

IN-MEDIUM EFFECTS FROM S-MATRIX

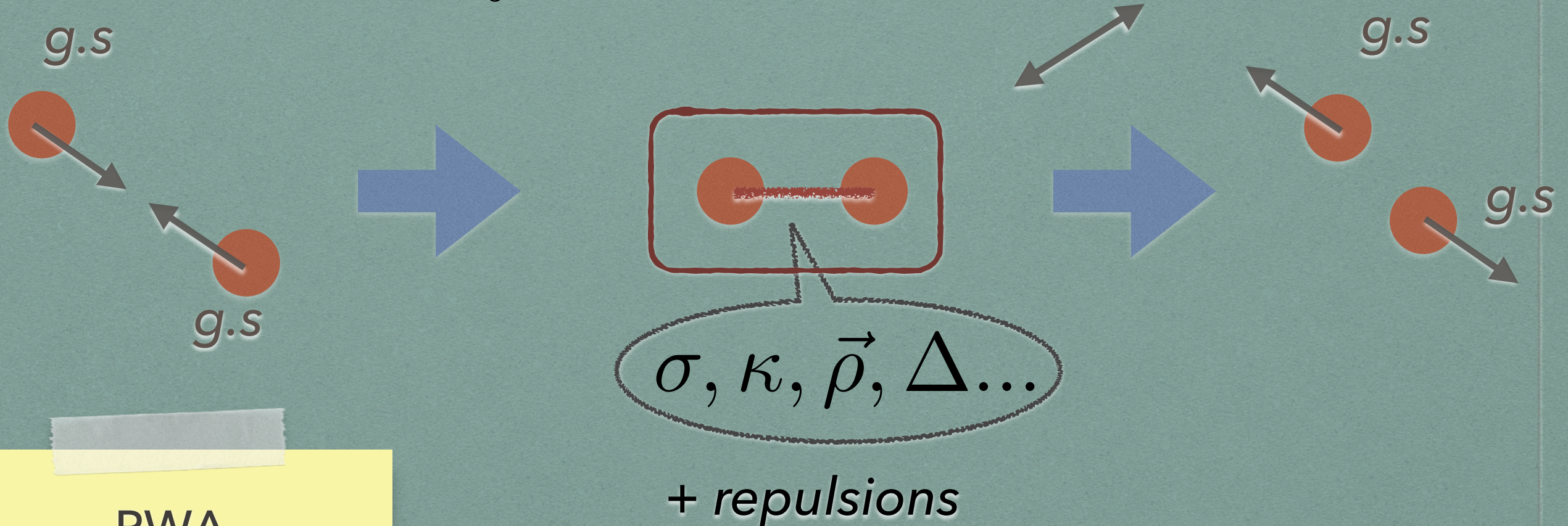
POK MAN LO (盧博文)

University of Wroclaw

PHD2025 (CCNU)
10.2025, WUHAN, CHINA

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \text{tr} (\delta_E) .$$



PWA

X

S-matrix thermo.

$$\delta \longrightarrow \mathcal{Q}(M) \equiv \frac{1}{2} \text{Im} (\text{tr} \ln S)$$

DENSITY OF STATE & S-MATRIX FORMULATION OF STATISTICAL MECHANICS

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969).

R. Venugopalan and M. Prakash,
Nucl. Phys. A546, 718 (1992).

(study notes) PML, EPJC **77** no.8 533 (2017)

PML PRD **102**, 034038 (2020)

PHASE SHIFT AND DENSITY OF STATES

particle in a box

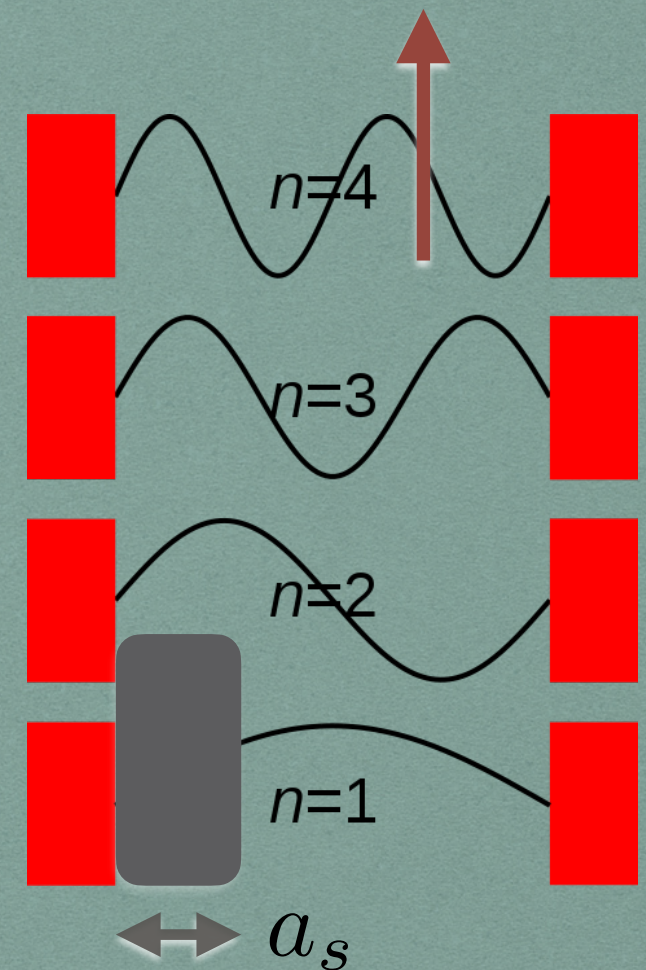
$$\psi \sim \sin(k^{(0)}x) \quad k^{(0)} = \frac{n\pi}{L}$$

in the presence of a scattering potential

$$\psi \sim \sin(kx + \delta(k))$$

density of states

$$kL + \delta(k) = n\pi \quad \longrightarrow \quad \frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi} \frac{d\delta}{dk}$$

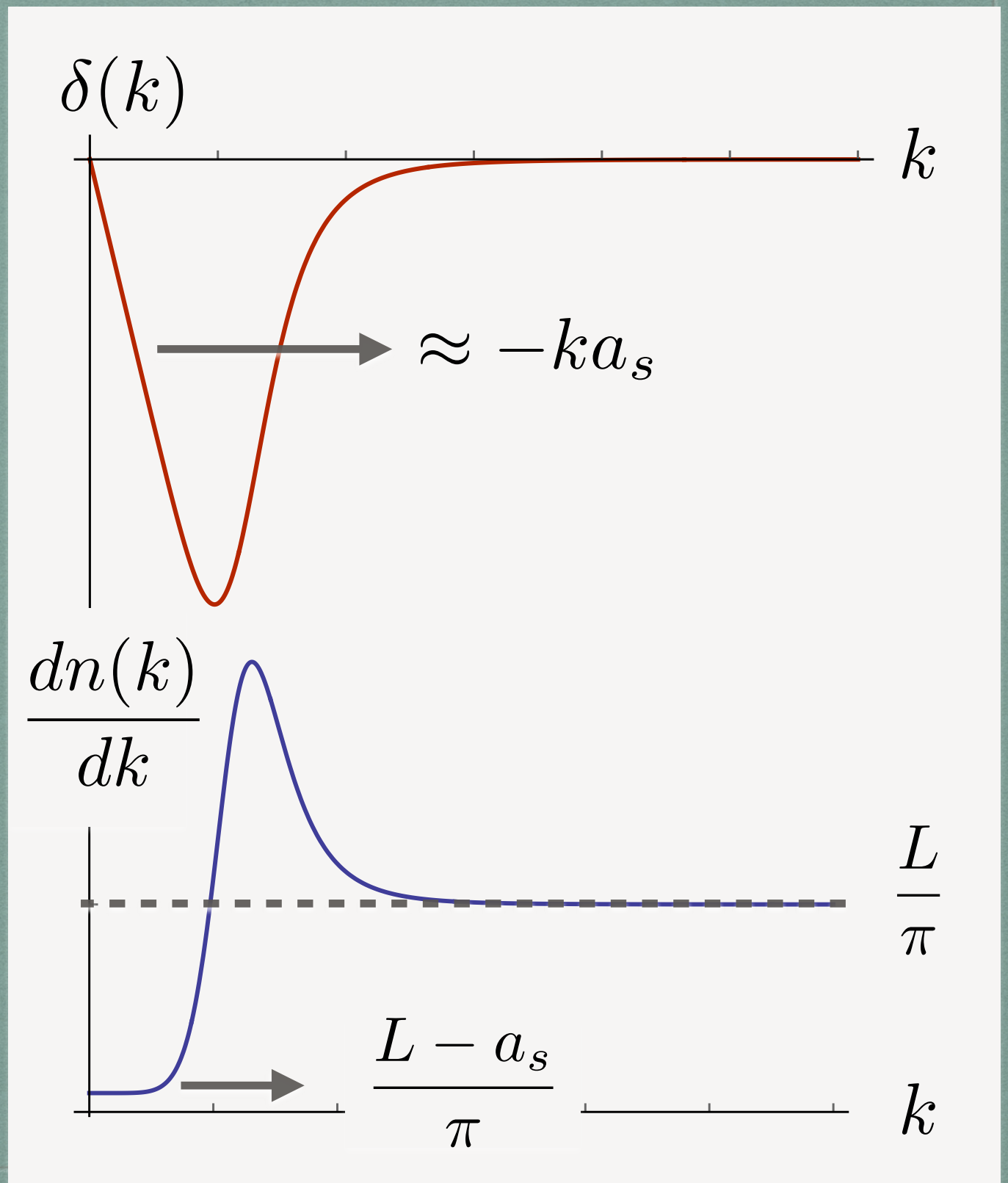


PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi} \frac{d\delta}{dk}$$

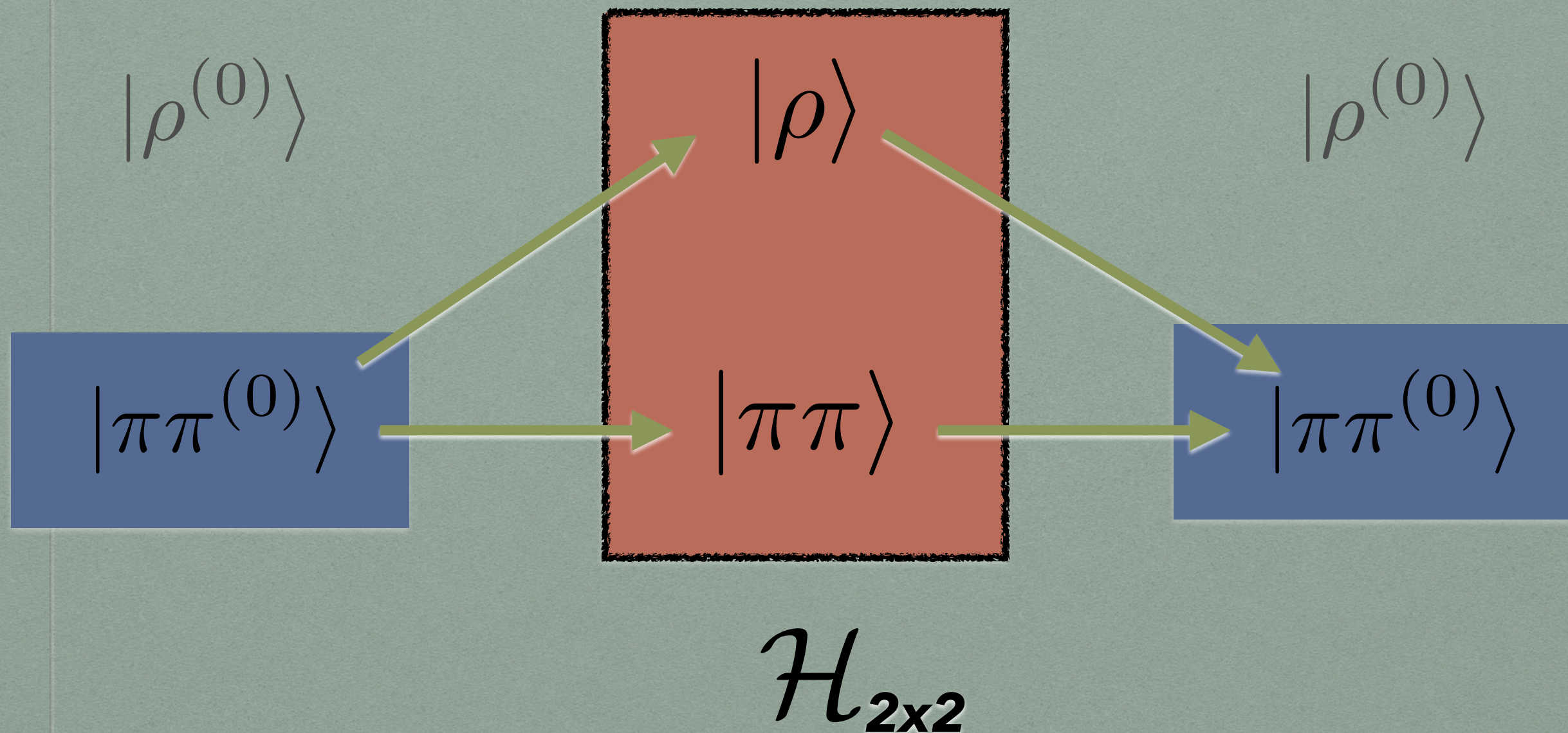
*change in d.o.s.
due to int.*

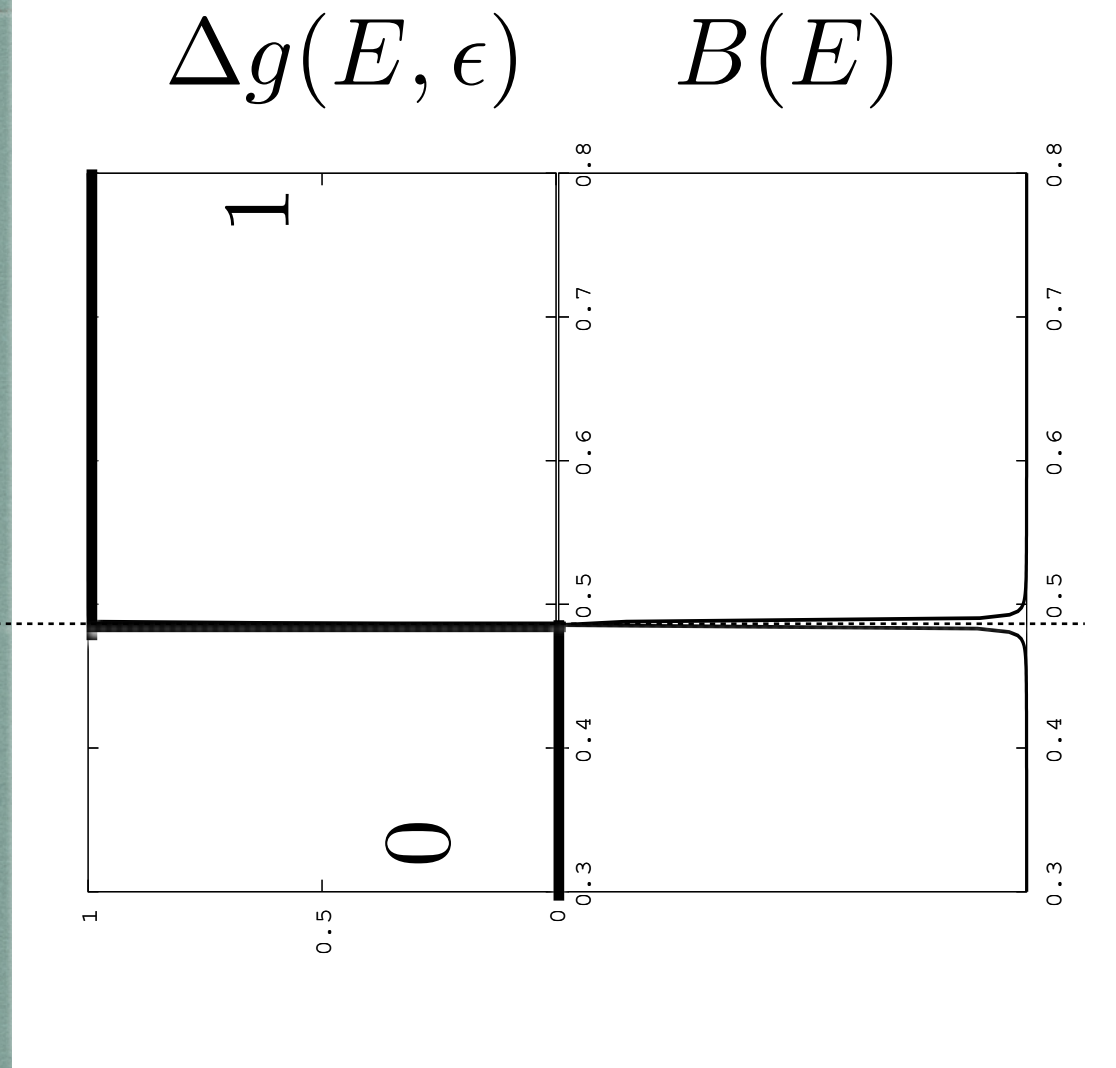
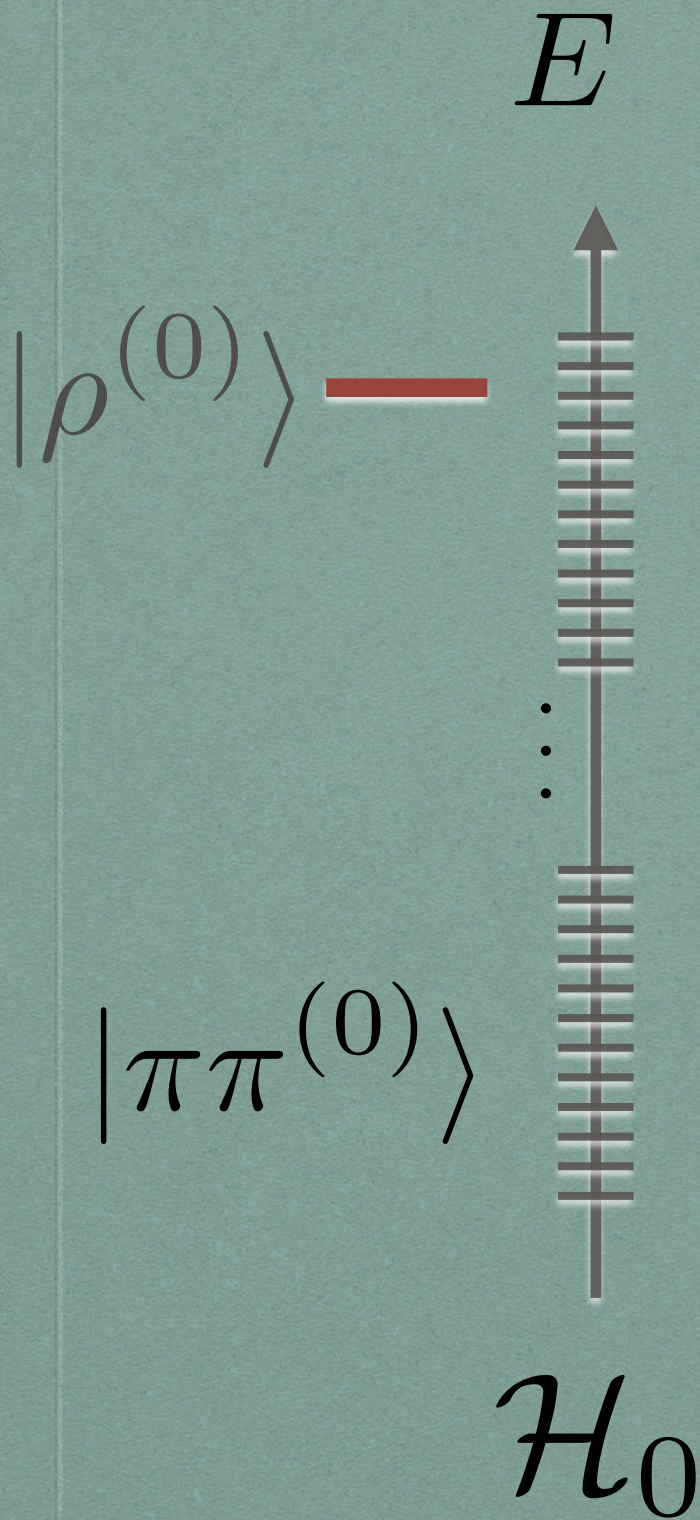
Effect of repulsive
interaction:
pushing states from low k
to high k



phase shift and d.o.s. (schematics)

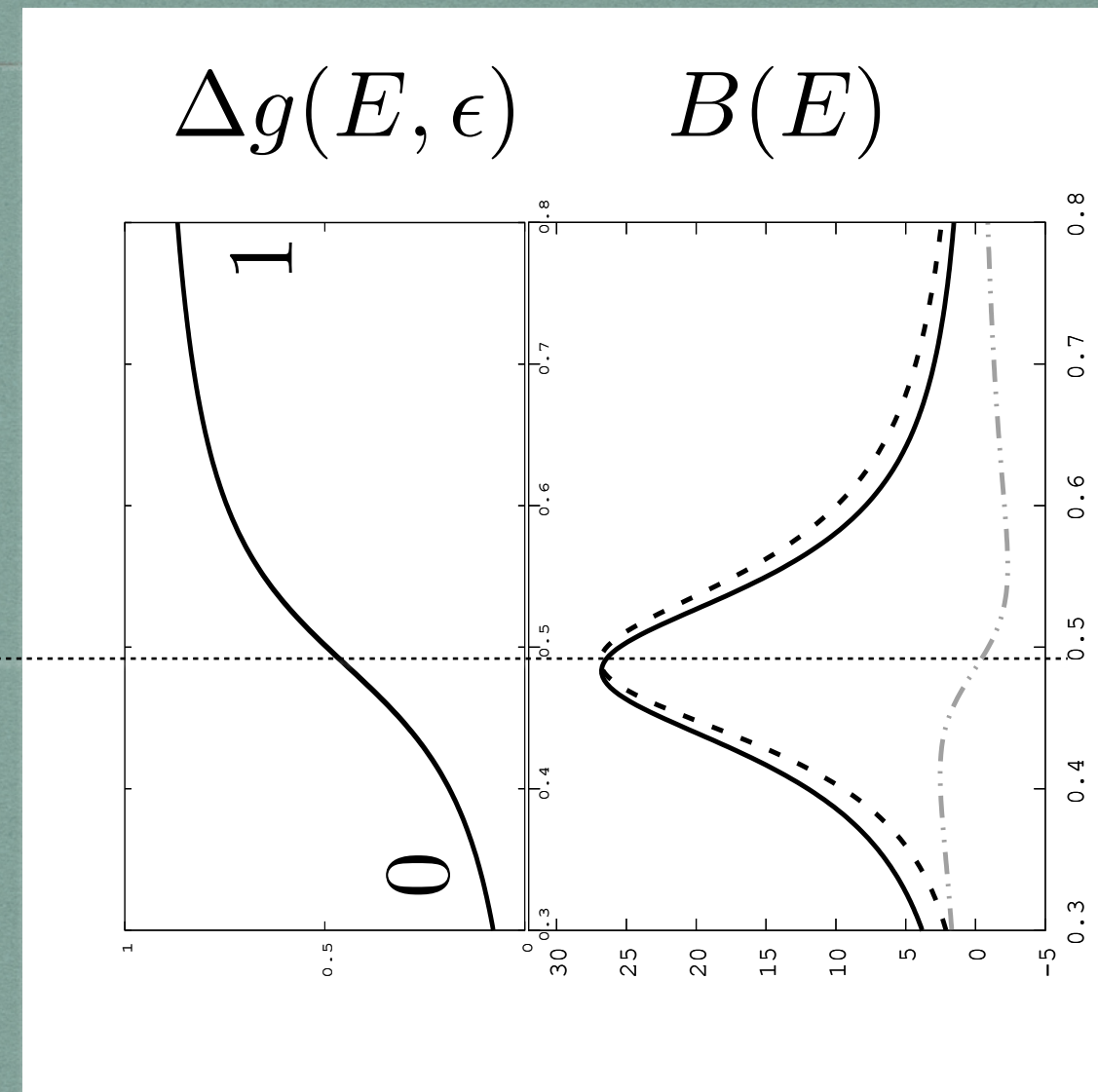
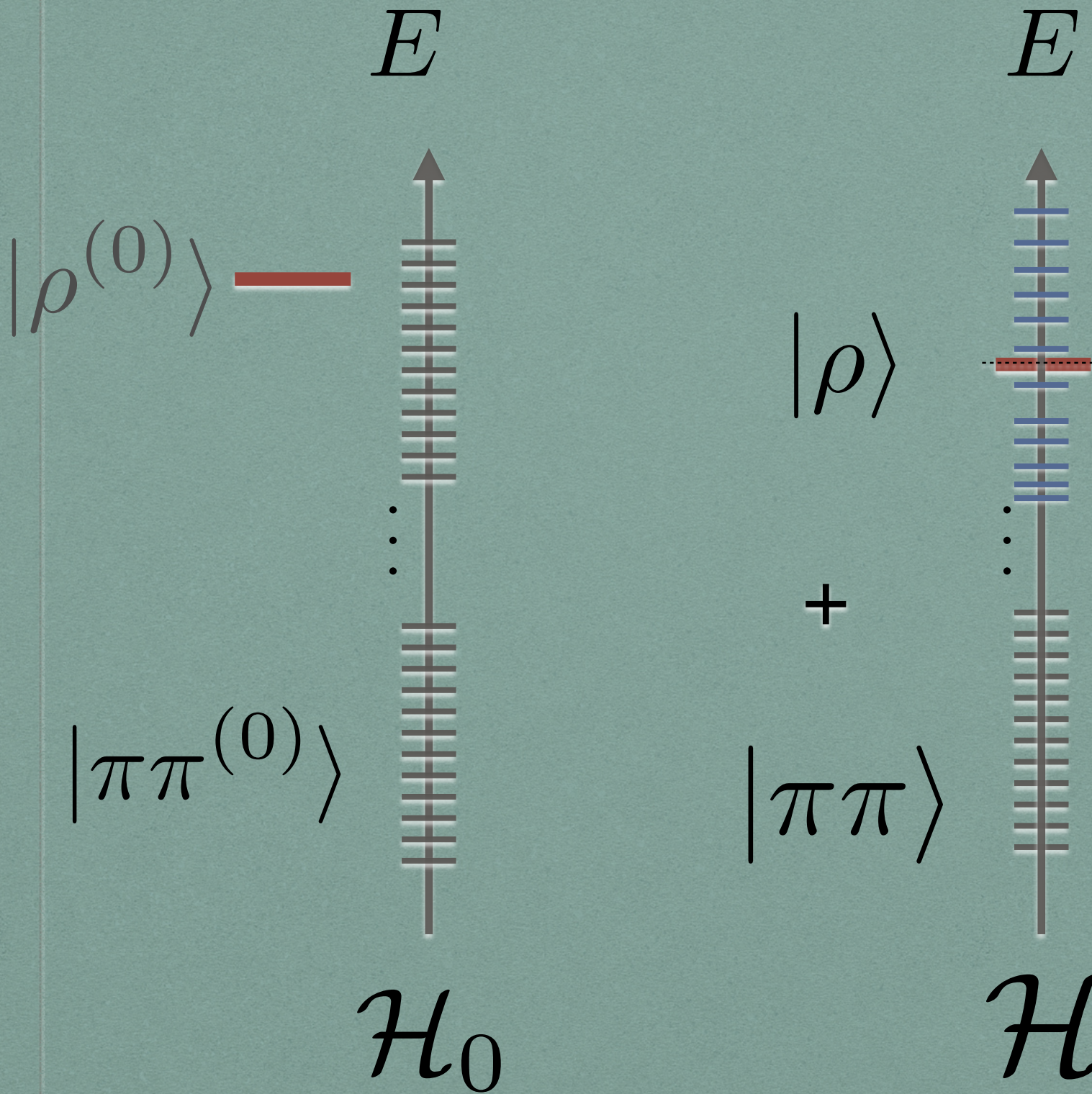
SCATTERING THEORY VS HAMILTONIAN (LEE MODEL)





$$g(E, \epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E, \epsilon)$$



$$g(E, \epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E, \epsilon)$$

$$\text{Tr } e^{-\beta \mathcal{H}_0} \quad \text{vs} \quad \text{Tr } e^{-\beta \mathcal{H}} = A_\rho + \Delta A_{\pi\pi}$$

PHYSICS OF B

$$\delta = -\text{Im Tr ln } G_{\rho}^{-1}$$

$$B = 2 \frac{\partial}{\partial E} \delta$$

$$= -2 \text{Im} \frac{\partial}{\partial E} \ln G_{\rho}^{-1}$$

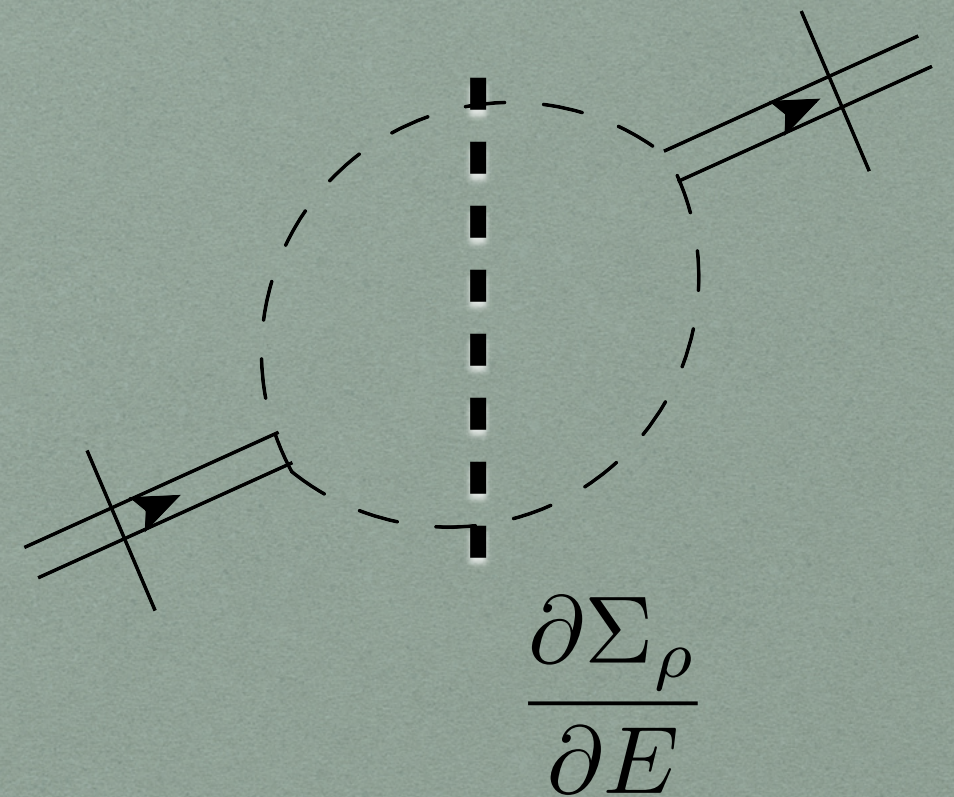
$$= -2 \text{Im}[G_{\rho}](2E) + 2 \text{Im}\left[\frac{\partial \Sigma_{\rho}}{\partial E} G_{\rho}\right]$$

$$= A_{\rho}(E) + \Delta A_{\pi\pi}$$

$$-\frac{\partial}{\partial E} \int d\phi_E T_{\text{re}} \quad \text{pipi} \rightarrow \text{pipi}$$

physical interpretation:

*contribution from
correlated pi pi pair*



PHYSICS OF B

to rho or not to rho?
that's OUT of the question!

resonance's picture:

$$B(E) = A_\rho(E) + \Delta A_{\pi\pi}$$

rho

scattering picture:

$$B_1 = \frac{\partial}{\partial E} \text{Tr } \hat{t}_{\text{re}}$$

pipi -> pipi

$$B_2 = \frac{1}{2} \text{Im Tr } \hat{t}^\dagger \overleftrightarrow{\partial_E} \hat{t}$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

$$-\frac{\partial}{\partial E} \int d\phi_E T_{\text{re}} \quad \text{pipi} \rightarrow \text{pipi}$$

$$\frac{\partial \Sigma_\rho}{\partial E}$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical

dynamical

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

$$\operatorname{tr}\{\dots\} \xLeftrightarrow{\text{QM}} \int d^3 q \, \langle q | \dots | q \rangle \xrightarrow{\text{N-body}} \int (d k) \, \langle k_1 k_2 | \dots | k_1 k_2 \rangle \xLeftrightarrow{\text{QFT}}$$

Fock Space Expansion

$$\int (d k) (\dots) \rightarrow \int \frac{d^3 p_1}{(2\pi)^3} \frac{1}{2E_1} \frac{d^3 p_2}{(2\pi)^3} \frac{1}{2E_2} (\dots)$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical

dynamical

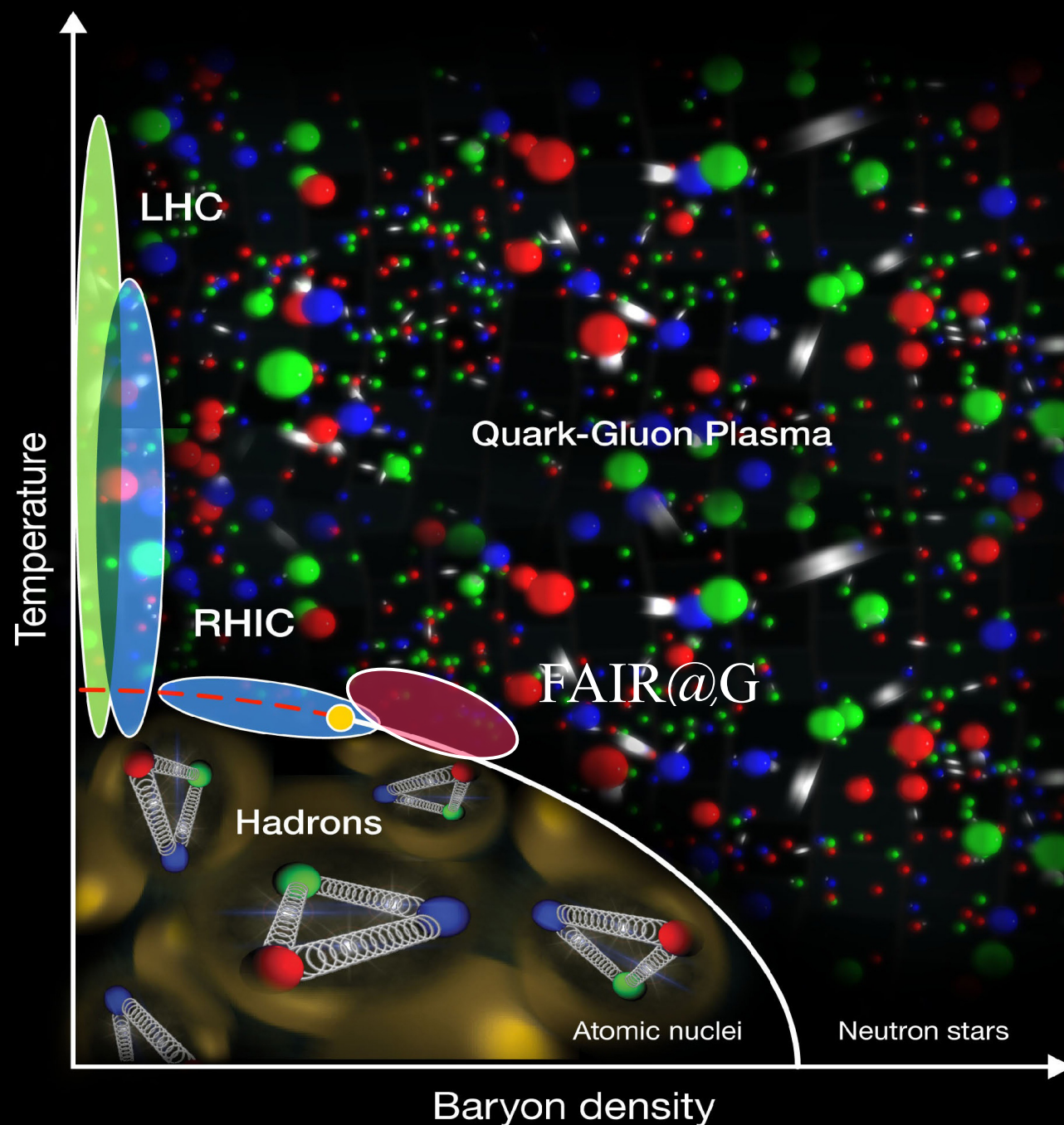
$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

***completeness assumption:
via asymptotic states***

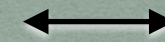
$$b_{\pi\pi} \xi_\pi^2 + b_{\pi K} \xi_\pi \xi_K + b_{\pi N} \xi_\pi \xi_N + b_{\pi\eta} \xi_\pi \xi_\eta + b_{K\bar{K}} \xi_K \xi_{\bar{K}} + \dots$$

$$b_{\pi N} = 2 \times b_{\pi N}^{I=1/2} + 4 \times b_{\pi N}^{I=3/2} \quad \text{orbital } L: \\ S, P, D, F, \text{ etc..}$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS



dynamical



$$\left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

completeness assumption:
asymptotic states

$$\pi\eta\xi\pi\xi\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \dots$$

$$- 4 \times b_{\pi N}^{I=3/2} \quad \text{orbital } L: \\ S, P, D, F, \text{ etc..}$$

S-M
STA

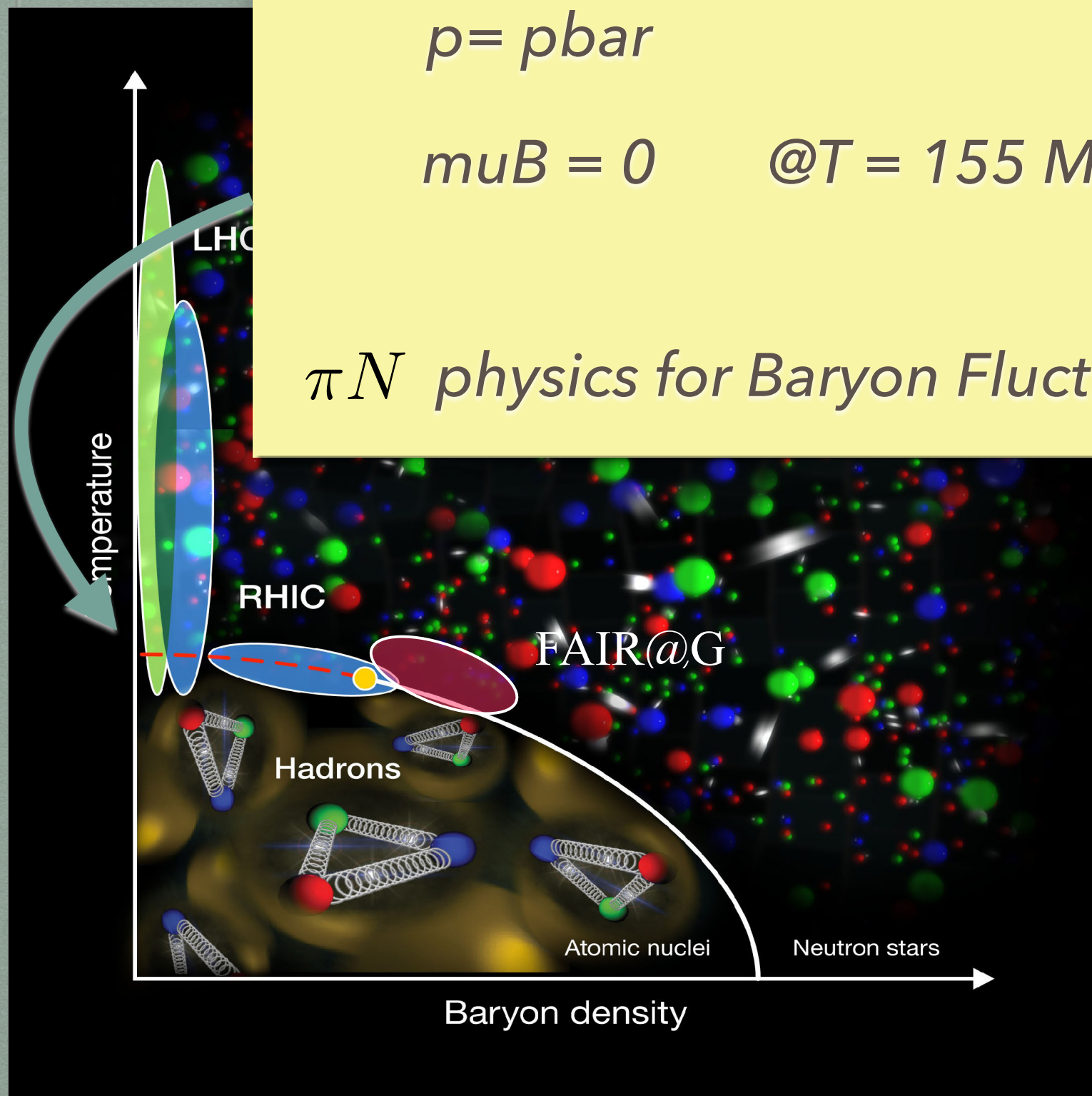
LHC conditions

$$\langle \pi \rangle / \langle p \rangle \approx 15$$

$$p = p_{\text{bar}}$$

$$\mu_B = 0 \quad @T = 155 \text{ MeV}$$

πN physics for Baryon Fluctns



F

ical

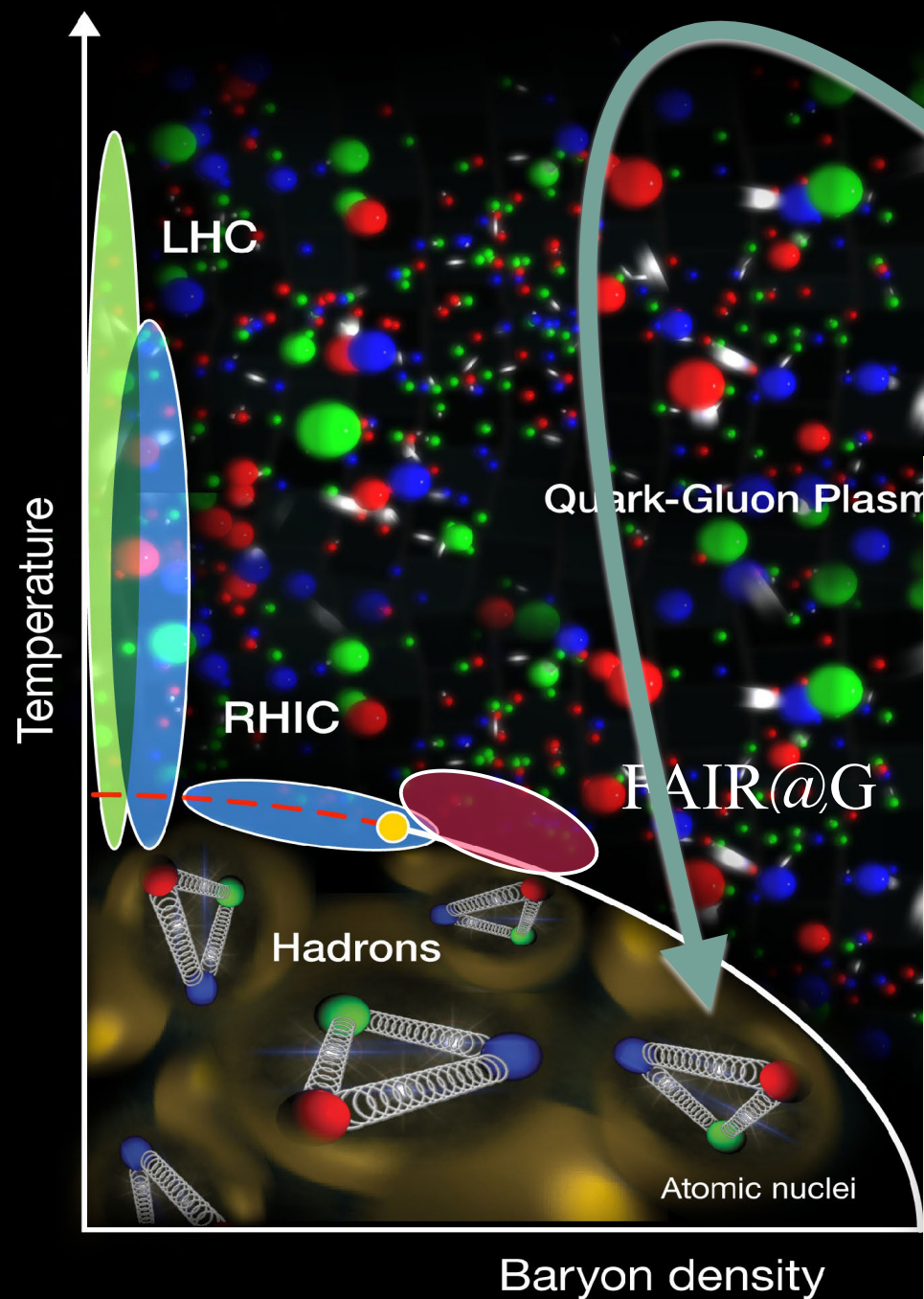
$$\left. \begin{matrix} \bar{E} \\ E \end{matrix} S_E \right\} c$$

completeness assumption:
asymptotic states

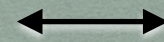
$$\pi\eta\xi\pi\xi\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \dots$$

$$- 4 \times b_{\pi N}^{I=3/2} \quad \text{orbital } L: \\ S, P, D, F, \text{ etc..}$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS



dynamical



$$\left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

NN guesstimate

$$a_S = 20 \text{ fm}$$

$$r \approx 0.0727 \quad \text{LHC}$$

$$r \approx 0.36 \quad \text{HADES}$$

$$r \approx 1.92$$

$$T = 60 \text{ MeV}$$

$$\mu_B = 700, 800 \text{ MeV}$$

HOW TO RELATE PHASE SHIFTS TO THERMODYNAMICS?

thermo-statistical

dynamical

$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

single channel, elastic

$$\frac{1}{\pi} \frac{d}{dE} \delta$$

N-body &

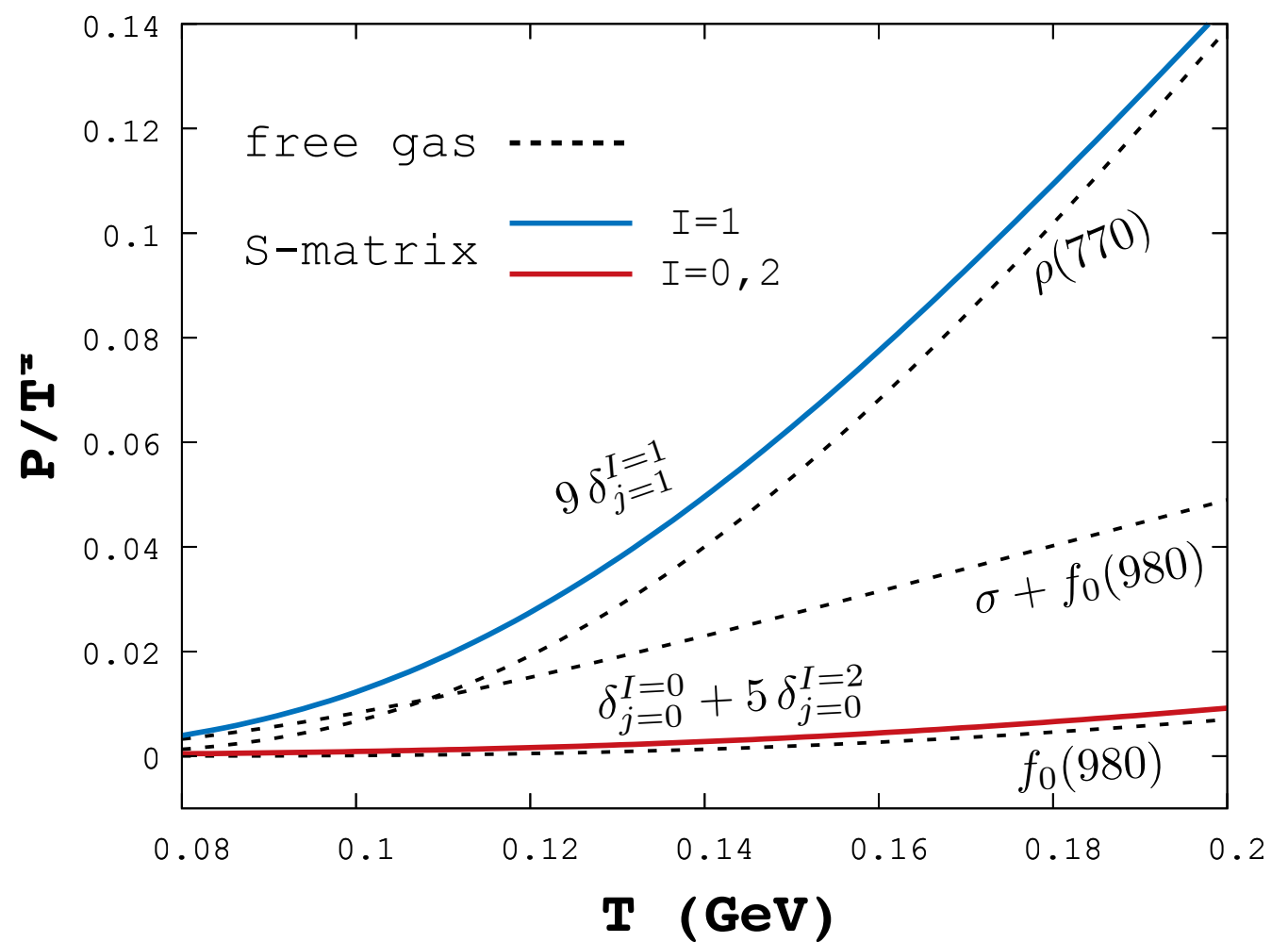
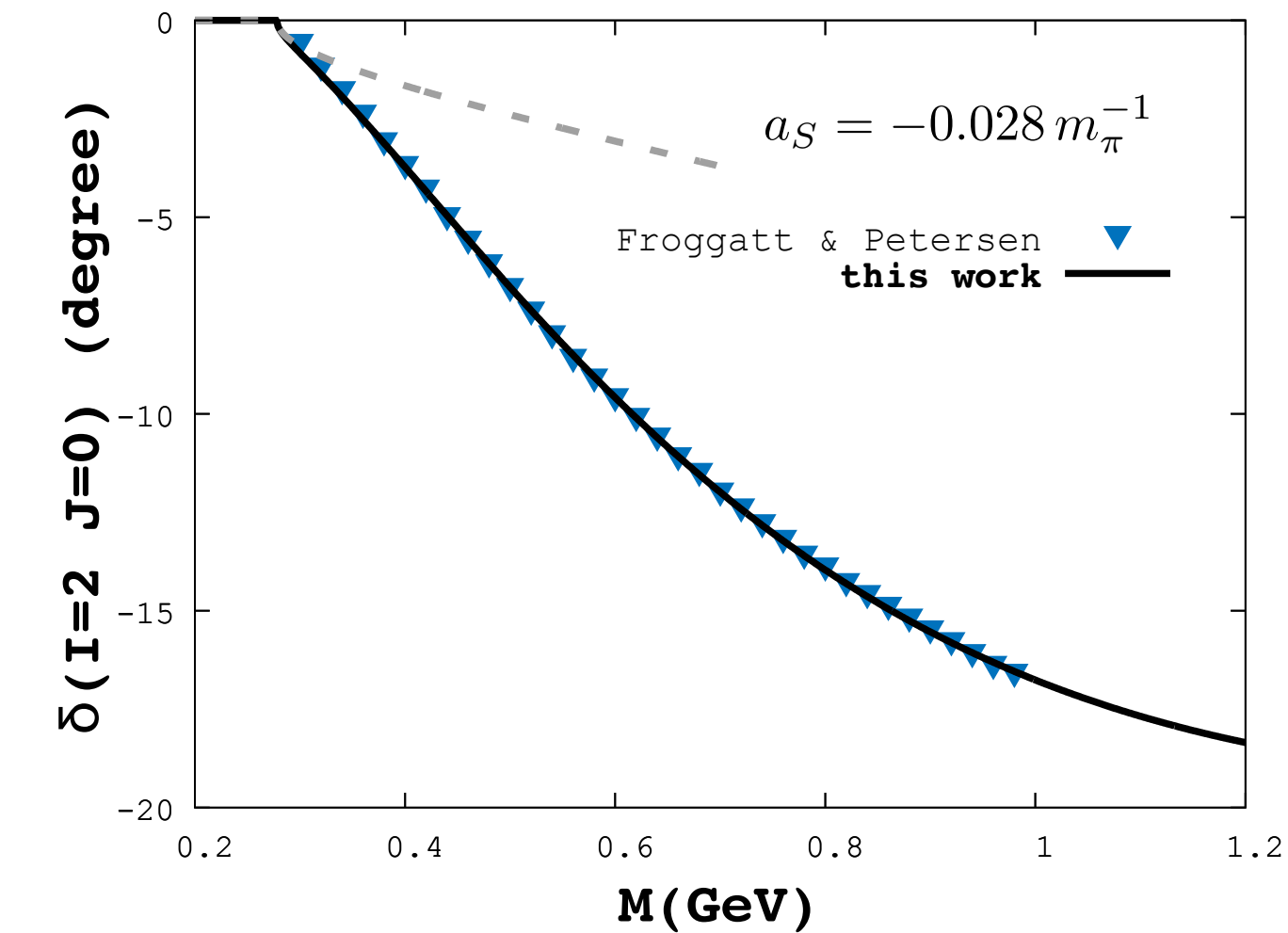
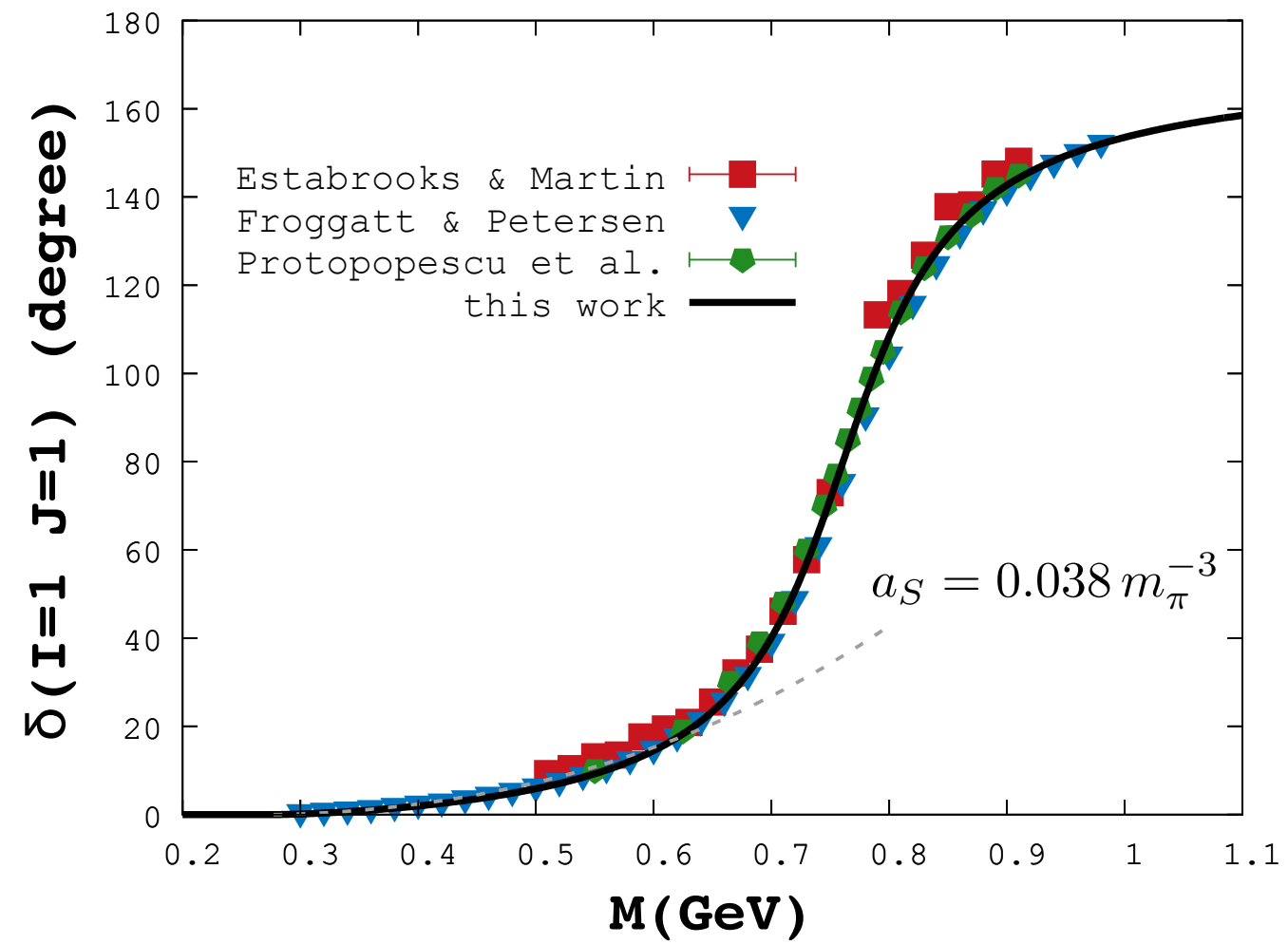
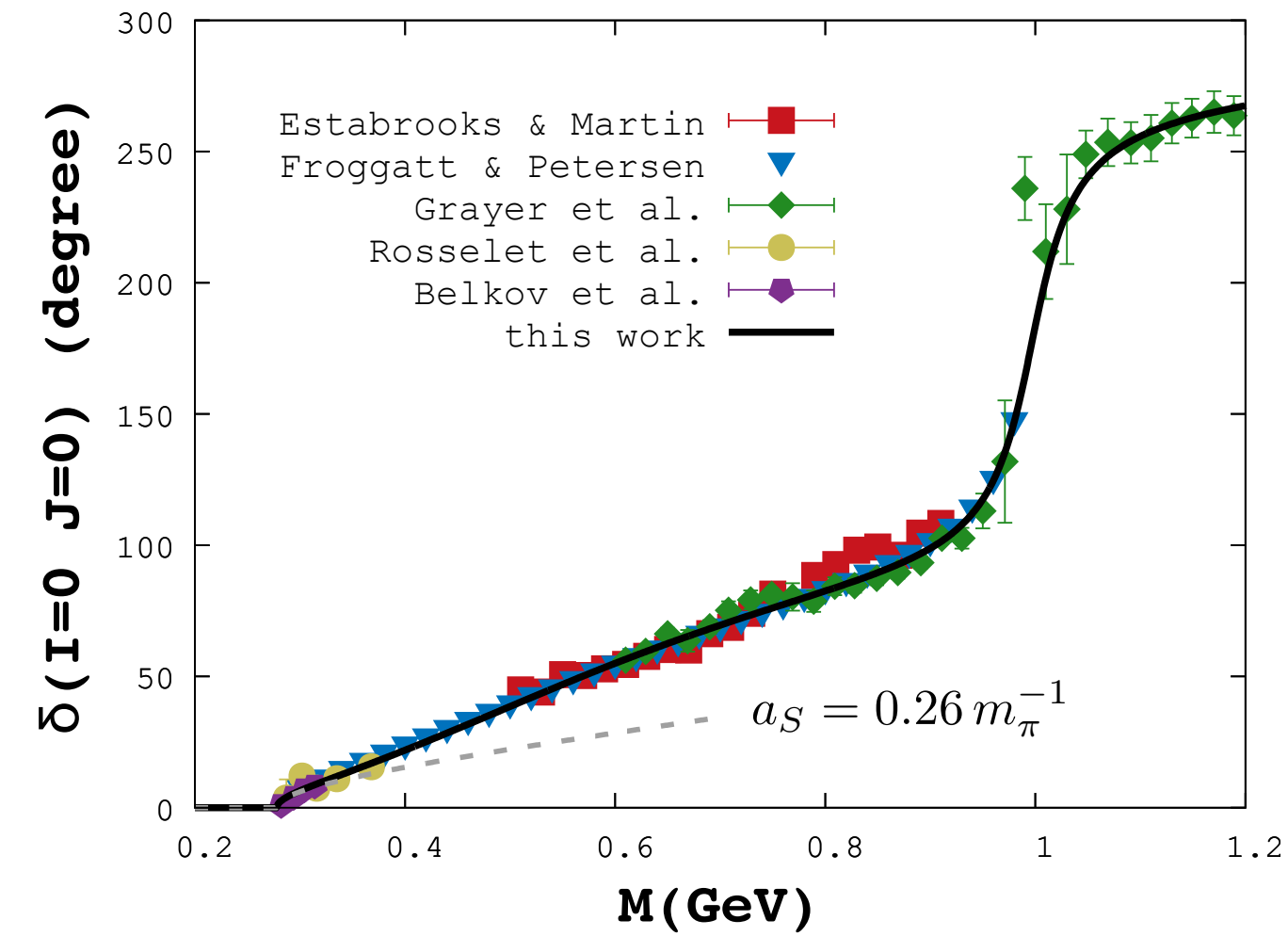
Coupled-Channel problem

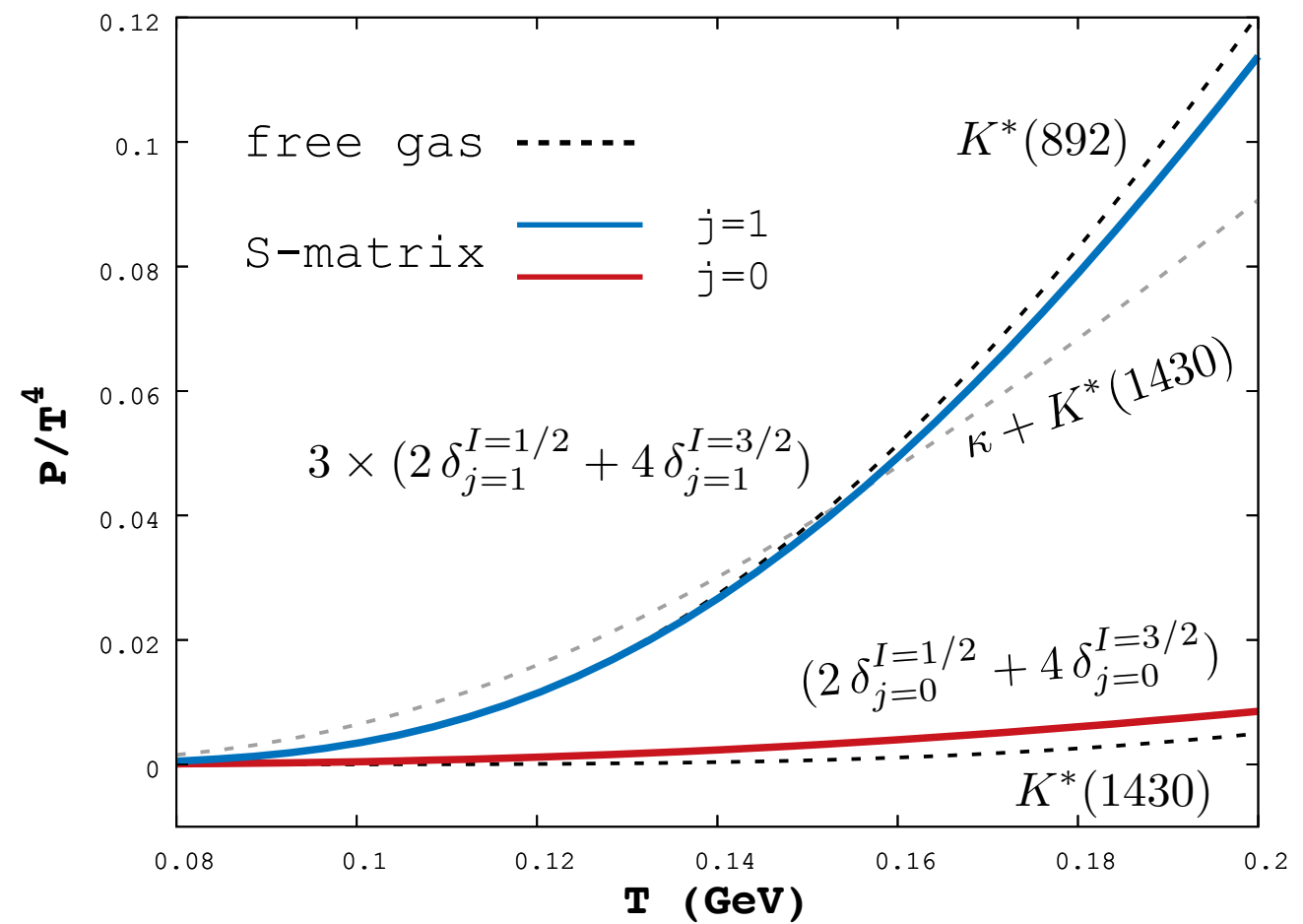
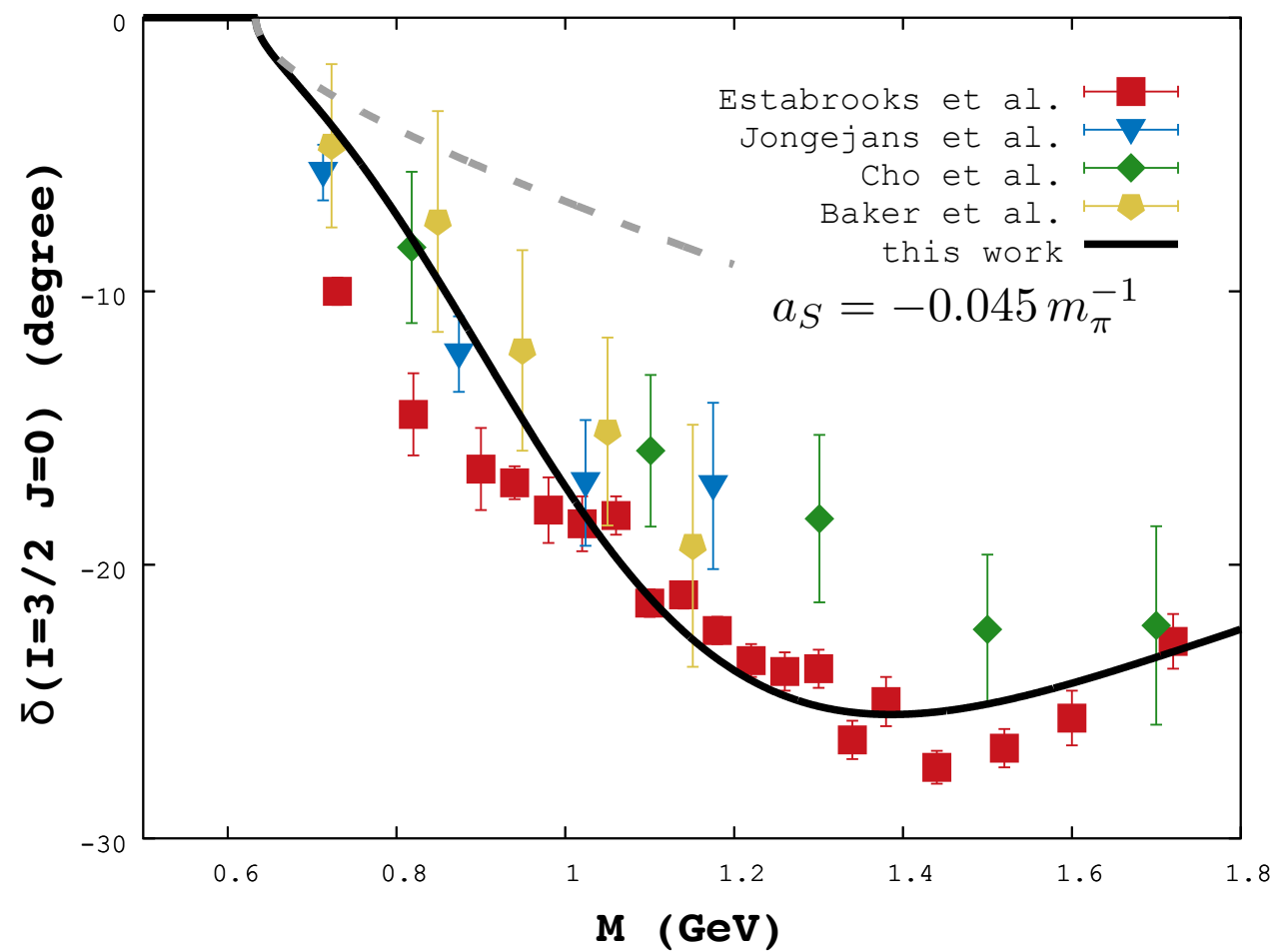
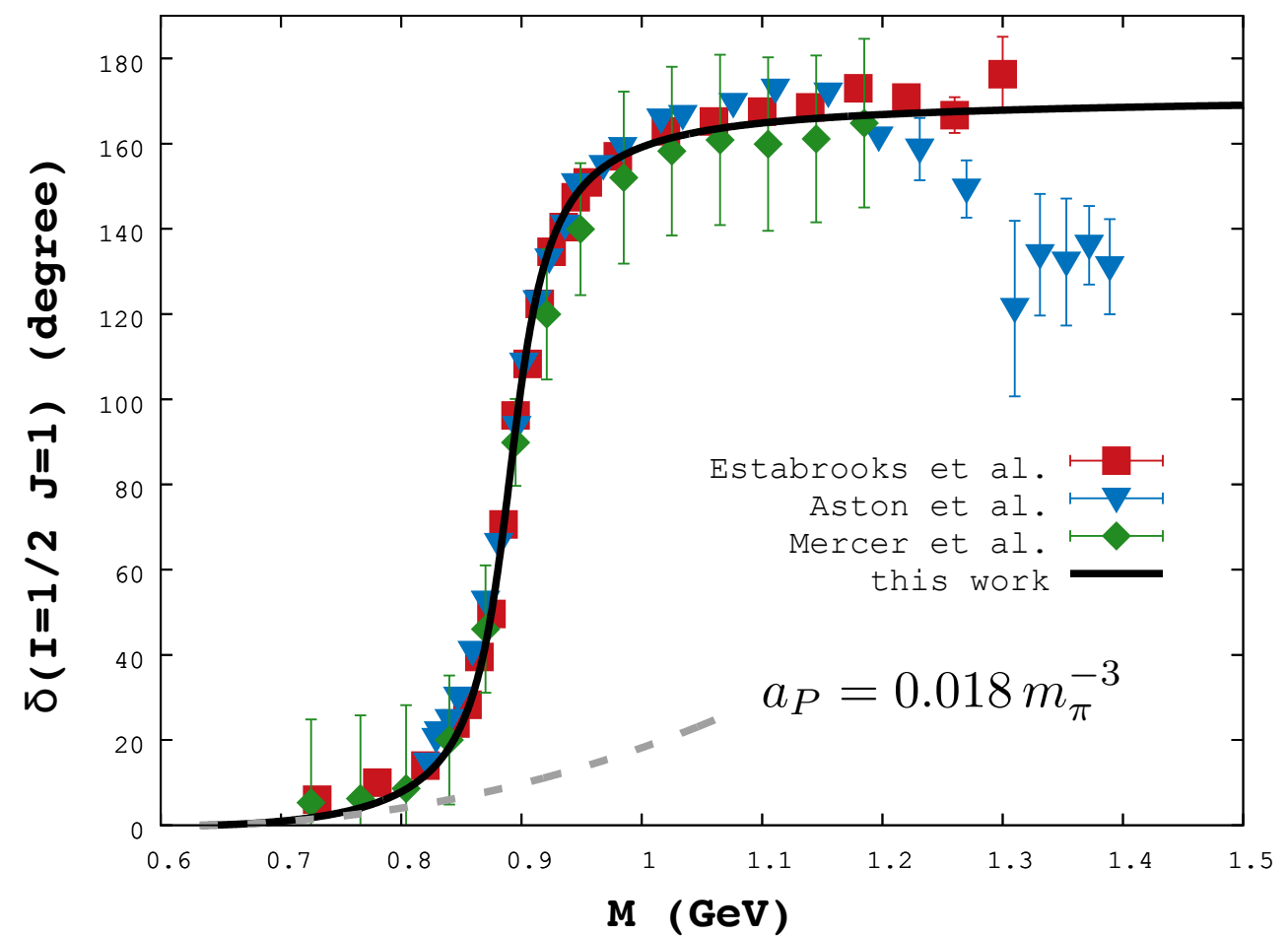
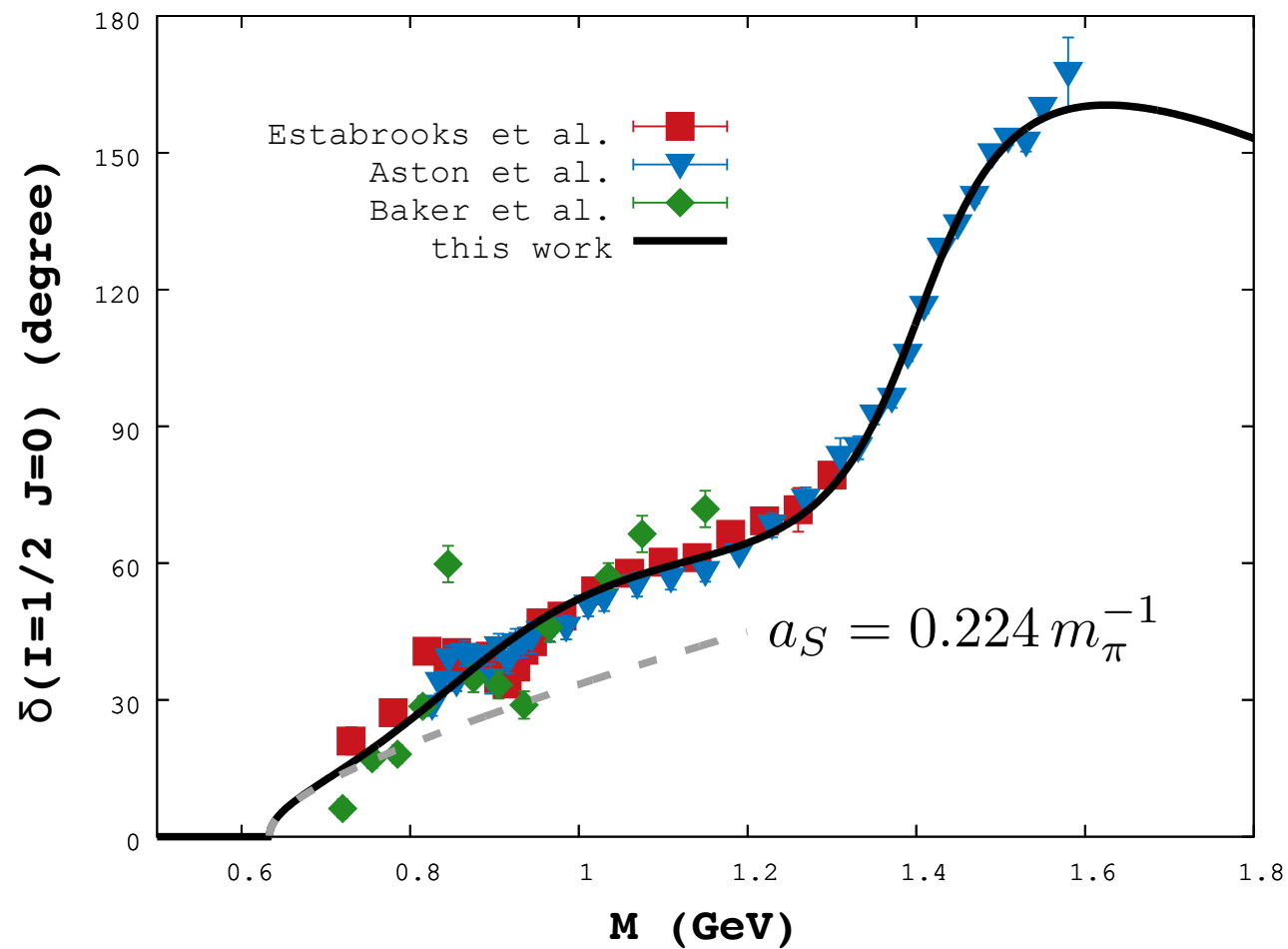
multi (coupled) channel

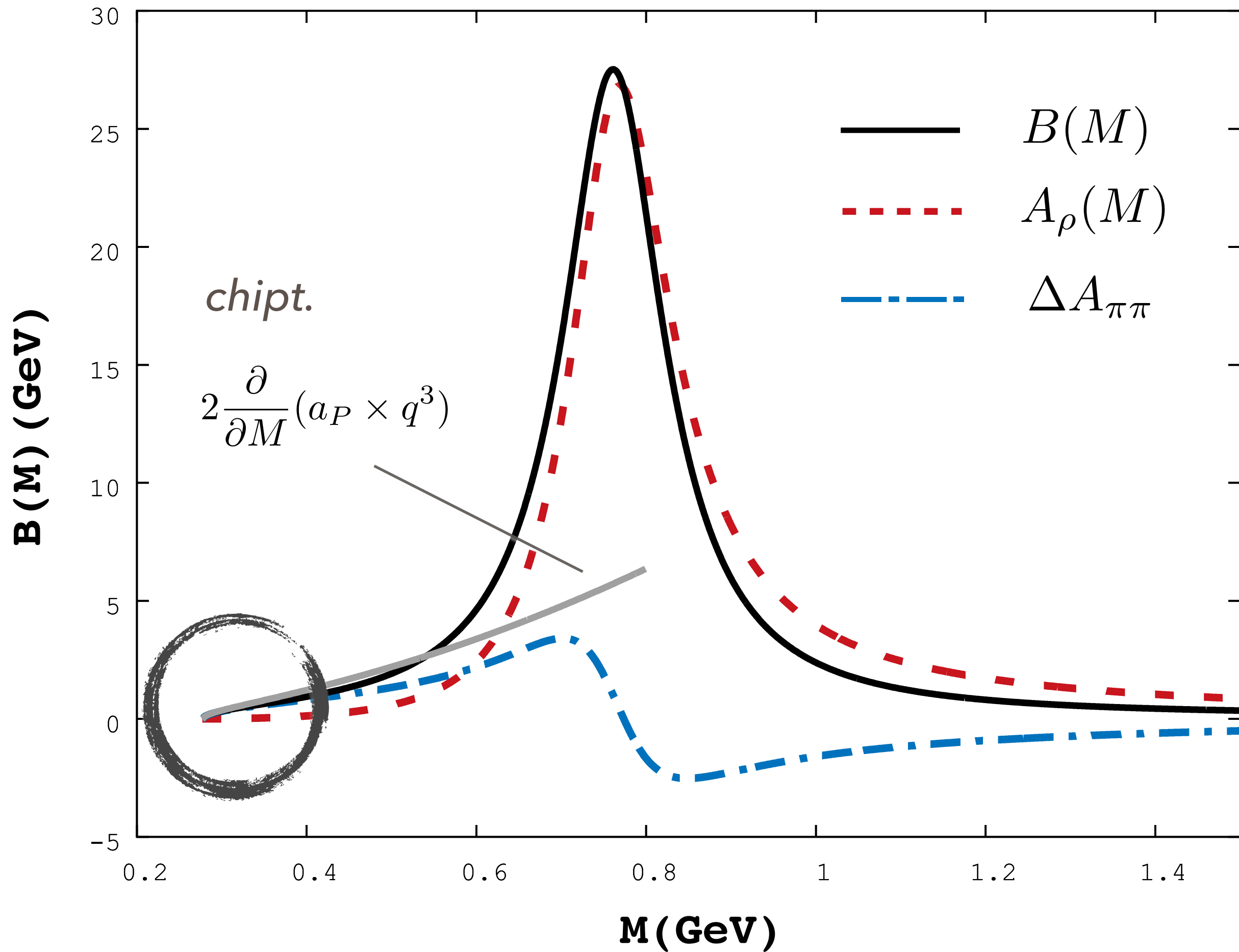
$$\frac{1}{\pi} \frac{d}{dE} Q$$

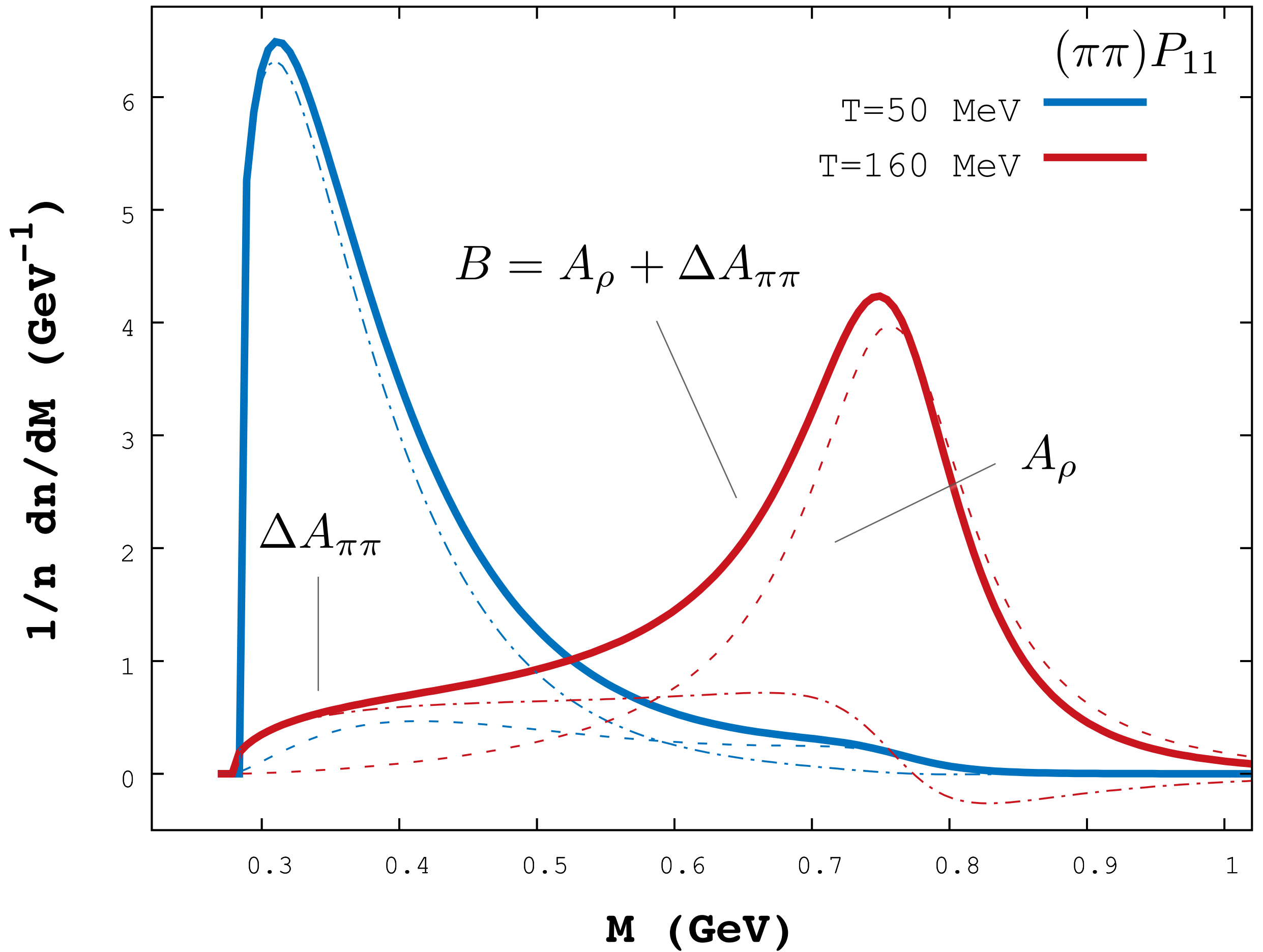
$$Q = \frac{1}{2} \text{Im Tr } \ln S$$

$$= \sum_{\text{channels}} \lambda_i$$









THE S-MATRIX PROGRAM

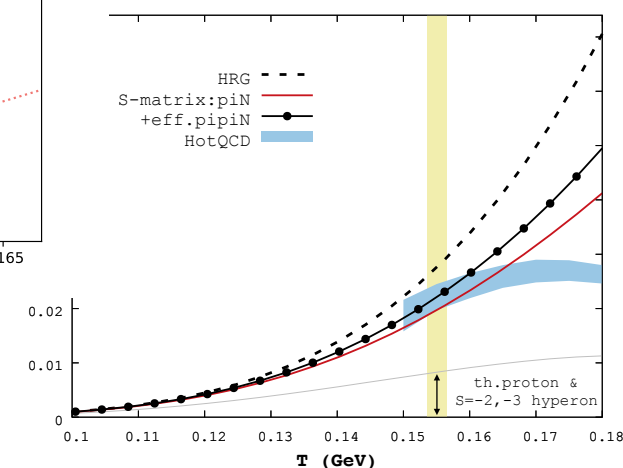
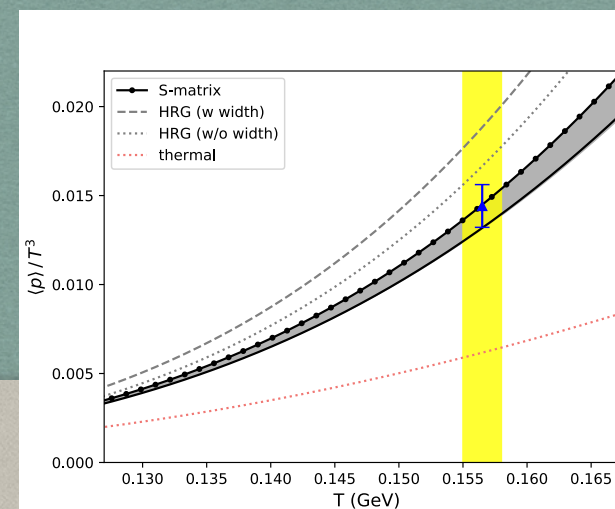
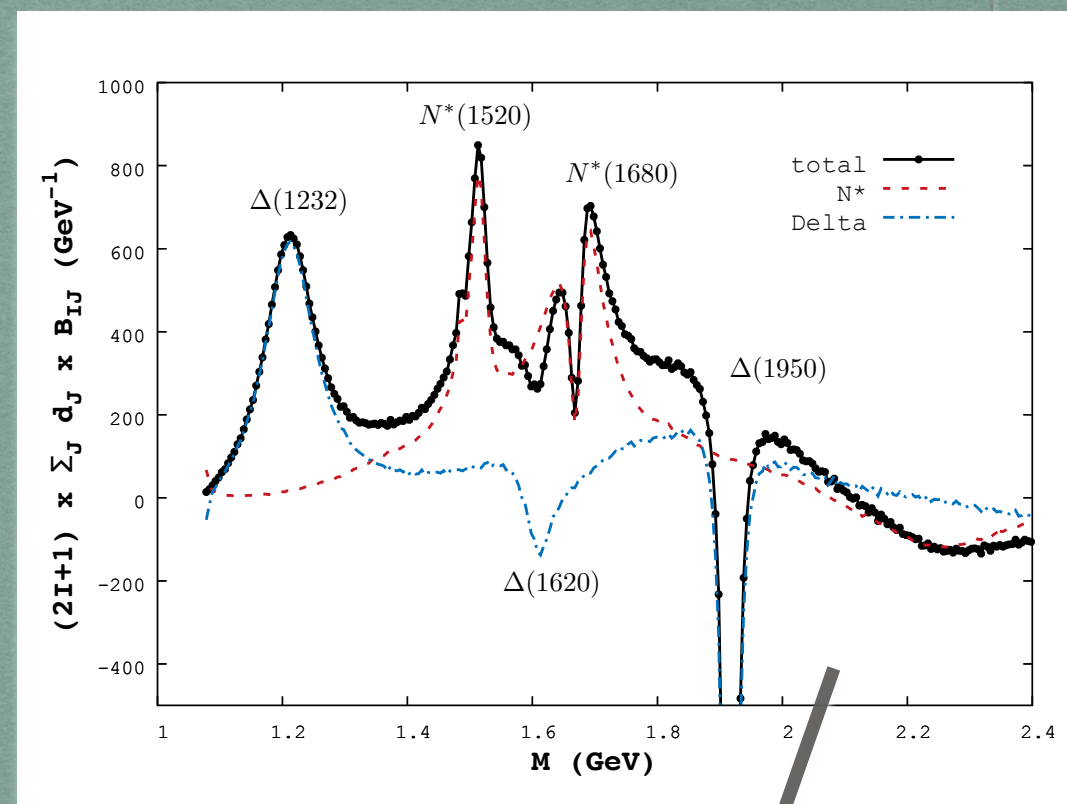
step 1: build a model for S-matrix

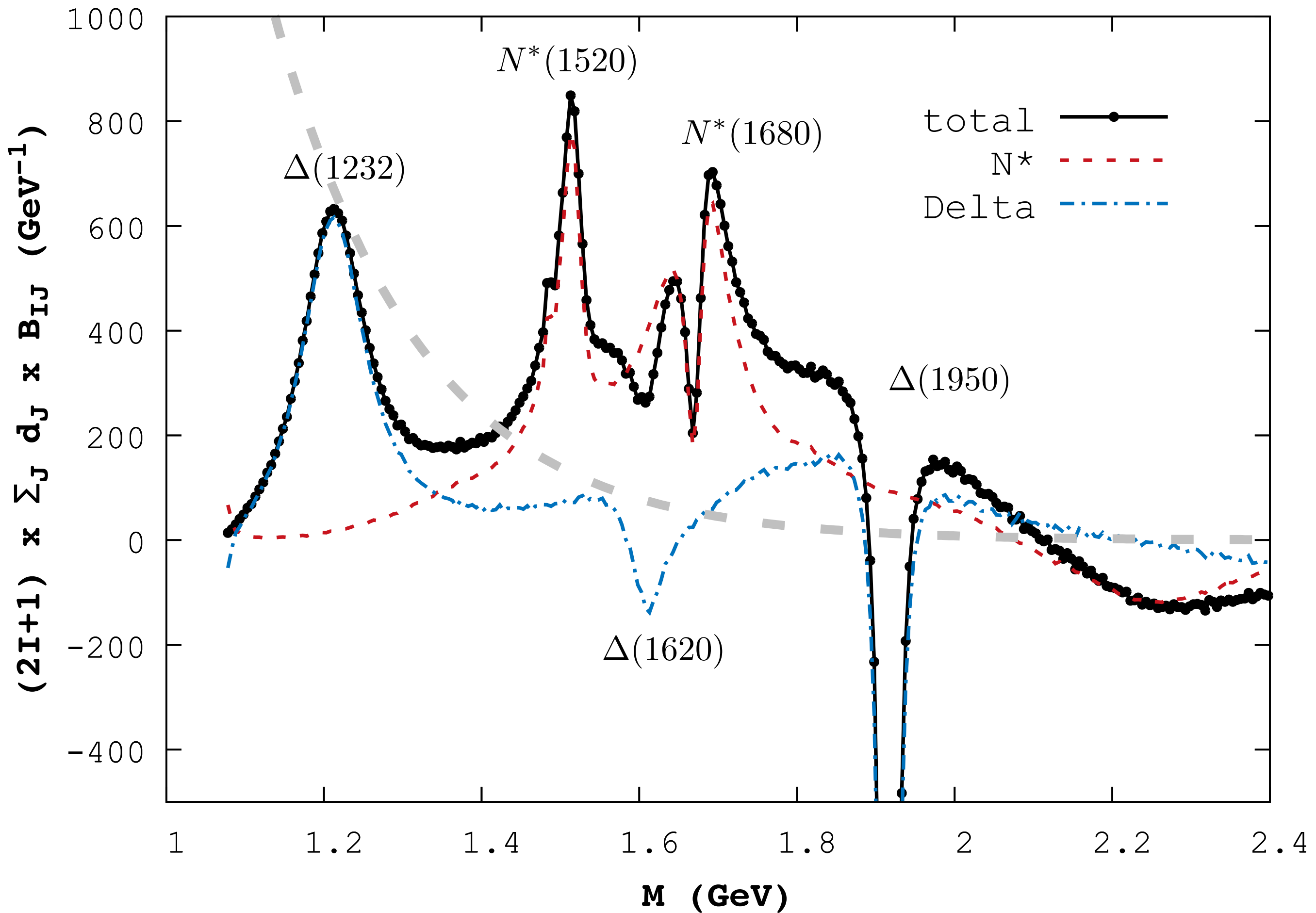
step 2: adjust model parameters to
match scattering experiments

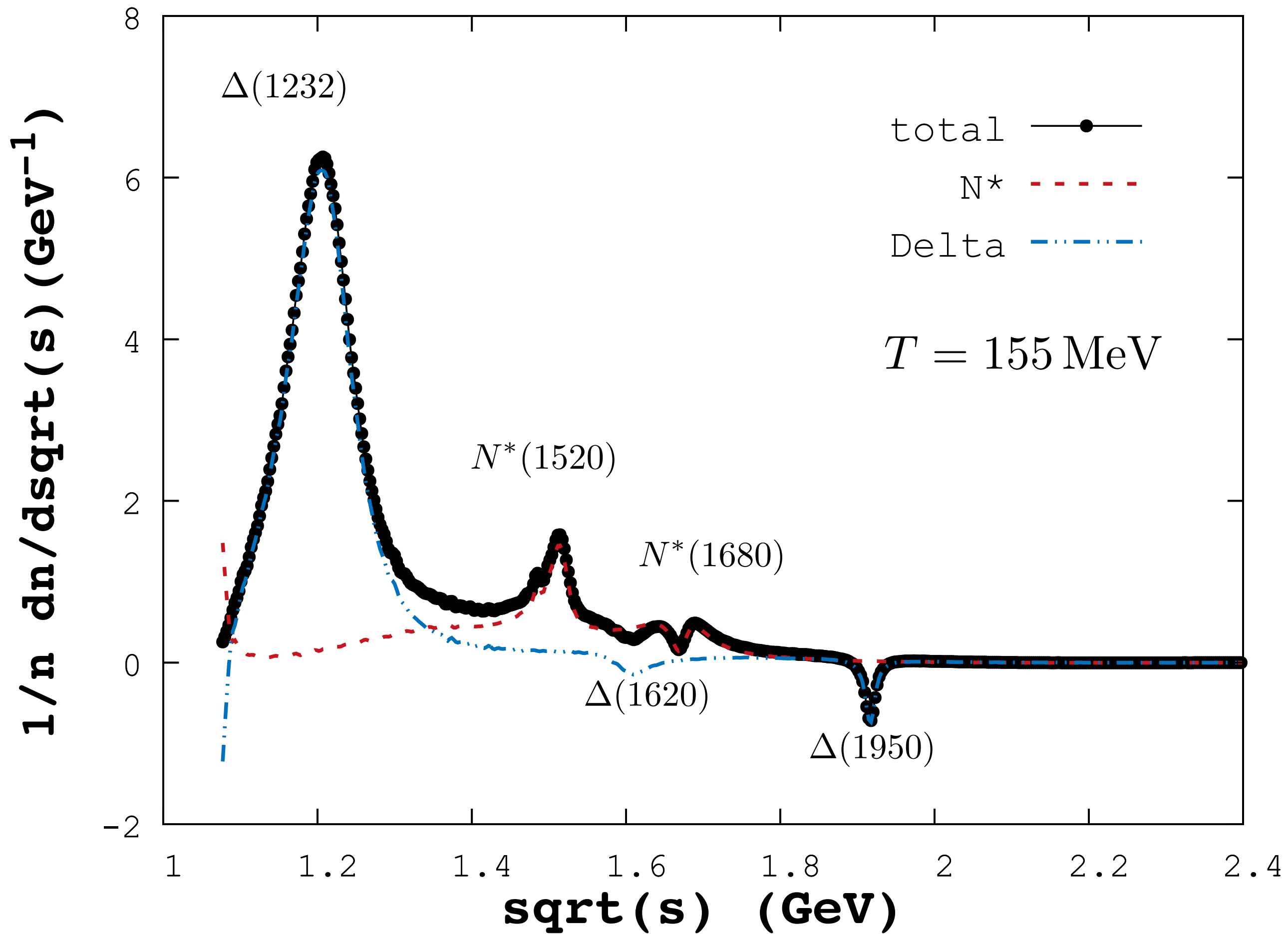
broad resonances & thresholds
coupled channel effects
energy dependent branchings

step 3: compute thermodynamics

HICs & LQCD







S = -1 HYPERONS COUPLED CHANNEL SYSTEM

JPAC, PRD **93**, 034029 (2016)

C. Fernandez-Ramirez, PML, and P. Petreczky,
PRC **98**, 044910 (2018)

J. Cleymans, PML, K. Redlich, and N. Sharma
PRC **103**, 014904 (2021)

PHASE SHIFT FROM PWA

Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)

Igor V. Danilkin (Jefferson Lab)

Vincent Mathieu (Indiana University)

Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)

This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

- 1 - This code follows the 'garbage in, garbage out' philosophy. If your parameters do not make sense, the output will not make sense either.
- 2 - You can use, share and modify this code under your own responsibility.
- 3 - This code is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY: without even the implied warranty of

MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.

- 4 - No PhD students or postdocs were severely damaged during the development of this project.
-

- $1 \rightarrow \bar{K}N,$

- $2 \rightarrow \pi\Sigma,$

- $3 \rightarrow \pi\Lambda,$

- $4 \rightarrow \eta\Lambda,$

- $5 \rightarrow \eta\Sigma,$

- $6 \rightarrow \bar{K}_1N,$

- $7 \rightarrow [\bar{K}_3N]_-,$

- $8 \rightarrow [\bar{K}_3N]_+,$

- $9 \rightarrow [\pi\Sigma^*]_-,$

- $10 \rightarrow [\pi\Sigma^*]_+,$

- $11 \rightarrow [\bar{K}\Delta]_-,$

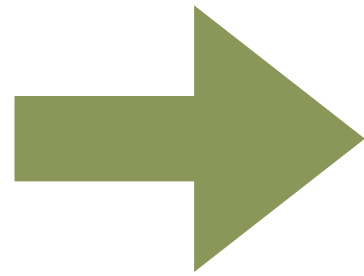
- $12 \rightarrow [\bar{K}\Delta]_+,$

- $13 \rightarrow [\pi\Lambda(1520)]_-,$

- $14 \rightarrow [\pi\Lambda(1520)]_+,$

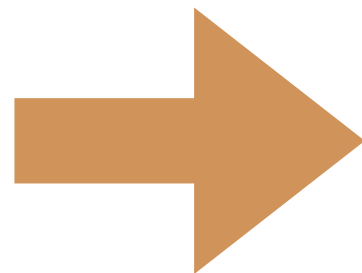
- $15 \rightarrow \pi\pi\Lambda,$

- $16 \rightarrow \pi\pi\Sigma.$

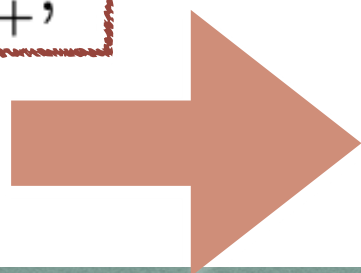


elastic scatterings (elementary)

Effective elementarity



quasi elastic scatterings



unitarity background

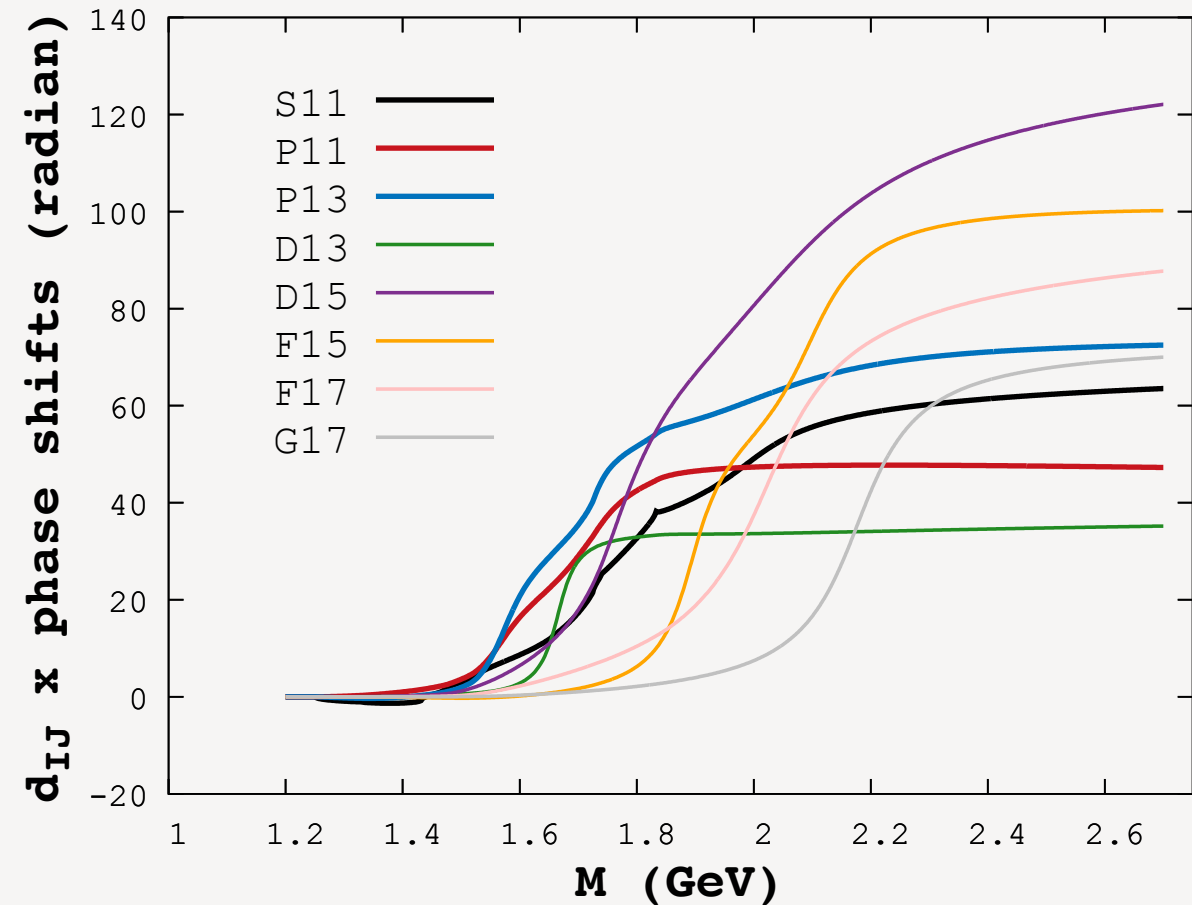
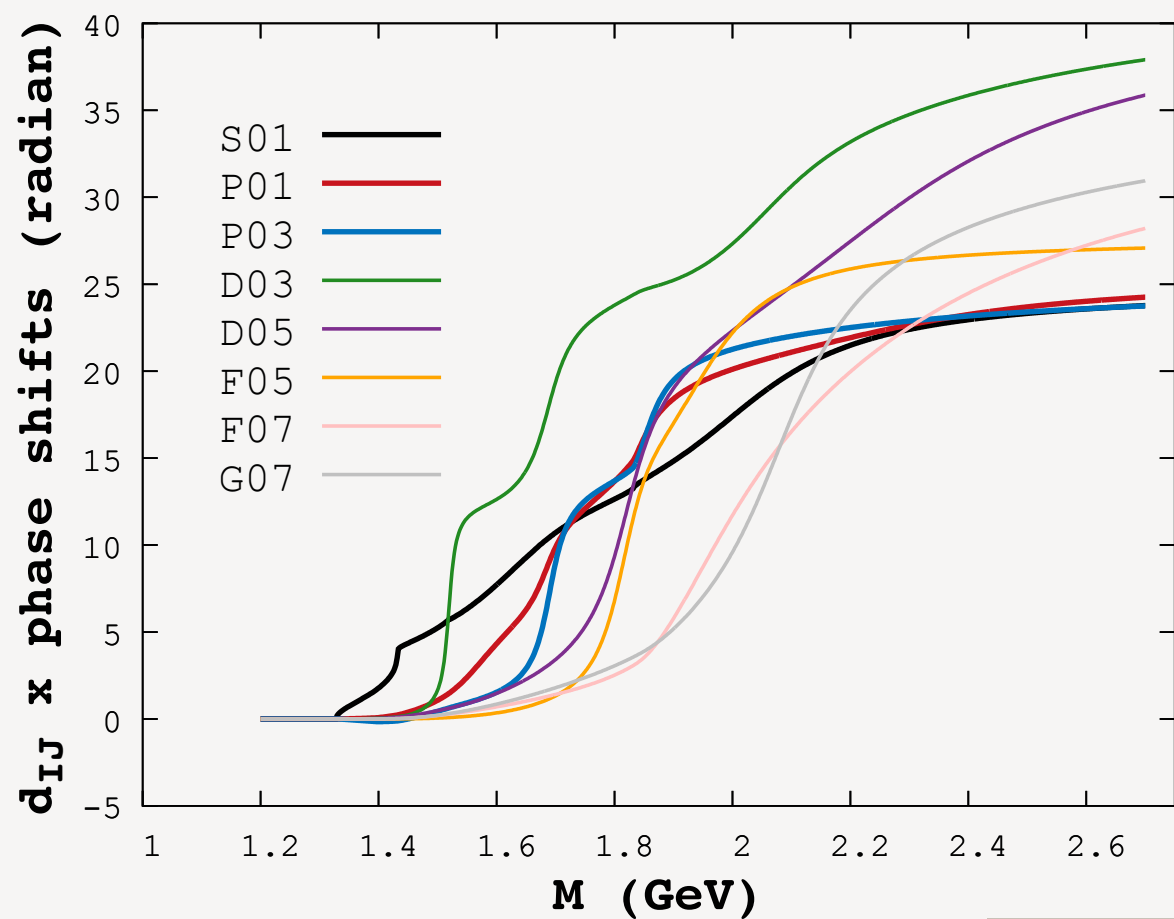
STRANGENESS CONTENT IN A HADRON GAS

- K-N system requires a coupled channel analysis

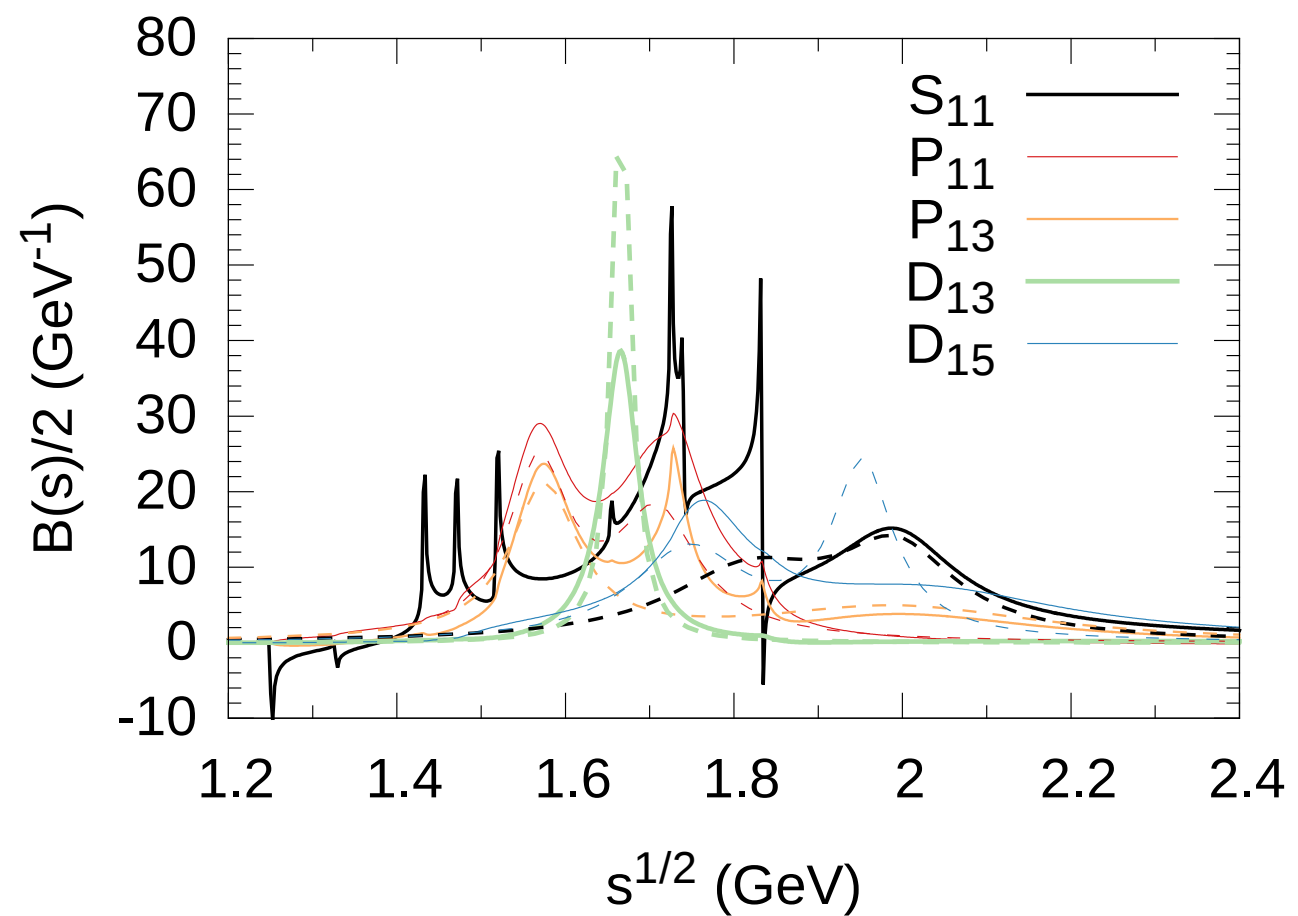
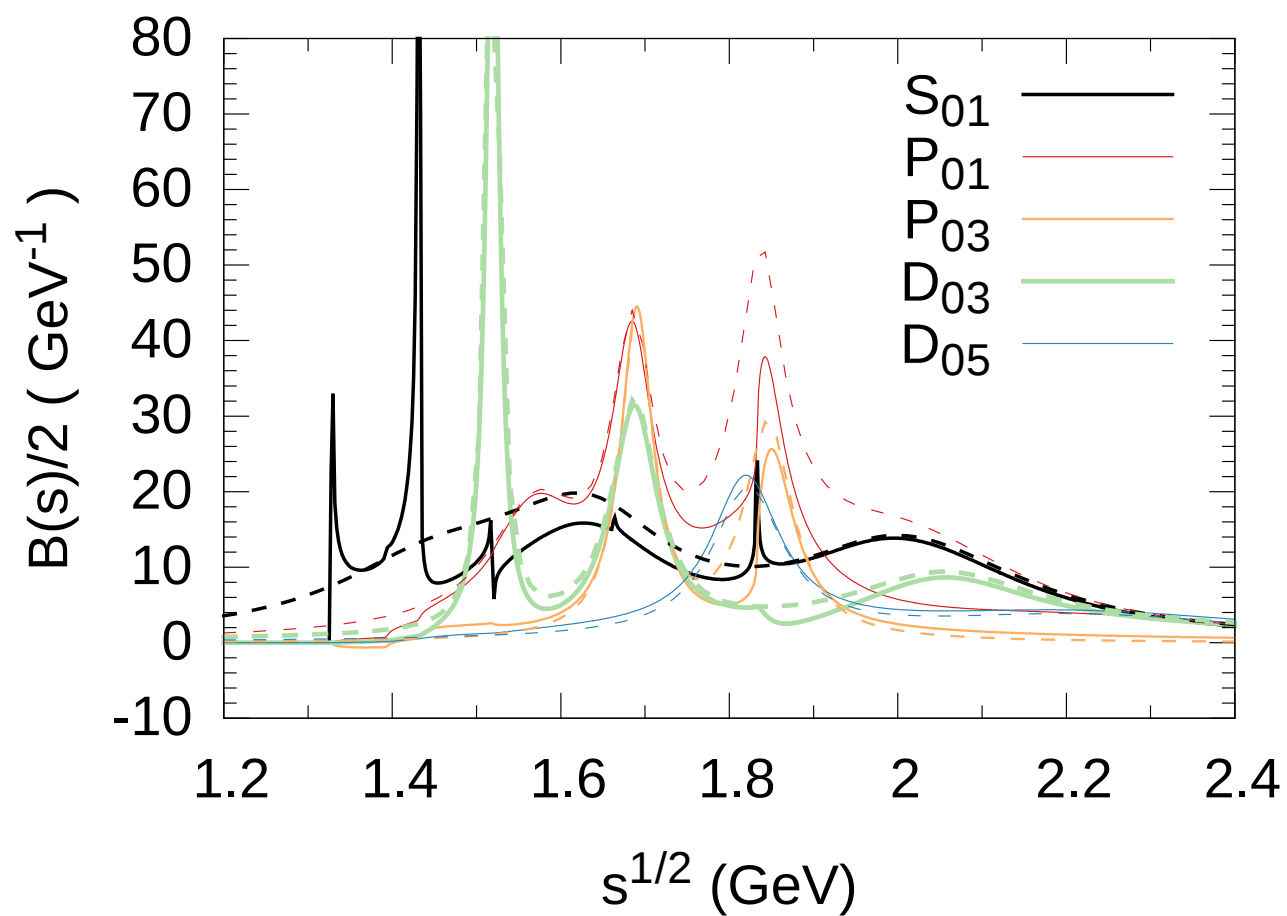
$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \dots$ *16 basis states*

$$\begin{aligned} Q(M) &\equiv \frac{1}{2} \text{Im} (\text{tr} \ln S) \\ &= \frac{1}{2} \text{Im} (\ln \det [S]) \\ &= \delta_{\bar{K}N} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \dots \end{aligned}$$

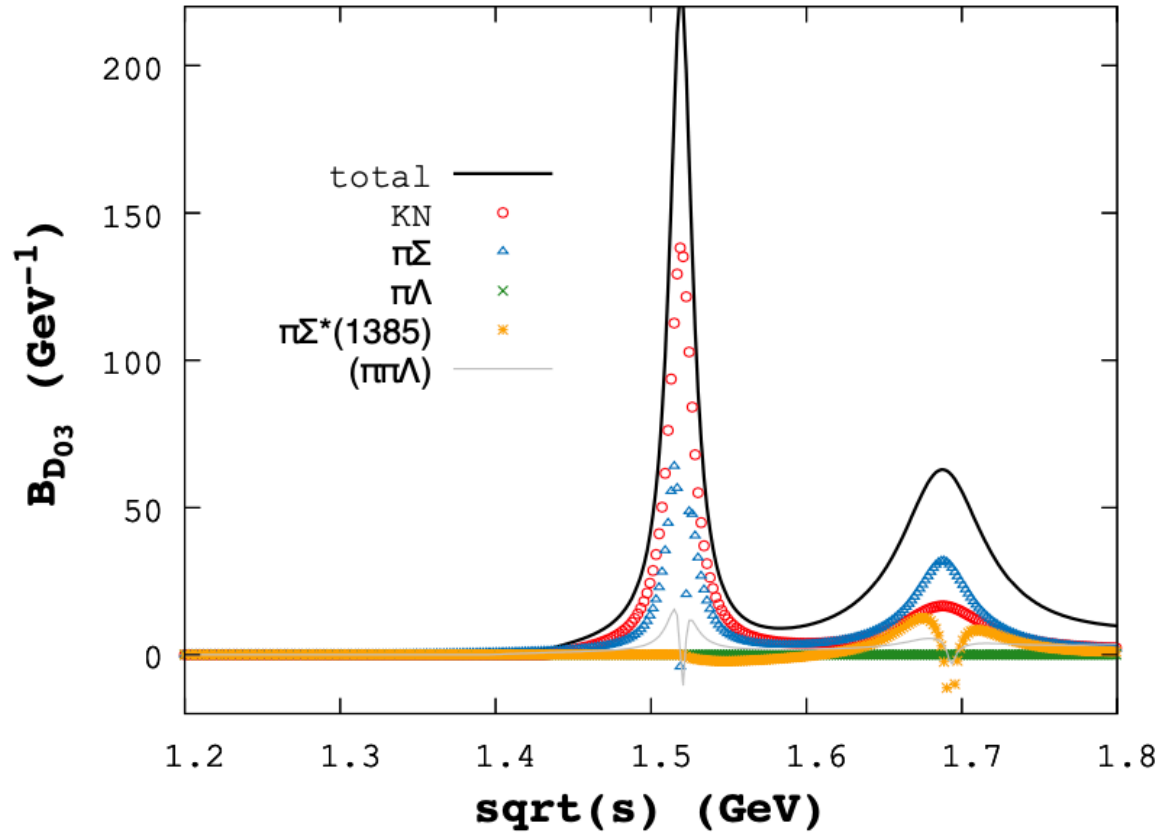
Compute $\det S$ for each
 $\sqrt{s}$ for each channel
isospin conserving



S=-1 Hyperon



1520, 1690



$\Lambda(1520) \ 3/2^-$

$$I(J^P) = 0(\frac{3}{2}^-)$$

Mass $m = 1519.5 \pm 1.0$ MeV [d]
 Full width $\Gamma = 15.6 \pm 1.0$ MeV [d]
 $p_{\text{beam}} = 0.39$ GeV/c $4\pi\chi^2 = 82.8$ mb

$\Lambda(1520)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\bar{K}$	$45 \pm 1\%$	243
$\Sigma\pi$	$42 \pm 1\%$	268
$\Lambda\pi\pi$	$10 \pm 1\%$	259
$\Sigma\pi\pi$	$0.9 \pm 0.1\%$	169
$\Lambda\gamma$	$0.85 \pm 0.15\%$	350

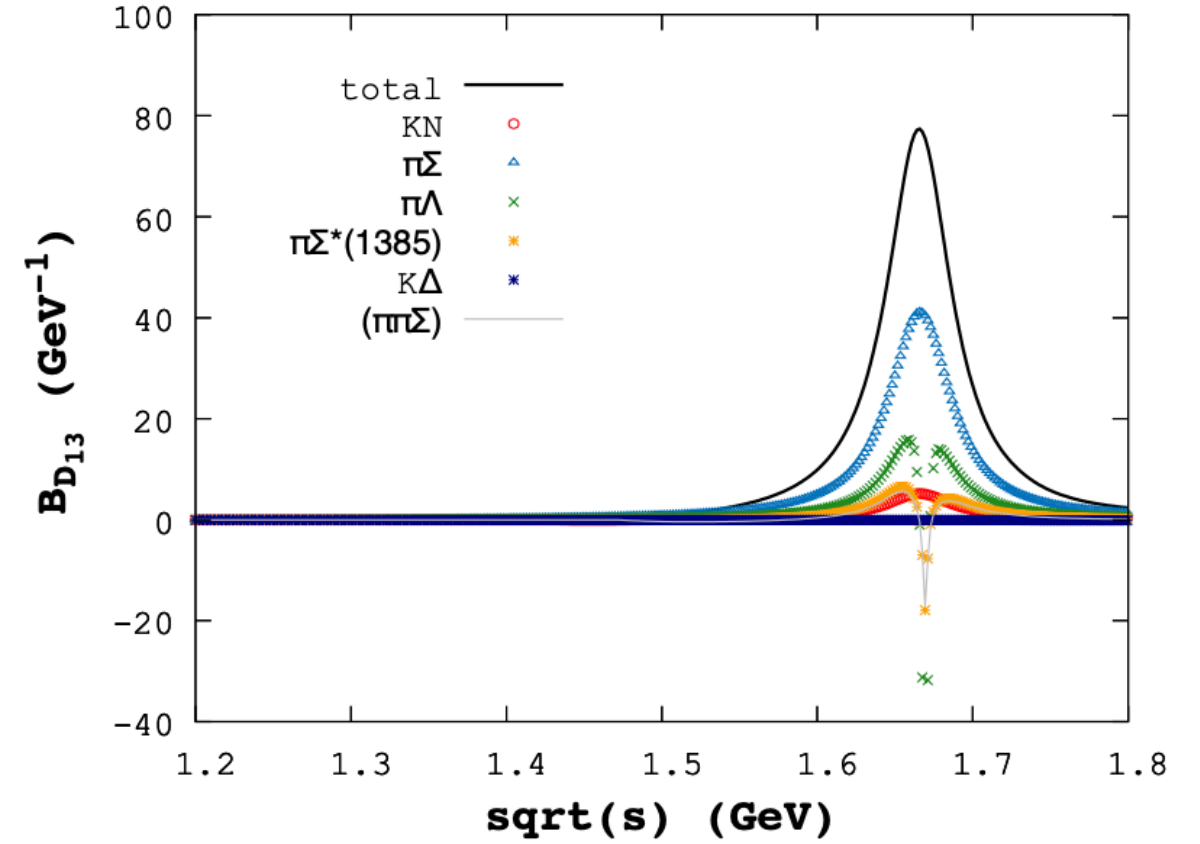
$\Lambda(1690) \ 3/2^-$

$$I(J^P) = 0(\frac{3}{2}^-)$$

Mass $m = 1685$ to 1695 (≈ 1690) MeV
 Full width $\Gamma = 50$ to 70 (≈ 60) MeV
 $p_{\text{beam}} = 0.78$ GeV/c $4\pi\chi^2 = 26.1$ mb

$\Lambda(1690)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\bar{K}$	20–30 %	433
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	~ 25 %	419
$\Sigma\pi\pi$	~ 20 %	358

1670

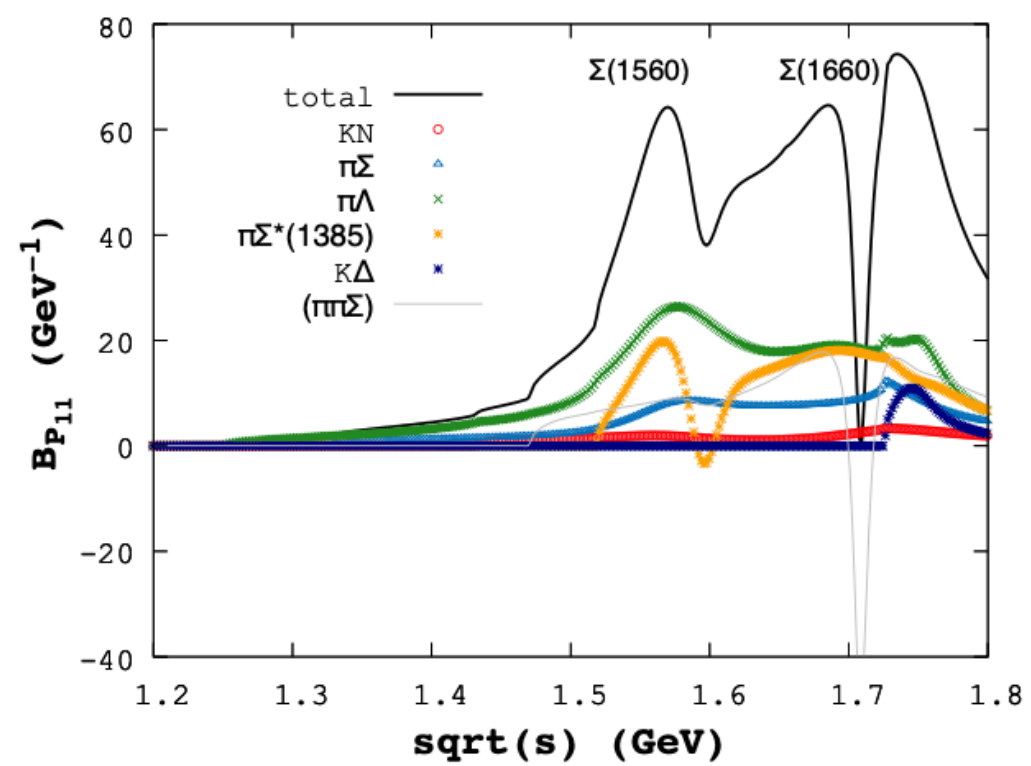
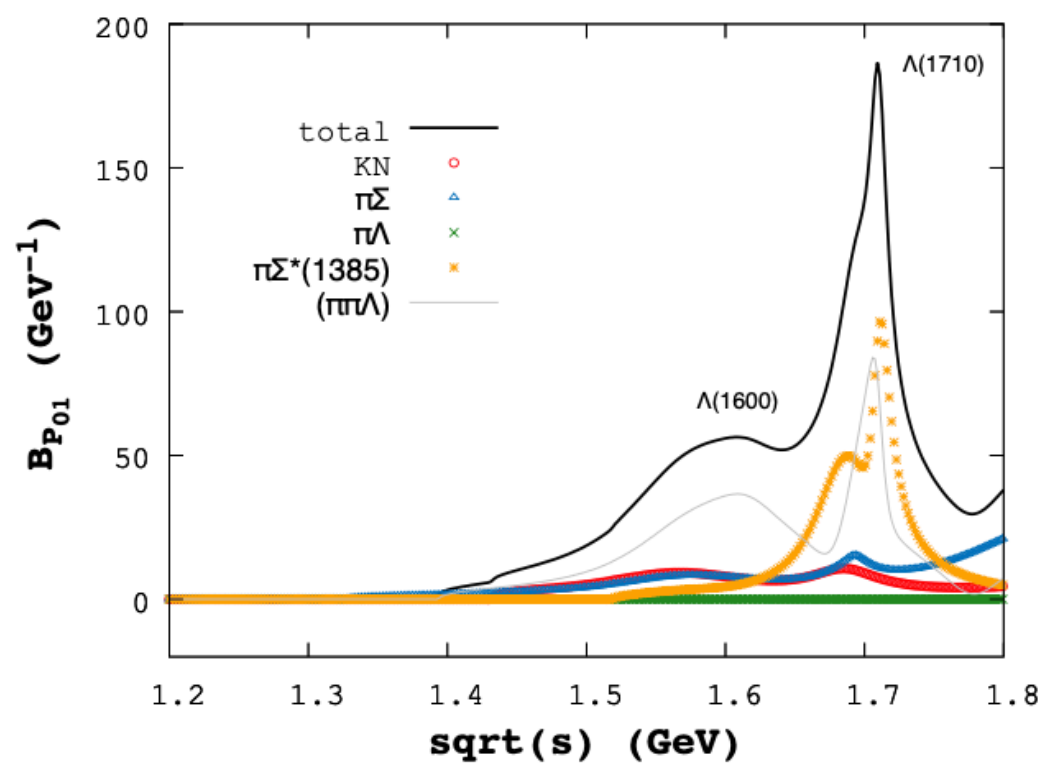
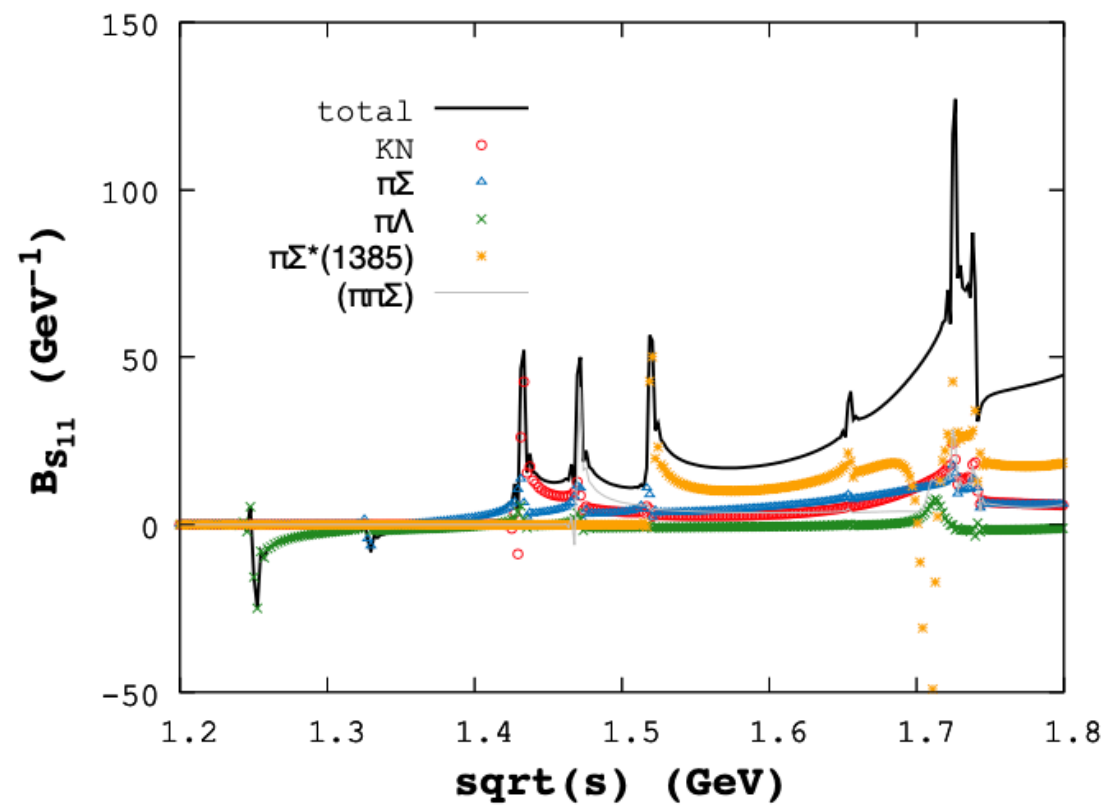
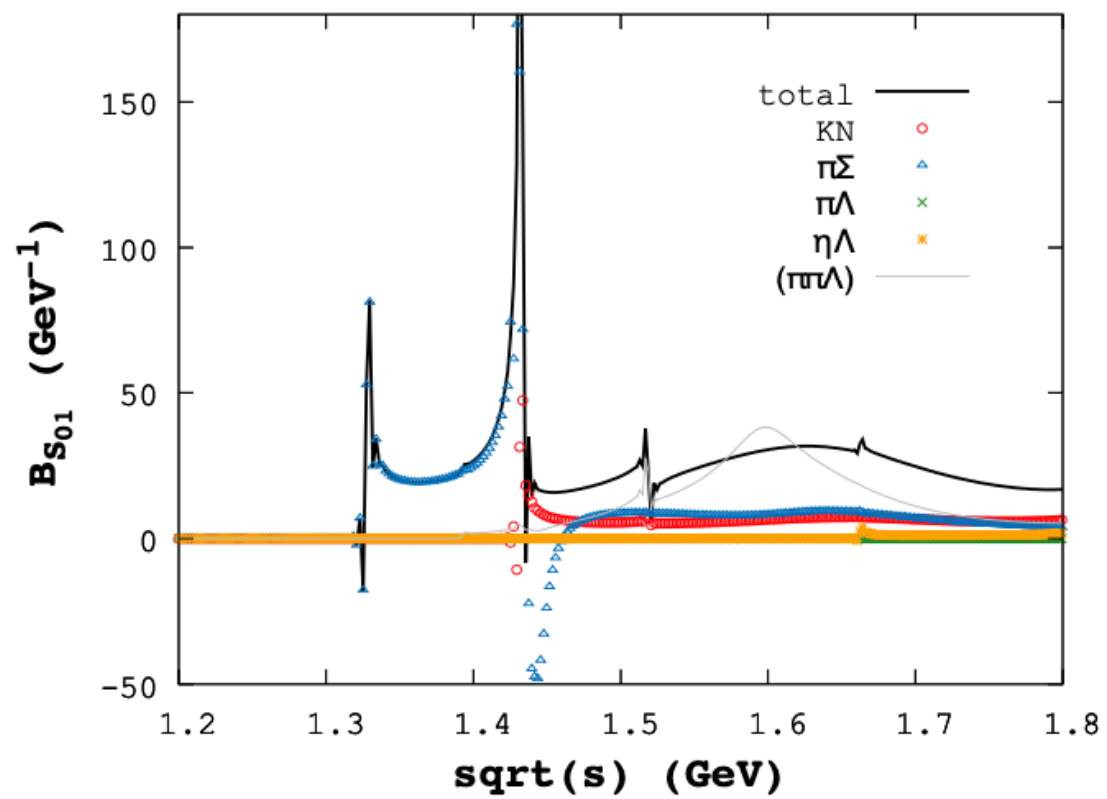


$\Sigma(1670) \ 3/2^-$

$$I(J^P) = 1(\frac{3}{2}^-)$$

Mass $m = 1665$ to 1685 (≈ 1670) MeV
 Full width $\Gamma = 40$ to 80 (≈ 60) MeV
 $p_{\text{beam}} = 0.74$ GeV/c $4\pi\chi^2 = 28.5$ mb

$\Sigma(1670)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\bar{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394



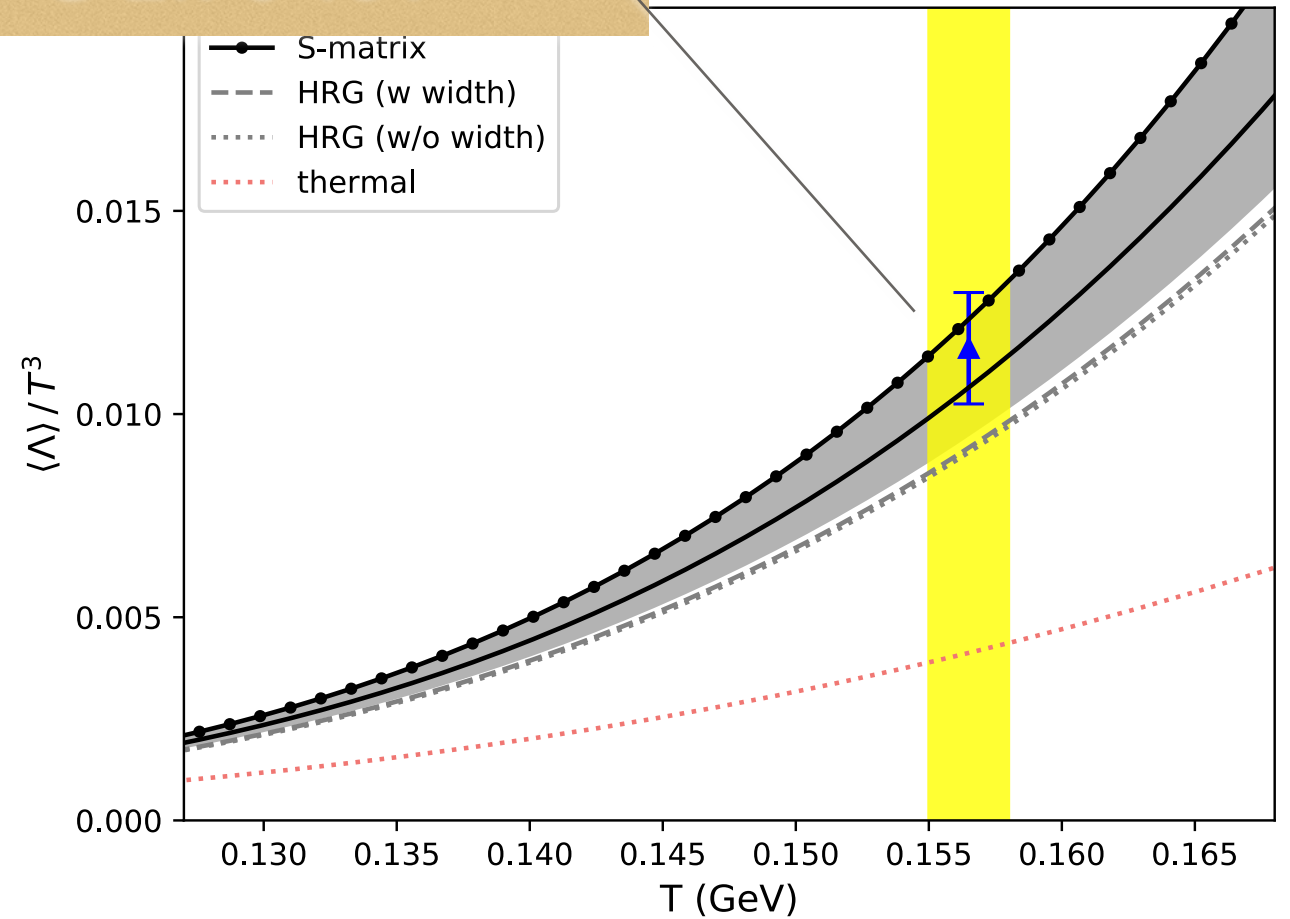
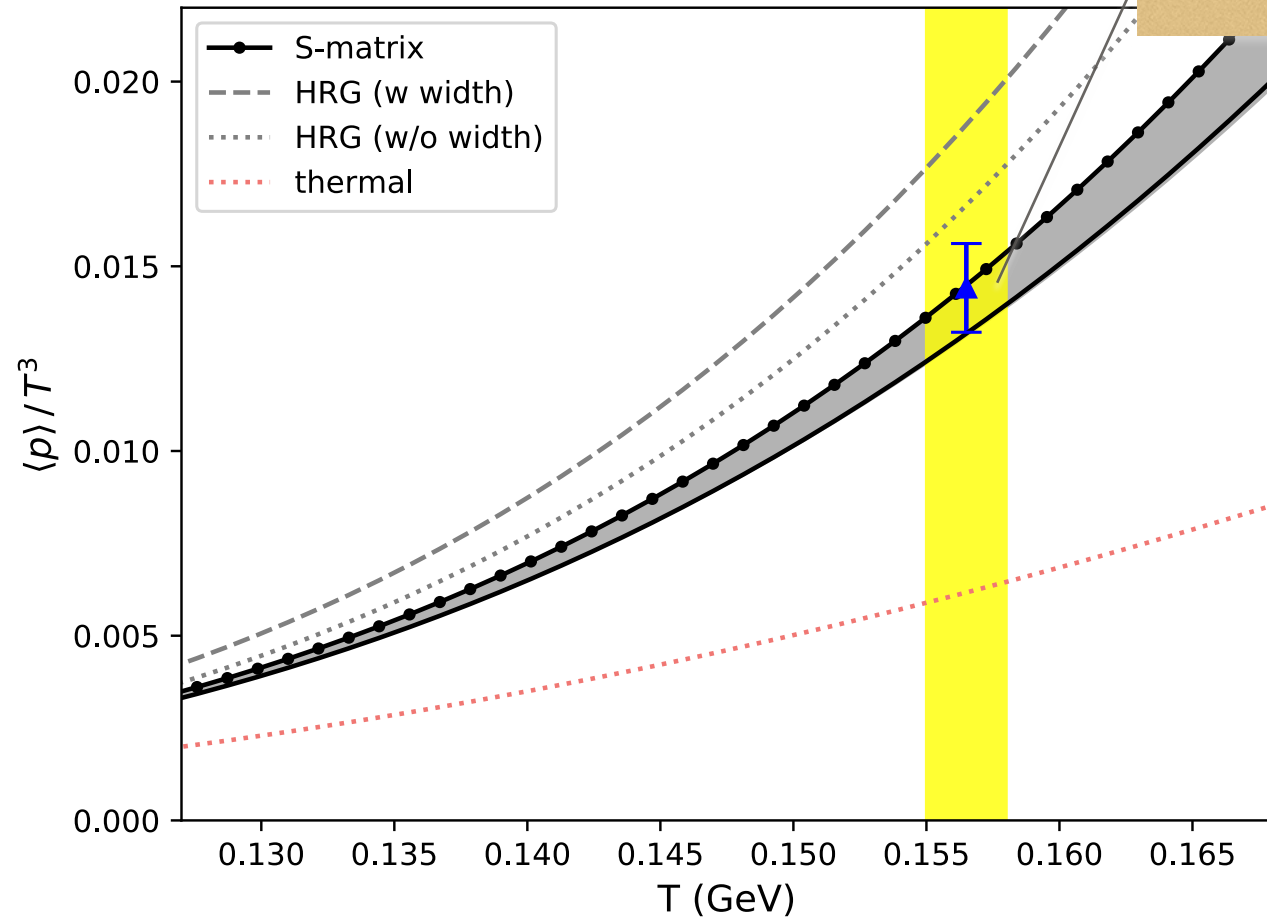
branching ratio?

S-matrix VS HRG

Will still go up!

Andronic et. al. NPA1010 (2021) 122176

ALICE proton yield
@ 2.76 TeV



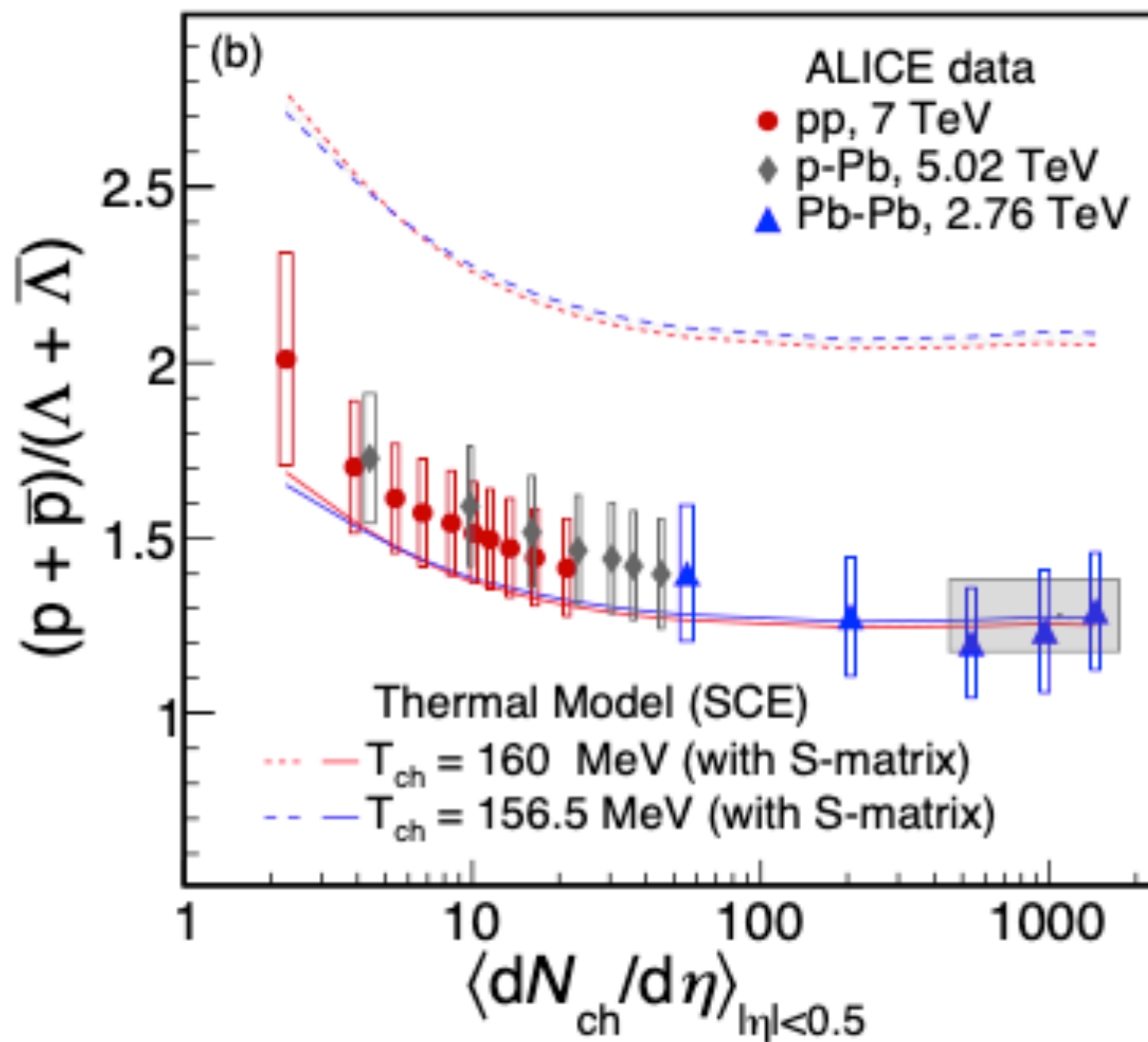
πN phase shifts
 $\pi \pi N$ BGs
hyperons



Coupled-Channel model:

$\bar{k}N, \pi\Lambda, \pi\Sigma, \dots$
extra hyperon states
beyond PDG
unitarity BGs

consistent treatment of res and non-res. int.



less protons

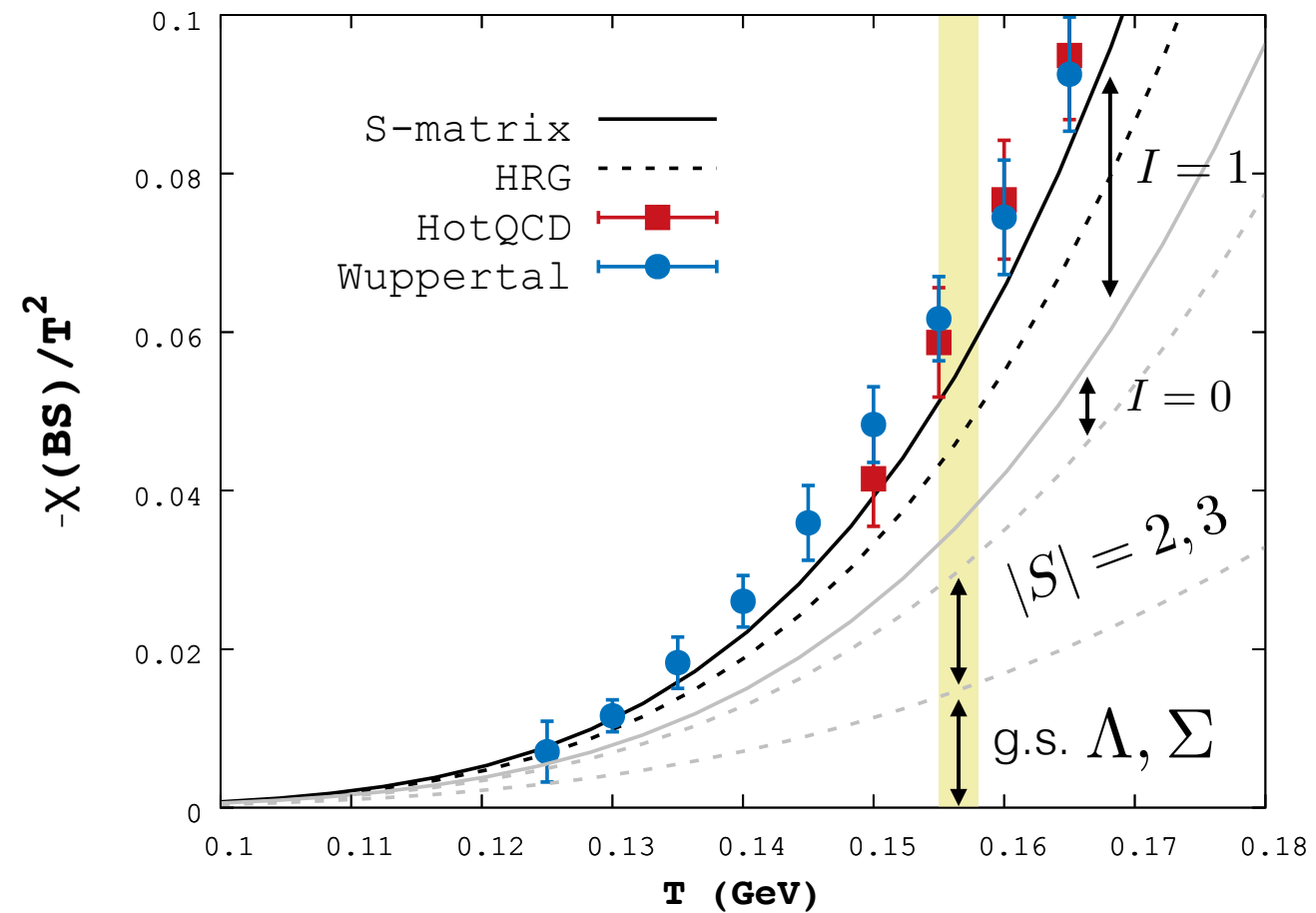
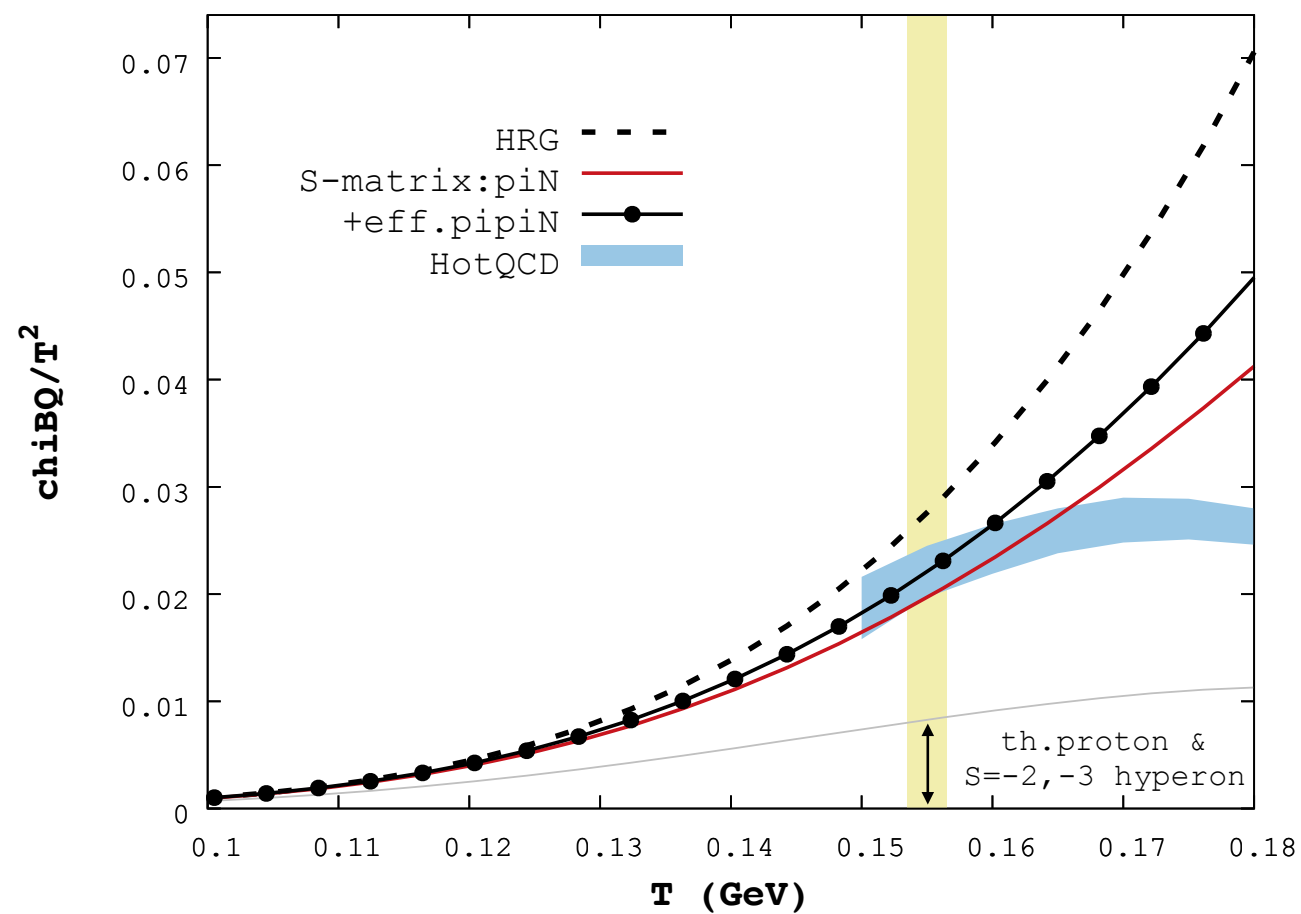
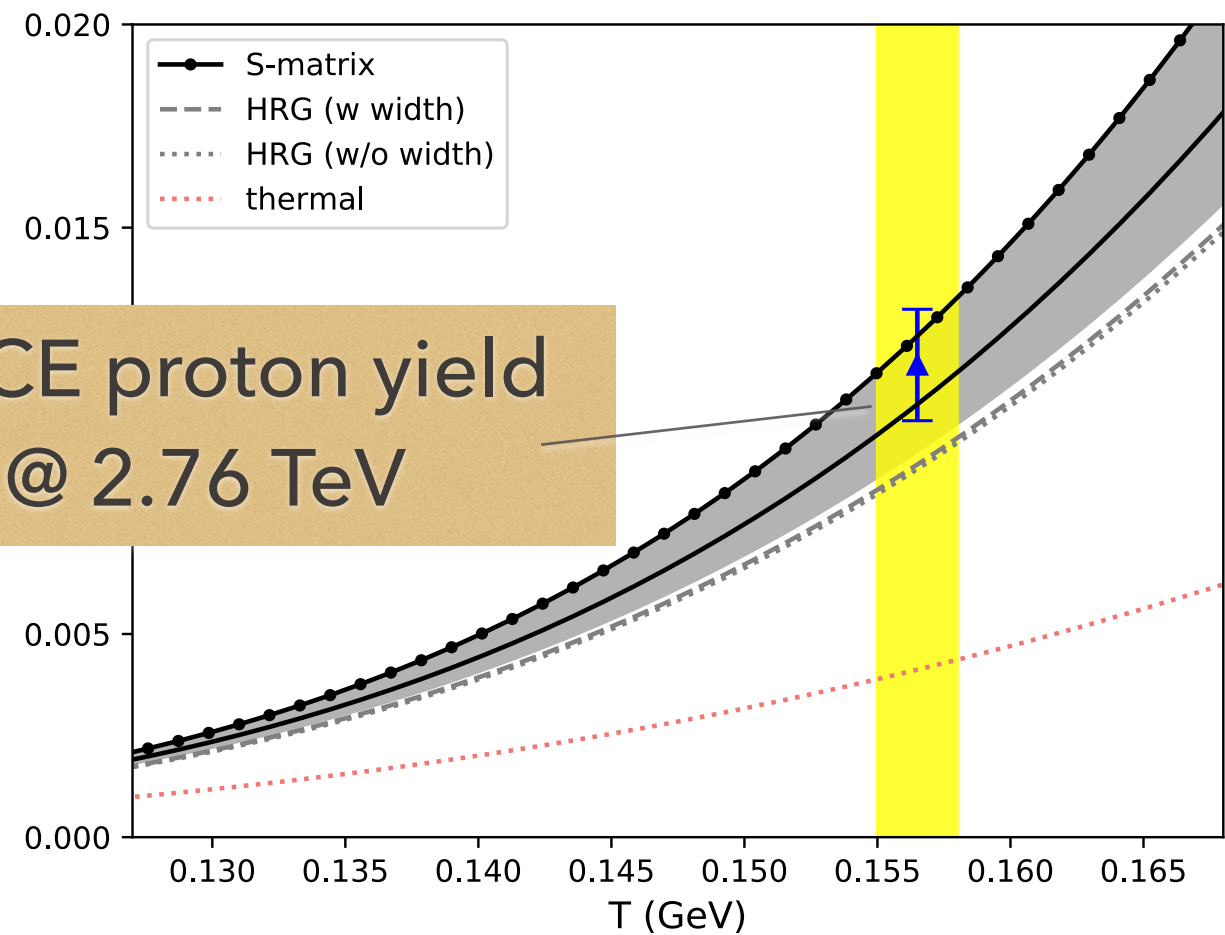
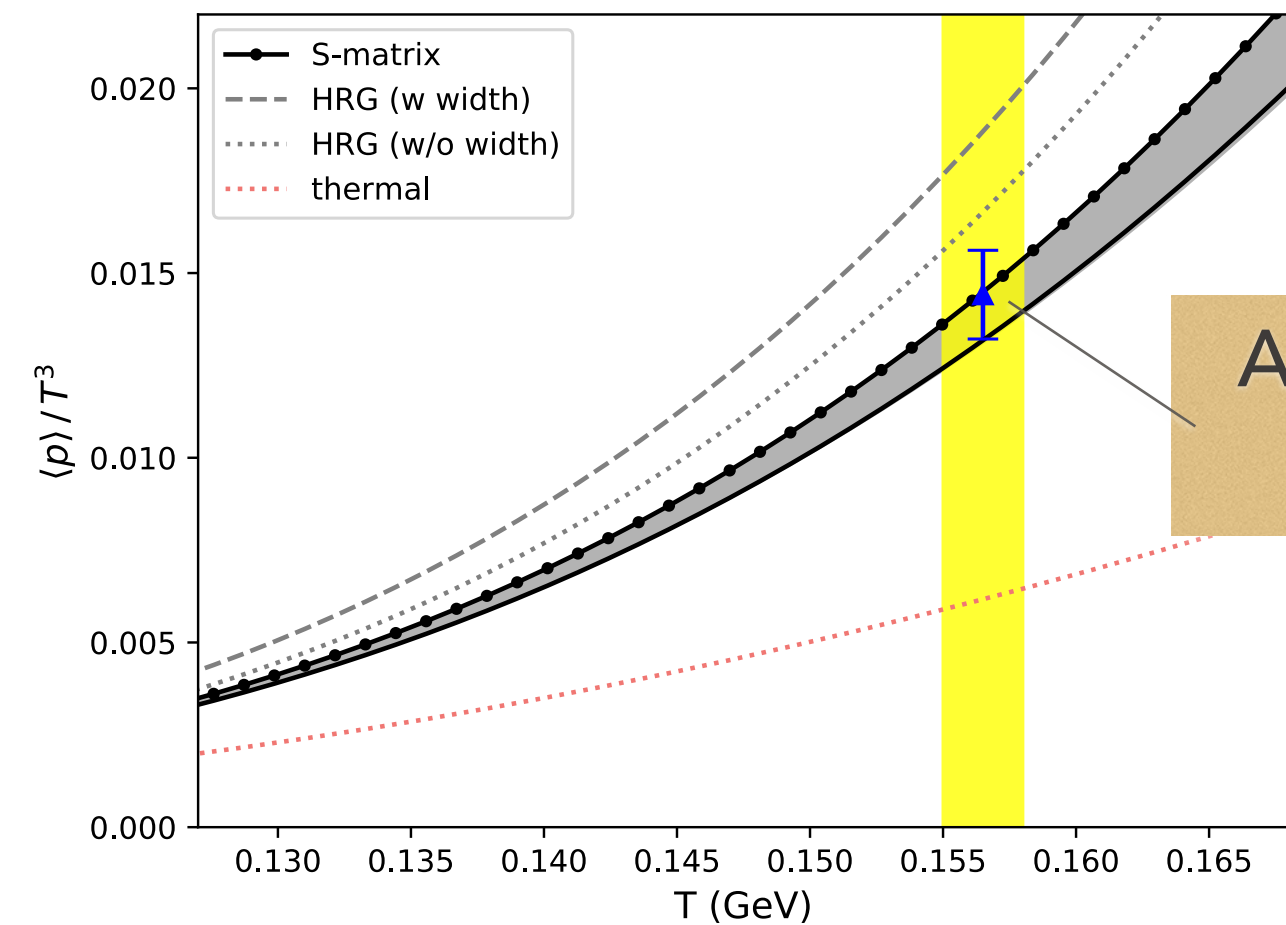
hrg

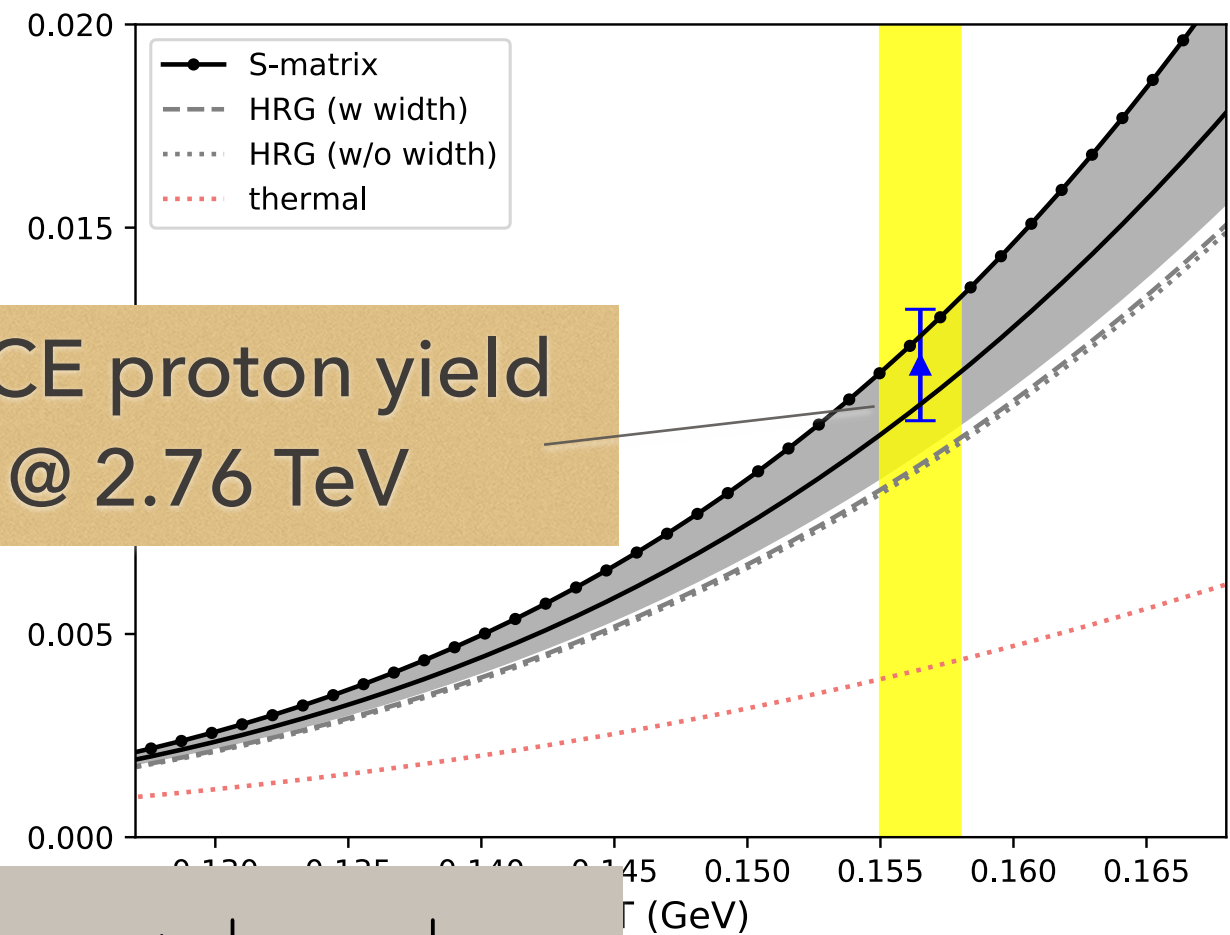
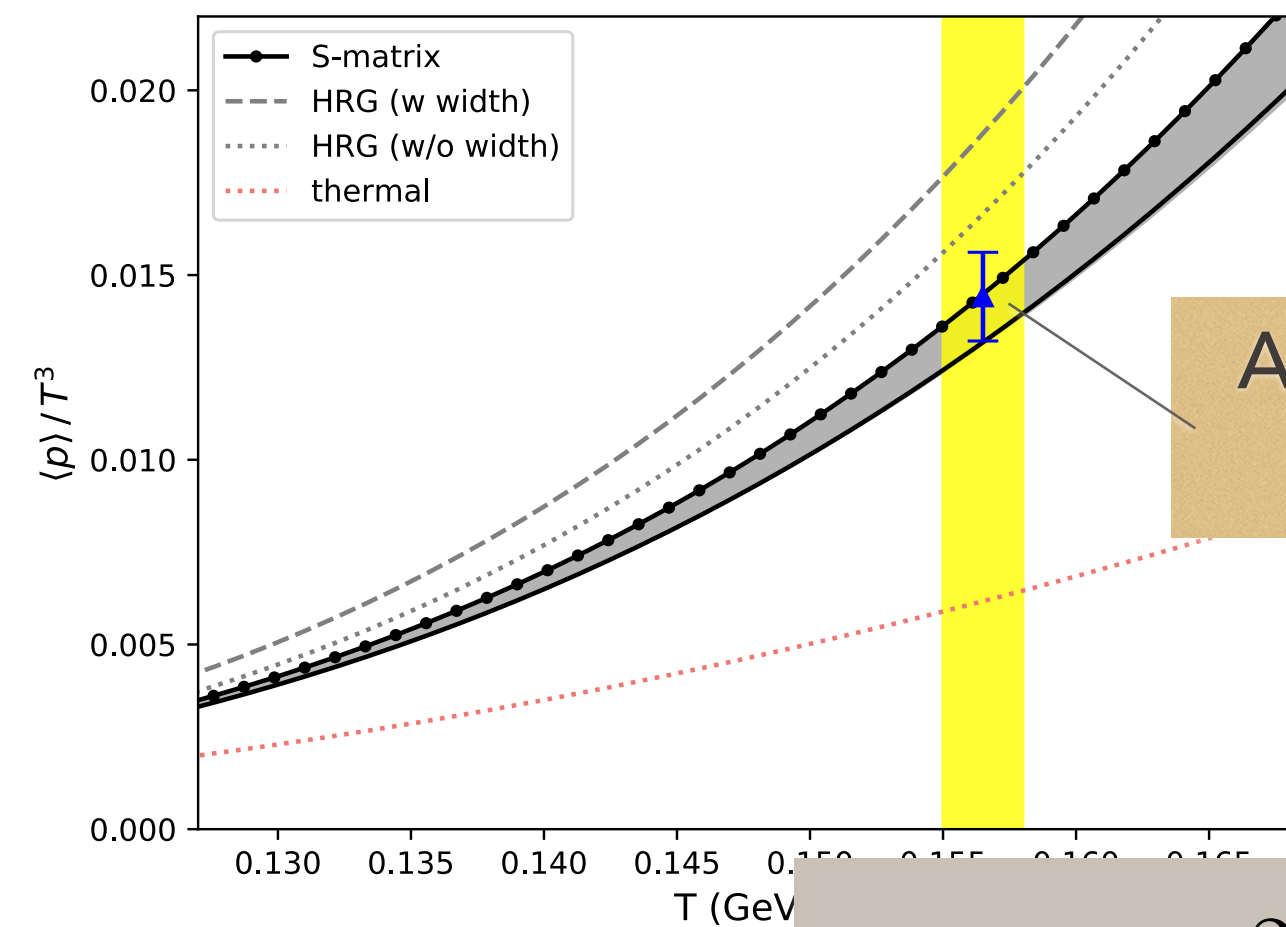
more lambdas

smat

Phys. Rev. C 103, 014904 (2021).

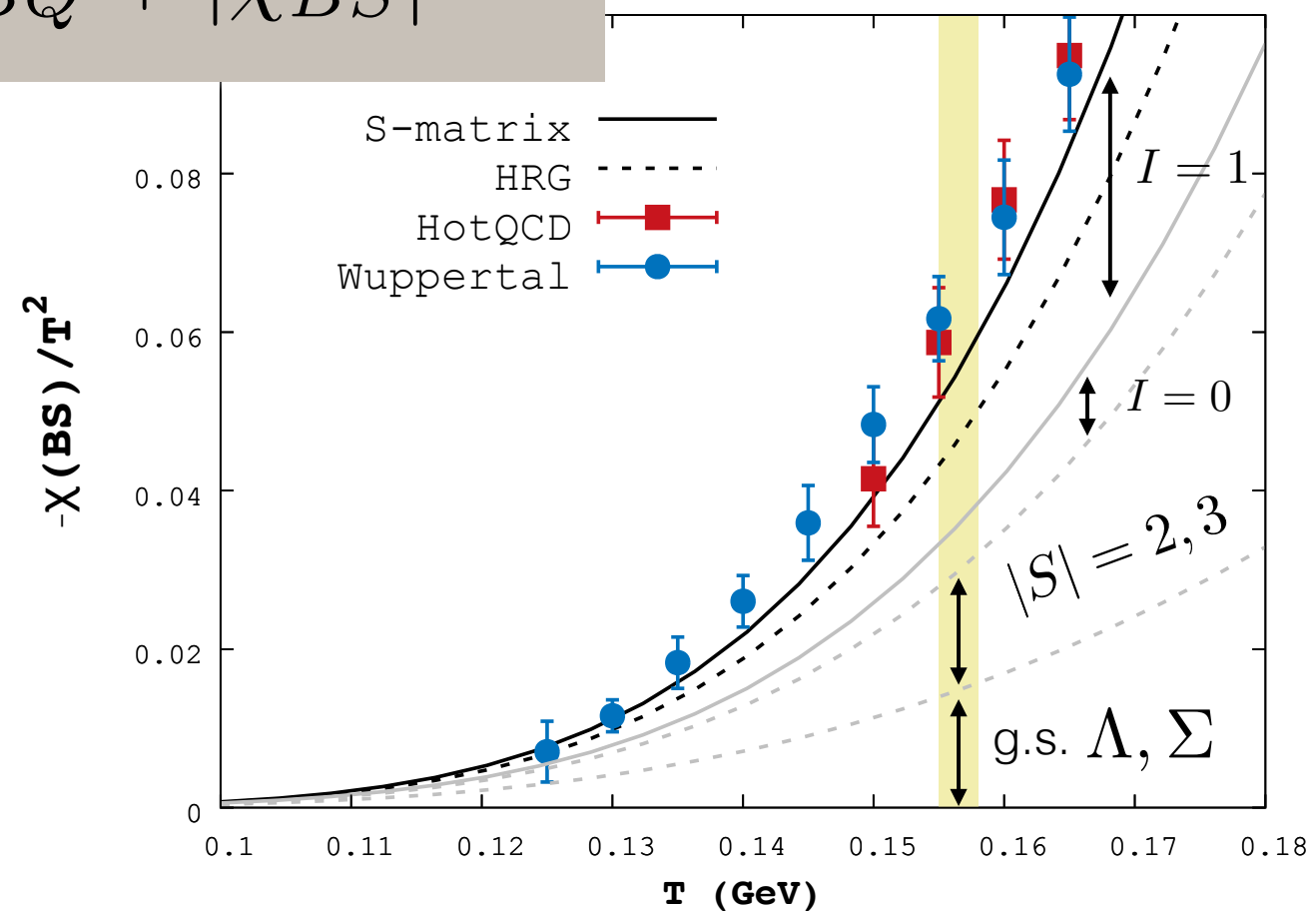
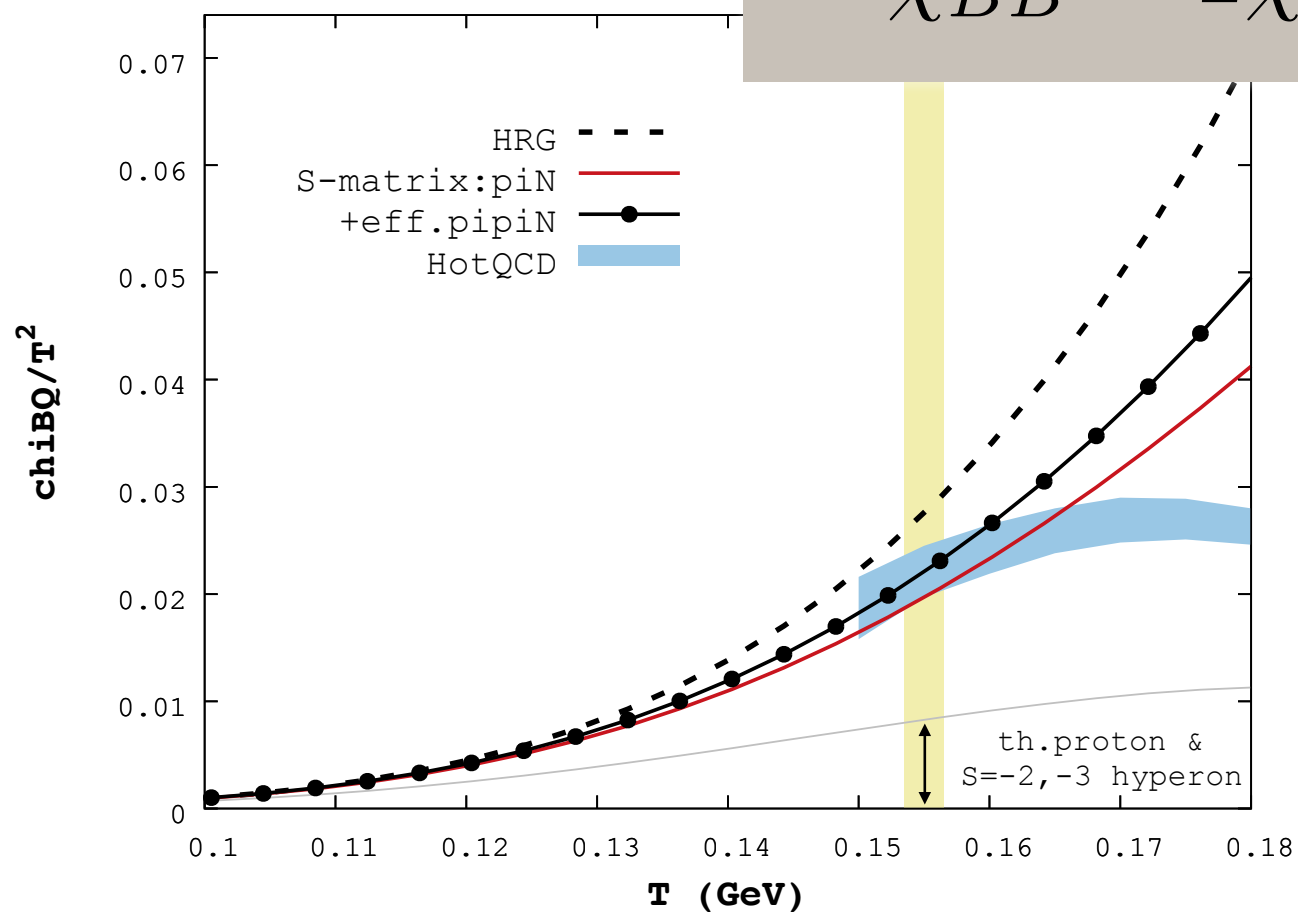
Phys. Lett. B 792, 304 (2019).

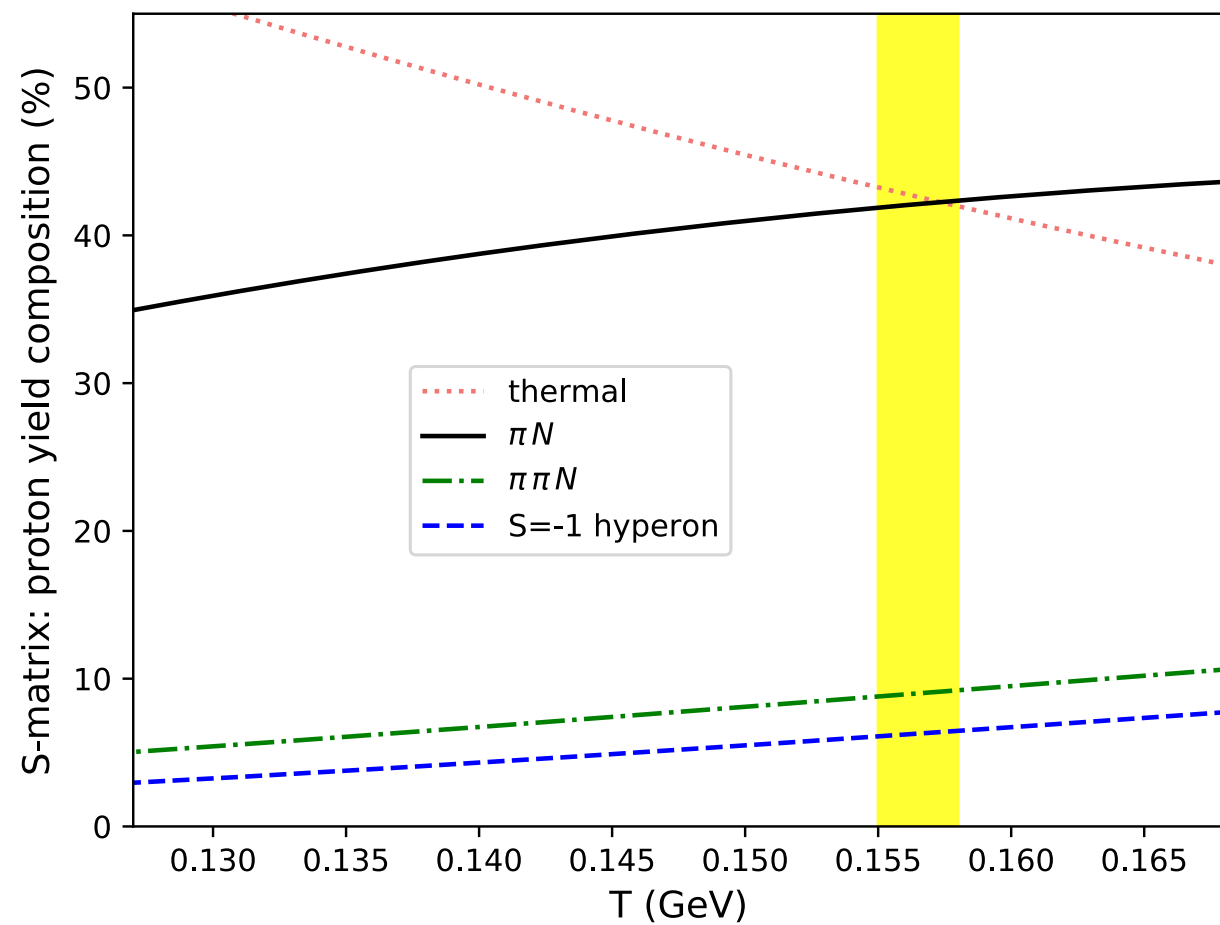




ALICE proton yield
@ 2.76 TeV

$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$

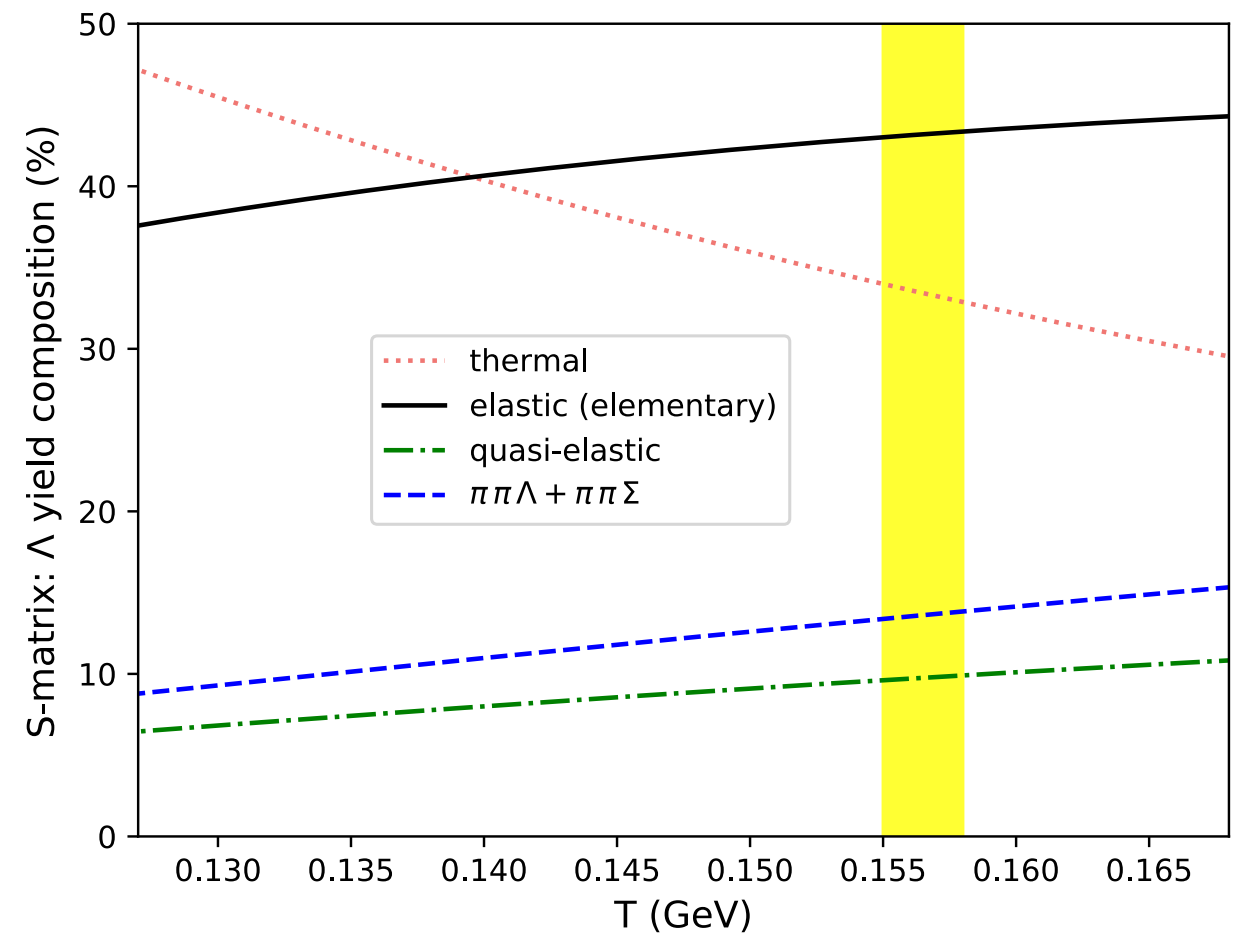




SAID GWU

πN phase shifts
 $\pi\pi N$ BGs
 hyperons

consistent treatment of res and non-res. int.



JPAC

Coupled-Channel system:
 $\bar{k}N, \pi\Lambda, \pi\Sigma, \dots$
extra hyperon states
beyond PDG
unitarity BGs

IN-MEDIUM EFFECTS

VACUUM PHYSICS?

Quantum statistical mechanics of gases in terms of dynamical filling fractions and scattering amplitudes

André LeClair

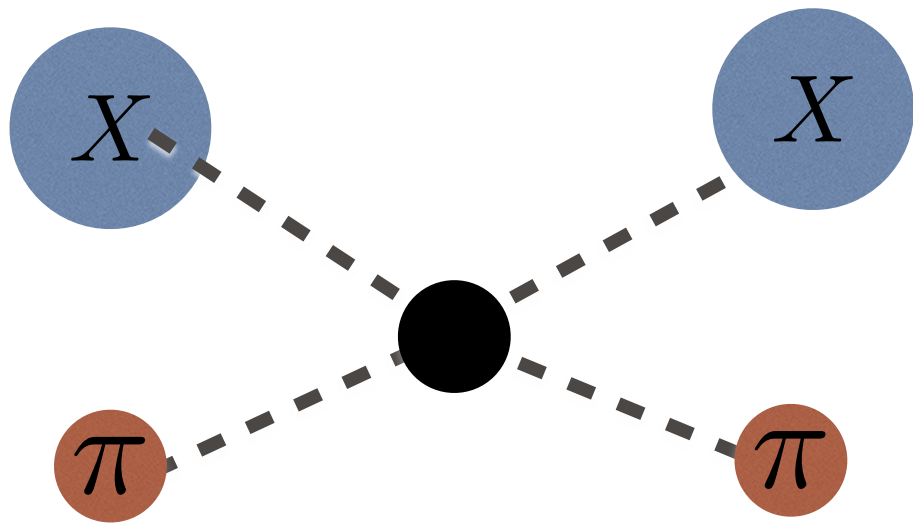
Newman Laboratory, Cornell University, Ithaca, NY, USA

Received 22 November 2006, in final form 3 May 2007

Published 19 July 2007

Online at stacks.iop.org/JPhysA/40/9655

It helps to realize that at least in principle it is possible to decouple the zero temperature dynamics and the quantum statistical sums. The argument is simple: the computation of the partition function $Z = \text{Tr}(e^{-\beta H})$ is in principle possible from the complete knowledge of the zero temperature eigenstates of the Hamiltonian H . In practice this is rather difficult and one resorts to perturbative methods such as the Matsubara method, which unfortunately entangles the zero temperature dynamics from the quantum statistical mechanics. However,



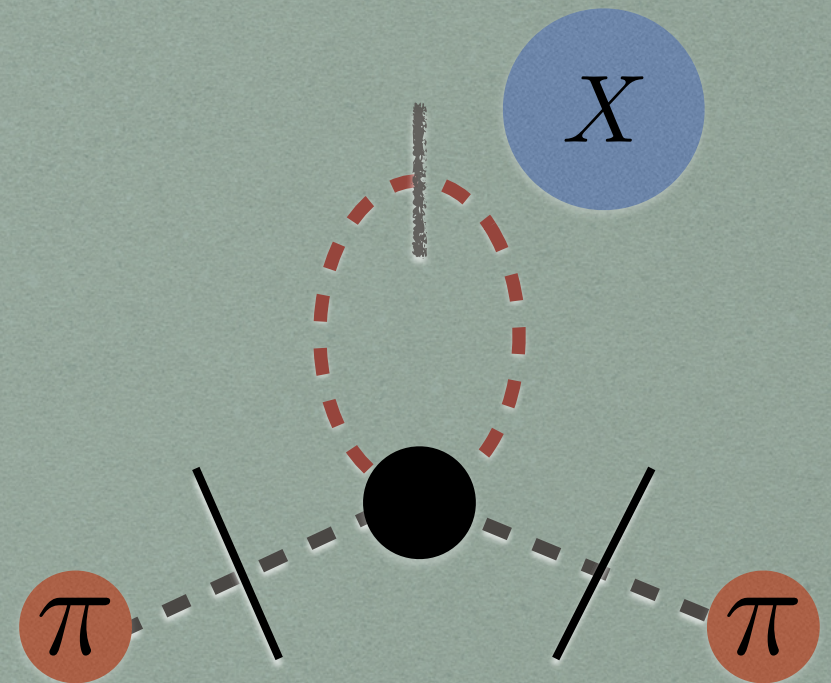
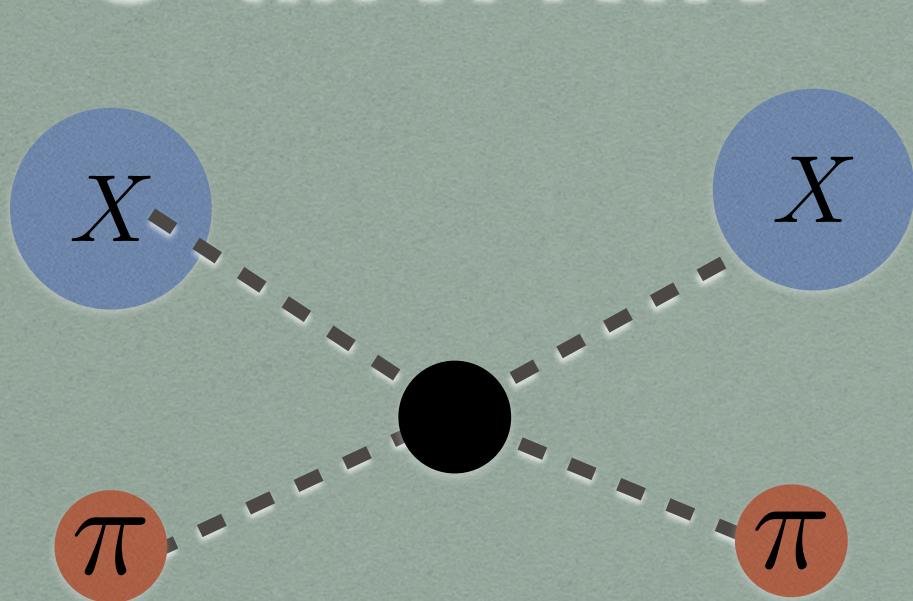
$$T_{\text{nr}} \approx -\frac{4\pi f}{2m_{\text{red}}}.$$

$$\begin{aligned}\Delta P &\approx \int \frac{d^3 P}{(2\pi)^3} \frac{dE'}{(2\pi)} e^{-\beta(m_{\text{tot}} + \frac{P^2}{2m_{\text{tot}}} + E')} 2Q(E') \\ &= \int \frac{d^3 P}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} e^{-\beta(m_{\text{tot}} + \frac{P^2}{2m_{\text{tot}}} + \frac{q^2}{2m_{\text{red}}})} (-T_{\text{nr}}) \\ &\approx N_{\text{th}}^A N_{\text{th}}^B \times (-T_{\text{nr}}).\end{aligned}$$

$$\begin{aligned}\Delta P &\approx T \int \frac{d^3 p_A}{(2\pi)^3} e^{-\beta(m_A + \frac{p_A^2}{2m_A})} (-\beta \Delta m_A) \\ &= -\Delta m_A N_{\text{th}}^A \\ &= N_{\text{th}}^A N_{\text{th}}^B \times \frac{4\pi f}{2m_{\text{red}}}.\end{aligned}$$

*Change of pressure to due
"Dressed mass"*

IN-MEDIUM EFFECTS FROM S-MATRIX



$$\Delta P = N_{\text{th}}^A N_{\text{th}}^B \times \frac{4\pi f}{2m_{\text{red}}}.$$

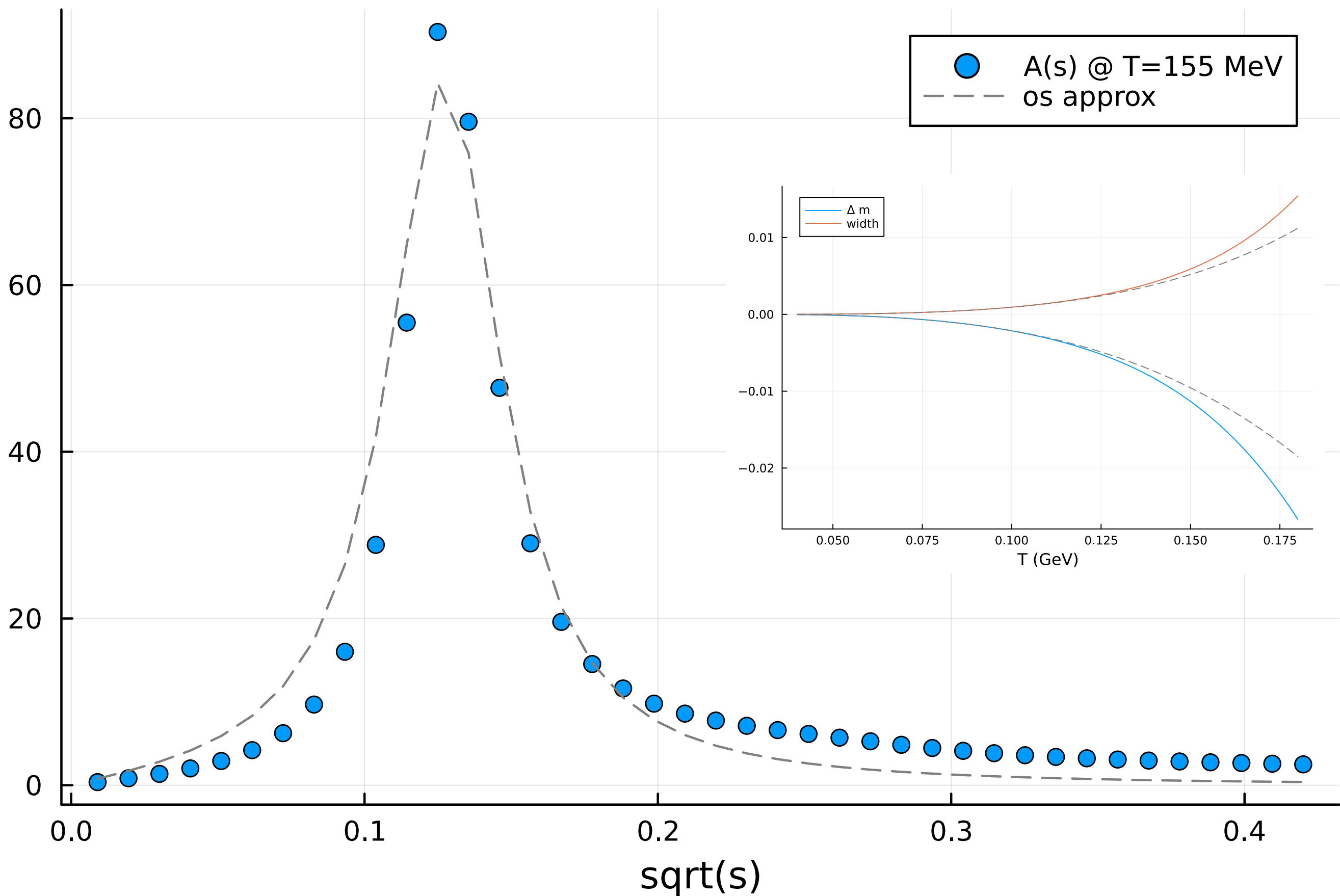
$$\Sigma_A(E_A) = \int \frac{d^3 k_B}{(2\pi)^3} \frac{1}{2E_B} n_{\text{th}}(E_B) T(AB \rightarrow AB).$$

$$\Delta m_A = \frac{1}{2E_A} \text{Re } \Sigma_A(p)$$

$$\approx N_{\text{th}}^B \times \frac{-4\pi f}{2m_{\text{red}}}.$$

A. Schenk NPB 363 (1991)

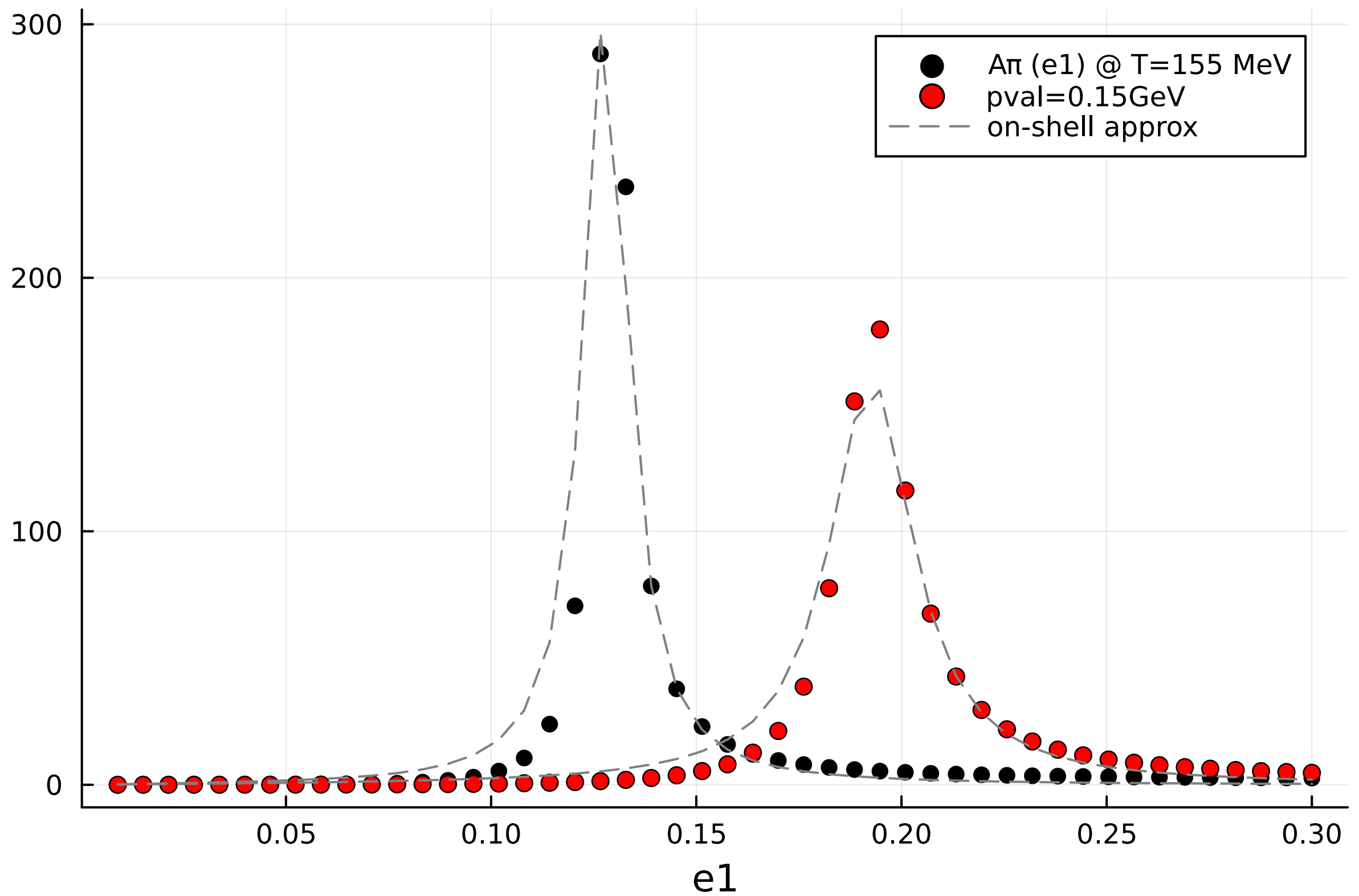
S. Jeon and P. J. Ellis PRD 58 045013 (1998)



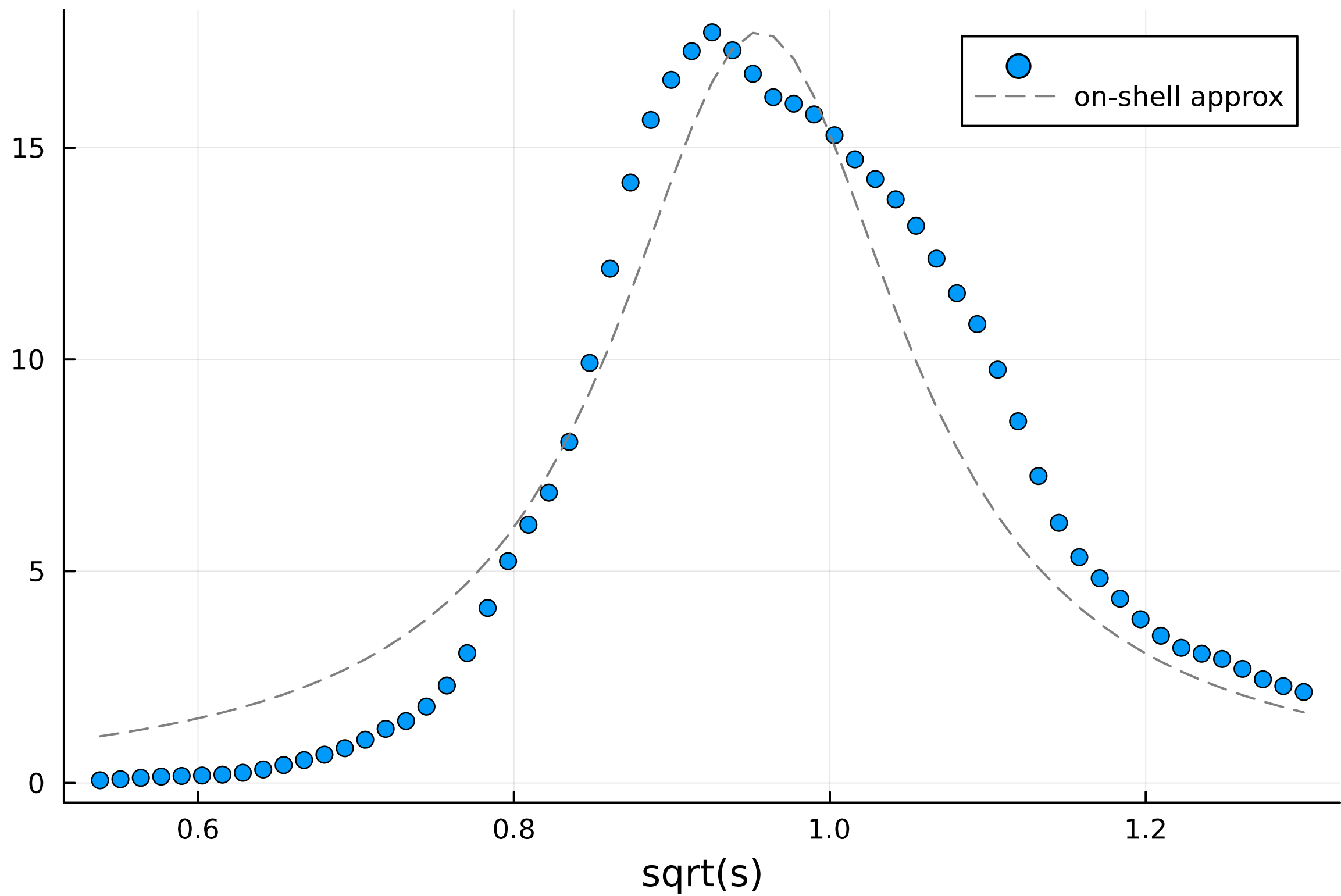
A. SCHERER AND J. J. DE SWART (1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

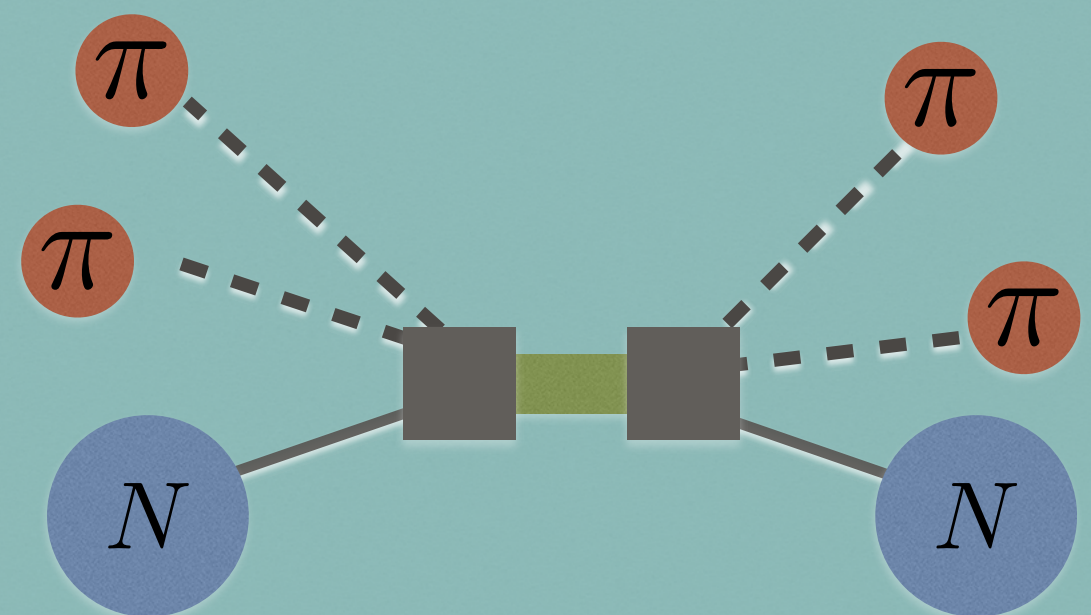
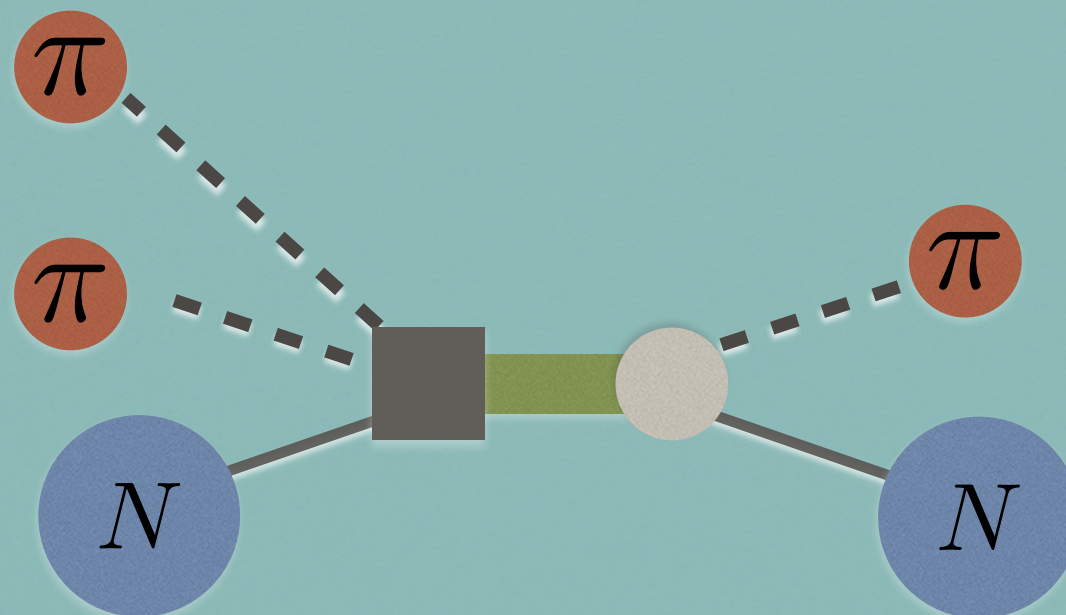
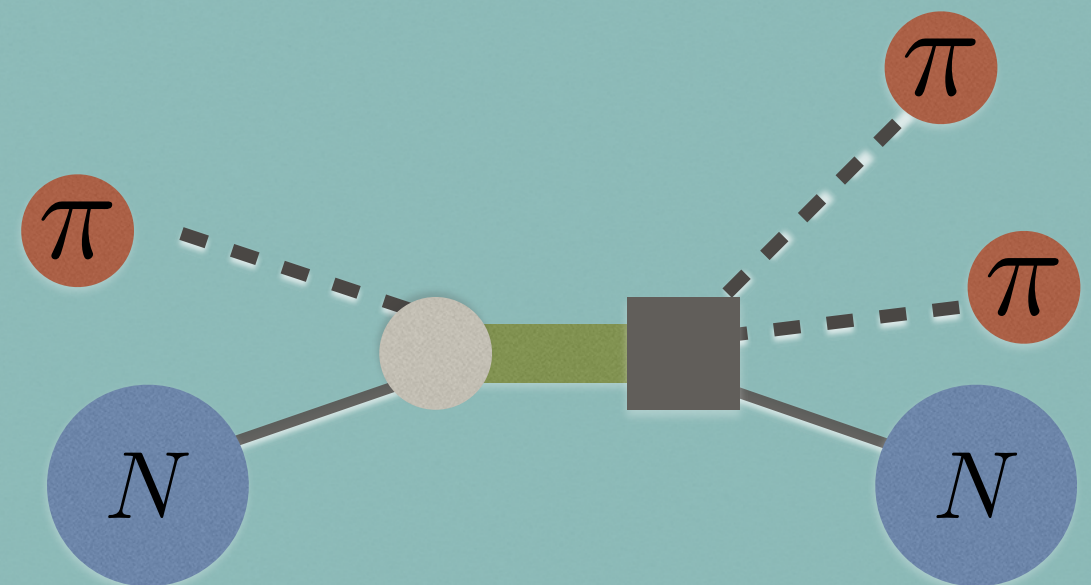
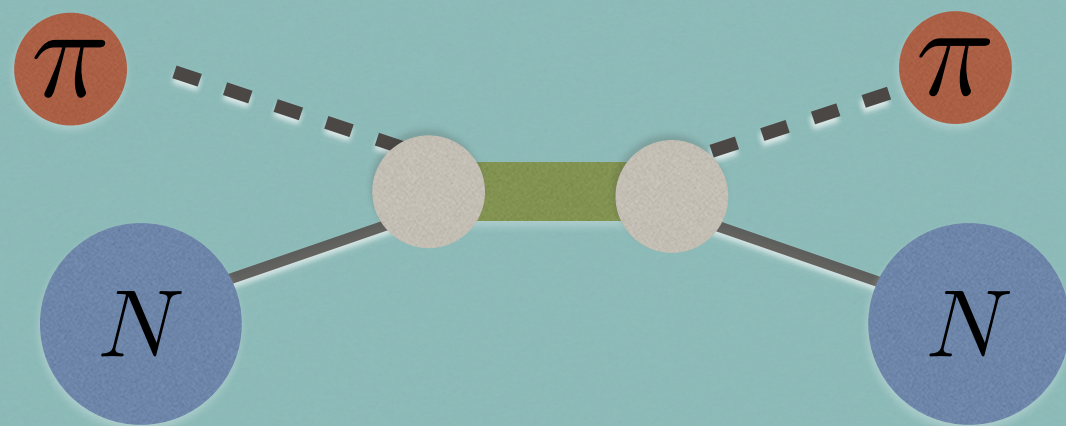
Pion spectral function



Proton spectral function

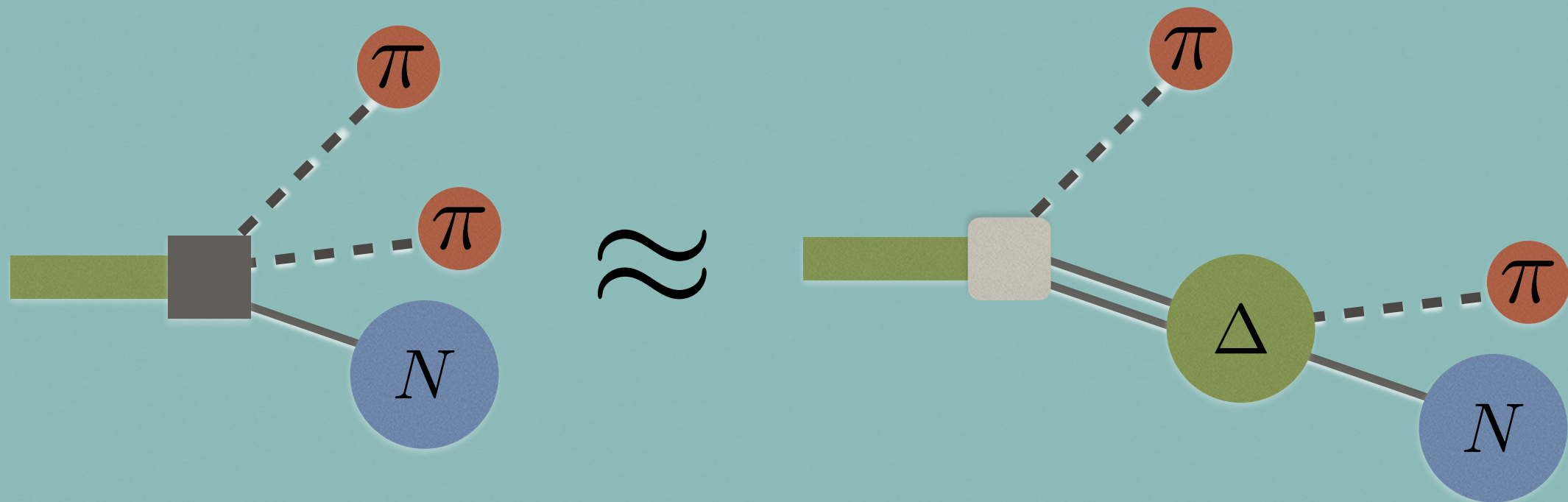


ISOBAR MODEL

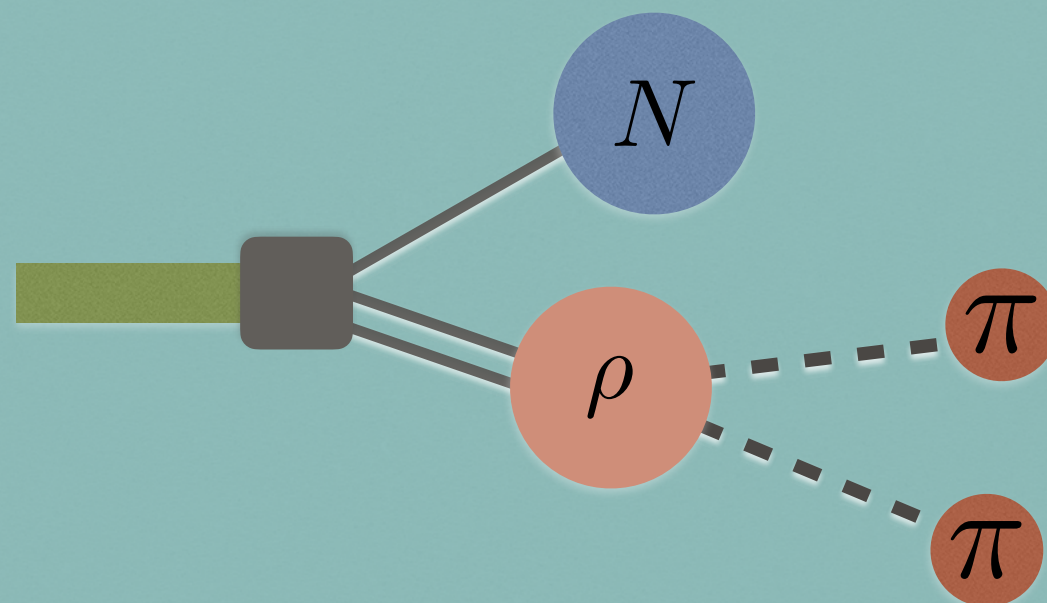


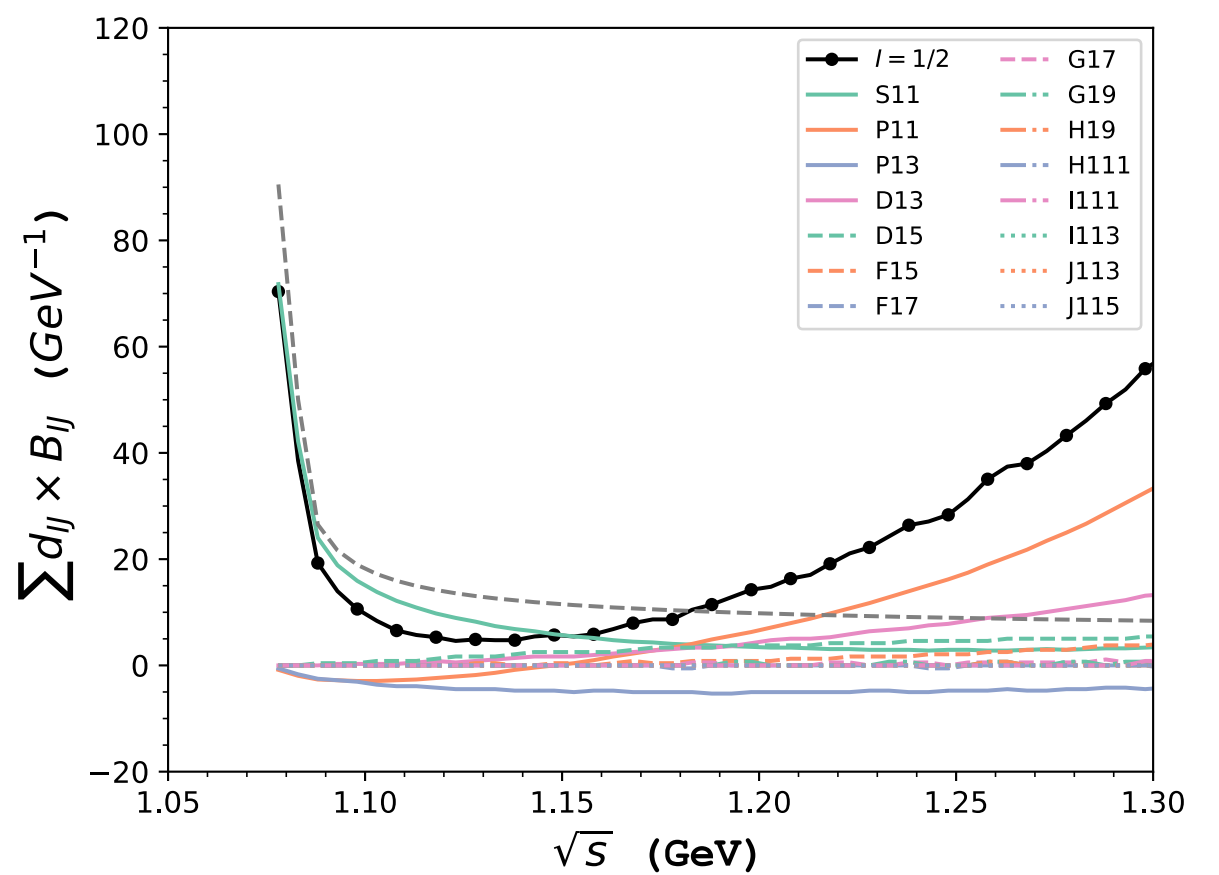
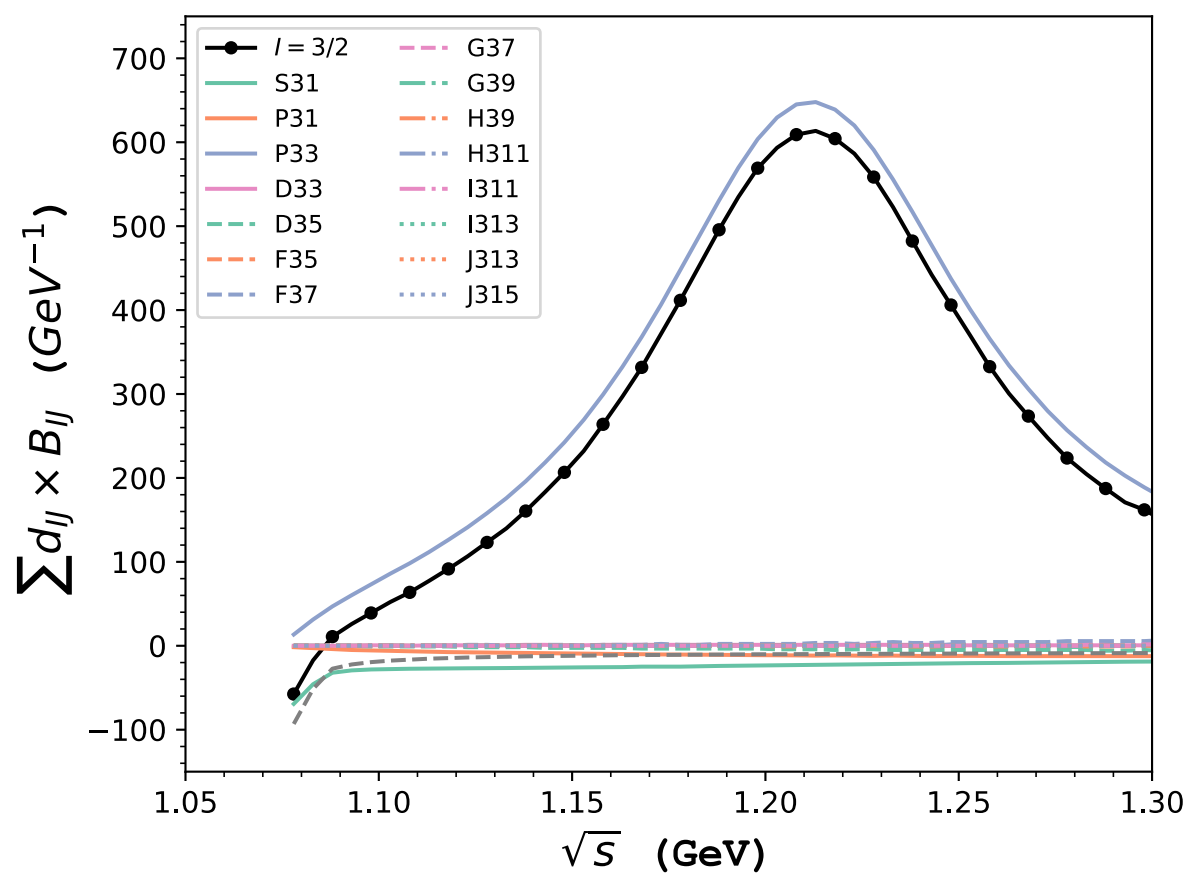
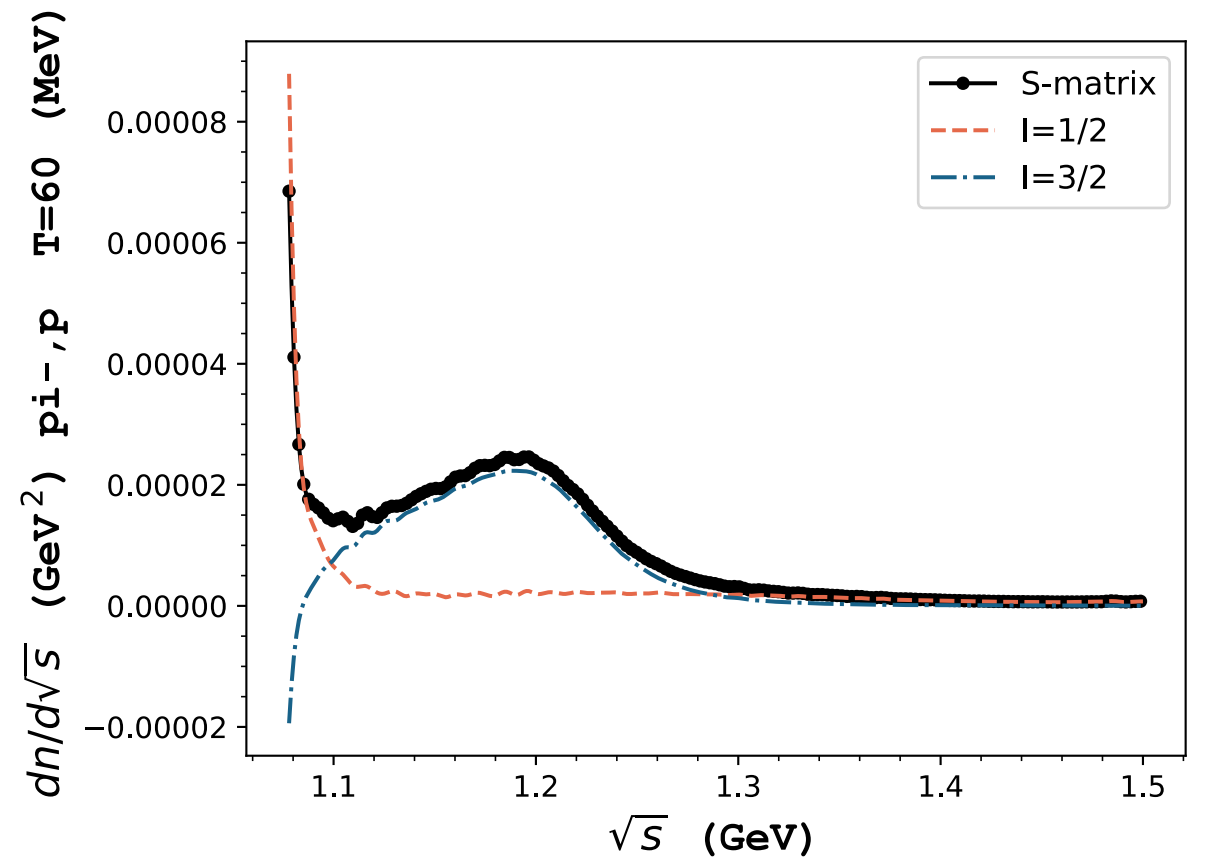
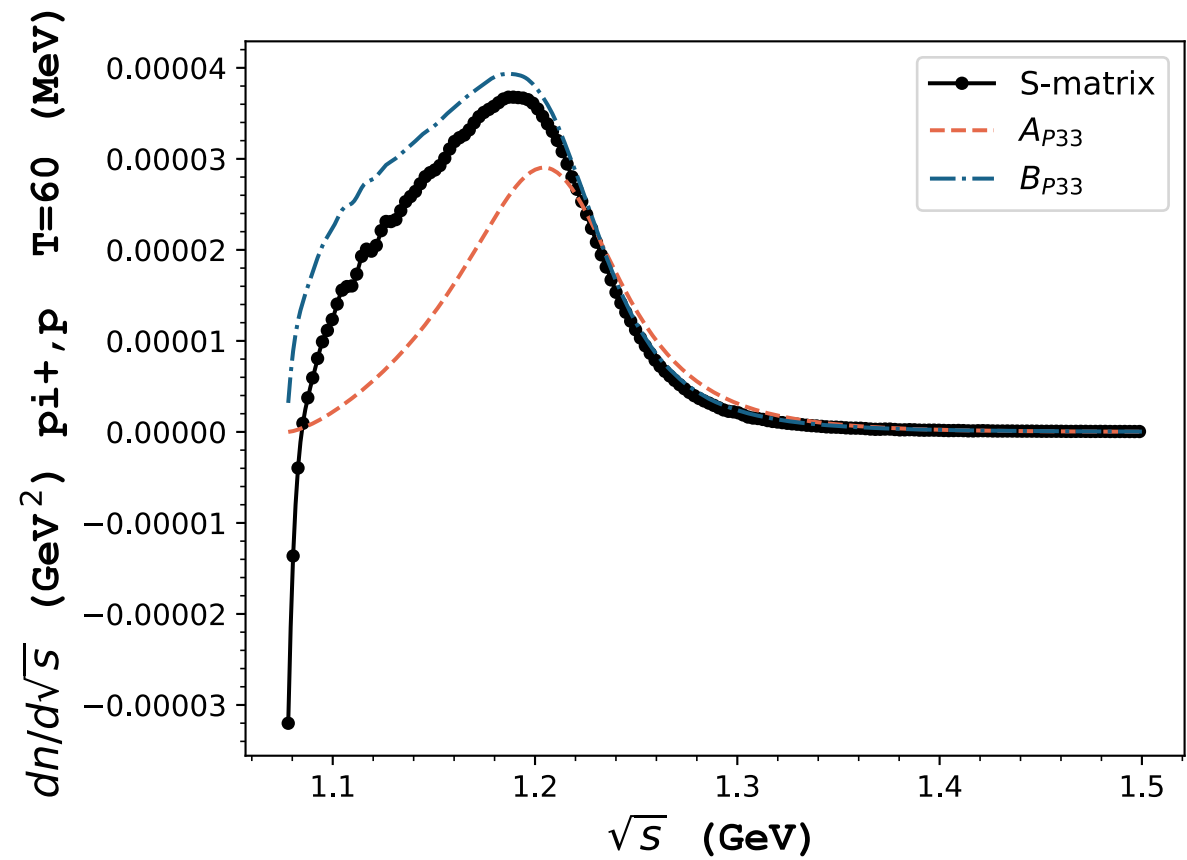
ISOBAR MODEL

sequential decay model



and / or





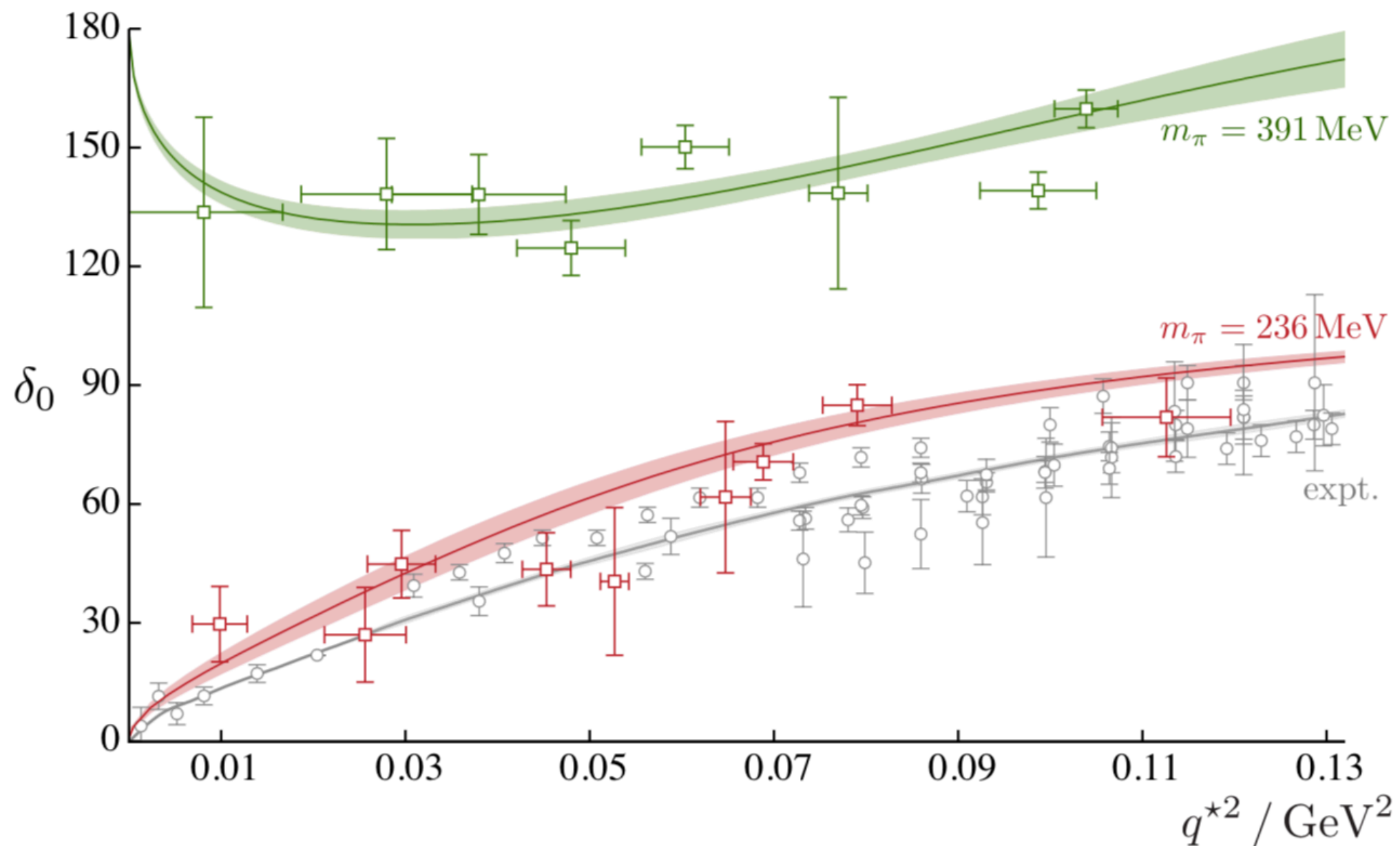
THE END (1)

ON-GOING PROJECTS

- Unquenching the Quark Model
- $S=-2,-3$ sectors; 3-body forces

LATTICE COMPUTATIONS ON PHASE SHIFT

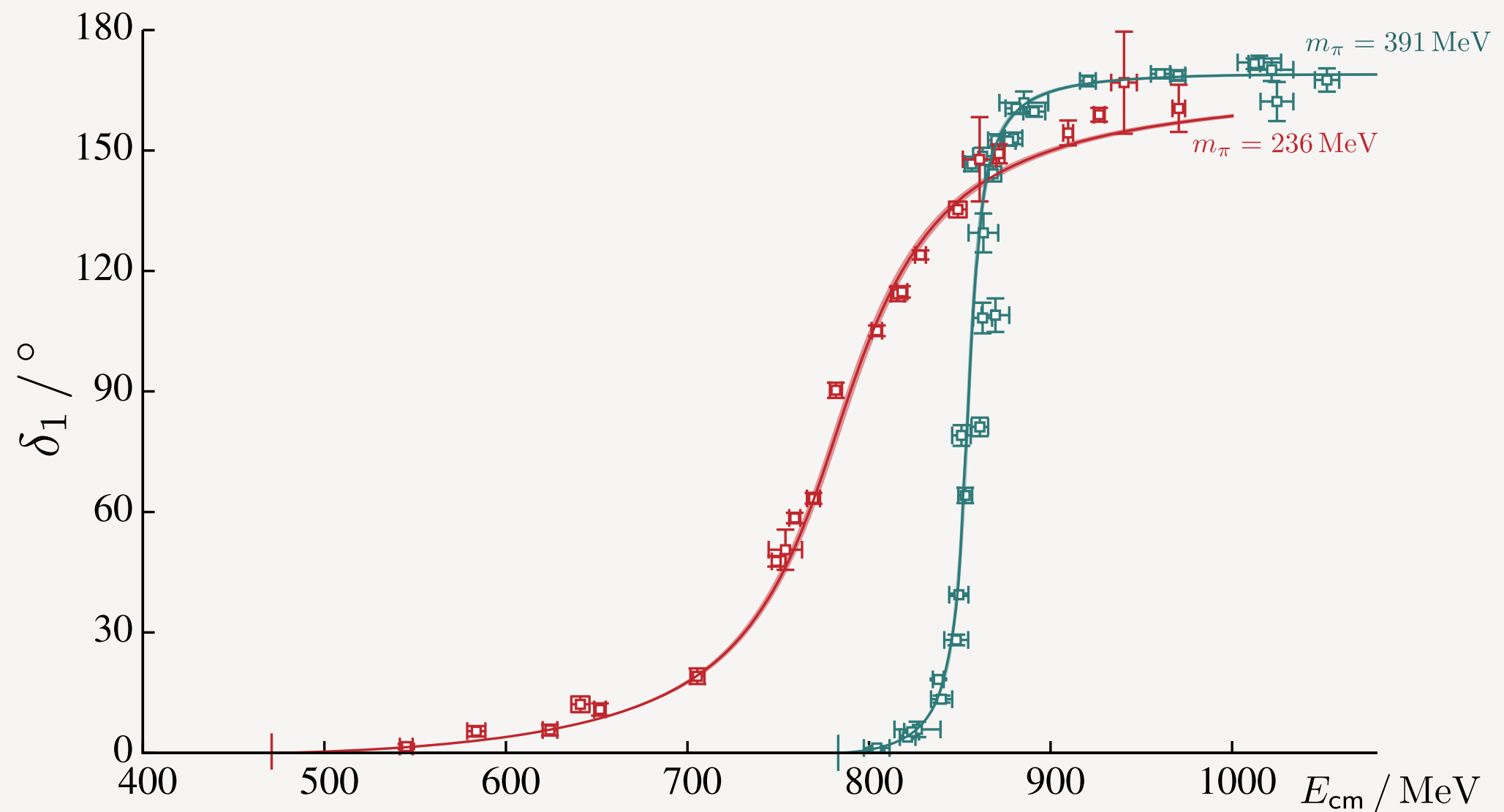
deuteron physics?



R. A. Briceno, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

LATTICE COMPUTATIONS ON PHASE SHIFT

WILSON *et al.*



NO SERIOUS MESON SPECTROSCOPY WITHOUT SCATTERING*

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SUSANA COITO

Institute of Modern Physics, CAS, Lanzhou 730000, China

(Received January 26, 2015)

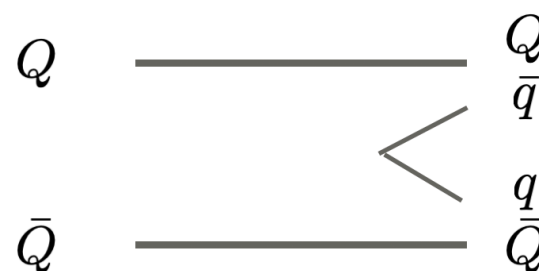
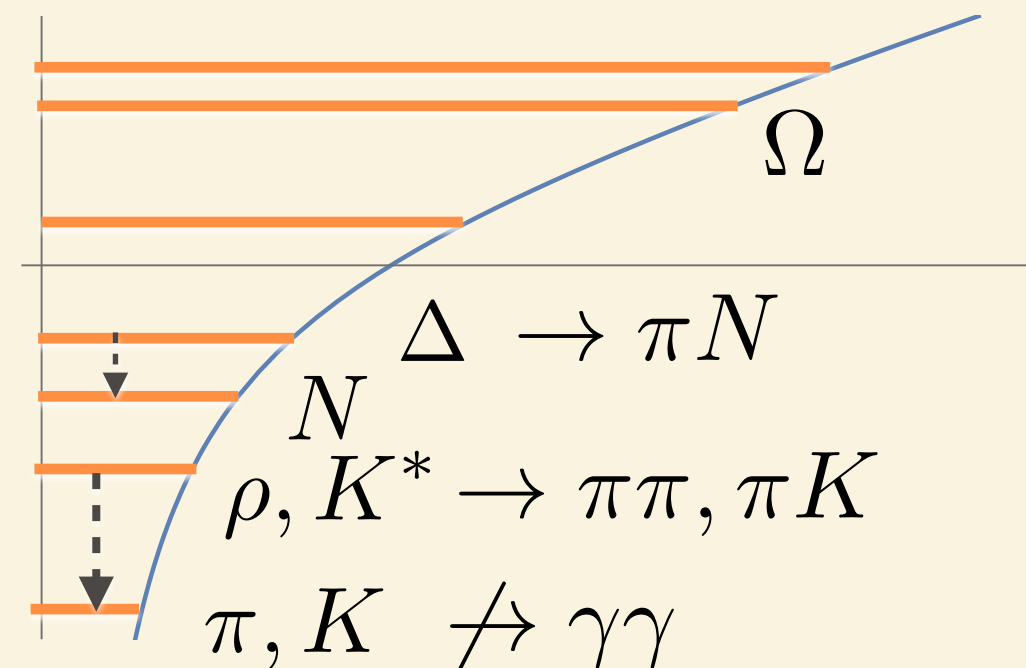
$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

meson loops effects:
shift in hadron masses

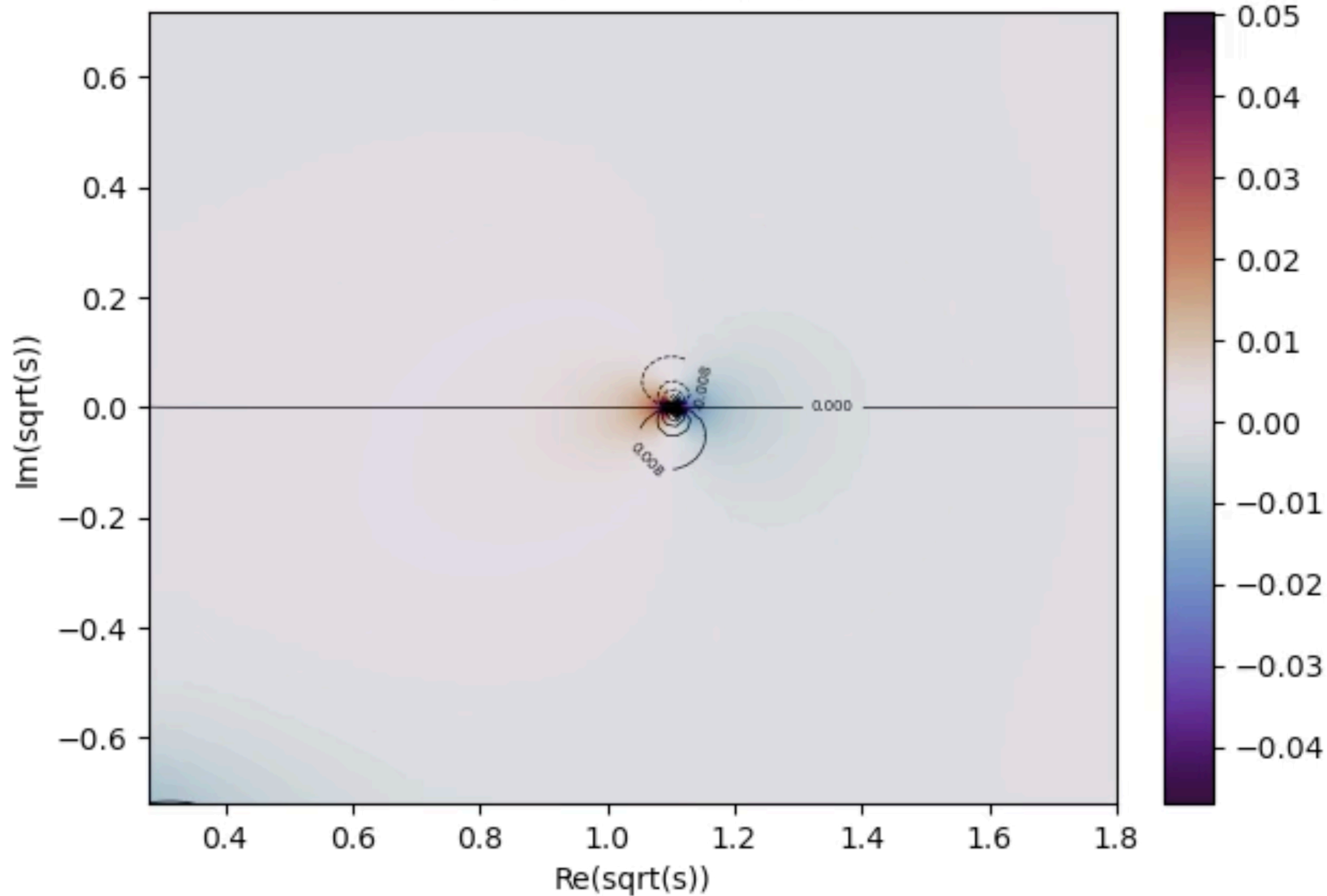
Quark Model States:
mixing with continuum

CONTINUUM

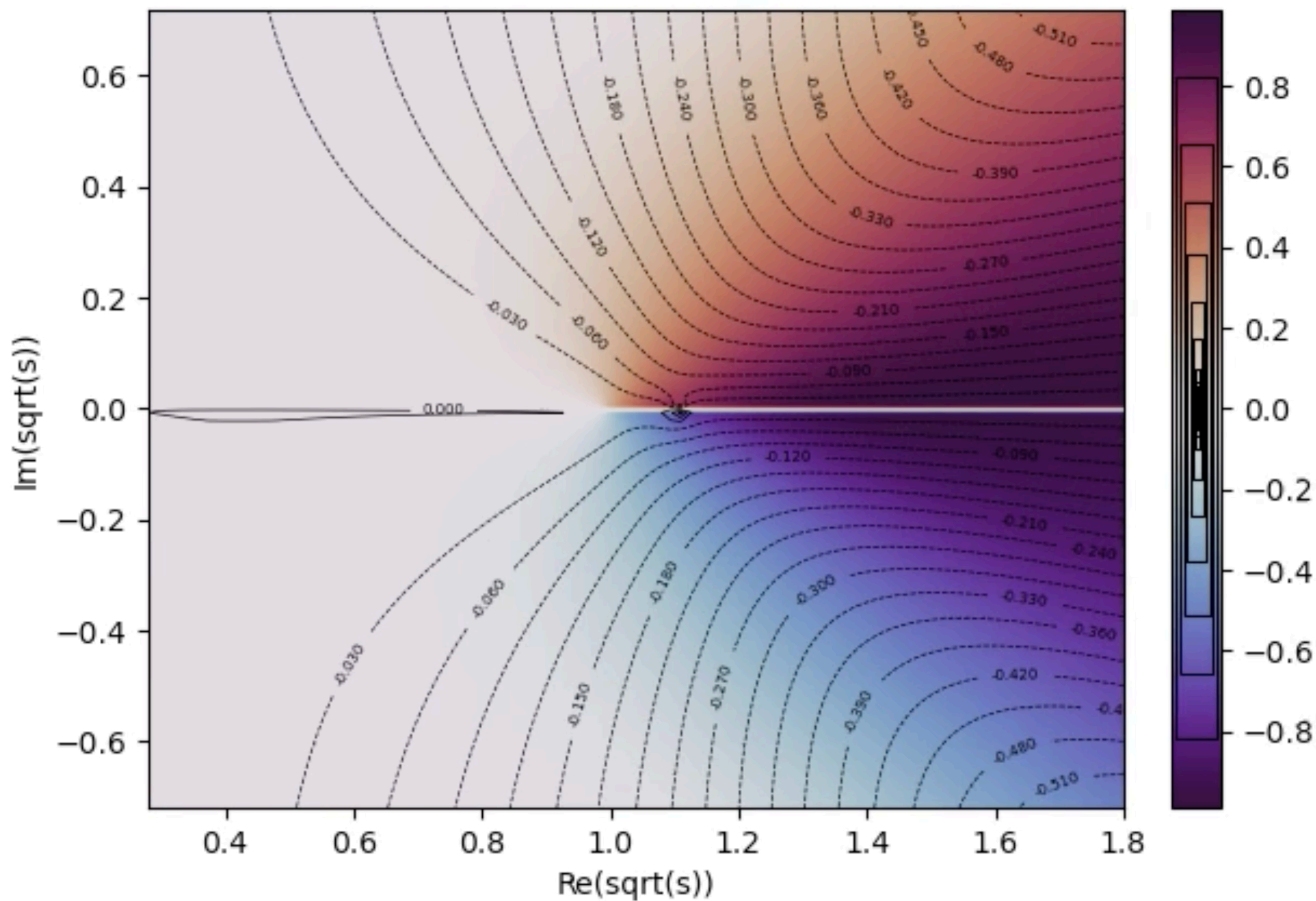
QCD spectrum



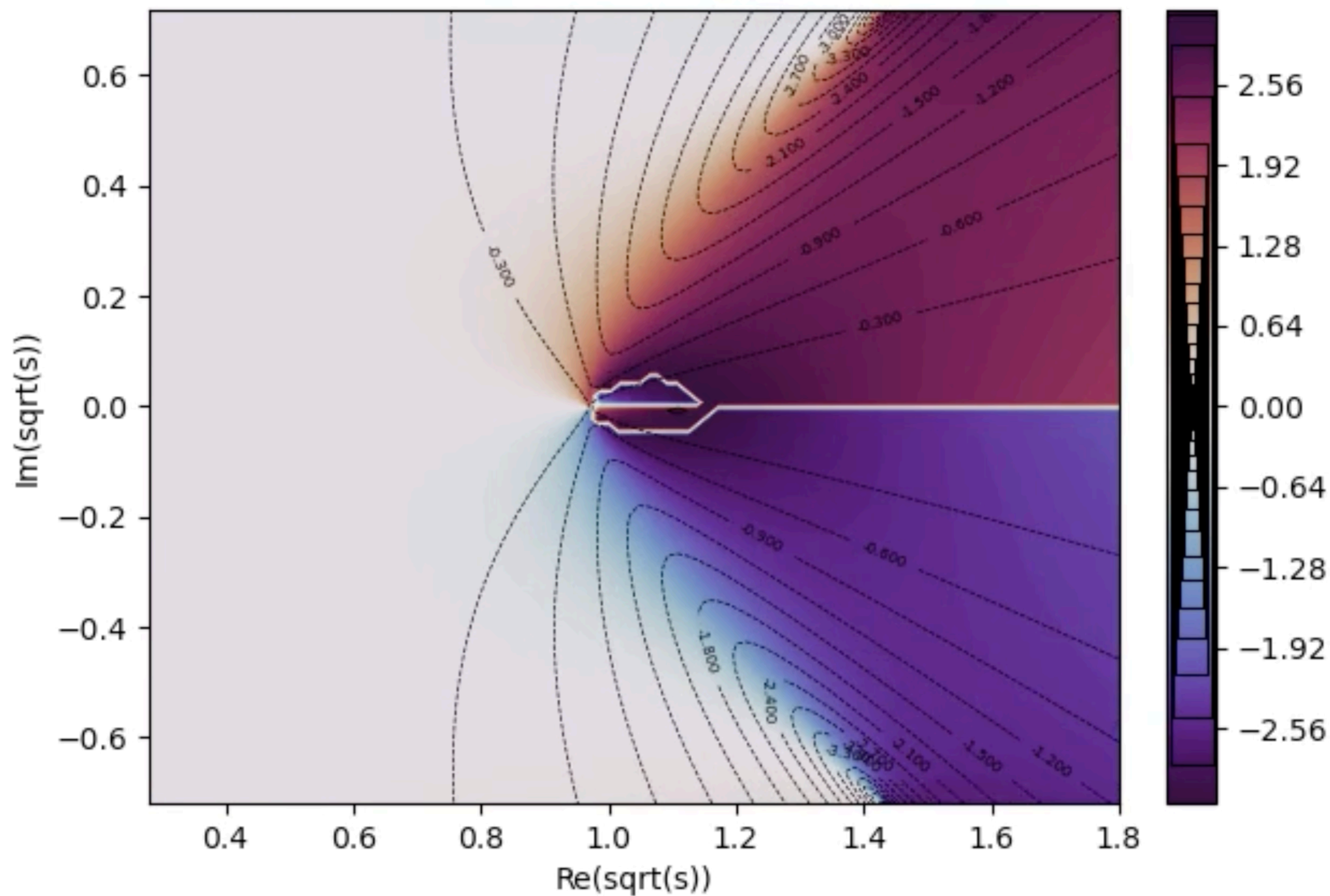
$(x,y)=(0.001, 0.001)$



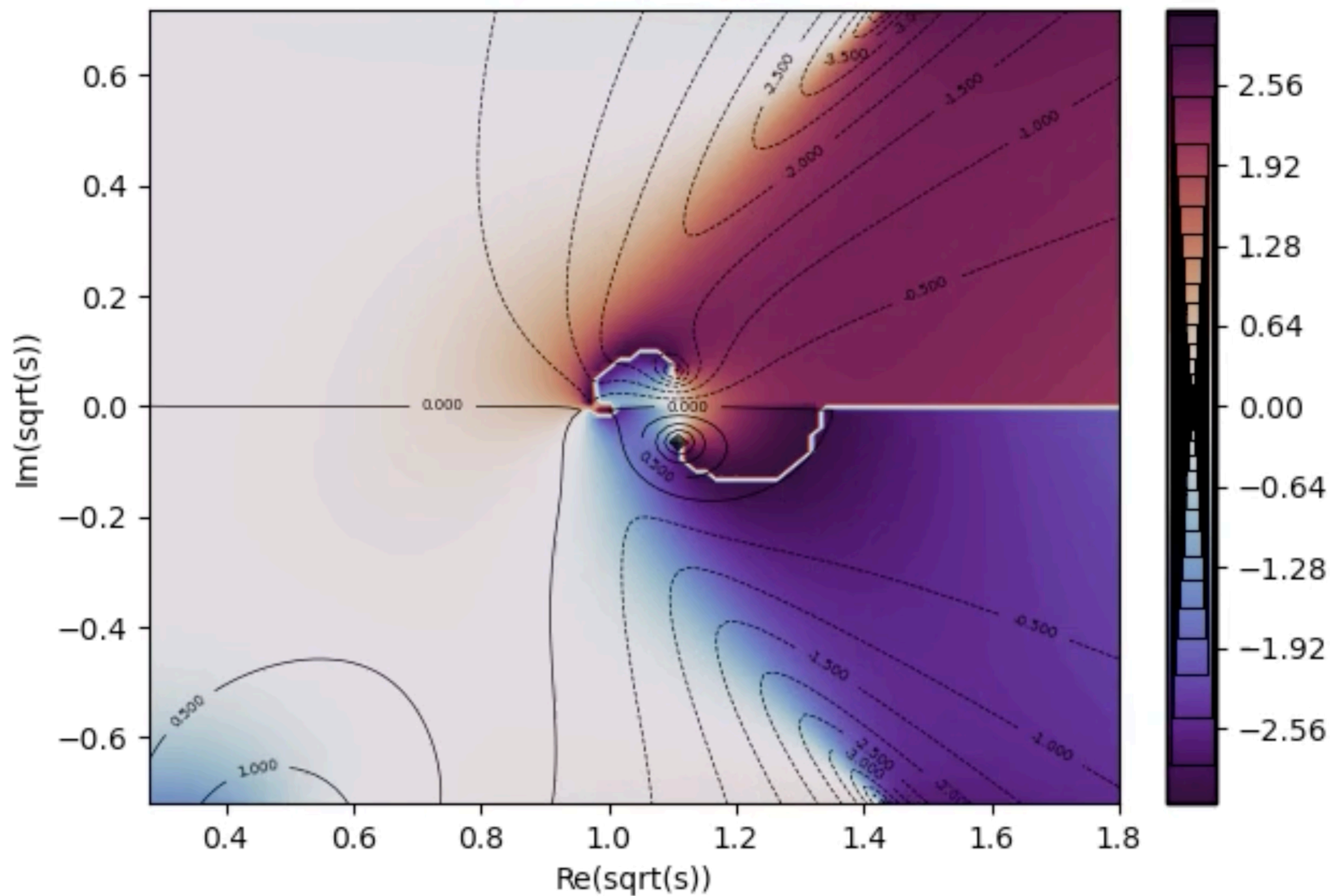
$(x,y)=(0.001, 0.527)$



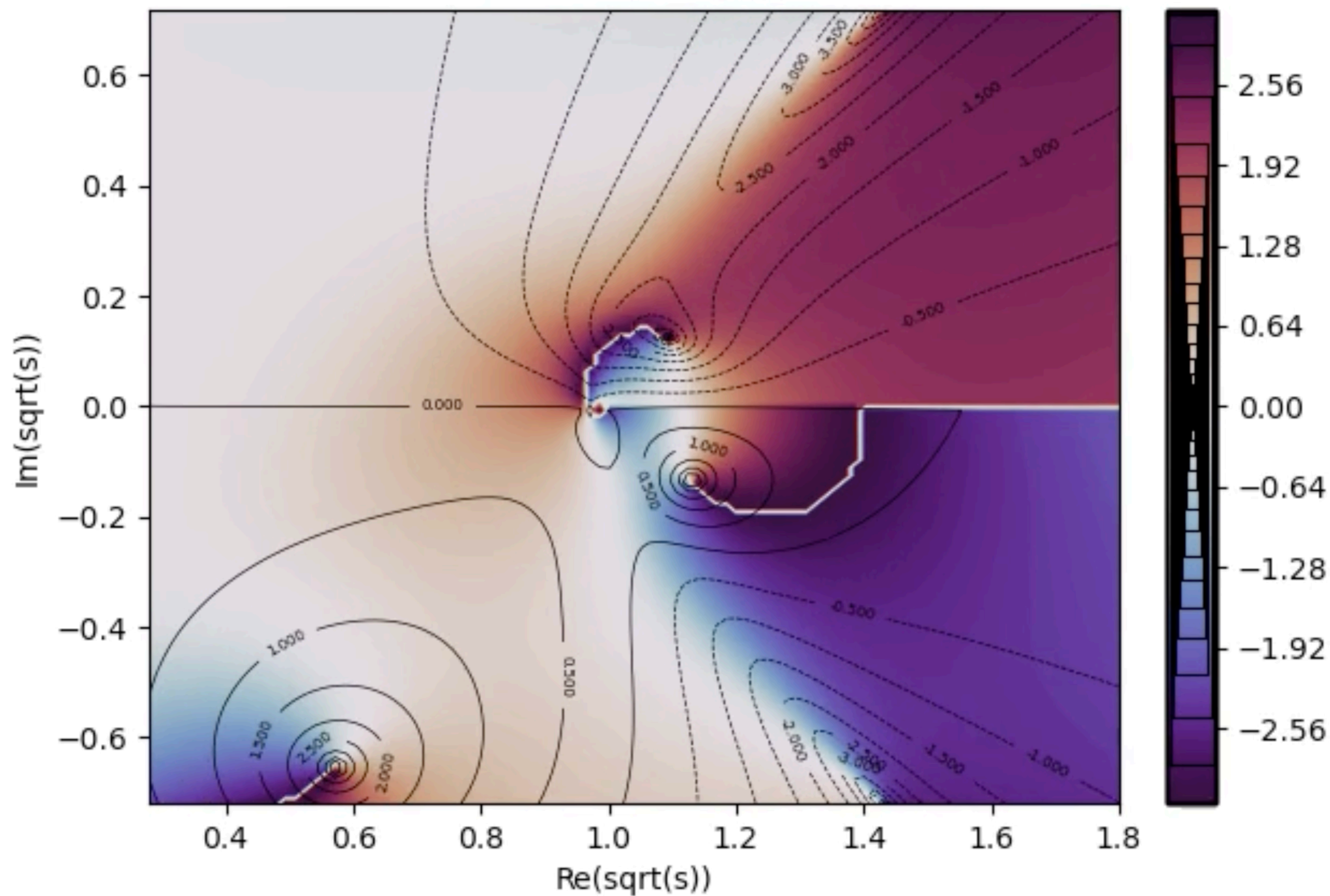
$(x,y)=(0.001, 1.0)$



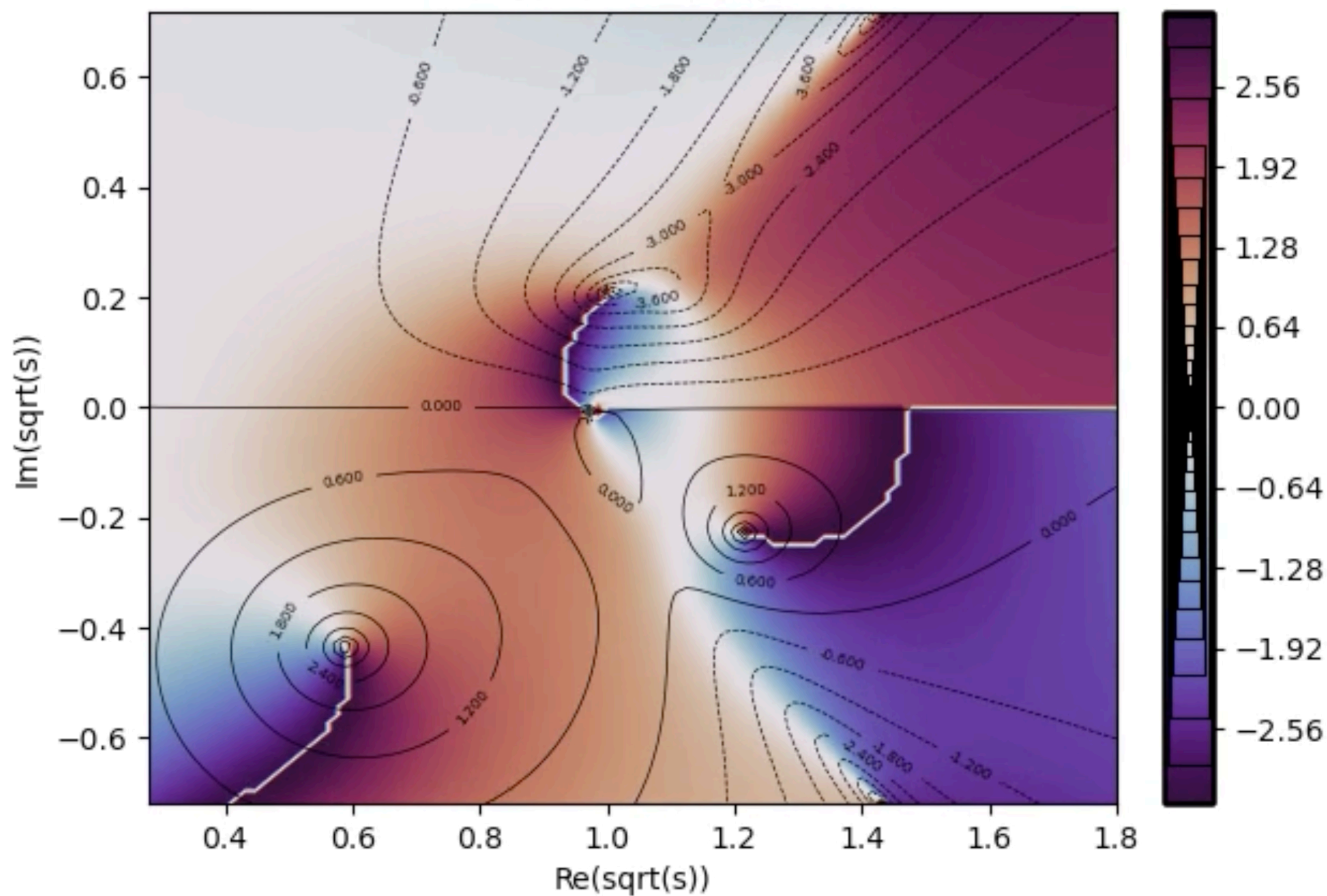
$(x,y)=(0.155, 1.0)$



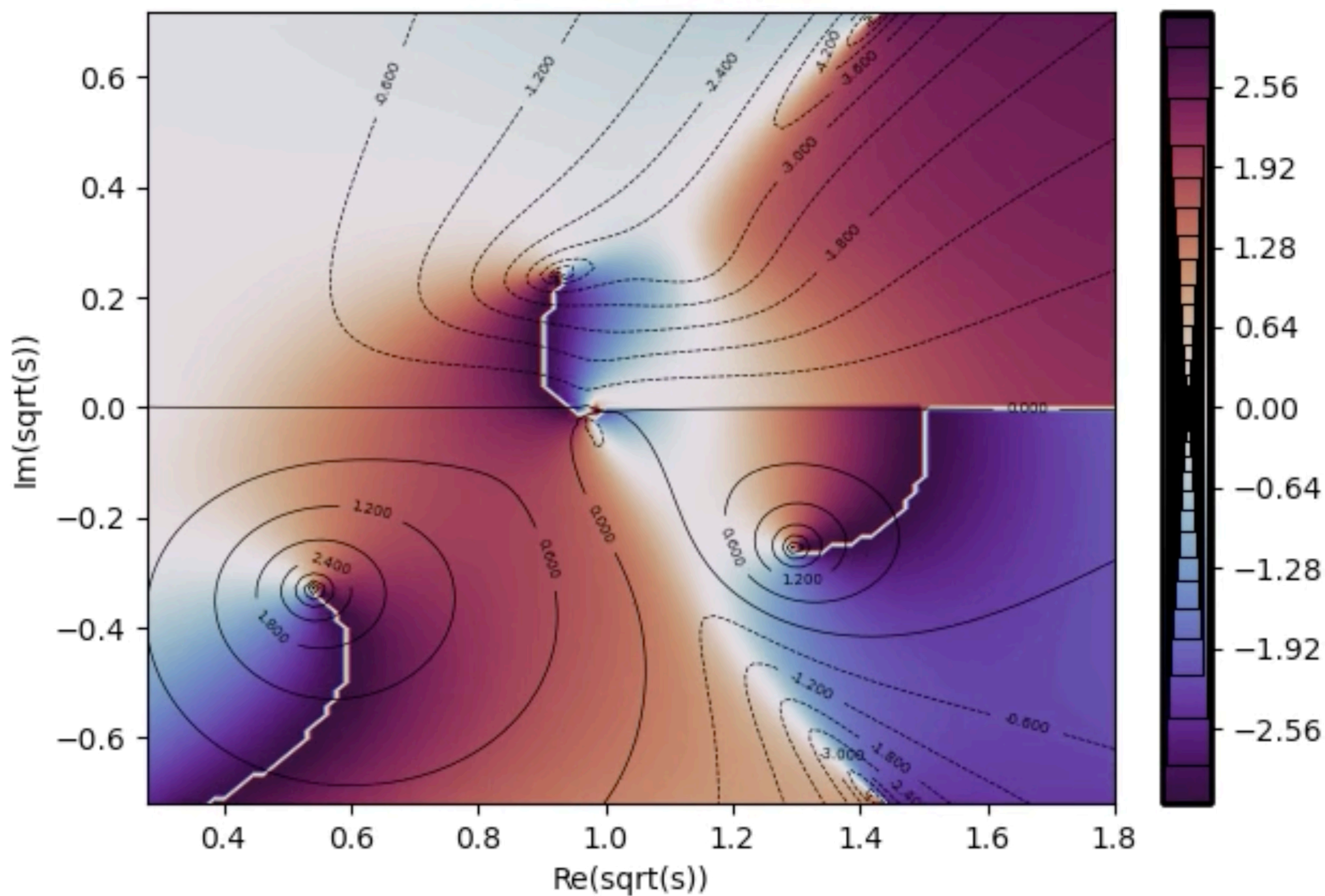
$(x,y)=(0.308, 1.0)$



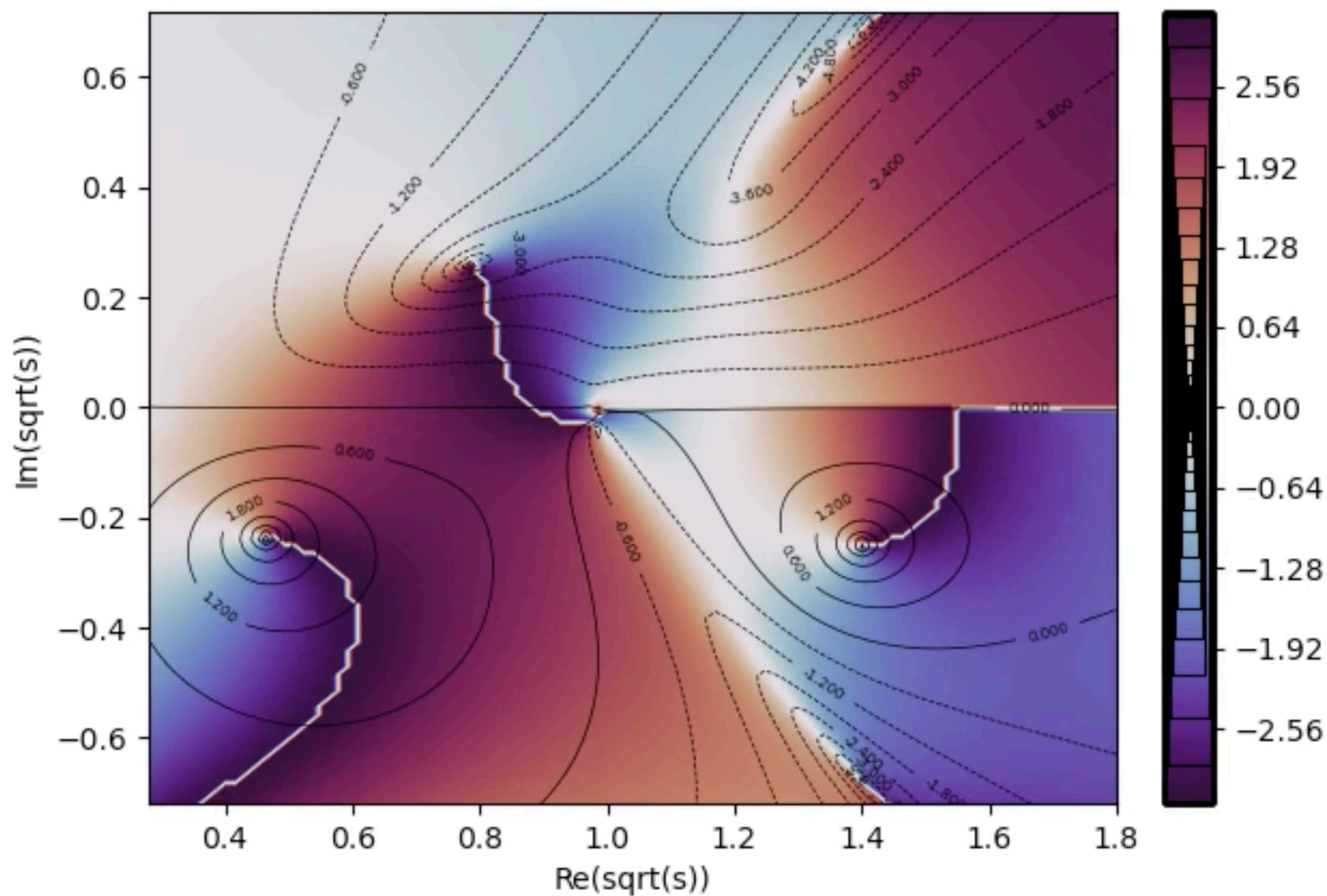
$(x,y)=(0.539, 1.0)$



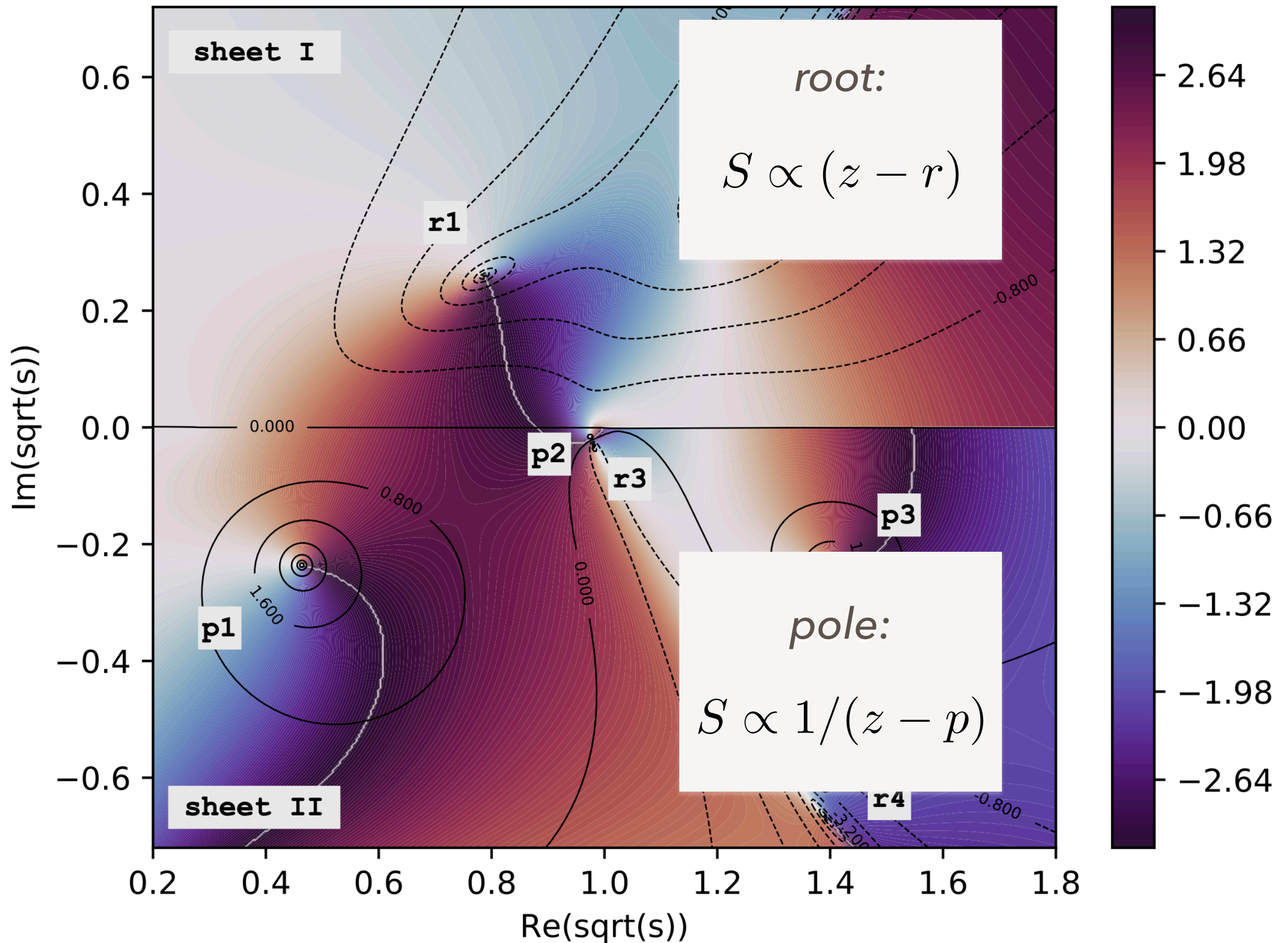
$(x,y)=(0.718, 1.0)$



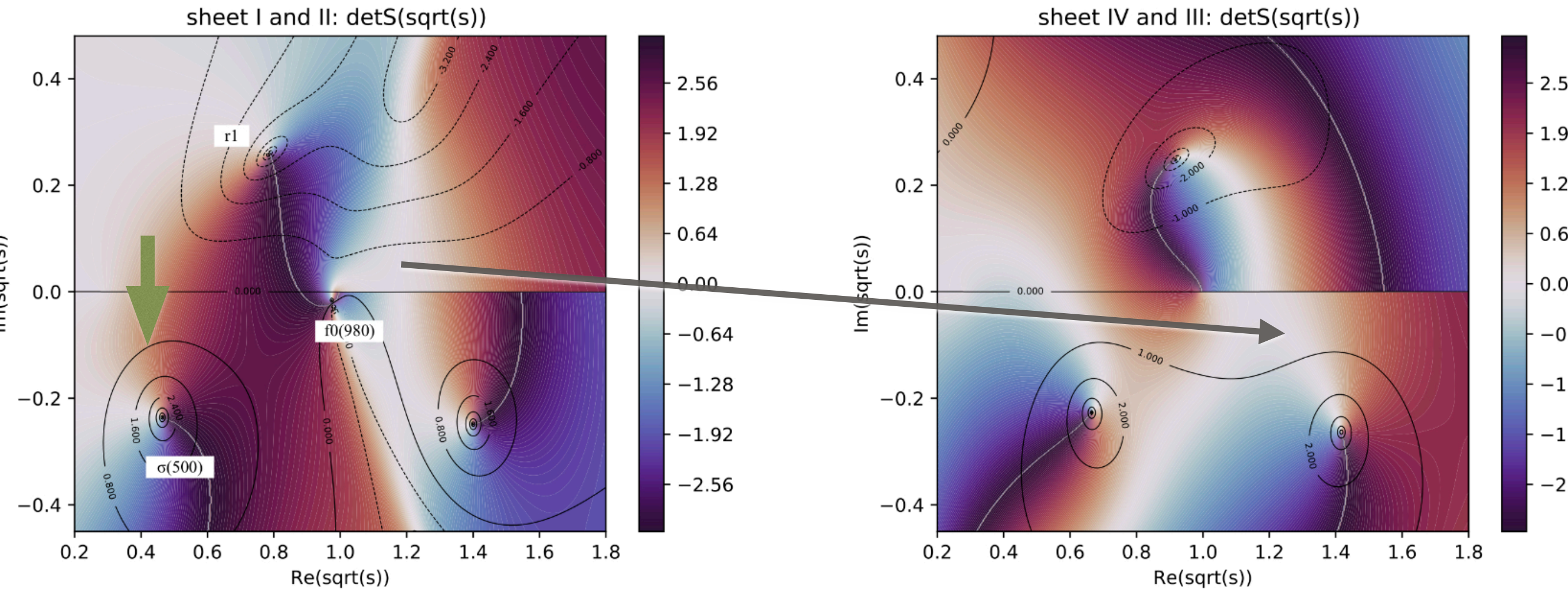
$(x,y)=(1.0, 1.0)$



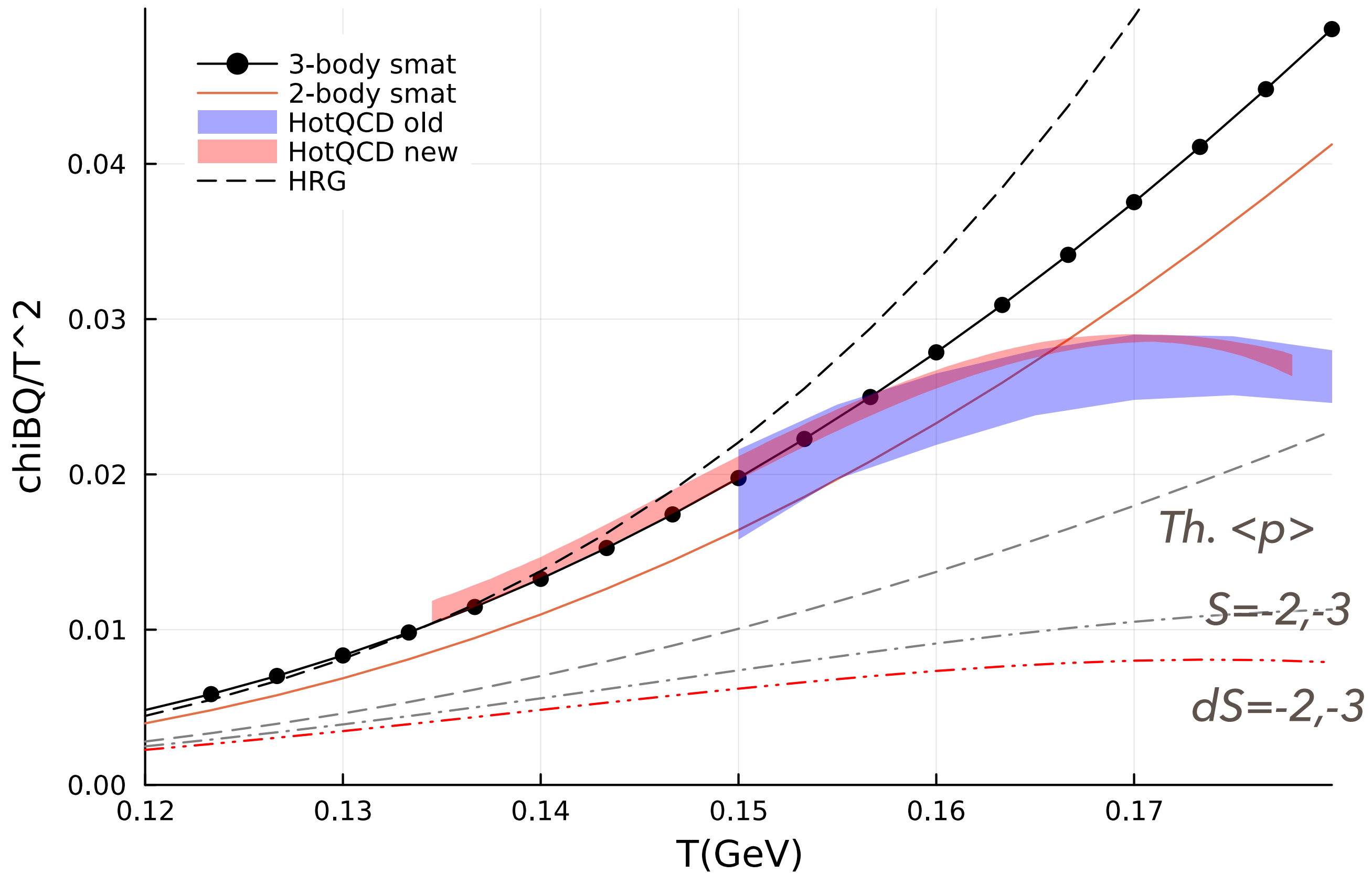
detS(sqrt(s))



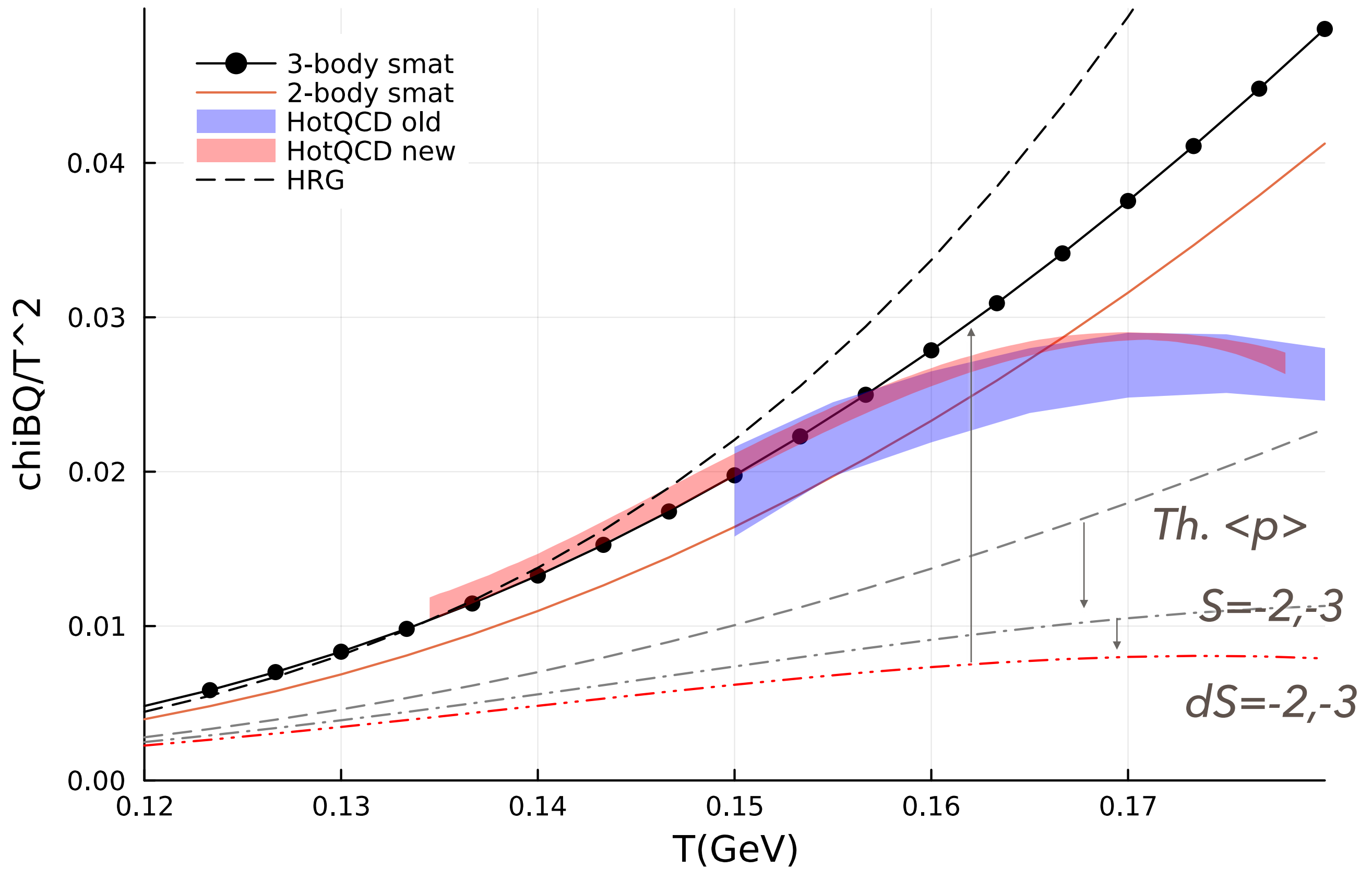
Continuity of Phase Change

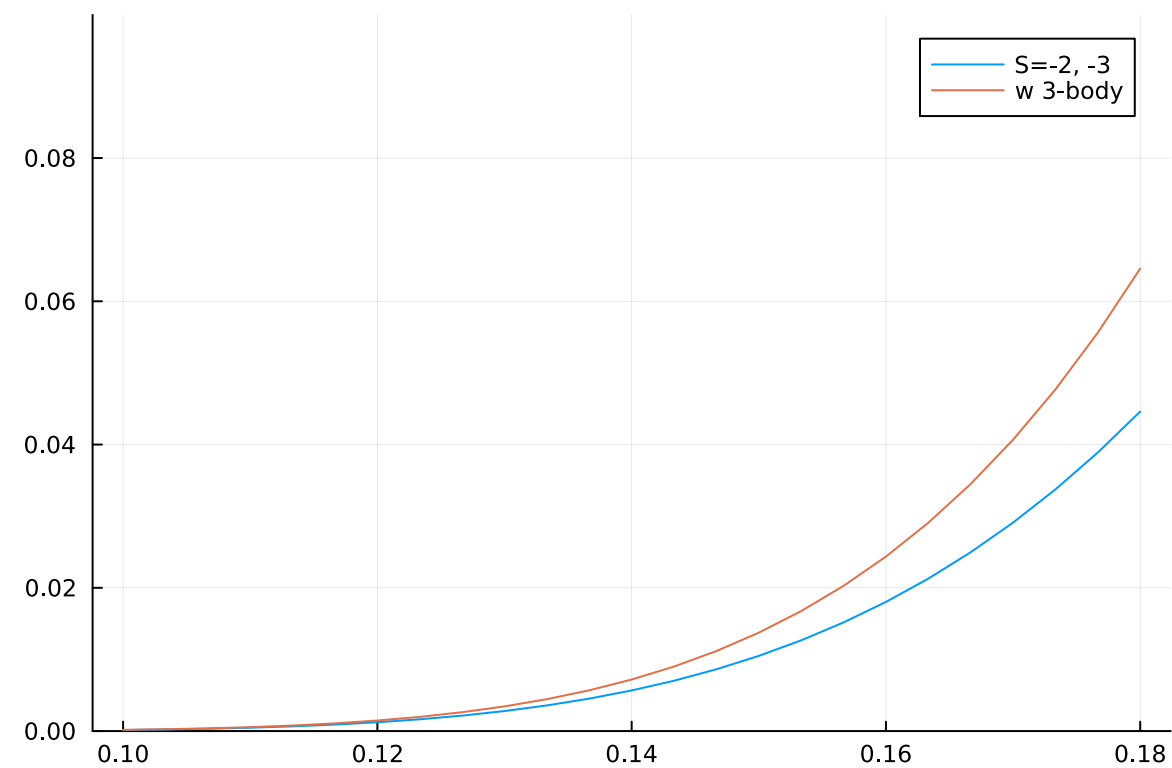
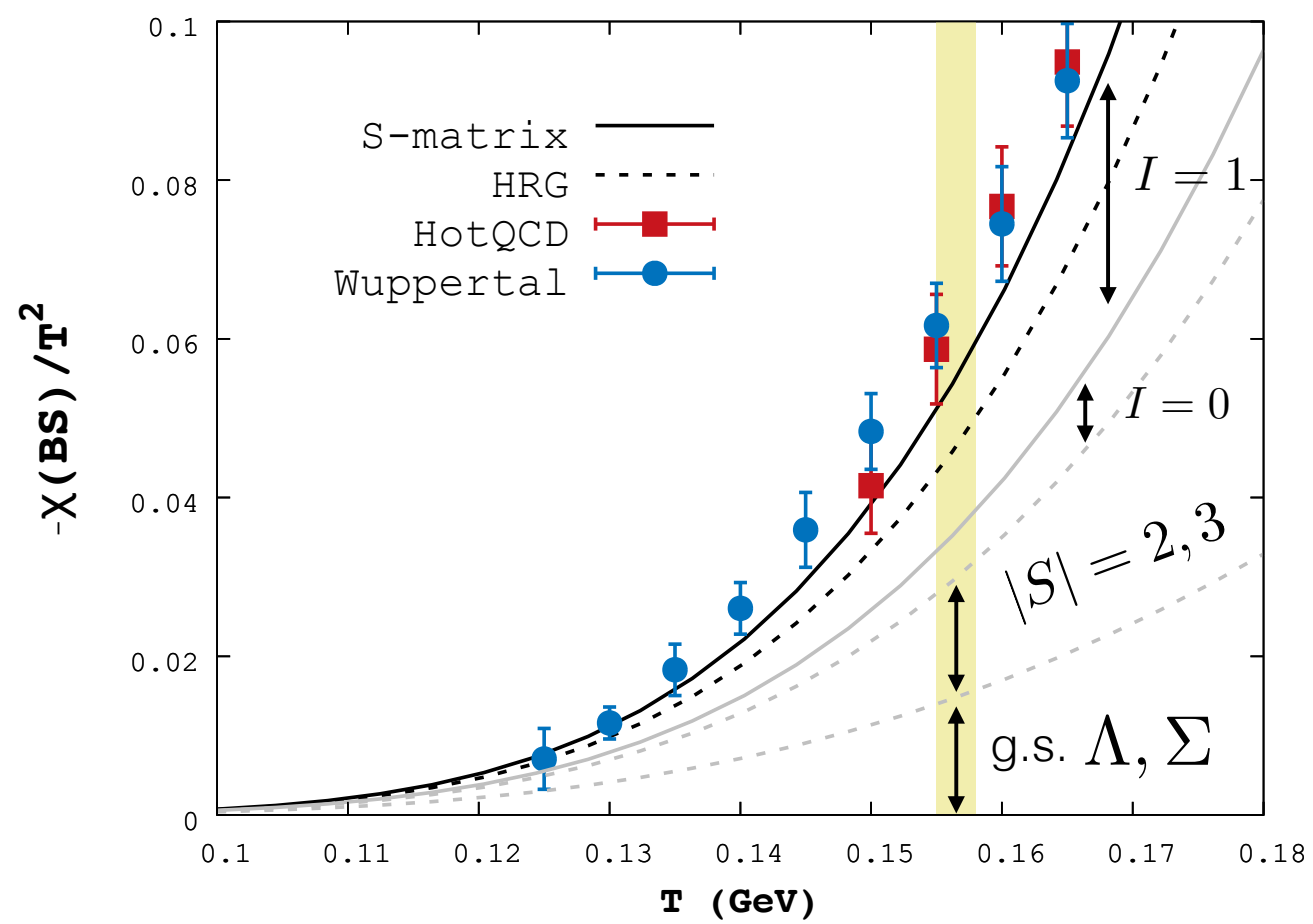


Prelim



Prelim





THANK YOU!

THE END (2)