathscr{R}$:

$$P = \frac{N_{\mathscr{R}}}{N_{total}}$$

$N_{\mathscr{R}}$: # of bootstrap samples yield $\mu_{B,c}$ in the region $\mathscr{R}$

N_{total}: total # of bootstrap samples used in the fit

Lower bound of $\mu_{B,c}$=350 MeV



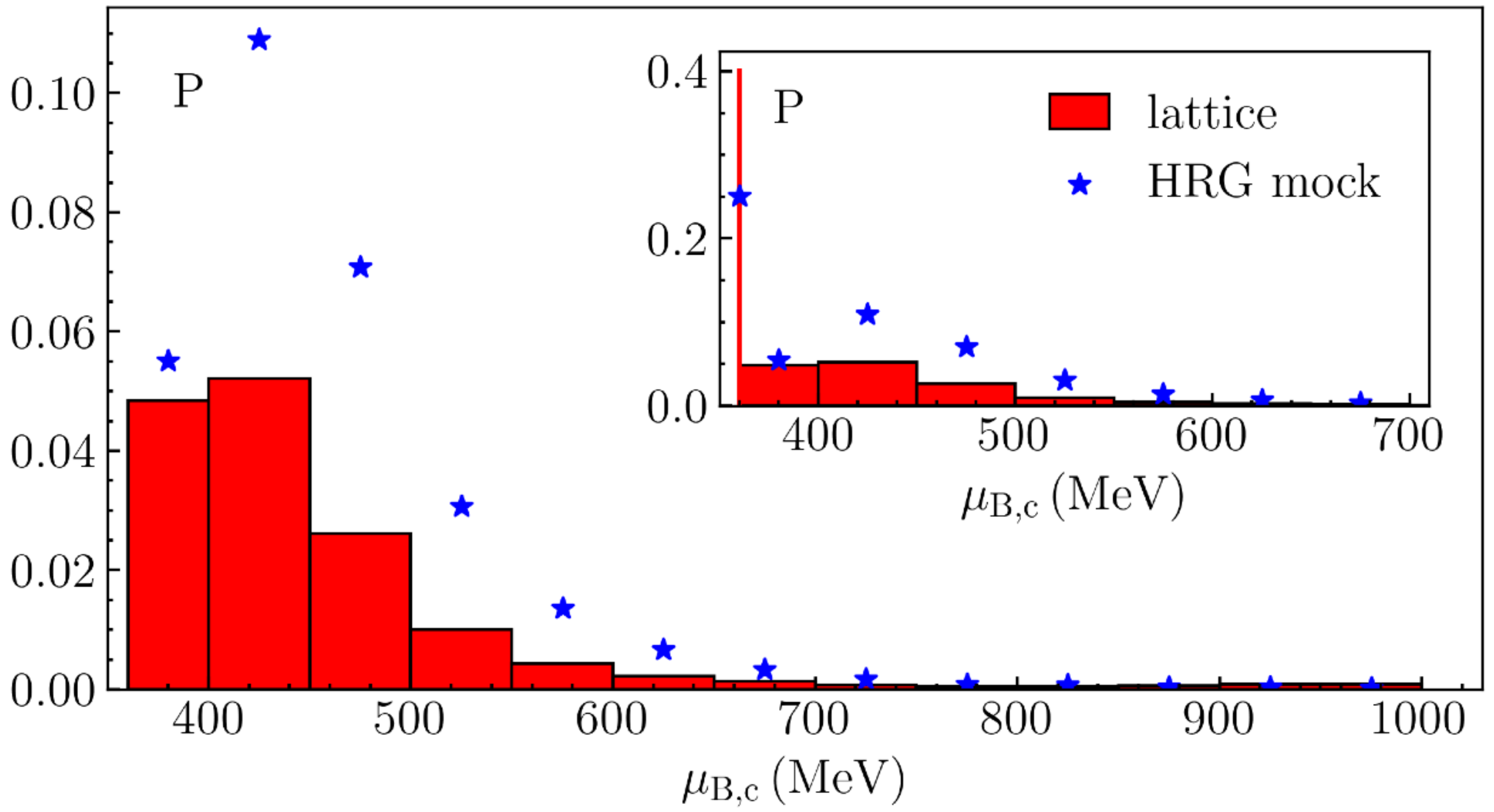
- Nonzero false-positive $P_{\mathrm{HRG}} > P_{\mathrm{LAT}}$!

Region $\mathcal{R}$ (MeV)	$P(\mathcal{R})$ lattice	$P(\mathcal{R})$ HRG mock
387–502 (Predicted Region I)	9.1%	20.4%
420–750 (Experimental Search Region I)	7.3%	19.4%
420–632 (Experimental Search Region II)	7.1%	18.7%
540.0–664.2 (Predicted Region II)	0.86%	2.6%

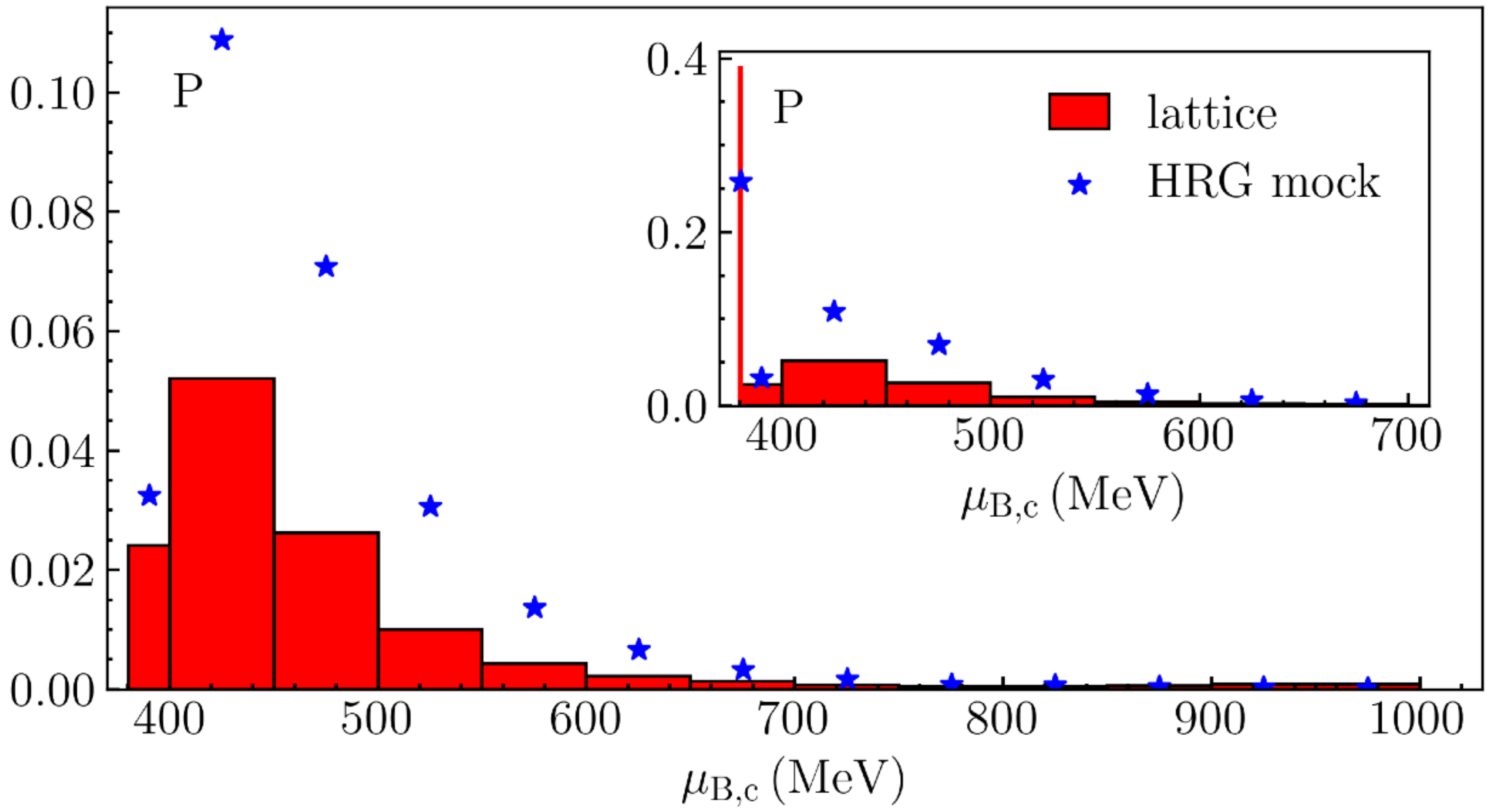
- Accumulation of large samples at lower bound

Dependence on the lower bound & # of bootstrap samples

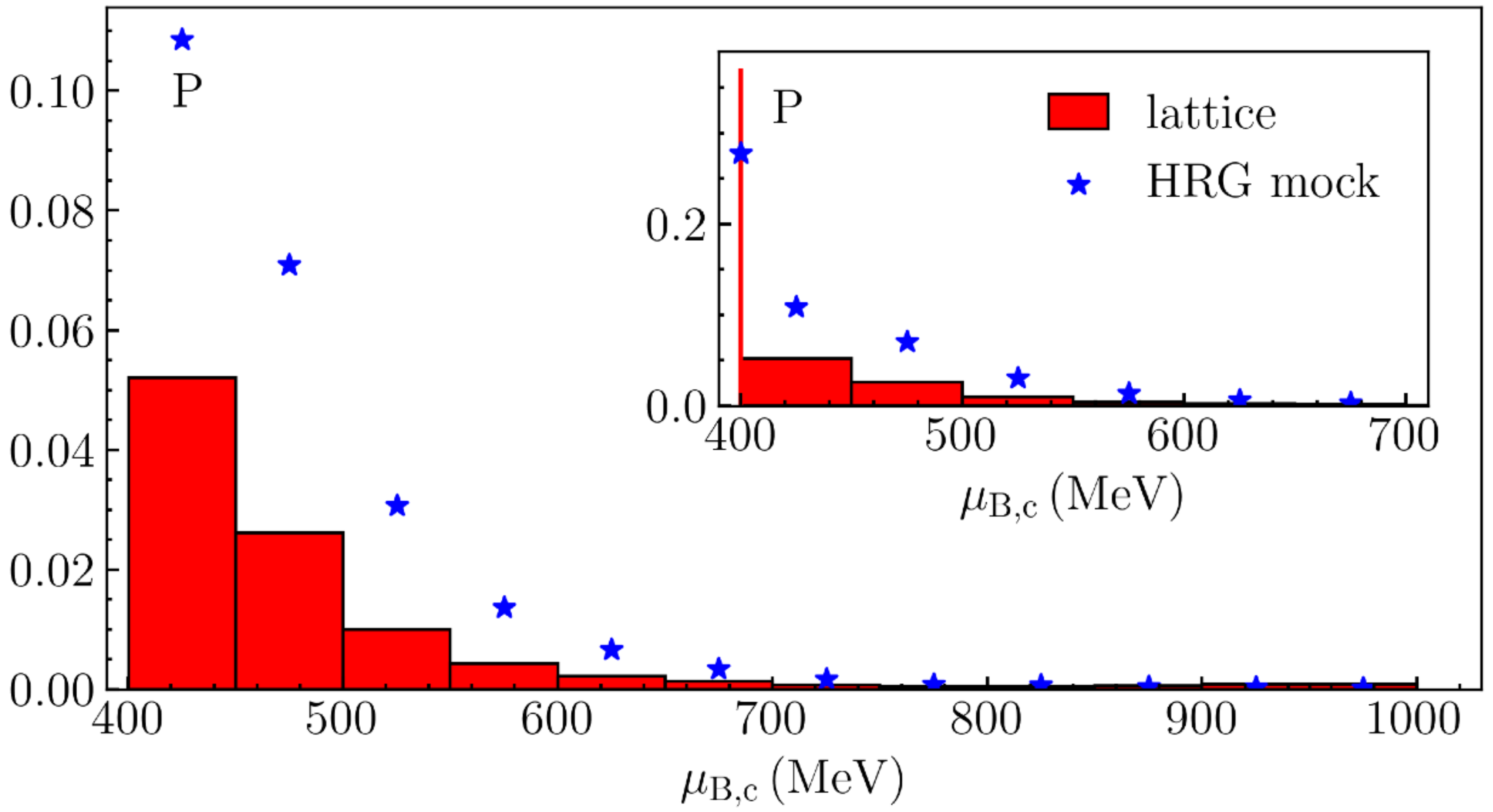
Lower bound 360 MeV



Lower bound 380 MeV

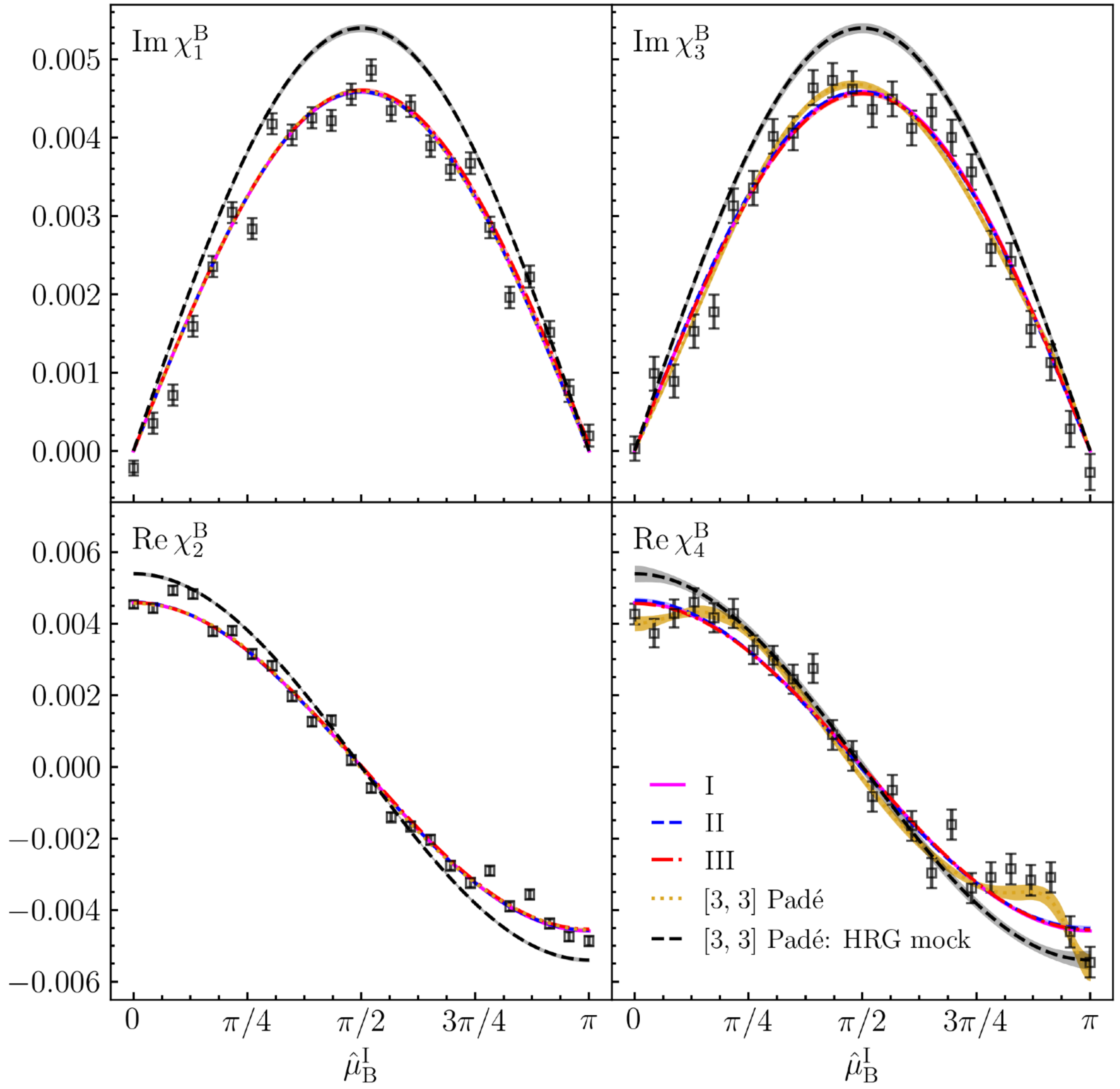


Lower bound 400 MeV



	$P(\mathcal{R})$											
	100K Samples				200K Samples				300K Samples			
Region $\mathcal{R}$ (MeV)	350	360	380	400	350	360	380	400	350	360	380	400
387–502	0.091	0.091	0.091	-	0.092	0.091	0.091	-	0.092	0.092	0.092	-
420–750	0.074	0.074	0.074	0.074	0.073	0.073	0.073	0.073	0.073	0.073	0.073	0.073
420–632	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071
540.0–664.2	0.0086	0.0086	0.0087	0.0087	0.0087	0.0087	0.0086	0.0086	0.0085	0.0085	0.0084	0.0085

Dependence on the reduced/full data sets



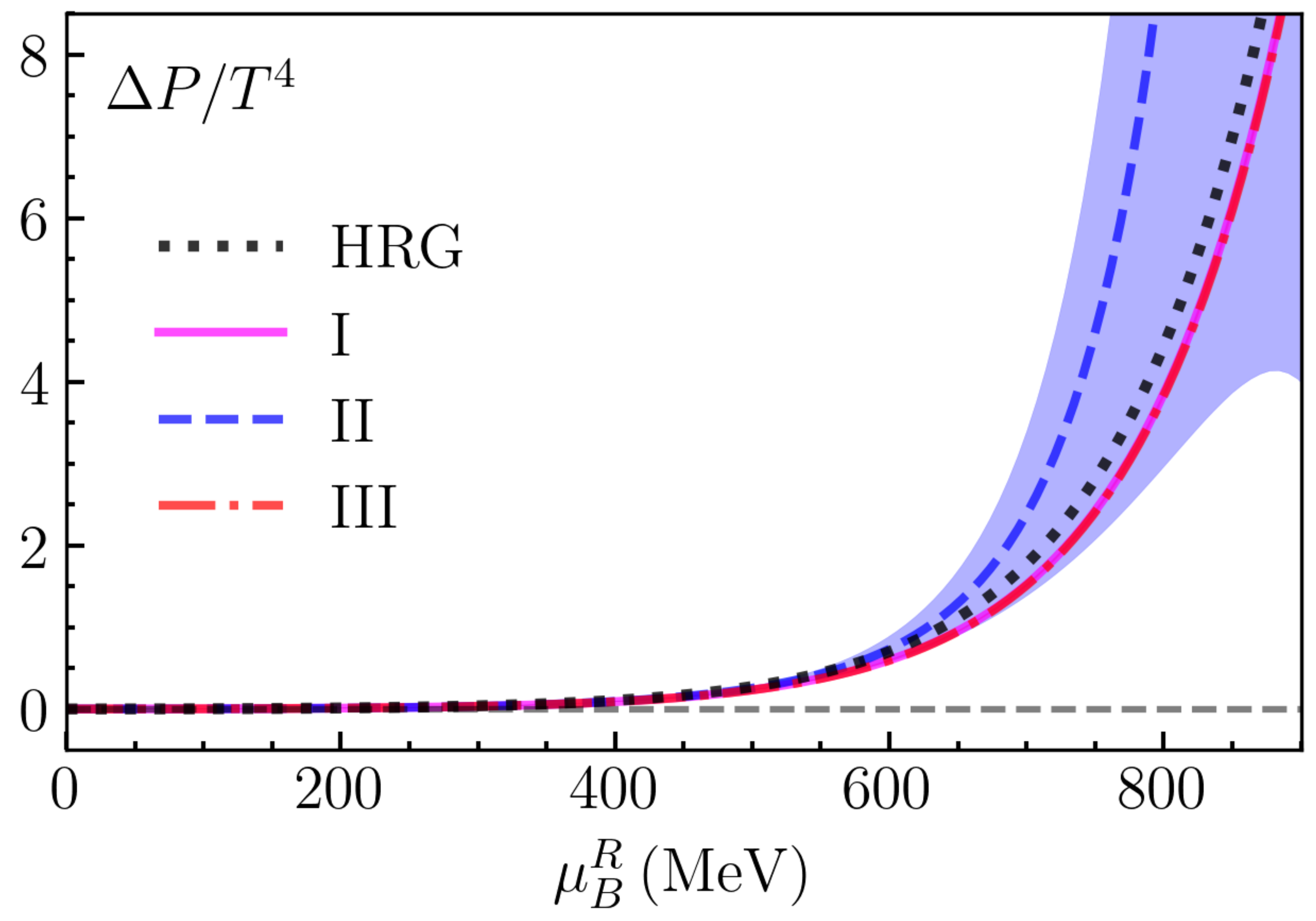
Lattice

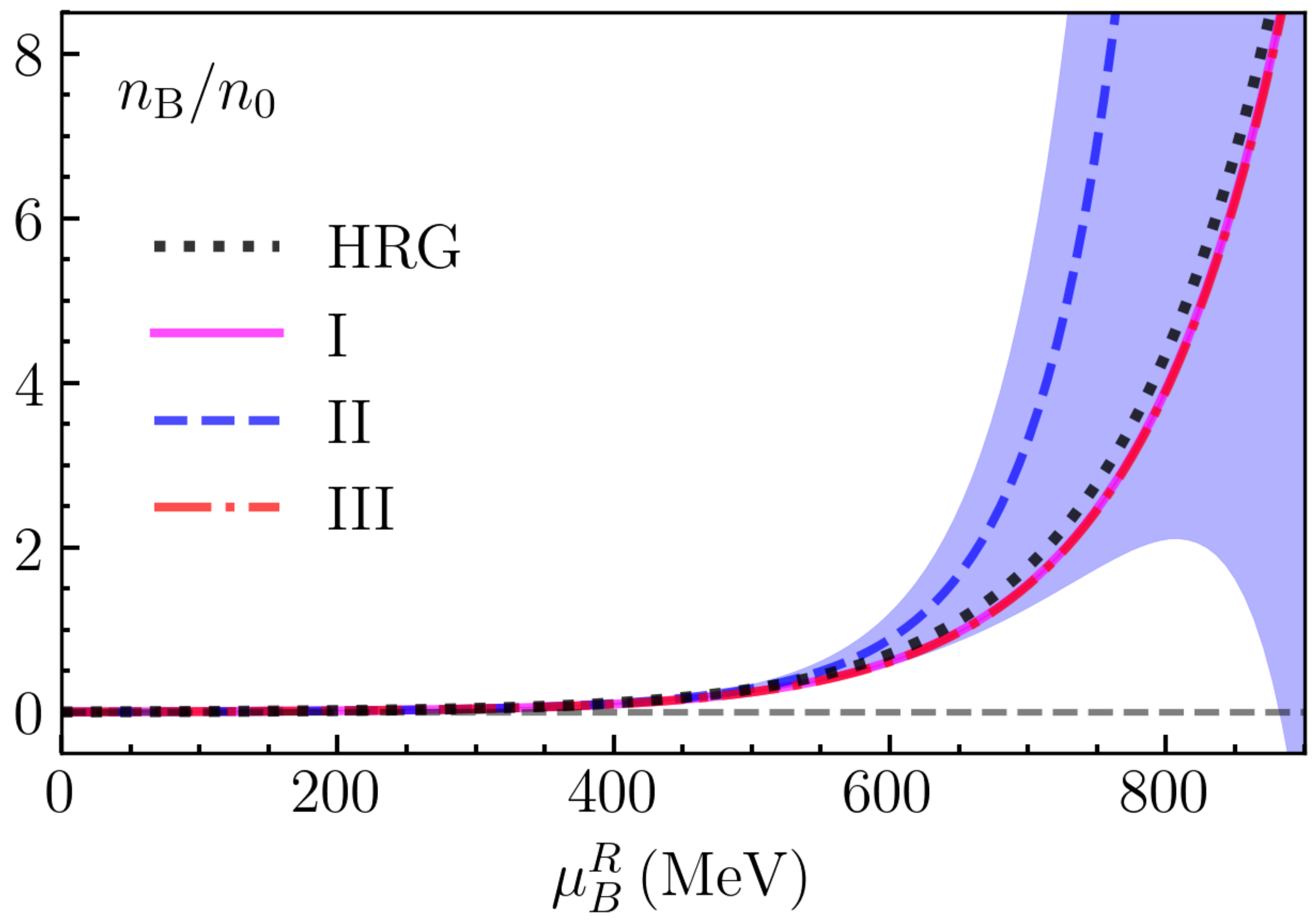
	$P(\mathcal{R})$							
	All 24 points				Random 12 points			
Region $\mathcal{R}$ (MeV)	350	360	380	400	350	360	380	400
387–502	0.092	0.092	0.092	-	0.124	0.124	0.124	-
420–750	0.073	0.073	0.073	0.073	0.133	0.133	0.132	0.133
420–632	0.071	0.071	0.071	0.071	0.126	0.126	0.126	0.126
540.0–664.2	0.0085	0.0085	0.0085	0.0085	0.023	0.023	0.022	0.023

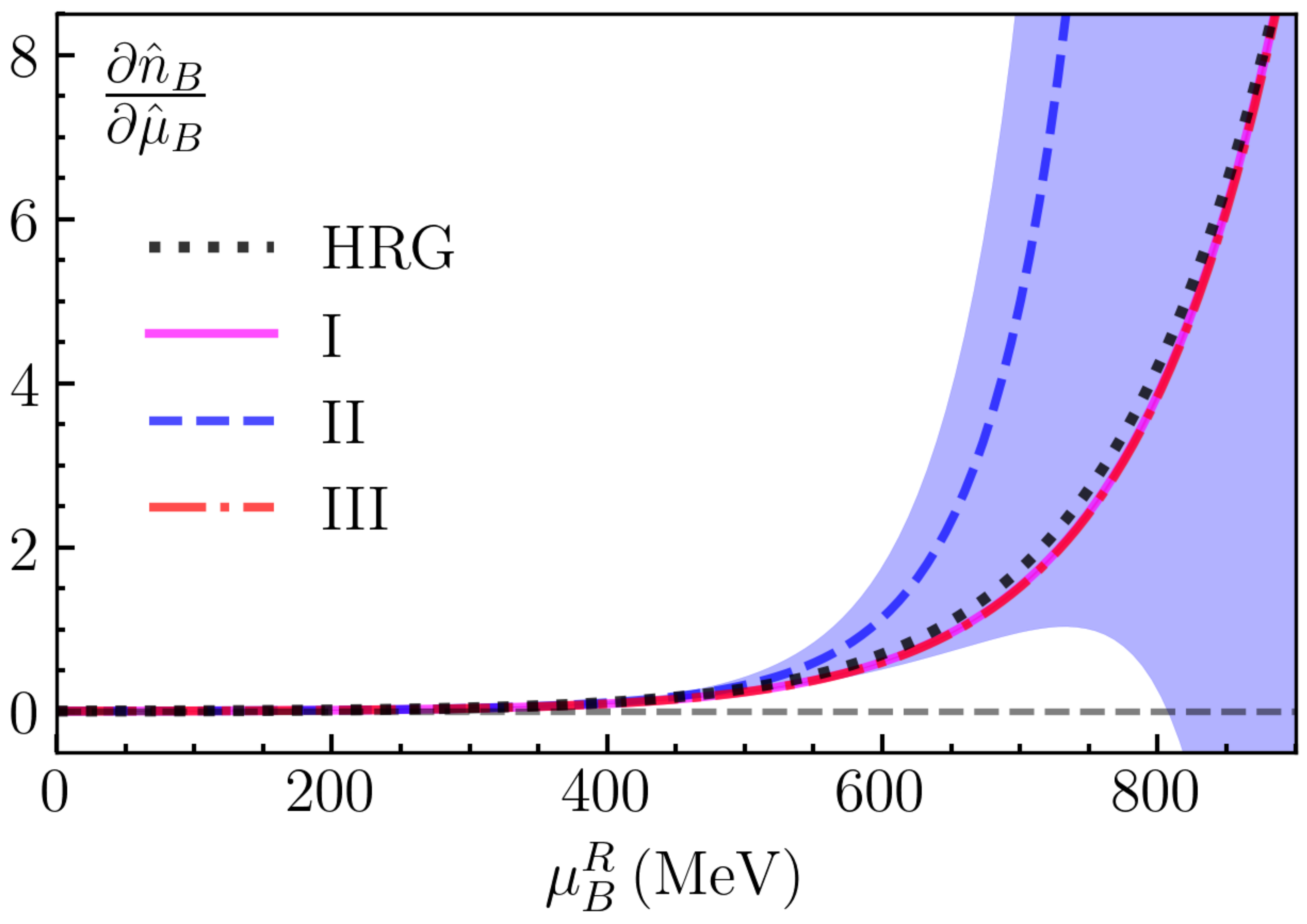
HRG

	$P(\mathcal{R})$							
	All 24 points				Random 12 points			
Region $\mathcal{R}$ (MeV)	350	360	380	400	350	360	380	400
387–502	0.204	0.204	0.204	-	0.263	0.225	0.197	-
420–750	0.194	0.194	0.194	0.194	0.301	0.218	0.175	0.184
420–632	0.187	0.187	0.187	0.187	0.293	0.211	0.168	0.177
540.0–664.2	0.0258	0.0259	0.0258	0.0258	0.0327	0.0266	0.0233	0.0247

Analytic continuation

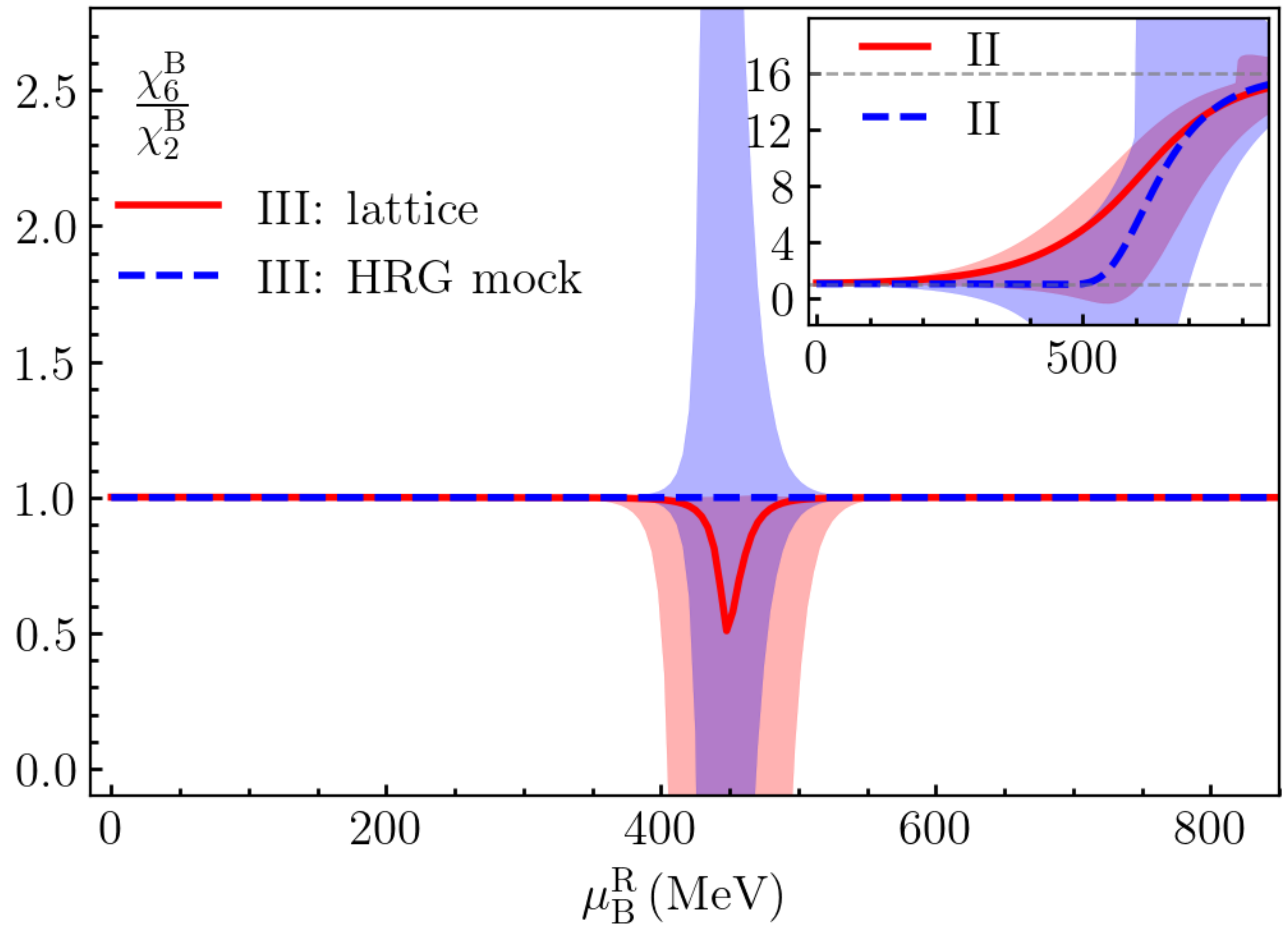


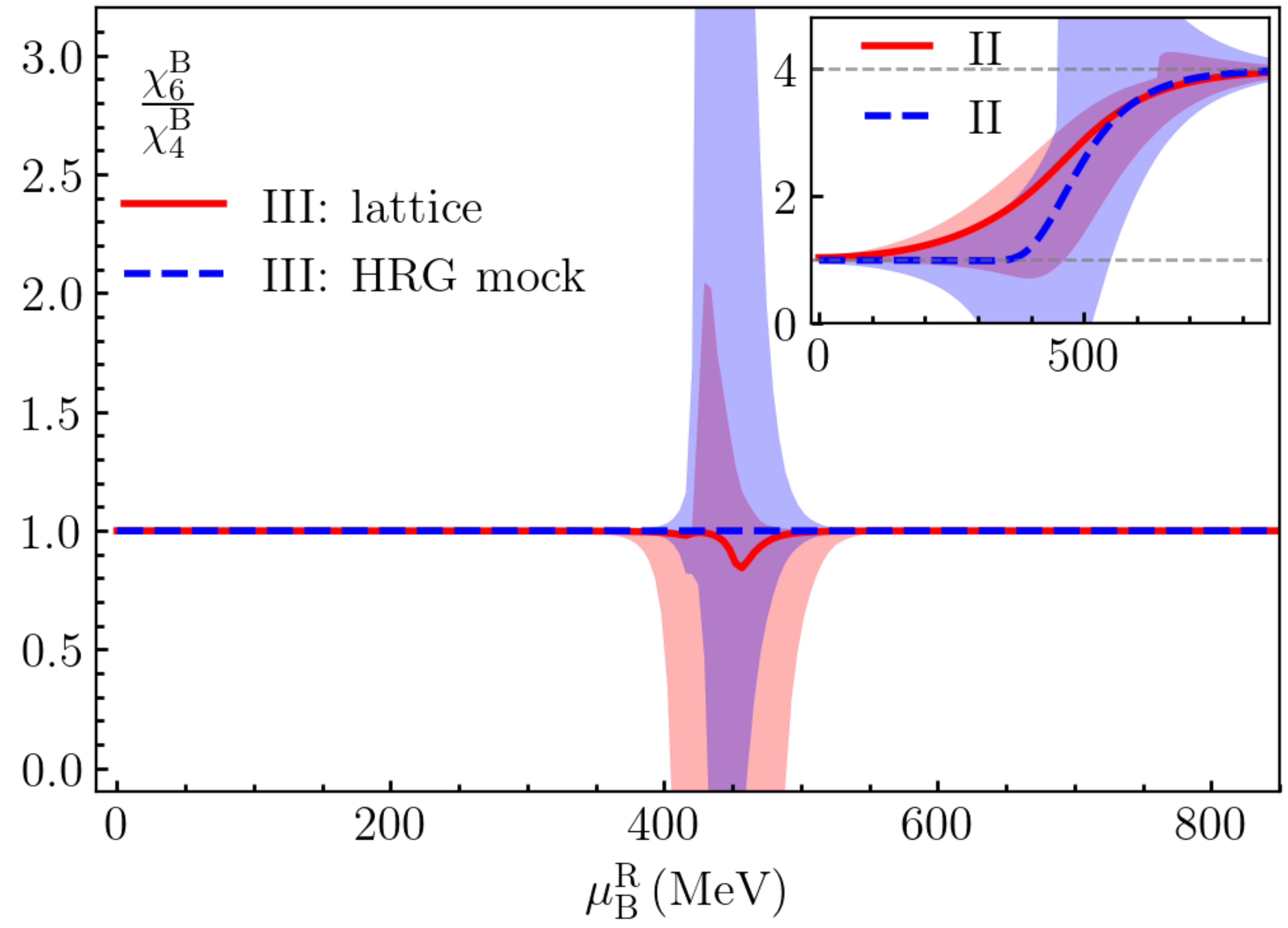




- Baryon number density $\hat{n}_B$ and $\partial\hat{n}_B/\partial\hat{\mu}_B$ shall be positive, which restricts $\mu_B^R \lesssim 800\text{MeV}$
- Results from ansatz I (non-interacting) and III (critical) are very much similar
- Results from ansatz II (interacting) differ from others at $\mu_B^R \gtrsim 500\text{MeV}$

Manifestation in the high order susceptibilities





Although about 7% bootstrap samples allowed to have $\mu_{B,c} \in (420,750)$ MeV
No significant sign change, deviation from HRG etc are observed

No signal in Exp

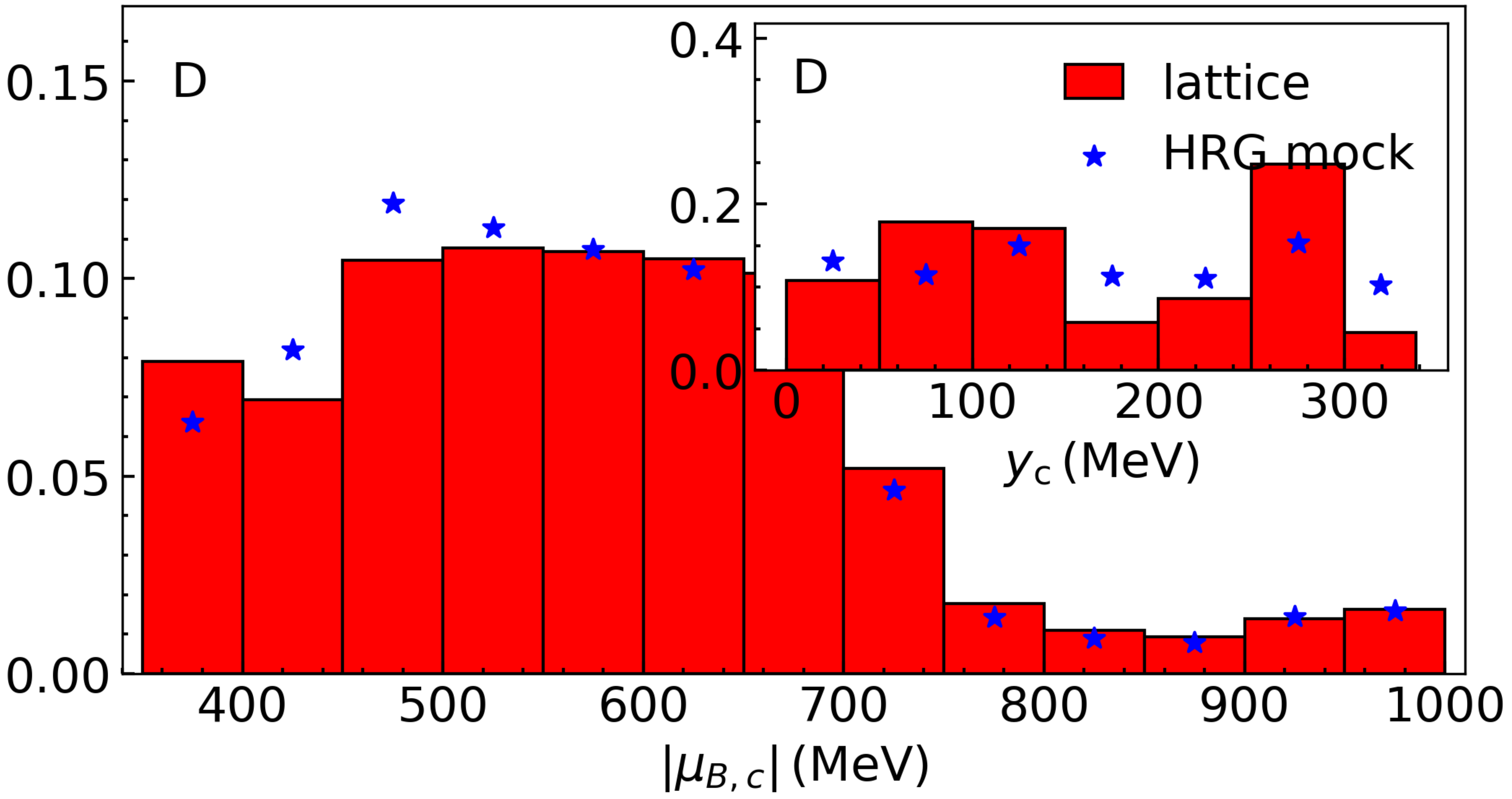
Criticality in the complex μ_B plane

$$f(\hat{\mu}_B) = A(T)\cosh(\hat{\mu}_B) \; + C\,(\hat{\mu}_B - \hat{\mu}_{B,c})^{\alpha_{\rm LY}} + C\,(-\hat{\mu}_B - \hat{\mu}_{B,c})^{\alpha_{\rm LY}} + C^*\,(\hat{\mu}_B - \hat{\mu}^*_{B,c})^{\alpha_{\rm LY}} + C^*\,(-\hat{\mu}_B - \hat{\mu}^*_{B,c})^{\alpha_{\rm LY}}$$

$C, \hat{\mu}_{B,c} \in \mathbb{C}$

$\alpha_{\rm LY} = 1.085$

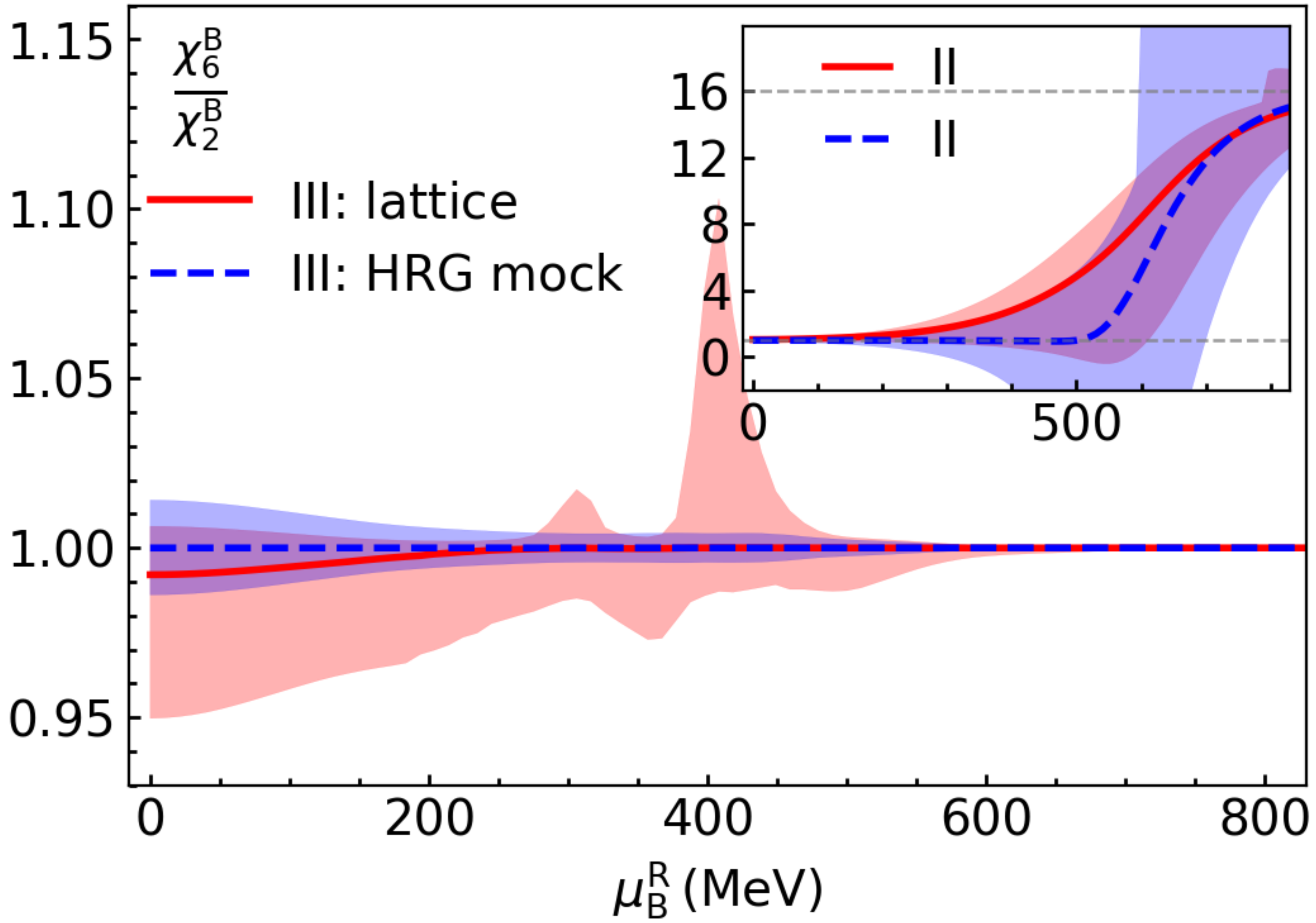
Gliozzi and Rago, JHEP 10 (2014) 042

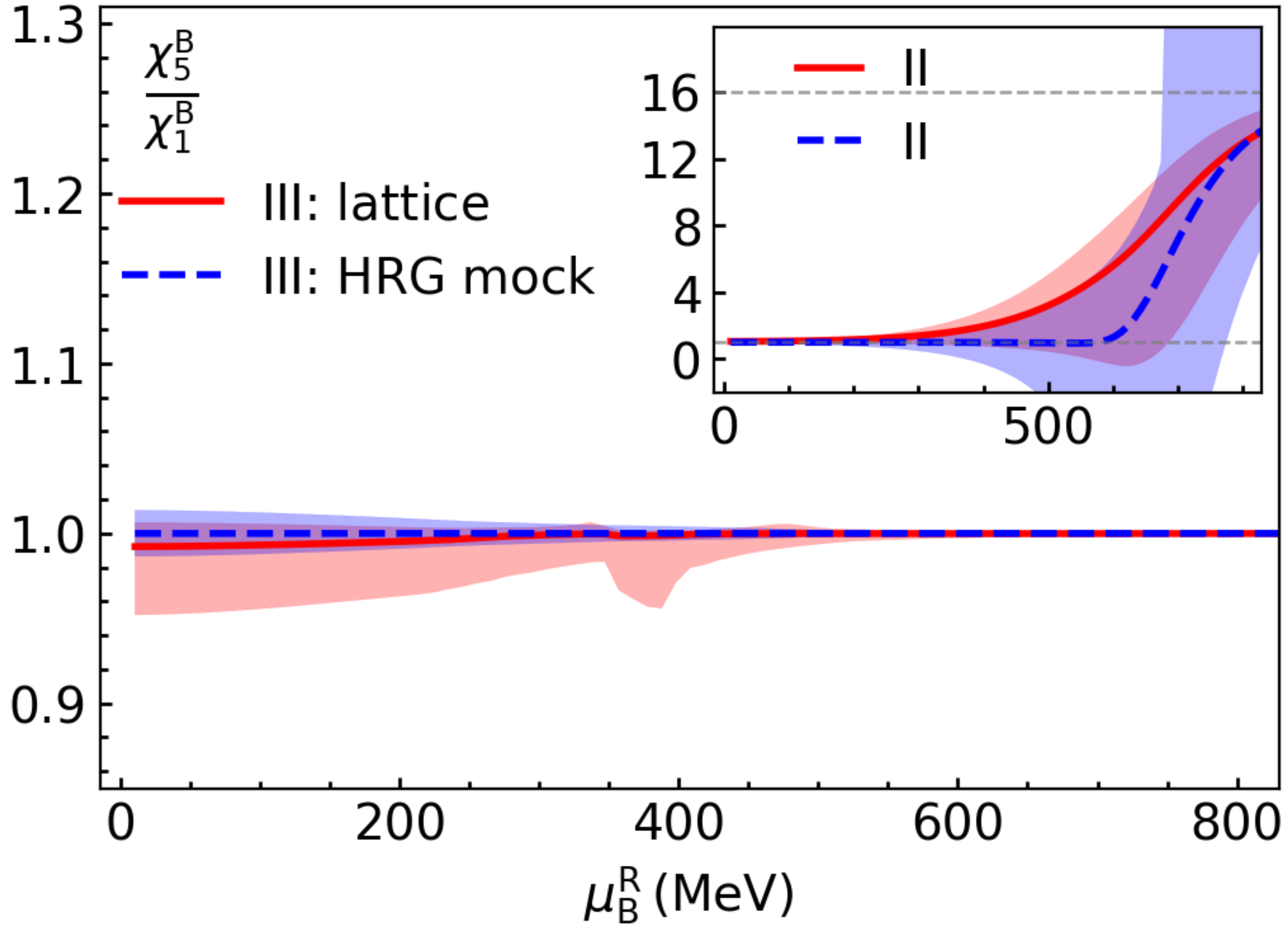


Kai-Fan Ye, work in progress

Ratios of susceptibilities at real μ_B

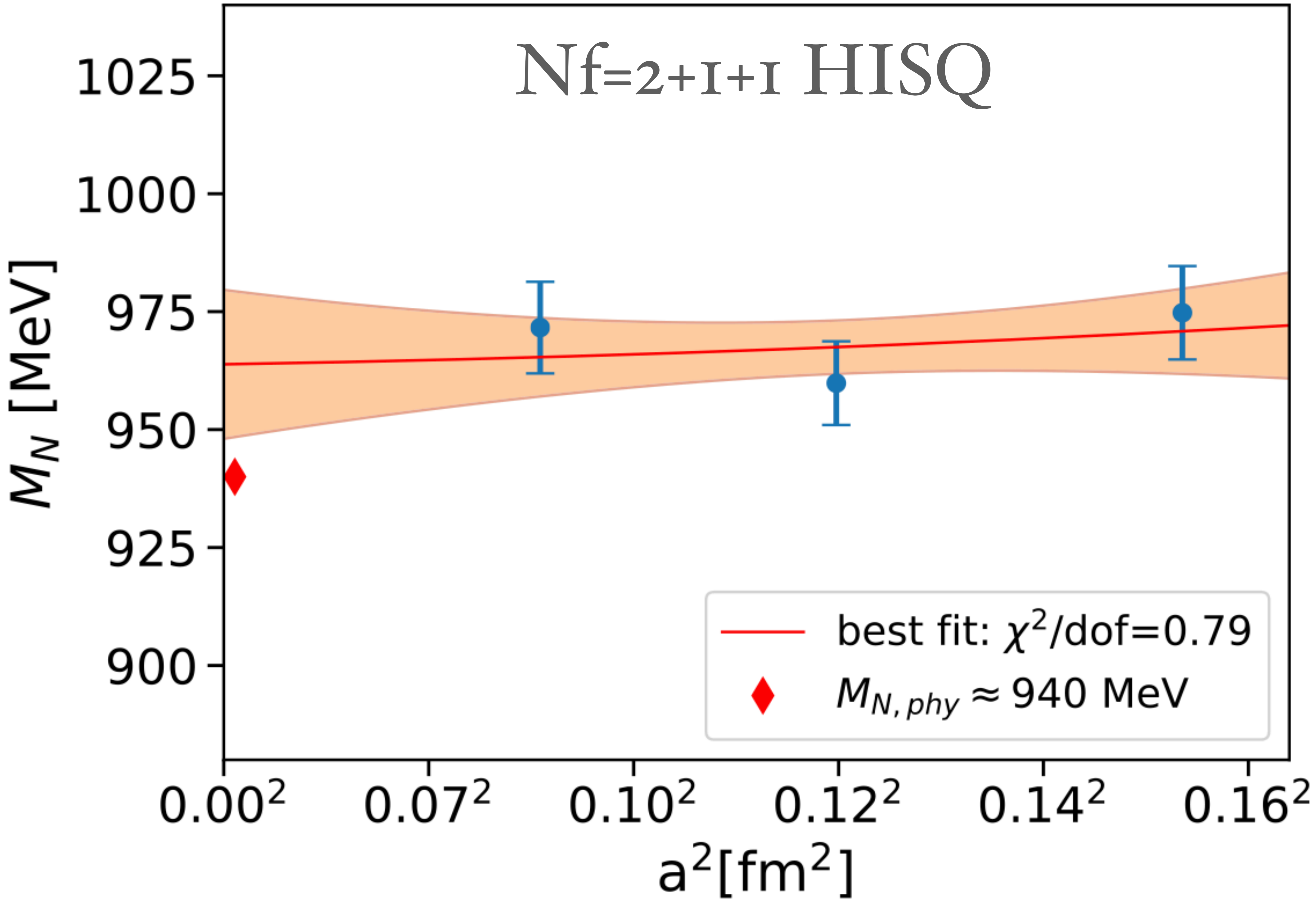
$$f(\hat{\mu}_B) = A(T)\cosh(\hat{\mu}_B) + C\,(\hat{\mu}_B - \hat{\mu}_{B,c})^{\alpha_{\mathrm{LY}}} + C\,(-\hat{\mu}_B - \hat{\mu}_{B,c})^{\alpha_{\mathrm{LY}}} + C^*\,(\hat{\mu}_B - \hat{\mu}^*_{B,c})^{\alpha_{\mathrm{LY}}} + C^*\,(-\hat{\mu}_B - \hat{\mu}^*_{B,c})^{\alpha_{\mathrm{LY}}}$$





Kai-Fan Ye, work in progress

Lattice cutoff effects



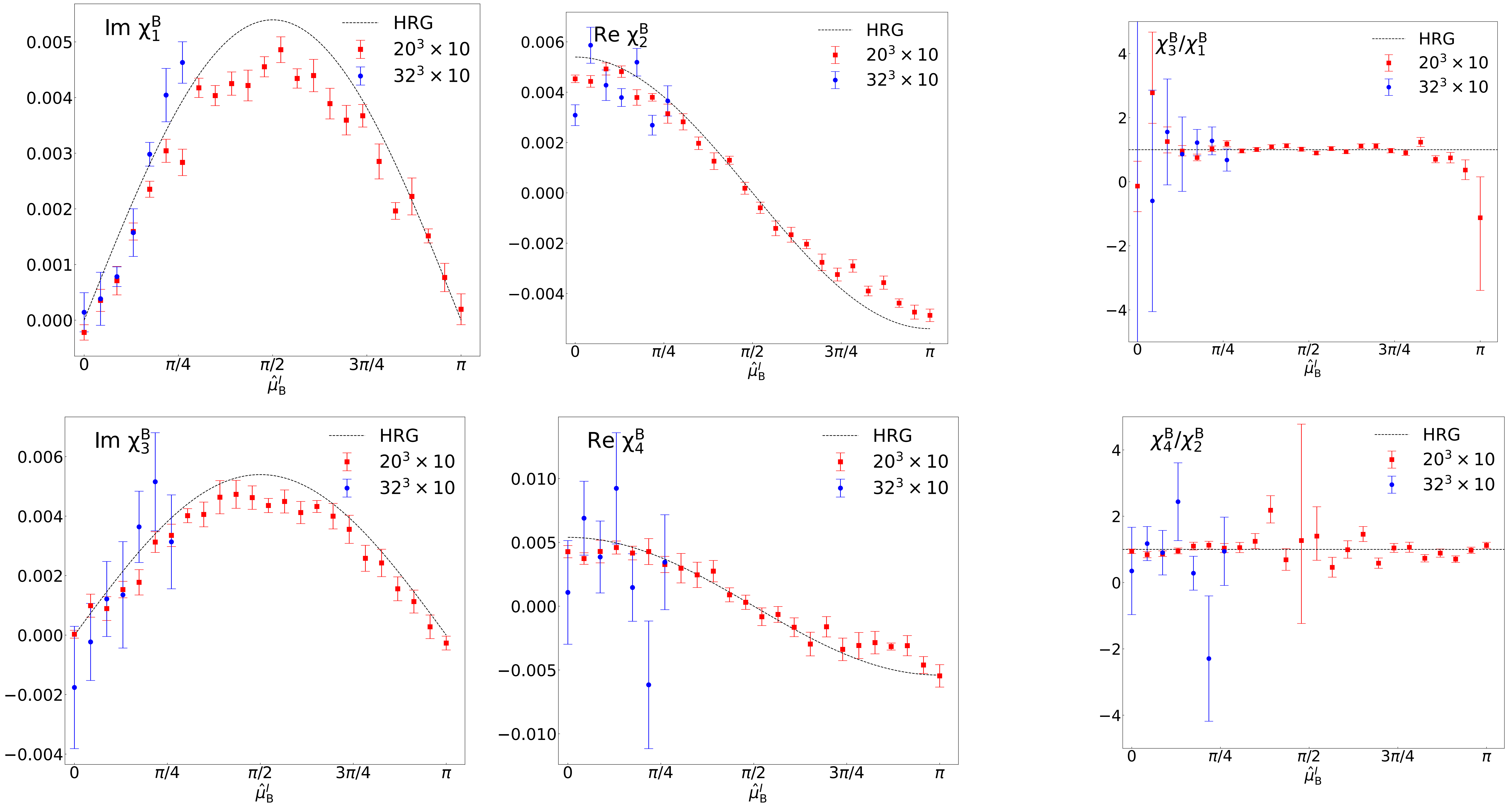
Yin Lin et al., *Phys.Rev.D* 103 (2021) 3, 034501

- Nt=10 at T=108 MeV⇒$a \approx 0.18\ fm$
- proton mass is inflated by about 4-5%, ~980MeV at $a \approx 0.18\ fm$, as estimated the from the difference in A fit parameter

A_{LAT}=4.584(35)
A_{HRG}=5.397(42)

- Consistent with direct computation in Nf=2+1+1 QCD

Volume dependences



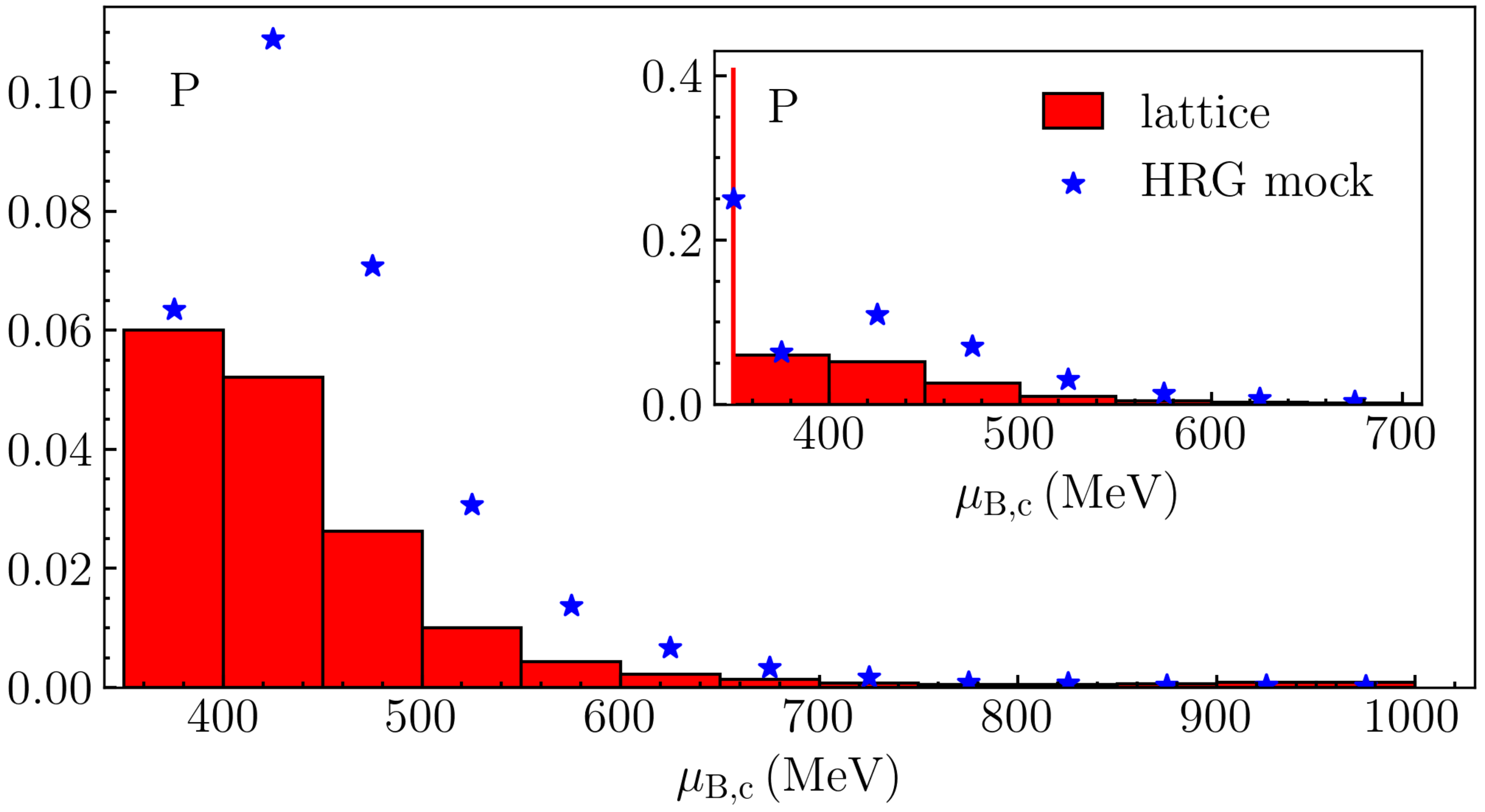
Summary

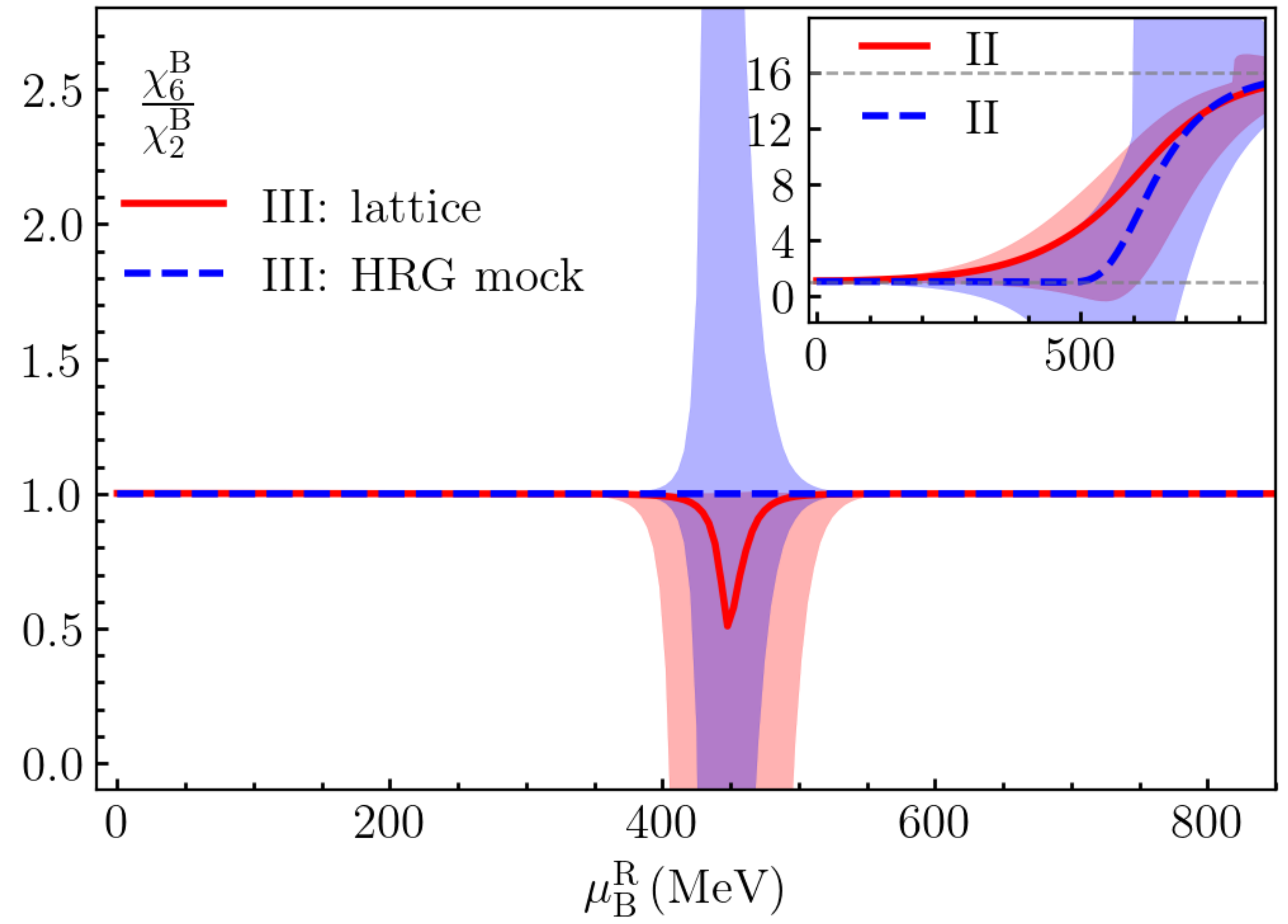
- Recent predictions on $T_c^{CEP} \approx 83 - 120$ MeV $< T_c^0$, extrapolated based on the lattice simulations at $T \gtrsim 120$ MeV

- Our LQCD simulations at Im(μ_B) @T=108 MeV: data compatible with smooth, analytic thermodynamics, obtained using fits with direct Padé approximants, physical motivated non-interacting, interacting and critical ansatz

- Only ~7% bootstrap samples gives for a CEP in $\mu_{B,c}^{CEP} \in$ 420-750MeV at T=108 MeV, however, it does manifest in the highest baryon susceptibilities that EXP can measure with reasonable precisions

- Where is the crossover point at T=108 MeV? Work in progress



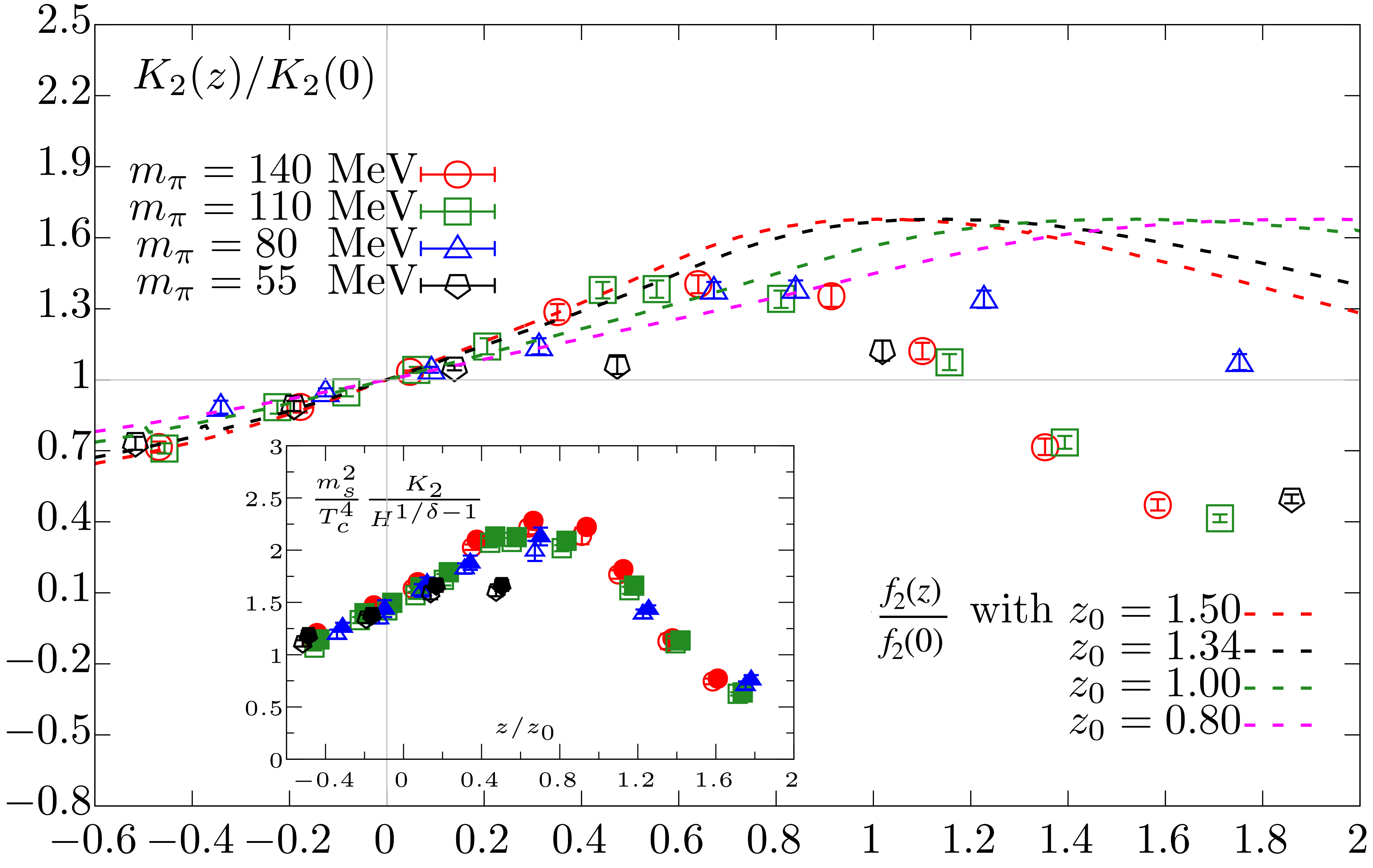


Backup

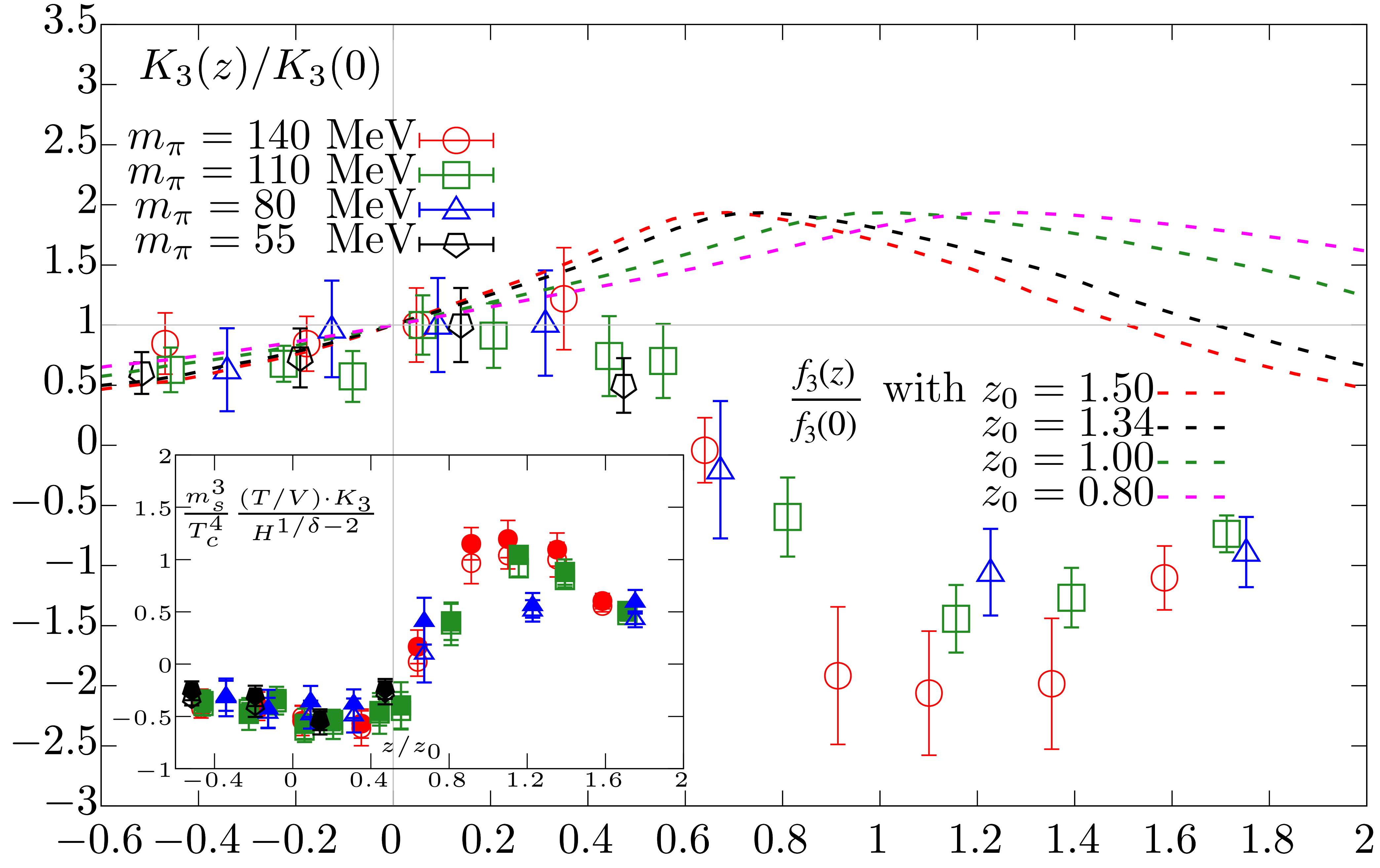
O(N) scaling behavior of $\mathbb{K}_n(\bar\psi\psi)$ at $\mu_B = 0$

HISQ, Nt=8, pion mass ranging from 140 to 55 MeV

$\mathbb{K}_2(z) \sim m_l^{1/\delta-1} f_2(z)$, $\mathbb{K}_2(z)/\mathbb{K}_2(z=0) \sim f_2(z)/f_2(0)$



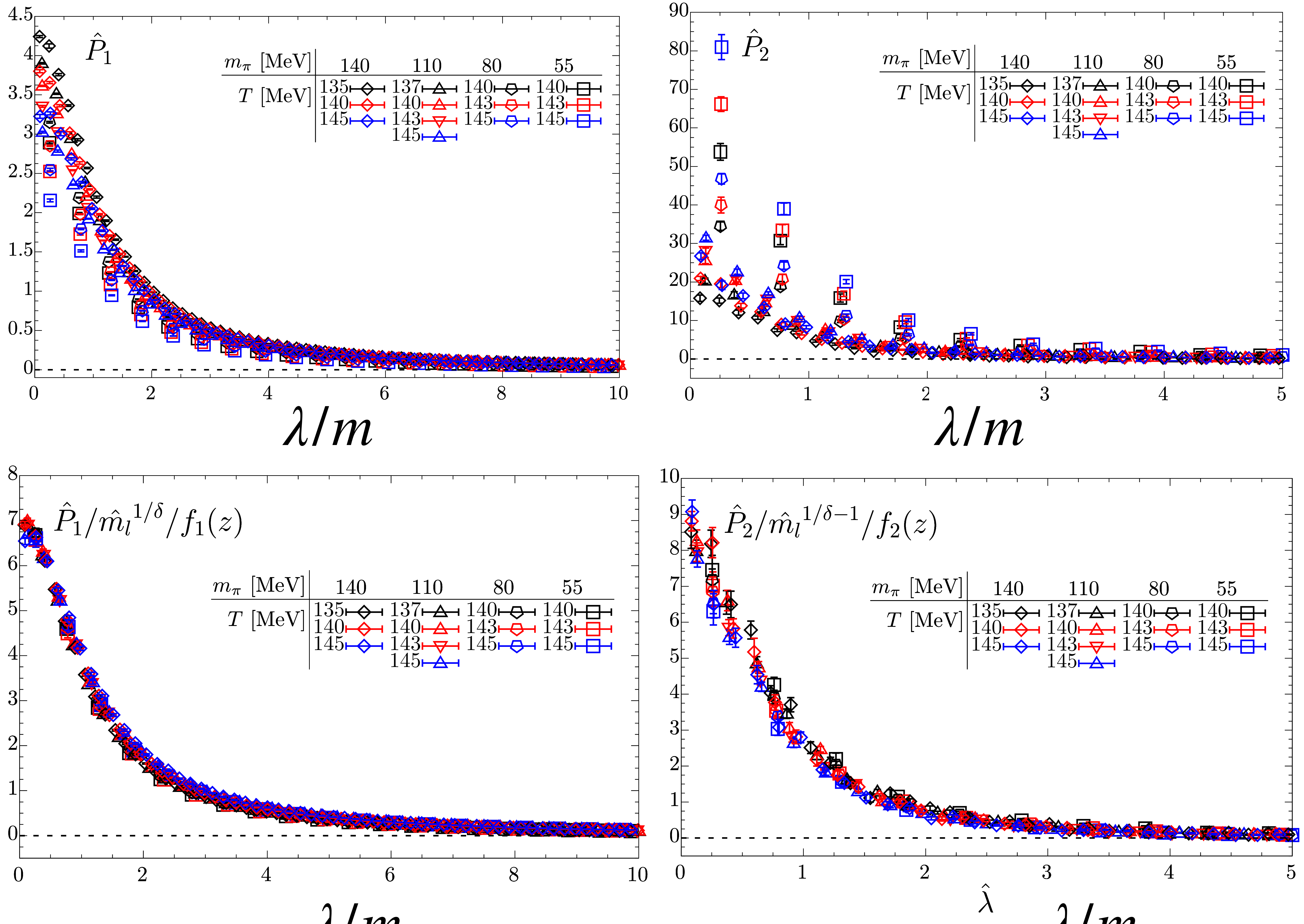
$\mathbb{K}_3(z) \sim m_l^{1/\delta-2} f_3(z)$, $\mathbb{K}_3(z)/\mathbb{K}_3(z=0) \sim f_3(z)/f_3(0)$



In the proximity of Tc: $\mathbb{K}_n(\bar\psi\psi) \sim m^{1/\delta-n+1} f_n(z)$!

HTD, W.-P. Huang, S. Mukherjee and P. Petreczky, PRL 131(2023)161903

Microscopic encoding of Macroscopic criticality



N_τ=8 lattices

Marco: $\mathbb{K}_n(\bar{\psi}\psi) \sim m^{1/\delta-n+1} f_n(z)$

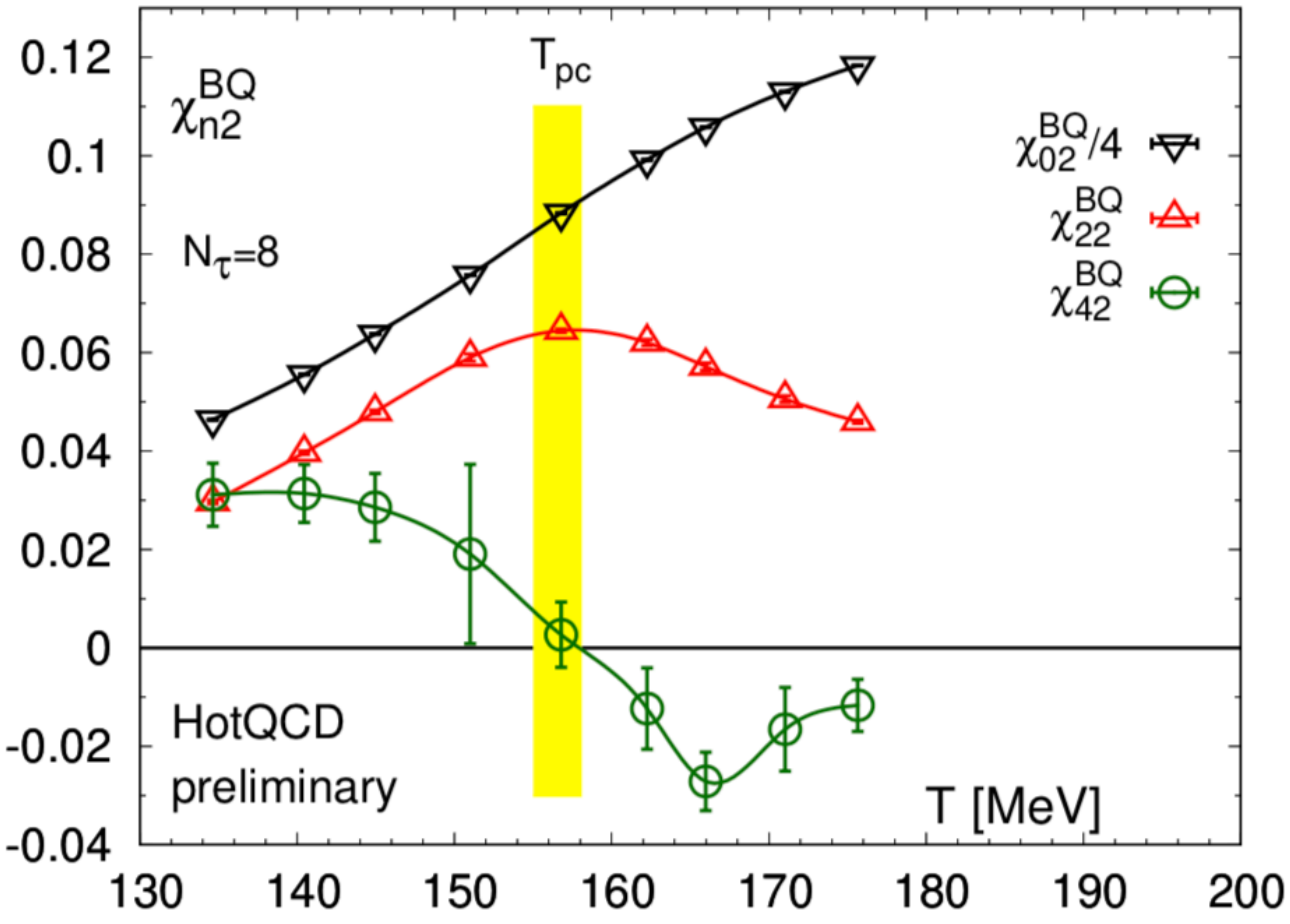
$$\mathbb{K}_n(\bar{\psi}\psi) = \int_0^\infty P_n(\lambda)\ \mathrm{d}\ \lambda$$

$$P_n(\lambda) = \int_0^\infty K_1[P_U(\lambda;m_l), P_U(\lambda_2;m_l), \cdots, P_U(\lambda_n;m_l)] \prod_{i=2}^{n} \mathrm{d}\lambda_i$$

Mirco: $\boxed{P_n(\lambda) = m^{1/\delta-n+1} f_n(z)\ g(\lambda/m)}$

HTD, W.-P. Huang, S. Mukherjee and P. Petreczky, PRL 131(2023)161903

Critical behavior from higher order cumulants at $\mu_B = 0$



$$\chi_{ijk}^{BQS}(T) = \frac{\partial^{i+j+k} P(T,\hat{\mu})/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Big|_{\hat{\mu}=0}$$

$$\boxed{t \sim \frac{T - T_c^0}{T_c^0} + \kappa_2^{B,0} \left(\frac{\mu_B}{T}\right)^2}$$

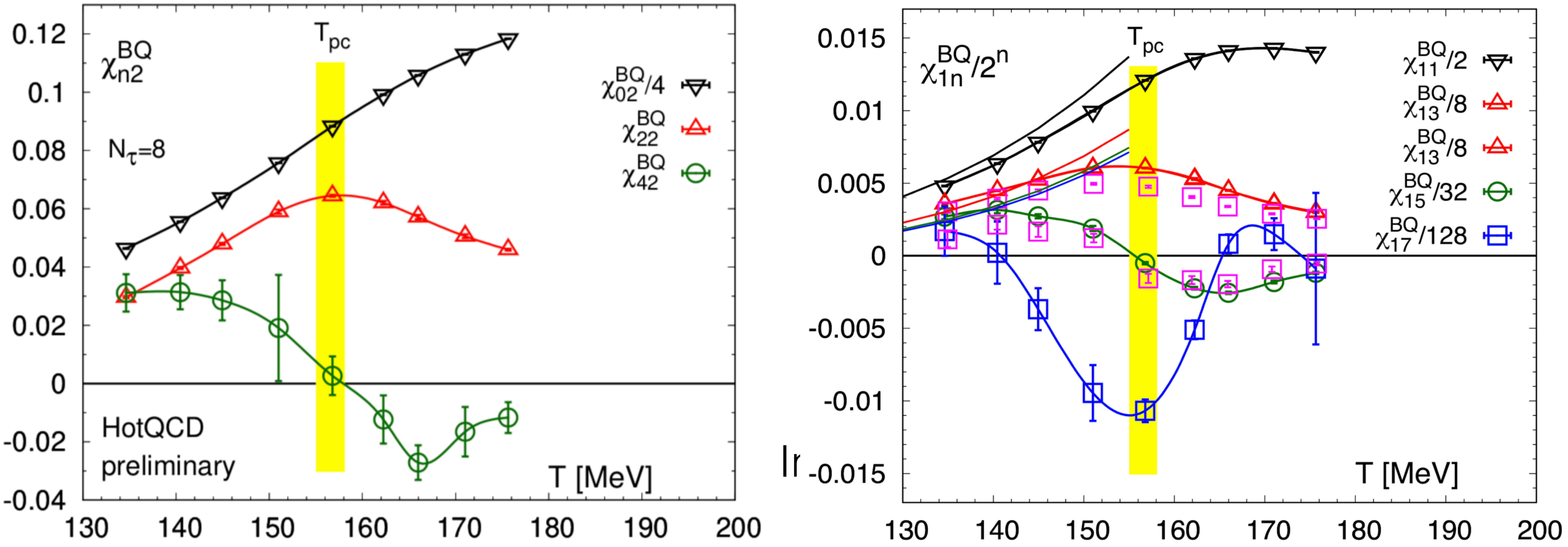
In the scaling regime: $\boxed{\frac{\partial}{\partial T} \simeq \frac{\partial^2}{\partial \mu_B^2}}$

F. Karsch, PoS CORFU2018 (2019) 163

Sign change seen at T>T_{pc} in χ_{42}^{BQ}

Sign change expected at T≳ 135 MeV in χ_{62}^{BQ}

Critical behavior from higher order cumulants at $\mu_B = 0$



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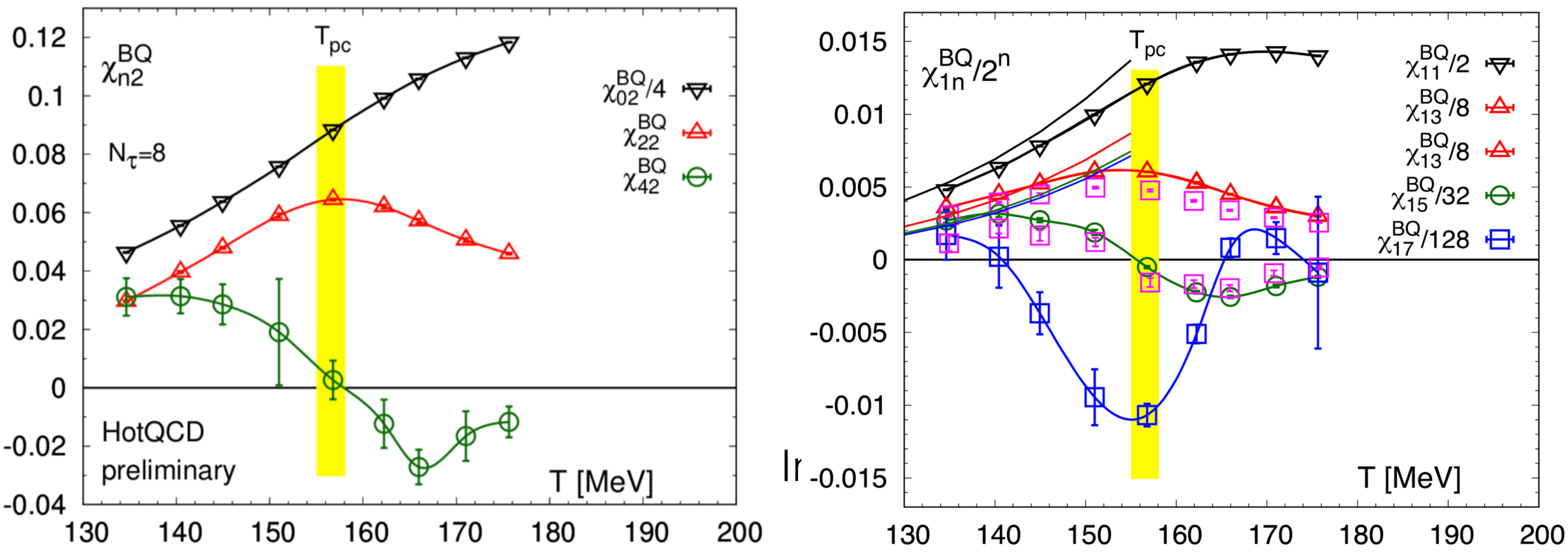
Many 8th order fluctuations turn to be negative at T ⩾ 135-140 MeV

Critical behavior from higher order cumulants at $\mu_B = 0$



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Many 8th order fluctuations turn to be negative at T ⩾ 135-140 MeV

Suggests singularity in the complex plane at T>135-140 MeV

More support for $T_{CEP} < T_c^0$

Multi-point Padé fits

$$P(x) \equiv R_n^m(x) = \frac{\sum_{i=0}^{m} a_i x^i}{1 + \sum_{j=1}^{n} b_j x^j}, x = \cosh(\mu_B) - 1$$

[3,3] [4,4] [5,5]