QCD phase structure at very high baryon density

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fQCD collaboration

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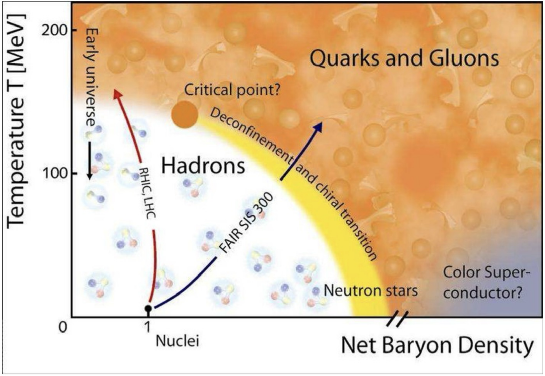
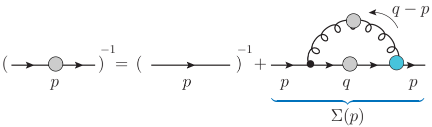
- Chiral phase transition with a CEP located at $(T, \mu_B) \sim (100 - 120, 600 - 700)$ MeV
  from
  functional QCD approaches(DSE and fRG),
  holographic QCD,
  extrapolation from lattice data
  machine learning from neutron star data....
- For extremely high density, liquid gas transition of nuclear matter, Chiral and deconfinement phase transition,
  quarkyonic phase?
  Color superconducting phase?
  Moat regime?
  ...

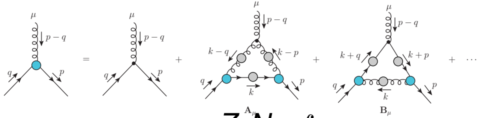


A Truncation scheme to close the DSEs

A closed set of DSEs that can well describe the QCD property in vacuum and also at finite temperature and chemical potential.(FG, *J. Papavassiliou, J. Pawlowski, PRD 103 (2021) 9, 094013*)



$$[S^{-1}(p)] = S_0^{-1} + g_s \int_q G_A^{\mu\nu}(q-p)\,(i\gamma_\mu)\,S(q)\,\Gamma_\nu^{A\bar{q}q}(q,-p$$



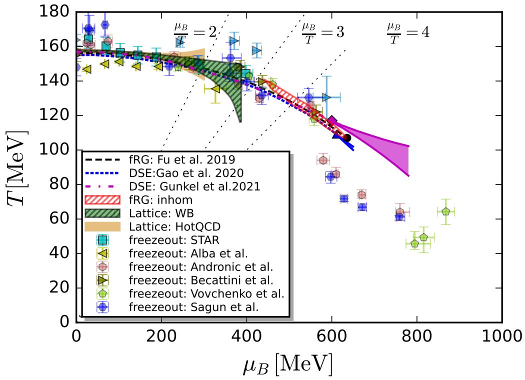
$$\Gamma_\mu^{A\bar{q}q}(q,-p) = g_s(i\gamma_\mu) + \mathbf{A}_\mu(q,-p) + \mathbf{B}_\mu(q,-p)$$

$$\mathbf{A}_\mu(q,-p) = \frac{Z_1 N_c}{2} \int_k \Gamma^{(0)}_{\mu\alpha\beta}\, G_A(k-q)\, G_A(k-p)\, \Gamma_\alpha^{A\bar{q}q}(k,-p)\, S(k)\, \Gamma_\beta^{A\bar{q}q}(q,-k)\,,$$

$$\mathbf{B}_\mu(q,-p) = -\frac{Z_1^f}{2N_c} \int_k G_A(k)\, \Gamma_\alpha^{A\bar{q}q}(k+p,-p) S(k+p)\,(i\gamma_\mu)\, S(k+q) \Gamma_\alpha^{A\bar{q}q}(q,-k-q)\,.$$

The DSE of gluon propagator is relatively separable which will be solved independently and applied as an input here.

Chiral Phase diagram for 2+1 flavour QCD can be directly obtained from quark propagator



The fQCD computations of chiral phase transition are converging:

- $T_c = 155\,\mathrm{MeV}$ and $\kappa \sim 0.015$
- Estimated range of CEP:
  $T \in (100, 120)$ MeV
  $\mu_B \in (600, 700)$ MeV

W.-j. Fu et al, PRD 101, 054032 (2020)

FG and J. Pawlowski, PRD 102, 034027 (2020)

FG and J. Pawlowski, PLB 820, 136584(2021)

P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

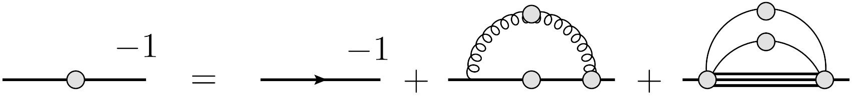
Towards the larger chemical potential region

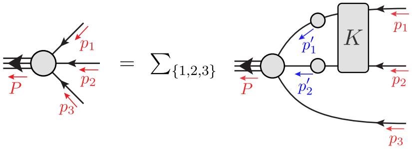
The hadron resonance channel becomes important at high density and the Faddeev equation for hadron is required.

The quark gap equation can be written as:

$$S^{-1}(p;\mu_q) = S_0^{-1}(\tilde{p}) + \Sigma^{\mathrm{G}}(p;\mu_q) + \Sigma^{\mathrm{N}}(p;\mu_q),$$

with diagrammatic representation as:





[1] FG, Yi Lu, Si-xue Qin, Bai Zhan, Lei Chang, Yu-xin Liu, PRD 111, L091503 (2025)

At zero temperature:

- Silver Blaze property at low chemical potential;
- Liquid gas transition of nuclear matter at $\mu_B \approx 922$ MeV;
- Chiral phase transition of QCD at $\mu_B \approx 1100$ MeV .

Silver Blaze property:
QCD matter observables remain unchanged for different chemical potentials up to the liquid gas transition

The gap equation and the Faddeev equation does not contain singularities and keep the analytic continuation from the vacuum:

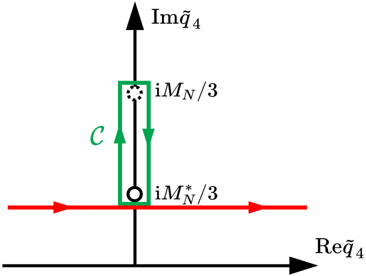
$$S(p;\mu_q) = S_{\text{vac}}(\tilde{p}), \quad \Gamma^{(3)}(p_1, p_2, P; \mu_q) = \Gamma^{(3)}_{\text{vac}}\left(\tilde{p}_1, \tilde{p}_2, \tilde{P}\right).$$

Liquid gas transition

The exploded view:

- **the happening of the liquid gas transition**:
  when μ_B exceeds the nucleon mass pole $M_N = 938$ MeV, the gap equation contains the first singularity and the analytic continuation is broken;
- **the nucleon mass shifting**:
  At the phase transition point, the Faddeev equation changes through the quark propagator in its kernel;
- **the finite shifting causing a first order phase transition**:
  This new “liquid solution” contains a new singularity at M_N^*, representing the in-medium nucleon pole mass and corresponding to the nuclear liquid-gas phase transition chemical potential at zero temperature.
- **the true phase transition at $\mu_B = M_N^* = 922$ MeV**:
  Essentially, the liquid solution represents a self-consistent solution of both the quark gap and the Faddeev equations in medium

The new solution is the vacuum solution with in medium nucleon mass M_N^* plus the contour contribution of the pole jumping from M_N to M_N^*:



The constructed liquid solution is:

$$S_{\text{liq}}^{-1}\left(p;\mu_q\right)=S_{\text{vac},M_N^*}^{-1}\left(\tilde{p}\right)-\delta f_{\mathcal{C}}\left(\tilde{p}\right)S_{\text{vac},M_N^*}^{-1}\left(\tilde{p}\right)$$

One can prove that the above construction satisfies the gap equation.

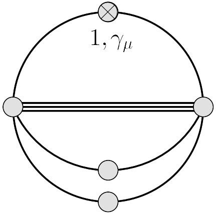
The proof of the validation of the liquid solution

For the self energy in Gap equation, for instance, Σ^{G}. One can deform the integral from $p_4 + iM_N$ to $p_4 + iM_N^*$

$$
\begin{aligned}
&\int \mathrm{d}^4 q D^{ab}_{\mu\nu}(\tilde{p}-\bar{q}) \frac{\lambda^a}{2}\gamma_\mu S_{\mathrm{vac},M_N^*}(\bar{q}) \Gamma^b_\nu(\bar{q},\tilde{p}) \\
=& \int \mathrm{d}^4 q D^{ab}_{\mu\nu}(\tilde{p}-\tilde{q}) \frac{\lambda^a}{2}\gamma_\mu S_{\mathrm{vac},M_N^*}(\tilde{q}) \Gamma^b_\nu(\tilde{q},\tilde{p}) \\
&+ \int \mathrm{d}^3 \vec{q} \oint_{\mathcal{C}} \mathrm{d}\tilde{q}_4 D^{ab}_{\mu\nu}(\tilde{p}-\tilde{q}) \frac{\lambda^a}{2}\gamma_\mu S_{\mathrm{vac},M_N^*}(\tilde{q}) \Gamma^b_\nu(\tilde{q},\tilde{p}),
\end{aligned}
$$

The contour in the third line is the difference δf between liquid solution and the vacuum solution, and for the full gap equation, we have:

$$
\begin{aligned}
\mathrm{RHS} = S_0^{-1}(\tilde{p}) + \Sigma^{\mathrm{G}}_{\mathrm{liq}} + \Sigma^{\mathrm{N}}_{\mathrm{liq}} =& S_0^{-1} + \Sigma^{\mathrm{G}}_{\mathrm{vac},M_N^*} + \Sigma^{\mathrm{N}}_{\mathrm{vac},M_N^*} - \delta f_{\mathcal{C}} S^{-1}_{\mathrm{vac},M_N^*} \\
=& S^{-1}_{\mathrm{vac},M_N^*} - \delta f_{\mathcal{C}} S^{-1}_{\mathrm{vac},M_N^*} \\
\equiv& S^{-1}_{\mathrm{liq}} = \mathrm{LHS}.
\end{aligned}
$$



The scalar and vector charge are:

$$\mathcal{S} = \frac{2M_N}{3}\int \frac{d^4q}{(2\pi)^4}\frac{g_s(q^2,0)}{q^2+M_N^2}, \quad g_s(-M_N^2) = \frac{\sigma_N}{2m_q}$$

$$\mathcal{V} = \frac{2M_N}{3}\int \frac{d^4q}{(2\pi)^4}\frac{g_v(q^2,0)}{q^2+M_N^2}, \quad g_v(-M_N^2) = 1$$

$$\delta_{\mathcal{C}}\mathcal{S} = \frac{2M_N^*}{3}\oint_{\mathcal{C}}\frac{dq_4}{2\pi}\int\frac{d^3q}{(2\pi)^3}\frac{g_s(-M_N^2,0)}{q^2+(M_N^*)^2} = \frac{1}{2}\oint_{\mathcal{C}}\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[S_{\mathrm{vac},M_N^*}\Sigma(q;\mu_q)S_{\mathrm{vac},M_N^*}]$$

$$= \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[S_{\mathrm{vac},M_N^*}\delta f_{\mathcal{C}}(q;\mu_q)] \cong \frac{1}{2}\langle\bar{q}q\rangle\delta\bar{f}$$

$$\delta_{\mathcal{C}}\mathcal{V} = \frac{2M_N^*}{3}\oint_{\mathcal{C}}\frac{dq_4}{2\pi}\int\frac{d^3q}{(2\pi)^3}\frac{g_v(-M_N^2,0)}{q^2+(M_N^*)^2} \cong \frac{1}{2}\langle\bar{q}\gamma_0 q\rangle\delta\bar{f} = \frac{1}{6}n_B^0\delta\bar{f}$$

From Faddeev equation as an eigenvalue problem, one has:

$$\lambda^{\mathrm{vac}}(p^2 = -M_N^{*2}) = 1 + Z\frac{M_N^{*2} - M_N^2}{M_N^2},$$

With the in-medium nucleon eigenvalue $\lambda(p^2 = -M_N^{*2}) = 1$ we have:

$$M_N^* = (1 - \delta\bar{f}/Z)M_N.$$

and one has the binding energy and also the location of the liquid gas transition of nucleon as:

$$\delta\bar{f} = \frac{81\pi^4 Z^3 M_\pi^4 f_\pi^4}{8M_N^6\sigma_N^2} = 0.0164, \quad \mu_B^* = M_N^* = 939 - 15.9 = 923.1\mathrm{MeV},$$

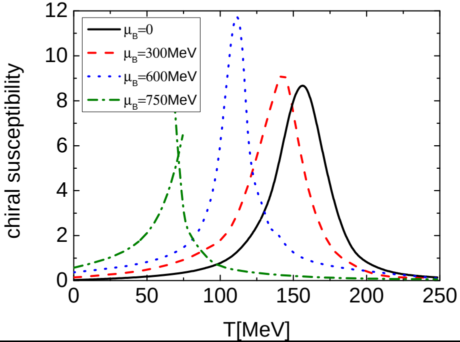
The saturation density is:

$$n_{\mathrm{B}}^0 = \frac{\sqrt{8\delta f/Z^3}}{27\pi^2}M_N^3 = \frac{M_\pi^2 f_\pi^2}{3\sigma_N} = 0.15\,\mathrm{fm}^{-3}.$$

These results are in precise agreement with the experiments.

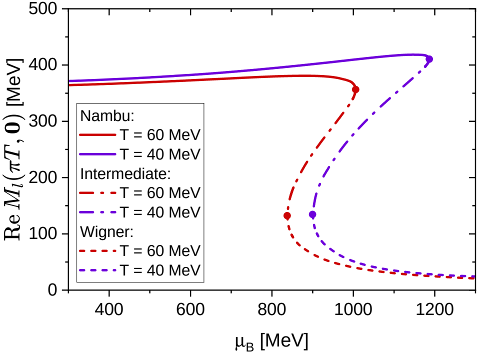
The on shell effect of nucleon in hadron resonance channel is negligible in quark gap equation, with $\Delta S/S \sim \delta f \approx 0.016$.

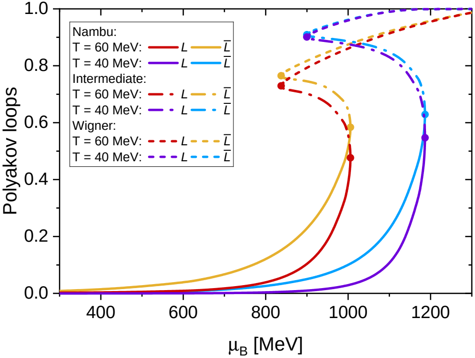
One can study the chiral phase transition in the current minimal scheme without considering the hadron resonance channel. Scanning the susceptibility in the whole temperature-chemical potential plane:



- low chemical potential, crossover
- large chemical potential, first order phase transition
- in first order phase transition region, what is the complete picture of the coexistence region?

Coexistence region: stable, meta stable and unstable phase.





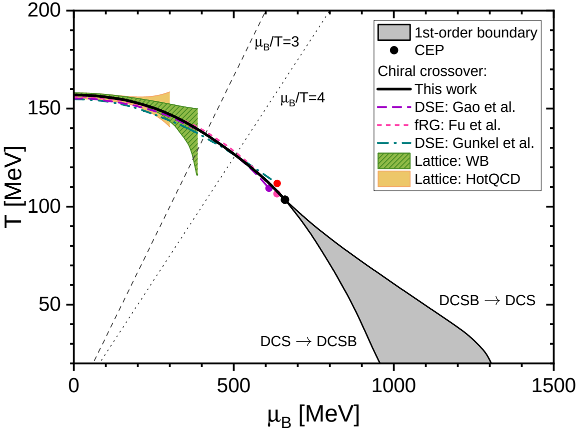
- Stable and Metastable phase: carefully choose the initial guess.
- More difficult for unstable phase, one requires Homotopy method:

$$G_q^{\text{init.}}(p;\eta) = \eta G_q^N(p) + (1-\eta) G_q^W(p)$$

Yi Lu, **FG**, Yu-xin Liu. arXiv:2509.02974.

QCD phase diagram

Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



- Coexistence region slightly above liquid gas transition: stable and meta stable phase (**nucleation**); unstable phase (**spinodal decomposition**)
- Ideal phase transition for Chiral PT at $\mu_B \approx 1100$ MeV which corresponds to 2-3 n_B^0

[1] Yi Lu, **FG**, Yu-xin Liu. arXiv:2509.02974.

Summary

The conclusions and discussions:

- The functional QCD approaches **calculate** the chiral PT and give the CEP located at $\mu_B \approx 600 - 700$ MeV.
- **Liquid gas transition** of nuclear matter can be described via incorporating **hadron resonance channel** in DSE.
- The on shell hadron has little impact on quark gap equation with $\delta S/S \sim 0.016$.
- Dynamics of first order phase transition in coexistence region:
  stable and metastable phase (**Nucleation**),
  unstable phase(**Spinodal decomposition**).
- An **ordering** of phase transitions at $T = 0$:
  liquid gas transition at $n_B^0 \sim 0.15\text{fm}^{-3} \lesssim$ Chiral PT at 2-3 n_B^0

Thank you!